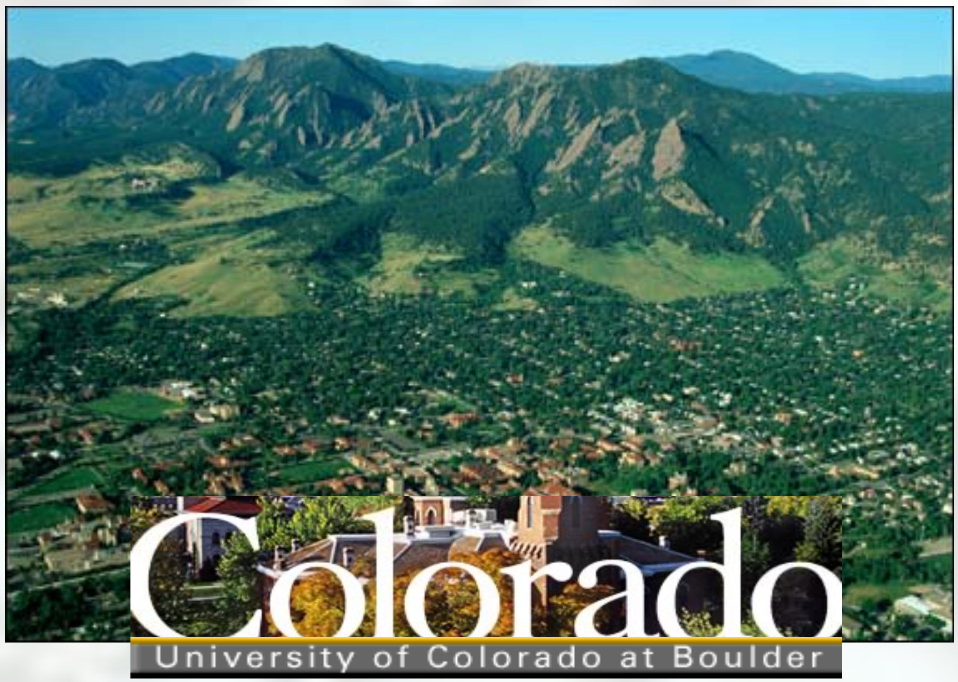
Critical Soft Matter

Soft Matter In and Out of Equilibrium



Leo Radzihovsky

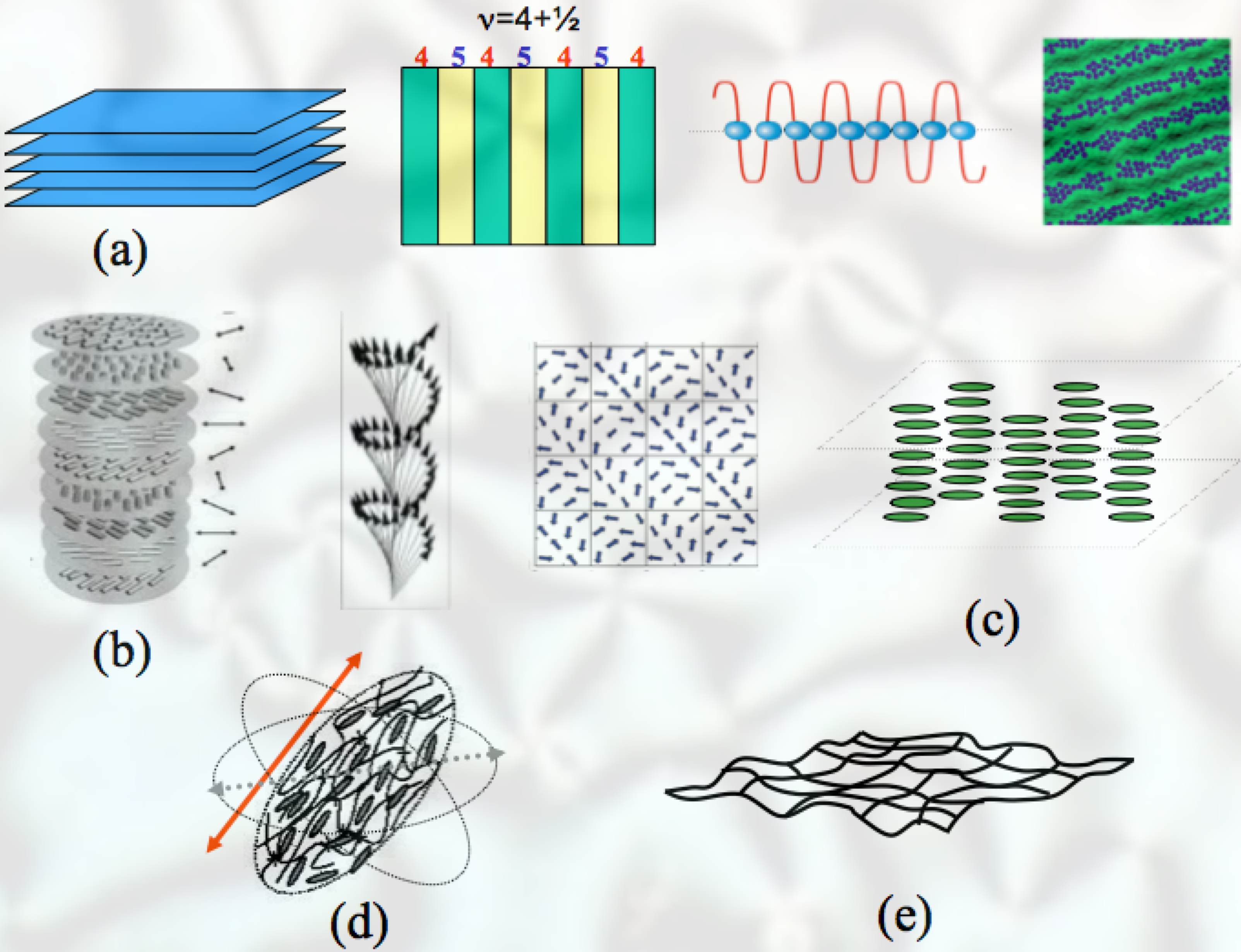
$: NSF, Simons Foundation

Boulder Summer School, July 2015

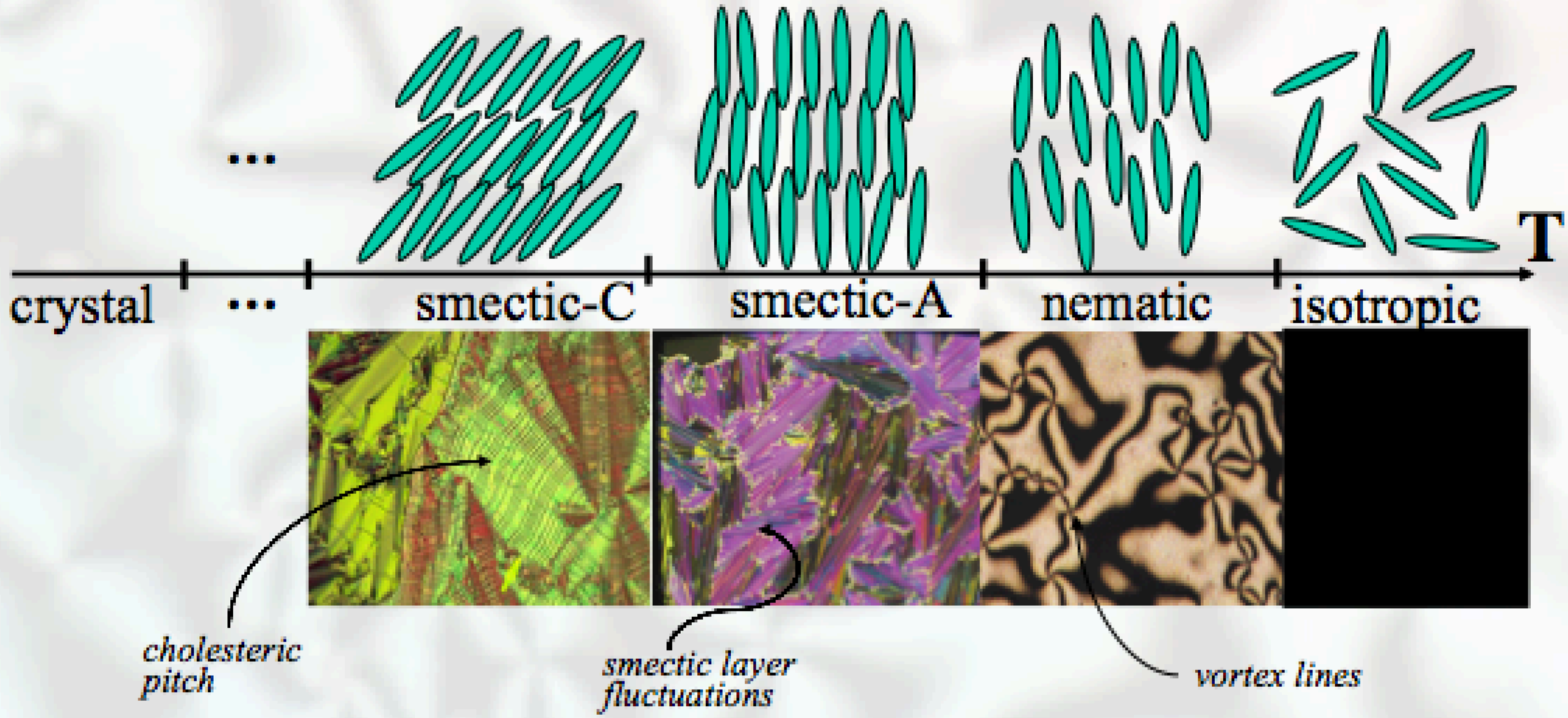
Outline

- Introduction and motivation
- Critical states of matter
- Smectics
- Cholesterics
- Columnar phase
- Polymerized membranes
- Elastomers

Critical soft matter



Liquid crystal phases



Undulation instability on dilation

Strain-induced instability of monodomain smectic A and cholesteric liquid crystals

Noel A. Clark and Robert B. Meyer*
Gordon McKay Laboratory, Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 9 February 1973)

A mechanism is proposed for the observed mechanical instability of monodomain smectic A and cholesteric liquid crystals subjected to uniaxial dilative stress. The threshold conditions for the instability are derived, and the possible roles of dislocations in controlling the instability and in producing large plastic distortions are discussed.

$$\mathcal{H} \approx \frac{B}{2}\left[\partial_z u + \frac{1}{2}(\nabla u)^2\right]^2 + \frac{\overline{K}}{2}(\nabla^2 u)^2$$

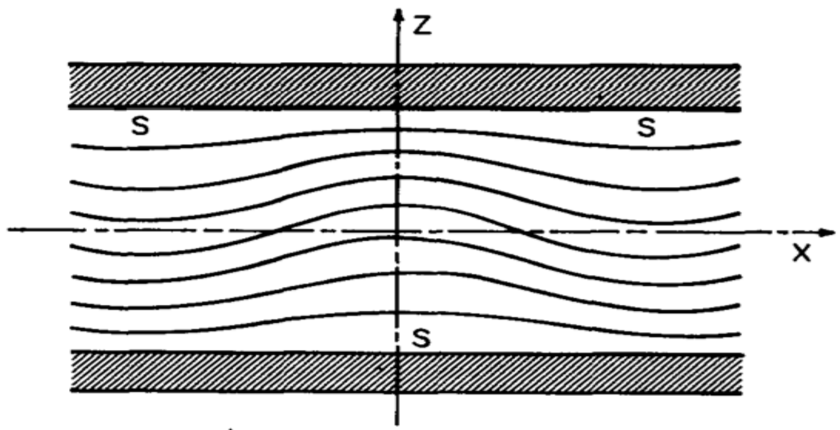
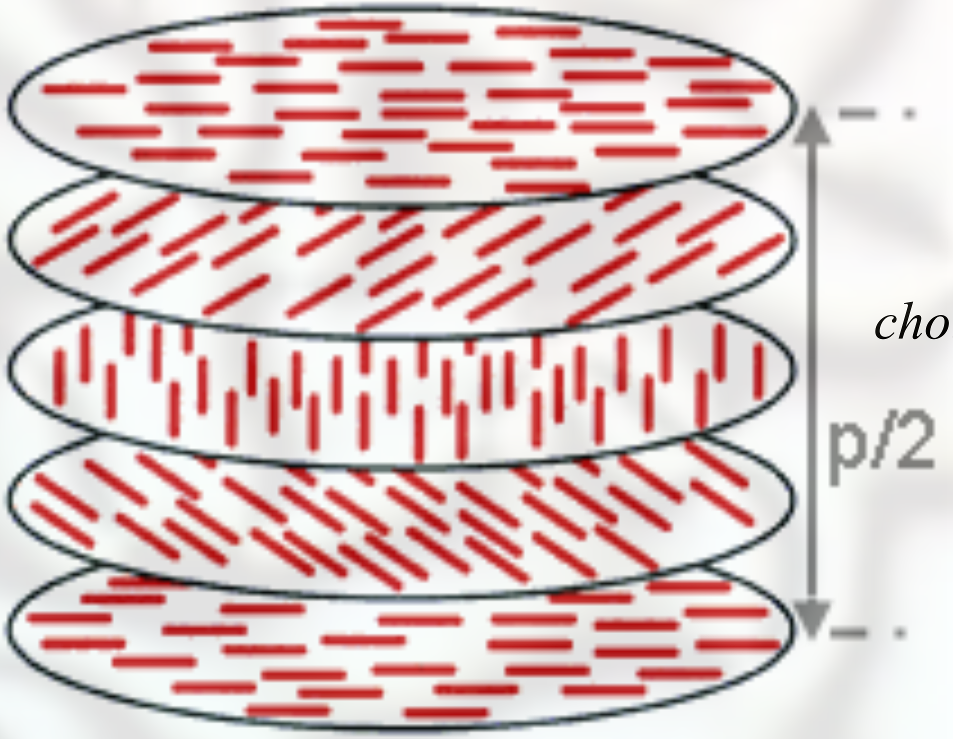


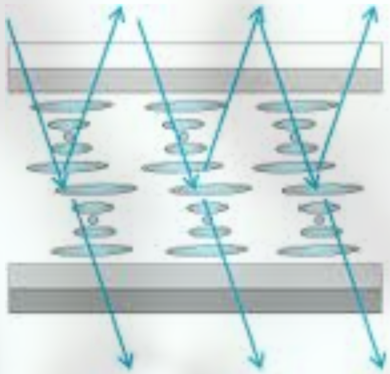
FIG. 1. Periodic undulation of the layers of a dilated smectic A liquid crystal. Regions of maximum dilation are marked S.

Chiral liquid crystals: cholesterics



- color selective Bragg reflection from cholesteric planes

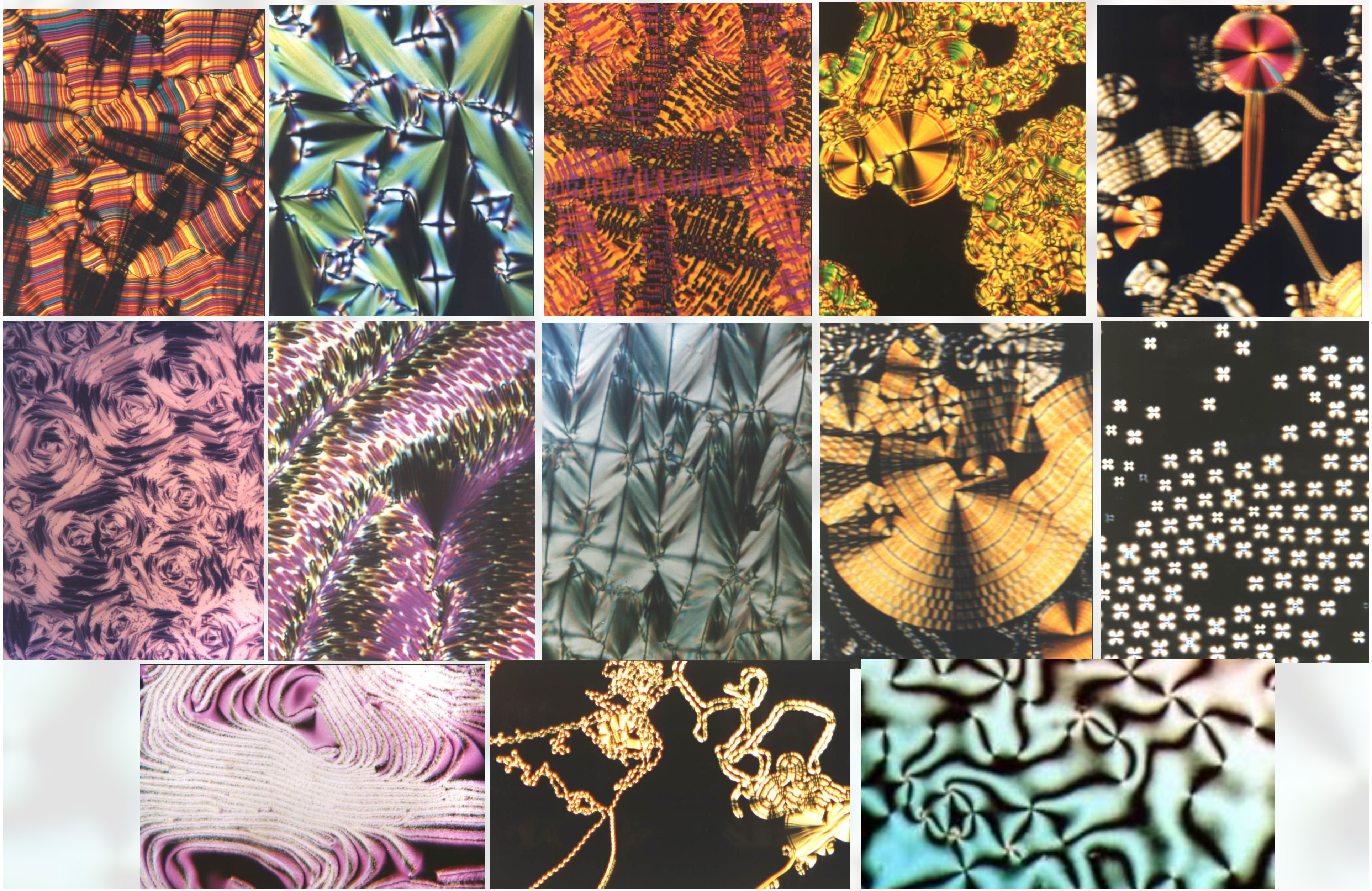
- temperature tunable pitch -> wavelength



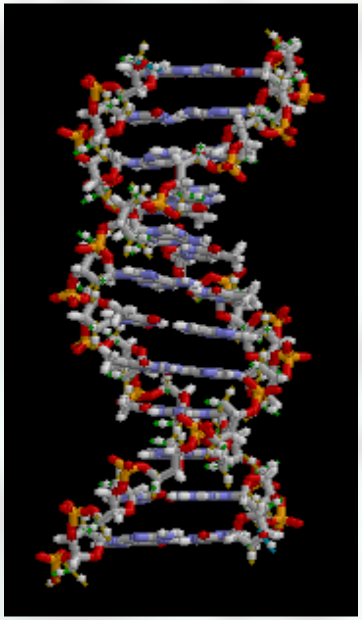
Chirality in liquid crystal

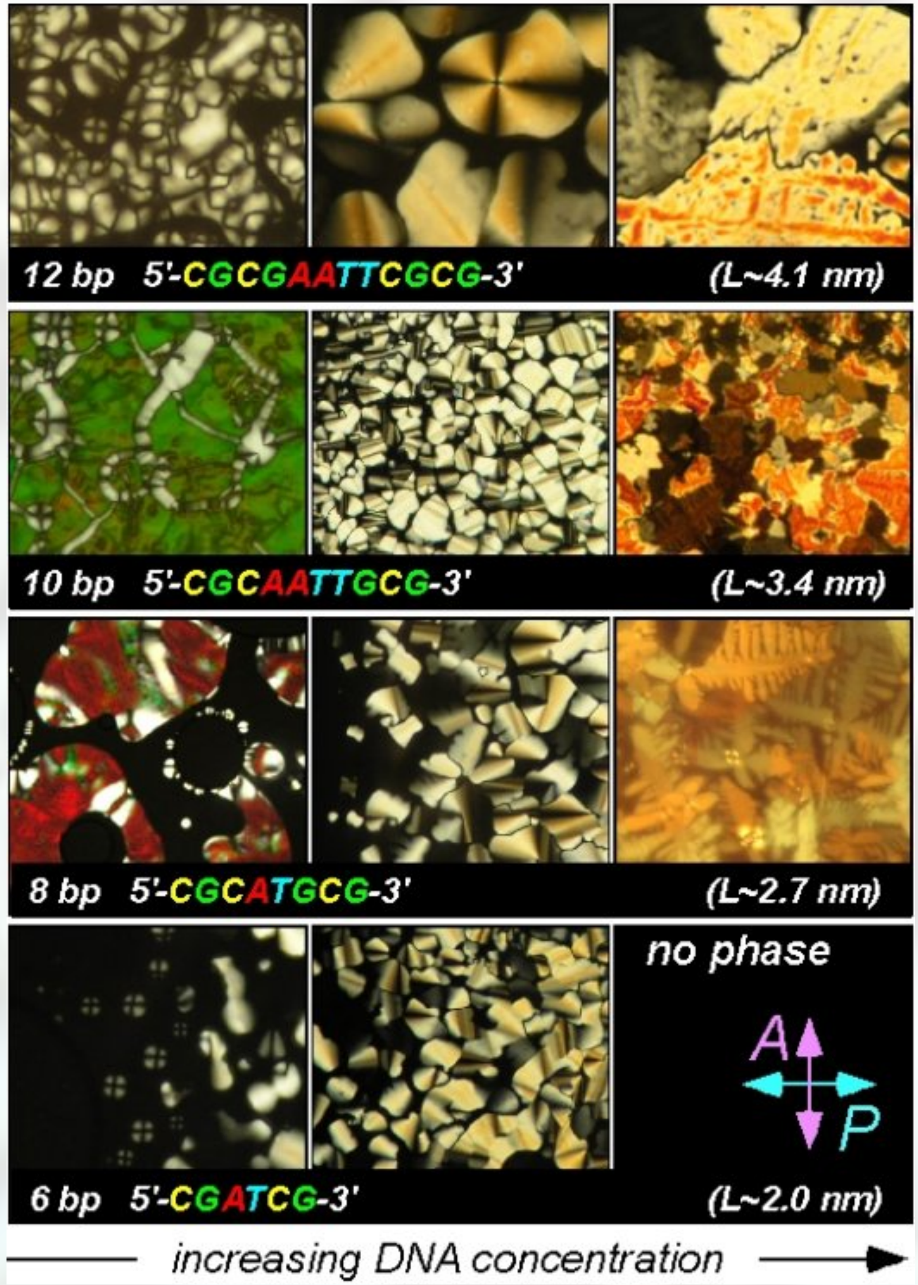
N. Clark

"Liquid crystals are beautiful and mysterious; I am fond of them for both reasons." – P.-G. De Gennes



Bio-polymer liquid crystals: DNA

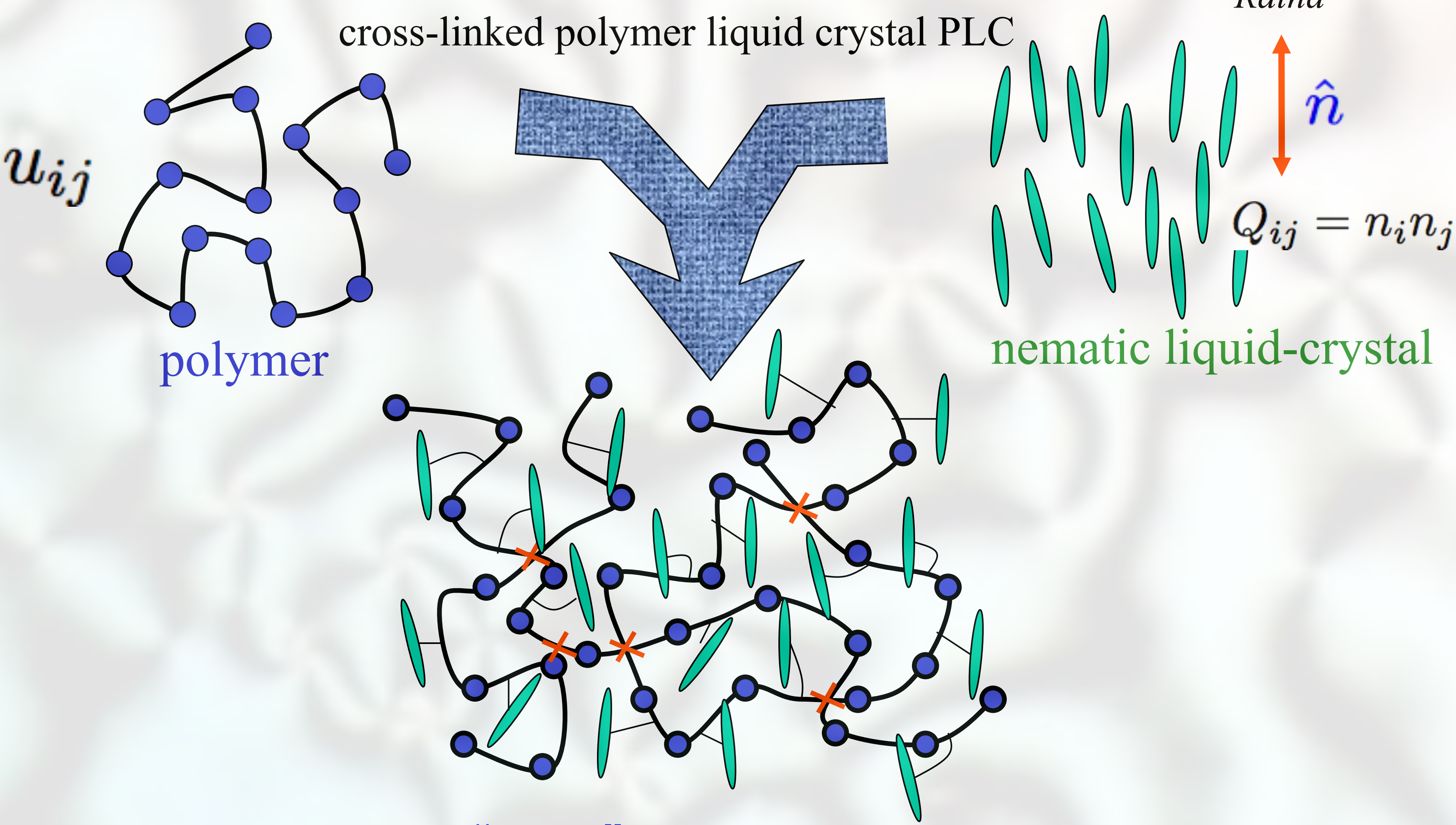




M. Nakata,
N. Clark, et al

Nematic Elastomer

Terentjev
Finkelmann
Ratna



"Solid" Liquid-Crystal

exhibits most conventional liquid-crystal phases (I, N, Sm-A, Sm-C,…)

Nematic Elastomer

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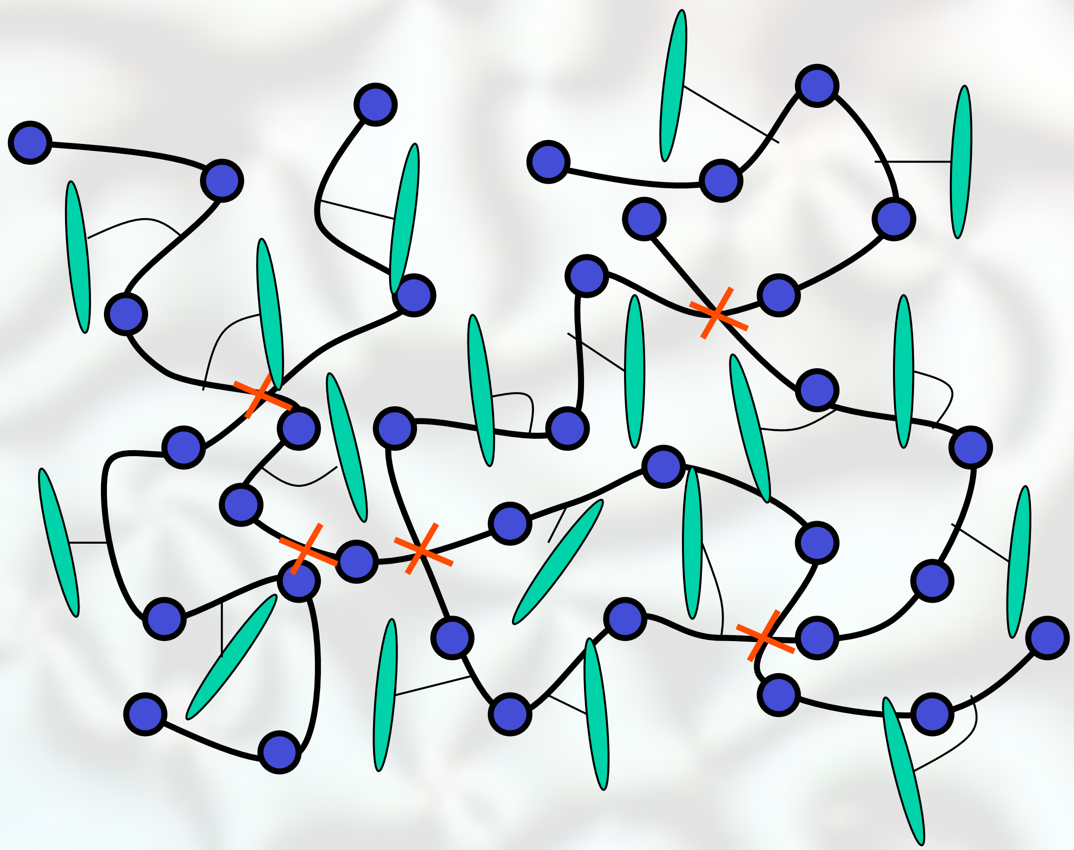
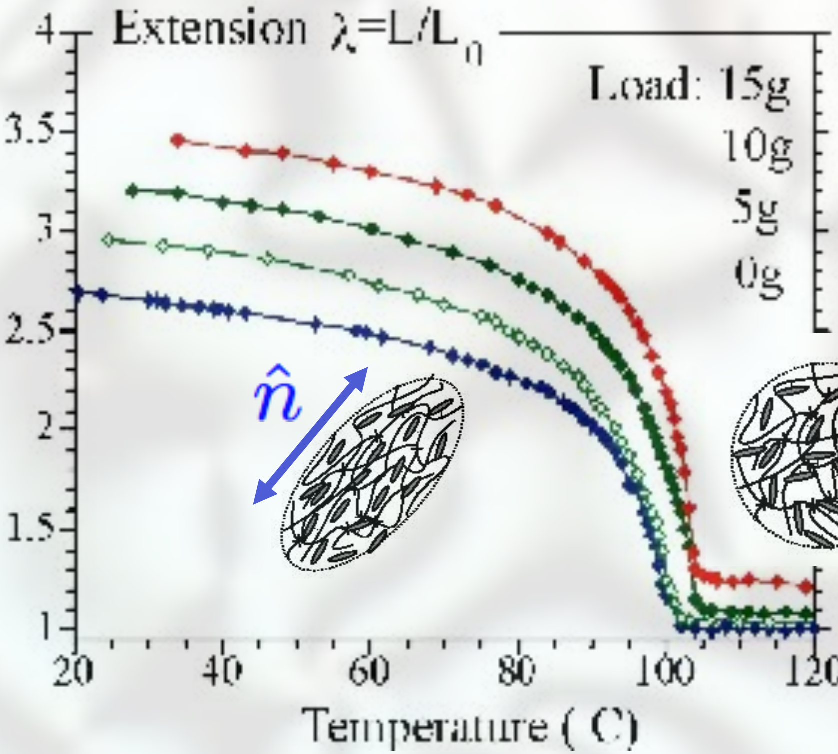
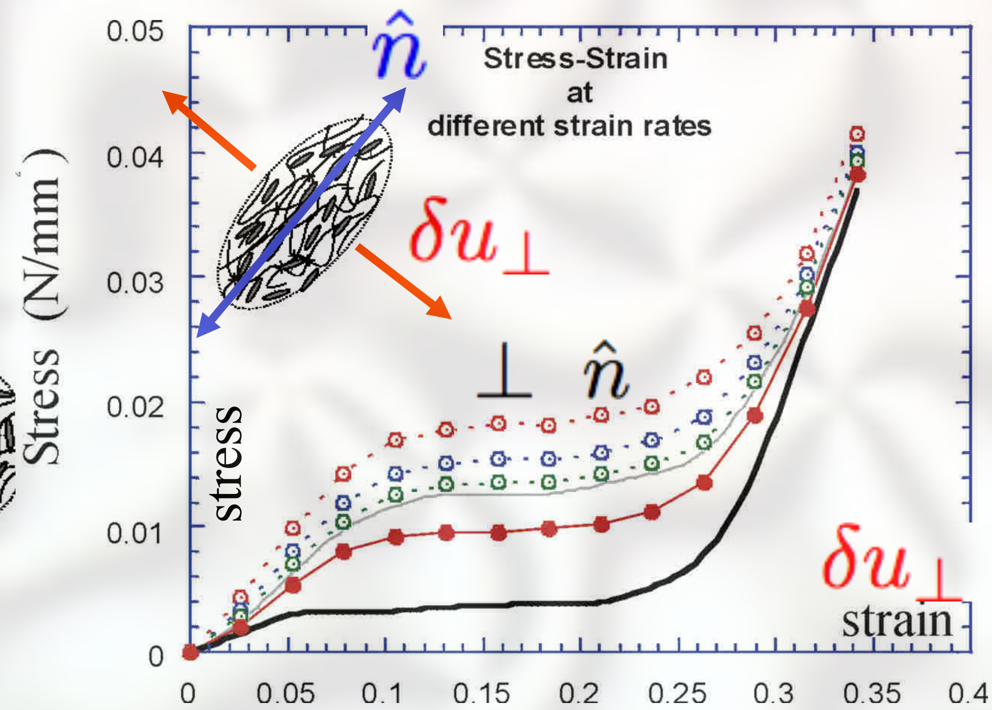
"Solid" Liquid-Crystal



Thermal response and stress-strain relation



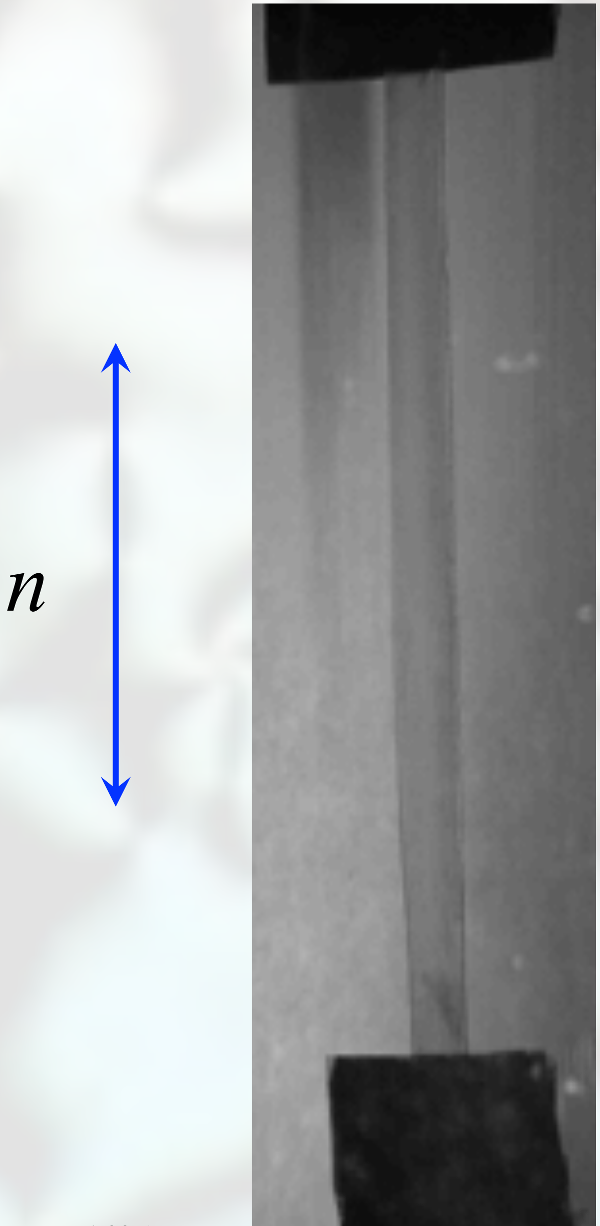


Properties: • *spontaneous distortion (~ 400%) at T_{IN} , thermoelastic*
• *"soft" elasticity*
• *giant electrostriction*

Applications: • *plastic displays*
• *switches*
• *actuators*
• *artificial muscle*

Terentjev, et al

Nematic elastomer as heat engine



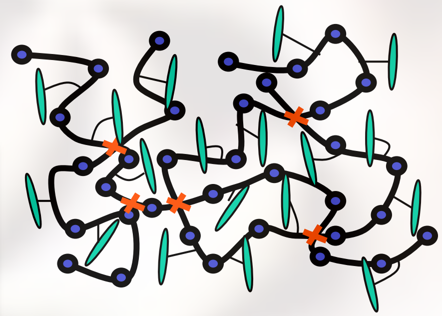
- monodomain nematic LCE
- 5cm x 5mm x 0.3mm
- lifts 30g wt. on heating, lowers it on cooling
- large strain (>400%)

$$\eta \simeq 10^5 \, Pa$$

(P. M-Palffy)

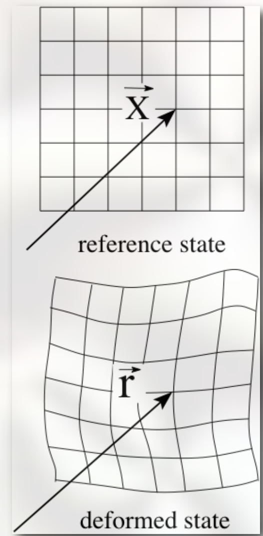
H. Finkelmann, Shahinpoor, et al

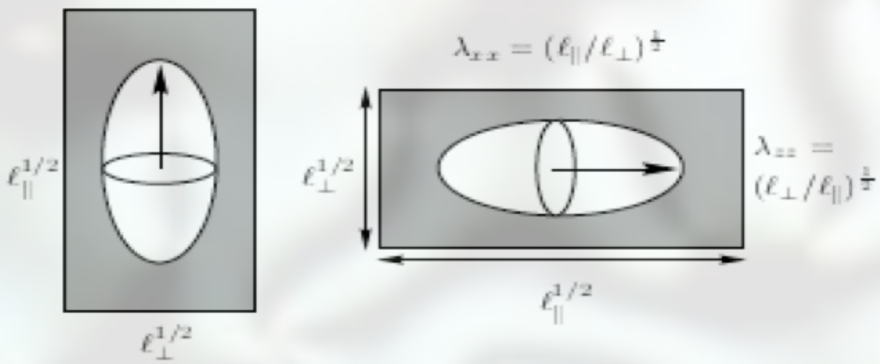
Visualization of soft deformation

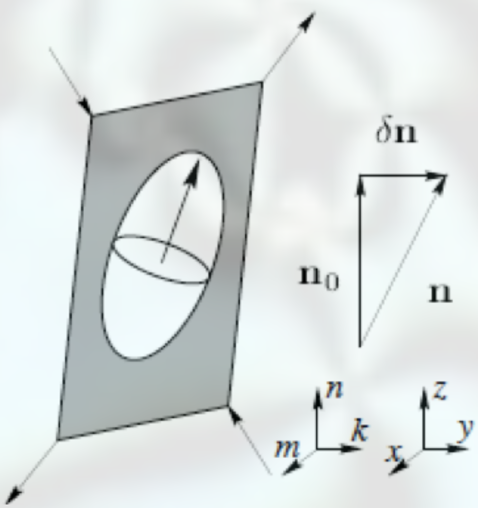


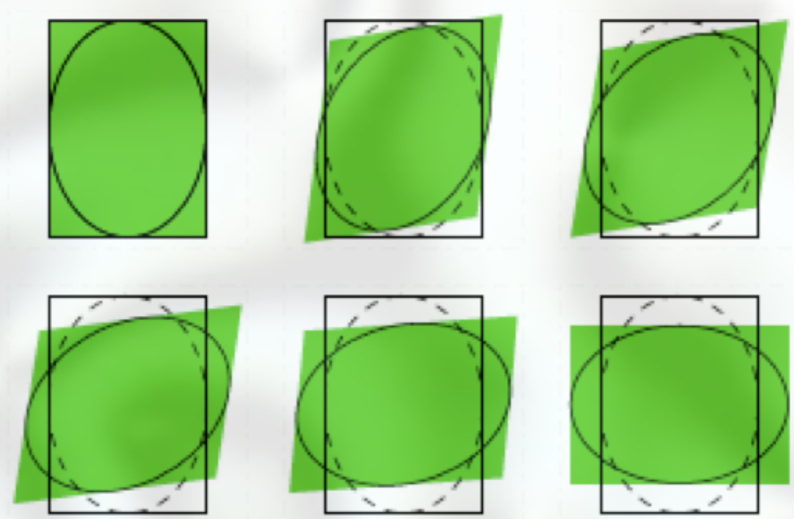
a "liquid" solid <--> a "solid"

liquid crystal









Warner, Terentjev '90
Olmsted '94

Elastic theory of NE

Xing + L.R., PRL, EPL, AOP
Lubensky + Stenull, EPL (2003)

- Construct rotationally invariant elastic theory of deformations about $\underline{\underline{u}}_0$
- Study fluctuations and heterogeneities about $\underline{\underline{u}}_0$

Must incorporate underlying rotational invariance of the nematic state

$\Longrightarrow$ *some distortions cost no energy:* ***"soft" uniaxial solid***

$$f[\vec{R}(\mathbf{x})] = f[O_T\vec{R}(O_R\mathbf{x})]$$

$$u' \approx \frac{(r-1)}{2\sqrt{r}}\begin{pmatrix} 0 & \theta \\ \theta & 0 \end{pmatrix} = (\Lambda_0^T)^{-1}\,\delta u\,\Lambda_0^{-1}$$

- Vanishing energy cost for: $\delta\underline{\underline{u}} = \underline{\underline{O}} \cdot \underline{\underline{u}}_0 \cdot \underline{\underline{O}}^T - \underline{\underline{u}}_0$

- Harmonic elasticity about nematic state: $\underline{\underline{\varepsilon}} = \underline{\underline{u}} - \underline{\underline{u}}_0$

$$\mathcal{H}^0_{NE} = \mu_{zi}\varepsilon^2_{zi} + B_z\varepsilon^2_{zz} + \mu_\perp\varepsilon^2_{ij} + \lambda\varepsilon^2_{ii} + \lambda_{zi}\varepsilon_{zz}\varepsilon_{ii}$$

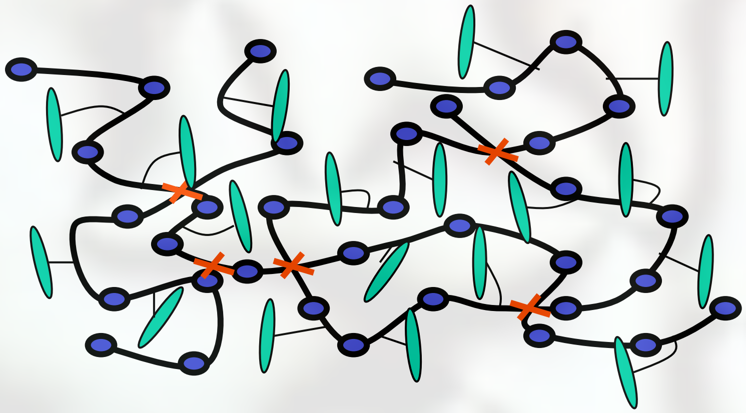
0 , *required by rotational invariance*

- Nonlinear elasticity about nematic state:

$$\mathcal{H}_{NE} = B_z w^2_{zz} + \mu_\perp w^2_{ij} + \lambda w^2_{ii} + \lambda_{zi} w_{zz} w_{ii}$$

$$w_{zz} = \partial_z u_z + \frac{1}{2}(\nabla u_z)^2 \qquad w_{ij} = \frac{1}{2}(\partial_{(i}u_{j)} - \partial_i u_z \partial_j u_z)$$

Fluctuations and heterogeneity



- Thermal fluctuations: $\mathcal{Z} = \mathrm{Trace}_u[e^{-\beta\mathcal{H}[u]}]$

- Heterogeneity ⟹ random torques and stresses:

nematic elastomers are only statistically homogeneous and isotropic

$$\mathcal{H}_{NE}^{real} = \mathcal{H}_{NE}[\underline{\underline{u}}] - \underbrace{\underline{\underline{u}} \cdot \underline{\underline{\sigma}}(\mathbf{r}) - (\hat{n} \cdot \vec{g}(\mathbf{r}))^2}_{\text{encodes heterogeneity}}$$

Elastic "softness" leads to strong qualitative effects of thermal fluctuations and network heterogeneity

Predictions

Xing + L.R., PRL (2003)

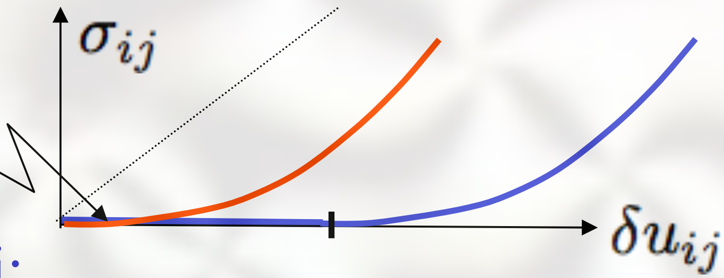
- *Universal* elasticity: $\overline{\langle|\delta u(q)|^2\rangle} \sim q_\perp^{-4+\eta}$, for $r_\perp > \xi_\perp \sim K^2/\Delta$

- Non-Hookean elasticity: $\sigma_{zz} \sim (u_{zz})^\delta \;,\;\; \delta > 1$

  (cf. non-Fermi liquid)

  vanishing slope

  no linear response

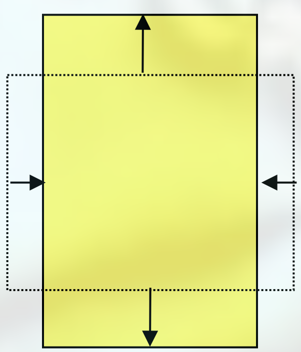


- Length-scale dependent elastic moduli:

$$K_{\text{eff}}(L) \sim L^\eta, \quad \mu_{\text{eff}}(L) \sim L^{-\eta_\mu}, \quad B_{\text{eff}}(L) \sim B_0$$

- Macroscopically incompressible: $\kappa_{\text{eff}} \sim \mu_{\text{eff}}(L)/B_{\text{eff}}(L) \to 0$

- Universal Poisson ratios:



$$u_{xx} > 0 \Longrightarrow \begin{cases} u_{yy} = \frac{5}{7} u_{xx} \\ u_{zz} = -\frac{12}{7} u_{xx} \end{cases}$$

$$u_{zz} > 0 \Longrightarrow u_{xx} = u_{yy} = -\frac{1}{2} u_{zz}$$