The Physics of Heavy Fermion Superconductivity

Lecture IV. Composite Pairing Hypothesis. Hastatic Order.



Piers Coleman Center for Materials Theory, Rutgers.

Boulder School 2014: Modern Aspects of Superconductivity June 30-July 25, 2014









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14-17 July 2014







A solvable model of composite pairing.

PC, Tsvelik, Kee, Andrei PRB 60, 3605 (1999).
Flint, Dzero, PC, Nature Physics 4, 643 (2008).
Flint, PC, PRL, 105, 246404 (2010).
Flint, Nevidomskyy, PC, PRB 84, 064514 (2011).

$$H = \sum_{k} \epsilon_{k} c_{k}^{\dagger} c_{k} + J_{1} \sum_{j} \psi_{1j\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1k\beta} \cdot \vec{S}_{j}$$





 $|\Gamma_7^+\rangle$

Wannier functions at site j:

$$\psi_{\Gamma j}^{\dagger} = \sum_{k} \Phi_{\Gamma k} \mathrm{e}^{i \vec{k} \cdot \vec{R}_{j}} c_{k}$$

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•
$$\Gamma_7^+$$
 $|0\rangle$



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cf Cox, Pang, Jarell (96) PC, Kee, Andrei, Tsvelik (98)

$$\psi_{\Gamma}(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma \mathbf{k}} \ c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_{j}}$$

$$Z = \int_{\text{history}} \mathcal{D}[\psi] e^{-S[\psi_{\sigma}]}$$

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Wild quantum fluctuations!

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Flint, Dzero Coleman Nature Physics '08 PRB 79, 014424(2009)

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Feynman

So how can we solve this model?

 $\sigma \in (-\frac{1}{2}, \frac{1}{2}) \longrightarrow$

$$\int S[\psi]$$

$$\int \tau, x$$

$$\psi(x, \tau)$$

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Large N

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$$\frac{1}{N} \sim \hbar_{eff}$$

 $N \rightarrow \infty$

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Single ES two channels

cf Cox, Pang, Jarell (96) PC, Kee, Andrei, Tsvelik (98)

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Feynman

$$= \int_{\text{history}} \mathcal{D}[\psi] e^{-\frac{S[\psi_{\sigma}]}{1/N}}$$

$$\sigma \in (-\frac{1}{2}, \frac{1}{2}) \longrightarrow (-\frac{N}{2}, \frac{N}{2})$$

$$\frac{1}{N} \sim h_{eff}$$

$$S^{ba} = f_b^{\dagger} f_a - \text{sgn}(a) \text{sgn}(b) f_{-b}^{\dagger} f_{-a} \longleftarrow ?$$

$$H = \sum_{k} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_1 \psi_{1a}^{\dagger}(j) \psi_{1b}(j) + J_2 \psi_{2a}^{\dagger}(j) \psi_{2b}(j) \right) S^{ba}(j)$$
Since FS, two channels.

 ΛT

 $\psi_{\Gamma}(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma \mathbf{k}} c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_{j}}$

 \sim

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$$S_{\alpha\beta} = f_{\alpha}^{\dagger} f_{\beta} + \operatorname{sgn}(\alpha\beta) f_{-\alpha} f_{-\beta}^{\dagger}$$

 $S_{\alpha\beta} = f_{\alpha}^{\dagger} f_{\beta} + \operatorname{sgn}(\alpha\beta) f_{-\alpha} f_{-\beta}^{\dagger}$

H= ZErcErcEr + ZJrcJra CJrp Spa(j) j, r=1,2 CITA IL SCHA Ever(F) e-iE.E. "I Form factor

.

 $S_{\alpha\beta} = f_{\alpha}^{\dagger} f_{\beta} + \operatorname{sgn}(\alpha\beta) f_{-\alpha} f_{-\beta}^{\dagger}$

H =
$$\sum_{j,r=1,2} \sum_{j,r=1,2} \int_{\Gamma} \sum_{j,r=1,2} \int_{\Gamma} \sum_{j,r=1,2} \int_{\Gamma} \sum_{j,r=1,2} \sum_{j,r=1,2} \int_{\Gamma} \sum_{j,r=1,2} \sum_{j,r=1,2} \int_{\Gamma} \sum_{j,r$$

more realistic with 5.0. C.

$$\Phi_{\Gamma}(\vec{k})_{\sigma x} = \sum_{m} Y^{3}_{m-\sigma}(\vec{k}) \langle \ell m \cdot \sigma \frac{1}{2} \sigma | \frac{5}{2} m \rangle \langle m | \alpha \rangle$$

 $\Gamma = \Gamma_6, \Gamma_7$

$$\Phi_{\Gamma}(\vec{k})_{\sigma\alpha} = \sum_{m} Y^{3}_{m-\sigma}(\vec{k}) \langle \ell m - \sigma \frac{1}{2} \sigma | \frac{5}{2} m \rangle \langle m | \alpha \rangle$$

 $\Gamma = \Gamma_6, \Gamma_1$

Γ.	B 127 - 21-3/27
Γ ₆ ====	1=27
Γ, † ====	d 1 27 + B -3/27

$$\frac{J}{2N} = c_j \frac{t}{\alpha} c_{j\beta} \left(f_{j\beta} f_{j\alpha} + z_{\beta} f_{j-\alpha} f_{j-\alpha} \right)$$

$$\frac{J}{2N} = \frac{J}{2N} \left(f_{jk}^{\dagger} f_{jk} + \lambda f_{jk} + \lambda f_{jk} f_{jk} + \lambda f_{jk} + \lambda$$

$$\frac{J}{2N} = c_{j} + c_{j\beta} \left(f_{j\beta} + f_{j\alpha} + z\beta f_{j-\alpha} f_{j\alpha} \right)$$

$$= -\frac{J}{2N} \int \left(c_{j\alpha} + f_{j\alpha} \right) \left(f_{j\alpha} + c_{j\alpha} + (zc_{j\alpha} + f_{j\alpha} + (zc_{j\alpha} + (zc_{j\alpha} + f_{j\alpha} + (zc_{j\alpha} + f_{j\alpha} + (zc_{j\alpha} + f_{j\alpha} + (zc_{j\alpha} + (zc_{j$$

I cja cja (fjafja + ZBfj-sfja) $-\frac{J}{2N}\left(c_{jx}^{\dagger}f_{a}\right)\left(f_{jk}^{\dagger}c_{j}\right) + \left(zc_{jx}^{\dagger}f_{j}^{\dagger}\right)\left(f_{jk}c_{j}\right)\right]$ $\sum_{\sigma} \left(\vec{V} c_{j-\sigma}^{\dagger} f_{\alpha} + H.c \right) + \left(\Delta c_{j\sigma}^{\dagger} f_{j-\sigma}^{\dagger} \vec{v} + H.c \right)$ $+2N\left(\frac{\overline{1}}{\overline{2}} + \frac{\overline{2}}{\overline{3}}\right)$ $= \left[\left(f_{j\alpha}^{\dagger}, f_{j-\alpha} \right) \left(V \right) \left(\Delta \right) \left(f_{j\alpha}^{\dagger} + u.c \right) \right]$ + $T_r \left(\begin{bmatrix} \overline{v} & \overline{o} \\ \overline{v} & -v \end{bmatrix} \right) \left(\begin{bmatrix} v & 0 \\ \overline{v} & -v \end{bmatrix} \right) \left(\begin{bmatrix} v \\ \overline{v} & -v \end{bmatrix} \right) \left(\begin{bmatrix} v \\ \overline{v} & -v \end{bmatrix} \right)$

I cja cja (fjafja + ZBfj-sfja) $-\frac{J}{2N}\left(c_{jx}^{\dagger}f_{a}\right)\left(f_{jk}^{\dagger}c_{j}\right) + \left(zc_{jx}^{\dagger}f_{j}^{\dagger}\right)\left(f_{jk}c_{j}\right)\right]$ $\sum_{\sigma} \left(\vec{V} c_{j-\sigma}^{\dagger} f_{\alpha} + H.c \right) + \left(\Delta c_{j\sigma}^{\dagger} f_{j-\sigma}^{\dagger} \vec{v} + H.c \right)$ $+2N\left(\frac{\overline{1}}{\overline{2}} + \frac{\overline{2}}{\overline{3}}\right)$ $= \left[\left(f_{j\alpha}^{\dagger}, f_{j-\alpha} \right) \left(V \right) \left(\Delta \right) \left(f_{j\alpha}^{\dagger} + u.c \right) \right]$ + $T_r \left(\begin{bmatrix} \overline{v} & \overline{o} \\ \overline{v} & -v \end{bmatrix} \right) \left(\begin{bmatrix} v & 0 \\ \overline{v} & -v \end{bmatrix} \right) \left(\begin{bmatrix} v \\ \overline{v} & -v \end{bmatrix} \right) \left(\begin{bmatrix} v \\ \overline{v} & -v \end{bmatrix} \right)$

$$= \sum_{n=0}^{\infty} \left(\left(f_{jx}^{+}, f_{j-x} \right) \left(V \quad \Delta \right) \left(f_{jx}^{+} + u.c \right) \\ \left(\overline{\Delta} \quad -\overline{V} \right) \left(f_{jx}^{+} + u.c \right) \\ + T_{r} \left(\left(\overline{\Delta} \quad -\overline{V} \right) \left(v \quad \overline{\Delta} \right) \right) \left(\frac{N}{\overline{a} \overline{J}} \right) \right)$$

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$$= \left\{ \left(\begin{array}{c} f_{jx}^{+}, f_{j-x} \end{array}\right) \left(\begin{array}{c} V & \Delta \\ \overline{\Delta} & -\overline{V} \end{array}\right) \left(\begin{array}{c} f_{jx}^{+} \\ \overline{\Delta} & -\overline{V} \end{array}\right) \left(\begin{array}{c} f_{jx}^{+} \\ \overline{\Delta} \end{array}\right) \right\} \\ + T_{r} \left(\left(\begin{array}{c} \overline{\nu} & \overline{\nu} \\ \overline{\nu} & -\overline{\nu} \end{array}\right) \left(\begin{array}{c} N \\ \overline{\nu} & \overline{\nu} \end{array}\right) \right) \left(\begin{array}{c} N \\ \overline{\nu} & \overline{\nu} \end{array}\right) \\ = \left(\begin{array}{c} \overline{f}_{j}^{+} & \overline{V} & c_{j} + c_{j}^{+} & \overline{V} & f_{j} \end{array}\right) + T_{r} \left(\begin{array}{c} \overline{V} & V \\ \overline{\nu} \end{array}\right) \left(\begin{array}{c} N \\ \overline{\nu} & \overline{\nu} \end{array}\right) \\ = \left(\begin{array}{c} \overline{f}_{j}^{+} & \overline{V} & c_{j} + c_{j}^{+} & \overline{V} & f_{j} \end{array}\right) + T_{r} \left(\begin{array}{c} \overline{V} & V \\ \overline{\nu} \end{array}\right) \left(\begin{array}{c} N \\ \overline{\nu} & \overline{\nu} \end{array}\right) \\ = \left(\begin{array}{c} \overline{f}_{j}^{+} & \overline{V} & c_{j} + c_{j}^{+} & \overline{V} & f_{j} \end{array}\right) + T_{r} \left(\begin{array}{c} \overline{V} & V \\ \overline{\nu} & \overline{\nu} \end{array}\right) \left(\begin{array}{c} N \\ \overline{\nu} & \overline{\nu} \end{array}\right) \\ = \left(\begin{array}{c} \overline{f}_{j}^{+} & \overline{V} & c_{j} + c_{j} & \overline{V} & f_{j} \end{array}\right) + T_{r} \left(\begin{array}{c} \overline{V} & V \\ \overline{\nu} & \overline{\nu} \end{array}\right) \left(\begin{array}{c} N \\ \overline{\nu} & \overline{\nu} \end{array}\right)$$

$$\mathcal{V} = \begin{pmatrix} \mathsf{v} & \mathsf{o} \\ +\bar{\mathsf{o}} & -\bar{\mathsf{v}} \end{pmatrix}$$

$$= \sum_{v \to o} \left(\left(\begin{array}{c} f_{j \neq v}^{+}, f_{j \rightarrow v} \right) \left(\begin{array}{c} V & \Delta \\ \overline{\Delta} & -\overline{V} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq u. v \\ \overline{\Delta} & -\overline{V} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq u. v \\ \overline{\Delta} & -\overline{V} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq u. v \\ \overline{\Delta} & -\overline{V} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq u. v \\ \overline{\Delta} & -\overline{V} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq u. v \\ \overline{\Delta} & -\overline{V} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq u. v \\ \overline{\Delta} & -\overline{V} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq u. v \\ \overline{\Delta} & -\overline{V} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq u. v \\ \overline{\Delta} & -\overline{V} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq u. v \\ \overline{\Delta} & -\overline{V} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq u. v \\ \overline{\Delta} & -\overline{V} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq u. v \\ \overline{\Delta} & -\overline{V} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq u. v \\ \overline{\Delta} & \overline{V} = \left(\begin{array}{c} V & \Delta \\ \overline{\Delta} & -\overline{V} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \neq v \\ \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} = \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \end{array} \right) \left(\begin{array}{c} \overline{J} \end{array}$$

$$H = \sum_{k} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_{1} \psi_{1a}^{\dagger}(j) \psi_{1b}(j) + J_{2} \psi_{2a}^{\dagger}(j) \psi_{2b}(j) \right) S^{ba}(j)$$

cf Cox, Pang, Jarell (96) PC, Kee, Andrei, Tsvelik (98) Single FS, two channels. $\psi_{\Gamma}(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma \mathbf{k}} c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_{j}}$ Impurity: quantum critical point for $J_1 = J_2$



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Nozieres and Blandin 1980



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Magnetic pair: intercell

 $\Psi_M^{\dagger} = \Delta_d (1-2) f_{\uparrow}^{\dagger}(1) f_{\downarrow}^{\dagger}(2)$



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Composite pair $\Psi_C^\dagger = c_{1\downarrow}^\dagger c_{2\downarrow}^\dagger S_+$

Abrahams, Balatsky, Scalapino, Schrieffer 1995

Andrei, Coleman, Kee & Tsvelik PRB (1998) Flint, Dzero, Coleman, Nat. Phys, (2008)

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Extreme Resilience to doping on Ce site.



Lei Shu et al, PRL, (2011)

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Lei Shu et al, PRL, (2011) M. Tanatar et al (unpublished) Erten and PC arXiv1402.7361





 $\Psi^{\dagger} = c_{1\downarrow}^{\dagger} c_{2\downarrow}^{\dagger} S_{+}$ $Q_{zz} \propto \Psi_{C}^{2}$



 $\Delta F \propto -Q_{zz} u_{tet}$



 $\Delta F \propto -Q_{zz} u_{tet}$ $\alpha_2 [T - (T_{c2} + \lambda u_{tet})] \Psi_C^2$



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 $\Rightarrow T_c = T_{c2} + \lambda u_{tet}$



$$\Delta F \propto -Q_{zz} u_{tet}$$
$$\alpha_2 [T - (T_{c2} + \lambda u_{tet})] \Psi_C^2$$

 $\Rightarrow T_c = T_{c2} + \lambda u_{tet}$

Strain expected to enhance Tc



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Strain expected to enhance Tc











円 48.770

48.769

48.768

48.767

48.766

48.765

48.764

∄ 48.763

2.0



$$\lambda_L(T) = \lambda_L(0) + aT^n$$



$$\lambda_L(T) = \lambda_L(0) + aT^n$$



STM: B. Zhou et al. Nature Phys. 9, 474 (2013)



Thermal cond: K. Izawa et al. PRL 87.



Torque magnetometry: H. Xiao et al. PRB 78,





Thermal cond: K. Izawa *et al.* PRL 87.

90

 θ (deg.)

45

0

 $\mu_0 H = 0.5 \text{ T}$

 $1 \mathrm{T}$

27

3 T

180

in the second

VVVV

135





Torque magnetometry: H. Xiao *et al.* PRB 78,



2. arXiv:1203.2189 [pdf, ps, other]

Magnetic field splitting of the spin-resonance in CeCoIn5 C. Stock, C. Broholm, Y. Zhao, F. Demmel, H.J. Kang, K. C. Rule, C. Petrovic Comments: 5 pages, 4 figures Subjects: Superconductivity (cond-mat.supr-con)



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1.5 • OSIRIS, E,=1.84 meV • SPINS, E,=3.7 meV • MACS, E,=3.5 meV • FLEX, E_f=3.5 meV • FLEX, E_f=3.5 meV • Gamma Barbon Strategy and the set of the



Conclusions

- Convergence of magnetism and superconductivity: require new concepts over and beyond spin fluctuation theory. TREMENDOUS POTENTIAL FOR DISCOVERY.
- 115 heavy fermion superconductors suggest a new kind of pairing: composite pairing, robust against disorder on magnetic site.

 Could the same phenomenon occur in d-electron materials, at much higher temperatures?

Hastatic Order in URu₂Si₂

Hasta: Spear (Latin). A new kind of spinor order.



Rebecca FlintPremi Chandra(MIT)(Rutgers)

P.Chandra, R. Flint and P.C arXiv:1404.5920, *PRB* 86, 155155 (2012). doi:10.1038/nature11820 Piers Coleman Center for Materials Theory Rutgers, USA Royal Holloway University of London

Wills Lab, Bristol U 19 Jun 2014



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- Motivation/Overview

- Hidden order in URu2Si2
- Giant Ising Anisotropy and implications
- Hastatic Order.



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Altarawneh et al., PRL 108, 066407 (2012)
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Take-home: Observation of (Perfect) Ising Quasiparticles







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Take-home:

Observation of (Perfect) Ising Quasiparticles Many body hybridization Spin 1/2 e \rightleftharpoons Integer spin (J_z=±1) Doublets

Proposal: Order parameter is a spinor

Altarawneh et al., PRL 108, 066407 (2012)

[001]







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[100]

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Piers Coleman Center for Materials Theory Rutgers, USA

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Wills Lab, Bristol U

19 Jun 2014

Roval Holloway

Jniversity of London



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90° Take-home: Observation of (Perfect) Ising Quasiparticles Many body hybridization Spin 1/2 e \rightleftharpoons Integer spin (J_z=±1) Doublets [100] Proposal: Order parameter is a spinor (mixing J & J+1/2, breaking <u>double</u> time reversal) [001] $\Theta^2 = (-1)^{2J}$ Altarawneh et al., PRL 108, 066407 (2012)

URu₂Si₂: The Hidden Order Mystery











A lot of action at the brink of localization.









 $\Delta S = \int_0^{T_0} \frac{C_V}{T} dT$





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=0.14 x 17.5 K =2.45 J/mol/K =0.42 R ln 2





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=0.42 R ln 2

Large entanglement entropy.





 $\Delta S = \int_{0}^{T_{0}} \frac{C_{V}}{T} dT = 0.14 \text{ x } 17.5 \text{ K} \\ = 2.45 \text{ J/mol/K}$

=0.42 R ln 2

Large entanglement entropy.



What is the nature of the hidden order?









25

20

15

10

5

0

T (K)



Rutgers Center for Materials Theory



(NMR,MuSR).





(NMR,MuSR).



25 Years of Theoretical Proposals

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Landau Theory Shah et al. ('00) "Hidden Order"

Landau Theory

Itinerant

25 Years of Theoretical Proposals

Shah et al. ('00) "Hidden Order"
Ramirez et al, '92 (Quadrupolar SDW)
Ikeda and Ohashi '98 (d-density wave)
Okuno and Miyake '98 (composite)
Tripathi, Chandra, PC and Mydosh, '02 (orbital afm)
Dori and Maki, '03 (Unconventional SDW)
Mineev and Zhitomirsky, '04 (SDW)
Varma and Zhu, '05 (Spin-nematic)
Ezgar et al '06 (Dynamic symmetry breaking)
Fujimoto, '11 (Spin-nematic)
Ikeda et al '12 (Rank 5 nematic)
Rau and Kee '12 (Rank 5 pseudo-spin vector)
Tanmoy Das '12 (Topological Spin-nematic)

Itinerant

Local

25 Years of Theoretical Proposals

Landau Theory Shah et al. ('00) "Hidden Order" Ramirez et al, '92 (Quadrupolar SDW) Ikeda and Ohashi '98 (d-density wave) Okuno and Miyake '98 (composite) Tripathi, Chandra, PC and Mydosh, '02 (orbital afm) Dori and Maki, '03 (Unconventional SDW) Mineev and Zhitomirsky, '04 (SDW) Varma and Zhu, '05 (Spin-nematic) Ezgar et al '06 (Dynamic symmetry breaking) Fujimoto, '11 (Spin-nematic) Ikeda et al '12 (Rank 5 nematic) Rau and Kee '12 (Rank 5 pseudo-spin vector) Tanmoy Das '12 (Topological Spin-nematic) Barzykin & Gorkov, '93 (three-spin correlation) Santini & Amoretti, '94, Santini ('98) (Quadrupole order) Amitsuka & Sakihabara (Γ_5 , Quadrupolar doublet, '94) Kasuya, '97 (U dimerization) Kiss and Fazekas '04, (Rank 3 octupolar order) Haule and Kotliar '09 (Rank 4 hexa-decapolar) Rau and Kee '12 (Rank 5 pseudo-spin vector)

25 Years of Theoretical Proposals

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k + QkMultipole

$$\begin{array}{c} k \\ \hline Multipole \\ \hline \\ \langle c_{\mathbf{k}+Q\alpha}^{\dagger}c_{\mathbf{k}} \rangle = \Psi_{\alpha\beta}(\mathbf{k}) = \begin{cases} \psi_0(\mathbf{k}) & \text{CDW} \\ \vec{d}(\mathbf{k}) \cdot \vec{\sigma}_{\alpha\beta} & \text{SDW} \end{cases}$$








$$\begin{array}{c} k \\ \hline Multipole \\ \hline \\ \langle c^{\dagger}_{\mathbf{k}+Q\alpha}c_{\mathbf{k}}\rangle = \Psi_{\alpha\beta}(\mathbf{k}) = \left\{ \begin{array}{c} \psi_{0}(\mathbf{k}) \\ \vec{d}(\mathbf{k}) \cdot \vec{\sigma}_{\alpha\beta} \end{array} \right. \begin{array}{c} \mathsf{CDW} \\ \mathsf{SDW} \end{array} \right.$$

 $\Psi(R\mathbf{k}) = D(R)\Psi(\mathbf{k})$

a)



 $\Psi(R\mathbf{k}) = D(R)\Psi(\mathbf{k})$

a)

OP	L	S	Name
1	0		Scalar (CDW)
m		1	AFM
k _x	1		Vector (c.f nematic)
k _x k _y k _z	3		Octopole
$k_x k_y (k_x^2 - k_y^2)$	4		Hexdecapole.

$$f_L[\psi] = \frac{1}{V}F[\psi] = \frac{r}{2}\psi^2 + \frac{u}{4}\psi^4.$$

Landau Theory

 $f_L[\psi] = \frac{1}{V}F[\psi] = \frac{r}{2}\psi^2 + \frac{u}{4}\psi^4.$ Landau Theory



 $f_L[\psi] = \frac{1}{V}F[\psi] = \frac{r}{2}\psi^2 + \frac{u}{4}\psi^4.$ Landau Theory





Broken Symmetry

 $f_L[\psi] = \frac{1}{V}F[\psi] = \frac{r}{2}\psi^2 + \frac{u}{4}\psi^4.$ Landau Theory









Broken Symmetry

 $f_L[\psi] = \frac{1}{V}F[\psi] = \frac{r}{2}\psi^2 + \frac{u}{4}\psi^4.$ Landau Theory







 $\begin{bmatrix} H, R \end{bmatrix} = 0$ Symmetry



Broken Symmetry

 $f_L[\psi] = \frac{1}{V}F[\psi] = \frac{r}{2}\psi^2 + \frac{u}{4}\psi^4.$ Landau Theory







 $\begin{bmatrix} H, R \end{bmatrix} = 0$
Symmetry

$$F[\psi_R] = F[\psi]$$
$$\psi \xrightarrow{R} \psi_R = D(R).\psi$$



Broken Symmetry

(a)

 $f_L[\psi] = \frac{1}{V}F[\psi] = \frac{r}{2}\psi^2 + \frac{u}{4}\psi^4.$ Landau Theory



 $\begin{bmatrix} H, R \end{bmatrix} = 0$
Symmetry

$$F[\psi_R] = F[\psi]$$
$$\psi \xrightarrow{R} \psi_R = D(R).\psi$$





Broken Symmetry

Can an electron OP- ever transform like a spinor, or 1/2 integer particle?

 $\Psi(R\mathbf{k}) = D(R)\Psi(\mathbf{k})$

a)



The Giant Ising Anisotropy.





$$M \propto \cos \left[2\pi \frac{\text{Zeeman}}{\text{cyclotron}} \right]$$













M. M. Altarawneh, N. Harrison, S. E. Sebastian, et al., PRL (2011). H. Ohkuni *et al.*, Phil. Mag. B 79, 1045 (1999).

16 spin zeros!

$$\frac{m^*}{m_e}g(\theta) = 2n+1$$



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Superconductivity: Giant Ising Anisotropy



$$\mu_0 H_{\rm p} = \frac{1}{\sqrt{2} \ \mu_{\rm B} g_{\rm eff}^*}$$

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Ising QP's pair condense.

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Electrons hybridize with Ising 5f state to form Landau quasiparticles. $\langle {\bf k}\sigma|J_{\pm}|{\bf k}\sigma'
angle=0$

"Ising Order: detailed analysis."

Giant Ising anisotropy indicates quasiparticle hybridization of electrons $|{\bf k}\sigma$) and Ising 5f doublet

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 $|\alpha\rangle \equiv |\pm\rangle$ =

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$$|+\rangle = \sum_{n} a_{n} |M - 4n\rangle$$
$$|-\rangle = \sum_{n} a_{n} |-M + 4n\rangle$$



4 fold symmetry mixes states differing by 4/h

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.: Integer spin M



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 $|\Gamma,\pm\rangle = a|\pm 3\rangle + b|\mp 1\rangle$ "\Gamma_5" non-Kramers doublet 5f²

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"Γ₅" non-Kramers doublet 5f²



```
|\Gamma,\pm\rangle = a|\pm 3\rangle + b|\mp 1\rangle
"\Gamma_5" non-Kramers doublet 5f<sup>2</sup>
```



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Kramers index K: quantum no of *double* time reversal $\theta x \theta = \theta^2$.

$$\Theta^2 |\psi\rangle = K |\psi\rangle$$



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Half-integer spins change sign, integer spins do not.

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V = -V

Hybridization is a spinor.

Hybridization transforms as a 1/2 integer spin.



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Hybridization transforms as a 1/2 integer spin. Unlike magnetism, it breaks **double time reversal. A new kind of order parameter.** Hybridization is a spinor.

 $V = -V^{2\pi}$

Support for hybridization as the order parameter.

Spectroscopy: H-gap in STM/C tics

NA





Spectroscopy H-gap in STM/Optics



















High Resolution ARPES

Chaterjee et al, arXiv 1211.5312



High Resolution ARPES

Chaterjee et al, arXiv 1211.5312



High Resolution ARPES



High Resolution ARPES



Heavy fermion bands come into focus at $T_{\rm HO}$
"Hastatic Order"

Landau Theory:

Conventional Landau theory of electron fluids involves the formation of two body bound-states. When the two body bound-state carries a quantum number, the corresponding two body wavefunction transforms non-trivially under the symmetries of the vacuum, and is promoted to an order parameter. Landau order parameters involve even numbers of electrons and carry integer spin.

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e.g.
$$\psi_{\alpha}^{\dagger}(x)\psi_{\beta}(x) = \vec{M}(x) \cdot \vec{\sigma}_{\alpha\beta}$$
 Ferromagnetism=2 body BS
Transforms as vector (S=1)

Microscopically, this results from the fractionalization of threefermion bound-states into an integer spin fermion and a halfinteger spin boson. The order parameter is the 3-body wavefunction, which transforms under a double group representation and carrying half-integer spin.

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$$\psi_{\sigma_1}^{\dagger}(1)\psi_{\sigma_2}^{\dagger}(2)\psi_{\bar{\sigma}_3}(3) = \Gamma_{\sigma_1\sigma_2\bar{\sigma}_3}^{\alpha}(1,2,3;x)\chi_{\alpha}^{\dagger}(x)$$

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$$\uparrow$$
Half integer OP
$$S=1 \text{ fermion}$$



In conventional heavy fermion materials a hybridization derives from virtual excitations between a Kramers doublet and an excited singlet.





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H. Amitsuka and T. Sakakibara, J. Phys. Soc. Japan 63, 736-47 (1994).

But if the ground-state is a non-Kramer's doublet, the Kondo effect occurs via an *excited Kramer's doublet*.



non-
Kramers
$$\Gamma_5$$
 ===== $|5f^2, \alpha\rangle = \hat{\chi}^{\dagger}_{\alpha}|0\rangle$ (K=+1)

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Kramers
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$$\Gamma_7 = [5f^3, \sigma\rangle = \hat{\Psi}^{\dagger}_{\sigma}|0\rangle$$

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"Hastatic" order.





$$5f^3, \sigma \rangle \langle 5f^2, \alpha | = \hat{\Psi}^{\dagger}_{\sigma} \hat{\chi}_{\alpha}$$





$$|5f^3,\sigma\rangle\langle 5f^2,\alpha|\longrightarrow \langle\hat{\Psi}^{\dagger}_{\sigma}\rangle\hat{\chi}_{\alpha}$$



("Magnetic Higgs Boson")

$$\Psi = \begin{pmatrix} \langle \Psi_{\uparrow} \rangle \\ \langle \Psi_{\downarrow} \rangle \end{pmatrix}$$

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Quasiparticles acquire the Ising anisotropy of the non-Kramers doublet.



The origin of the anisotropy in the non-linear susceptibility.

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A. Ramirez, P. Coleman, P. Chandra, A. Menovsky, E. Bruck, Z. Fisk and E. Bucher, Physical Review Letters, 68, 2680, (1992).



$$\Psi = \begin{pmatrix} \Psi_{\uparrow} \\ \Psi_{\downarrow} \end{pmatrix}$$

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AFM: $P > P_c$ $\Psi_A \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \Psi_B \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Large f-moment

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HO: P<Pc

$$\Psi_A \sim \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\phi/2} \\ e^{i\phi/2} \end{pmatrix}, \Psi_B \sim \frac{1}{\sqrt{2}} \begin{pmatrix} -e^{-i\phi/2} \\ e^{i\phi/2} \end{pmatrix}$$

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 $f[T, P, B_z] = [\alpha(T_c - T) - \eta_z B_z^2] |\Psi|^2 + \beta |\Psi|^4 - \gamma (\Psi^{\dagger} \sigma_z \Psi)^2$

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Microscopic theory of Hastatic Order

$$|\Gamma_{7}^{+},\sigma\rangle \equiv \Psi_{\sigma}^{\dagger}|0\rangle = \uparrow$$
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$$H_{VF} = \sum_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} \mathcal{V}_{6}(\mathbf{k}) \chi_{\mathbf{k}} + c_{\mathbf{k}}^{\dagger} \mathcal{V}_{7}(\mathbf{k}) \chi_{\mathbf{k}+\mathbf{Q}} + \text{h.c.}$$

$$H = \sum_{\mathbf{k}} \left(c_{\mathbf{k}}^{\dagger}, c_{\mathbf{k}+\mathbf{Q}}^{\dagger}, \chi_{\mathbf{k}}^{\dagger}, \chi_{\mathbf{k}+\mathbf{Q}}^{\dagger} \right) \underbrace{\begin{pmatrix} \epsilon_{\mathbf{k}} & 0 & \mathcal{V}_{6}(\mathbf{k}) & \mathcal{V}_{7}(\mathbf{k}) \\ 0 & \epsilon_{\mathbf{k}+\mathbf{Q}} & -\mathcal{V}_{7}(\mathbf{k}) & -\mathcal{V}_{6}(\mathbf{k}) \\ \mathcal{V}_{6}^{\dagger}(\mathbf{k}) & -\mathcal{V}_{7}^{\dagger}(\mathbf{k}) & \lambda_{\mathbf{k}} & 0 \\ \mathcal{V}_{7}^{\dagger}(\mathbf{k}) & -\mathcal{V}_{6}^{\dagger}(\mathbf{k}) & 0 & \lambda_{\mathbf{k}+\mathbf{Q}} \end{pmatrix}}_{\mathcal{X}_{\mathbf{k}+\mathbf{Q}}} \begin{pmatrix} c_{\mathbf{k}} \\ c_{\mathbf{k}+\mathbf{Q}} \\ \chi_{\mathbf{k}} \\ \chi_{\mathbf{k}+\mathbf{Q}} \end{pmatrix}$$

$$5f^2 \rightleftharpoons 5f^1 + e^ \psi^{\dagger}_{\Gamma\sigma}(j) = \sum_{\mathbf{k}} \left[\Phi^{\dagger}_{\Gamma}(\mathbf{k}) \right]_{\sigma\tau} c^{\dagger}_{\mathbf{k}\tau} \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{R}_j}$$

$$H_{VF}(j) = V_6 \psi_{\Gamma_6 \pm}^{\dagger}(j) \Psi_{j\pm}^{\dagger} \chi_{j\pm} + V_7 \psi_{\Gamma_7 \mp}^{\dagger}(j) \Psi_{j\mp}^{\dagger} \chi_{j\pm} + \text{H.c.}$$

$$\langle \Psi_{j}^{\dagger} \rangle = |\Psi| \begin{pmatrix} e^{i(\mathbf{Q} \cdot \mathbf{R}_{j} + \phi)/2} \\ e^{-i(\mathbf{Q} \cdot \mathbf{R}_{j} + \phi)/2} \end{pmatrix}, \qquad (\phi = \pi/4).$$

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Consistency with experiment.



Computed g-factor Anisotropy





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Nematicity: anisotropic X_{xy} consistent with observed susceptibility anomaly.





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(c) S ~ 1/2 ln (2) natural consequence of Majorana zero mode in two channel Kondo physics.



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(b)

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Predictions.



(b) Giant non-linear susceptibility anomaly.







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(c) Collapse of gap to Ising fluctuations at 1st order transition line.

(d) Resonant Nematicity in STM



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