The analytical computations will be derived for the \( p \)-spin:

- \( p = 2 \) \rightarrow spherical \ domain growth
- \( p > 3 \) \rightarrow fragile glasses

**COARSENING**

<table>
<thead>
<tr>
<th>ORDERED</th>
<th>DISORDERED</th>
</tr>
</thead>
<tbody>
<tr>
<td>equilibrium phases</td>
<td>( \phi )</td>
</tr>
<tr>
<td>( T/H )</td>
<td>( T_f )</td>
</tr>
<tr>
<td>( 1 )</td>
<td>( 2^{nd} ) ( \phi ) ( \text{creep} )</td>
</tr>
</tbody>
</table>

Prepare sample at \( T > T_c \)

Quench:

- simplest example: 2D Ising model on lattice
  \( H(s) = -J \sum_{<i,j>} s_i s_j \)
  \( s_i = \pm 1 \)

From config \( p(s) = \frac{1}{2} \) run Monte Carlo Metropolis Chain

Observe the formation of domains (1 small fluctuations)

With one domain growing along time for \( T > T_c \)

- 2 types of domain as \( T \) \( \neq \) equilibrium states \( \pm \phi \)
- additional extra energy is in the boundary
- \( T \rightarrow T_c \) by closing boundaries

2\(^{nd} \) order phase transition

For the spins

Consider Kramers-Heisenberg dynamics: \( s_i \rightarrow -s_i \); \( \text{accepted} \)

For the field

Consider Langevin dynamics: \( m \vec{\phi}(\vec{r}, t) + \chi \vec{\dot{\phi}}(\vec{r}, t) = \vec{F}(\phi) + \vec{\xi}(\vec{r}, t) \)

- \( \phi \rightarrow \text{non conserved order parameter} \)
- \( \phi \rightarrow \text{phase separation} \)

\( \phi = 0 \) \( \text{outside} \)

Homogeneous \( \phi \) \( \rightarrow \text{energy} \) \( \phi = \phi_0 \)
over damped limit: \( X_0 \phi(a_t) = \frac{\partial^2 \phi(a_t)}{\partial a_t^2} - \frac{S V}{S a_t} + \xi(a_t) \)

PK: conserve order parameter:
\[ \langle \phi(x,T) \rangle = \langle \phi \rangle + \xi(x,T) \]
and \( \text{HOM}: \phi = \xi \)

**Critical/subcritical quenches**

As we already saw differences between \( T < T_c, T > T_c \).
A common property: 

\[ \text{factorization} \lim_{L \to \infty} \langle \phi_{qL} \rangle \to \text{true ergodicity, natured states as Gibbs} \]

\[ \omega \to \omega \]

\[ \phi \to \phi \]

\[ \phi > 0 \]

**Dynamical scaling**

Correlation function: \( C(L,t) = \langle s_i(t) s_j(t) \rangle \)

\[ \xi \]

at equilibrium: \( C(q) = \langle s_{i+L} s_j(t) \rangle \)

**AEE:** \( C(q) \propto 2^{-d/\eta} \)

The can extract from \( C(L, t) \) curves a first estimate of \( R(L, t) \)

**Dynamic scaling:** \( C(L, t) \propto \left( \frac{R(L, t)}{R(L, t)} \right)^z \)

At \( T < T_c \): repeat the procedure with the \( (L, t) \) curves.

**Dynamical scaling:** \( C(L, t) \propto \left[ C_{eq}(L) - n_{eq} \right] + n_{eq} f\left( \frac{L}{R(L, t)} \right) \)

\[ \xi \]

**Critical quenching**

Short length/scale equilibrate faster than long scales

**Critical quenching**

\[ \log L \]

\[ \xi \]

\[ n_{eq} \]

\[ \text{Localized domain wall motion} \]

\[ \text{independent spin} \]

\[ \text{Non zero spin parameter with temperature dependence} \]

\[ \text{Domain wall motion} \]

\[ \text{Critical quenching} \]

\[ \text{C}(L) \to \left( \frac{s}{2} \right)^2 = \text{meq}^2 \]
A flavor of the computation of the dynamic scaling:

\[ \frac{d \phi}{dt} (x,t) = \psi^2 \phi(x,t) - \frac{1}{2} \phi^2 \phi'' \]

\[ \psi = -\frac{1}{2} \psi^2 + \frac{1}{4} \phi^4 \]

**Finite wall**

\[ \phi(x) = \pm \phi_0 \cosh \sqrt{\lambda/2} (x-0) \]

*Allen-Cahn* expanded around this solution: velocity proportional to the curvature.

*Only static if wall is vertical.*

*Can expand and shrink domains.*

\[ \frac{d A}{dt} = 2 \pi R \frac{d R}{dt} = -2 \pi \sqrt{\lambda(t)} \frac{d R}{dt} \]

\[ \Rightarrow R(t) = R_0 - \lambda(t) \frac{t}{2} \rightarrow \text{linearly vanishing} \]

(any form of domain)

in 2d

**Weak disorder**

\[ h_v(0) = h_x(0) \]

The domain wall is pinned by disorder -> hard to move.

**Anharmonic time scale:**

\[ t_{an} \rightarrow e^{-B/R(t)} \]

As expected argument: \( \Delta F(R) = -2 R^2 + \frac{1}{2} \Phi \Phi'^2 \rightarrow \]

\[ \Rightarrow R = \frac{\Delta F(0)}{\Phi} \rightarrow \text{some jumpdown} \]

\[ \Rightarrow R = \frac{\Delta F(0)}{\Phi} \rightarrow \text{much slower!} \]

**Summary and discussion on coarsening systems**

Instead of looking at \( C(t, t') \) we can look at the 2 time self correlations:

\[ \langle s(t,t') \rangle = \langle s(t) s(t) \rangle \]

**Critical \( T = T_c \)**

\[ C(t, t') \rightarrow \text{critical} \]

\[ \text{subcritical} \]

\[ \text{multiplcative} \]

**Separation of time scales**

\[ qca(t) = m(t) \]