→ The analytical computations will be derived for the p-spin:

$p = 2$ → spherical → domain growth.
→ Ising → spin glasses.

$p \geq 3$ → Fragile glasses.

13/07/2017

COARSENING

equilibrium phases: ORDERED FM — T_c/J — DISORDERED PM → T/J ↘ energy scale

T_c/J: 2nd ORDER

PREPARE SAMPLE À $T_i >> T_c$

INSTANTANEOUS QUENCH!

→ statics as a function of order parameter well understood

→ now we want to investigate the dynamics
. start in high T configuration quenched
. study evolution → $T = T_c$ dyn at criticality
→ $T < T_c$

simplest example: 2d Ising model on lattice

$H(s) = -J \sum_{\langle ij \rangle} s_i s_j$ $\left\{ \begin{array}{l} s_i = \pm 1 \\ \text{n.n interactions} \\ J > 0 \end{array} \right.$

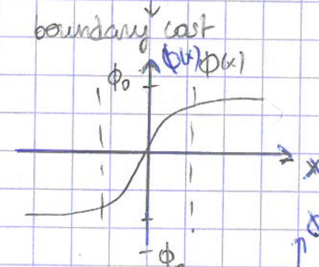
from config $p(s_i = 1) = 1/2$ run Monte Carlo Markov Chains.
↳ observe the formation of domains (+ small fluctuations)
. with size domain growing along time for $T < T_c$.
↳ 2 types of domain as 2 FM equivalent states ↑↓.
↳ additional extra energy is in the boundaries!
↳ $\varepsilon_{syst}(t)$ ↘ by erasing boundaries
. at $T = T_c$, different behavior looks like a lot more fluctuations.

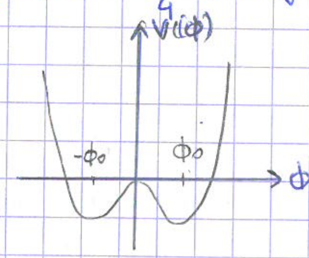
theoretical setting:

Ising model in 2d → to ease the analytics: coarse-grain spins → GINZBURG LANDAU.

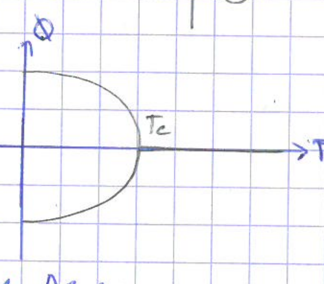
$$\phi(\vec{r}) = \frac{1}{V_{\vec{r}}} \sum_{i \in V_{\vec{r}}} S_i \quad \rightarrow F(\phi) = \int d^d r \left\{ \frac{1}{2}[\nabla \vec{\phi}(\vec{r})]^2 + \frac{T-J}{2} \vec{\phi}^2(\vec{r}) + \frac{g}{4} \vec{\phi}^4(\vec{r}) \right\}$$

boundary cost





→ 2nd order phase transition



For the spins

Consider Metropolis MMC dynamics: $s_i \rightarrow -s_i$ accepted $\left\{ \begin{array}{ll} p = 1 & \Delta\varepsilon < 0 \\ p = e^{-\Delta\varepsilon} & \Delta\varepsilon > 0 \\ p = 1/2 & \Delta\varepsilon = 1/2 \end{array} \right.$

→ non conserved order param dynamics "NCOP-DYN".
↳ $\Delta m = 0$ or ± 2.
⚠ need other rule for phase separation pbs.

For the field

Consider Langevin dynamics: $m \ddot{\vec{\phi}}(\vec{r}, t) + \gamma_0 \dot{\vec{\phi}}(\vec{r}, t) = \vec{F}(\vec{\phi}) + \vec{\xi}(\vec{r}, t)$

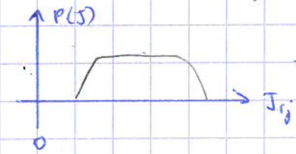
inertia dissipation deterministic noise

overdamped limit: $\gamma_0 \dot{\phi}(\vec{r},t) = \nabla^2 \phi(\vec{r},t) - \frac{\delta V}{\delta \phi(\vec{r},t)} + \xi(\vec{r},t)$

RK: conserve order parameter:

$\partial_t \phi(\vec{r},t) = \nabla^2 \delta_\phi \mathcal{F}[\phi] + \xi(\vec{r},t)$ and MCMC: $\uparrow\downarrow \leftrightarrow \downarrow\uparrow$ COP KAWASAKI: only exchange particles.

we introduce quenched disorder: $J_{ij} \sim$ pdf P



→ no frustration, but regions more or less magnetic.

$g, \dots \sim$ pdf

↳ there are universality classes in this models: $\begin{cases} \text{dimension of order parameter } (\phi, \vec{\phi}, \dots) \\ \text{dyn rule} \leftrightarrow \text{conservation law} \end{cases}$

critical / subcritical quenches

As we already saw differences between $T<T_c$, $T=T_c$.

A common property $t_{relaxation}(L) \xrightarrow[L\to+\infty]{} \infty \ll t_{eq}(L)$ ↳ true ergodicity, visited states as Gibbs Boltzmann

→ we'll show that the linear sizes of equilibrium patch grows with time.

↳ $R(t, T/J) \to t_{relax}$ as $R(t_{relax}, T/J) \sim L$.

≡ characterize universality classes.

already reaches $\phi > 0$.

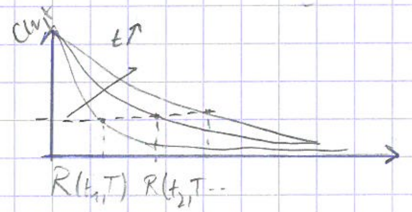
dynamical scaling.

Correlation function $C(r,t) \equiv \langle s_i(t) s_j(t) \rangle \big|_{|\vec{r}_i - \vec{r}_j| = r}$

at equilibrium: $C_{eq}(r) = \langle s_i(t) s_j(t) \rangle_{|\vec{r}_i - \vec{r}_j| = r}$

At T_c: $C_{eq}(r) \propto r^{2-d-\eta}$

↳ we can extract from $C(r,t)$ curves a first estimate of $R(t,T)$



→ $R(t,T) \propto t^{1/z_{eq}}$

for 2d Ising $z_{eq} = 2.17$

dynamic critical scaling.

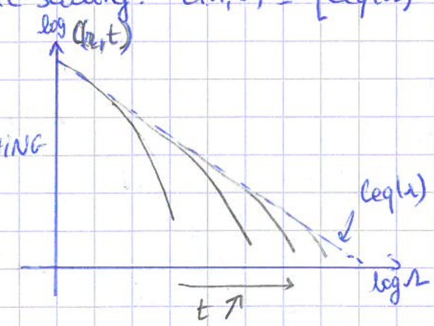
dynamic scaling: $\begin{cases} C(r,t) \simeq f_c\left(\frac{r}{R(t,T)}\right) C_{eq}(r) \\ C(t,t_w) \propto f_c\left(\frac{R(t,T)}{R(t_w,T)}\right) \end{cases}$

At $T<T_c$: repeat the procedure with the $C(r,t)$ curves $R(t, T/J) \sim \lambda(T/J)\, t^{1/z_d}$ $z_d = 2$ $\forall T < T_c$.

dynamic scaling: $C(r,t) \simeq [C_{eq}(r) - m_{eq}^2] + m_{eq}^2 f\left(\frac{r}{R(t,T)}\right)$ → here $C_{eq}(r)$ is an additive term!

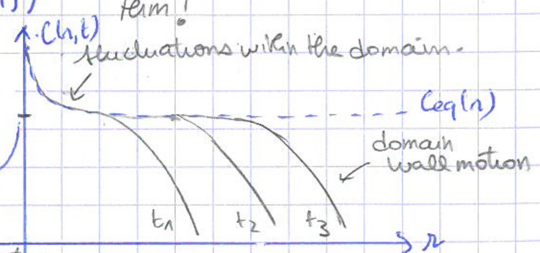
differences:

CRITICAL QUENCHING



short length scale equilibrate faster than long scales

SUBCRITICAL QUENCHING



$m_{eq}^2(T)$

↳ non zero order param with Temperature dependence.

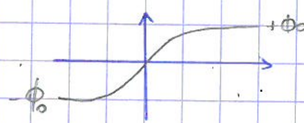
$C_{eq}(r) \xrightarrow[r\to+\infty]{} \langle s_i \rangle^2 = m_{eq}^2$ independent spins

A flavor of the computation of the dynamic scaling:

$T=0$ → no noise: $\frac{\partial \phi}{\partial t}(\vec{r},t) = \nabla^2\phi(\vec{r},t) - \frac{\delta V}{\delta\phi(\vec{r},t)}$ $V = -\frac{\mu}{2}\phi^2 + \frac{g}{4}\phi^4$

* trivial solution $\phi(r,t) = \pm\phi_0 \ \forall r,t$.

* finite wall



$\phi(x) = \pm\phi_0 \tanh\sqrt{\frac{\mu}{2}}(x-O)$ ← position

ALLEN-CAHN expanded around this solution: velocity interior direction of the curvature

↳ only static if wall is vertical!

↳ rounds and shrinks domain: (curvature $k = 1/R$)

$$\frac{dA}{dt} = \frac{d(\pi R^2)}{dt} = 2\pi R\frac{dR}{dt} = -\frac{2\pi\lambda^2(T)}{2\pi}Rk = -\lambda(T)$$

tend to round

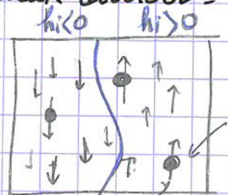
⇒ $A(t) = A_0 - \lambda^2(T)t$ → linearly vanishing (any form of domain) in 2d

Collapse the domain wall

↳ $R(t) \sim t^{1/2}$

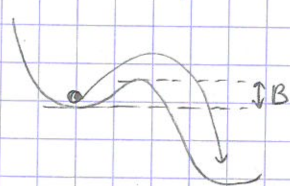
HULL ENCLOSED AREAS -

weak disorder -



fixed spins imposing random field.

→ the domain wall is pinned by disorder → hard to move



Arrhenius time scale: $t_A \sim t_0 e^{B/k_BT}$

B ~ length of the domain wall we would like to move $B(R)$.

↳ droplet argument: $\Delta F(R) \sim -\varepsilon R^d + \sigma R^{d-1}$

↳ $R^* = \max \Delta F(R)$ as barrier → $R^* \sim \frac{\sigma}{\varepsilon}$ → $\Delta F(R^*) \sim \varepsilon R^{*d}$

$B(R) \sim \varepsilon R^{\psi}$ ← some power law

↳ $t \sim t_0 e^{\varepsilon R^\psi / k_BT}$ ⇒ $R = \frac{k_BT}{\varepsilon}(\ln t/t_0)^{1/\psi}$ → much slower!

14/07/2017

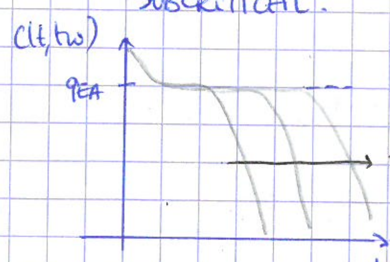
summary & discussions on coarsening systems -   2-
3-

Instead of looking at $C(r,t)$ we can look at the 2 time self correlations $C(t,t_w) = \langle s_i(t)s_i(t_w)\rangle$ { local in space, non-local in time -

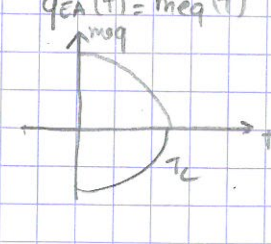
↳ the picture is equivalent

CRITICAL $T=T_c$

SUBCRITICAL.



$q_{EA}(T) = m_{eq}(T)$



still looking at $t < t_{RELAX} \sim L^{z_d=2}$

MULTIPLICATIVE   SEPARATION OF TIME SCALES $f(\theta)$↑ ADDITIVE -