

The Physics of Heavy Fermion Superconductivity

Lecture II. BCS meets Kondo: mean-field approach to the Kondo Lattice.

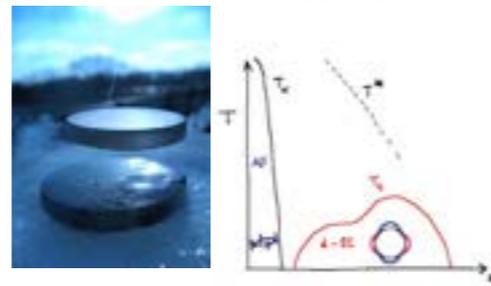


Piers Coleman

Center for Materials Theory, Rutgers.

Boulder School 2014: Modern Aspects of Superconductivity

June 30-July 25, 2014



14-17 July 2014



The Physics of Heavy Fermion Superconductivity

1. Introduction: Heavy Fermions and the Kondo Lattice.
2. BCS meets Kondo: mean-field approach to the Kondo Lattice.
3. *Glue vs Fabric: Good, Bad and Ugly Heavy Fermion Superconductors.*
4. *Composite vs AFM induced pairing.*

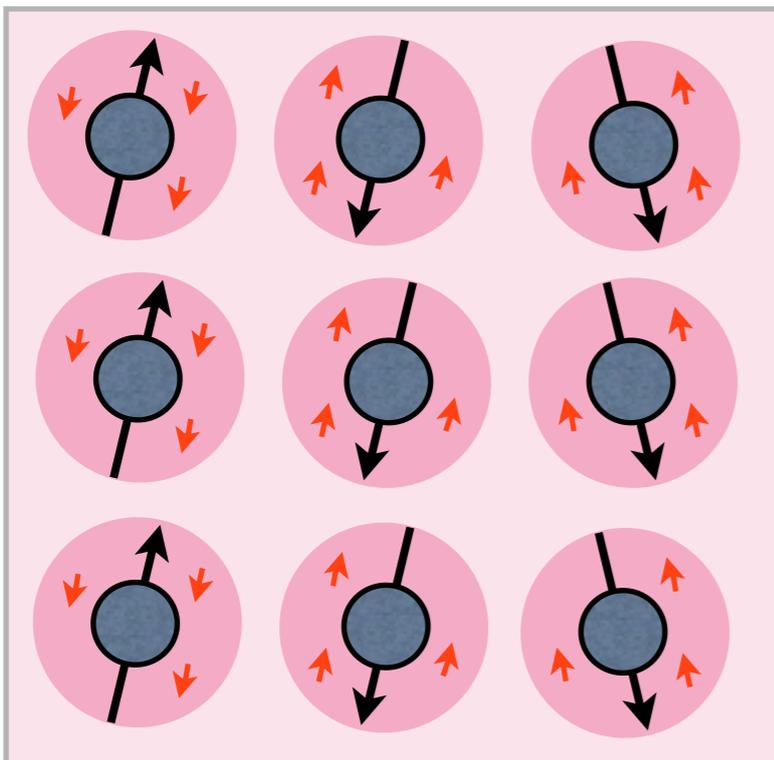
Last Time: Lecture 1 Introduction to Heavy Fermions and the Kondo Lattice.

1. Magnetism and SC: a remarkable convergence.
2. Electrons on the Brink of Localization.
3. Cartoon introduction to Heavy Fermions.
4. Lev Landau versus Ken Wilson: Criticality as a driver of Superconductivity.
5. Anderson, Kondo and Doniach.

THE KONDO LATTICE (From Lecture I)

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \frac{J}{\mathcal{N}} \sum_j \vec{S}_j \cdot c_{\mathbf{k}\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}'\beta} e^{i(\mathbf{k}' - \mathbf{k}) \cdot \mathbf{R}_j}$$

T. Kasuya (1951)



“Kondo Lattice”

Note: can also write Kondo interaction in the “Coqblin Schrieffer” form

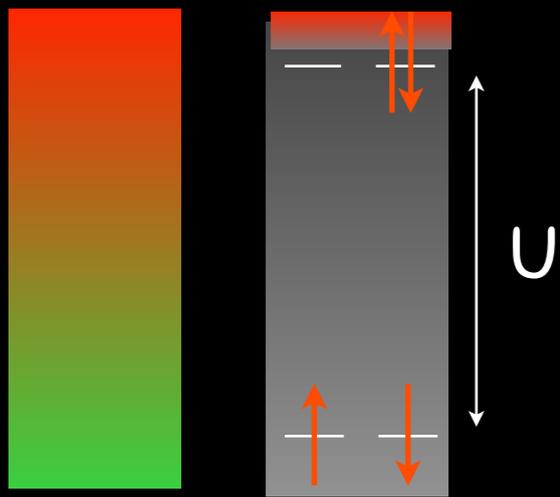
$$H_K = -J \sum_{j,\alpha,\beta} (c_{j\alpha}^\dagger f_{j\alpha})(f_{j\beta}^\dagger c_{j\beta})$$

Physics of Heavy Fermion Superconductivity Lecture II:

1. The large N approach to the Kondo lattice.
2. Heavy Fermion Metals.
3. Optical Conductivity of Heavy Fermion Metals
4. Kondo Insulators

Gauge Theories and Strong Correlation.

Strong correlation \leftrightarrow Constrained Hilbert space
 \leftrightarrow Gauge theories



e^-
 Φ
 $U(1)_{\text{global}} \times U(1)_{\text{local}}$

e.g. $\vec{S}_j = f_{j\alpha}^\dagger \left(\frac{\vec{\sigma}}{2} \right)_{\alpha\beta} f_{j\beta},$

$f_j \rightarrow e^{i\phi_j} f_j, \quad U(1)_{\text{local}}$

$$H = \sum_k \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \frac{J}{N} \sum_j c_{j\alpha}^\dagger c_{j\beta} S_{\beta\alpha}(j) + H_g$$

$$S_{\alpha\beta} = f_\alpha^\dagger f_\beta - \delta_{\alpha\beta} n_f / N$$

$$H_g = (\Phi - \mu) c_j^\dagger c_j + \lambda_j (f_j^\dagger f_j - Q),$$

$$(Q = qN = 1)$$

Strongly correlated electron physics: no small parameter

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Large N : family of models with “ N ” spin components, which retain the key physics and can be solved in the large N limit.

Strongly correlated electron physics: no small parameter

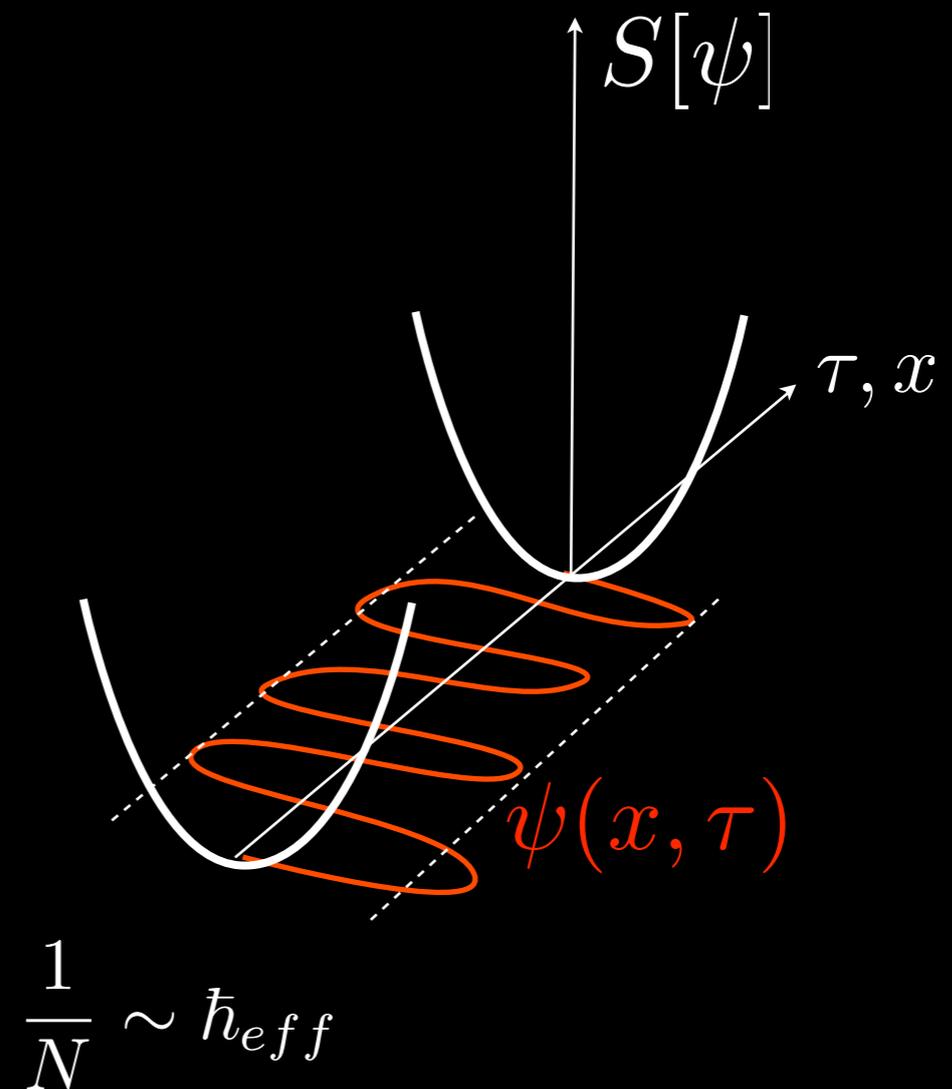
Large N : family of models with “N” spin components, which retain the key physics and can be solved in the large N limit.

$$Z = \int \mathcal{D}[\psi] e^{-NS[\psi, \dot{\psi}]}$$

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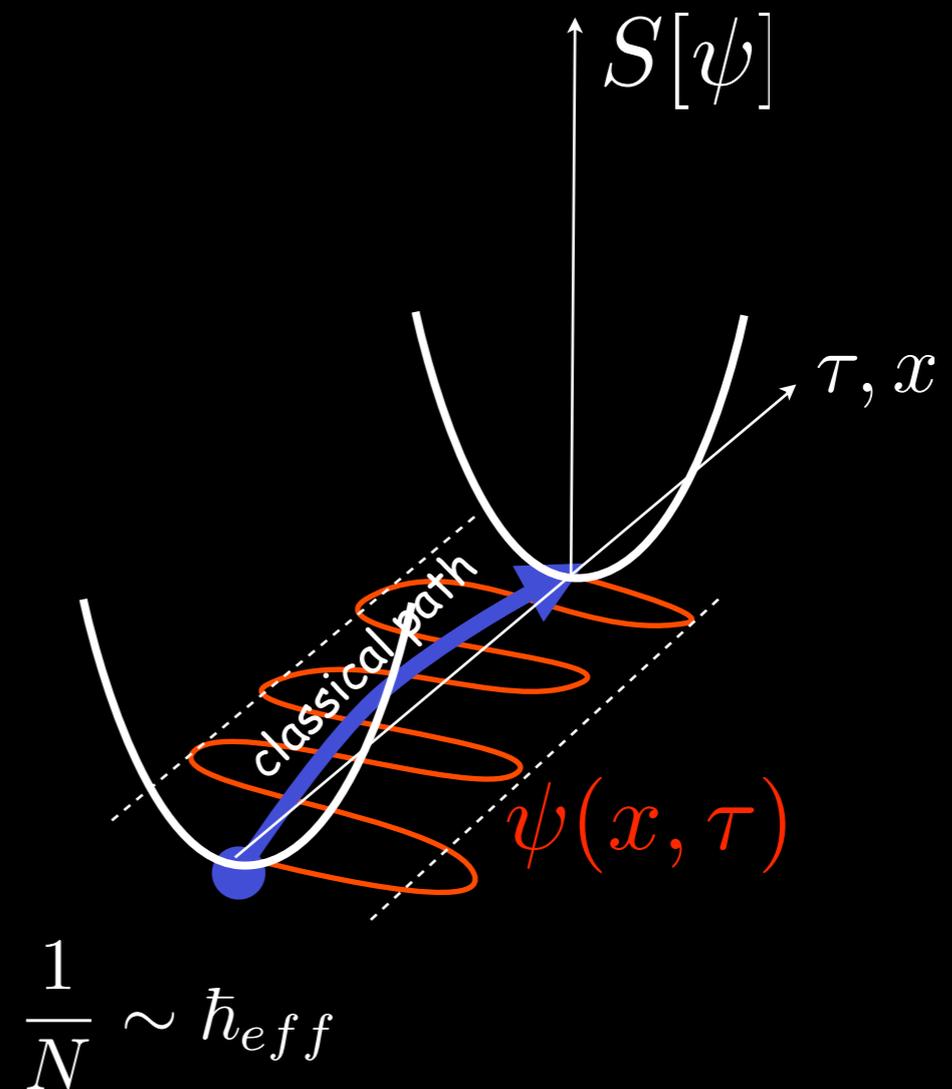
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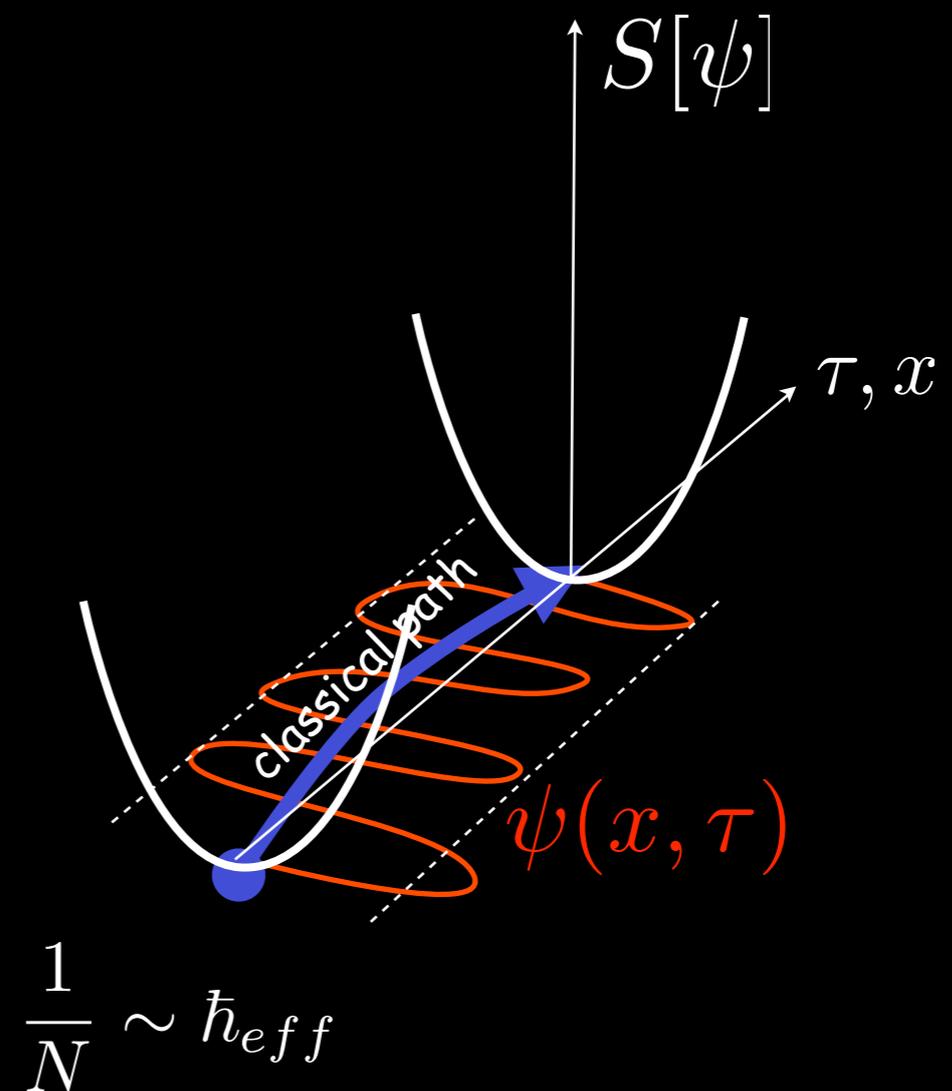
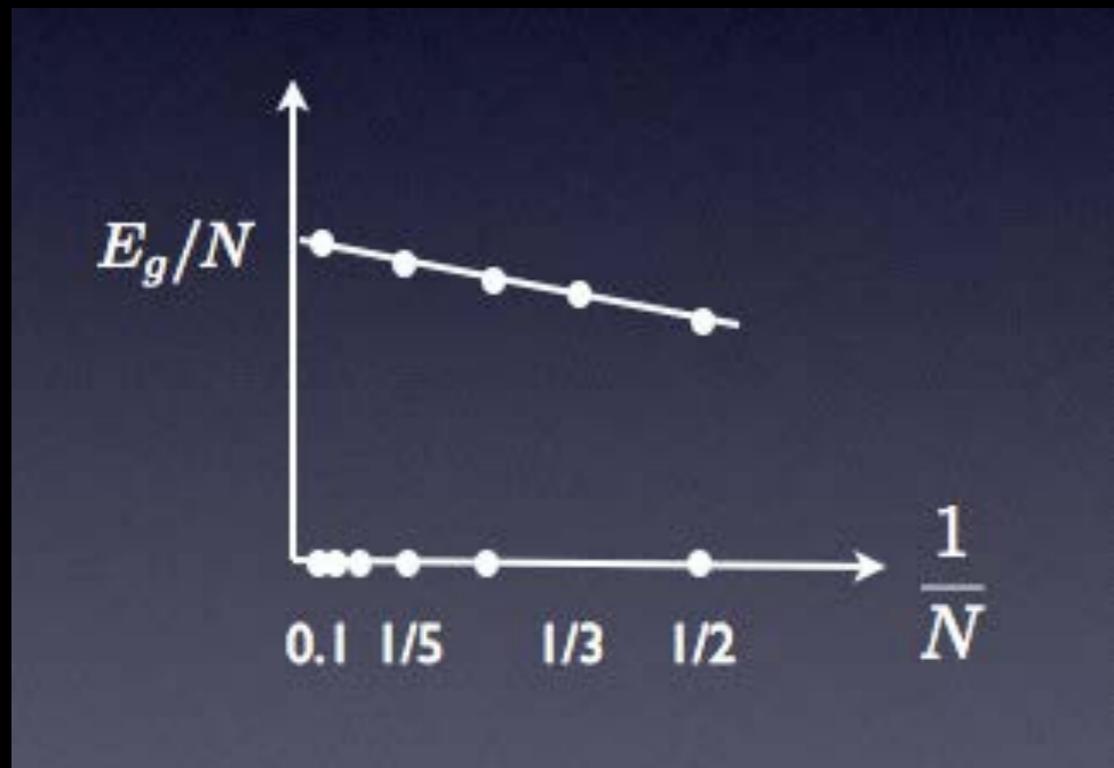
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Large N Approach.

Read and Newns '83.

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \sum_j H_I(j)$$

$$H_I(j) = -\frac{J}{N} \left(c_{j\beta}^\dagger f_{j\beta} \right) \left(f_{j\alpha}^\dagger c_{j\alpha} \right)$$

Large N Approach.

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$$c_{j\alpha}^\dagger = \frac{1}{\sqrt{\mathcal{N}_s}} \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger e^{-i\mathbf{k}\cdot\vec{R}_j}$$

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Constraint $n_f = Q=qN$
all terms extensive in N

Large N Approach.

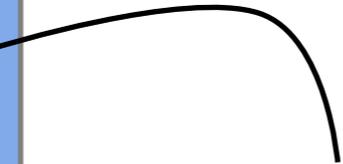
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$$-gA^\dagger A \rightarrow A^\dagger V + \bar{V} A + \frac{\bar{V}V}{g}$$

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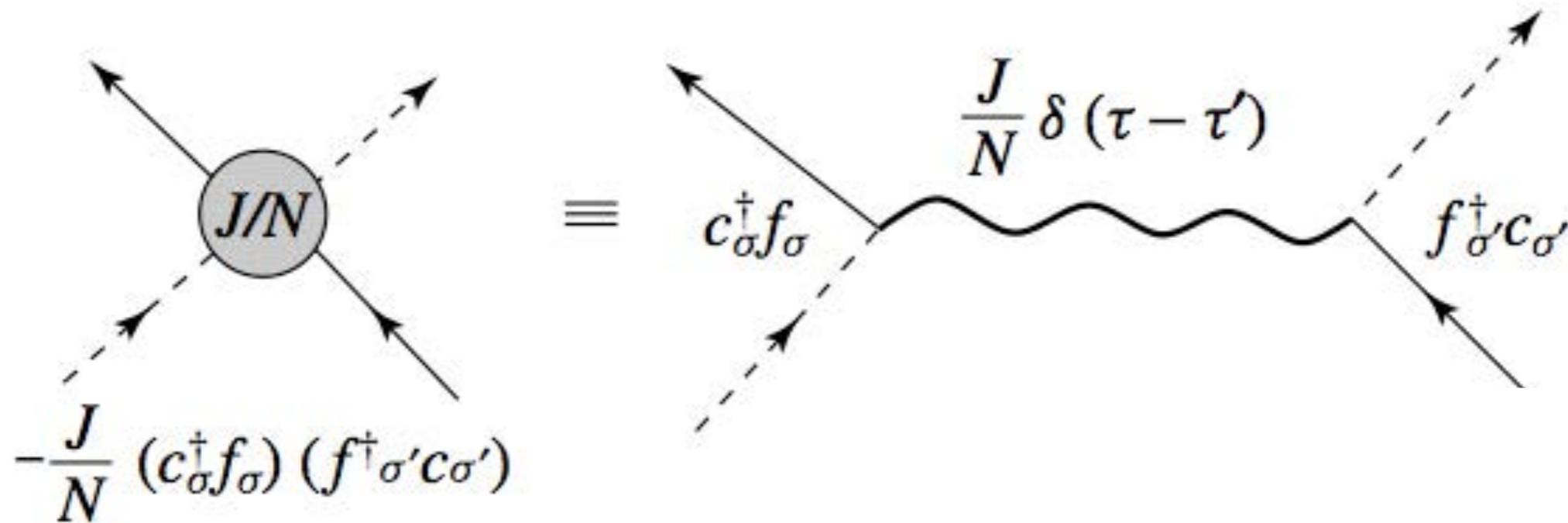
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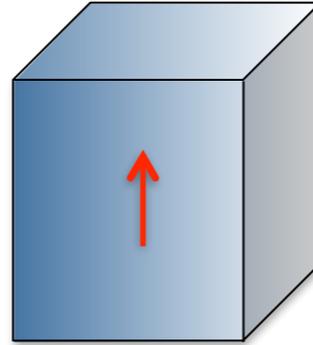
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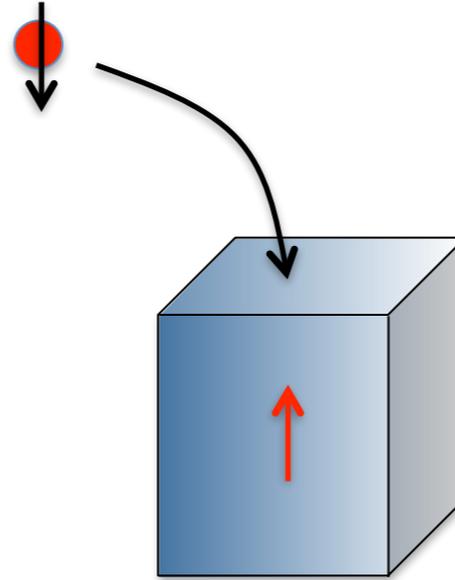
$$H_I(j) \rightarrow H_I[V, j] = \bar{V}_j \left(c_{j\alpha}^\dagger f_{j\alpha} \right) + \left(f_{j\alpha}^\dagger c_{j\alpha} \right) V_j + N \frac{\bar{V}_j V_j}{J}.$$



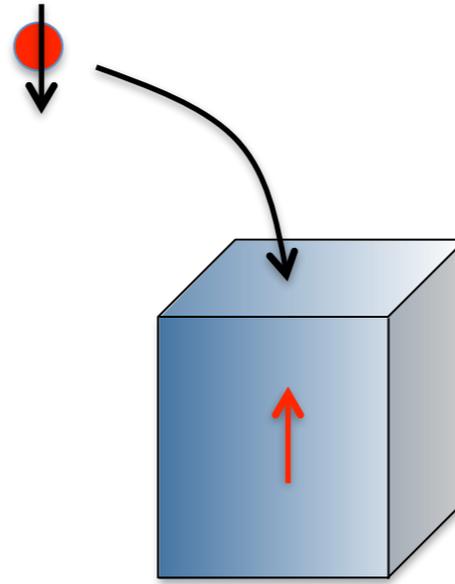
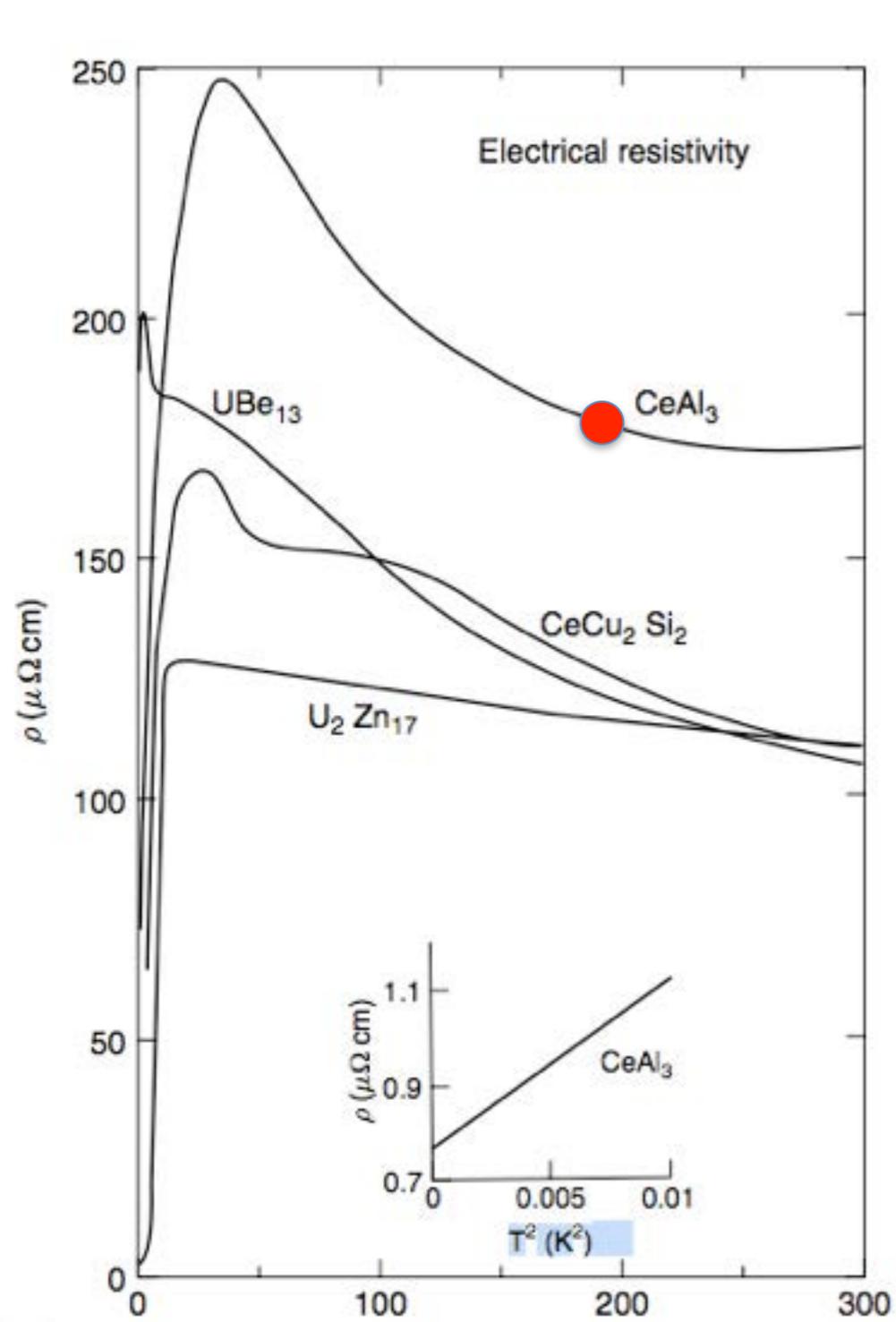
Coherence and composite fermions



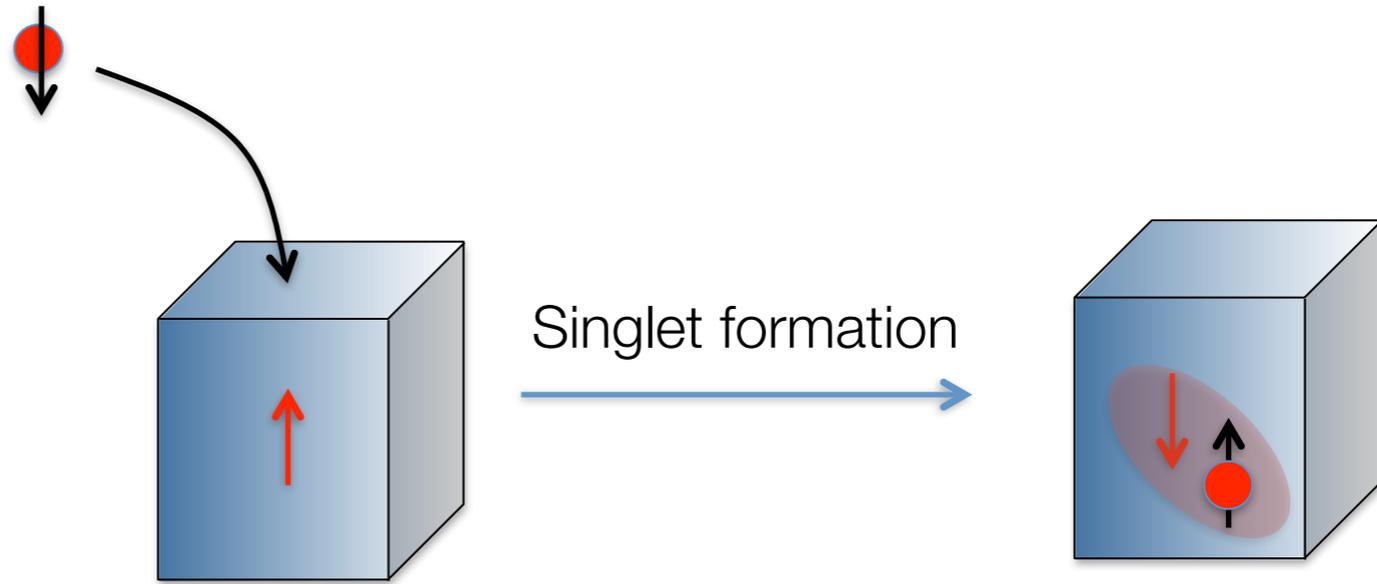
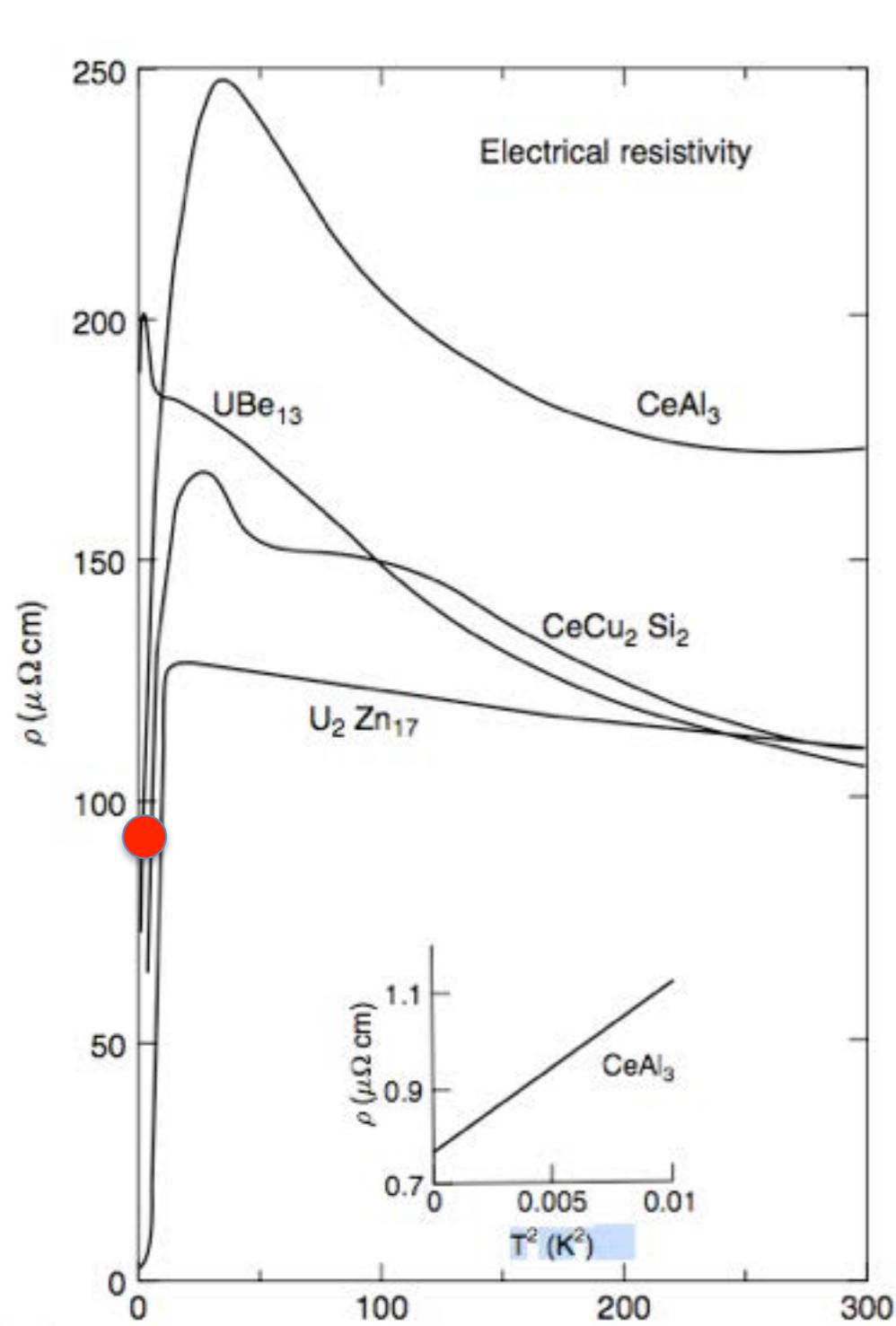
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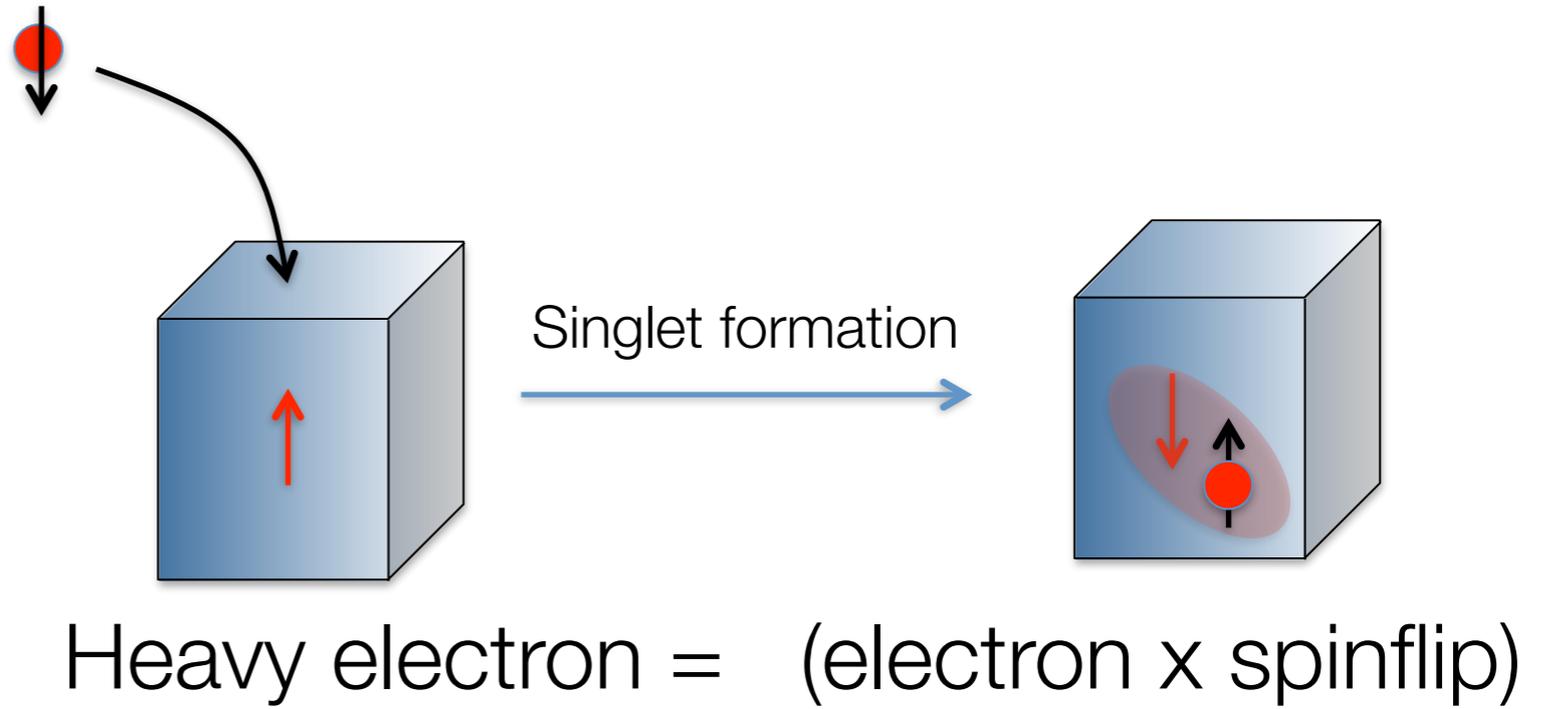
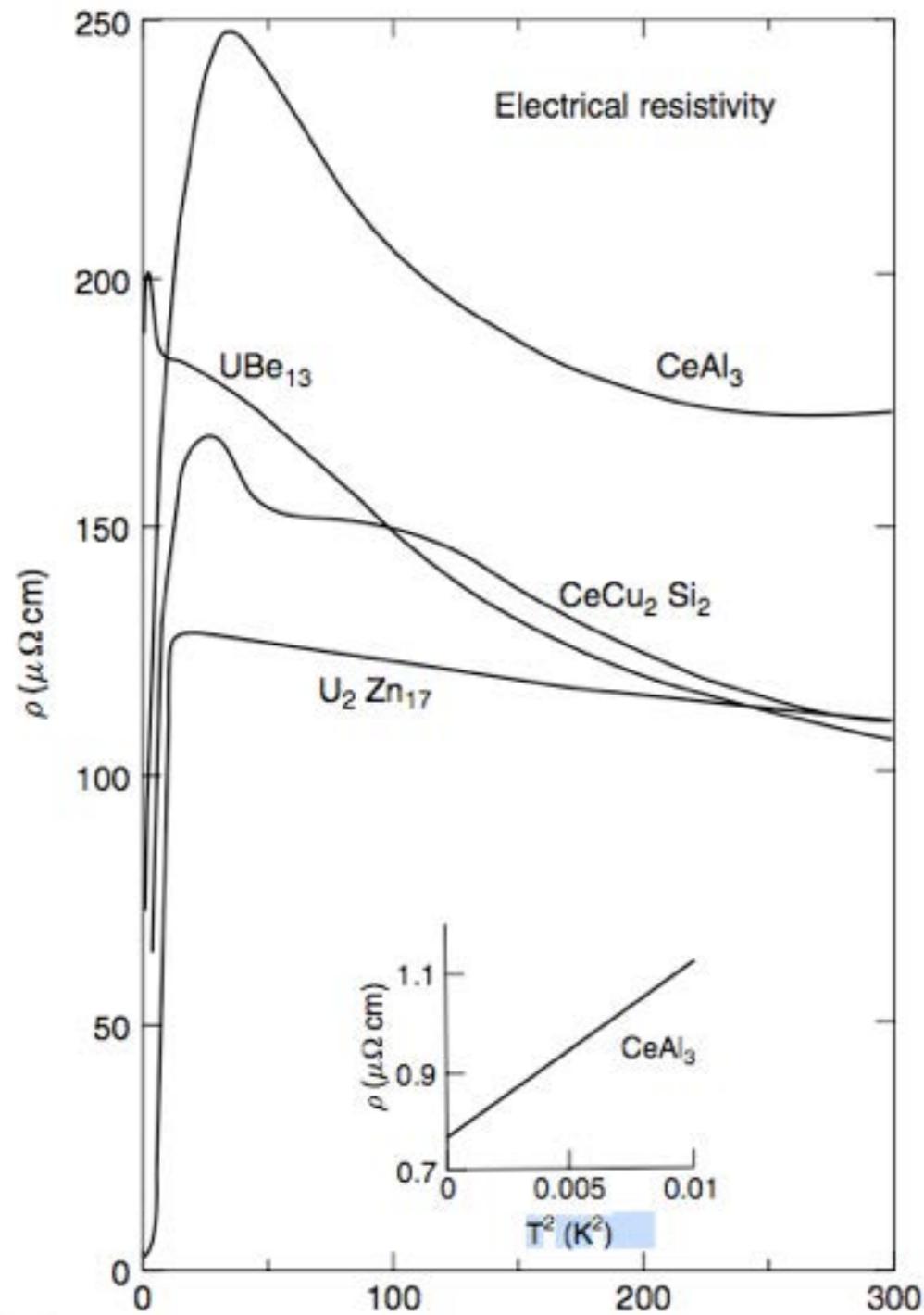
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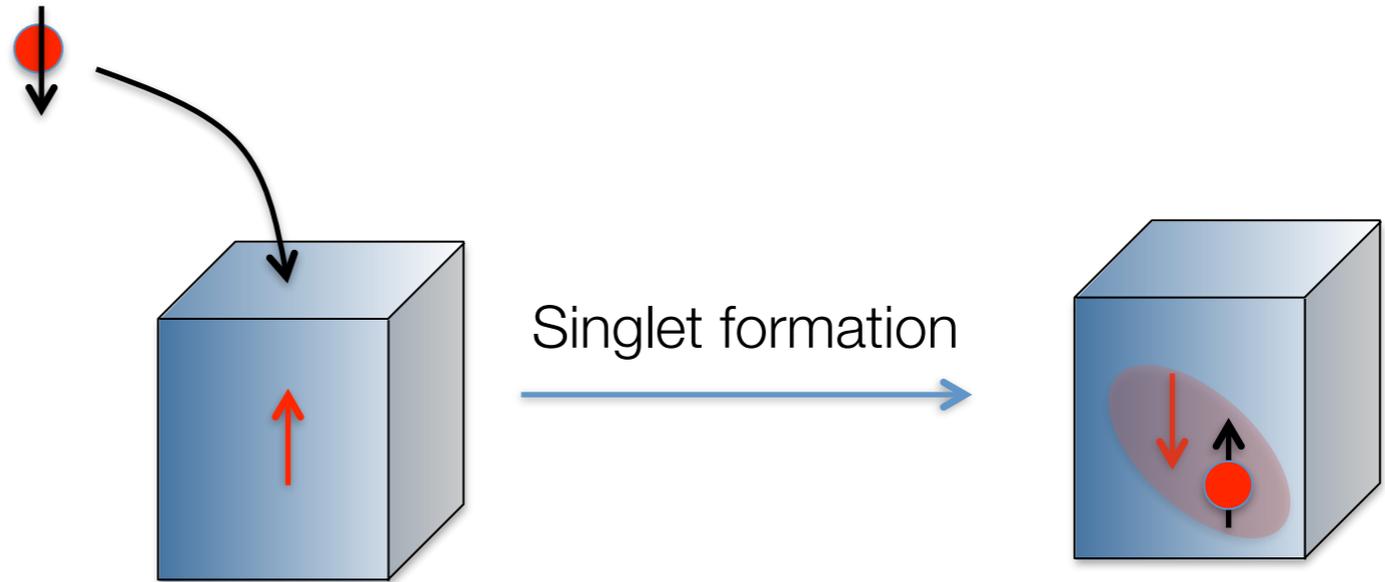
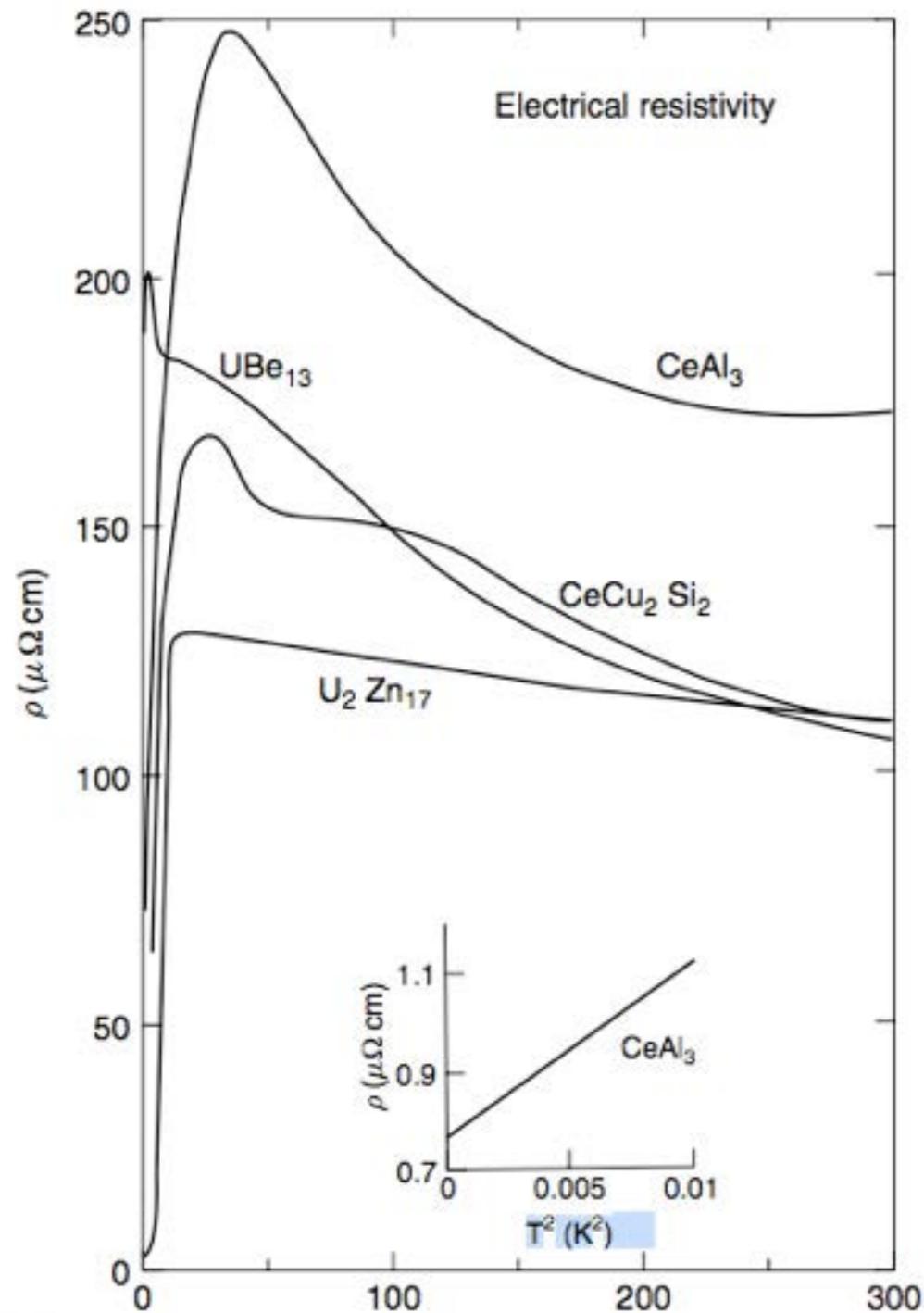
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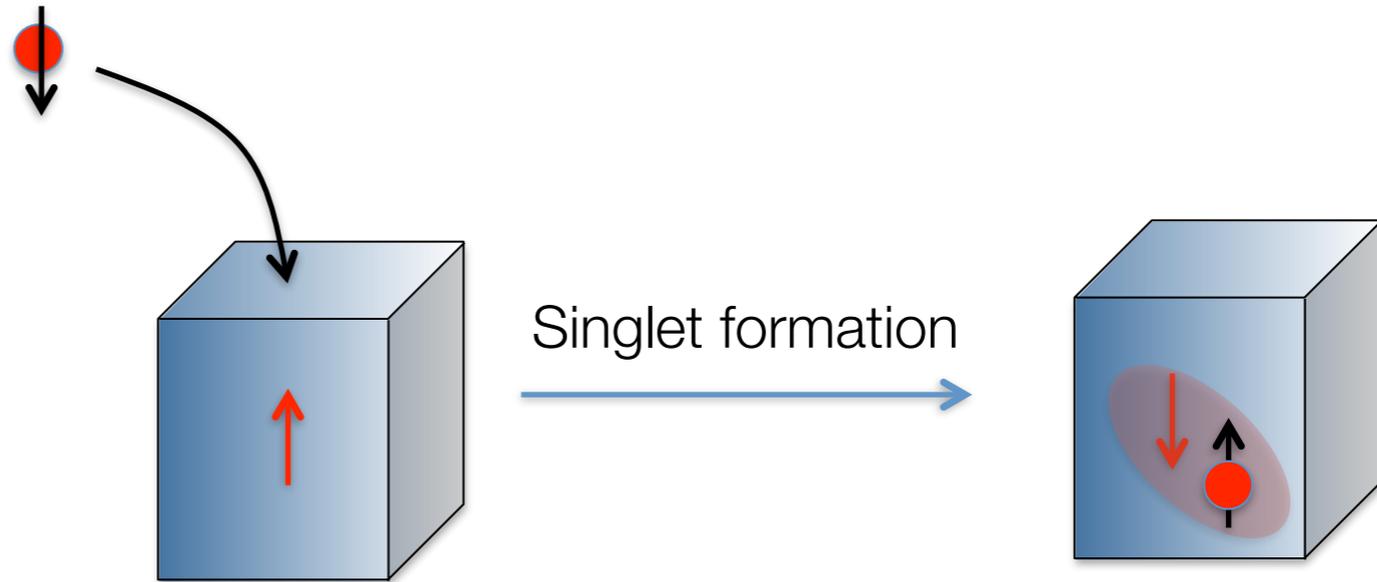
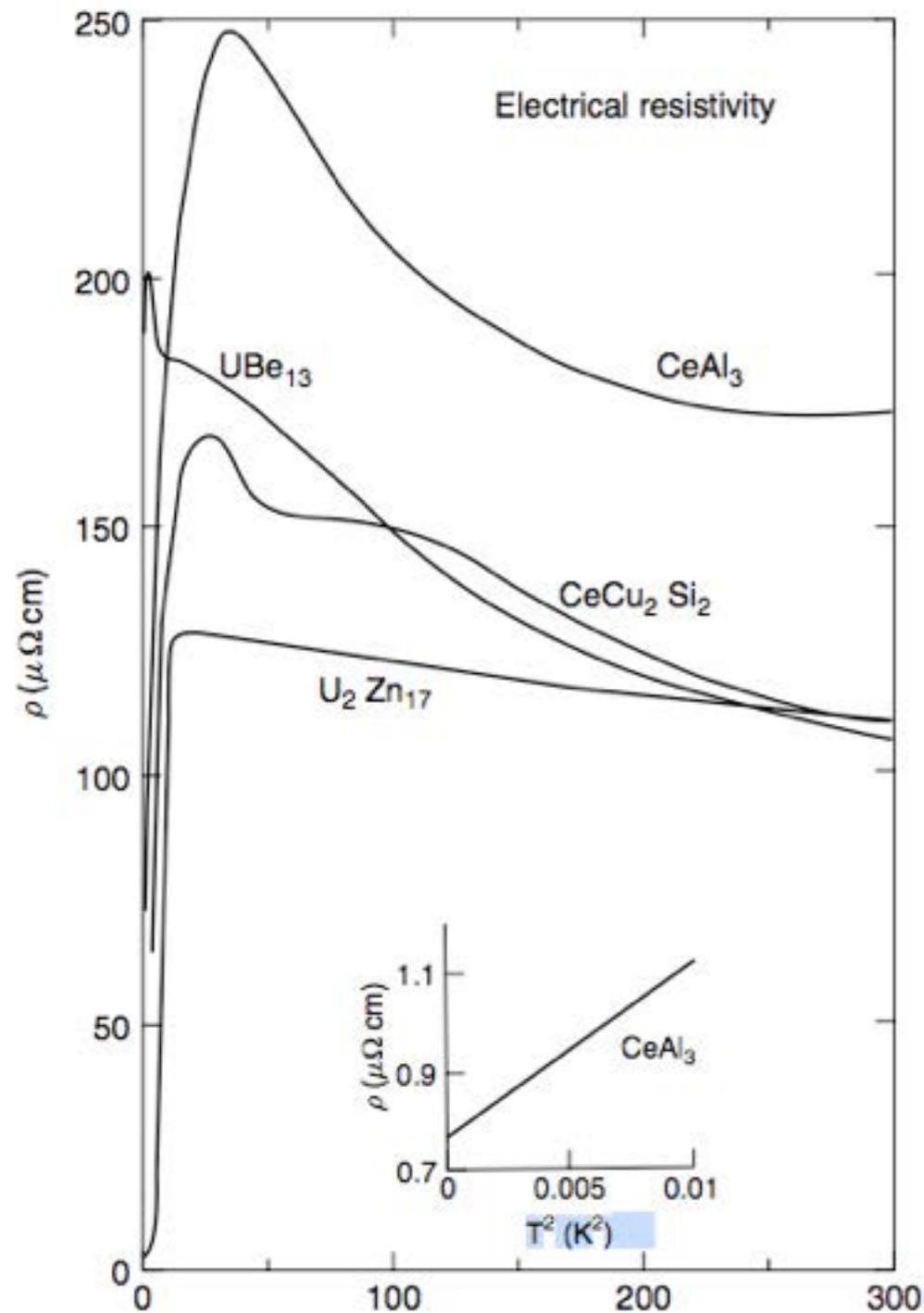
Coherence and composite fermions



Heavy electron = (electron x spinflip)

- The large N approach to the Kondo lattice.
Spin x conduction = composite fermion

Coherence and composite fermions

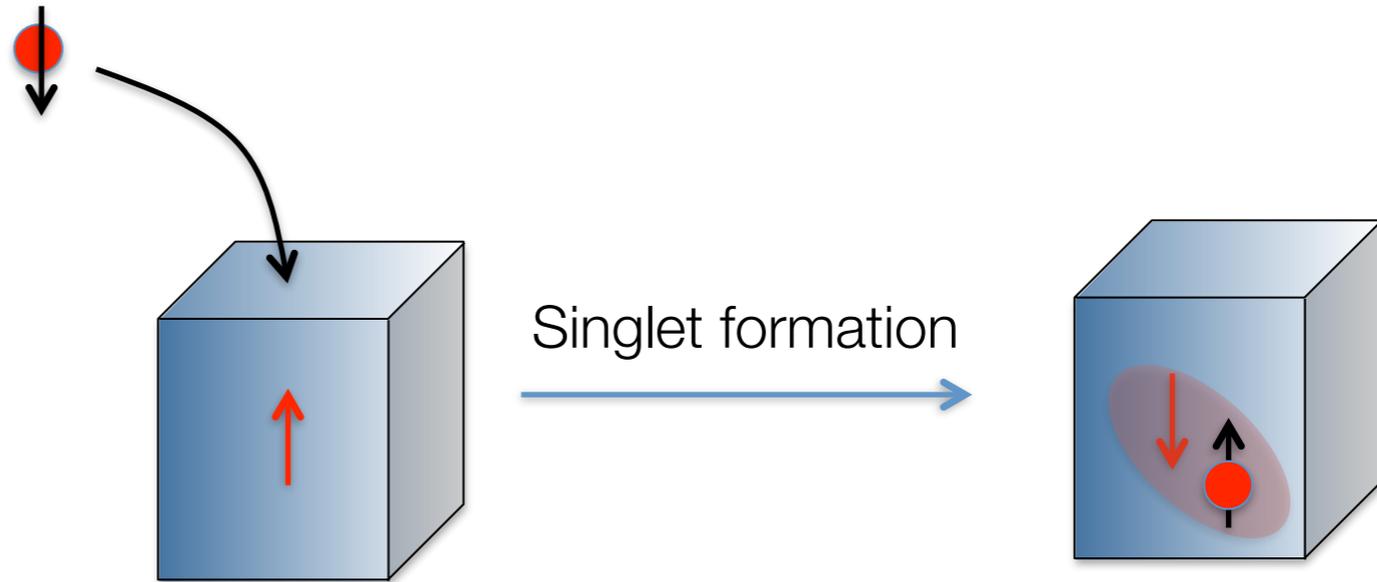
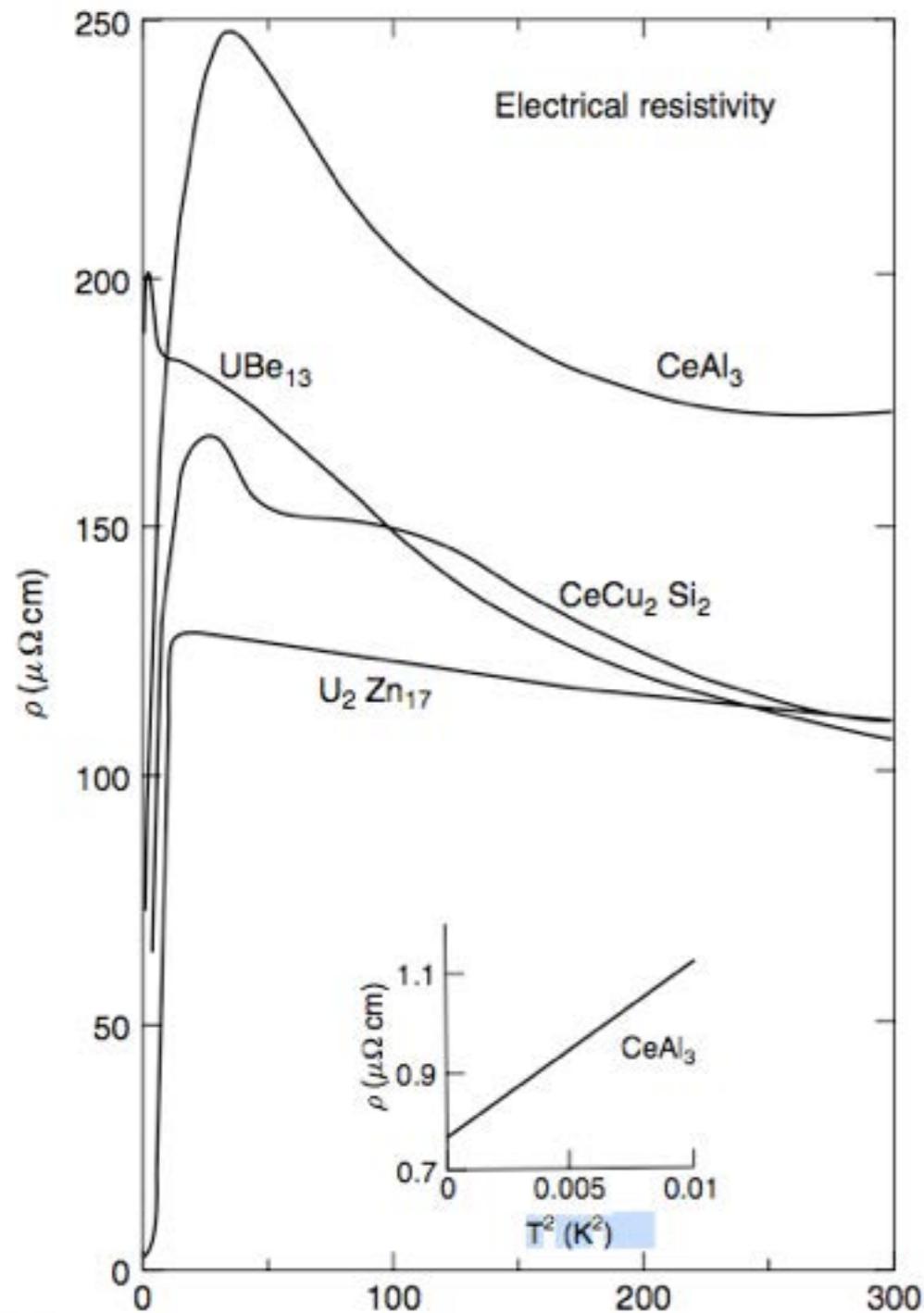


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$$\frac{J}{N} c^\dagger_\beta S_{\alpha\beta} c_\alpha \rightarrow \bar{V} (c^\dagger_\alpha f_\alpha) + (f^\dagger_\alpha c_\alpha) V + N \frac{\bar{V}V}{J},$$

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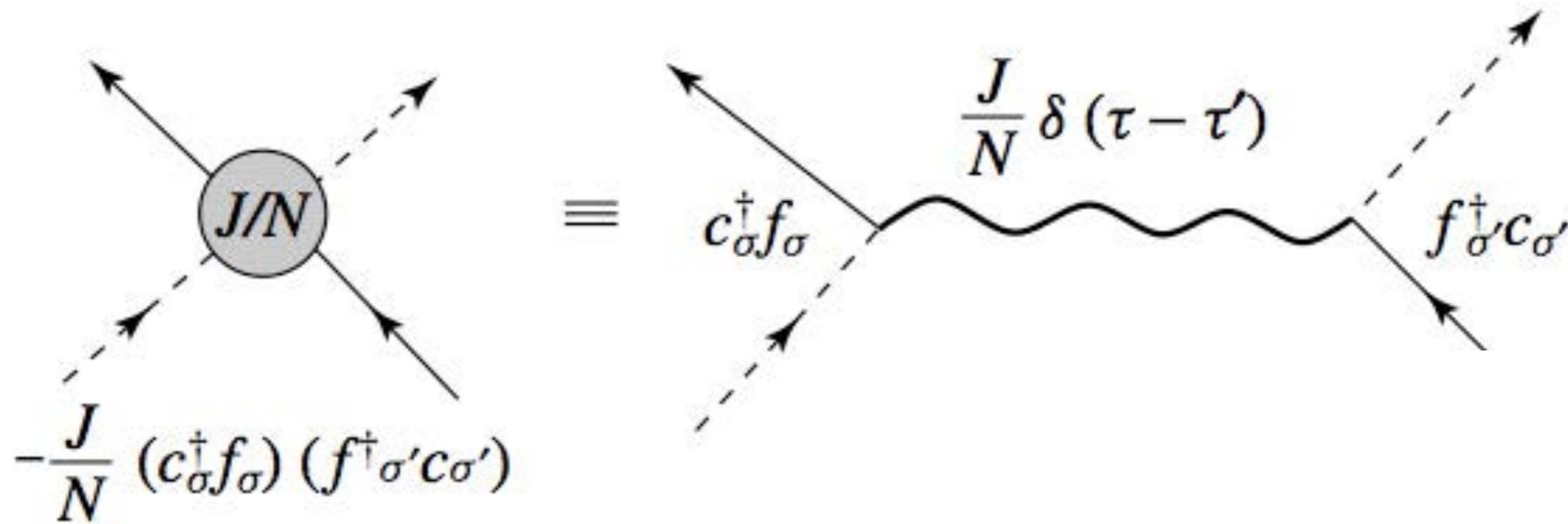
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$$\frac{J}{N} c^\dagger_{j\alpha} S_{\alpha\beta} \equiv \bar{V} f^\dagger_{j\beta}$$

Composite Fermion

Large N Approach

Read and Newns '83.

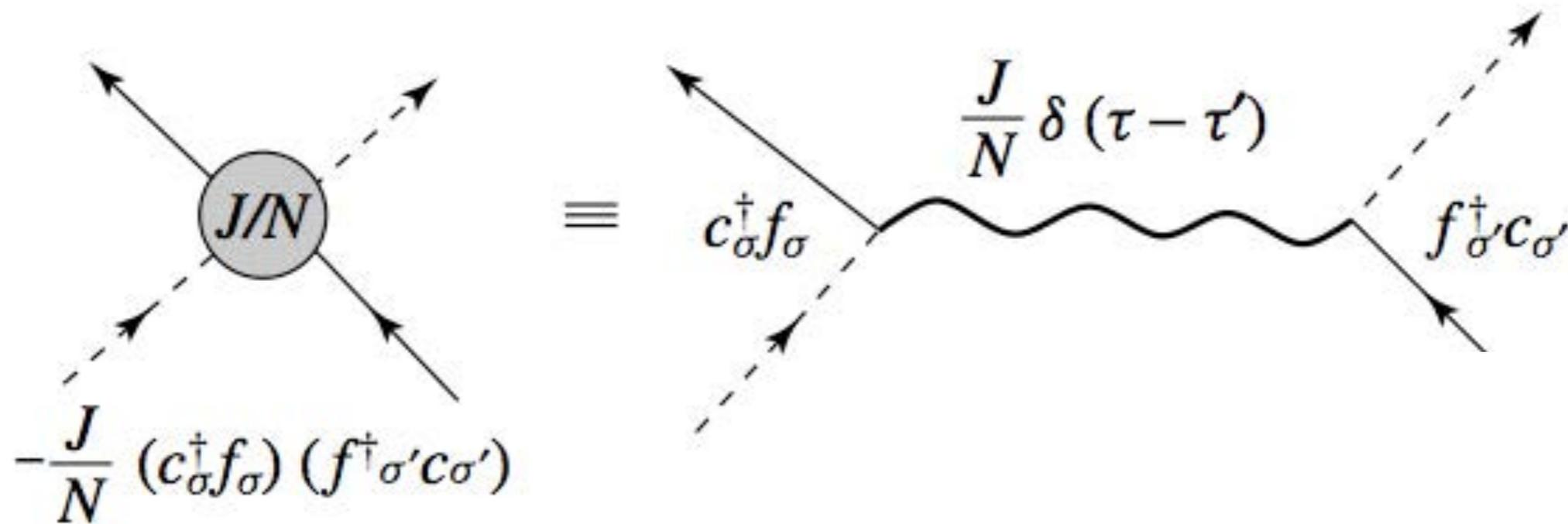


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$$H[V, \lambda] = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \sum_j (H_I[V_j, j] + \lambda_j [n_f(j) - Q]),$$

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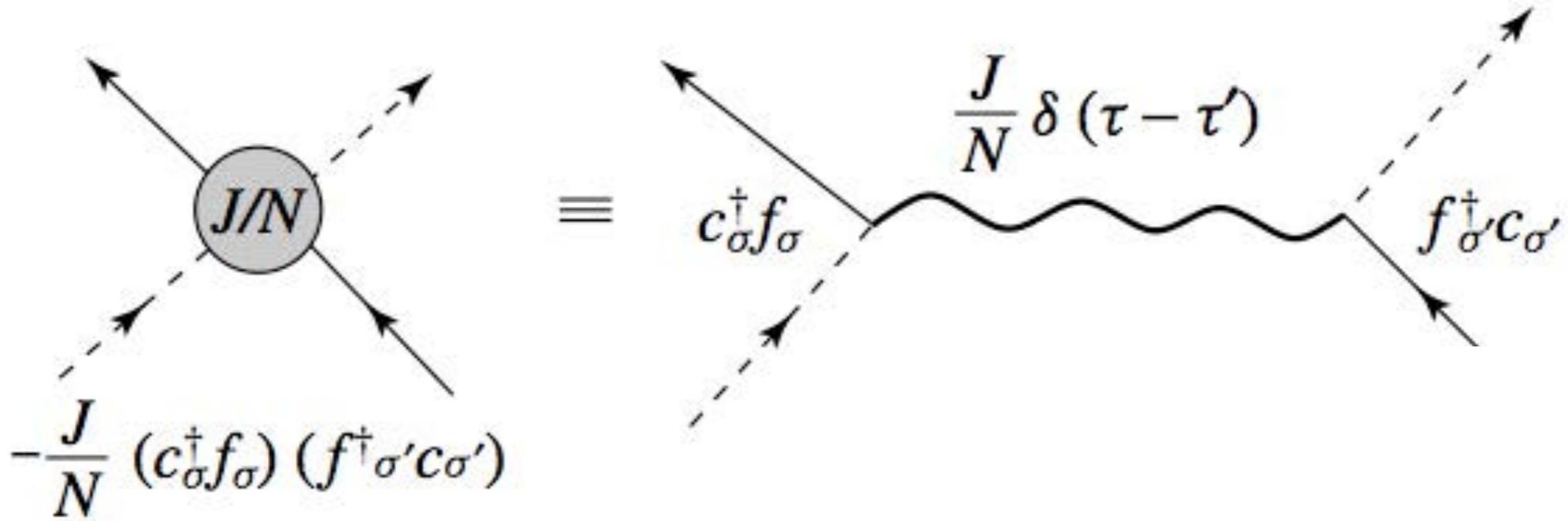
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U(1) constraint: note $n_f = Q = (qN)$



Large N Approach

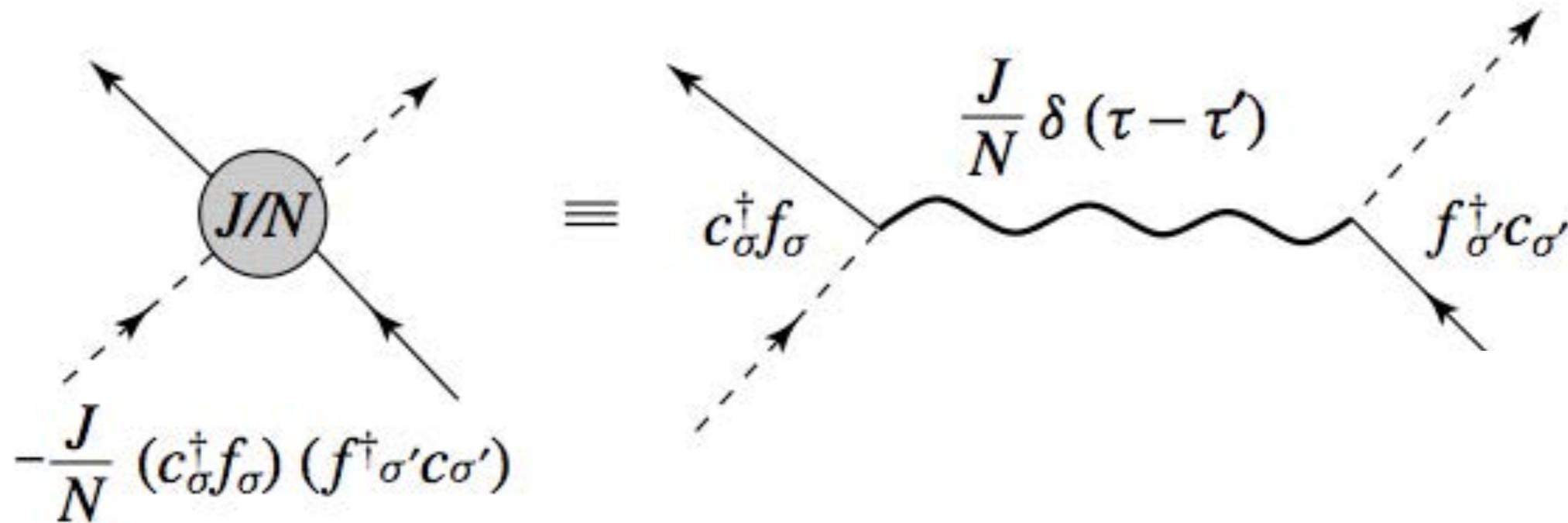
Read and Newns '83.

$$Z = \int \mathcal{D}[V, \lambda] \int \mathcal{D}[c, f] \exp \left[- \int_0^\beta \left(\sum_{k\sigma} c_{k\sigma}^\dagger \partial_\tau c_{k\sigma} + \sum_{j\sigma} f_{j\sigma}^\dagger \partial_\tau f_{j\sigma} + H[V, \lambda] \right) \right] = \text{Tr} \left[T \exp \left(- \int_0^\beta H[V, \lambda] d\tau \right) \right]$$

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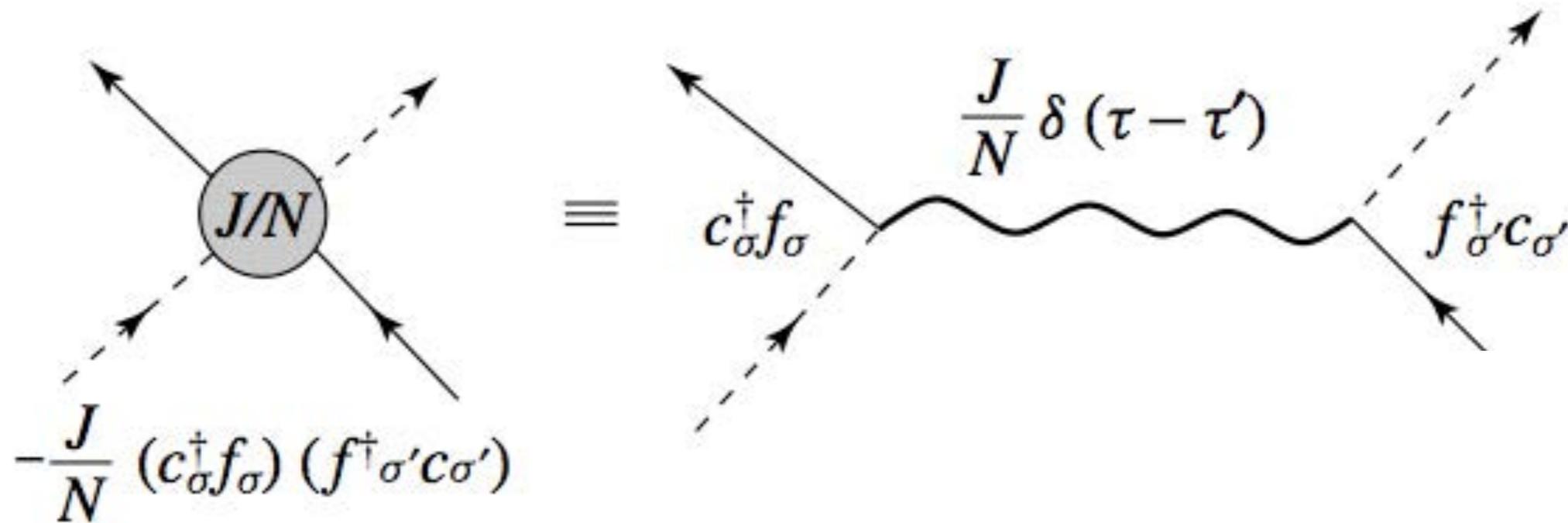
$=\text{Tr} \left[T \exp \left(- \int_0^\beta H[V, \lambda] d\tau \right) \right]$ **Extensive in N**

$$Z = \int \mathcal{D}[V, \lambda] \int \mathcal{D}[c, f] \exp \left[- \int_0^\beta \left(\sum_{k\sigma} c_{k\sigma}^\dagger \partial_\tau c_{k\sigma} + \sum_{j\sigma} f_{j\sigma}^\dagger \partial_\tau f_{j\sigma} + H[V, \lambda] \right) \right]$$

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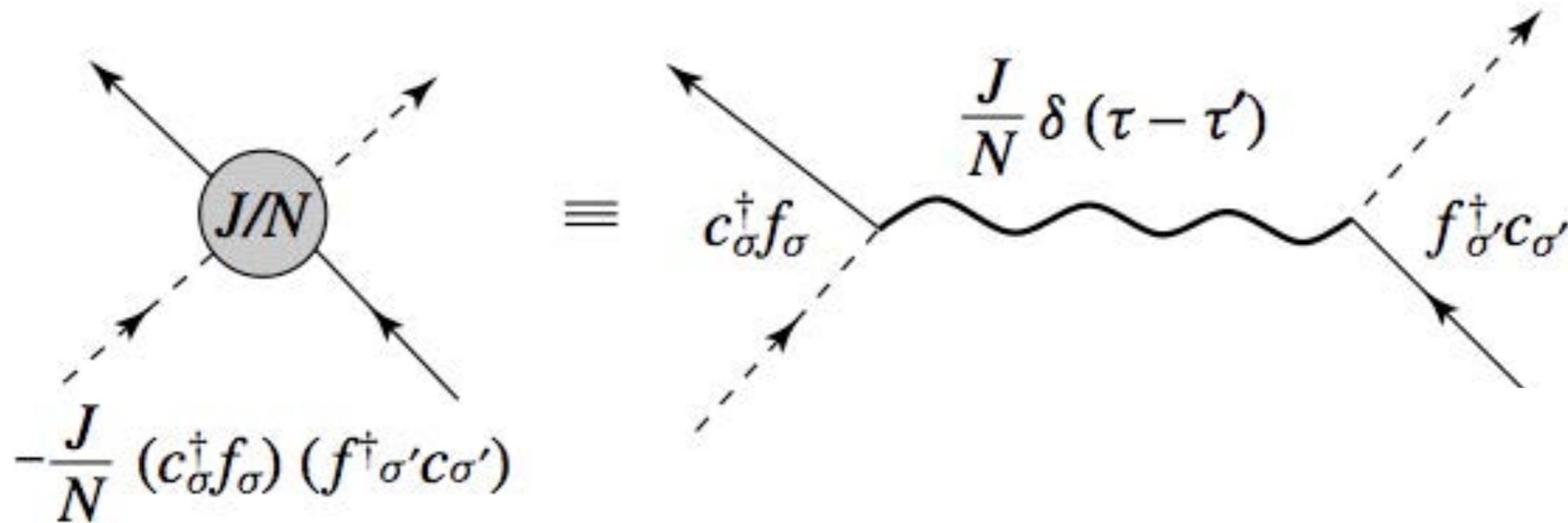
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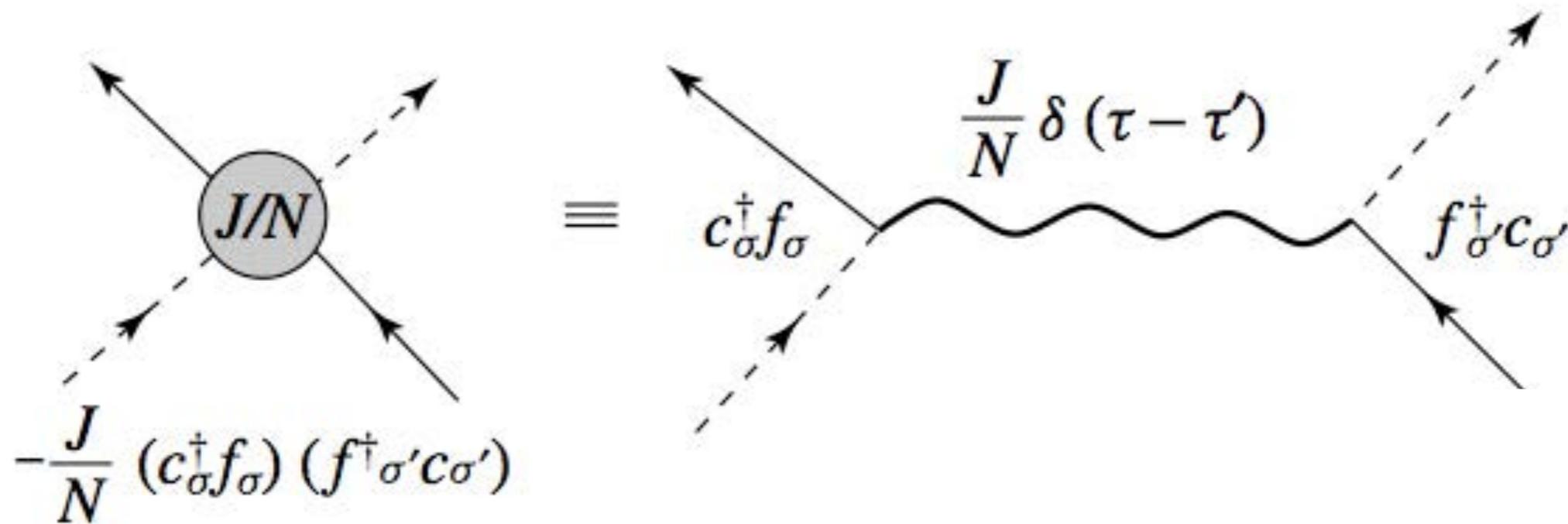


Large N Approach.

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$$Z = \text{Tr} e^{-\beta H_{MFT}}, \quad (N \rightarrow \infty)$$

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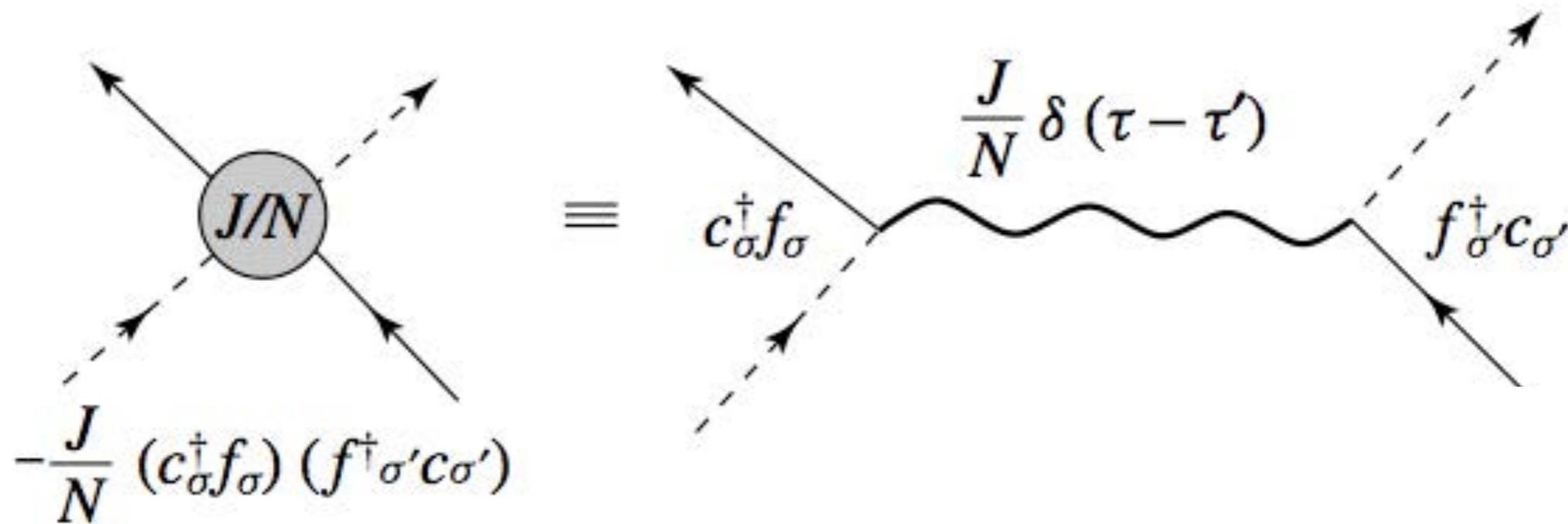


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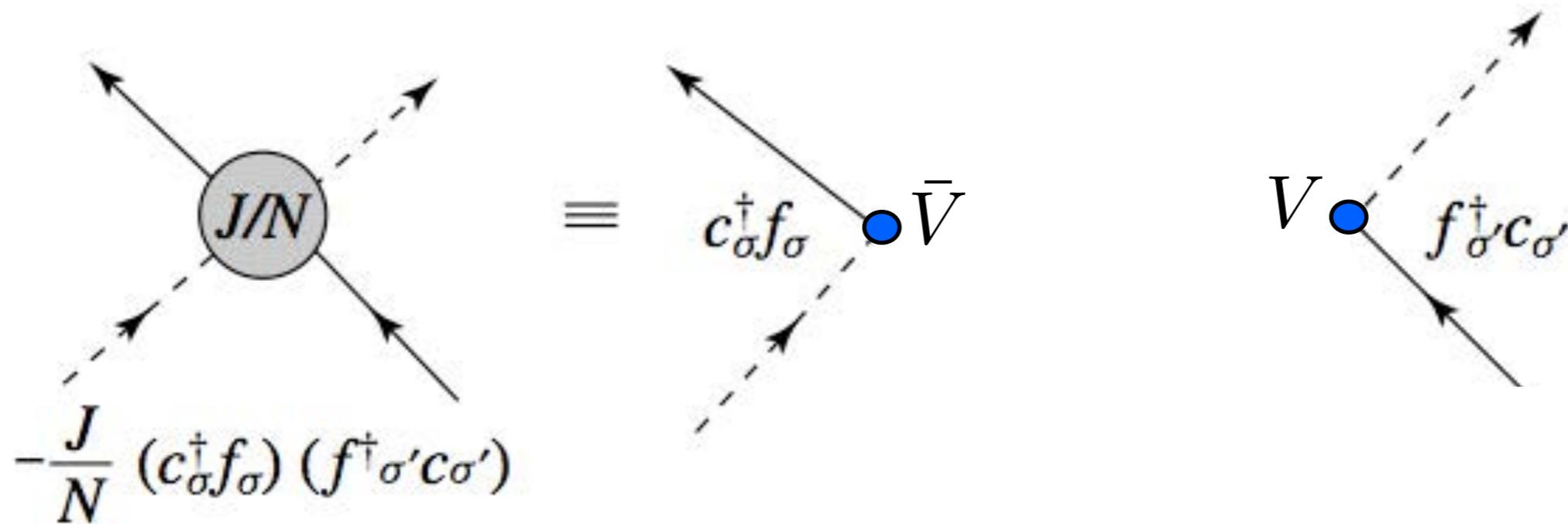
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$V_j = V$
at each site

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Detailed calcn.

$$\begin{aligned} H_{MFT} &= \sum_{\mathbf{k}\sigma} (c_{\mathbf{k}\sigma}^\dagger, f_{\mathbf{k}\sigma}^\dagger) \overbrace{\begin{pmatrix} \epsilon_{\mathbf{k}} & V \\ \bar{V} & \lambda \end{pmatrix}}^{h(\mathbf{k})} \begin{pmatrix} c_{\mathbf{k}\sigma} \\ f_{\mathbf{k}\sigma} \end{pmatrix} + N\mathcal{N}_s \left(\frac{|V|^2}{J} - \lambda q \right) \\ &= \sum_{\mathbf{k}\sigma} \psi_{\mathbf{k}\sigma}^\dagger \underline{h}(\mathbf{k}) \psi_{\mathbf{k}\sigma} + N\mathcal{N}_s \left(\frac{|V|^2}{J} - \lambda q \right). \end{aligned}$$

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$$H_{MFT} = \sum_{\mathbf{k}\sigma} \left(a_{\mathbf{k}\sigma}^\dagger, b_{\mathbf{k}\sigma}^\dagger \right) \begin{pmatrix} E_{\mathbf{k}^+} & 0 \\ 0 & E_{\mathbf{k}^-} \end{pmatrix} \begin{pmatrix} a_{\mathbf{k}\sigma} \\ b_{\mathbf{k}\sigma} \end{pmatrix} + Nn \left(\frac{\bar{V}V}{J} - \lambda q \right).$$

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 &= \sum_{\mathbf{k}\sigma} \psi_{\mathbf{k}\sigma}^\dagger \underline{h}(\mathbf{k}) \psi_{\mathbf{k}\sigma} + N\mathcal{N}_s \left(\frac{|V|^2}{J} - \lambda q \right).
 \end{aligned}$$

$$f_{\vec{k}\sigma}^\dagger = \frac{1}{\sqrt{n}} \sum_j f_{j\sigma}^\dagger e^{i\vec{k}\cdot\vec{R}_j}$$

$$H_{MFT} = \sum_{\mathbf{k}\sigma} (a_{\mathbf{k}\sigma}^\dagger, b_{\mathbf{k}\sigma}^\dagger) \begin{pmatrix} E_{\mathbf{k}^+} & 0 \\ 0 & E_{\mathbf{k}^-} \end{pmatrix} \begin{pmatrix} a_{\mathbf{k}\sigma} \\ b_{\mathbf{k}\sigma} \end{pmatrix} + Nn \left(\frac{\bar{V}V}{J} - \lambda q \right).$$

$$\text{Det} \left[E_{\mathbf{k}\pm} - \begin{pmatrix} \epsilon_{\mathbf{k}} & V \\ \bar{V} & \lambda \end{pmatrix} \right] = (E_{\mathbf{k}\pm} - \epsilon_{\mathbf{k}})(E_{\mathbf{k}\pm} - \lambda) - |V|^2 = 0,$$

Detailed calcn.

$$\begin{aligned}
 H_{MFT} &= \sum_{\mathbf{k}\sigma} (c_{\mathbf{k}\sigma}^\dagger, f_{\mathbf{k}\sigma}^\dagger) \overbrace{\begin{pmatrix} \epsilon_{\mathbf{k}} & V \\ \bar{V} & \lambda \end{pmatrix}}^{h(\mathbf{k})} \begin{pmatrix} c_{\mathbf{k}\sigma} \\ f_{\mathbf{k}\sigma} \end{pmatrix} + N\mathcal{N}_s \left(\frac{|V|^2}{J} - \lambda q \right) \\
 &= \sum_{\mathbf{k}\sigma} \psi_{\mathbf{k}\sigma}^\dagger \underline{h}(\mathbf{k}) \psi_{\mathbf{k}\sigma} + N\mathcal{N}_s \left(\frac{|V|^2}{J} - \lambda q \right).
 \end{aligned}$$

$$f_{\vec{k}\sigma}^\dagger = \frac{1}{\sqrt{n}} \sum_j f_{j\sigma}^\dagger e^{i\vec{k}\cdot\vec{R}_j}$$

$$H_{MFT} = \sum_{\mathbf{k}\sigma} (a_{\mathbf{k}\sigma}^\dagger, b_{\mathbf{k}\sigma}^\dagger) \begin{pmatrix} E_{\mathbf{k}^+} & 0 \\ 0 & E_{\mathbf{k}^-} \end{pmatrix} \begin{pmatrix} a_{\mathbf{k}\sigma} \\ b_{\mathbf{k}\sigma} \end{pmatrix} + Nn \left(\frac{\bar{V}V}{J} - \lambda q \right).$$

$$\text{Det} \left[E_{\mathbf{k}\pm} 1 - \begin{pmatrix} \epsilon_{\mathbf{k}} & V \\ \bar{V} & \lambda \end{pmatrix} \right] = (E_{\mathbf{k}\pm} - \epsilon_{\mathbf{k}})(E_{\mathbf{k}\pm} - \lambda) - |V|^2 = 0,$$

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

Detailed calcn.

$$\begin{aligned}
 H_{MFT} &= \sum_{\mathbf{k}\sigma} (c_{\mathbf{k}\sigma}^\dagger, f_{\mathbf{k}\sigma}^\dagger) \overbrace{\begin{pmatrix} \epsilon_{\mathbf{k}} & V \\ \bar{V} & \lambda \end{pmatrix}}^{h(\mathbf{k})} \begin{pmatrix} c_{\mathbf{k}\sigma} \\ f_{\mathbf{k}\sigma} \end{pmatrix} + N\mathcal{N}_s \left(\frac{|V|^2}{J} - \lambda q \right) \\
 &= \sum_{\mathbf{k}\sigma} \psi_{\mathbf{k}\sigma}^\dagger \underline{h}(\mathbf{k}) \psi_{\mathbf{k}\sigma} + N\mathcal{N}_s \left(\frac{|V|^2}{J} - \lambda q \right).
 \end{aligned}$$

$$f_{\vec{k}\sigma}^\dagger = \frac{1}{\sqrt{n}} \sum_j f_{j\sigma}^\dagger e^{i\vec{k}\cdot\vec{R}_j}$$

$$H_{MFT} = \sum_{\mathbf{k}\sigma} (a_{\mathbf{k}\sigma}^\dagger, b_{\mathbf{k}\sigma}^\dagger) \begin{pmatrix} E_{\mathbf{k}^+} & 0 \\ 0 & E_{\mathbf{k}^-} \end{pmatrix} \begin{pmatrix} a_{\mathbf{k}\sigma} \\ b_{\mathbf{k}\sigma} \end{pmatrix} + Nn \left(\frac{\bar{V}V}{J} - \lambda q \right).$$

$$\text{Det} \left[E_{\mathbf{k}\pm} - \begin{pmatrix} \epsilon_{\mathbf{k}} & V \\ \bar{V} & \lambda \end{pmatrix} \right] = (E_{\mathbf{k}\pm} - \epsilon_{\mathbf{k}})(E_{\mathbf{k}\pm} - \lambda) - |V|^2 = 0,$$

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

$$a_{\mathbf{k}\sigma}^\dagger = u_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger + v_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger$$

$$b_{\mathbf{k}\sigma}^\dagger = -v_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger$$

Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

$$\begin{aligned} a_{\mathbf{k}\sigma}^\dagger &= u_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger + v_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger \\ b_{\mathbf{k}\sigma}^\dagger &= -v_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger \end{aligned} \quad \begin{Bmatrix} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{Bmatrix} = \left[\frac{1}{2} \pm \frac{(\epsilon_{\mathbf{k}} - \lambda)/2}{2 \sqrt{(\frac{\epsilon_{\mathbf{k}} - \lambda}{2})^2 + |V|^2}} \right]^{\frac{1}{2}}.$$

Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

$$|MF\rangle = \prod_{|\mathbf{k}| < k_F \sigma} b_{\mathbf{k}\sigma}^\dagger |0\rangle = \prod_{|\mathbf{k}| < k_F \sigma} (-v_{\mathbf{k}} c_{\mathbf{k}\sigma} + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger) |0\rangle.$$

$$\begin{aligned} a_{\mathbf{k}\sigma}^\dagger &= u_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger + v_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger \\ b_{\mathbf{k}\sigma}^\dagger &= -v_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger \end{aligned} \left\{ \begin{array}{l} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{array} \right\} = \left[\frac{1}{2} \pm \frac{(\epsilon_{\mathbf{k}} - \lambda)/2}{2 \sqrt{\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2}\right)^2 + |V|^2}} \right]^{\frac{1}{2}}.$$

Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

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$$|MF\rangle = \prod_{|\mathbf{k}| < k_F \sigma} b_{\mathbf{k}\sigma}^\dagger |0\rangle = \prod_{|\mathbf{k}| < k_F \sigma} (-v_{\mathbf{k}} c_{\mathbf{k}\sigma} + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger) |0\rangle.$$

$$|GW\rangle = P_Q \prod_{|\mathbf{k}| < k_F \sigma} (-v_{\mathbf{k}} c_{\mathbf{k}\sigma} + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger) |0\rangle,$$

“Gutzwiller” wavefunction

Detailed calcn.

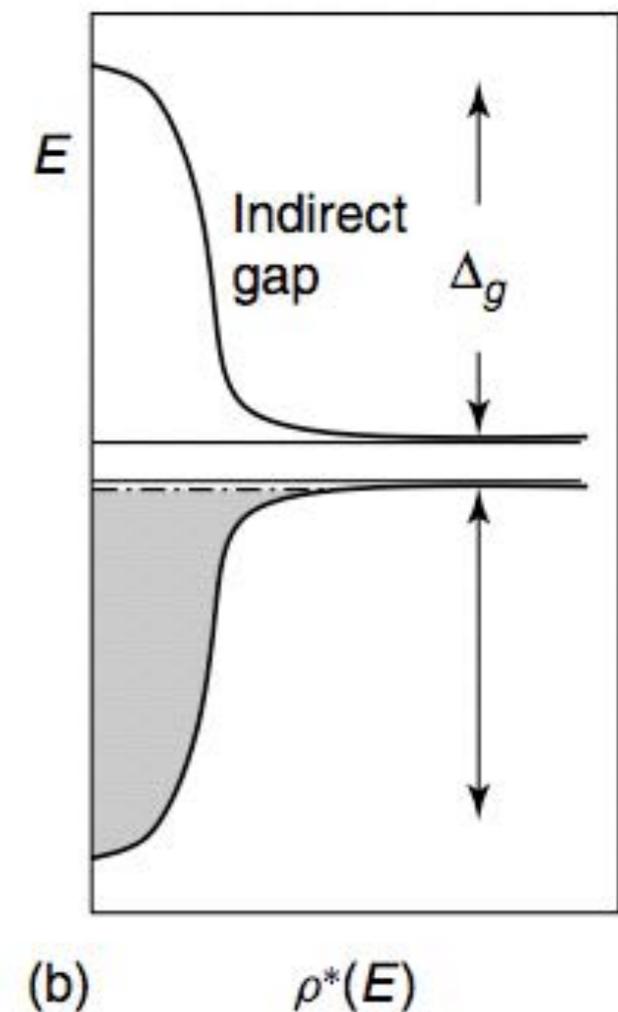
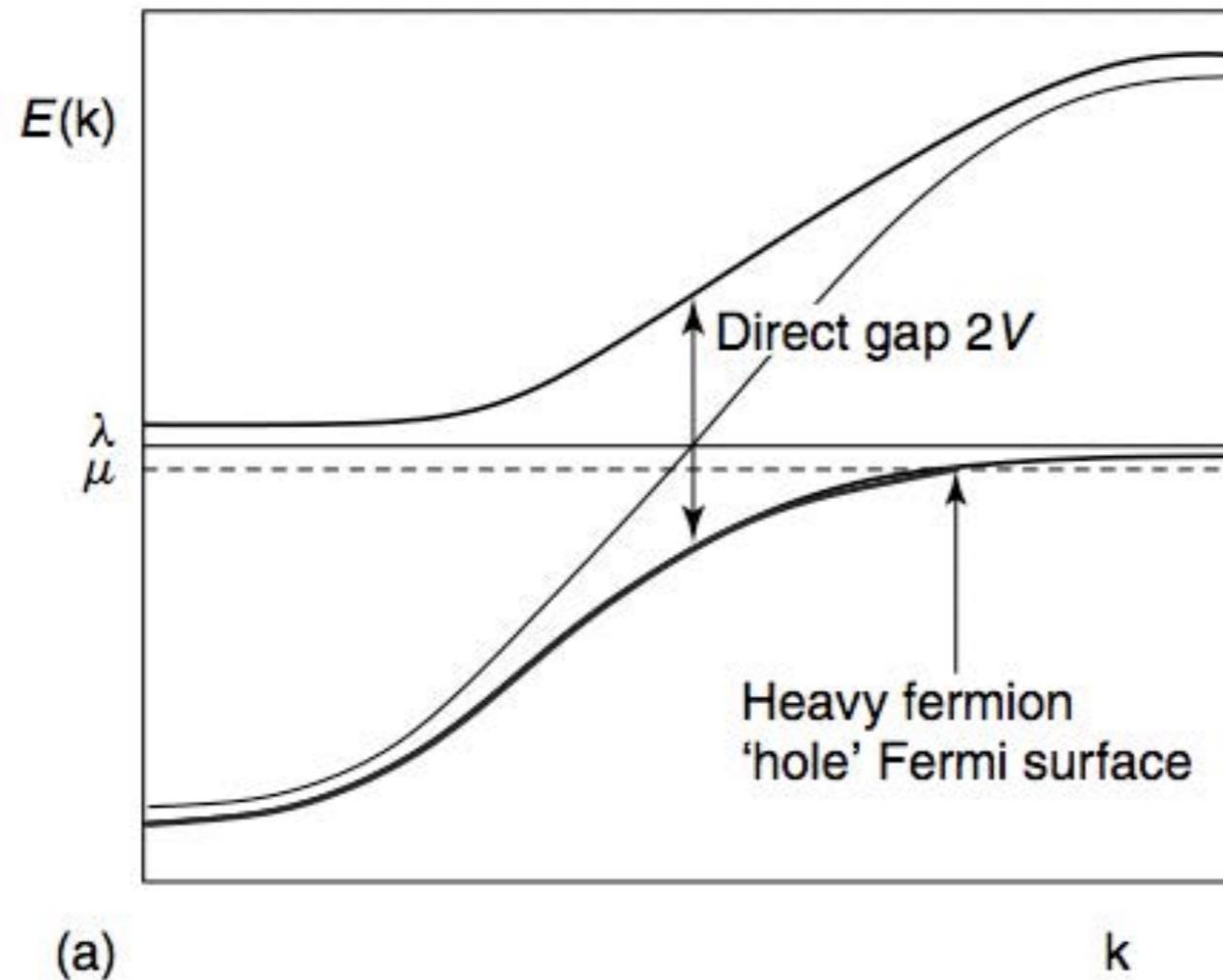
$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

$$|MF\rangle = \prod_{|\mathbf{k}| < k_F \sigma} b_{\mathbf{k}\sigma}^\dagger |0\rangle = \prod_{|\mathbf{k}| < k_F \sigma} (-v_{\mathbf{k}} c_{\mathbf{k}\sigma} + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger) |0\rangle.$$

$$\begin{cases} a_{\mathbf{k}\sigma}^\dagger = u_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger + v_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger \\ b_{\mathbf{k}\sigma}^\dagger = -v_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger \end{cases} \left\{ \begin{matrix} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{matrix} \right\} = \left[\frac{1}{2} \pm \frac{(\epsilon_{\mathbf{k}} - \lambda)/2}{2 \sqrt{(\frac{\epsilon_{\mathbf{k}} - \lambda}{2})^2 + |V|^2}} \right]^{\frac{1}{2}}$$

$$|GW\rangle = P_Q \prod_{|\mathbf{k}| < k_F \sigma} (-v_{\mathbf{k}} c_{\mathbf{k}\sigma} + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger) |0\rangle,$$

“Gutzwiller” wavefunction



Detailed calcn.

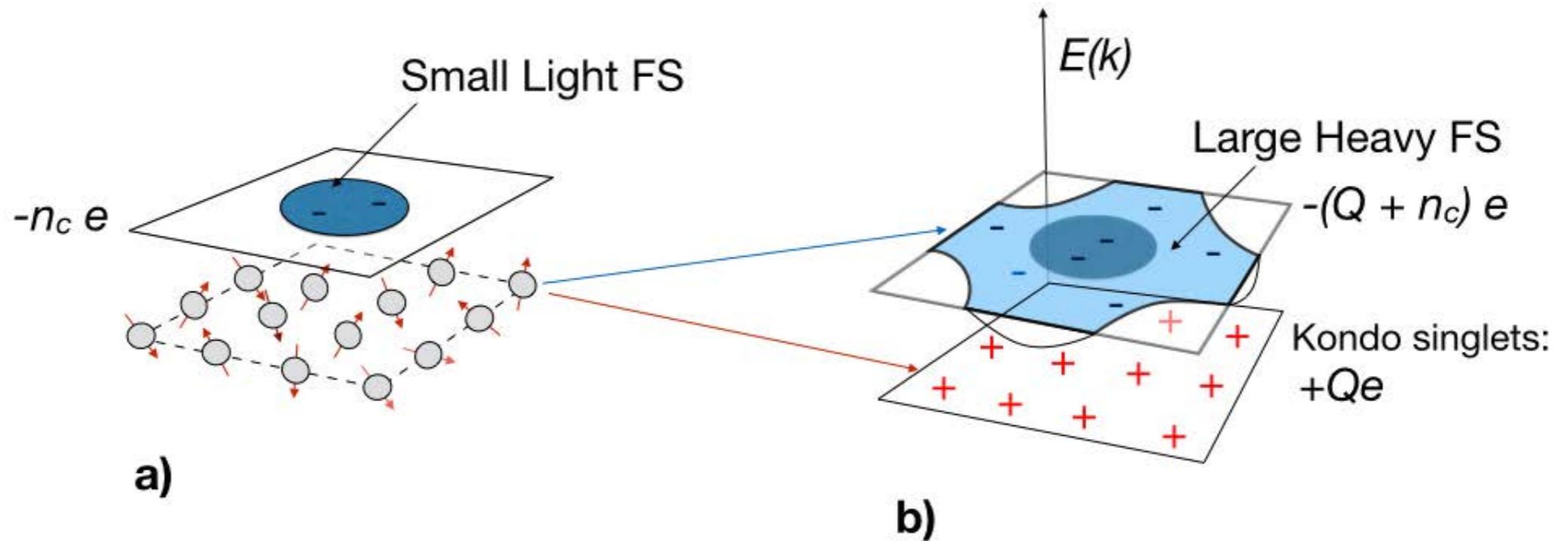
$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

$$\begin{aligned} a_{\mathbf{k}\sigma}^\dagger &= u_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger + v_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger \\ b_{\mathbf{k}\sigma}^\dagger &= -v_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger \end{aligned} \quad \left\{ \begin{array}{l} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{array} \right\} = \left[\frac{1}{2} \pm \frac{(\epsilon_{\mathbf{k}} - \lambda)/2}{2 \sqrt{(\frac{\epsilon_{\mathbf{k}} - \lambda}{2})^2 + |V|^2}} \right]^{\frac{1}{2}}$$

$$|MF\rangle = \prod_{|\mathbf{k}| < k_F \sigma} b_{\mathbf{k}\sigma}^\dagger |0\rangle = \prod_{|\mathbf{k}| < k_F \sigma} (-v_{\mathbf{k}} c_{\mathbf{k}\sigma} + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger) |0\rangle.$$

$$|GW\rangle = P_Q \prod_{|\mathbf{k}| < k_F \sigma} (-v_{\mathbf{k}} c_{\mathbf{k}\sigma} + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger) |0\rangle,$$

“Gutzwiller” wavefunction



Detailed calcn.

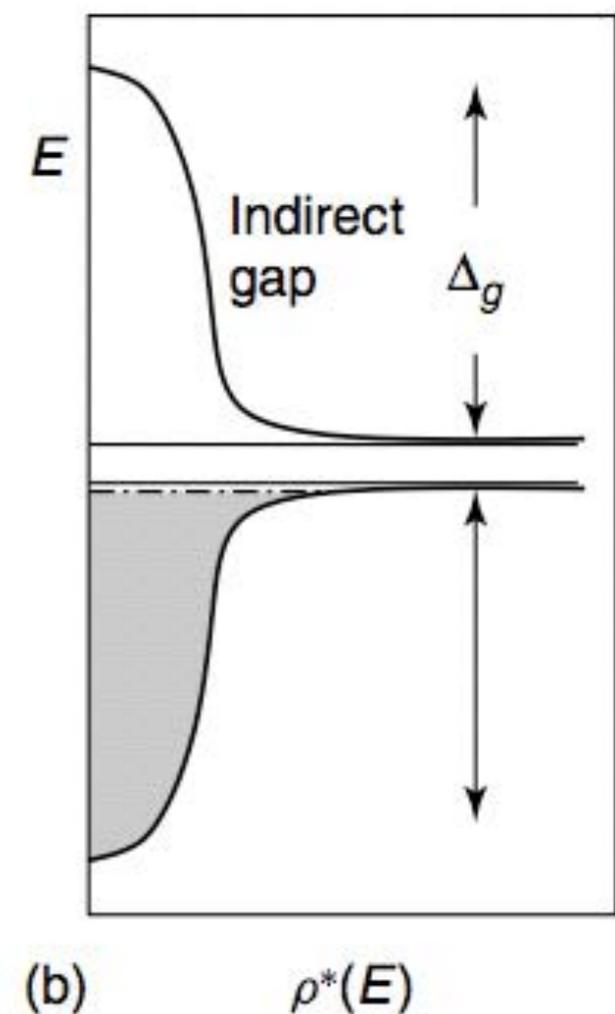
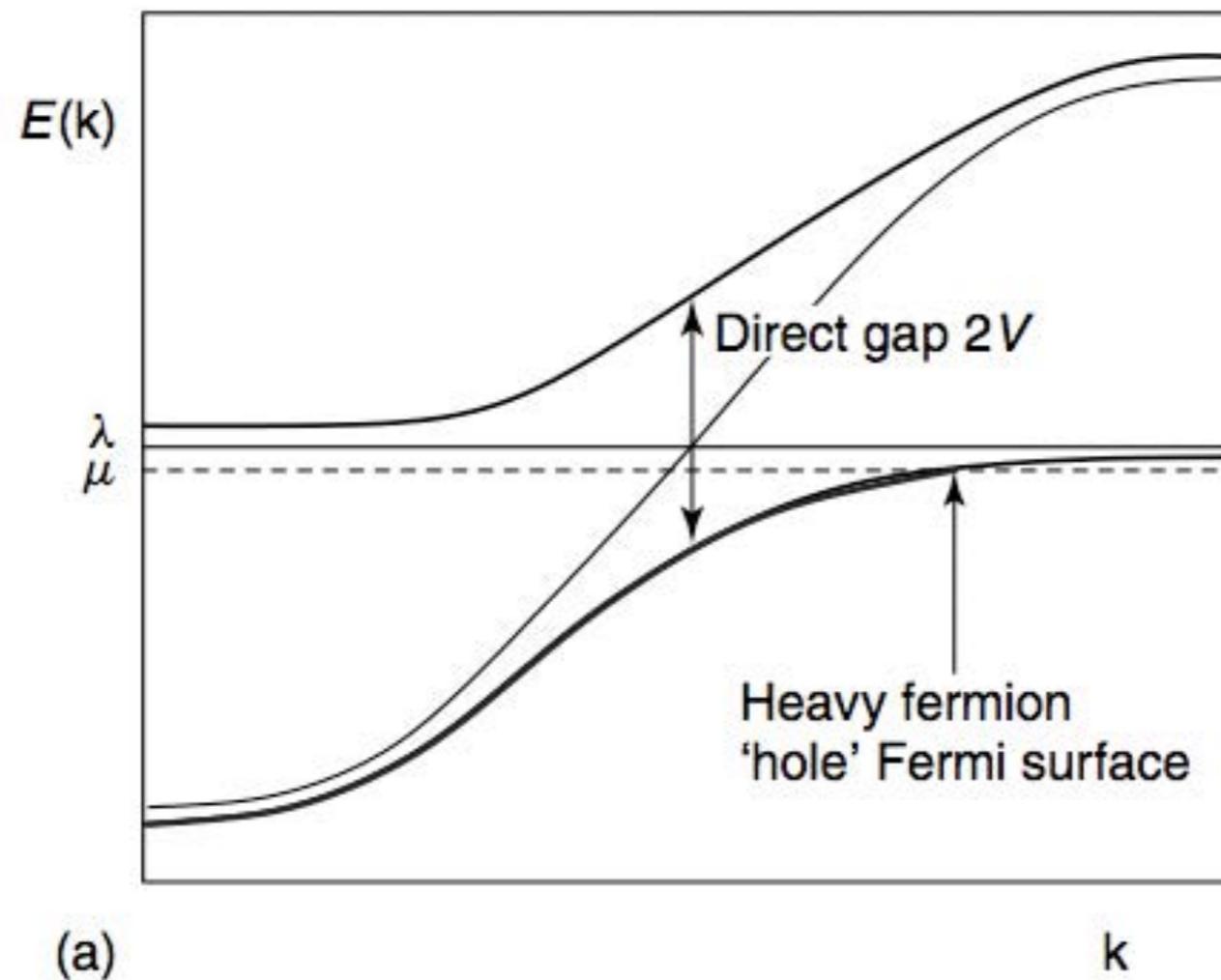
$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

$$|MF\rangle = \prod_{|\mathbf{k}| < k_F \sigma} b_{\mathbf{k}\sigma}^\dagger |0\rangle = \prod_{|\mathbf{k}| < k_F \sigma} (-v_{\mathbf{k}} c_{\mathbf{k}\sigma} + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger) |0\rangle.$$

$$\begin{cases} a_{\mathbf{k}\sigma}^\dagger = u_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger + v_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger \\ b_{\mathbf{k}\sigma}^\dagger = -v_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger \end{cases} \left\{ \begin{matrix} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{matrix} \right\} = \left[\frac{1}{2} \pm \frac{(\epsilon_{\mathbf{k}} - \lambda)/2}{2 \sqrt{(\frac{\epsilon_{\mathbf{k}} - \lambda}{2})^2 + |V|^2}} \right]^{\frac{1}{2}}$$

$$|GW\rangle = P_Q \prod_{|\mathbf{k}| < k_F \sigma} (-v_{\mathbf{k}} c_{\mathbf{k}\sigma} + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger) |0\rangle,$$

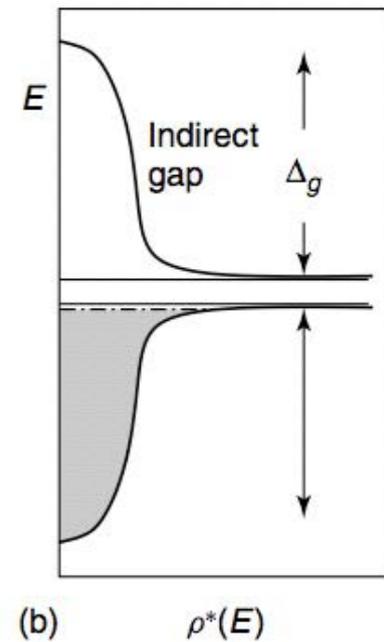
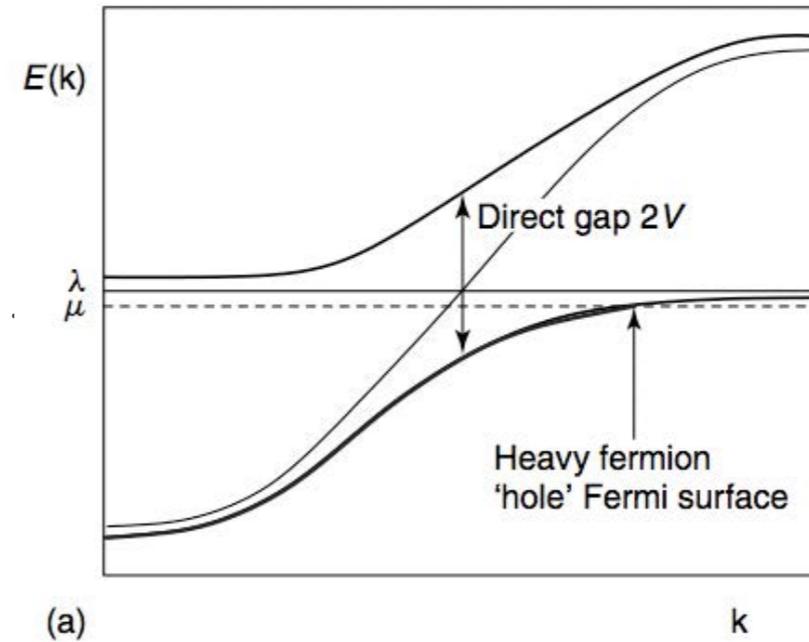
“Gutzwiller” wavefunction



Detailed calcn.

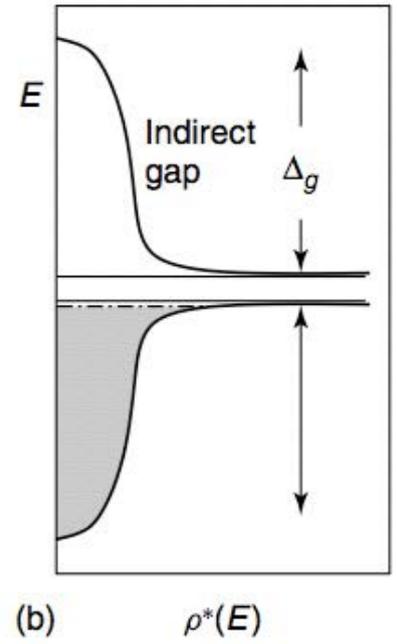
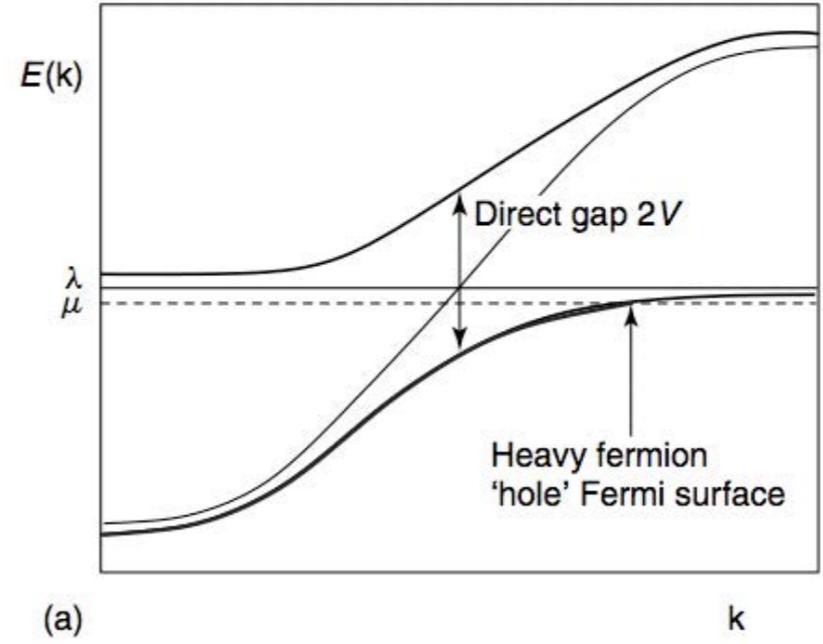
$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

$$\begin{aligned} a_{\mathbf{k}\sigma}^\dagger &= u_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger + v_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger \\ b_{\mathbf{k}\sigma}^\dagger &= -v_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger + u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger \end{aligned} \quad \left\{ \begin{array}{l} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{array} \right\} = \left[\frac{1}{2} \pm \frac{(\epsilon_{\mathbf{k}} - \lambda)/2}{2 \sqrt{\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2}\right)^2 + |V|^2}} \right]^{\frac{1}{2}}$$



Detailed calcn.

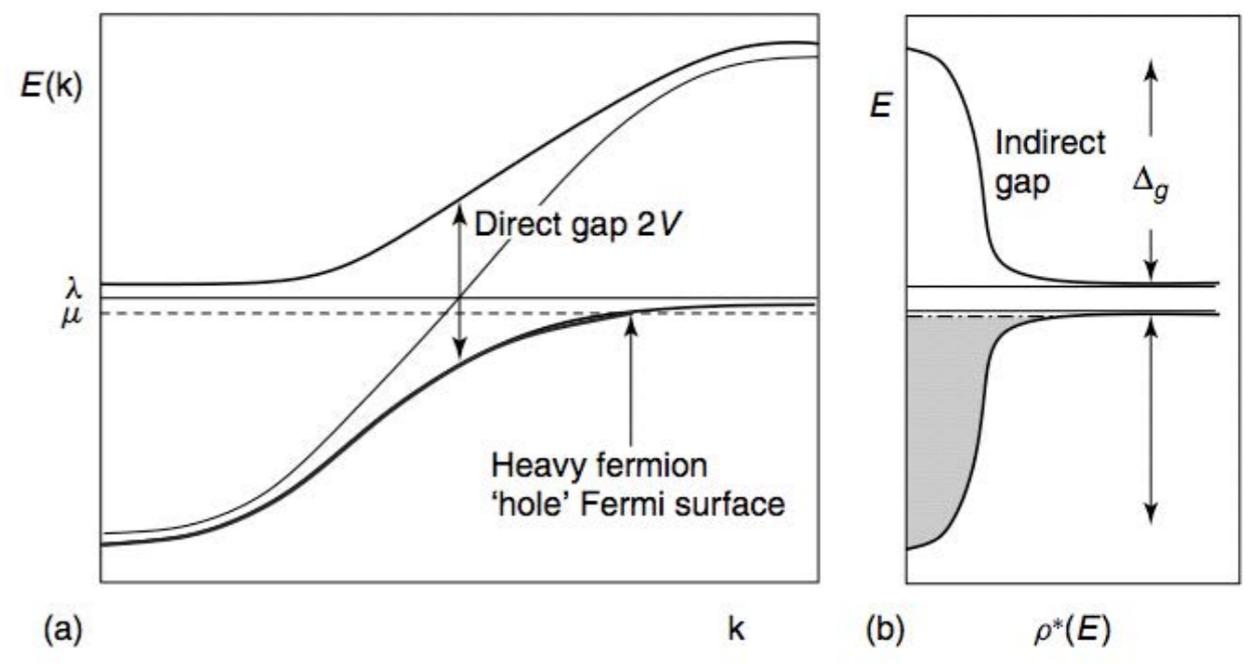
$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$



Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

$$\frac{F}{N} = -T \sum_{\mathbf{k}, \pm} \ln \left[1 + e^{-\beta E_{\mathbf{k}\pm}} \right] + \mathcal{N}_s \left(\frac{V^2}{J} - \lambda q \right).$$

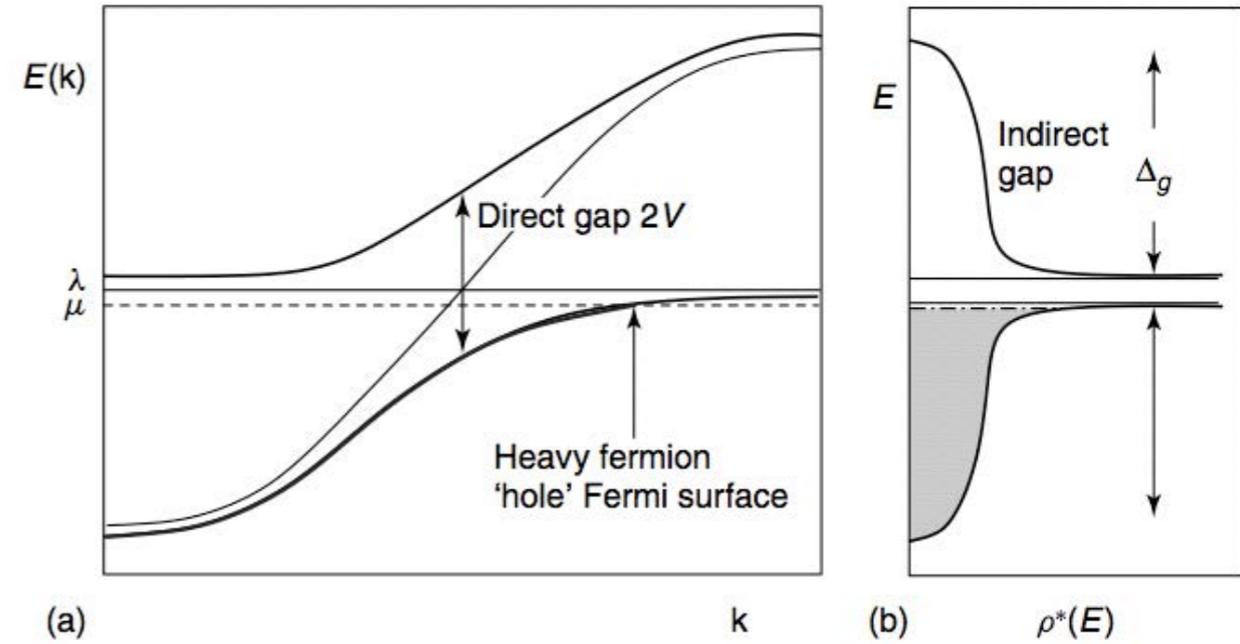


Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

$$\frac{F}{N} = -T \sum_{\mathbf{k}, \pm} \ln \left[1 + e^{-\beta E_{\mathbf{k}\pm}} \right] + \mathcal{N}_s \left(\frac{V^2}{J} - \lambda q \right).$$

$$\frac{E_o}{N \mathcal{N}_s} = \int_{-\infty}^0 dE \rho^*(E) E + \left(\frac{V^2}{J} - \lambda q \right)$$



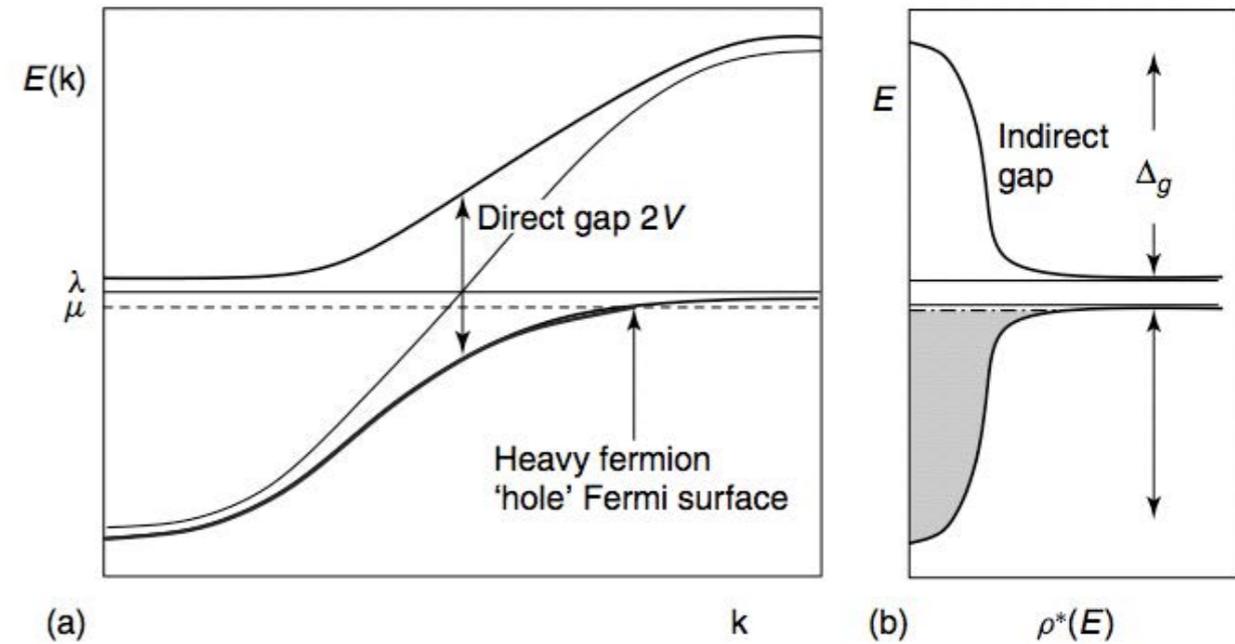
Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

$$\frac{F}{N} = -T \sum_{\mathbf{k}, \pm} \ln \left[1 + e^{-\beta E_{\mathbf{k}\pm}} \right] + \mathcal{N}_s \left(\frac{V^2}{J} - \lambda q \right).$$

$$\frac{E_o}{N \mathcal{N}_s} = \int_{-\infty}^0 dE \rho^*(E) E + \left(\frac{V^2}{J} - \lambda q \right)$$

$$E = \epsilon + \frac{V^2}{E - \lambda}$$



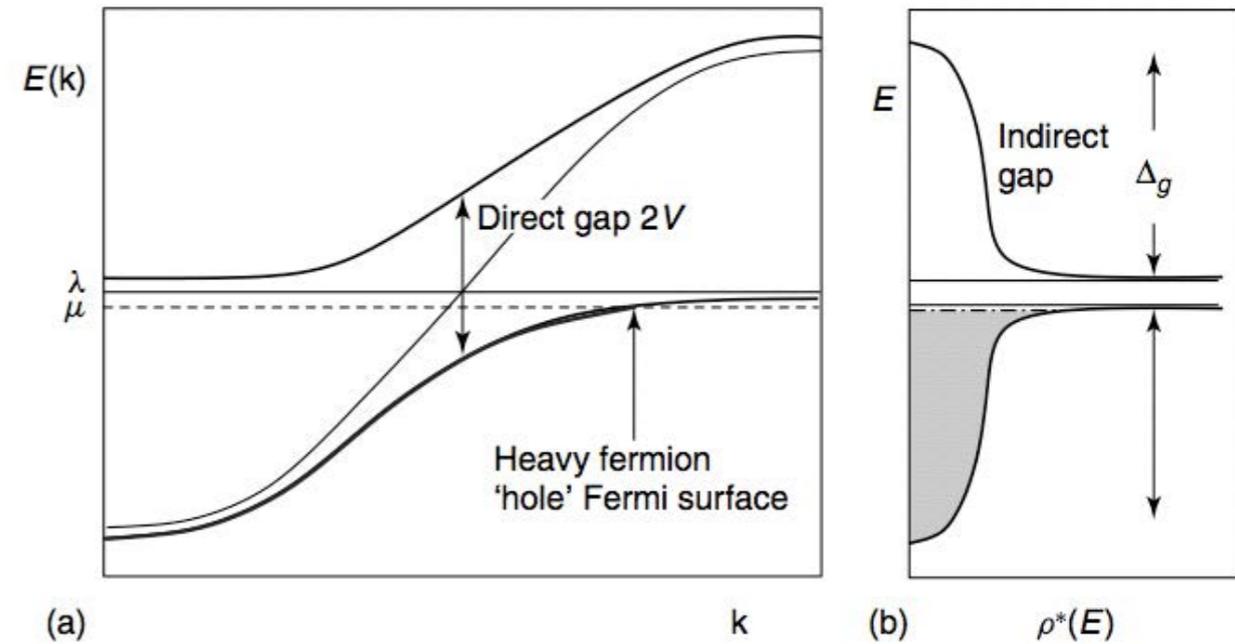
Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

$$\frac{F}{N} = -T \sum_{\mathbf{k}, \pm} \ln \left[1 + e^{-\beta E_{\mathbf{k}\pm}} \right] + \mathcal{N}_s \left(\frac{V^2}{J} - \lambda q \right).$$

$$\frac{E_o}{N \mathcal{N}_s} = \int_{-\infty}^0 dE \rho^*(E) E + \left(\frac{V^2}{J} - \lambda q \right)$$

$$E = \epsilon + \frac{V^2}{E - \lambda} \quad \rho^*(E) = \rho \frac{d\epsilon}{dE} = \rho \left(1 + \frac{V^2}{(E - \lambda)^2} \right)$$



Detailed calcn.

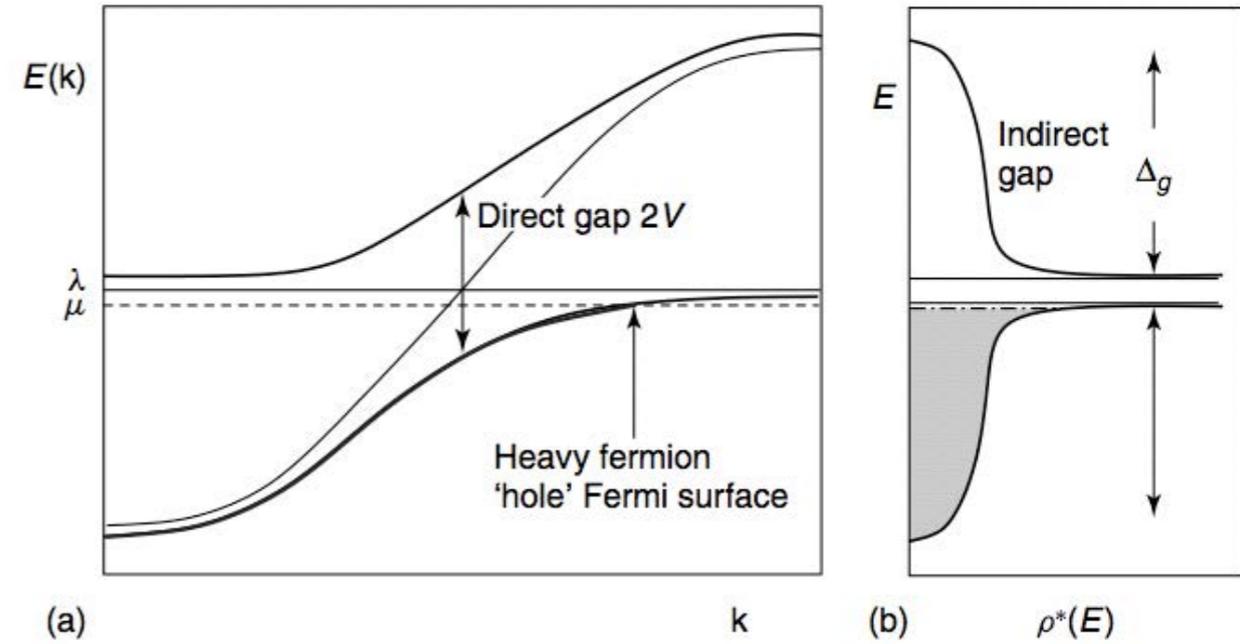
$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

$$\frac{F}{N} = -T \sum_{\mathbf{k}, \pm} \ln \left[1 + e^{-\beta E_{\mathbf{k}\pm}} \right] + \mathcal{N}_s \left(\frac{V^2}{J} - \lambda q \right).$$

$$\frac{E_o}{N\mathcal{N}_s} = \int_{-\infty}^0 dE \rho^*(E) E + \left(\frac{V^2}{J} - \lambda q \right)$$

$$E = \epsilon + \frac{V^2}{E - \lambda} \quad \rho^*(E) = \rho \frac{d\epsilon}{dE} = \rho \left(1 + \frac{V^2}{(E - \lambda)^2} \right)$$

$$\frac{E_o}{N\mathcal{N}_s} = \rho \int_{-D-V^2/D}^0 dE E \left(1 + \frac{V^2}{(E - \lambda)^2} \right) + \left(\frac{V^2}{J} - \lambda q \right)$$



Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

$$\frac{F}{N} = -T \sum_{\mathbf{k}, \pm} \ln \left[1 + e^{-\beta E_{\mathbf{k}\pm}} \right] + \mathcal{N}_s \left(\frac{V^2}{J} - \lambda q \right).$$

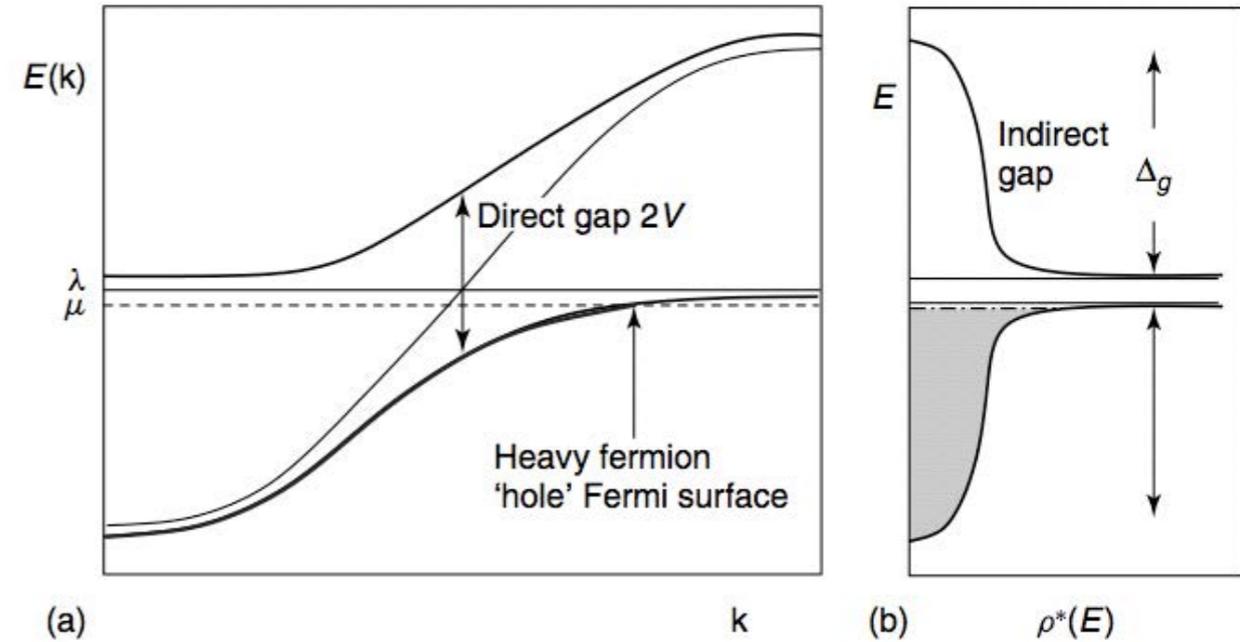
$$\frac{E_o}{N\mathcal{N}_s} = \int_{-\infty}^0 dE \rho^*(E) E + \left(\frac{V^2}{J} - \lambda q \right)$$

$$E = \epsilon + \frac{V^2}{E - \lambda} \quad \rho^*(E) = \rho \frac{d\epsilon}{dE} = \rho \left(1 + \frac{V^2}{(E - \lambda)^2} \right)$$

$$\frac{E_o}{N\mathcal{N}_s} = \rho \int_{-D-V^2/D}^0 dE E \left(1 + \frac{V^2}{(E - \lambda)^2} \right) + \left(\frac{V^2}{J} - \lambda q \right)$$

$$\begin{aligned} \frac{E_o}{N\mathcal{N}_s} &= -\frac{\rho}{2} \left(D + \frac{V^2}{D} \right)^2 + \frac{\Delta}{\pi} \int_{-D}^0 dE \left(\frac{1}{E - \lambda} + \frac{\lambda}{(E - \lambda)^2} \right) + \left(\frac{V^2}{J} - \lambda q \right) \\ &= -\frac{D^2 \rho}{2} + \frac{\Delta}{\pi} \ln \left(\frac{\lambda}{D} \right) + \left(\frac{V^2}{J} - \lambda q \right) \end{aligned}$$

$$(\Delta = \pi \rho |V|^2)$$



Detailed calcn.

$$E_{\mathbf{k}\pm} = \frac{\epsilon_{\mathbf{k}} + \lambda}{2} \pm \left[\left(\frac{\epsilon_{\mathbf{k}} - \lambda}{2} \right)^2 + |V|^2 \right]^{\frac{1}{2}},$$

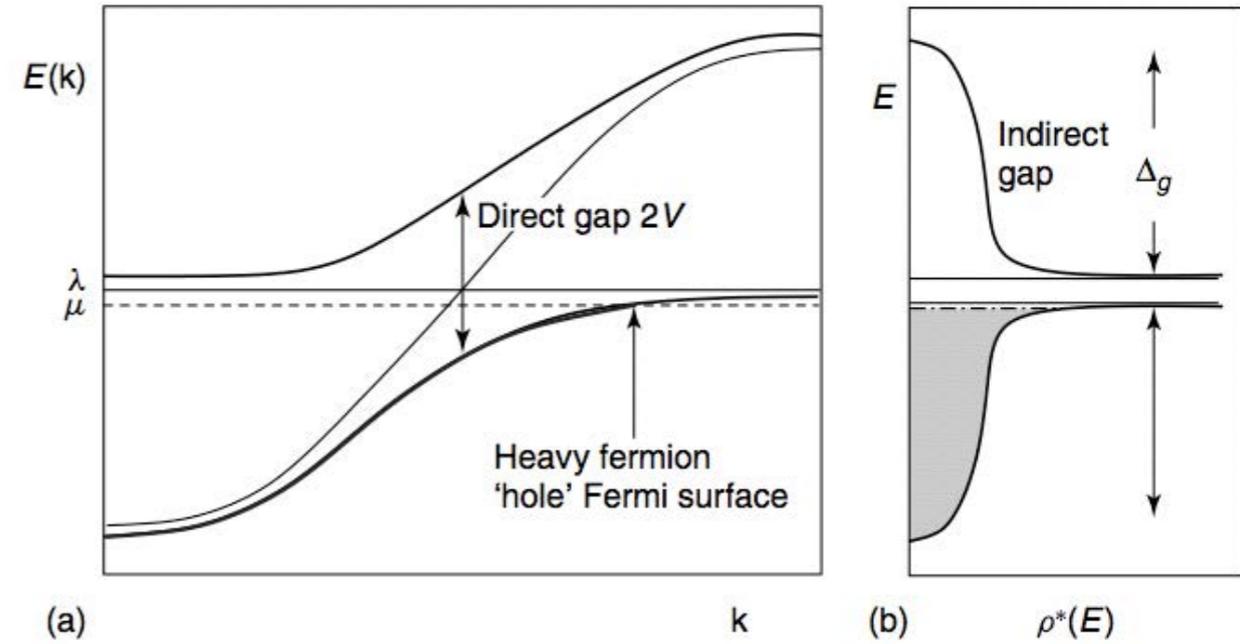
$$\frac{F}{N} = -T \sum_{\mathbf{k}, \pm} \ln \left[1 + e^{-\beta E_{\mathbf{k}\pm}} \right] + \mathcal{N}_s \left(\frac{V^2}{J} - \lambda q \right).$$

$$\frac{E_o}{N\mathcal{N}_s} = \int_{-\infty}^0 dE \rho^*(E) E + \left(\frac{V^2}{J} - \lambda q \right)$$

$$E = \epsilon + \frac{V^2}{E - \lambda} \quad \rho^*(E) = \rho \frac{d\epsilon}{dE} = \rho \left(1 + \frac{V^2}{(E - \lambda)^2} \right)$$

$$\frac{E_o}{N\mathcal{N}_s} = \rho \int_{-D-V^2/D}^0 dE E \left(1 + \frac{V^2}{(E - \lambda)^2} \right) + \left(\frac{V^2}{J} - \lambda q \right)$$

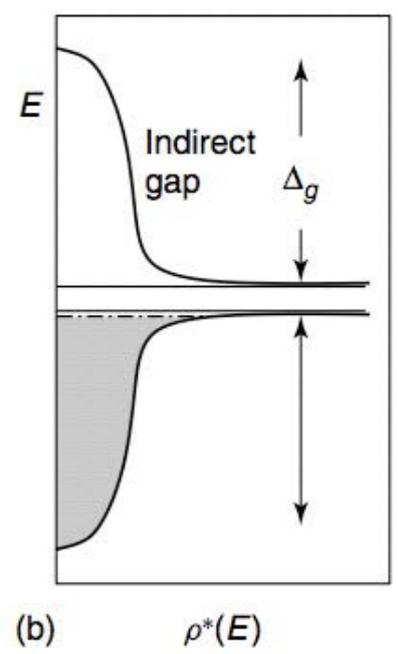
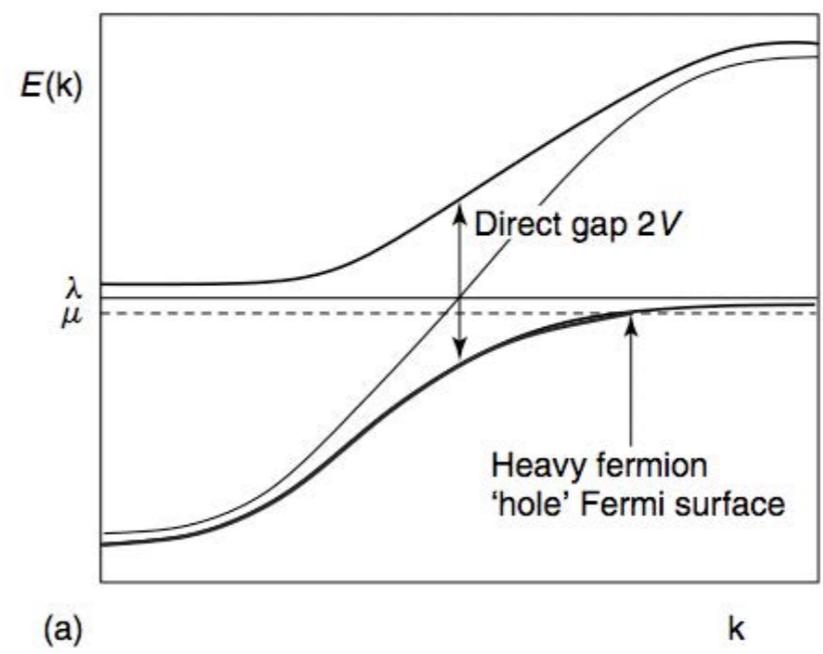
$$\begin{aligned} \frac{E_o}{N\mathcal{N}_s} &= -\frac{\rho}{2} \left(D + \frac{V^2}{D} \right)^2 + \frac{\Delta}{\pi} \int_{-D}^0 dE \left(\frac{1}{E - \lambda} + \frac{\lambda}{(E - \lambda)^2} \right) + \left(\frac{V^2}{J} - \lambda q \right) \\ &= -\frac{D^2 \rho}{2} + \frac{\Delta}{\pi} \ln \left(\frac{\lambda}{D} \right) + \left(\frac{V^2}{J} - \lambda q \right) \end{aligned}$$



Detailed calcn

$$\frac{E_F}{NN_s} - \frac{V^2}{2\left(D + \frac{V^2}{D}\right)} + \frac{\Delta}{\pi} \int_{-D}^0 dE \left(\frac{1}{E - \lambda} + \frac{\lambda}{(E - \lambda)^2} \right) + \left(\frac{V^2}{J} - \lambda q \right)$$

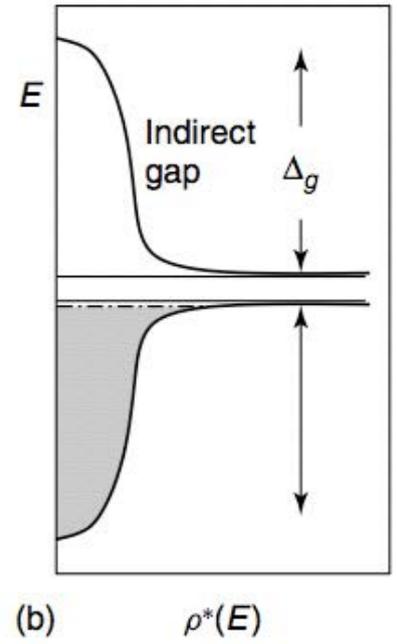
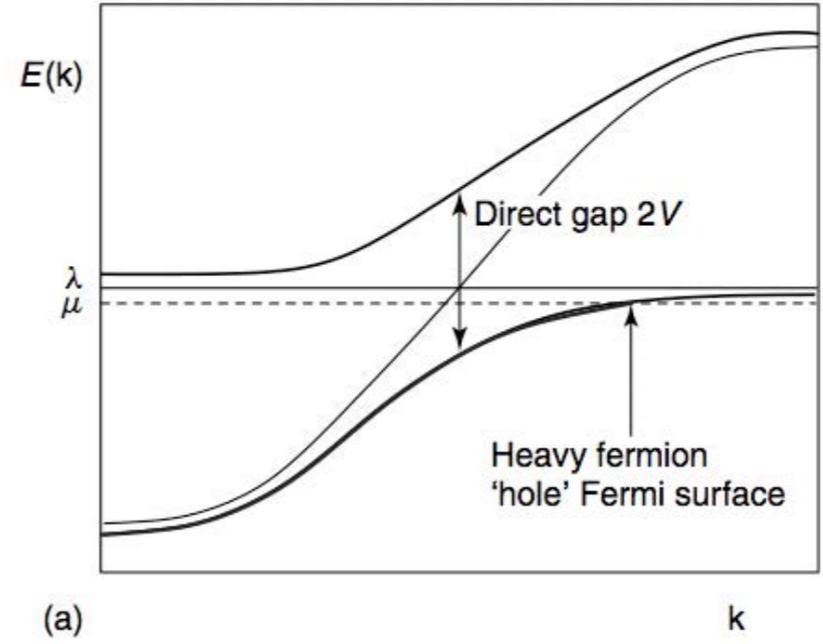
$$= -\frac{D^2 \rho}{2} + \frac{\Delta}{\pi} \ln \left(\frac{\lambda}{D} \right) + \left(\frac{V^2}{J} - \lambda q \right)$$



Detailed calcn.

$$(\Delta = \pi\rho|V|^2)$$

$$\frac{E_0}{N\mathcal{N}_s} = -\frac{D^2\rho}{2} + \frac{\Delta}{\pi} \ln\left(\frac{\lambda}{D}\right) + \left(\frac{\pi\rho V^2}{\pi\rho J} - \lambda q\right)$$

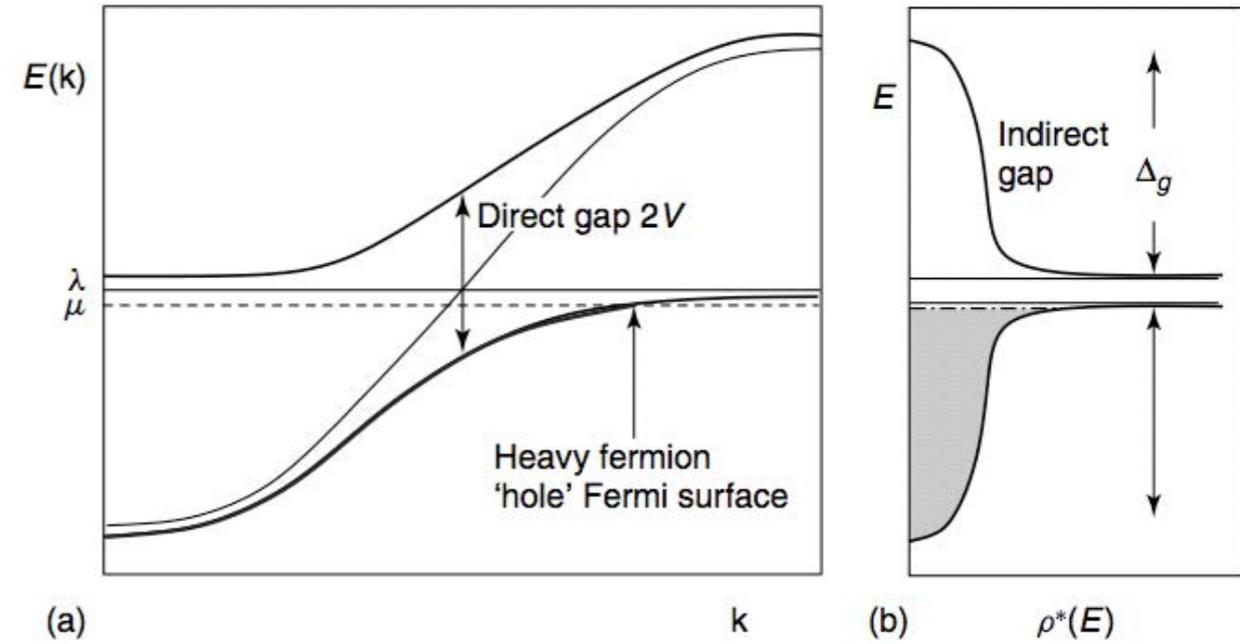


Detailed calcn.

$$(\Delta = \pi\rho|V|^2)$$

$$\begin{aligned} \frac{E_0}{NN_s} &= -\frac{D^2\rho}{2} + \frac{\Delta}{\pi} \ln\left(\frac{\lambda}{D}\right) + \left(\frac{\pi\rho V^2}{\pi\rho J} - \lambda q\right) \\ &= -\frac{D^2\rho}{2} + \frac{\Delta}{\pi} \ln\left(\frac{\lambda}{D}\right) + \left(\frac{\Delta}{\pi\rho J} - \lambda q\right) \\ &= -\frac{D^2\rho}{2} + \frac{\Delta}{\pi} \ln\left(\frac{\lambda}{De^{-\frac{1}{J\rho}}}\right) - \lambda q \\ &= -\frac{D^2\rho}{2} + \frac{\Delta}{\pi} \ln\left(\frac{\lambda}{T_K}\right) - \lambda q. \end{aligned}$$

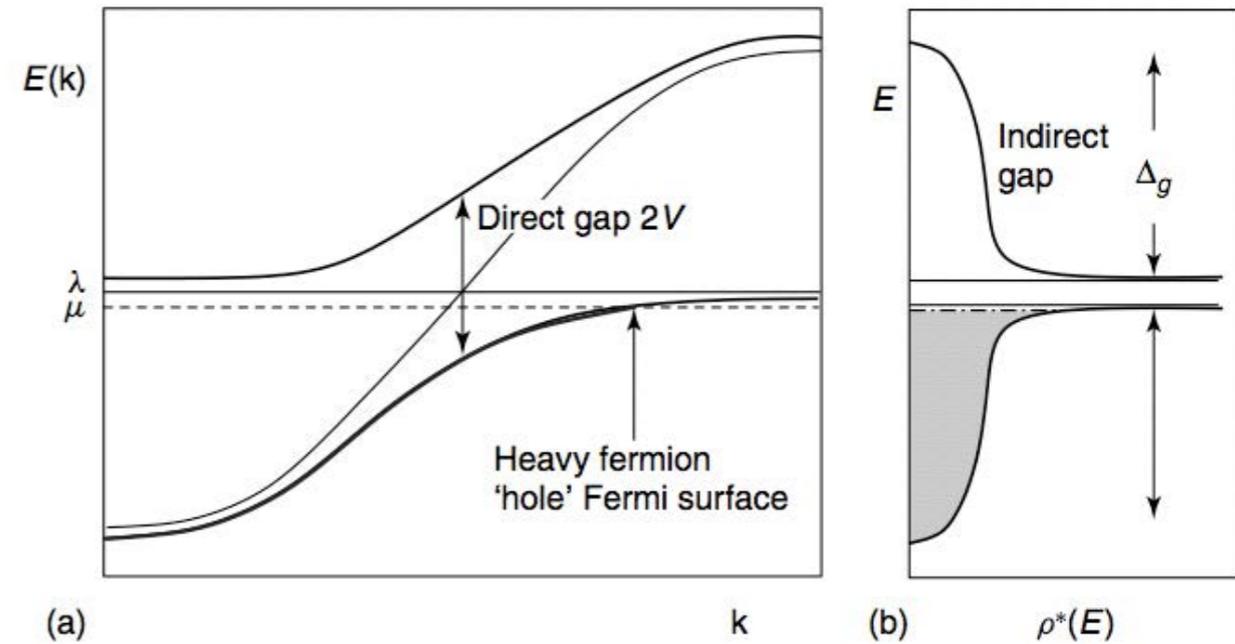
$$T_K = De^{-\frac{1}{J\rho}}$$



Detailed calcn.

$$(\Delta = \pi\rho|V|^2)$$

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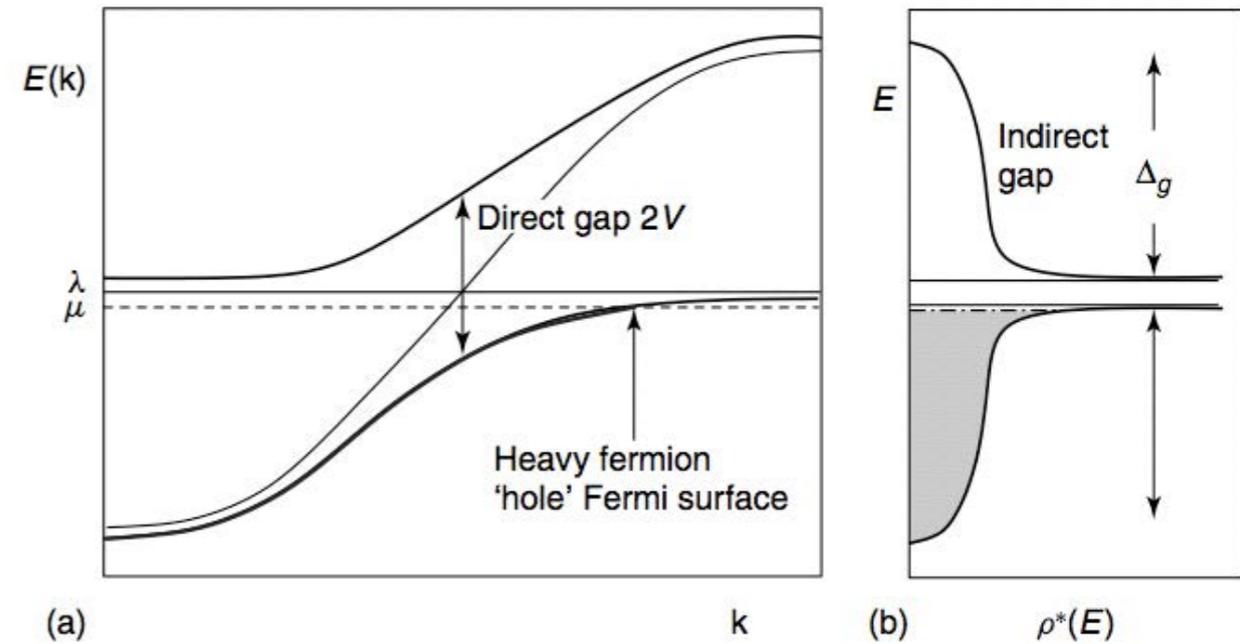
$$T_K = De^{-\frac{1}{J\rho}}$$

$$\frac{\partial E_0}{\partial \lambda} = \langle n_f \rangle - Q = 0$$

Detailed calcn.

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$$T_K = De^{-\frac{1}{J\rho}}$$

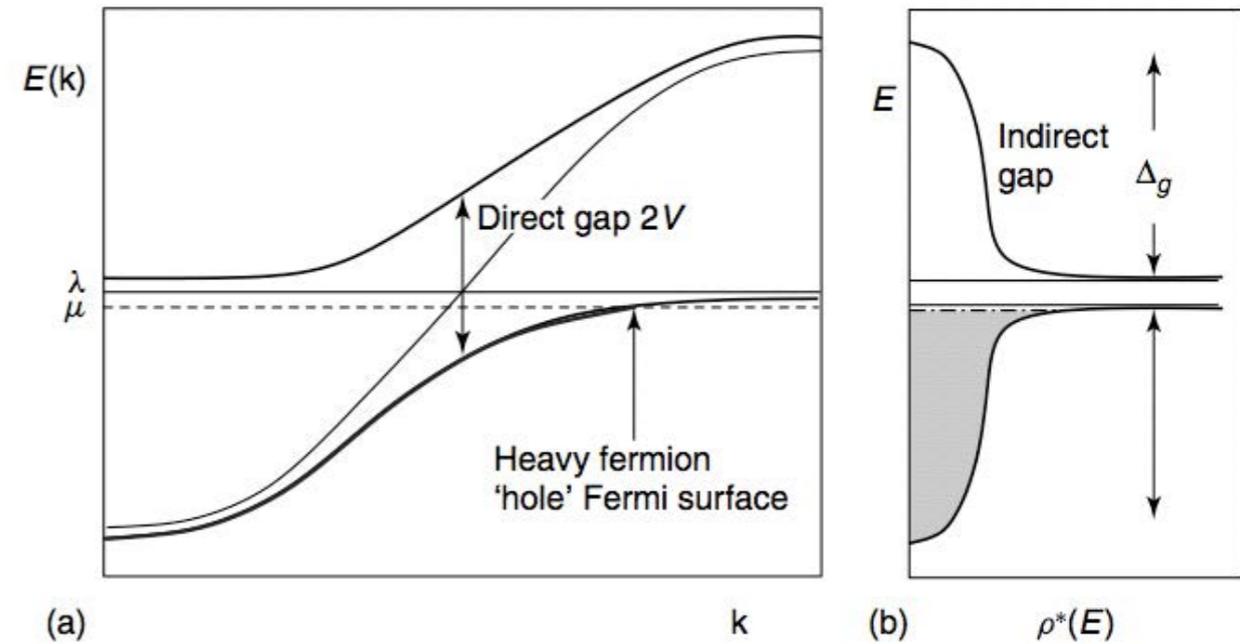
$$\frac{\partial E_0}{\partial \lambda} = \langle n_f \rangle - Q = 0 \quad \frac{\Delta}{\pi\lambda} - q = 0$$

$$\frac{E_o(V)}{N\mathcal{N}_s} = \frac{\Delta}{\pi} \ln\left(\frac{\Delta}{\pi q e T_K}\right) - \frac{D^2\rho}{2},$$

Detailed calcn.

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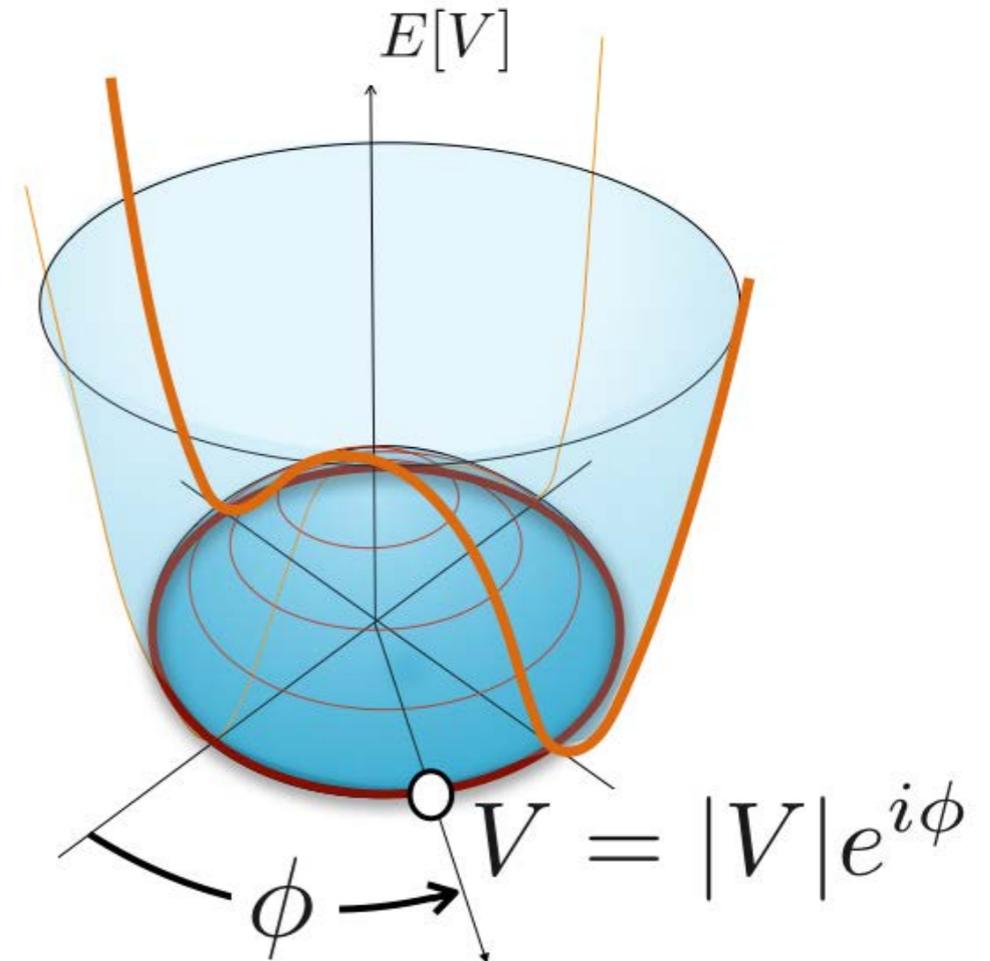
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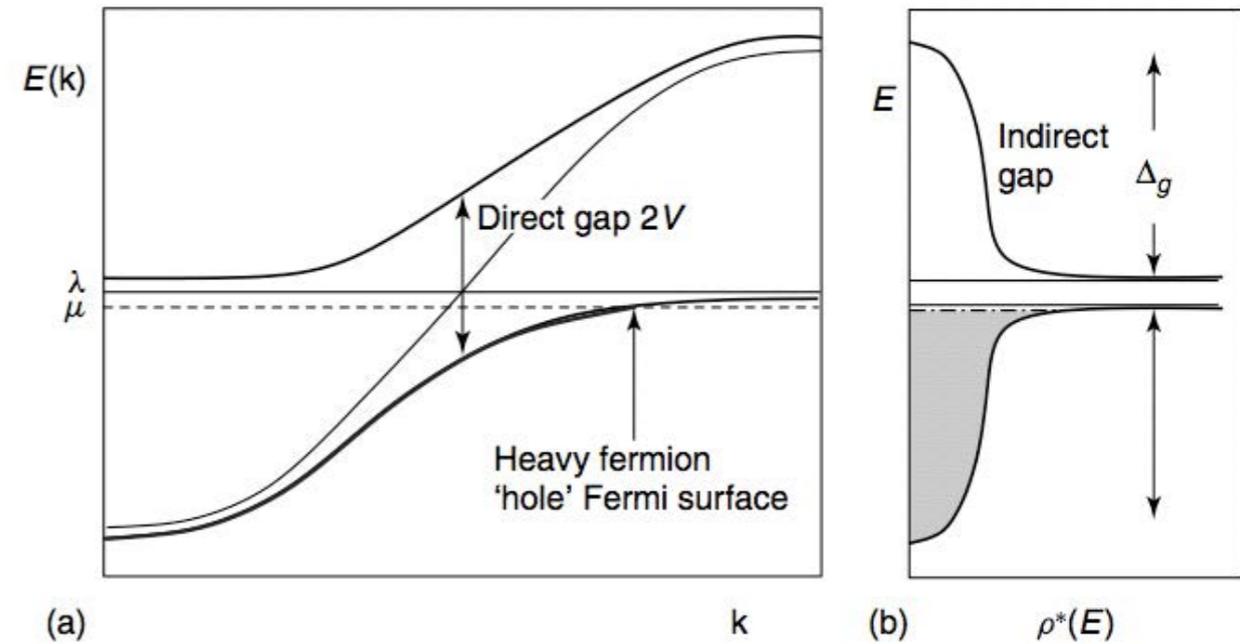
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Detailed calcn.

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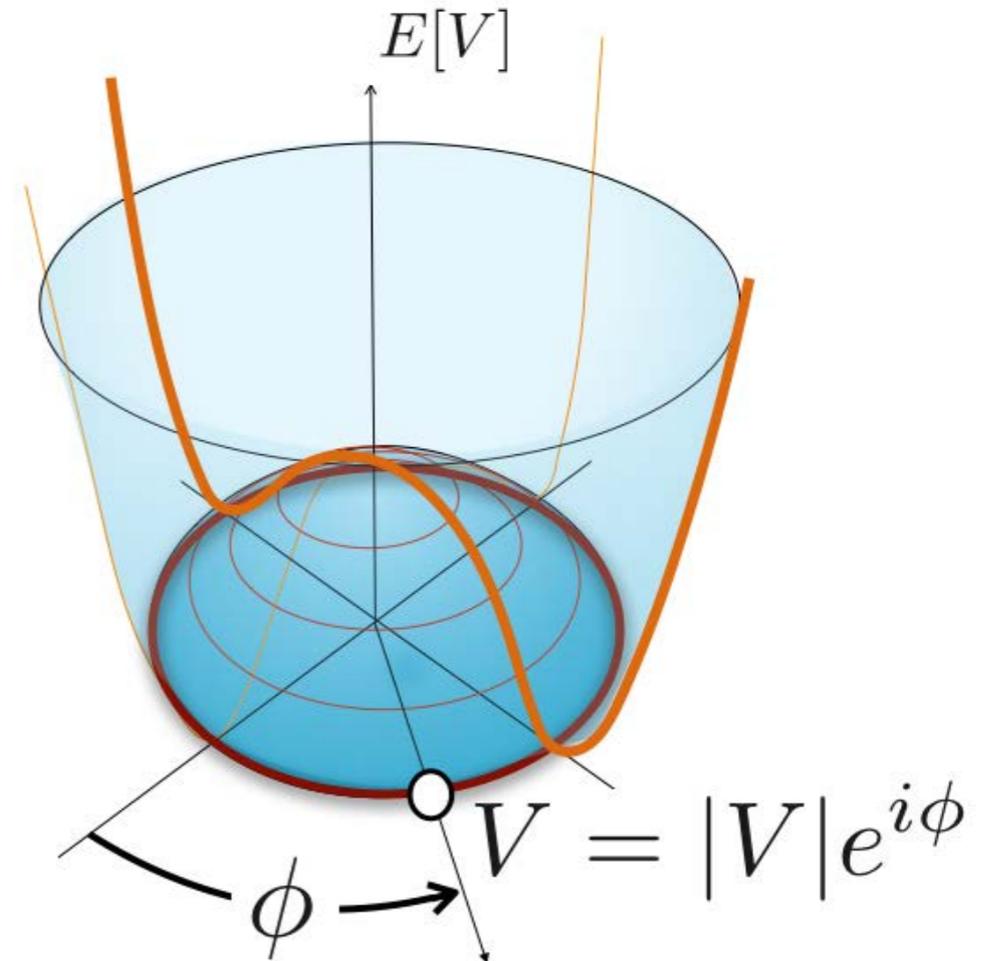


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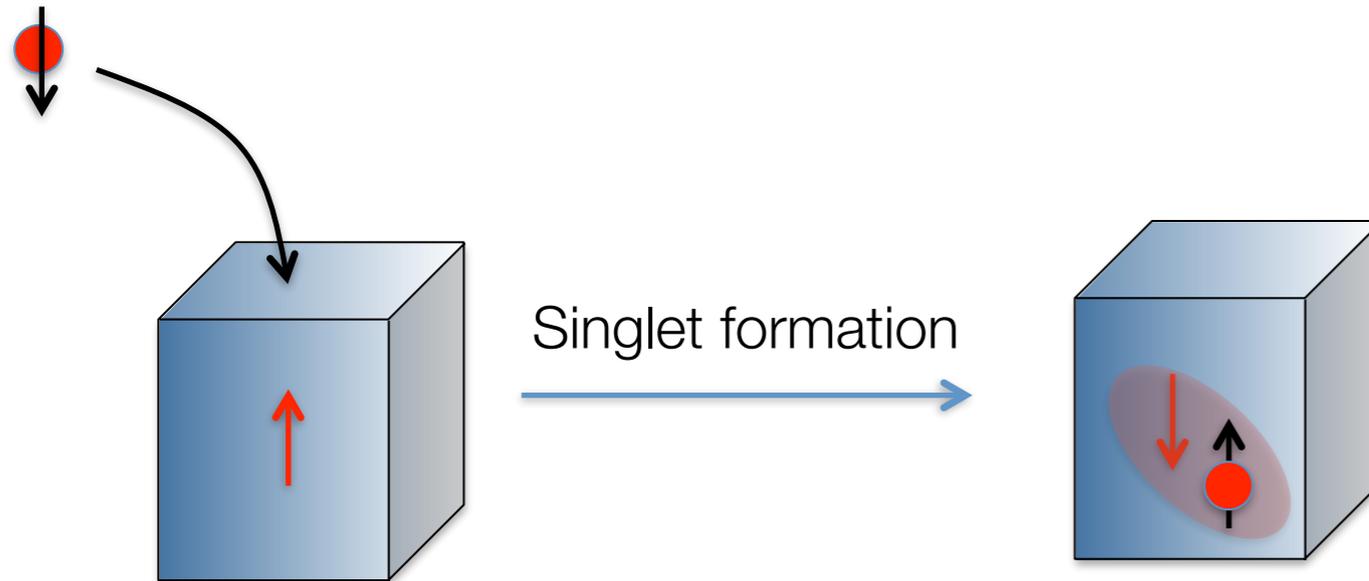
$$\frac{\partial E_0}{\partial \Delta} = 0 \quad 0 = \frac{1}{\pi} \ln\left(\frac{\Delta e^2}{\pi q T_K}\right)$$

$$\Delta = \frac{\pi q}{e^2} T_K$$



Composite nature of the Heavy Fermion.

Coherence and composite fermions (again)

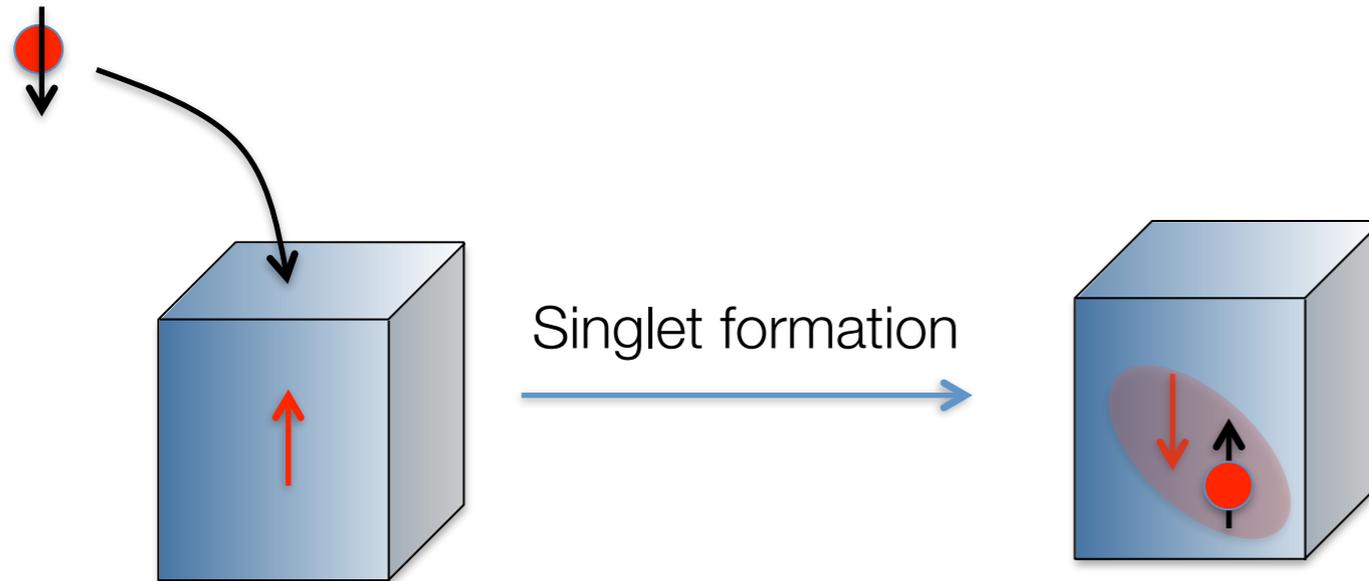


- The large N approach to the Kondo lattice.
Spin x conduction = composite fermion

$$\frac{J}{N} \overline{\square} c_{j\alpha}^\dagger S_{\alpha\beta} \equiv \bar{V} f_{j\beta}^\dagger$$

Composite Fermion

Coherence and composite fermions (again)



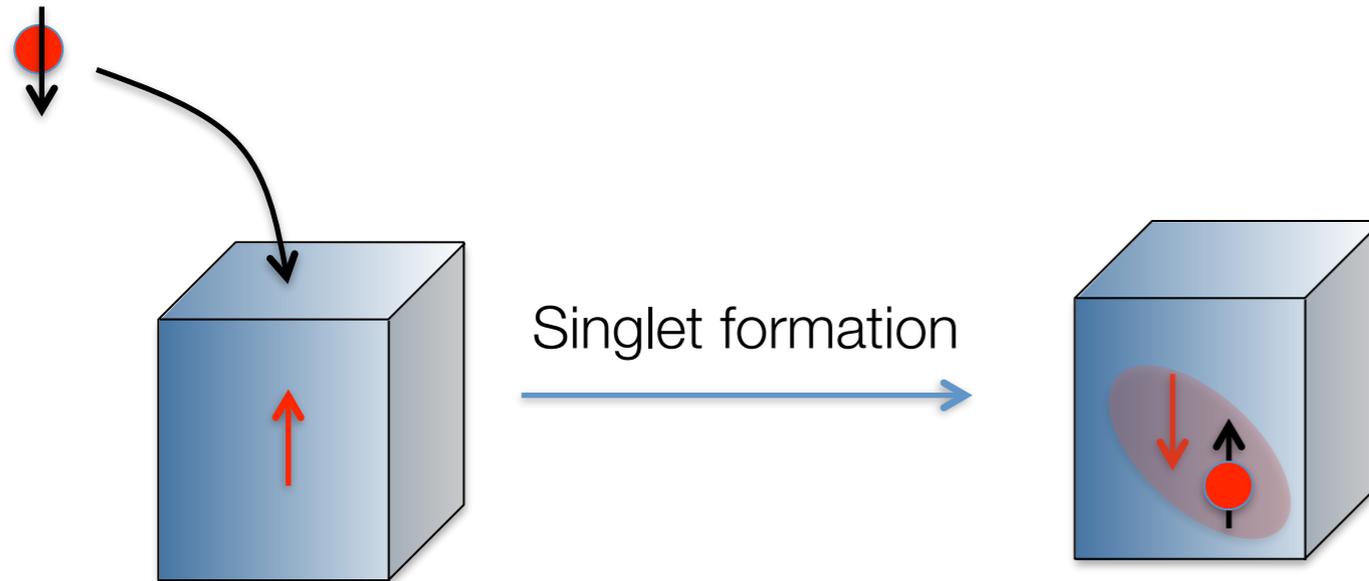
$$\frac{1}{N} \sum_{\beta} \overline{c_{\beta}(\tau)} S_{\beta\alpha}(\tau') = g(\tau - \tau') \hat{f}_{\alpha}(\tau').$$

- The large N approach to the Kondo lattice.
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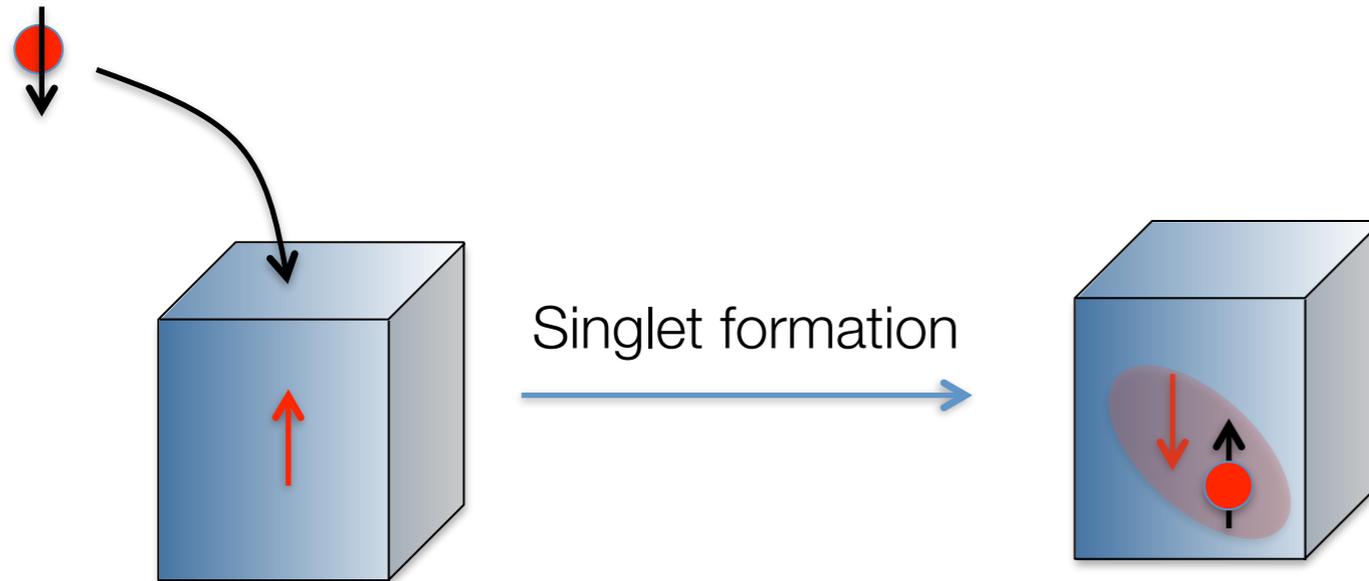
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Composite Fermion

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$$\begin{aligned} \frac{1}{N} \sum_{\beta} \overline{c_{\beta}(\tau) S_{\beta\alpha}(\tau')} &= \frac{1}{N} \sum_{\beta} \overline{c_{\beta}(\tau) f_{\beta}^{\dagger}(\tau') f_{\alpha}(\tau')} \\ &= \frac{1}{N} \sum_{\beta} \langle T c_{\beta}(\tau) f_{\beta}^{\dagger}(\tau') \rangle f_{\alpha}(\tau') \\ &= -G_{cf}(\tau - \tau') f_{\alpha}(\tau'). \end{aligned}$$

Coherence and composite fermions (again)



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Spin x conduction = composite fermion

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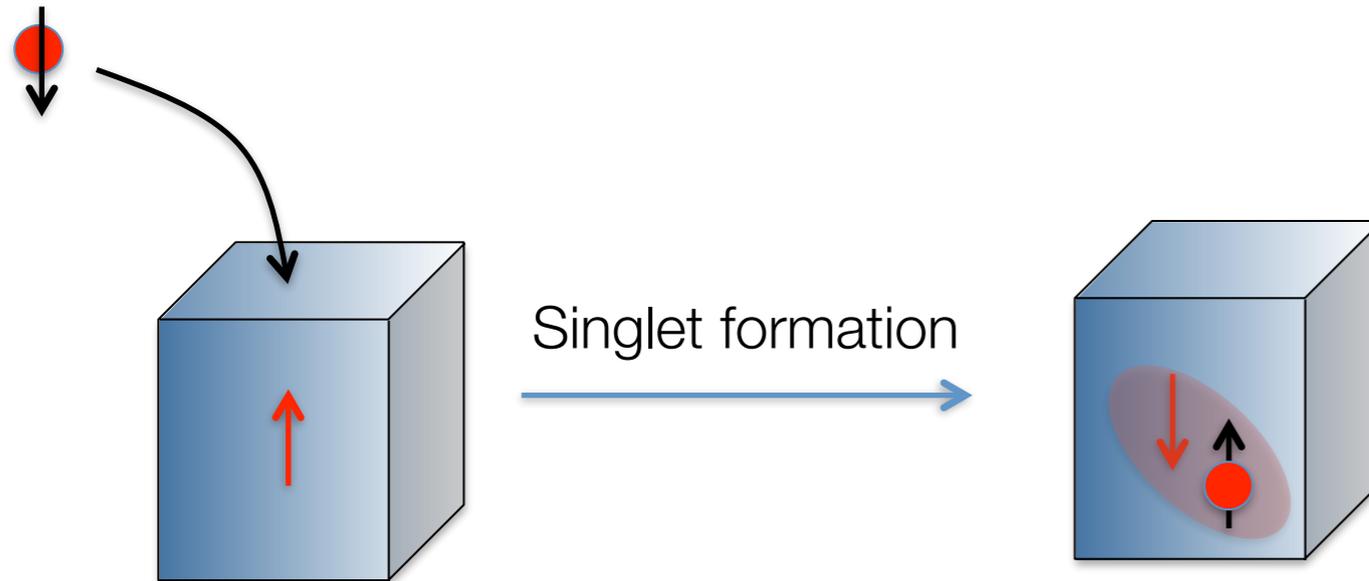
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Coherence and composite fermions (again)



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$$g(\tau - \tau') = \langle T c_{\beta}(\tau) f_{\beta}^{\dagger}(\tau') \rangle = -G_{cf}(\tau - \tau')$$

$$g(\tau) \sim \begin{cases} \rho V \ln\left(\frac{T_K \tau}{\hbar}\right) & (\hbar/D \ll \tau \ll \hbar/T_K) \\ \frac{1}{\tau} & (\tau \gg \hbar/T_K) \end{cases}$$

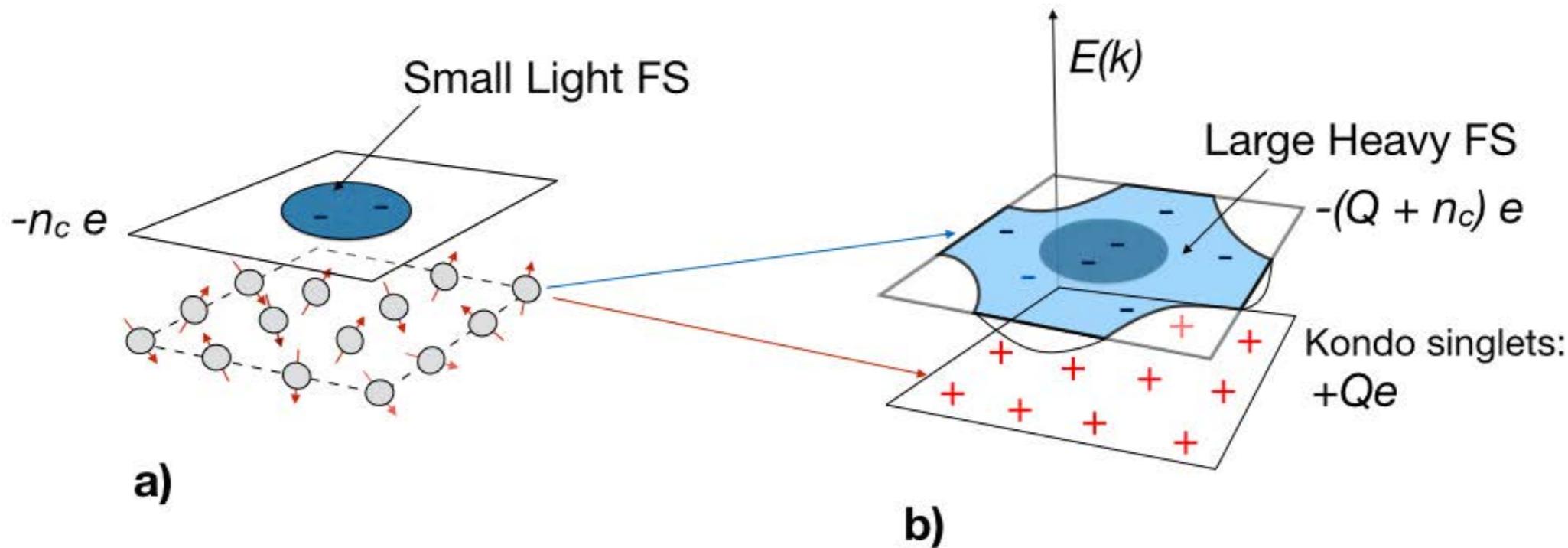
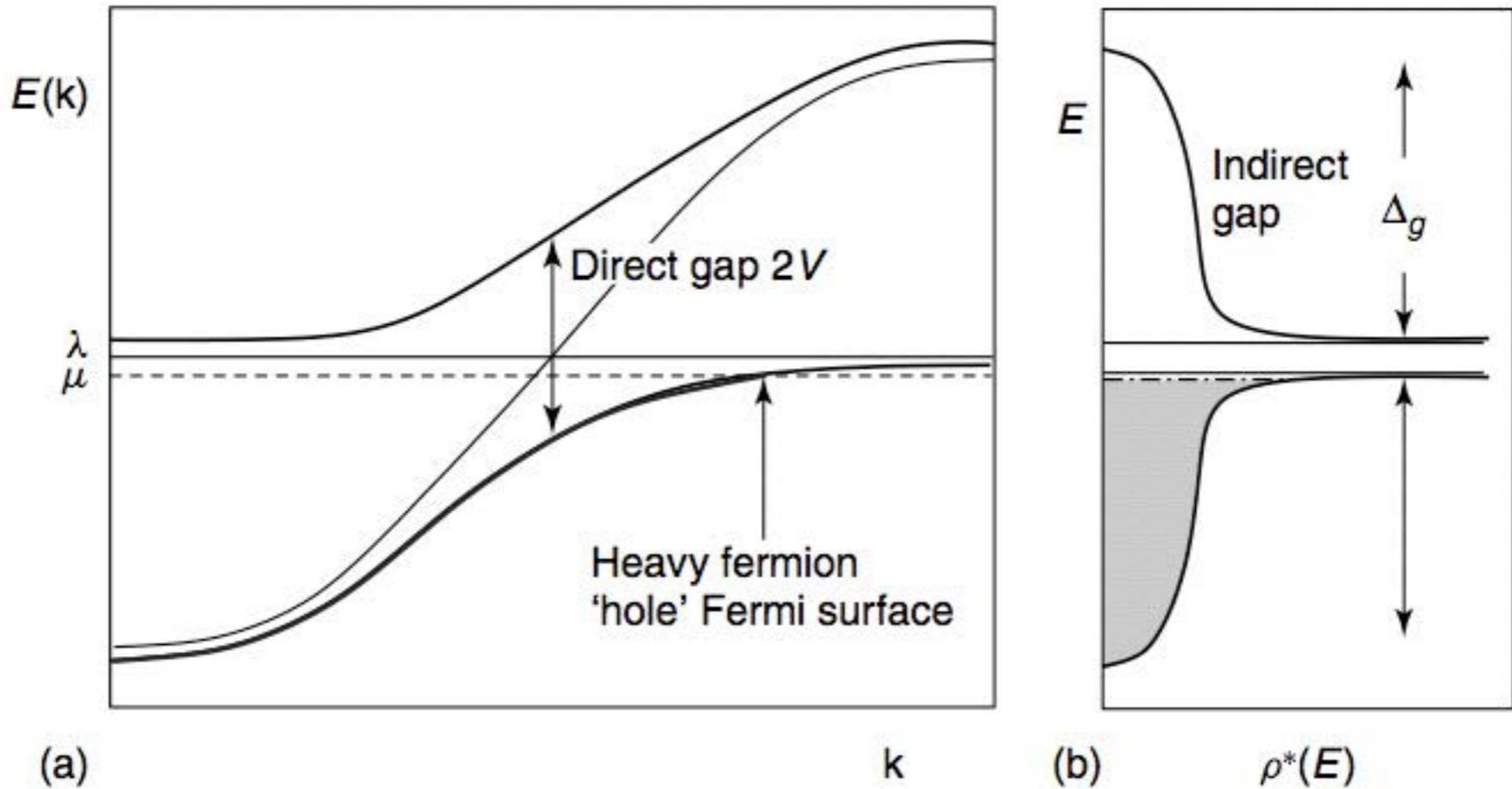
Bound-state built from electrons spanning decades of energy out to the Band-Width.

Physics of Heavy Fermion Superconductivity Lecture II:

1. The large N approach to the Kondo lattice.
2. Heavy Fermion Metals.
3. Optical Conductivity of Heavy Fermion Metals
4. Kondo Insulators

Large N Approach.

Read and Newns '83.



Physics of Heavy Fermion Superconductivity Lecture II:

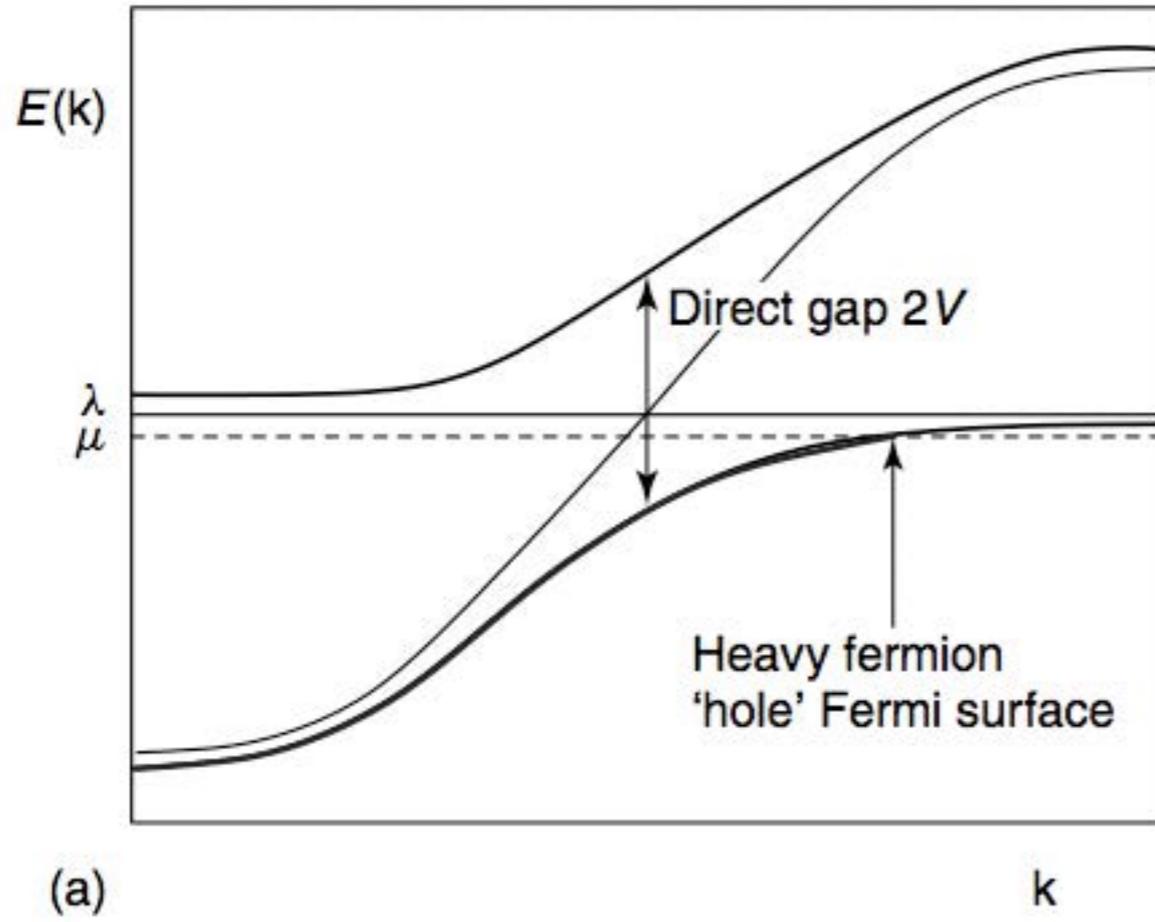
1. The large N approach to the Kondo lattice.

2. Heavy Fermion Metals.

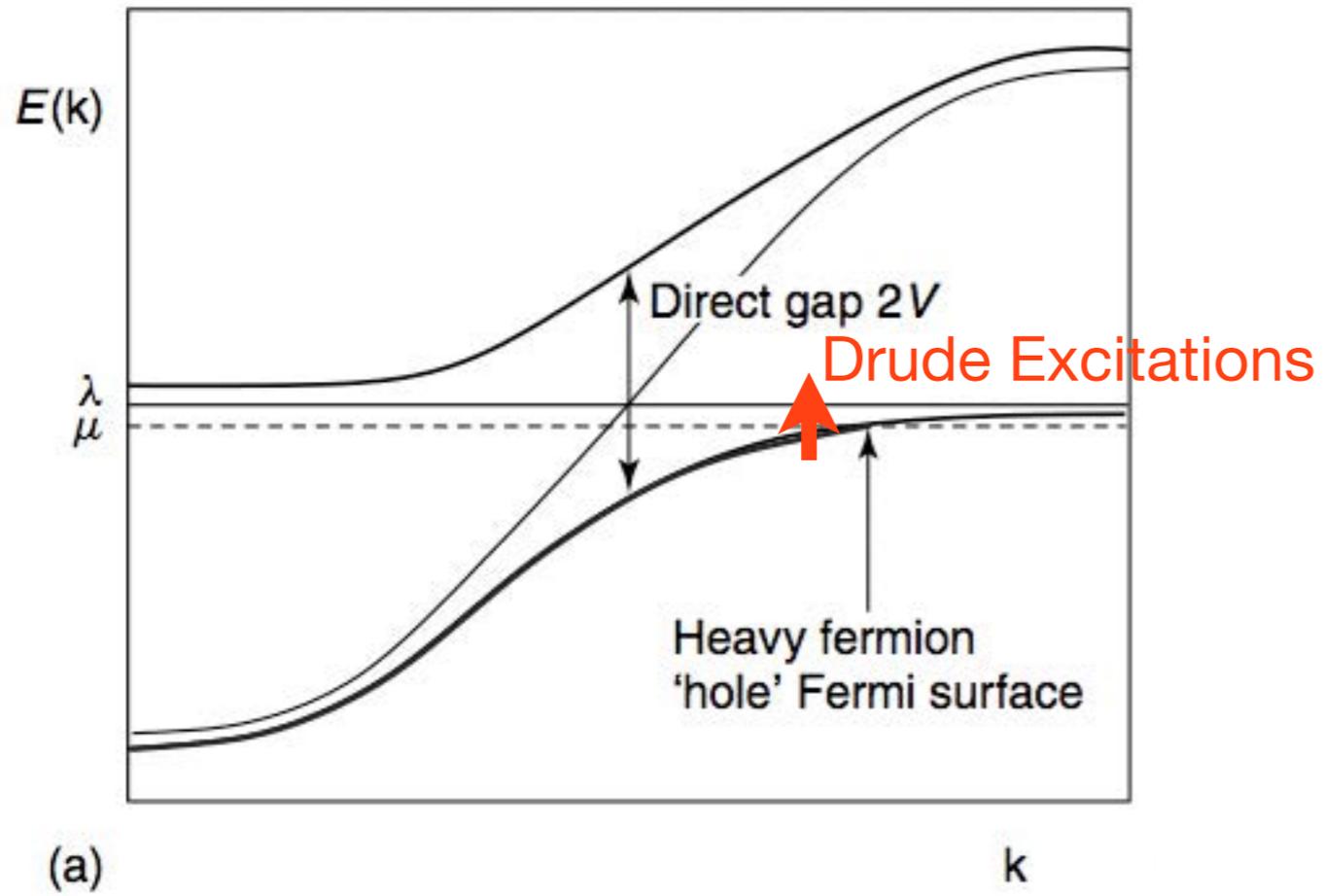
3. Optical Conductivity of Heavy Fermion Metals

4. Kondo Insulators

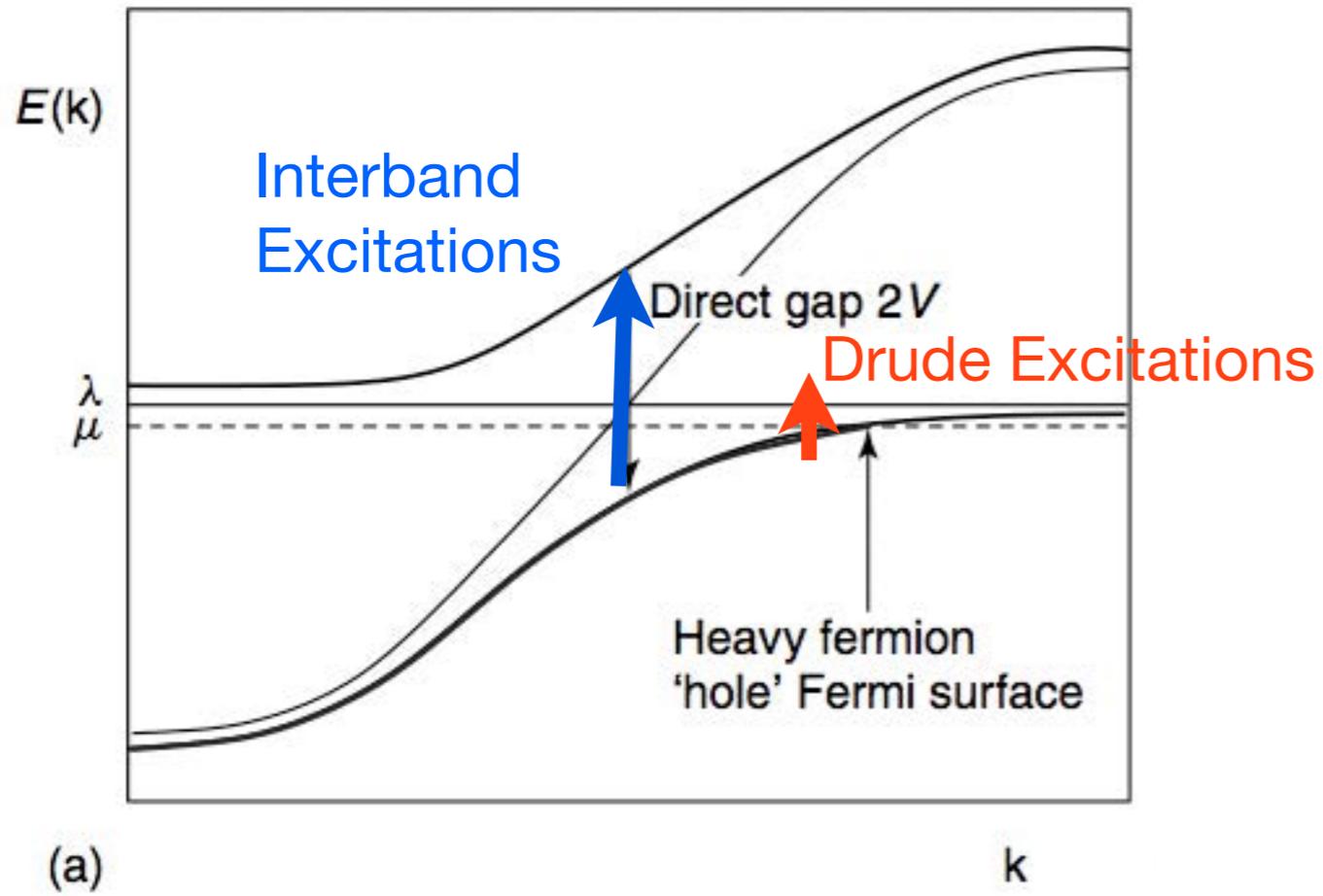
Optical Conductivity.



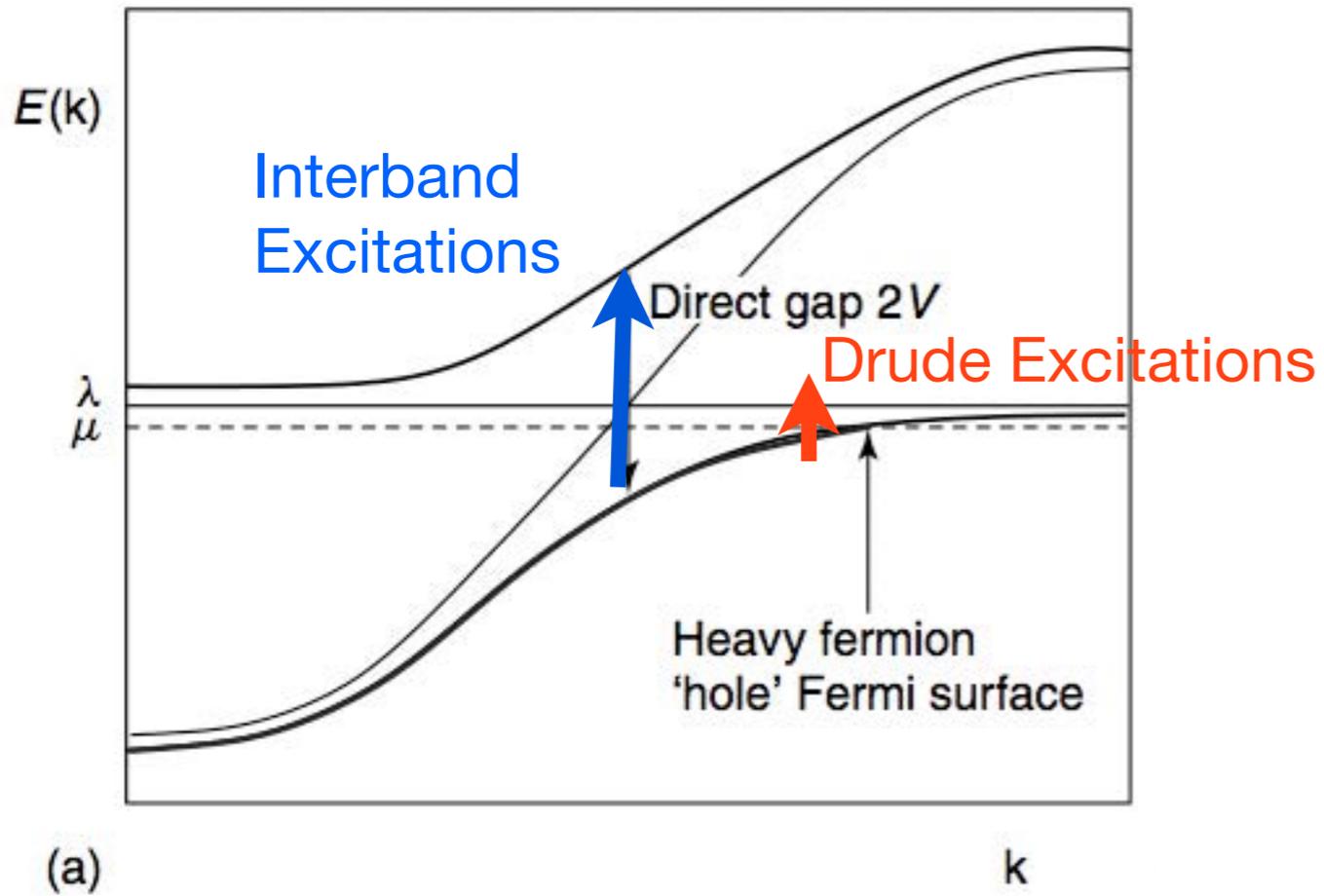
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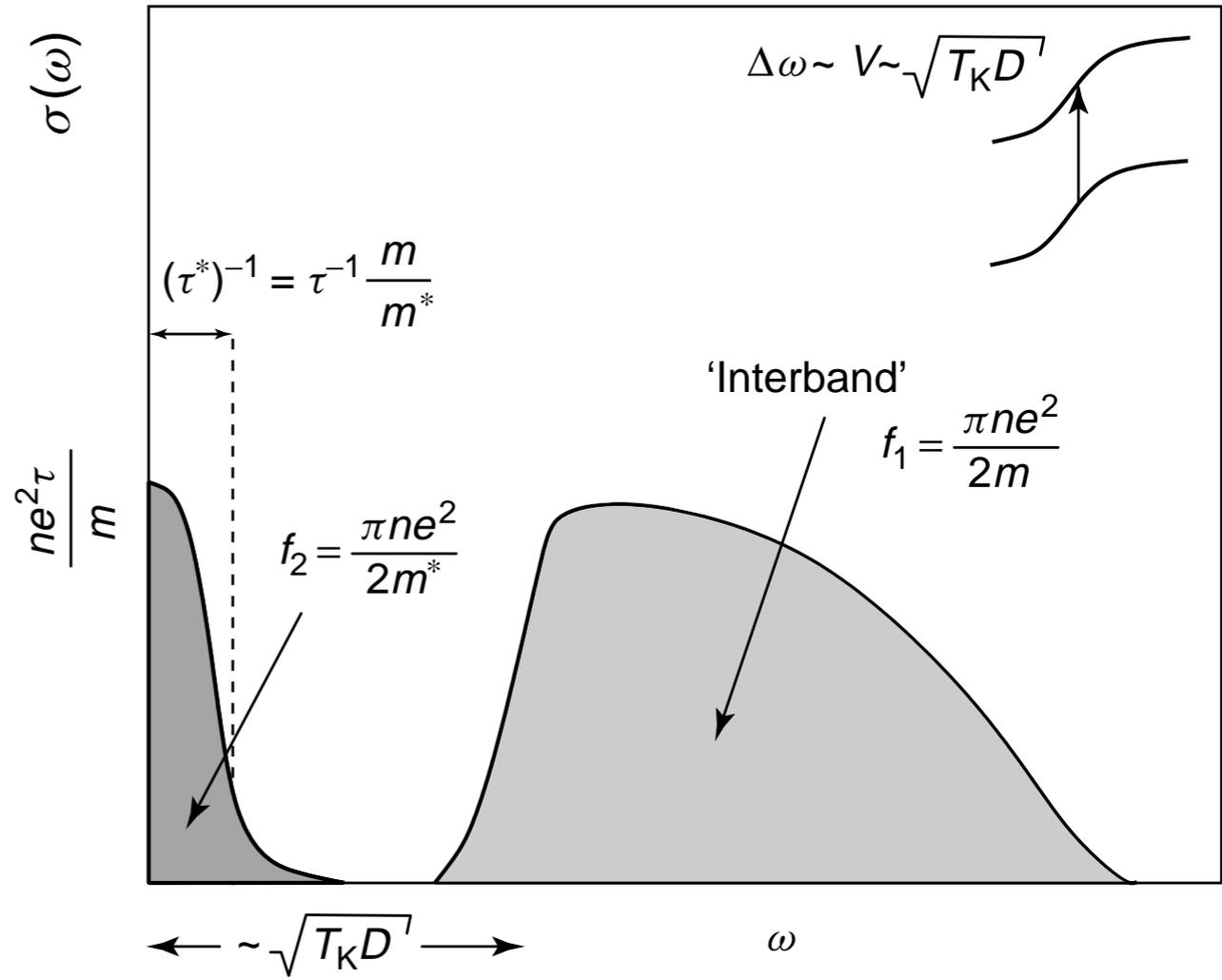
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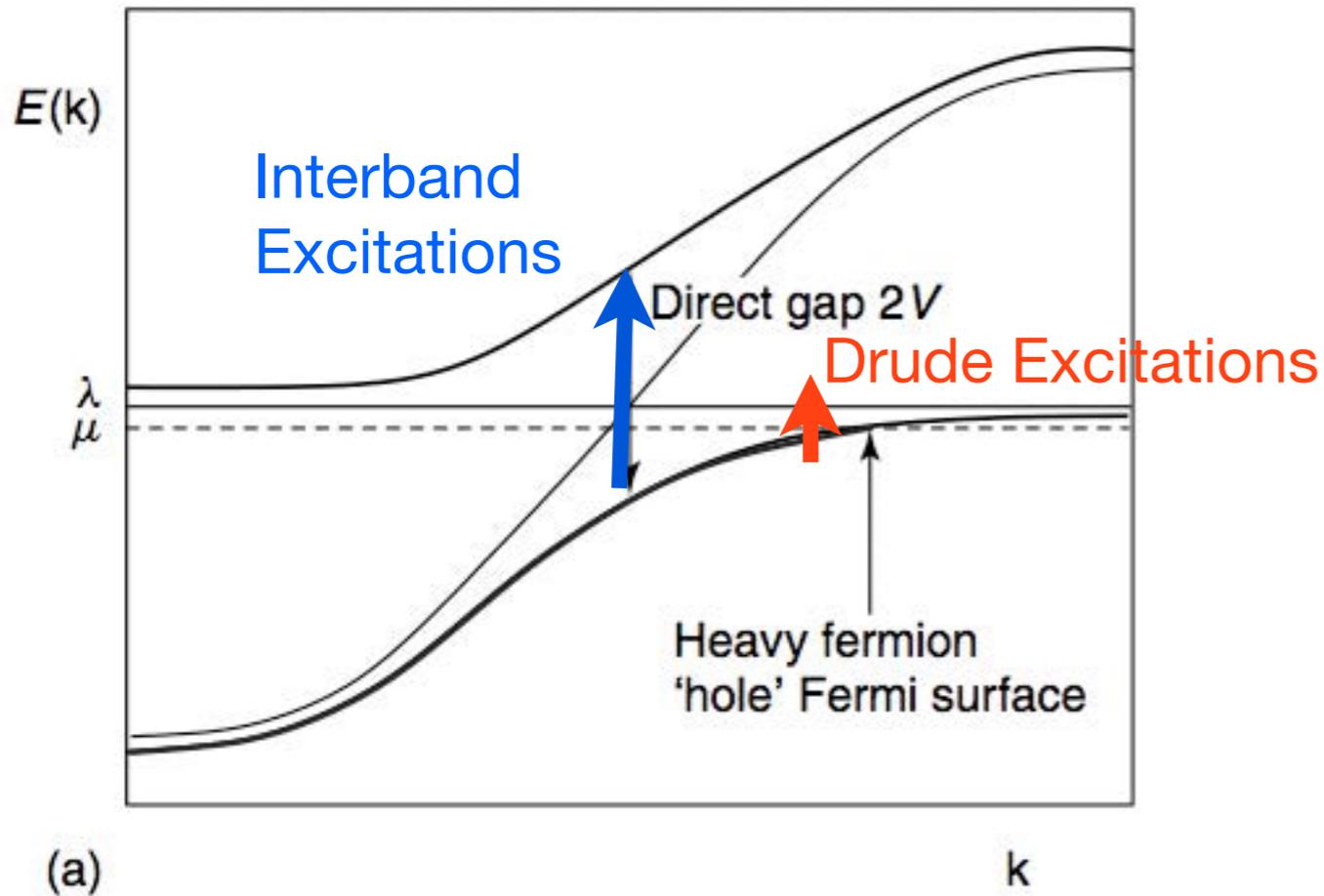
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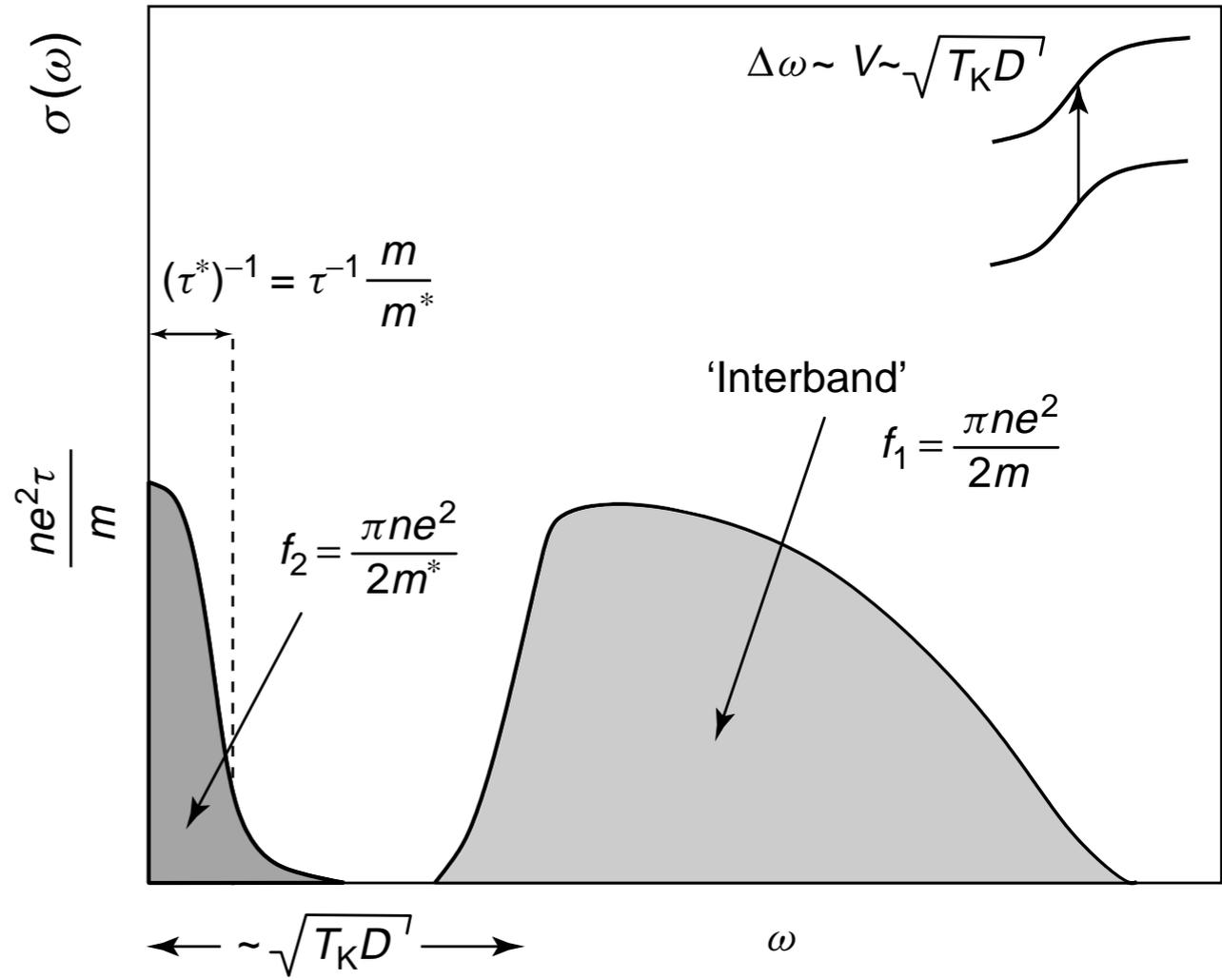
Millis and Lee, 1987



Optical Conductivity.

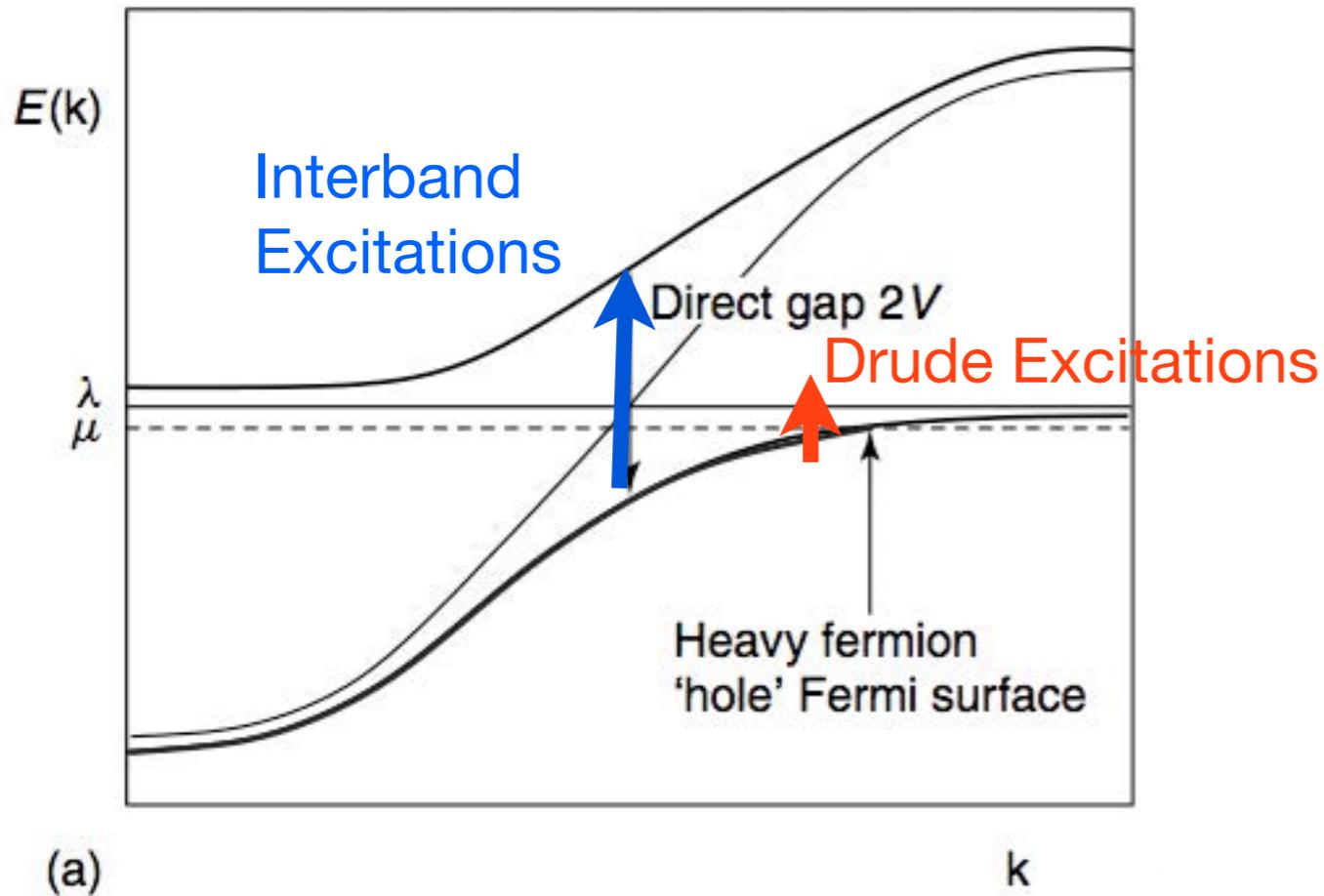


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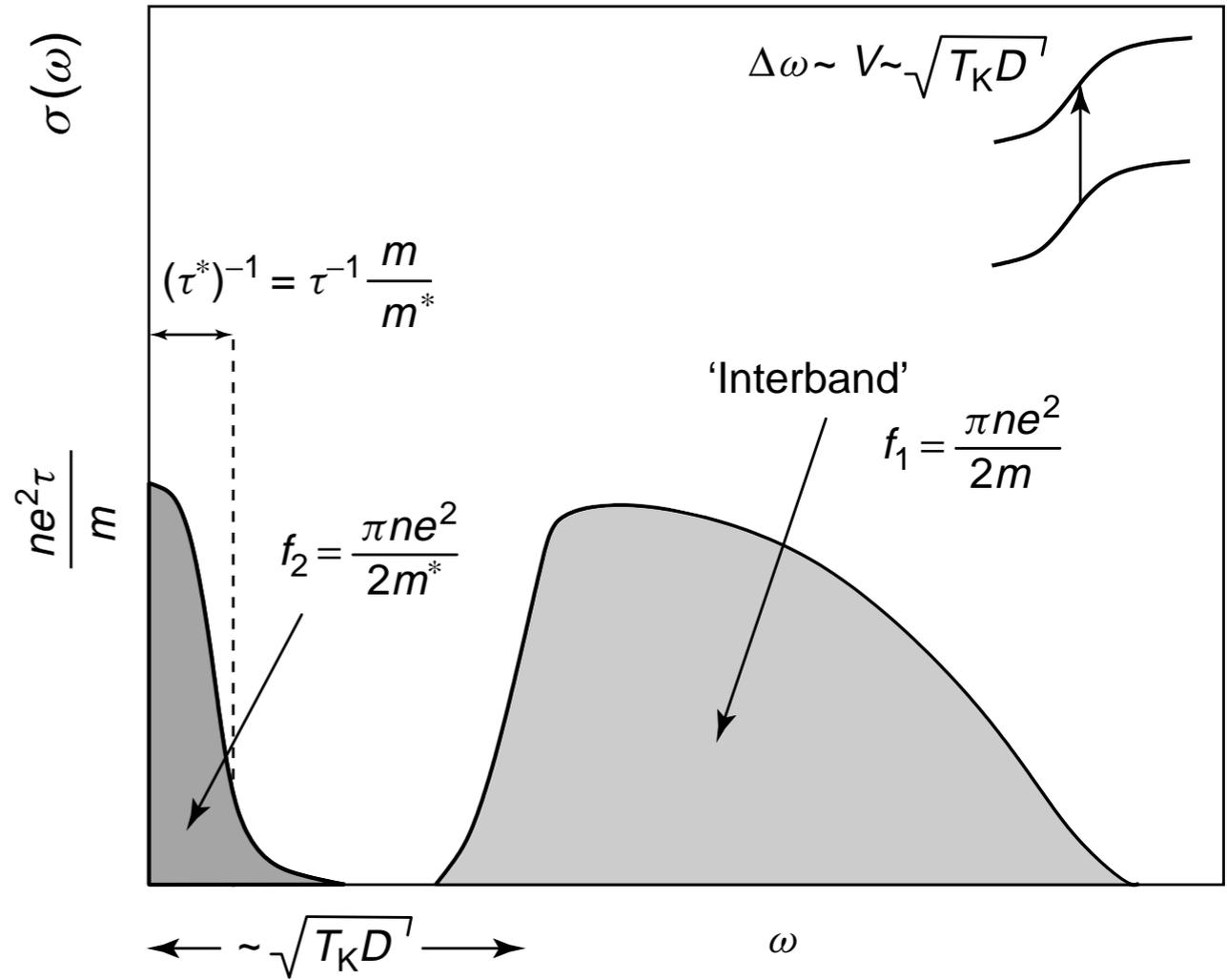


$$V^2 / D \sim T_K \Rightarrow V \sim \sqrt{T_K D}$$

Optical Conductivity.



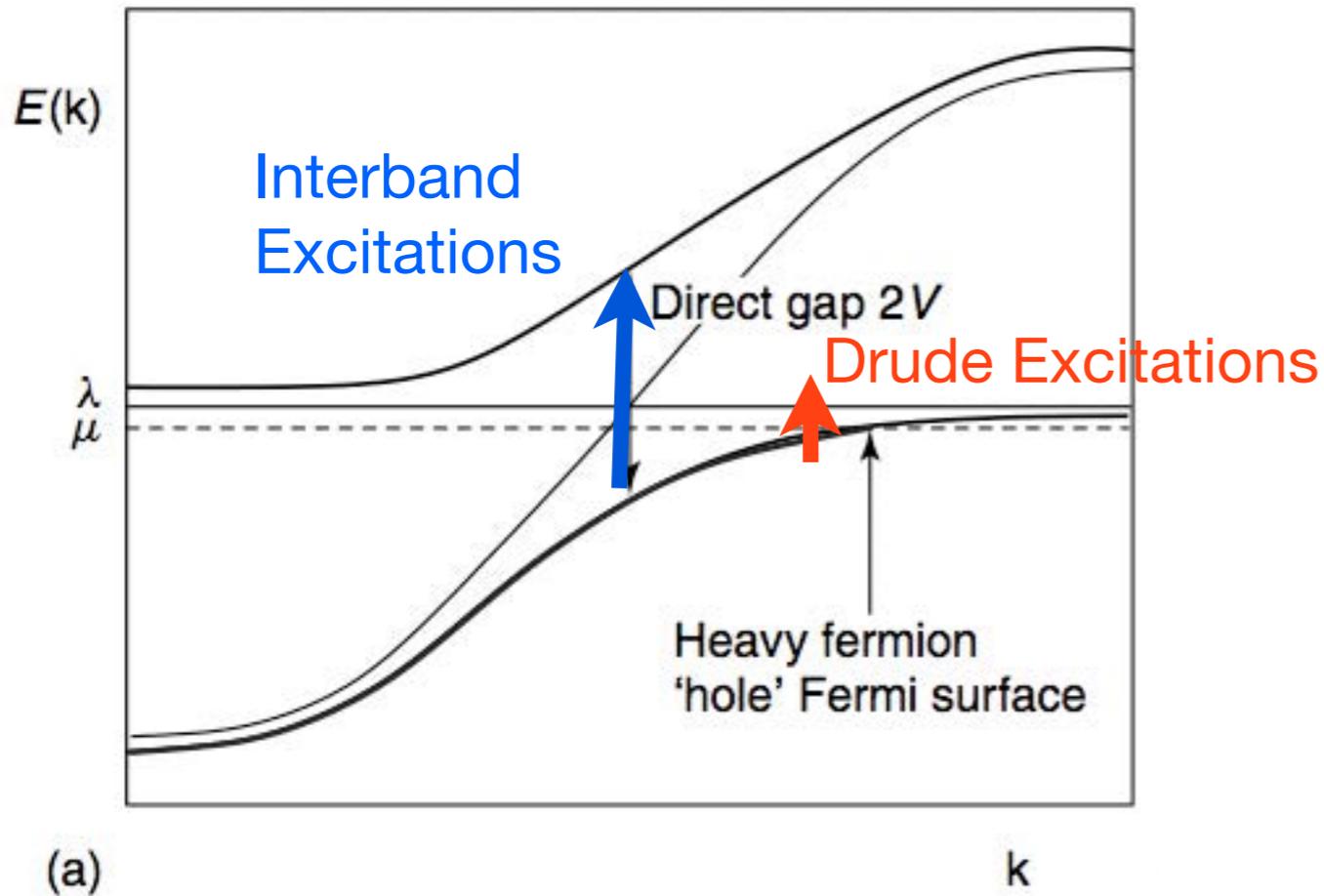
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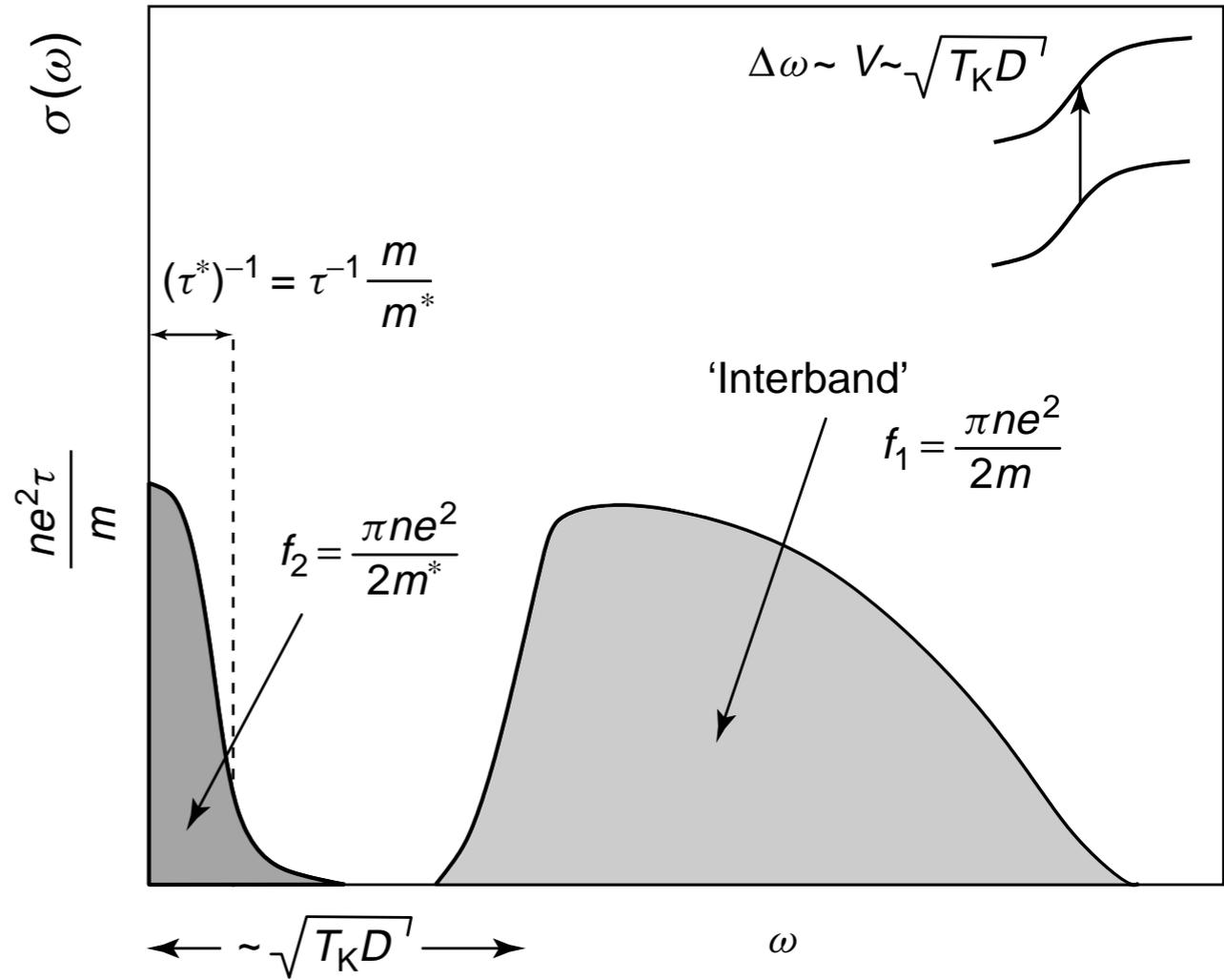
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$$\int_0^\infty \frac{d\omega}{\pi} \sigma(\omega) = f_1 = \frac{\pi}{2} \left(\frac{n_c e^2}{m} \right)$$

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Millis and Lee, 1987

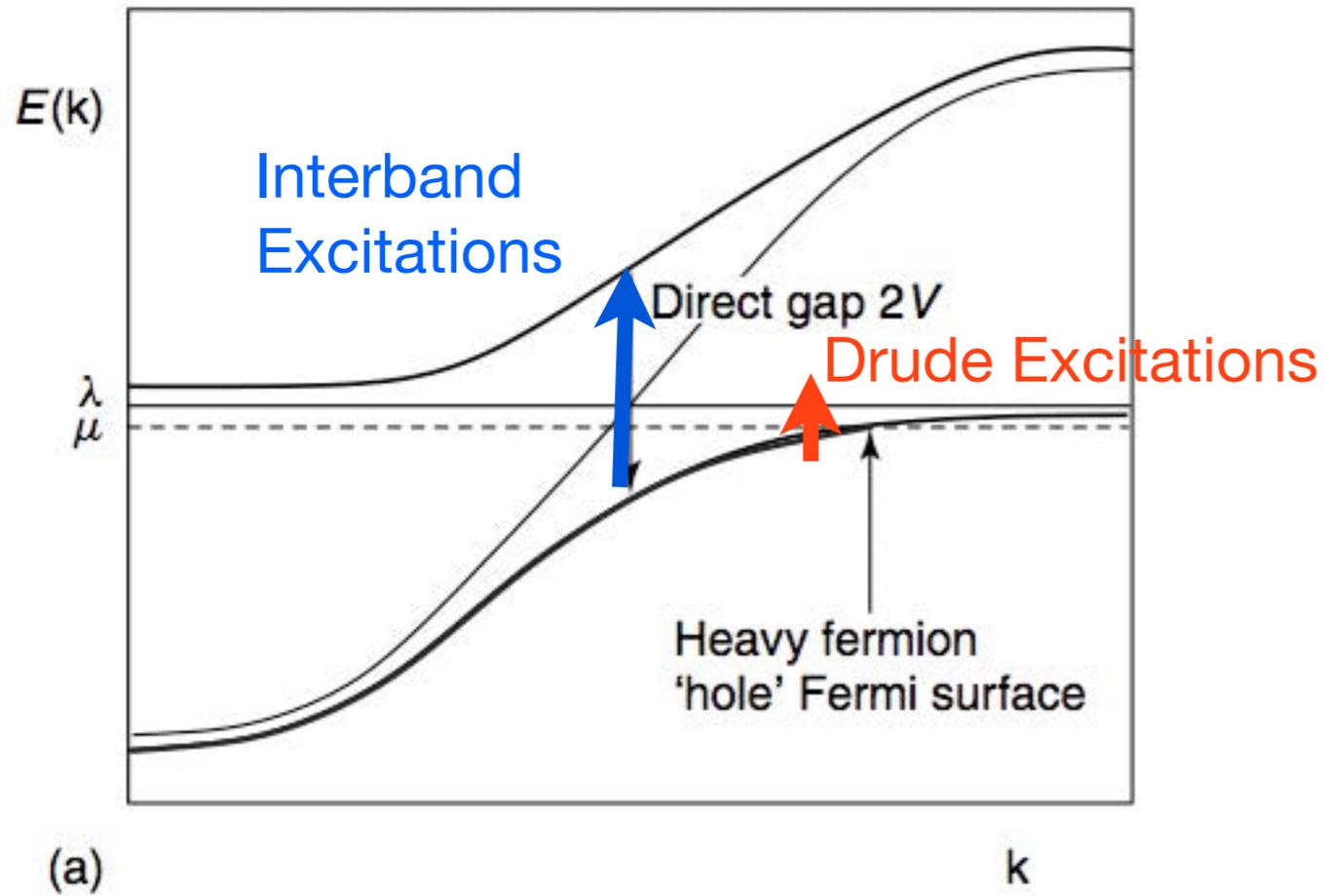


$$V^2 / D \sim T_K \Rightarrow V \sim \sqrt{T_K D}$$

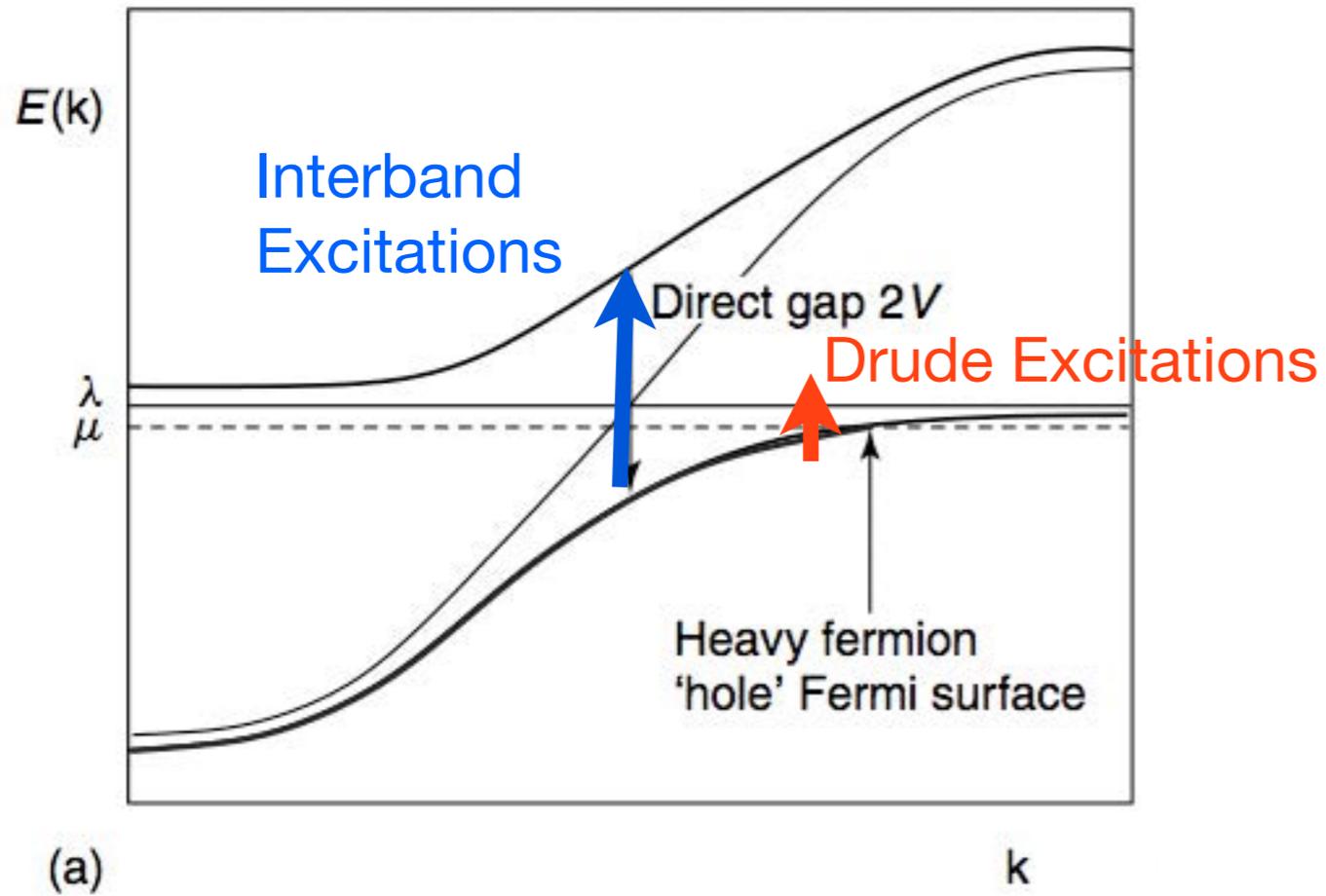
$$\int_0^\infty \frac{d\omega}{\pi} \sigma(\omega) = f_1 = \frac{\pi}{2} \left(\frac{n_c e^2}{m} \right)$$

$$\int_0^{\sim V} d\omega \sigma(\omega) = f_2 = \frac{\pi}{2} \frac{n_{\text{HFE}} e^2}{m^*}$$

Optical Conductivity: Details



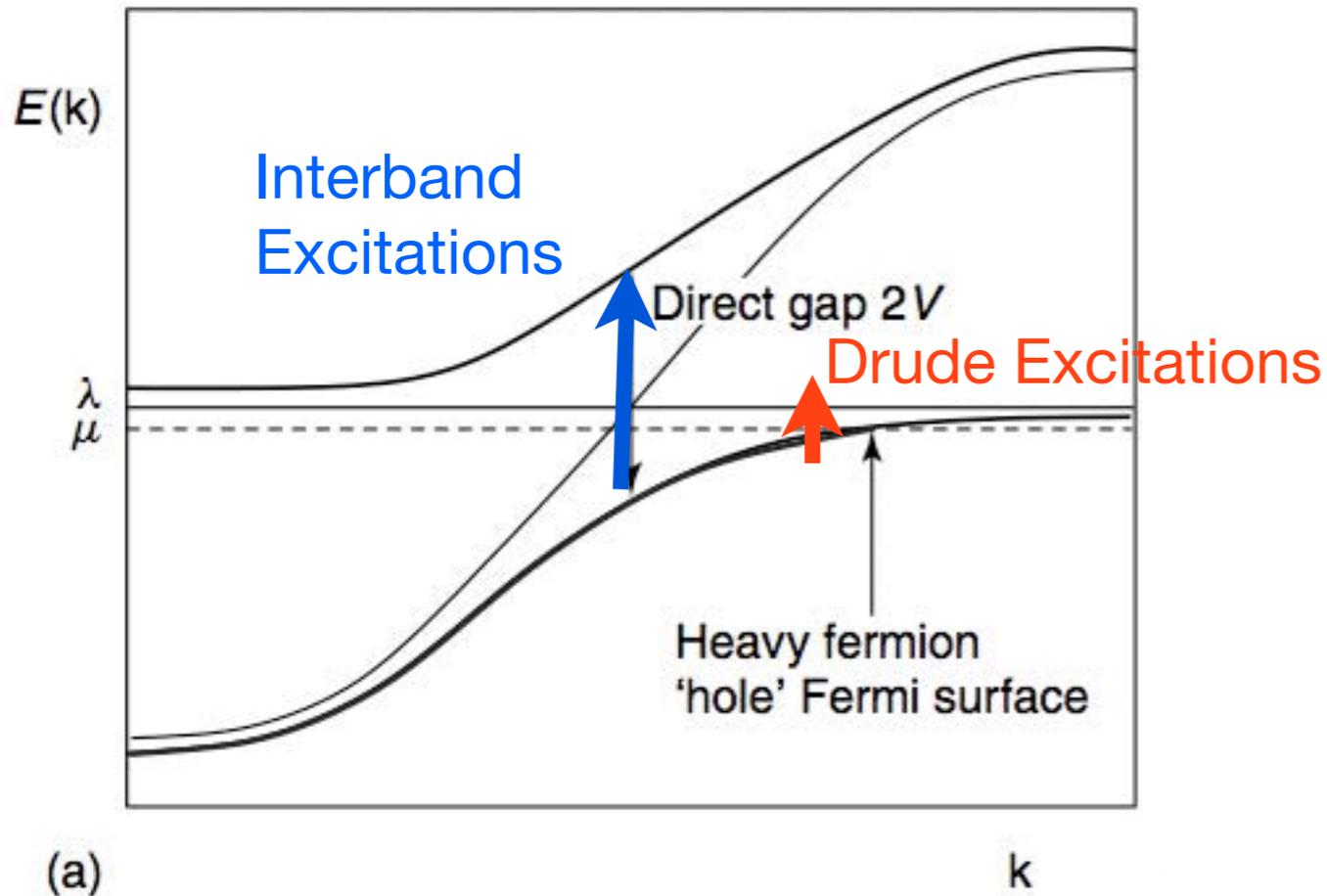
Optical Conductivity: Details



$$\vec{j} = e \sum_{\mathbf{k}\sigma} \vec{\nabla}_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma}$$

Optical Conductivity: Details

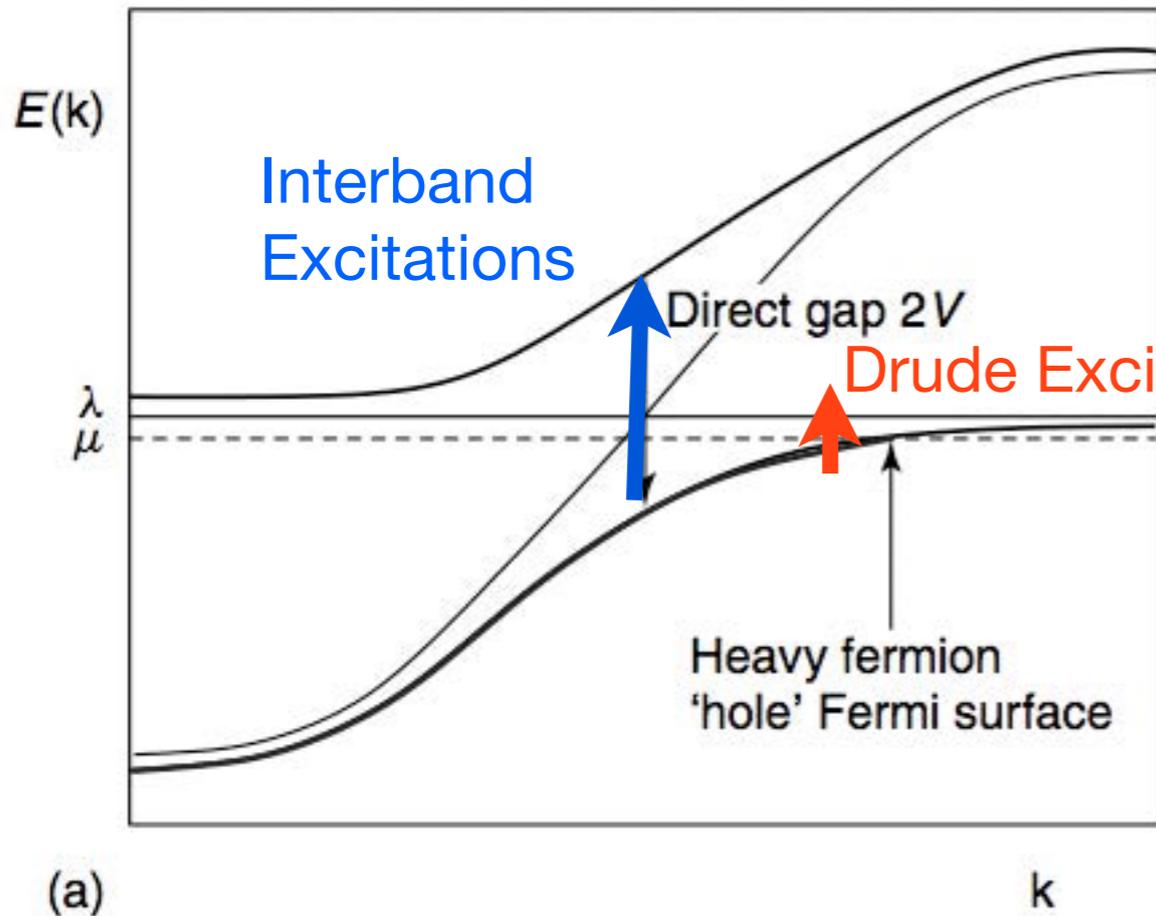
$$\vec{j} = e \sum_{\mathbf{k}\sigma} \vec{\nabla}_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}$$



$$\sigma(i\nu_n)\delta_{ab} = -\frac{e^2}{\nu_n} \left[a \text{ --- } \text{loop} \text{ --- } b \right]_{0}^{i\nu_n}$$

$$= -\frac{e^2}{\nu_n} \left[T \sum_{\kappa=(\mathbf{k}, i\omega_r)} \left(\nabla_a \epsilon_{\mathbf{k}} \nabla_b \epsilon_{\mathbf{k}} G_c(\mathbf{k}, i\omega_r + i\nu_n) G_c(\mathbf{k}, i\omega_r) \right) - (i\nu_n \rightarrow 0) \right].$$

Optical Conductivity: Details



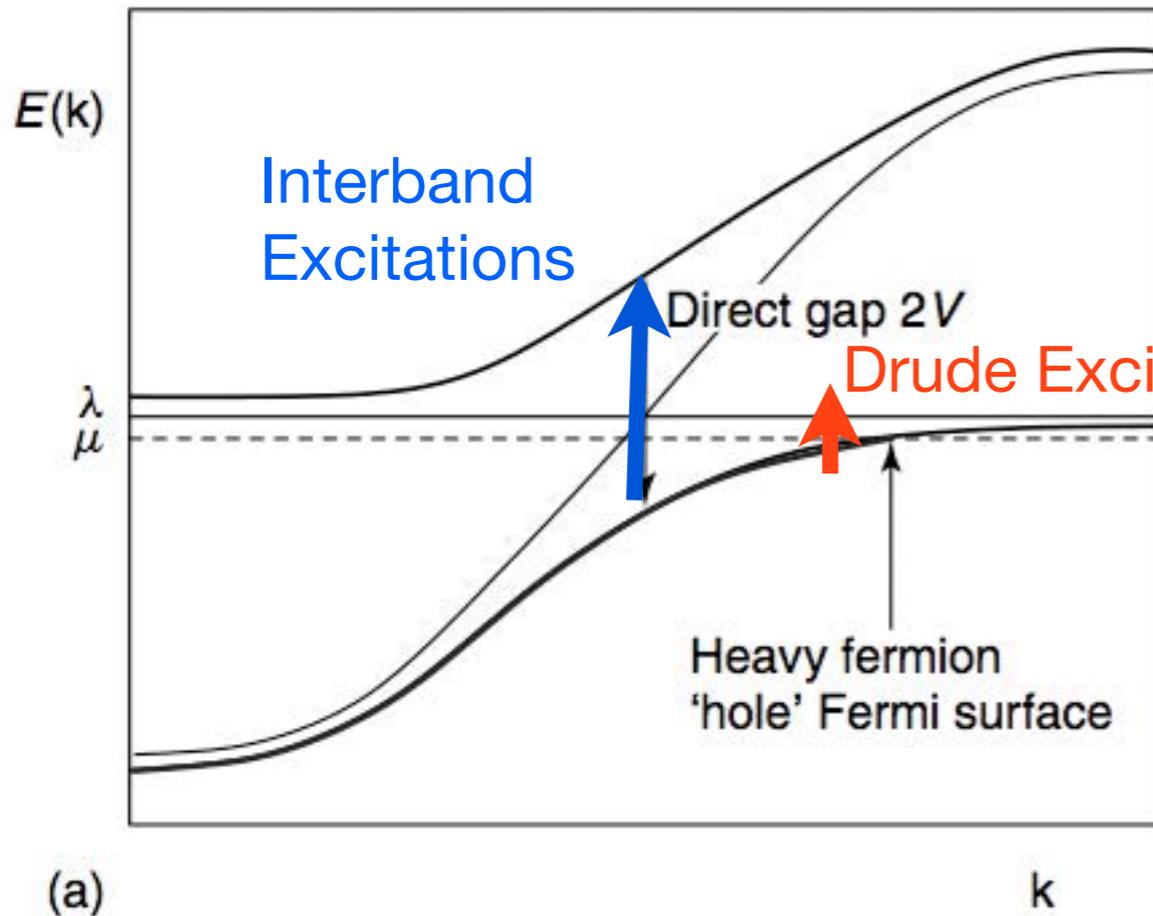
$$\vec{j} = e \sum_{\mathbf{k}\sigma} \vec{\nabla}_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma}$$

$$G_c(\mathbf{k}, i\omega_n) = \frac{1}{i\tilde{\omega}_n - \epsilon_{\mathbf{k}} - \Sigma_c(i\tilde{\omega}_n)}, \quad \Sigma_c(z) = \frac{V^2}{i\omega_n - \lambda}$$

$$i\tilde{\omega}_n = i\omega_n + i \operatorname{sgn}(\omega_n) \frac{\Gamma}{2}$$

$$\begin{aligned} \sigma(i\nu_n) \delta_{ab} &= -\frac{e^2}{\nu_n} \left[a \text{---} \text{---} \text{---} b \right]_{0}^{i\nu_n} \\ &= -\frac{e^2}{\nu_n} \left[T \sum_{\kappa=(\mathbf{k}, i\omega_r)} \left(\nabla_a \epsilon_{\mathbf{k}} \nabla_b \epsilon_{\mathbf{k}} G_c(\mathbf{k}, i\omega_r + i\nu_n) G_c(\mathbf{k}, i\omega_r) \right) - (i\nu_n \rightarrow 0) \right]. \end{aligned}$$

Optical Conductivity: Details



$$\vec{j} = e \sum_{\mathbf{k}\sigma} \vec{\nabla}_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma}$$

$$G_c(\mathbf{k}, i\omega_n) = \frac{1}{i\tilde{\omega}_n - \epsilon_{\mathbf{k}} - \Sigma_c(i\tilde{\omega}_n)}, \quad \Sigma_c(z) = \frac{V^2}{i\omega_n - \lambda}$$

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$$= -\frac{e^2}{\nu_n} \left[T \sum_{\kappa=(\mathbf{k}, i\omega_r)} \left(\nabla_a \epsilon_{\mathbf{k}} \nabla_b \epsilon_{\mathbf{k}} G_c(\mathbf{k}, i\omega_r + i\nu_n) G_c(\mathbf{k}, i\omega_r) \right) - (i\nu_n \rightarrow 0) \right].$$

$$\sigma(i\nu_n) = -\left(\frac{ne^2}{m} \right) \frac{1}{\nu_n} \left[T \sum_{i\omega_r} \int_{-\infty}^{\infty} \frac{d\epsilon}{(i\tilde{\omega}_r^+ - \epsilon - \Sigma_c(i\omega_r^+))(i\tilde{\omega}_r^- - \epsilon - \Sigma_c(i\omega_r^-))} - (i\nu_n \rightarrow 0) \right]$$

Optical Conductivity: Details

$$\sigma(i\nu_n) = \left(\frac{ne^2}{m}\right) \frac{1}{\nu_n} \left[2\pi iT \sum_{|\omega_r| < \nu_n/2} \frac{1}{i\nu_n + i\Gamma - (\Sigma^+ - \Sigma^-)} \right], \quad (\nu_n > 0)$$

Optical Conductivity: Details

$$\sigma(i\nu_n) = \left(\frac{ne^2}{m}\right) \frac{1}{\nu_n} \left[2\pi iT \sum_{|\omega_r| < \nu_n/2} \frac{1}{i\nu_n + i\Gamma - (\Sigma^+ - \Sigma^-)} \right], \quad (\nu_n > 0)$$

$$\sigma(i\nu) = \left(\frac{ne^2}{m}\right) \frac{1}{\nu} \int_{-i\nu/2}^{i\nu/2} dz \left(\frac{1}{i\nu + i\Gamma - (\Sigma(z + i\nu/2) - \Sigma(z - i\nu/2))} \right),$$

Optical Conductivity: Details

$$\sigma(i\nu_n) = \left(\frac{ne^2}{m}\right) \frac{1}{\nu_n} \left[2\pi iT \sum_{|\omega_r| < \nu_n/2} \frac{1}{i\nu_n + i\Gamma - (\Sigma^+ - \Sigma^-)} \right], \quad (\nu_n > 0)$$

$$\sigma(i\nu) = \left(\frac{ne^2}{m}\right) \frac{1}{\nu} \int_{-i\nu/2}^{i\nu/2} dz \left(\frac{1}{i\nu + i\Gamma - (\Sigma(z + i\nu/2) - \Sigma(z - i\nu/2))} \right),$$

$$\int_{-i\nu/2}^{i\nu/2} dz \left(\frac{1}{i\nu - (\Sigma(z + i\nu/2) - \Sigma(z - i\nu/2))} \right) = \frac{i\nu}{i\tilde{\nu}} \left[1 - \frac{V^2}{i\tilde{\nu}(z_+ - z_-)} \left(\ln \left[\frac{\frac{i\nu}{2} - z_+}{-\frac{i\nu}{2} - z_+} \right] - \ln \left[\frac{\frac{i\nu}{2} - z_-}{-\frac{i\nu}{2} - z_-} \right] \right) \right]$$

$$z_{\pm} = \lambda \pm \sqrt{\left(\frac{i\nu}{2}\right)^2 - V^2[i\nu]}, \quad V^2[i\nu] = V^2 \frac{i\nu}{i\nu + i\Gamma}$$

Optical Conductivity: Details

$$\sigma(i\nu_n) = \left(\frac{ne^2}{m}\right) \frac{1}{\nu_n} \left[2\pi iT \sum_{|\omega_r| < \nu_n/2} \frac{1}{i\nu_n + i\Gamma - (\Sigma^+ - \Sigma^-)} \right], \quad (\nu_n > 0)$$

$$\sigma(i\nu) = \left(\frac{ne^2}{m}\right) \frac{1}{\nu_n} \int_{-i\nu/2}^{i\nu/2} dz \left(\frac{1}{i\nu + i\Gamma - (\Sigma(z + i\nu/2) - \Sigma(z - i\nu/2))} \right),$$

$$\int_{-i\nu/2}^{i\nu/2} dz \left(\frac{1}{i\nu - (\Sigma(z + i\nu/2) - \Sigma(z - i\nu/2))} \right) = \frac{i\nu}{i\tilde{\nu}} \left[1 - \frac{V^2}{i\tilde{\nu}(z_+ - z_-)} \left(\ln \left[\frac{\frac{i\nu}{2} - z_+}{-\frac{i\nu}{2} - z_+} \right] - \ln \left[\frac{\frac{i\nu}{2} - z_-}{-\frac{i\nu}{2} - z_-} \right] \right) \right]$$

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$$\sigma(i\nu) = \left(\frac{ne^2}{m}\right) \frac{1}{\Gamma - i(i\nu)} \left[1 - \frac{V^2}{i(\nu + \Gamma)(z_+ - z_-)} \left(\ln \left[\frac{\frac{i\nu}{2} - z_+}{-\frac{i\nu}{2} - z_+} \right] - \ln \left[\frac{\frac{i\nu}{2} - z_-}{-\frac{i\nu}{2} - z_-} \right] \right) \right]$$

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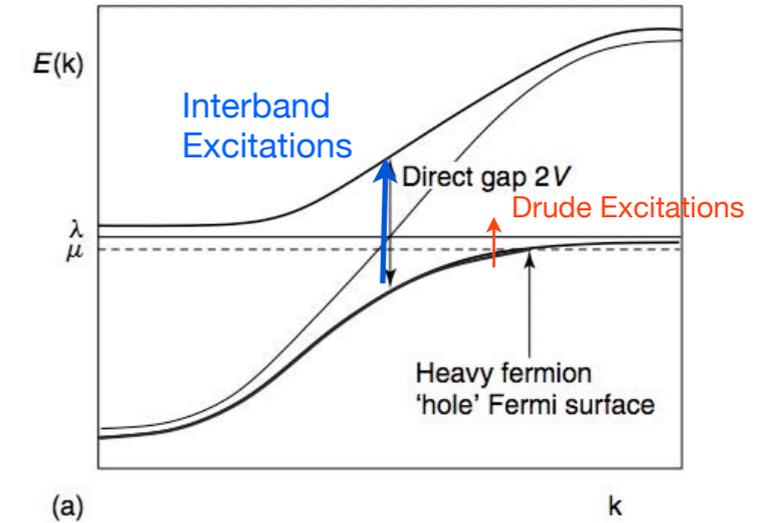
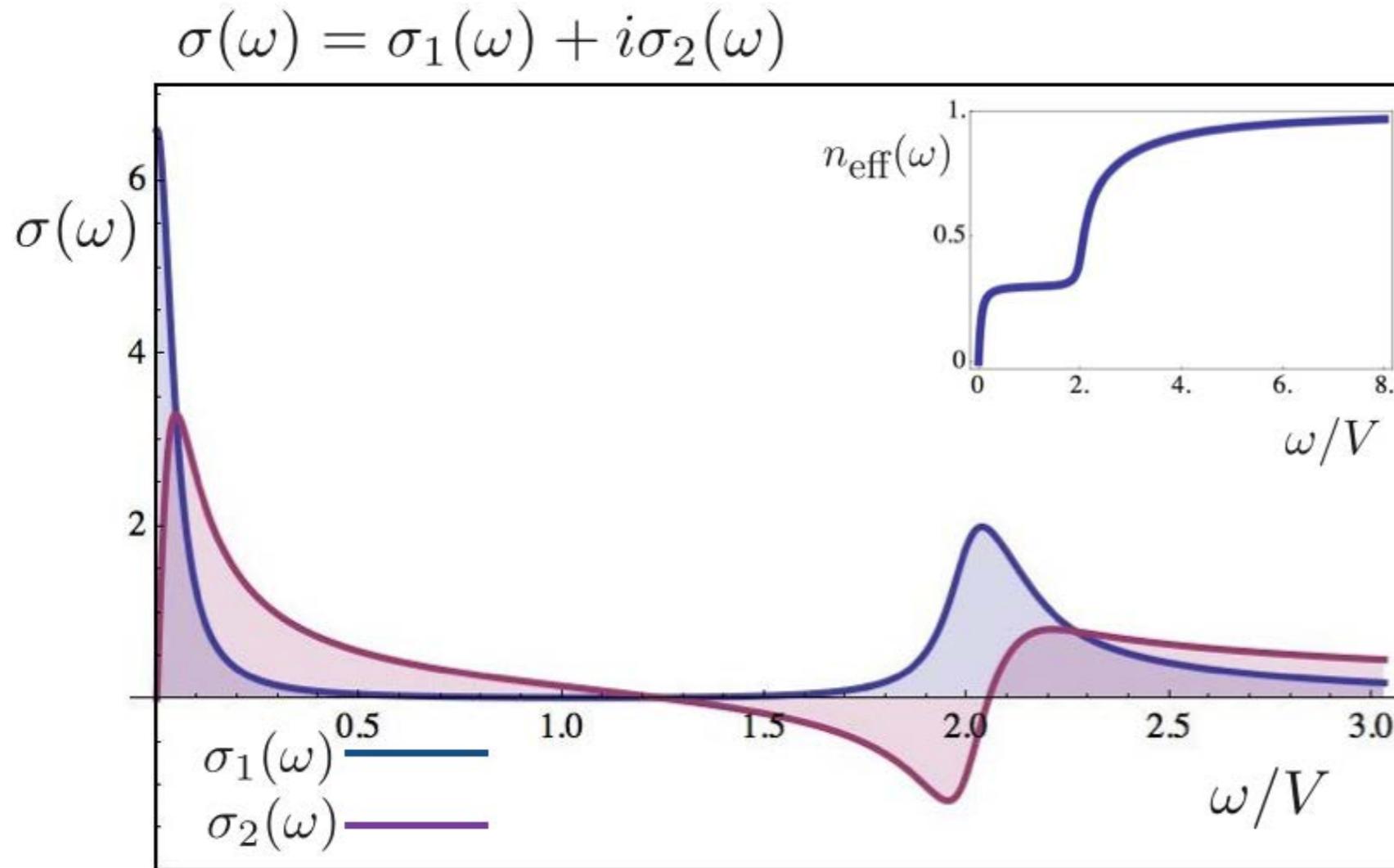
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$$\sigma(\omega + i\delta) = \left(\frac{ne^2}{m}\right) \frac{1}{\Gamma - i\omega} \left[1 + \frac{V^2}{(\omega + i\Gamma)(z_+ - z_-)} \left(\ln \left[\frac{z_+ + \frac{\omega}{2}}{z_+ - \frac{\omega}{2}} \right] - \ln \left[\frac{z_- + \frac{\omega}{2}}{z_- - \frac{\omega}{2}} \right] \right) \right],$$

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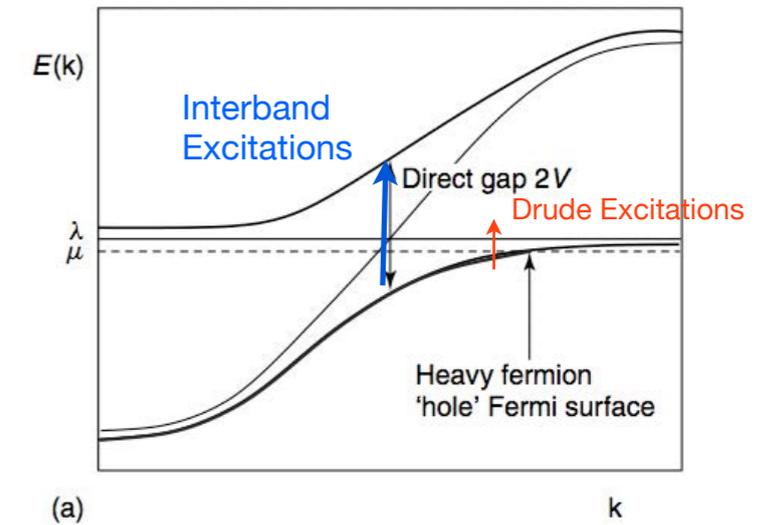
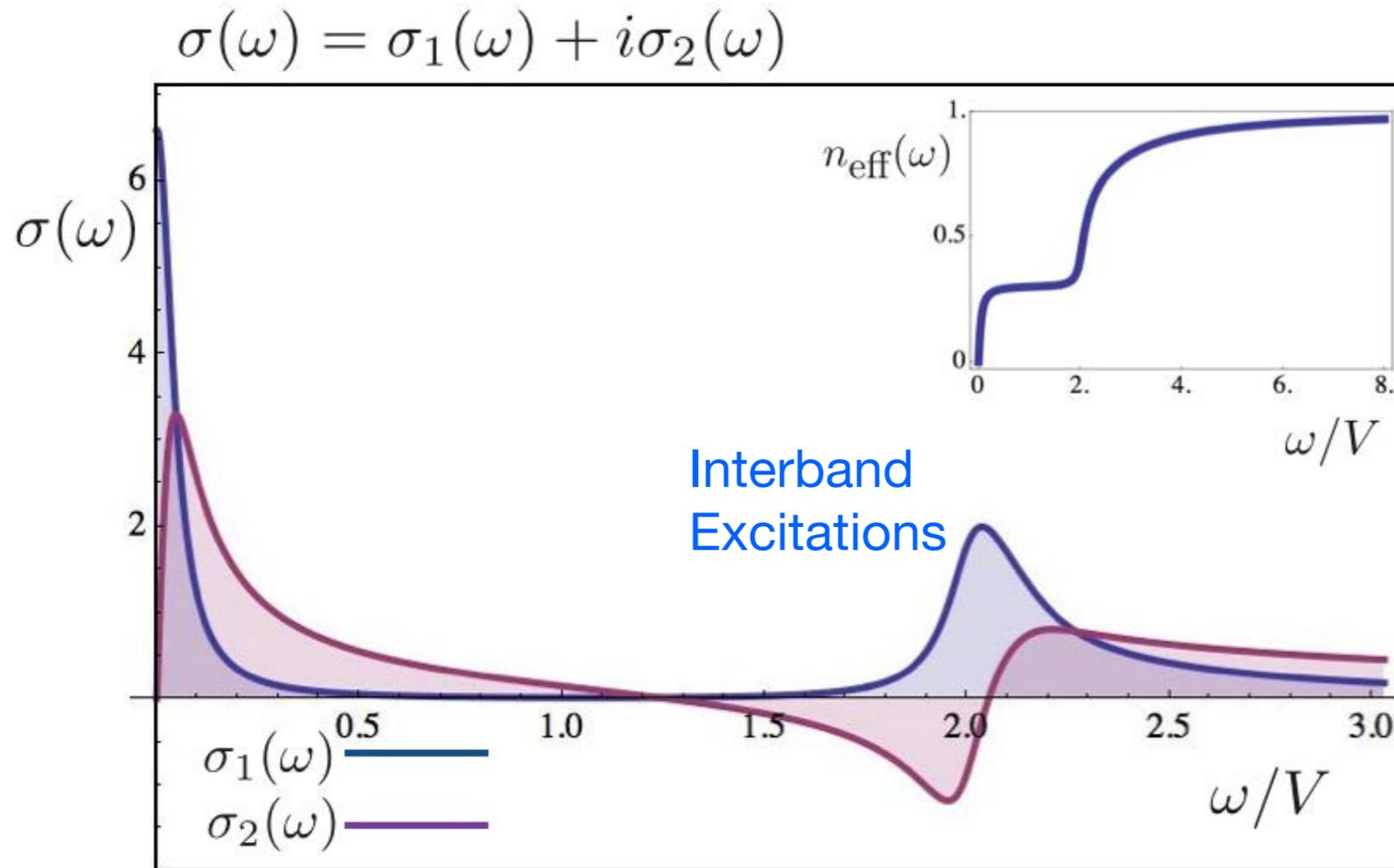
Optical Conductivity: Details



$$\sigma(\omega + i\delta) = \left(\frac{ne^2}{m}\right) \frac{1}{\Gamma - i\omega} \left[1 + \frac{V^2}{(\omega + i\Gamma)(z_+ - z_-)} \left(\ln \left[\frac{z_+ + \frac{\omega}{2}}{z_+ - \frac{\omega}{2}} \right] - \ln \left[\frac{z_- + \frac{\omega}{2}}{z_- - \frac{\omega}{2}} \right] \right) \right],$$

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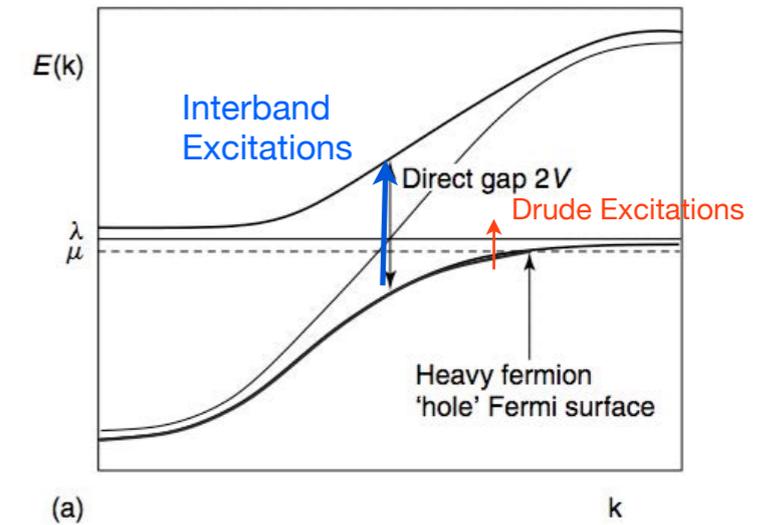
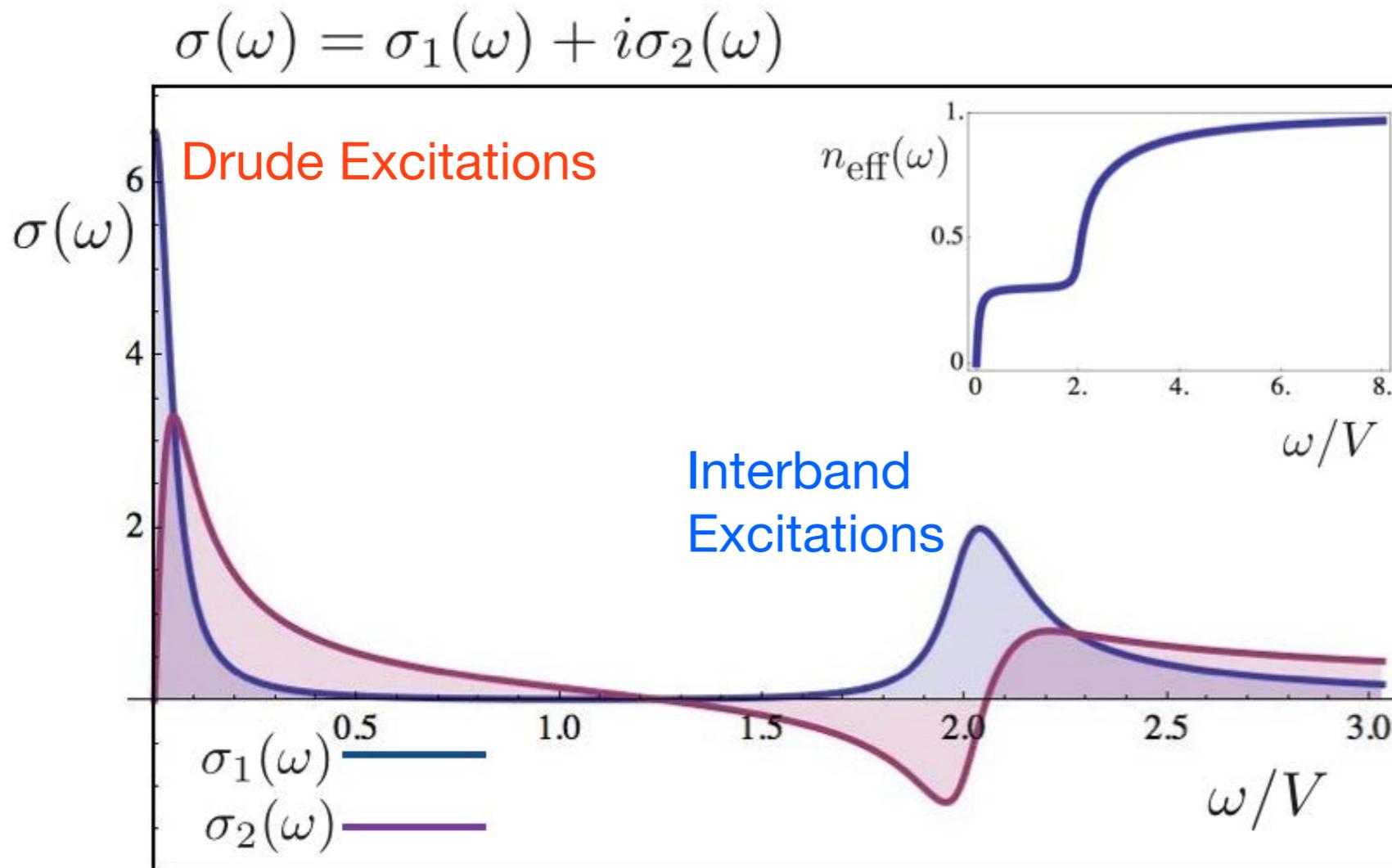
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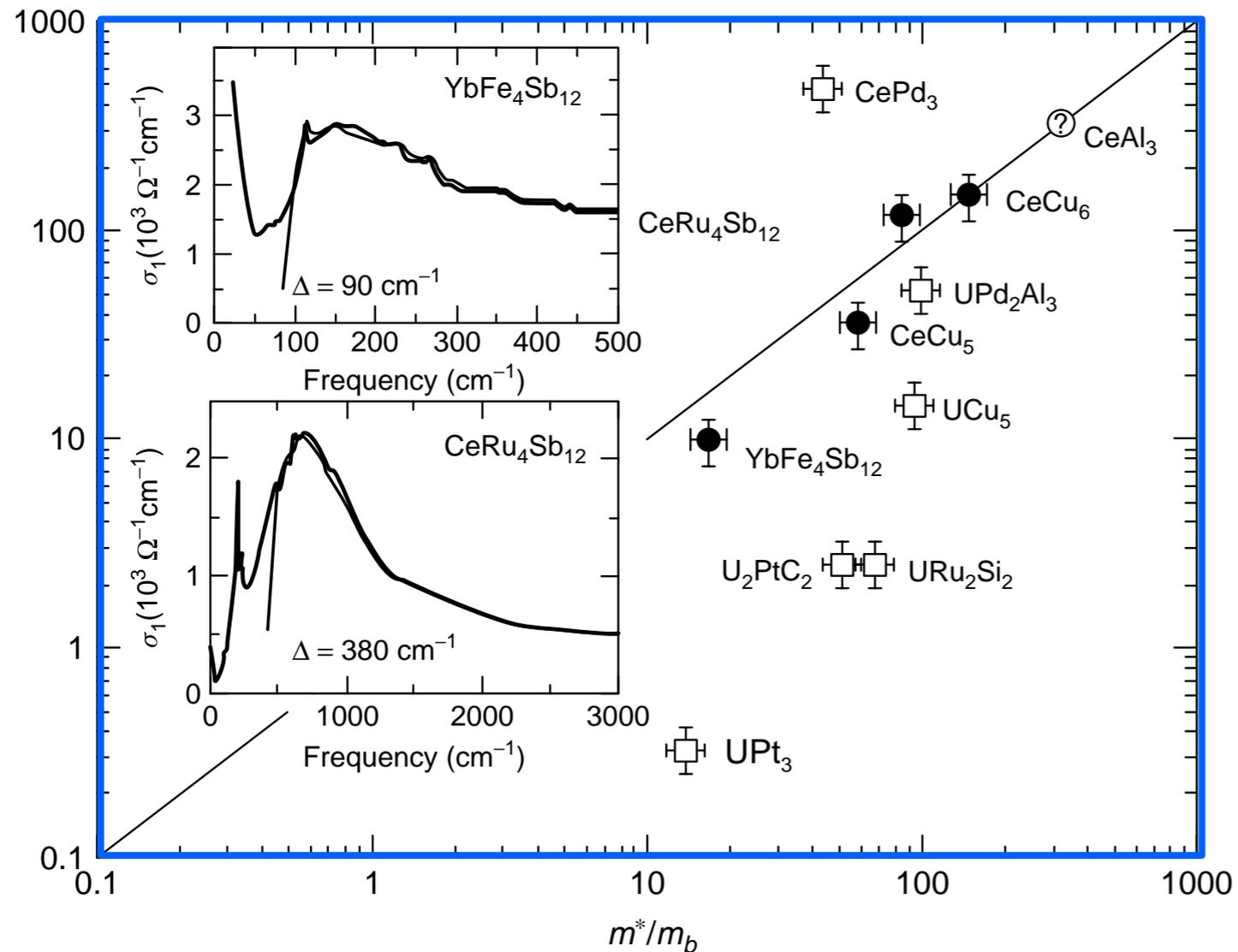


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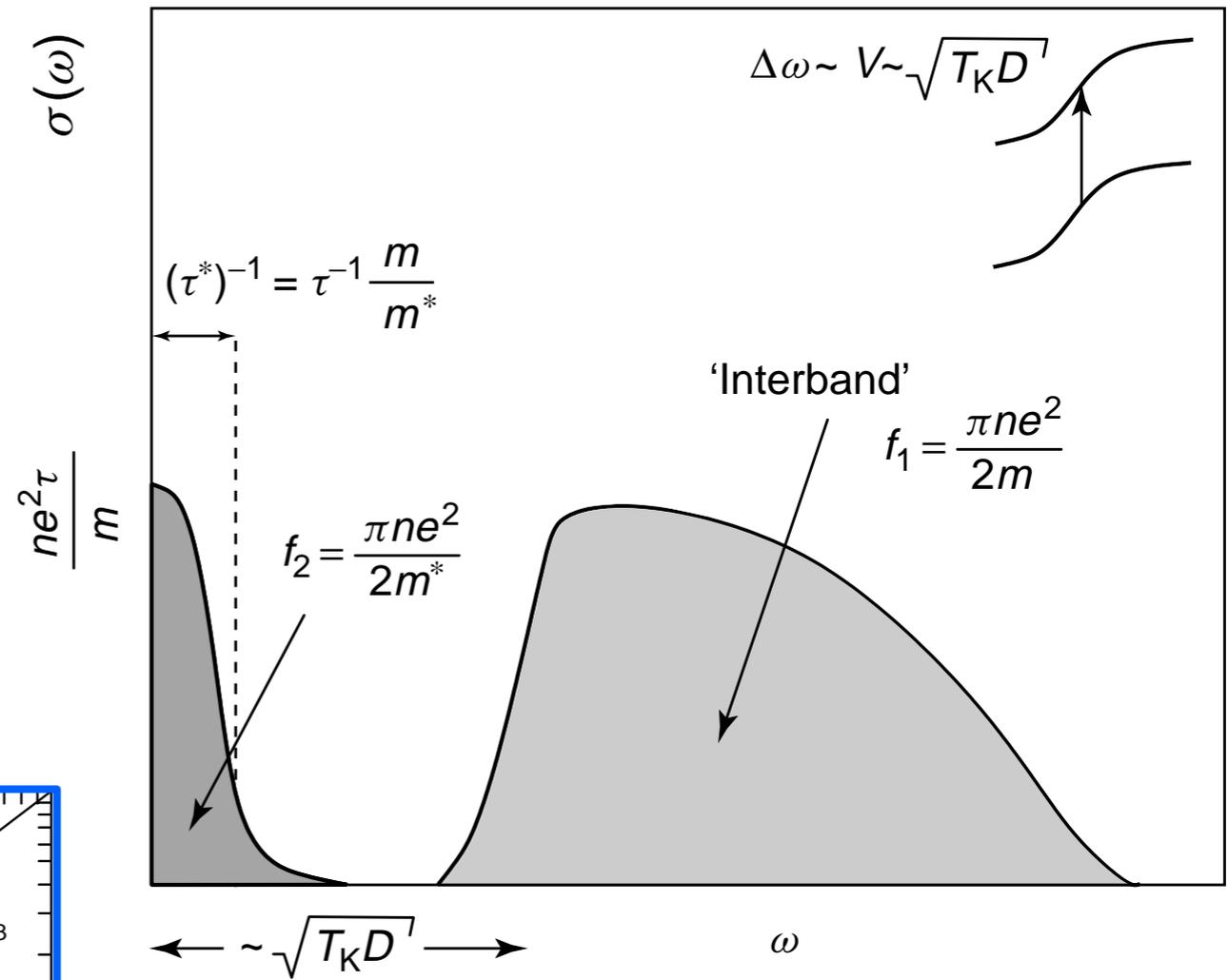
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Optical Conductivity.

S.V. Dordevic, D.N. Basov, N.R. Dilley, E.D. Bauer, and M.B. Maple,
Phys. Rev. Lett. **86**, 2001, 684,



Millis and Lee, 1987



$$V^2 / D \sim T_K \Rightarrow V \sim \sqrt{T_K D}$$

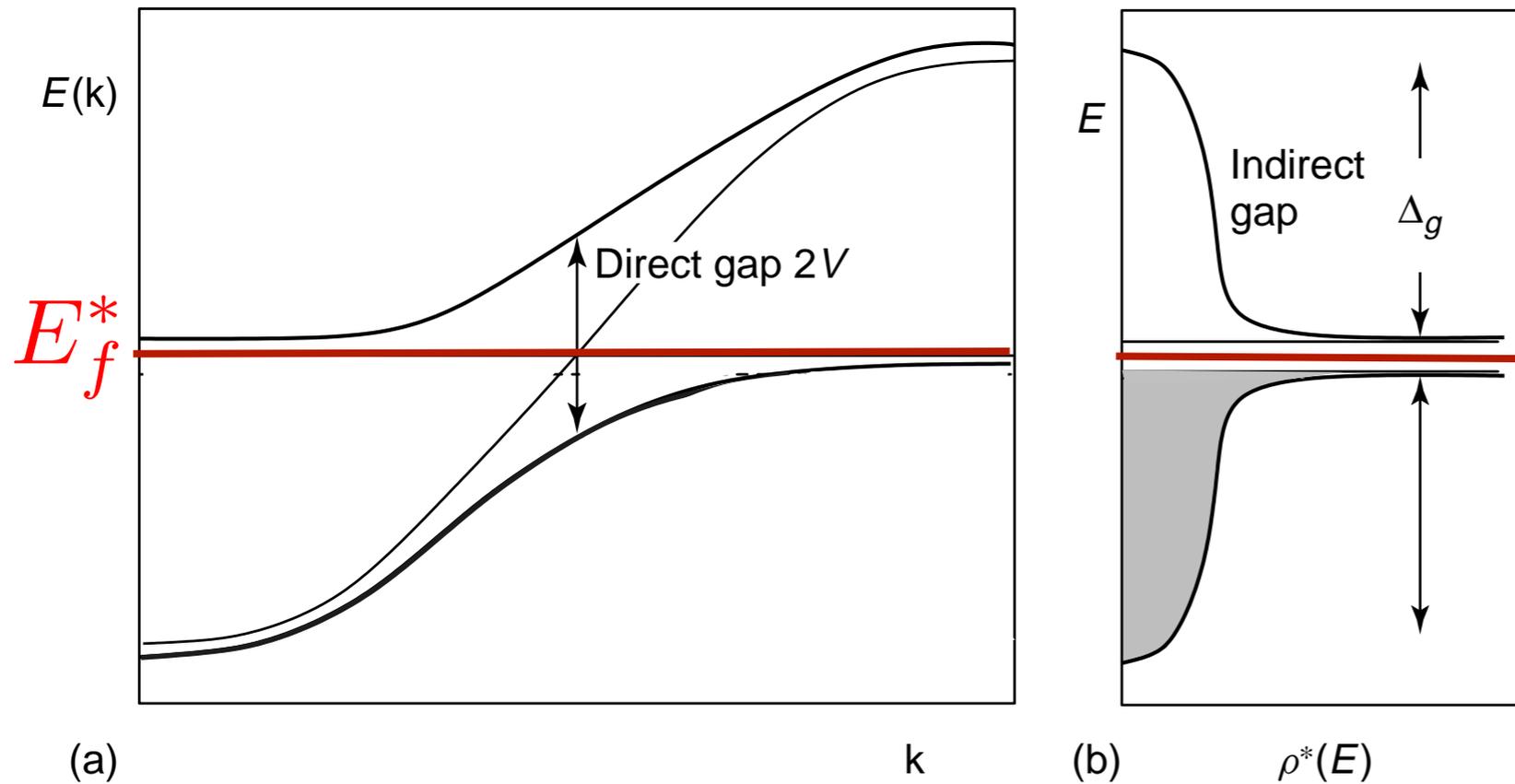
$$\int_0^{\infty} \frac{d\omega}{\pi} \sigma(\omega) = f_1 = \frac{\pi}{2} \left(\frac{n_c e^2}{m} \right)$$

$$\int_0^{\sim V} d\omega \sigma(\omega) = f_2 = \frac{\pi}{2} \frac{n_{\text{HF}} e^2}{m^*}$$

Physics of Heavy Fermion Superconductivity Lecture II:

1. The large N approach to the Kondo lattice.
2. Heavy Fermion Metals.
3. Optical Conductivity of Heavy Fermion Metals
4. Kondo Insulators

“Kondo Insulator”



Mott Phil Mag, 30,403,1974



pressure are discussed. It is suggested that the low-pressure form of SmS is an excitonic insulator. In SmB₆ and high-pressure SmS a very small gap separates occupied from unoccupied states, this in our view being due to hybridization of 4f and 5d bands. The electrical properties are discussed ; if kT is greater than the gap

energy, then the gap does not affect the metallic behaviour. Finally metallic compounds such as CeAl₃ are described, in which there is no magnetic ordering at

MAGNETIC AND SEMICONDUCTING PROPERTIES OF SmB_6

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and

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Department of Applied Physics, Stanford University, Stanford, California,
and Bell Telephone Laboratories, Murray Hill, New Jersey

(Received 21 November 1968)

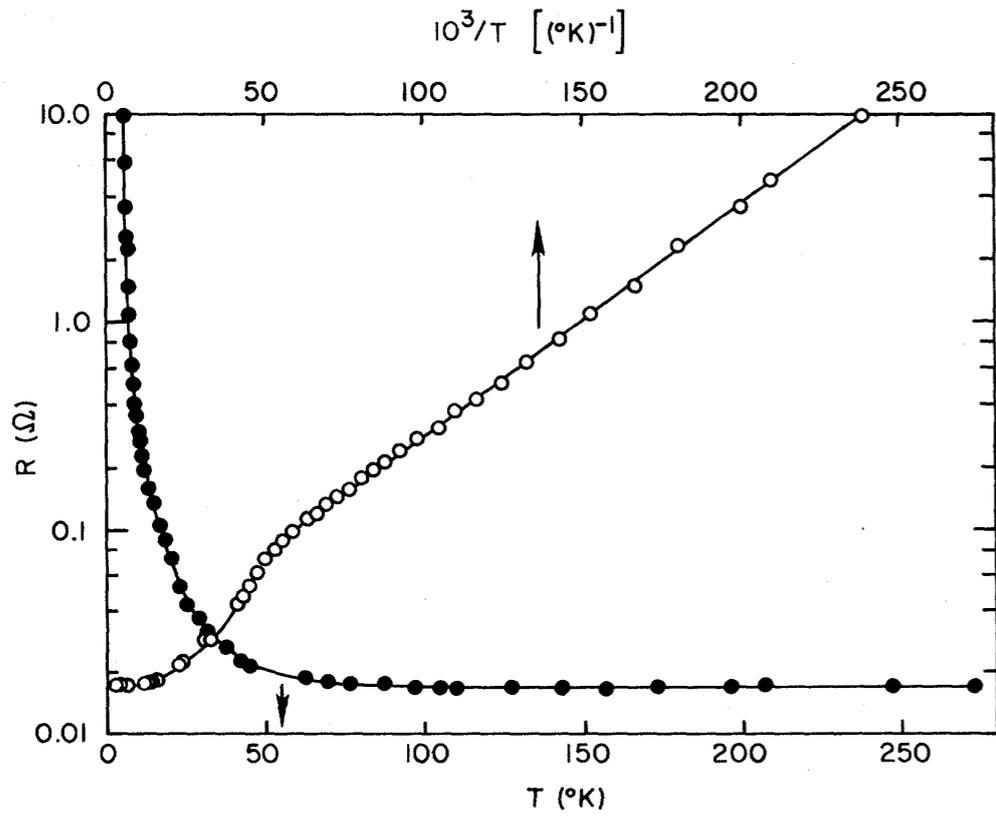


FIG. 1. Resistance of SmB_6 as a function of temperature. Closed circles: resistance versus T ; open circles: resistance versus $10^3/T$.

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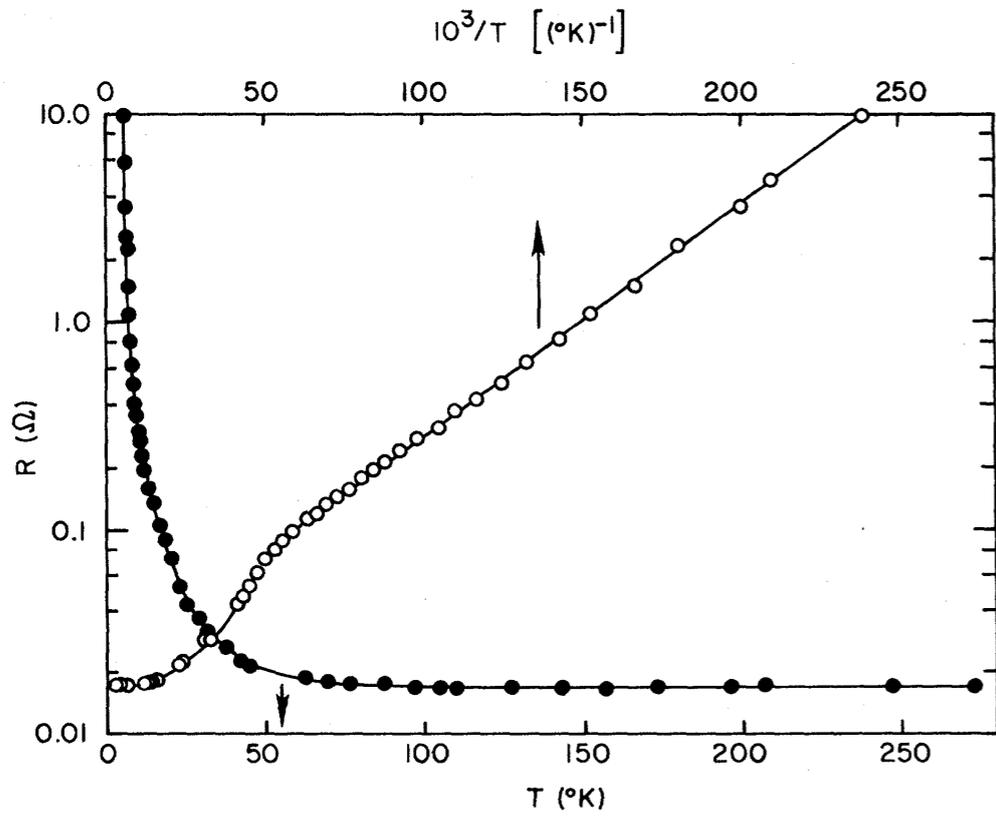
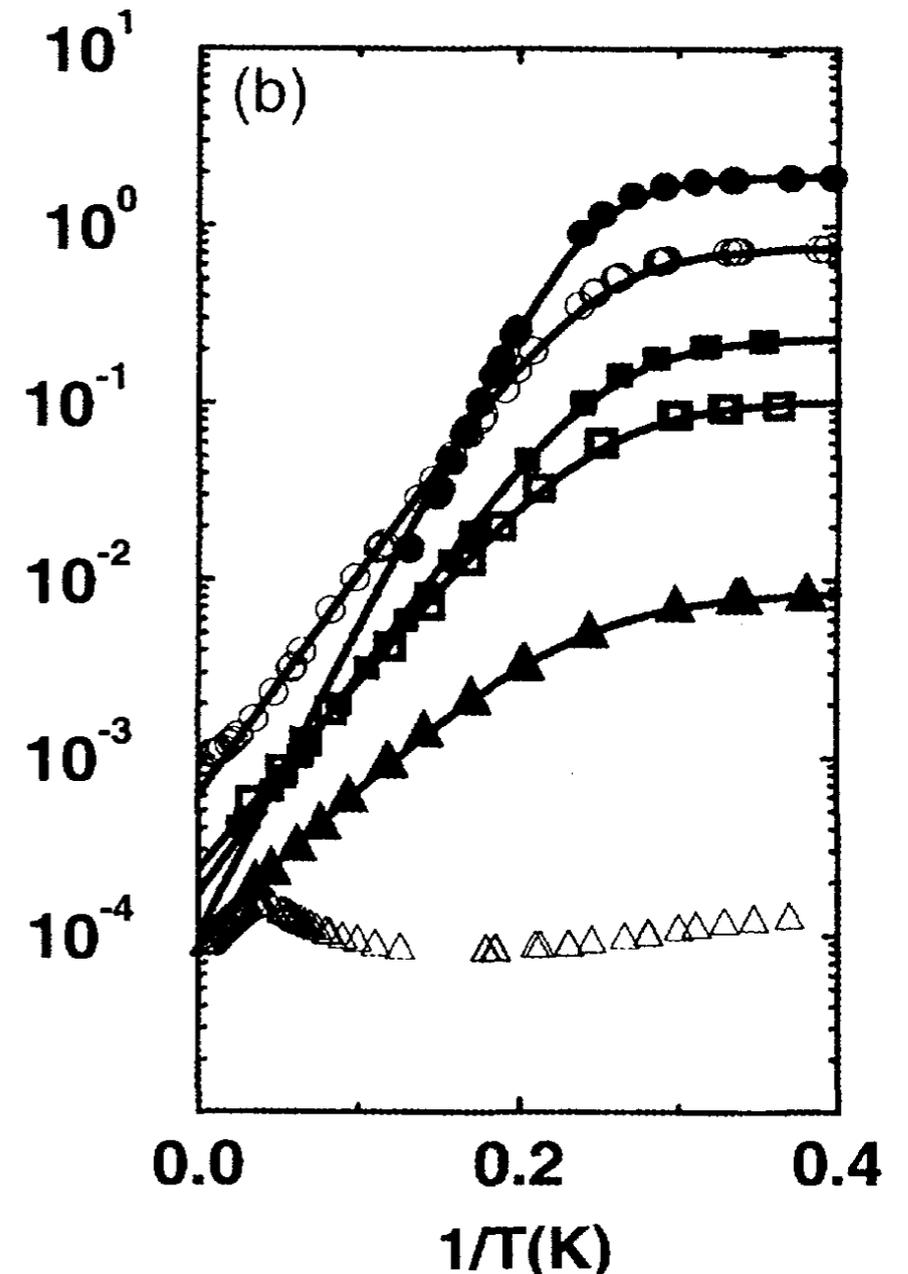


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Persistent conductivity
Plateau



But what about Superconductivity?

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Frustrated magnetism: pairing of spinons $SP(N)$.

Read and Sachdev, PRL, 66, 1773 (1991)

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“Symplectic Large N” R. Flint and PC '08

$$S^{ba} = f_b^\dagger f_a - \text{sgn}(a)\text{sgn}(b) f_{-b}^\dagger f_{-a}$$

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