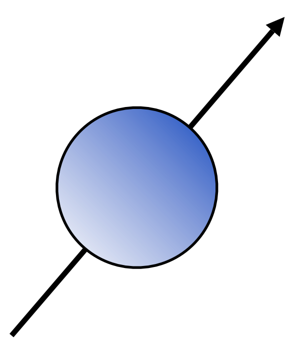
$$\overrightarrow{M} = g\mu_B \overrightarrow{S}$$



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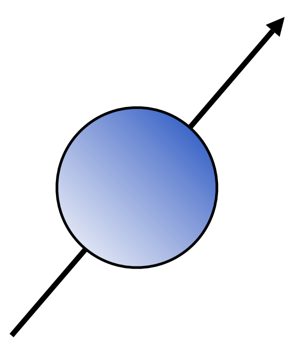
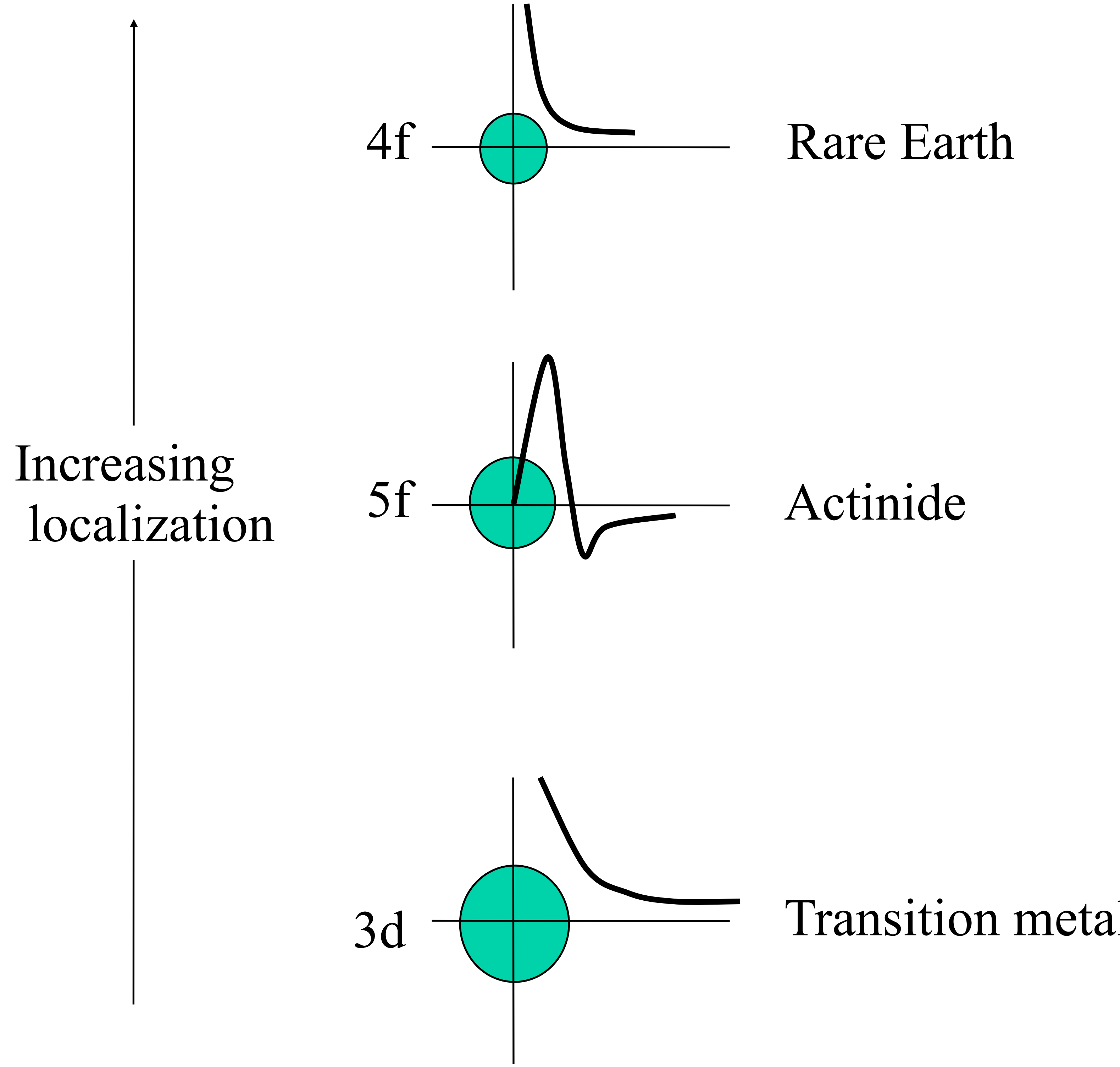


Localized Moment

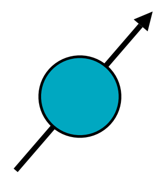


Outline:

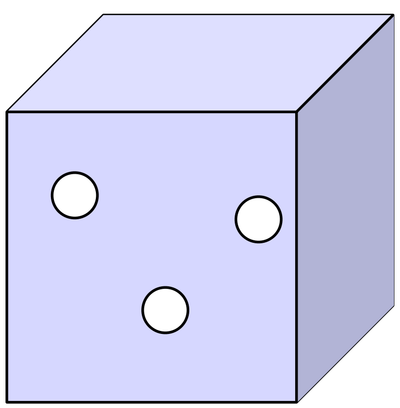
1. Chemistry of Kondo.
2. Key Properties.
3. Tour of Heavy Fermions.
4. Anderson model: the atomic limit.
5. Adiabaticity: the heavy Fermi liquid.
6. Scaling: the Kondo effect.
7. Doniach's Kondo Lattice Concept.

Moment Formation

$$\overrightarrow{M} = g\mu_B \vec{S}$$



Fe



$x<0.4$

Moment Formation

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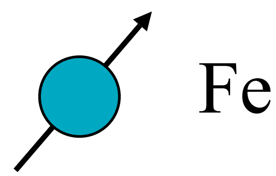

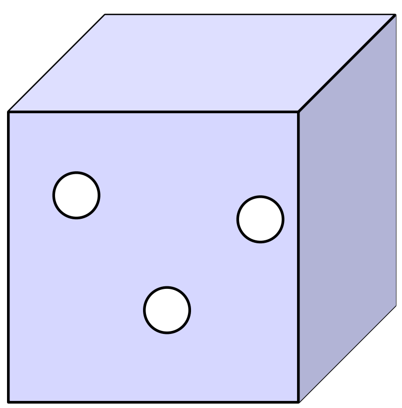

x<0.4

$$Nb_{1-x}Mo_x$$

Clogston, Mathias et al, 1962

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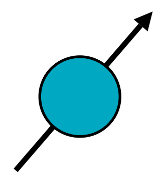

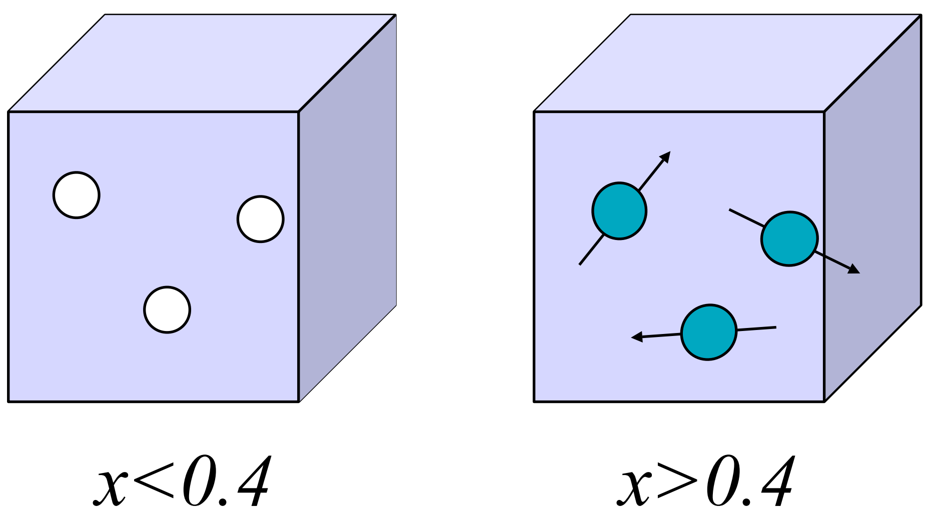

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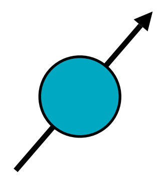

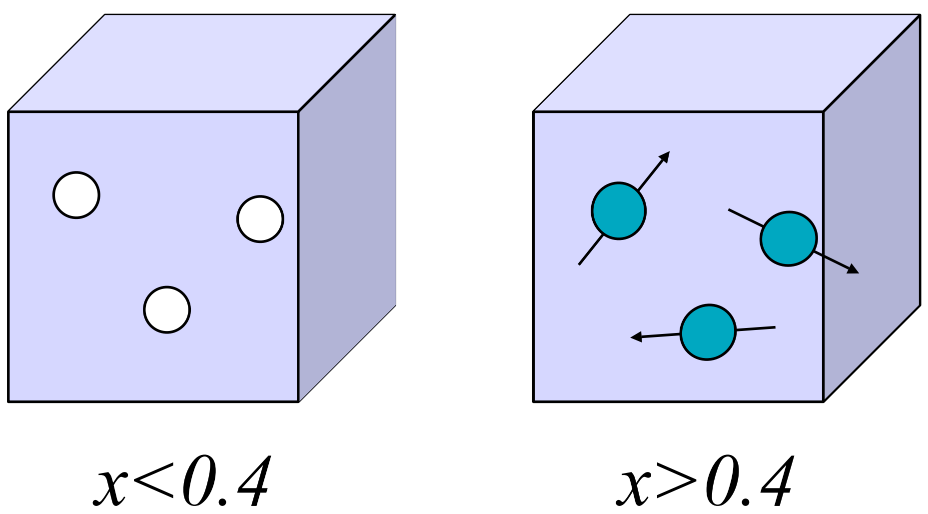

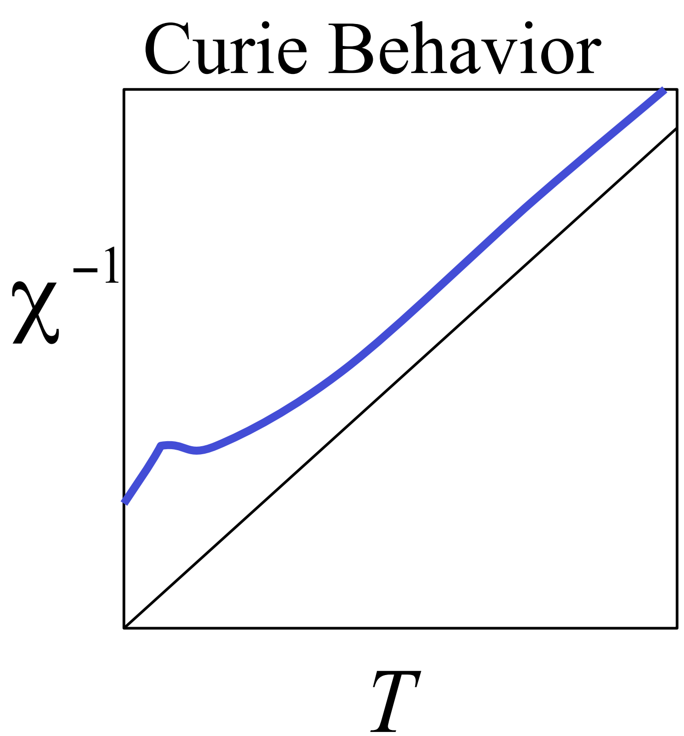

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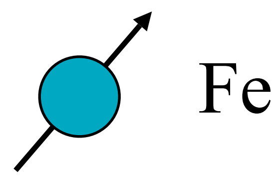
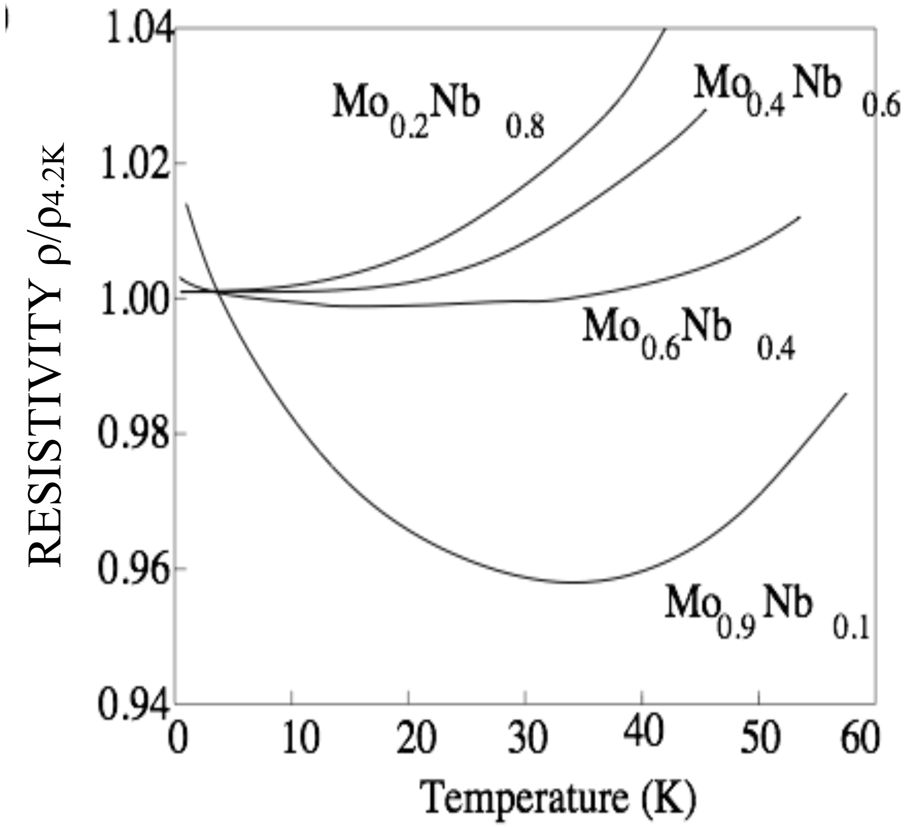
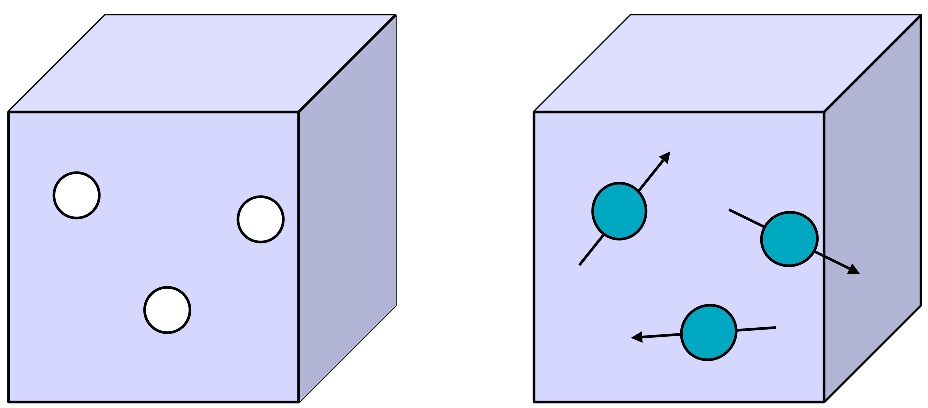
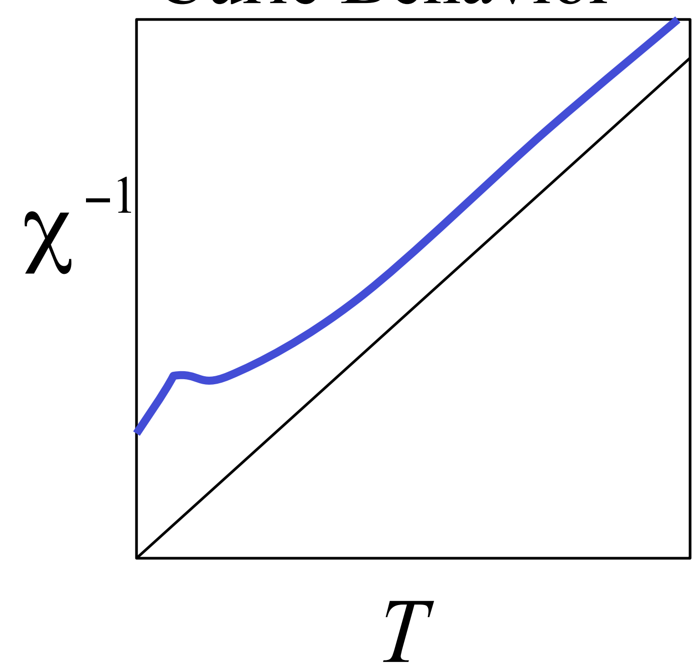






x<0.4 *x>0.4*

Curie Behavior



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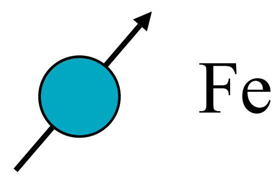

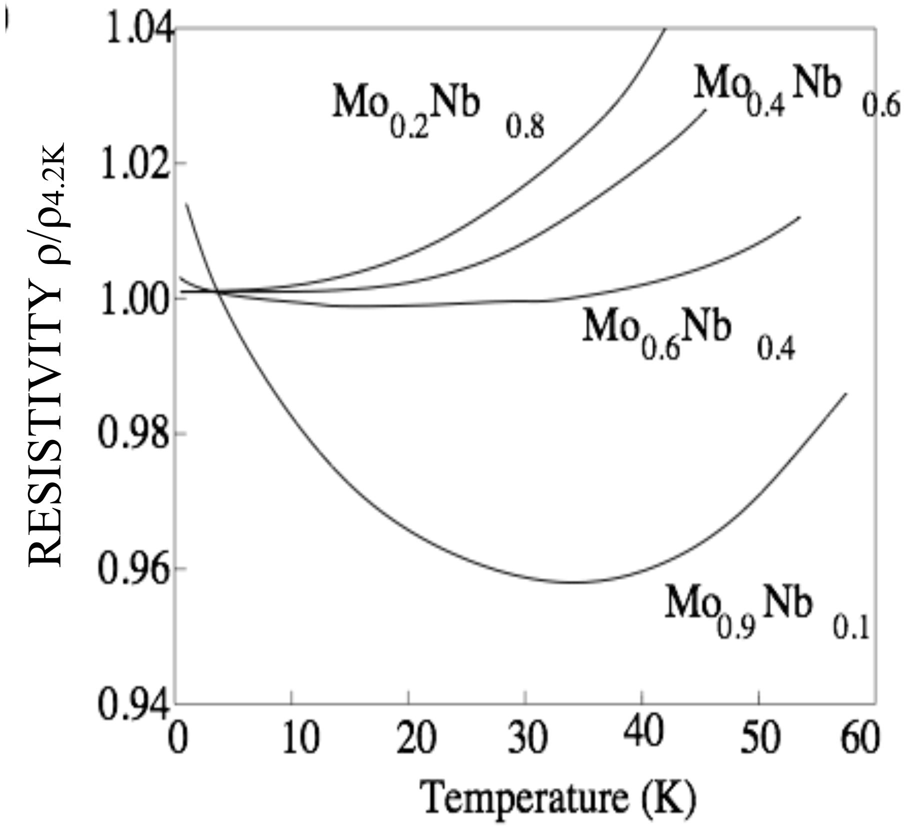

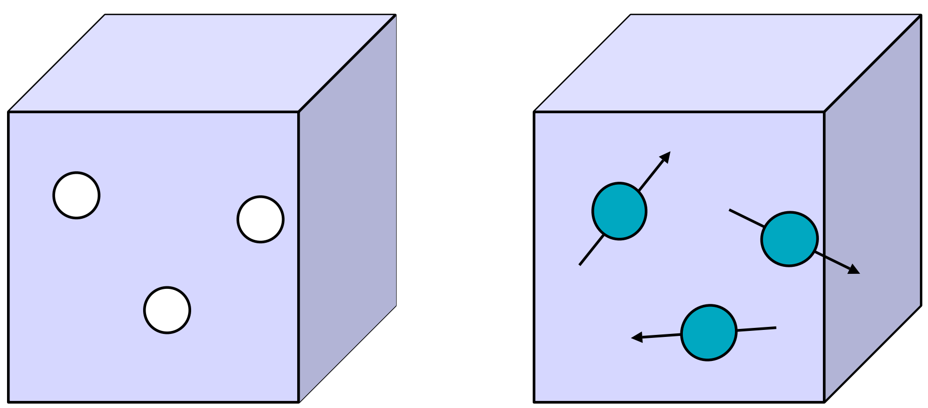

$x<0.4$ $x>0.4$

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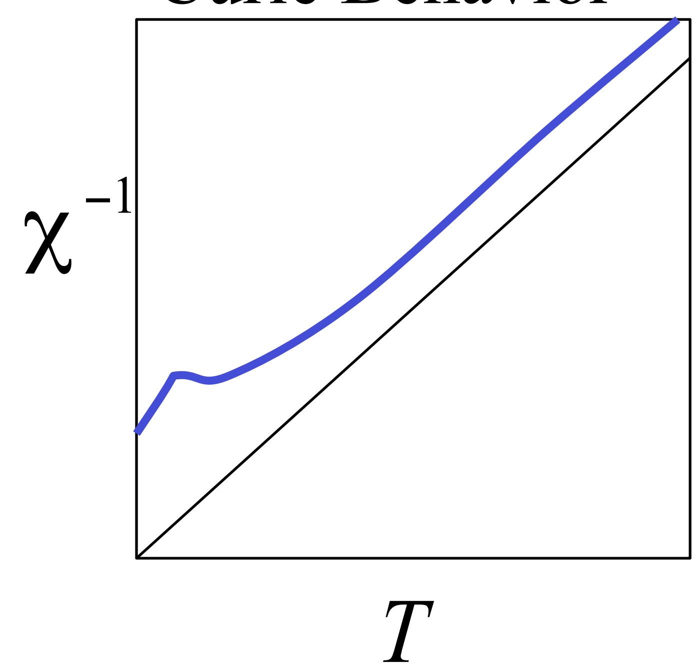
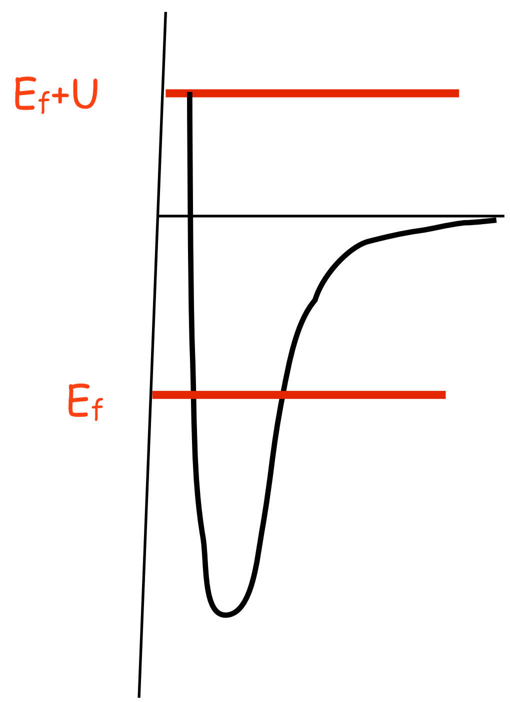


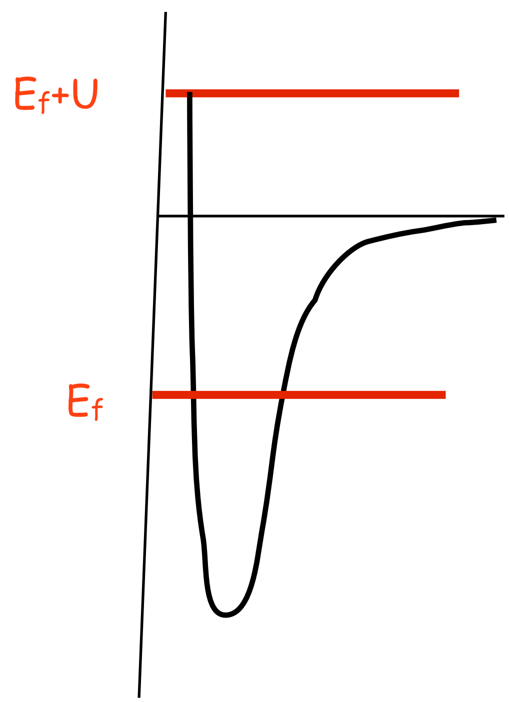
How?

Anderson Model



Anderson Model

$$H = \underbrace{E_f n_f + U n_{f\uparrow} n_{f\downarrow}}_{H_{atomic}}$$



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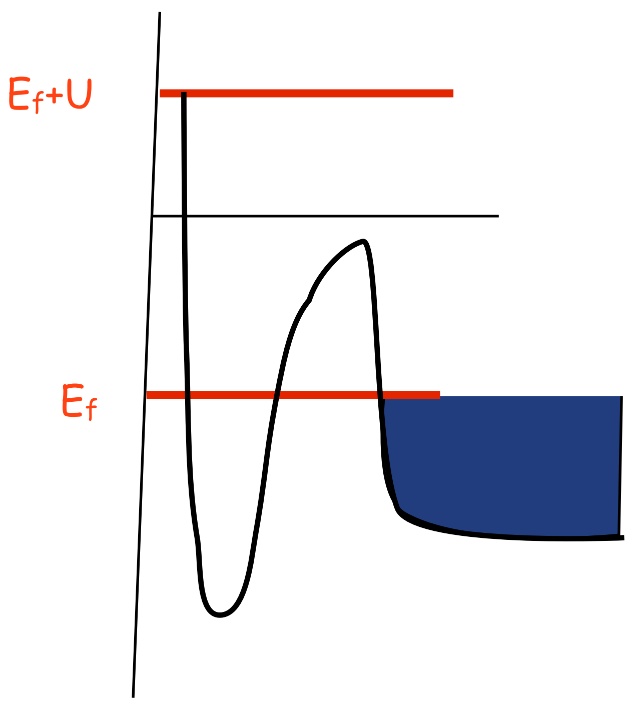
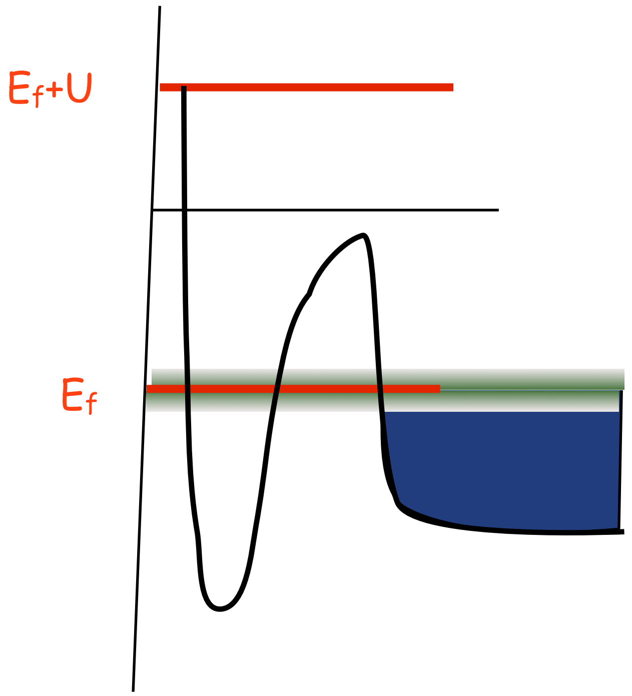


Anderson Model

$$H = \overbrace{\sum_{k,\sigma} \epsilon_k n_{k\sigma} + \sum_{k,\sigma} V(k) \left[c^\dagger_{k\sigma} f_\sigma + f^\dagger_\sigma c_{k\sigma} \right]}^{H_{resonance}} + \underbrace{E_f n_f + U n_{f\uparrow} n_{f\downarrow}}_{H_{atomic}}$$



Anderson Model

$$V(k) = \langle k|V_{atomic}|f\rangle = 4\pi i^l \int_0^\infty r^2 dr j_l(kr)V(r)R_f(r)$$

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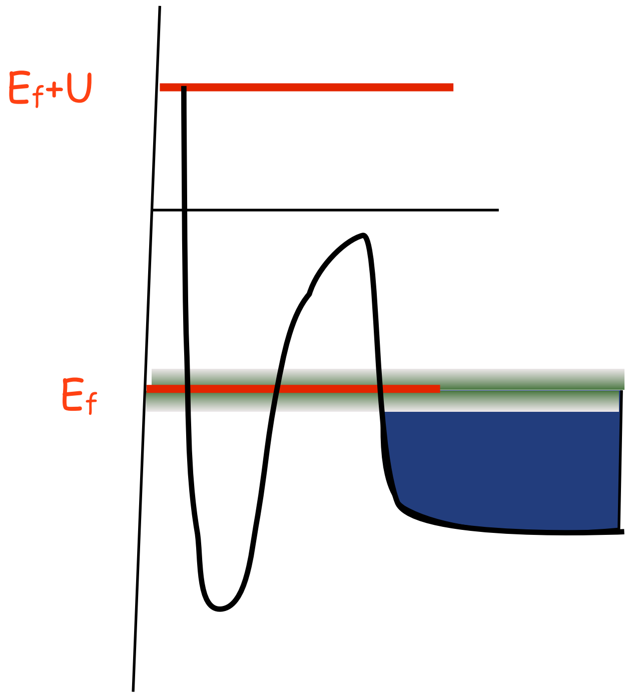

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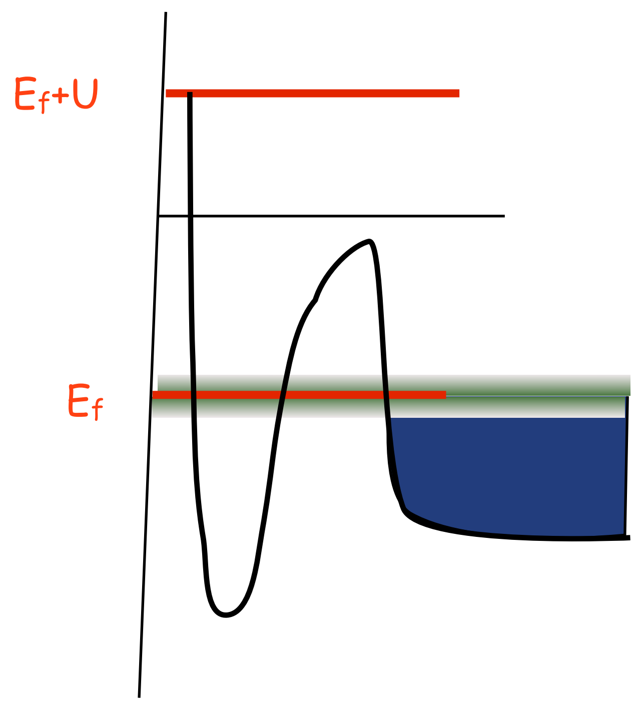

$$U = \int d^3x d^3x' V(\mathbf{x} - \mathbf{x}') |\psi_f(\mathbf{x})|^2 |\psi_f(\mathbf{x}')|^2$$

- Atomic approach : Start with V(k)=0, then dial up the hybridization

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Local moment, states at Ef and Ef+U

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Friedel-Abrikosov-Suhl Resonance.

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Local moment, states at Ef and Ef+U

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Friedel-Abrikosov-Suhl Resonance.

How to reconcile the two approaches?

Anderson Model of Moment Formation

$$H_{atomic} = E_d n_d + U n_{d\uparrow} n_{d\downarrow}$$

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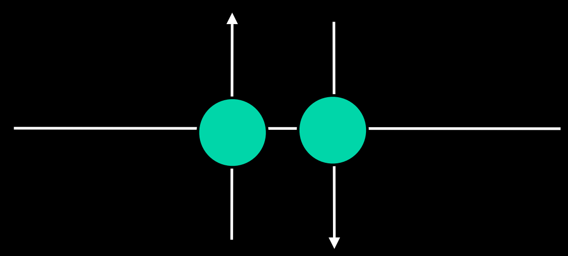
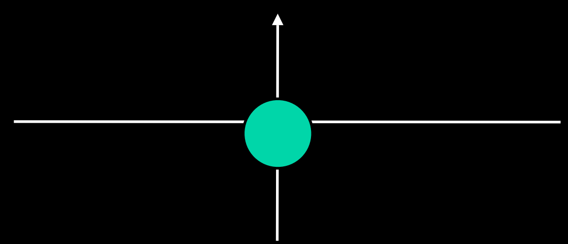


$E_d + U$



E_d

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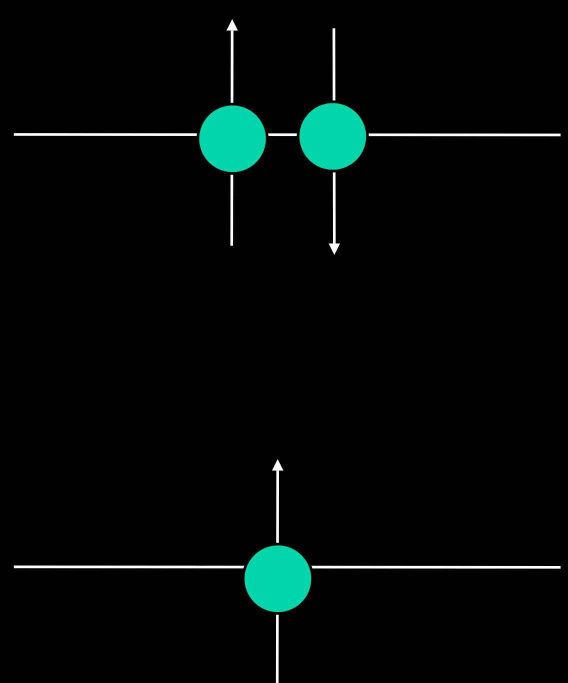

$E_d + U$

E_d

$$U/2 > \left| E_d + U/2 \right|$$

Anderson Model of Moment Formation

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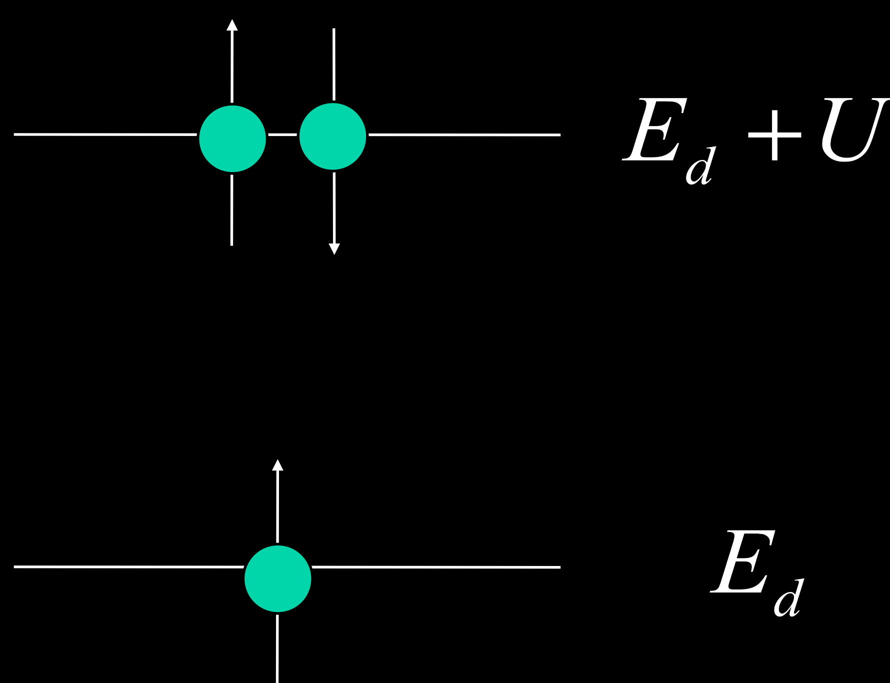

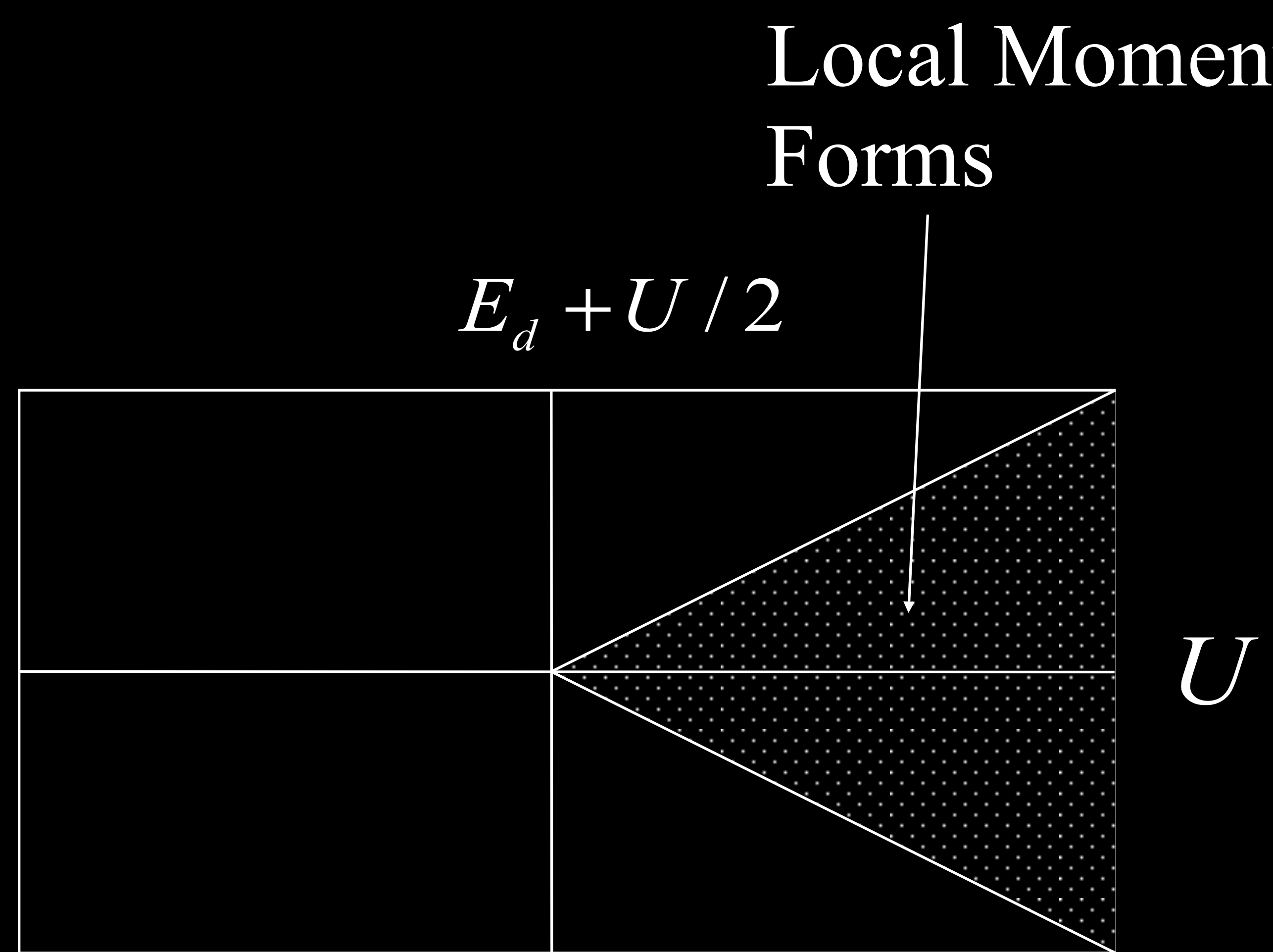

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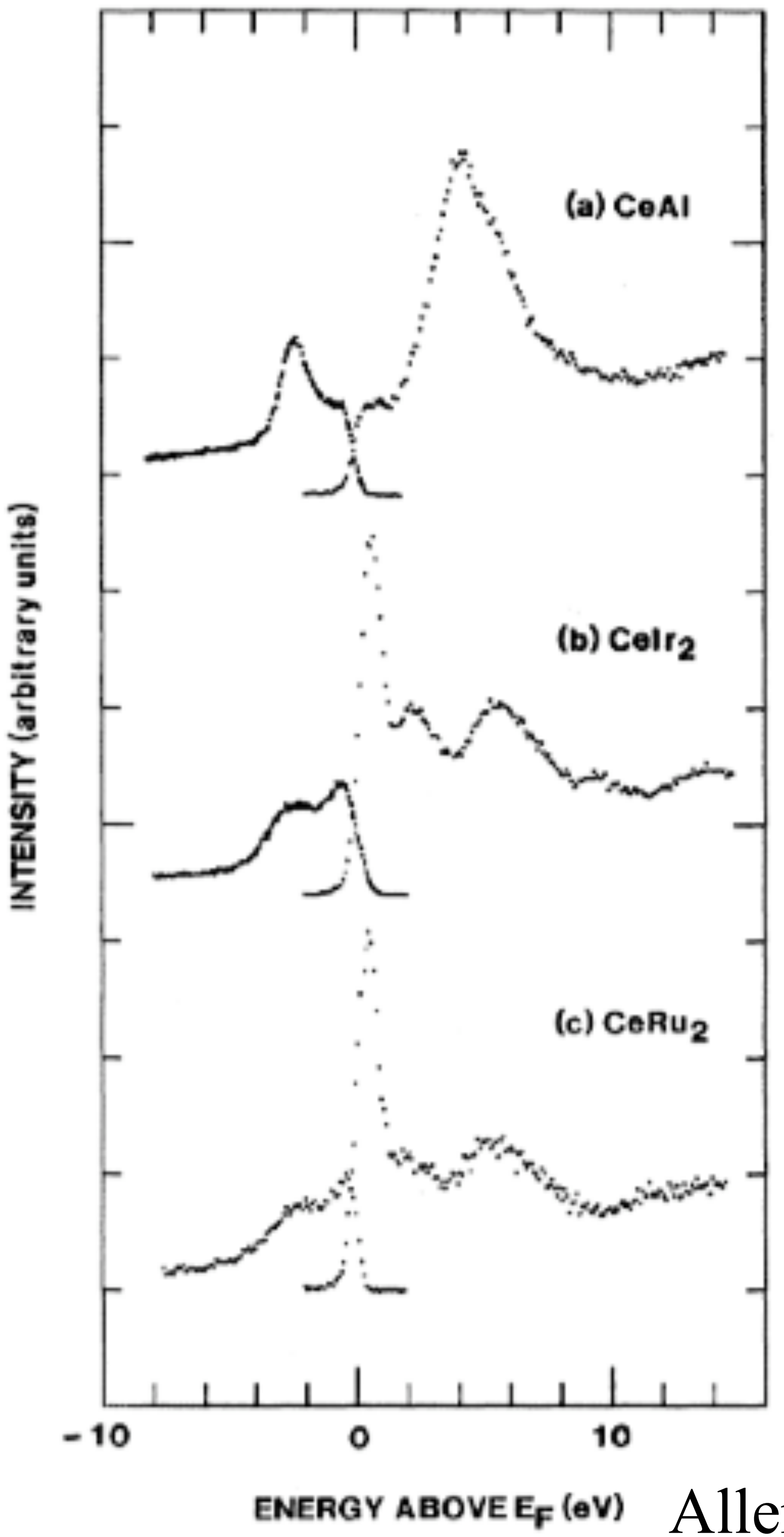
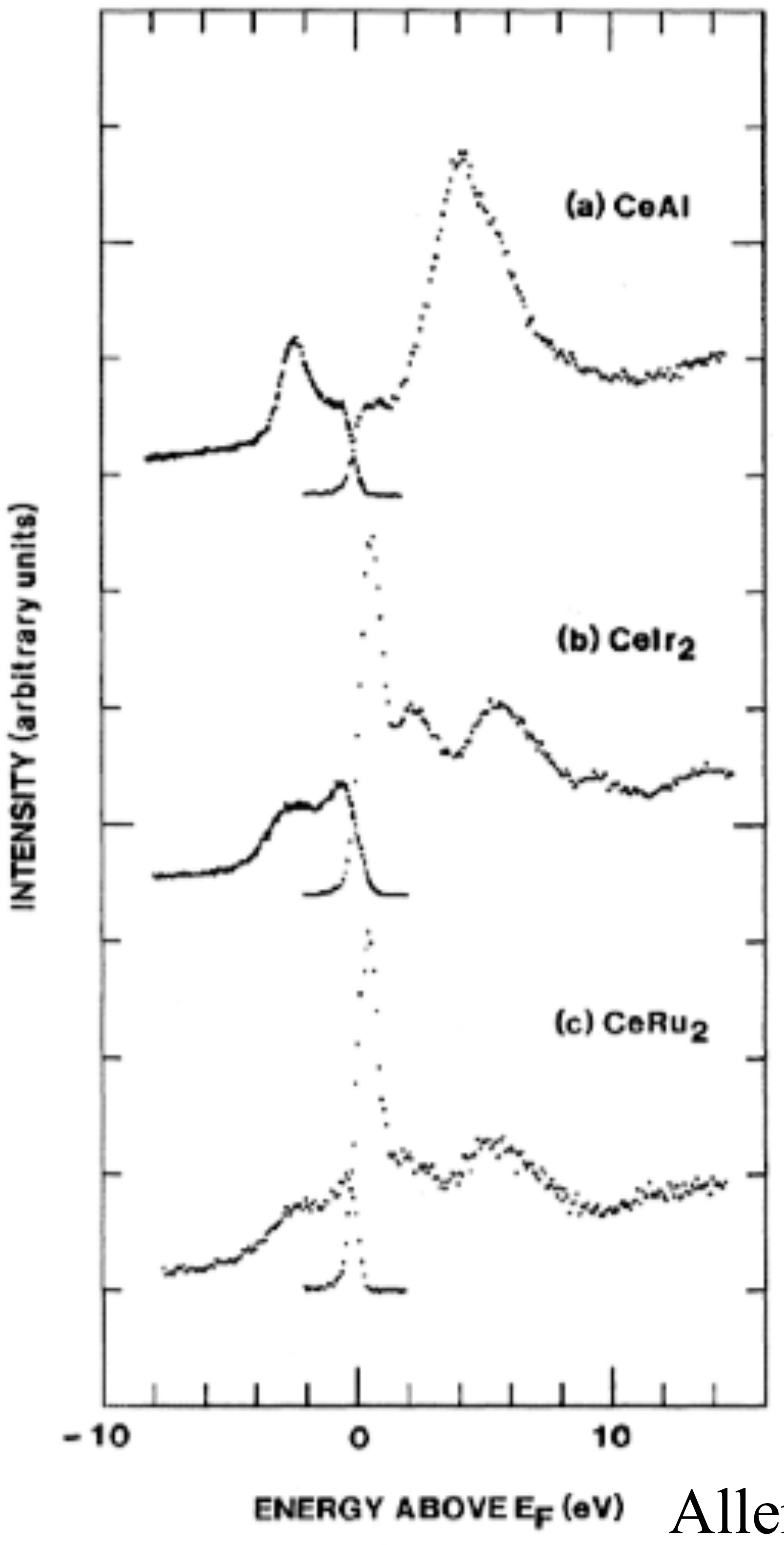


$$A_f(\omega) = \frac{1}{\pi}\mathrm{Im}G_f(\omega - i\delta)$$

Allen et al (1983)



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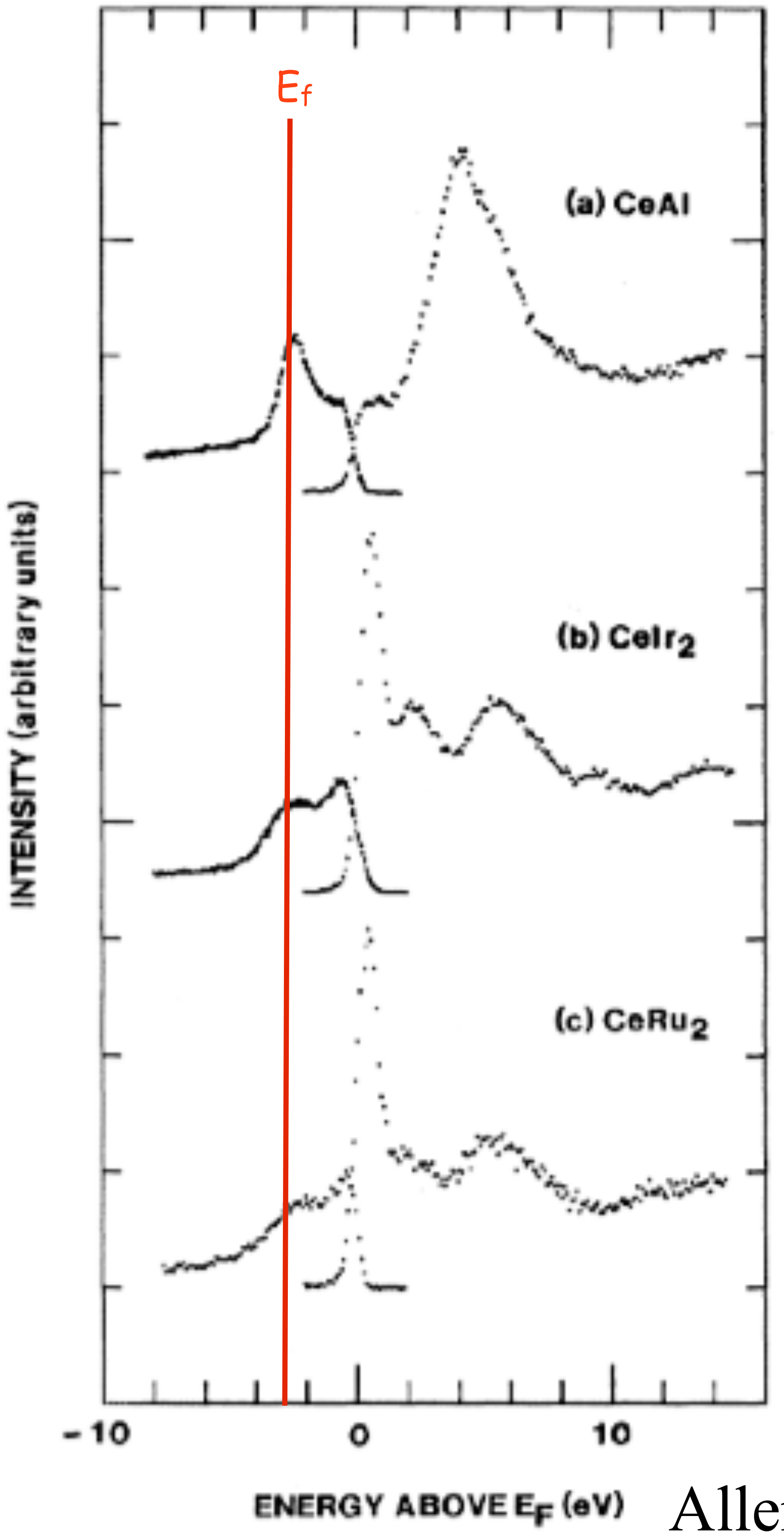
$$A_f(\omega) = \frac{1}{\pi}\mathrm{Im}G_f(\omega - i\delta)$$

$$G_f(\omega) = -i\int_{-\infty}^{\infty} dt\langle \mathrm{T} f_\sigma(t) f^\dagger_\sigma(0)\rangle e^{i\omega t}$$



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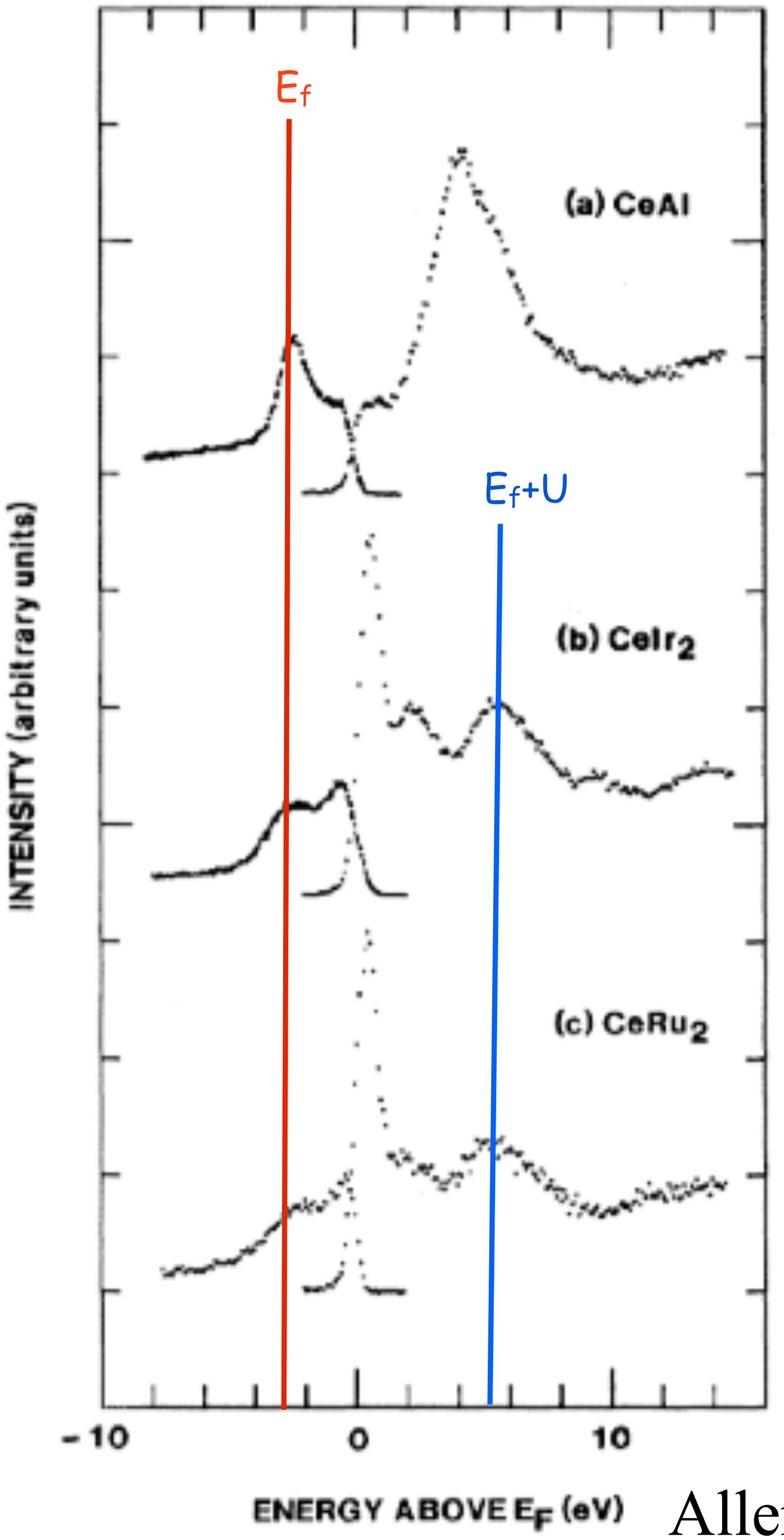

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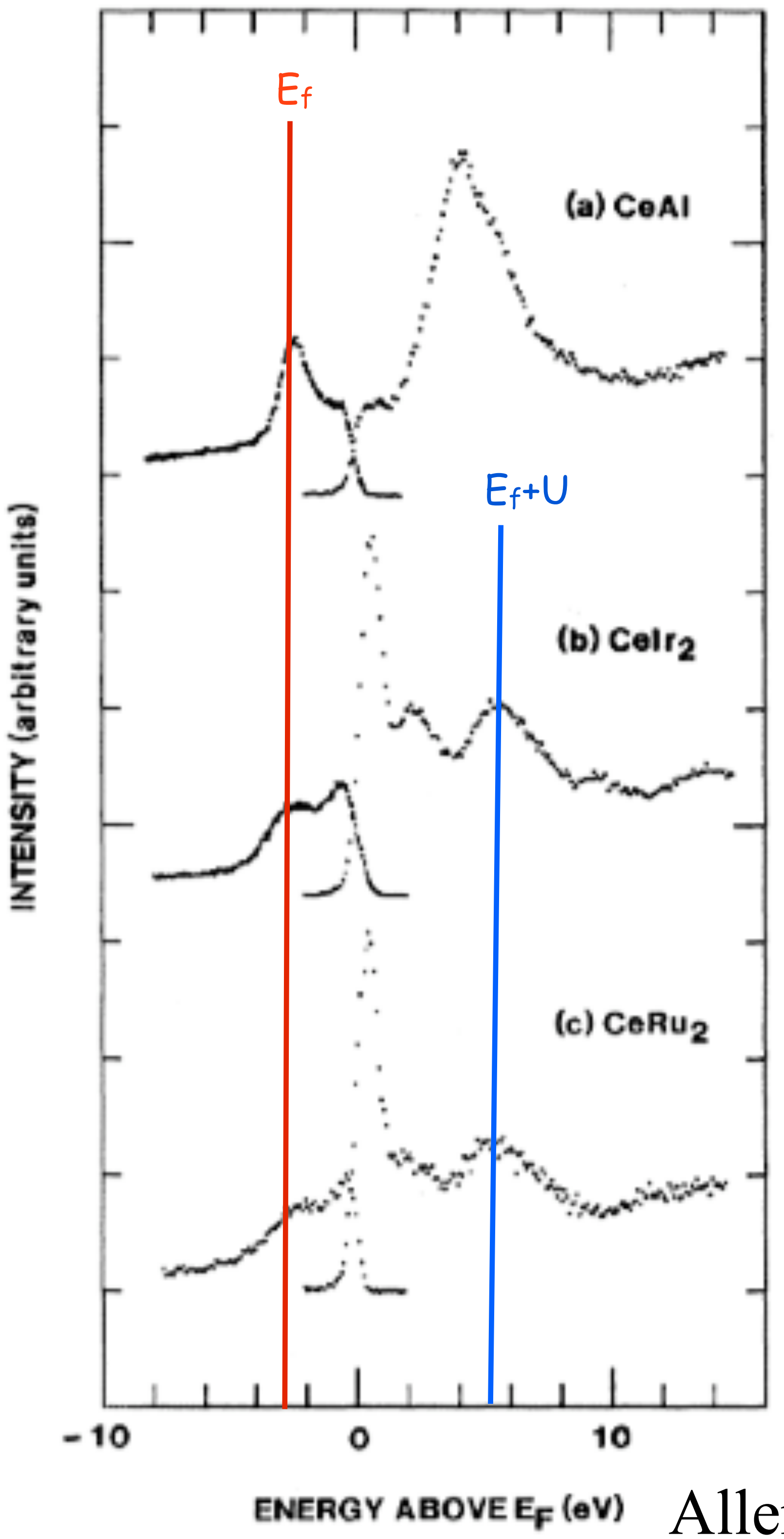

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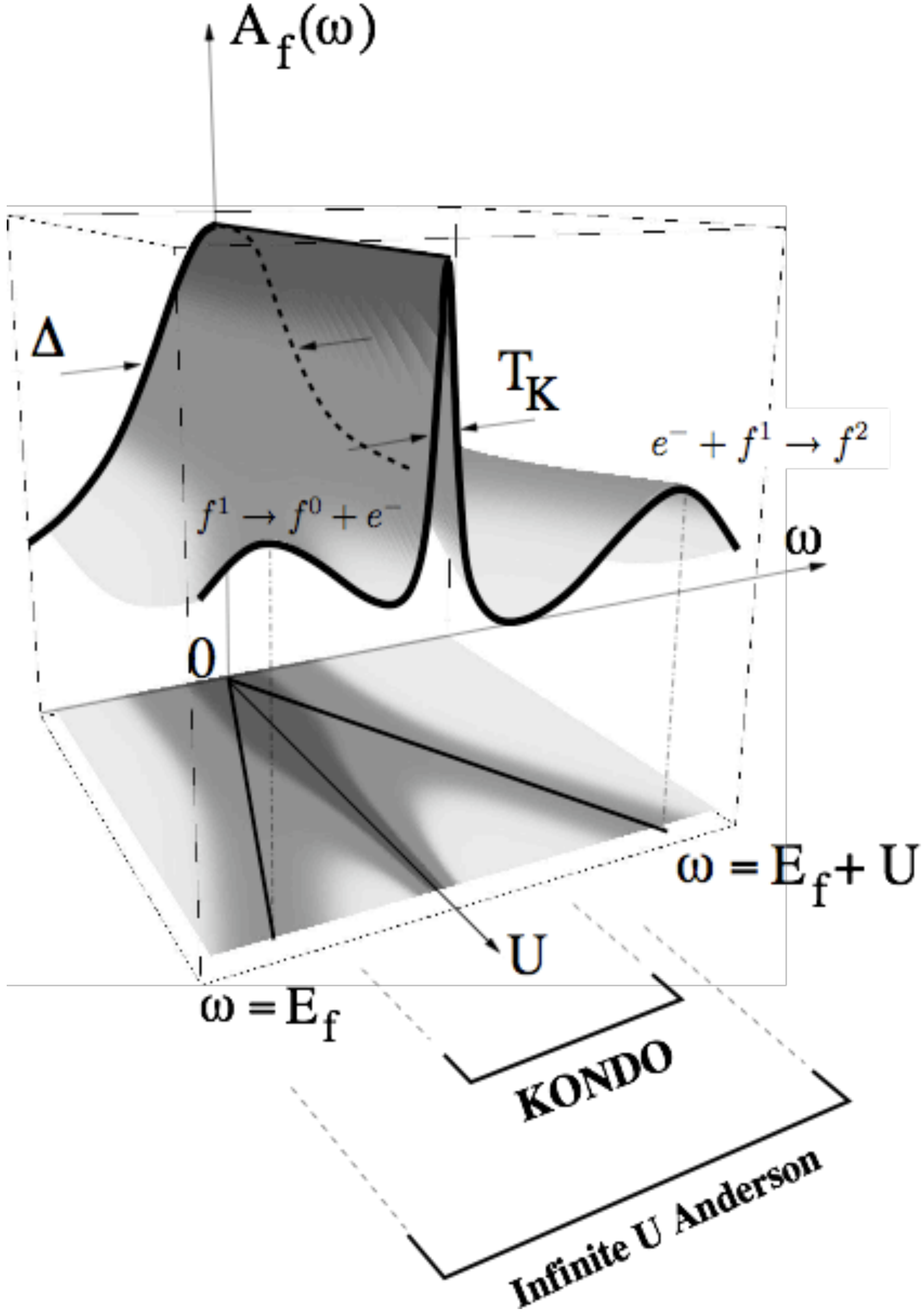
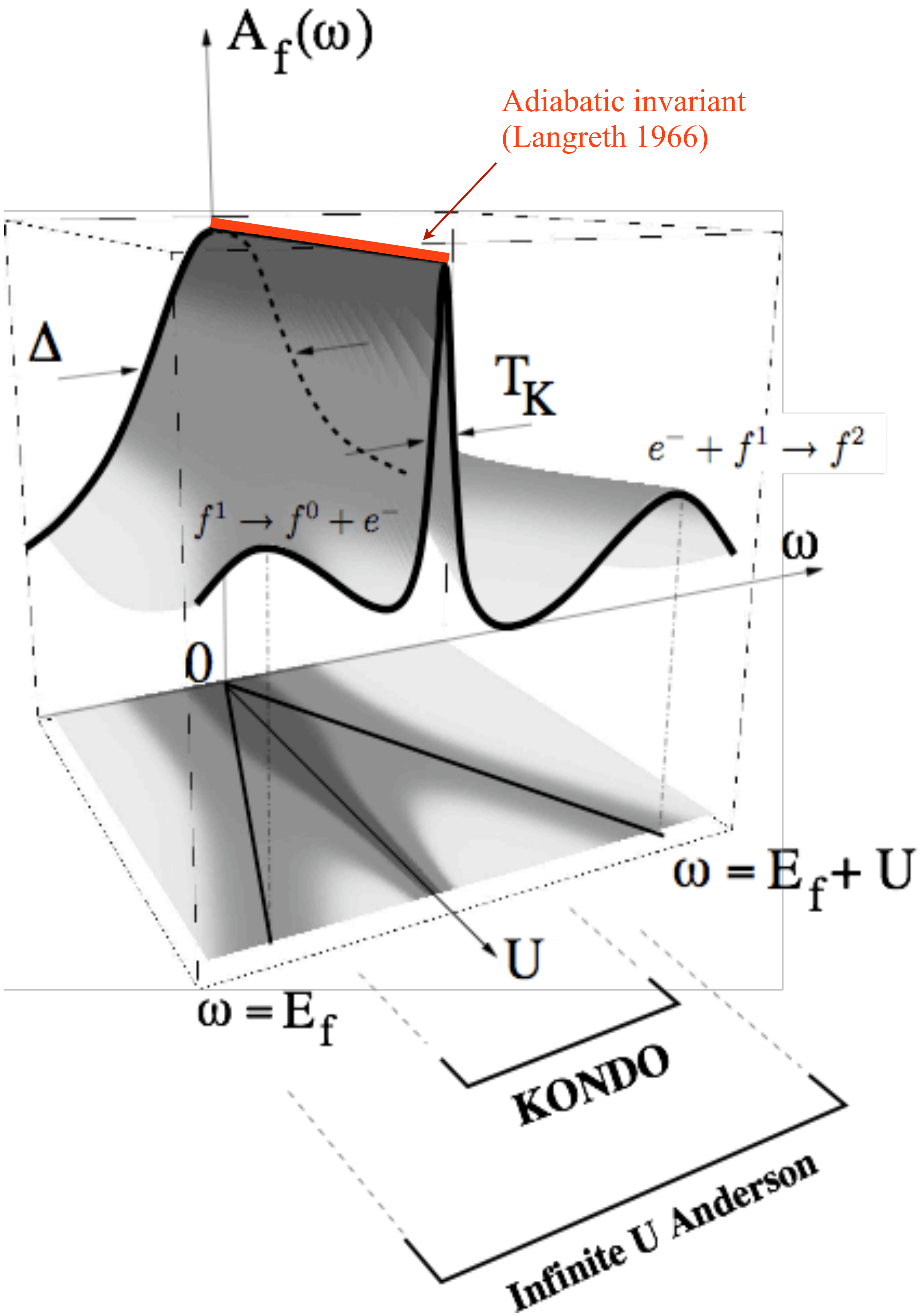
$$A_f(\omega) = \begin{cases} \overbrace{\sum_\lambda \left|\langle\lambda|f_\sigma^\dagger|\phi_0\rangle\right|^2 \delta(\omega - [E_\lambda - E_0]),}^{\text{Energy distribution of state formed by adding one f-electron.}} & (\omega > 0) \\ \underbrace{\sum_\lambda \left|\langle\lambda|f_\sigma|\phi_0\rangle\right|^2 \delta(\omega - [E_0 - E_\lambda]),}_{\text{Energy distribution of state formed by removing an f-electron}} & (\omega < 0) \end{cases}$$

Adiabatic approach & Kondo Resonance



Adiabatic approach & Kondo Resonance



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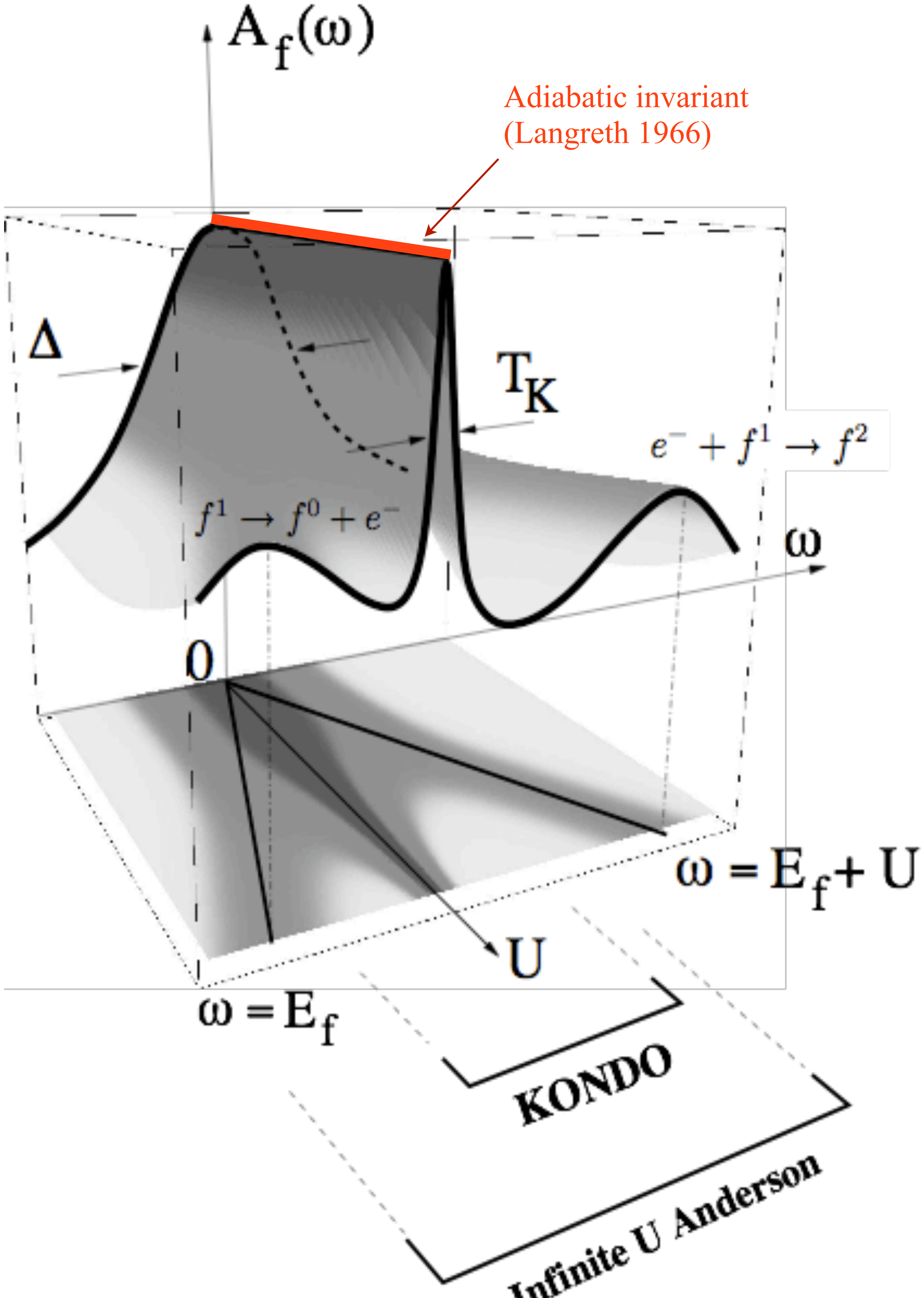

$$A_f(\omega = 0) = \frac{\sin^2 \delta_f}{\pi \Delta}$$

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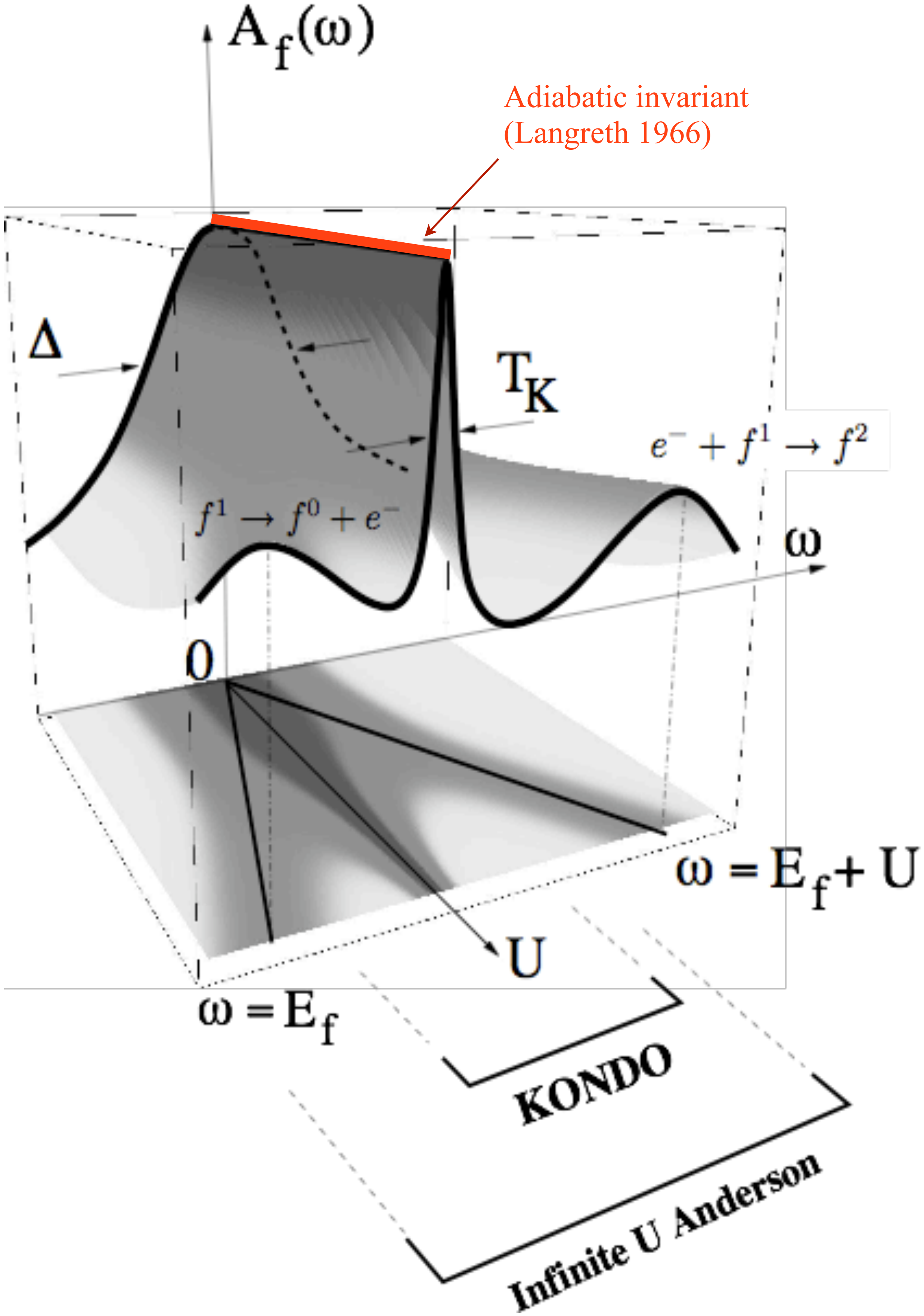
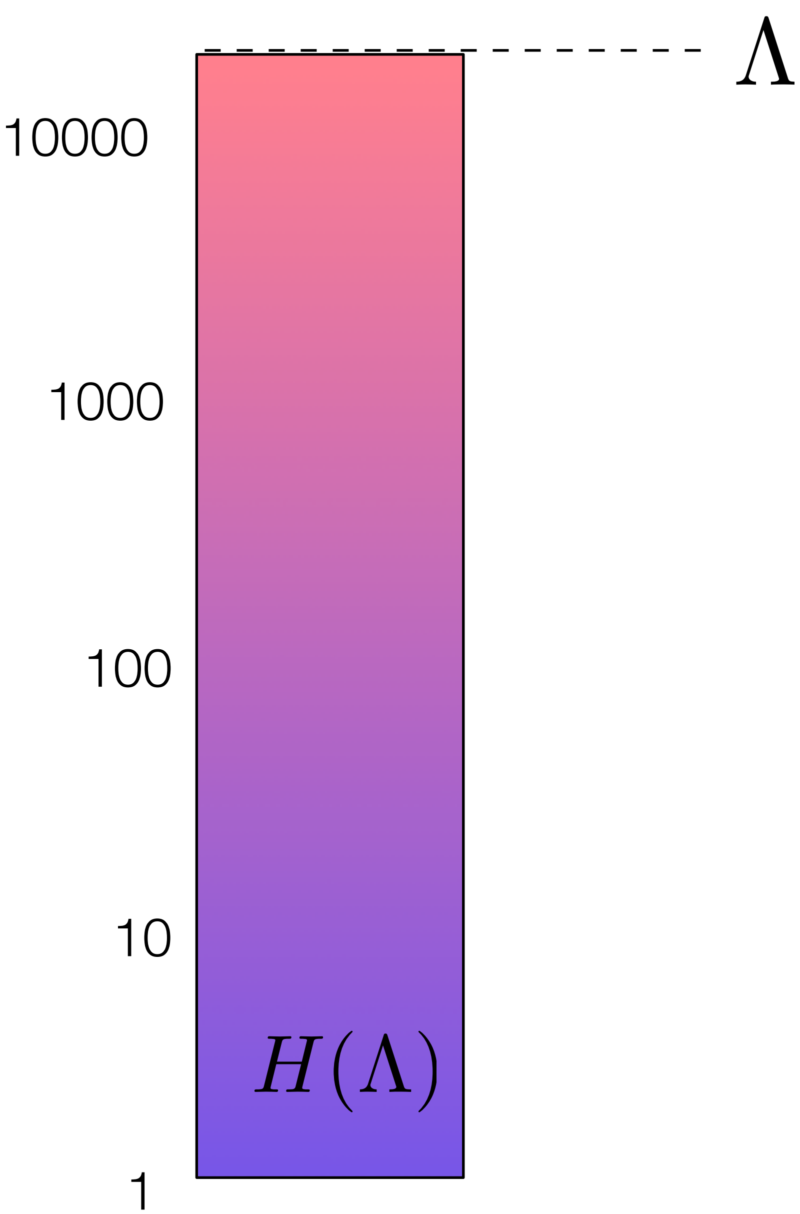


$$A_f(\omega = 0) = \frac{\sin^2 \delta_f}{\pi \Delta}$$

$$\sum_\sigma \frac{\delta_{f\sigma}}{\pi} = 2\frac{\delta}{\pi} = n_f$$

Hierachies of Energy scales: Renormalization concept.

(Anderson, Wilson, ...)



Hierachies of Energy scales:
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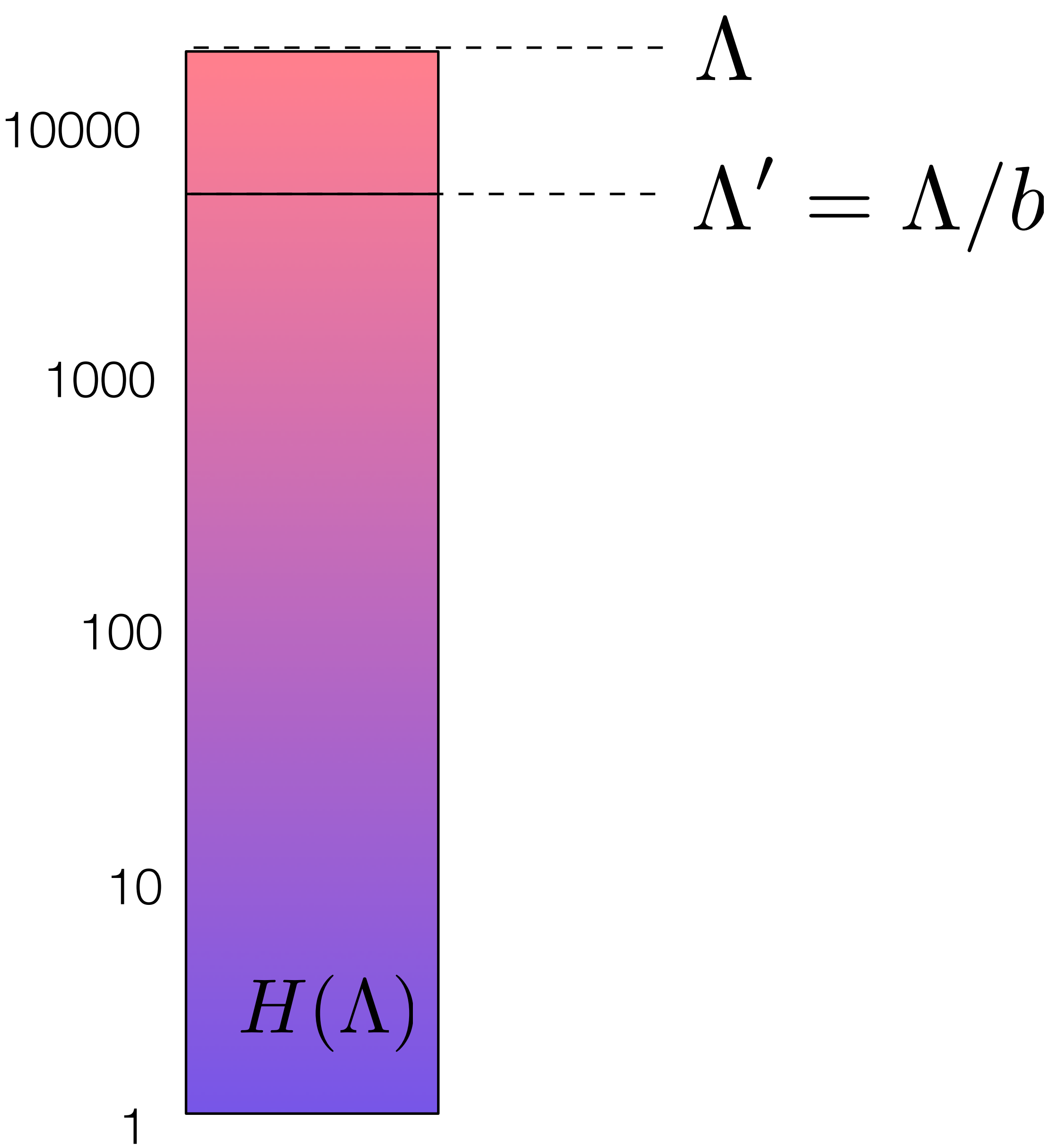

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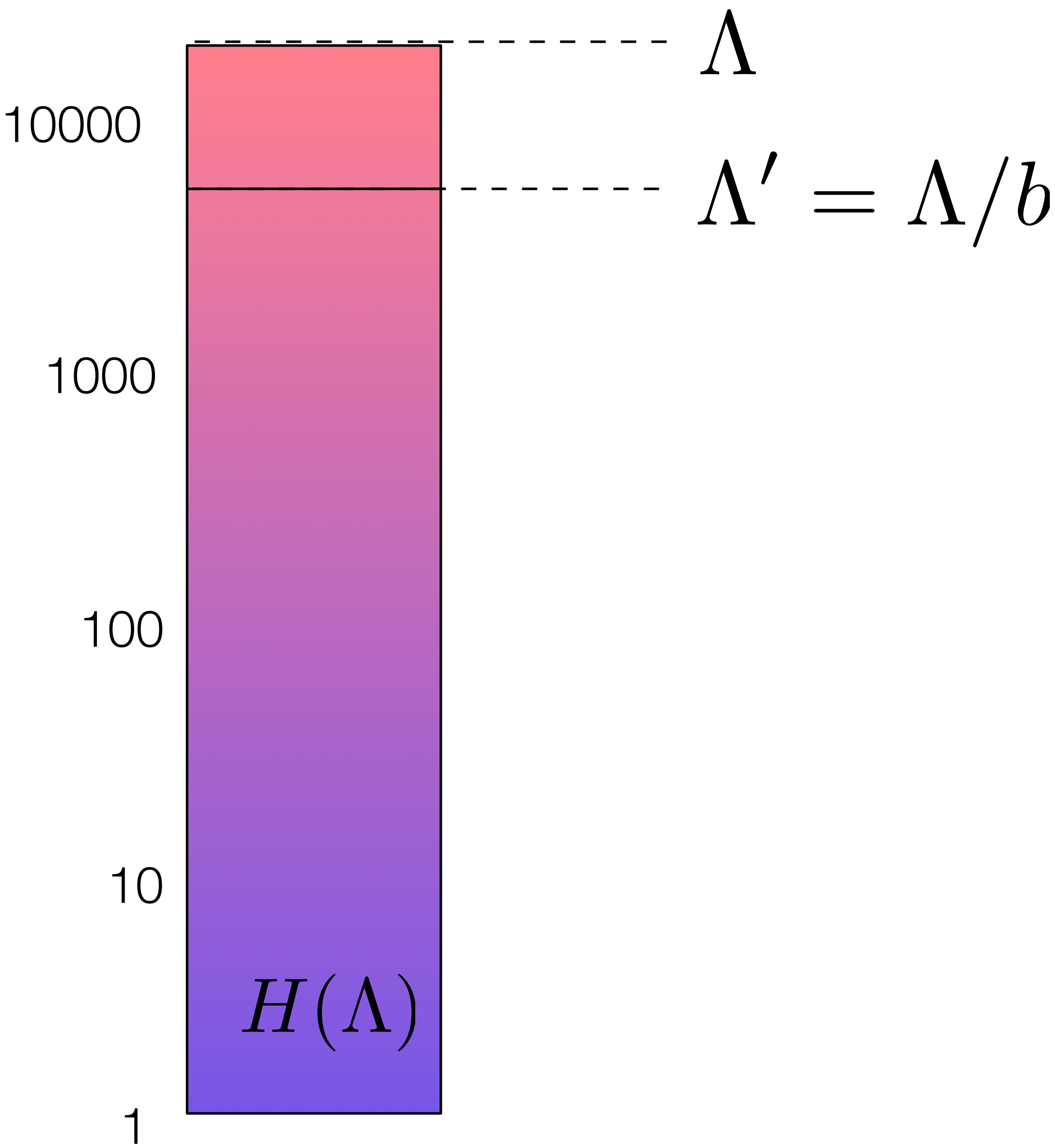

$$H(\Lambda) = \left[\begin{array}{c|c} H_L & V^\dagger \\ \hline V & H_H \end{array} \right],$$

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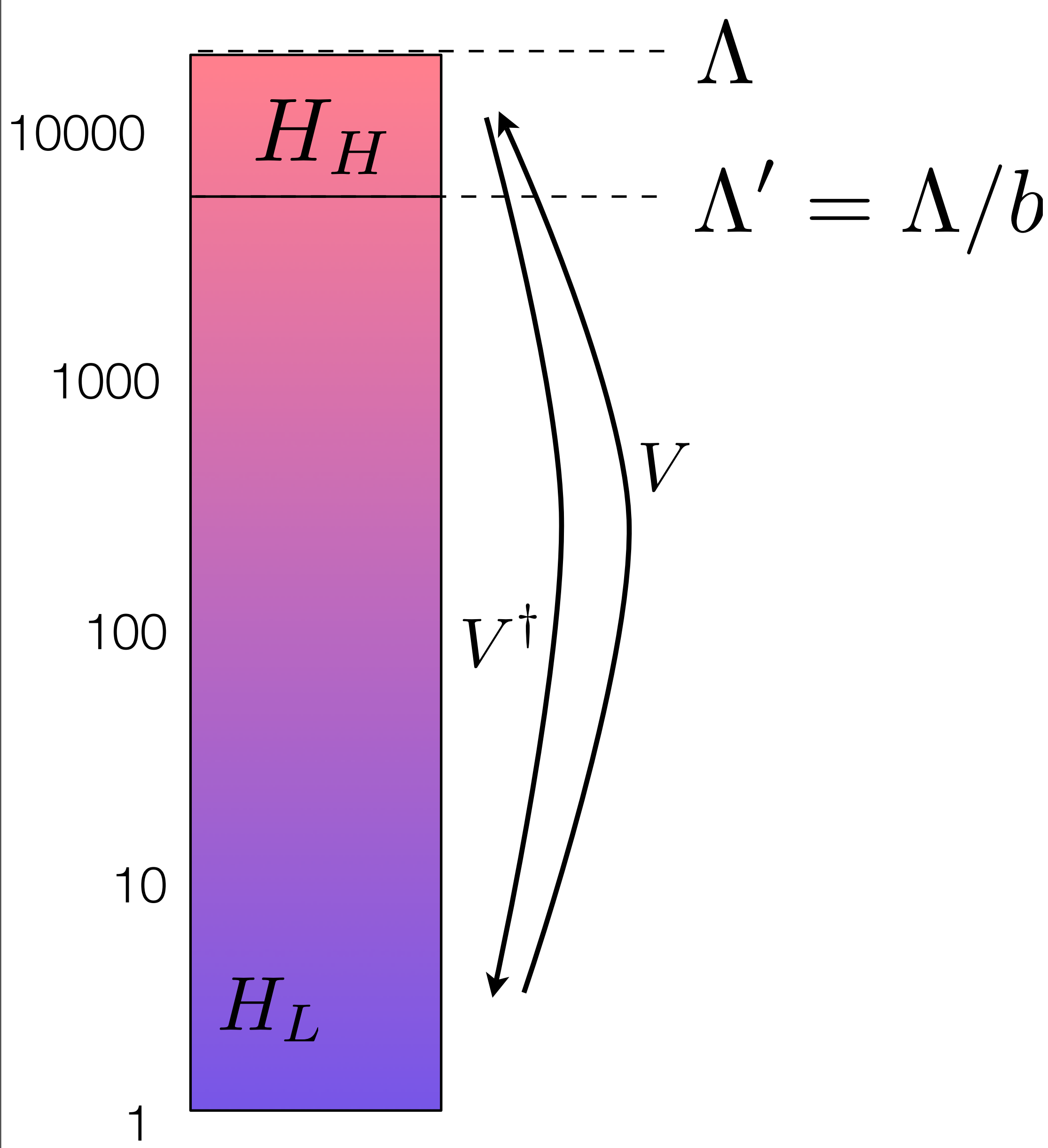

$$H(\Lambda) = \left[\begin{array}{c|c} \frac{H_L}{V} & \frac{V^\dagger}{H_H} \end{array} \right],$$

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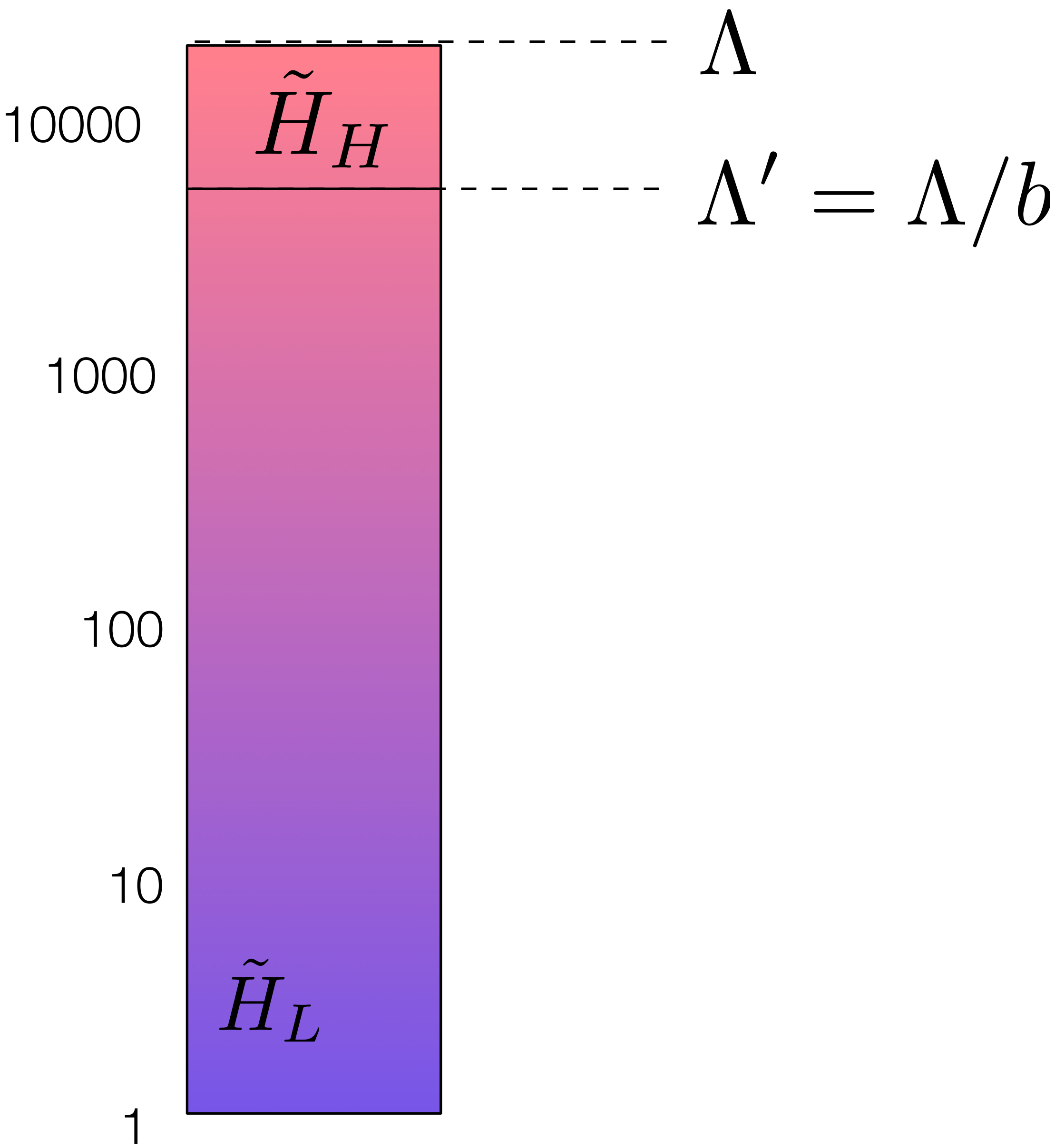

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$$H(\Lambda) \rightarrow U H(\Lambda) U^\dagger = \left[\begin{array}{c|c} \tilde{H}_L & 0 \\ \hline 0 & \tilde{H}_H \end{array} \right]$$

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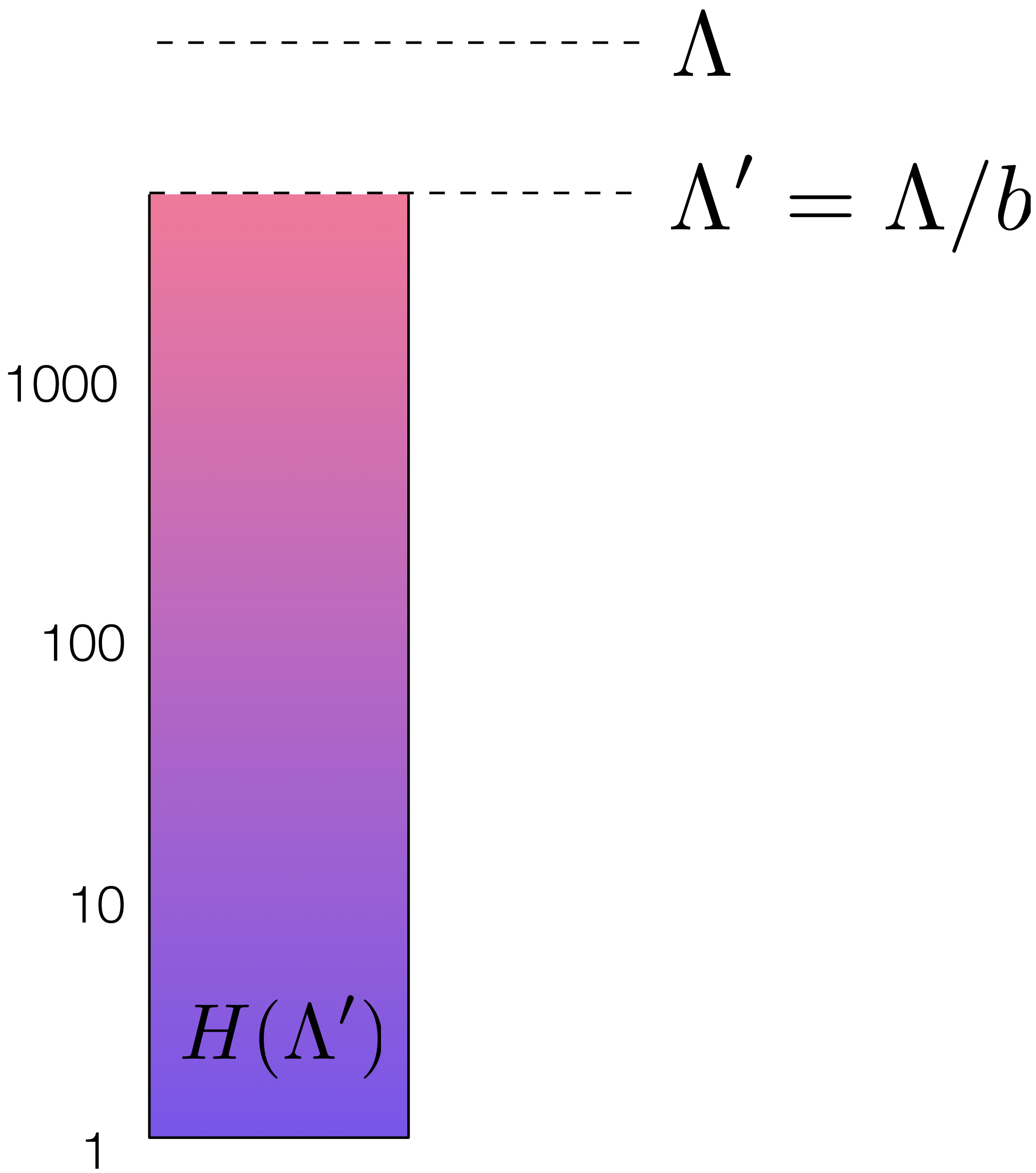

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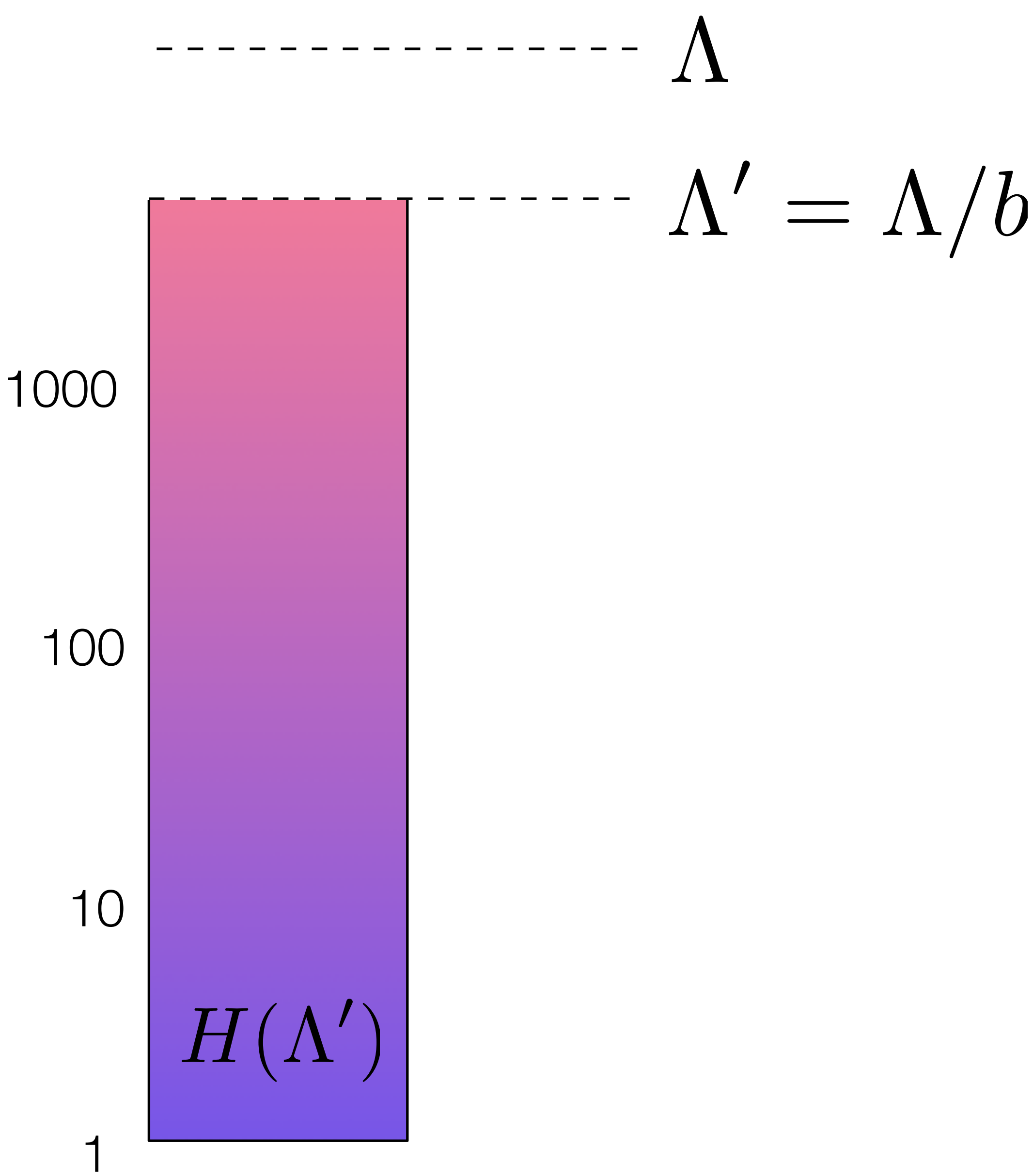

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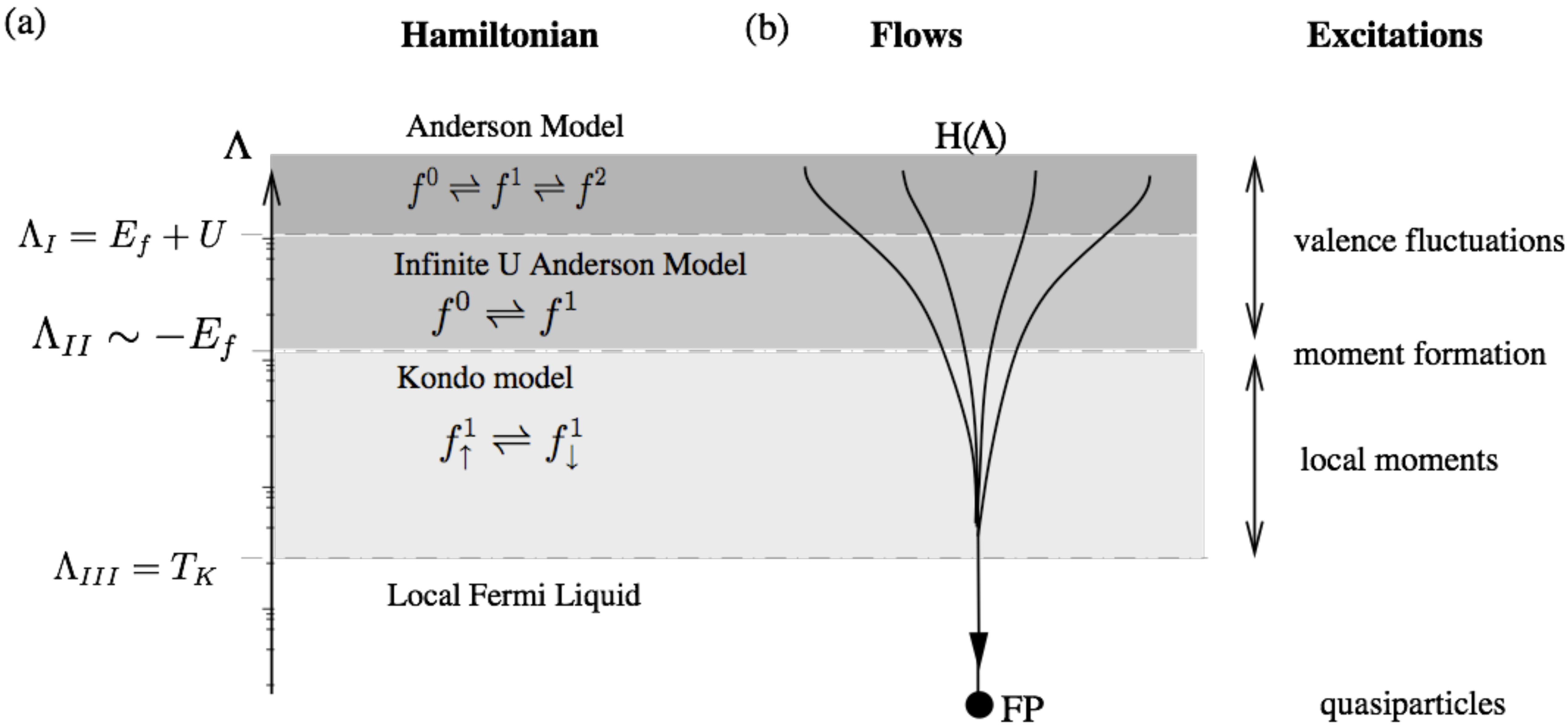
$$\delta H = P_L V^\dagger \left(\frac{1}{E - \tilde{H}_H} \right) V P_L \Big|_{E \sim E_L}$$


(a)
Hamiltonian
(b)
Flows
Excitations
Anderson Model
Λ
H(Λ)
$f^0 \rightleftharpoons f^1 \rightleftharpoons f^2$
$\Lambda_I = E_f + U$
valence fluctuations
Infinite U Anderson Model
$f^0 \rightleftharpoons f^1$
$\Lambda_{II} \sim -E_f$
moment formation
Kondo model
$f^1_\uparrow \rightleftharpoons f^1_\downarrow$
local moments
$\Lambda_{III} = T_K$
Local Fermi Liquid
FP
quasiparticles

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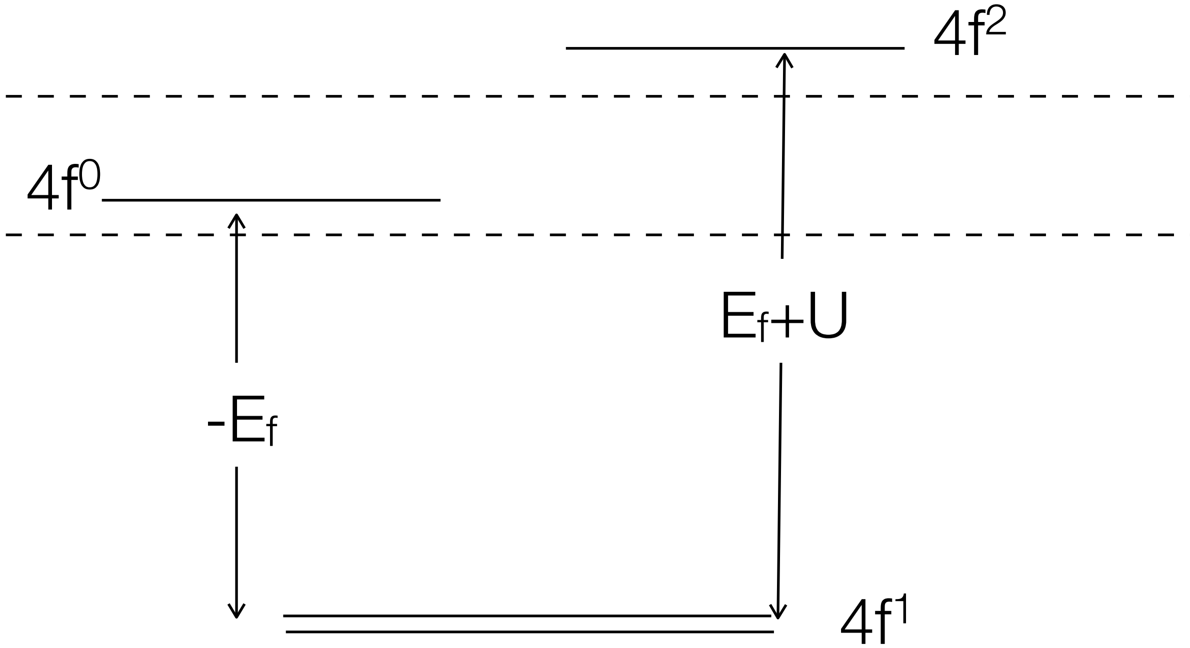
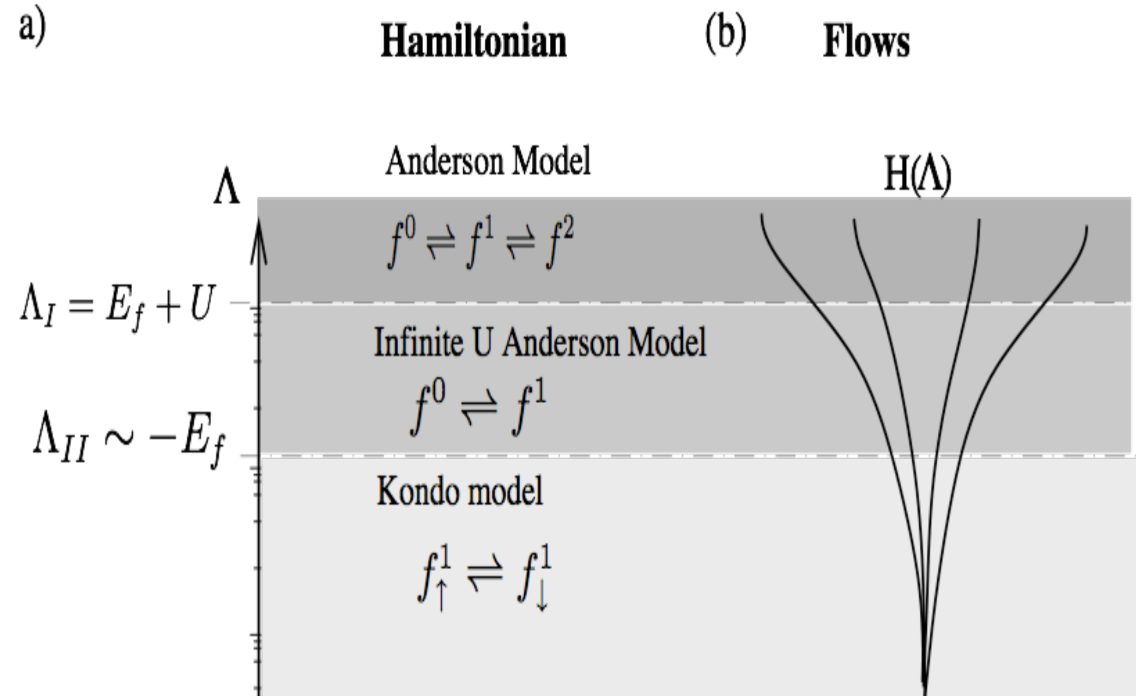
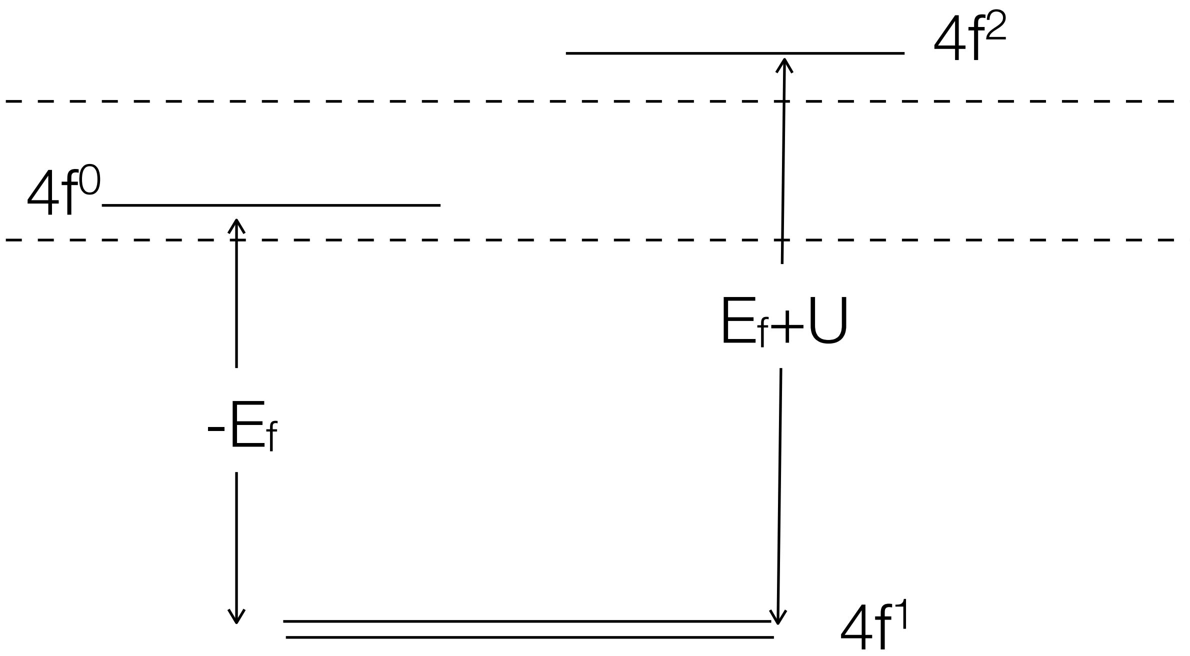
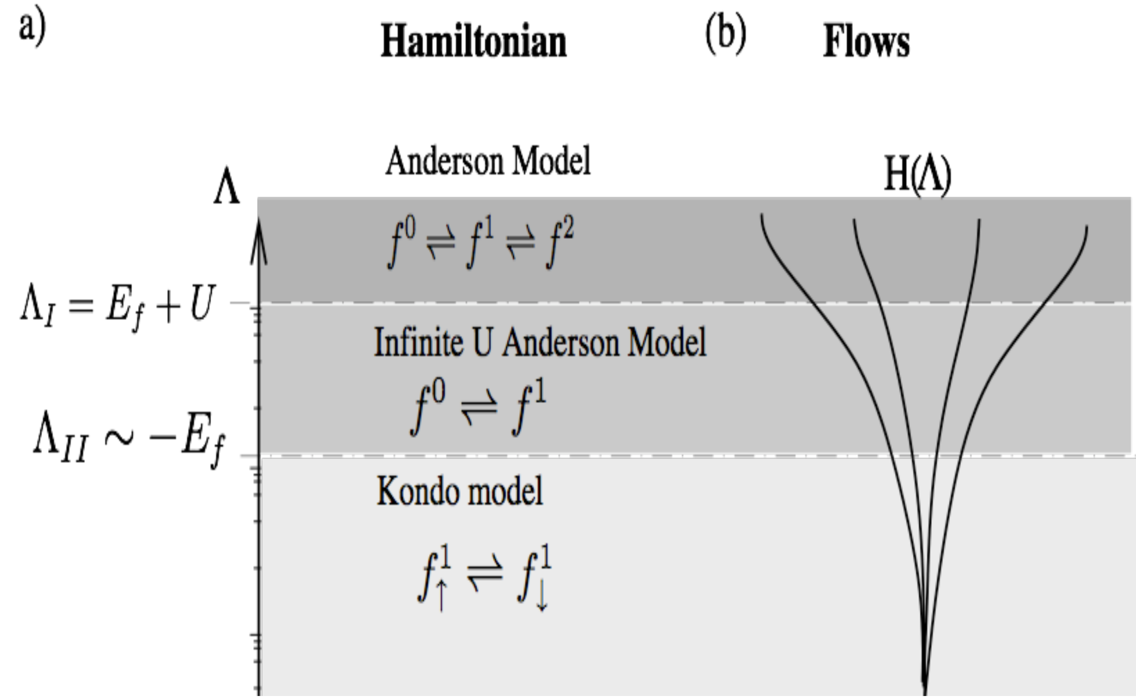
Schrieffer-Wolff Transformation.





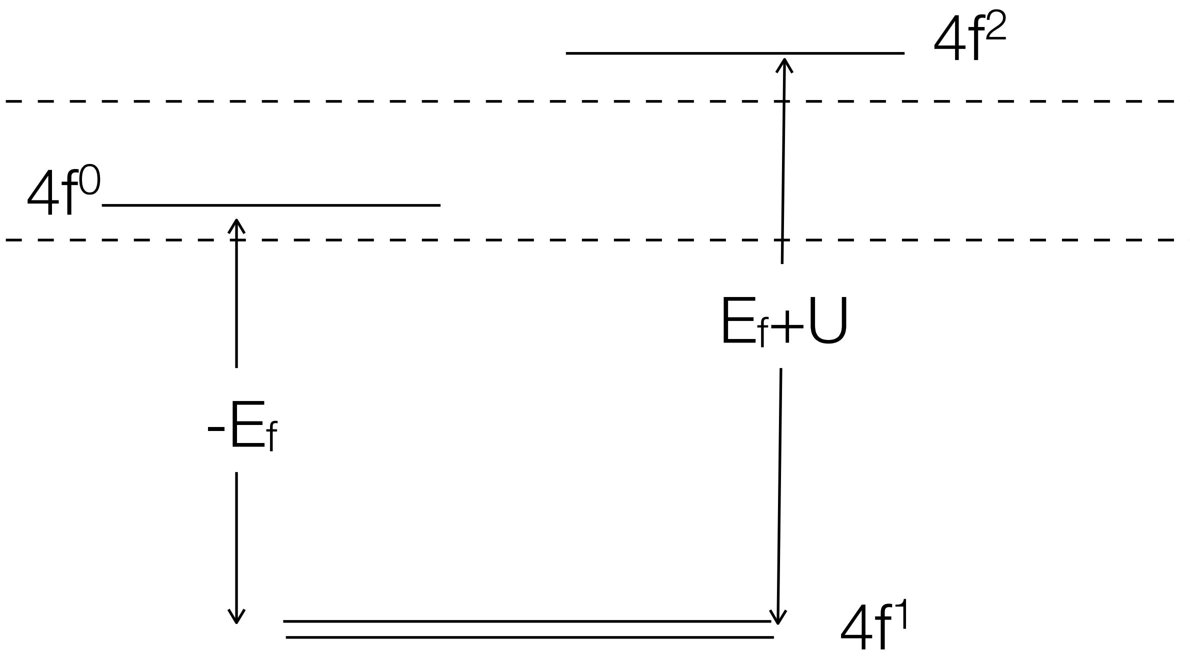
Schrieffer-Wolff Transformation.

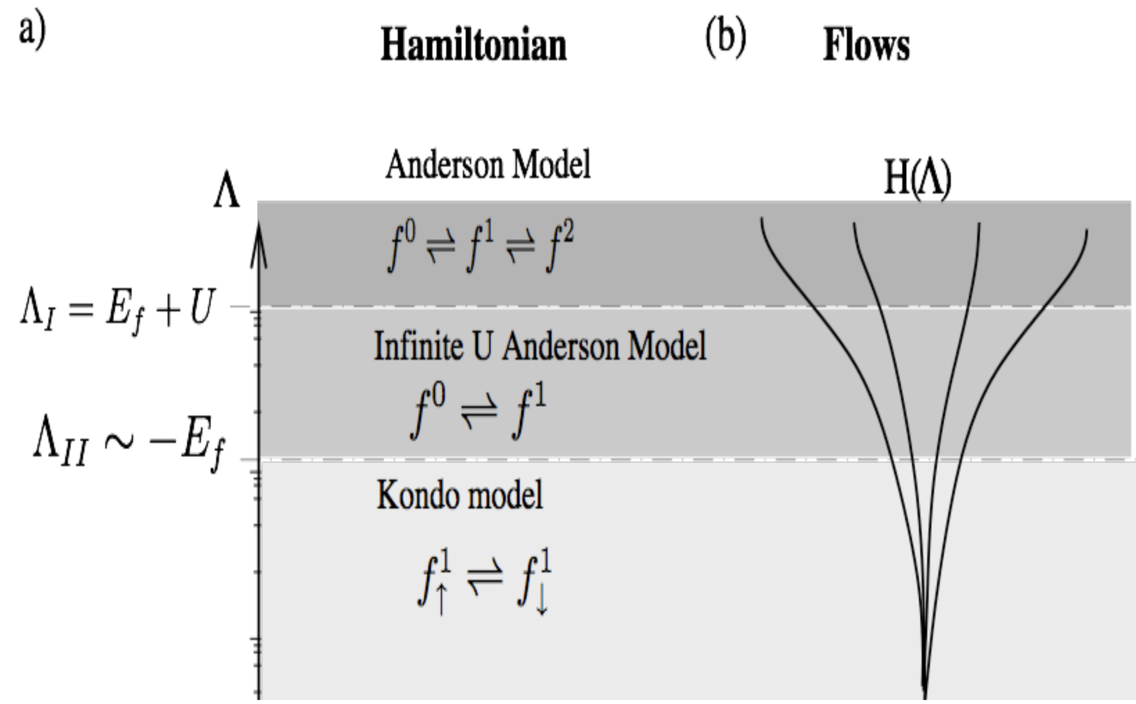




$$H_{int} = -\sum_{k\sigma,k'\sigma'} V_{k'}^* V_k \left[\overbrace{\frac{(c^\dagger_{k\sigma} f_\sigma)(f^\dagger_{\sigma'} c_{k'\sigma'})}{E_f + U}}^{f^1 + e^- \leftrightarrow f^2} + \overbrace{\frac{(f^\dagger_{\sigma'} c_{k'\sigma'})(c^\dagger_{k\sigma} f_\sigma)}{-E_f}}^{f^1 \leftrightarrow f^0 + e^-} \right]$$

Schrieffer-Wolff Transformation.

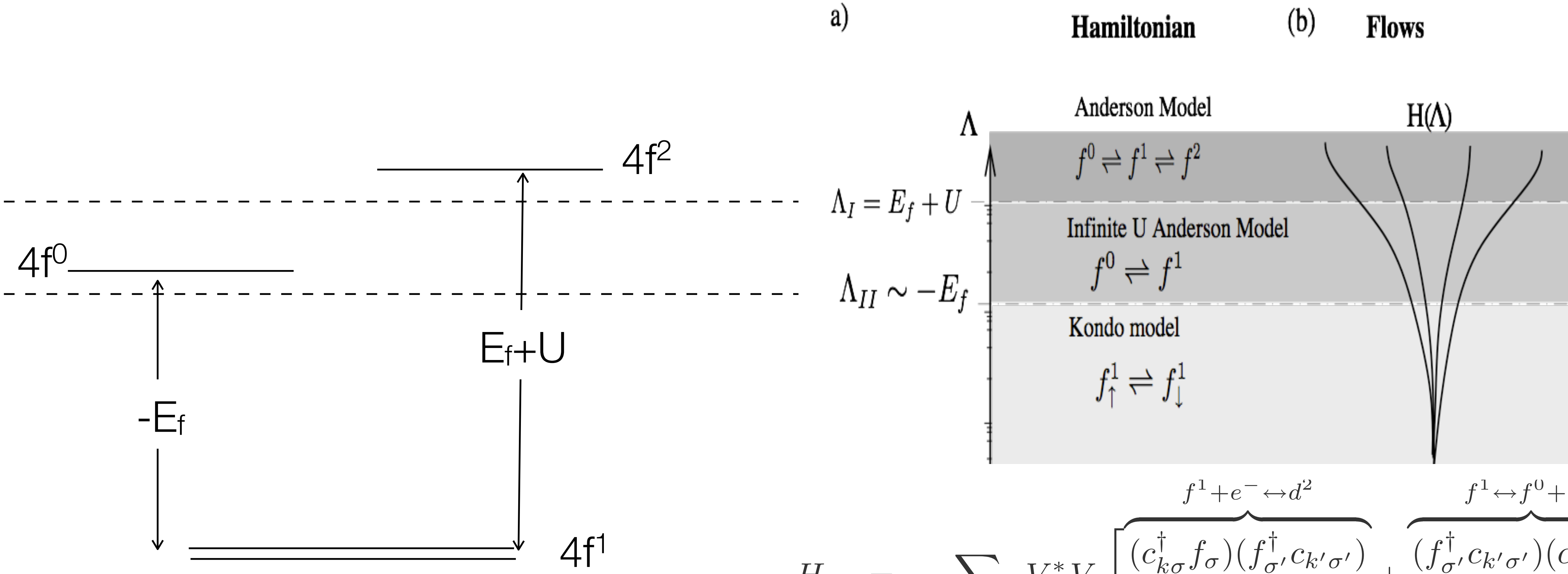




$\delta_{ab}\delta_{cd} + \vec{\sigma}_{ab} \cdot \vec{\sigma}_{cd} = 2\delta_{ad}\delta_{bc}$

$$H_{int} = -\sum_{k\sigma,k'\sigma'} V^*_{k'} V_k \left[\overbrace{\frac{(c^\dagger_{k\sigma} f_\sigma)(f^\dagger_{\sigma'} c_{k'\sigma'})}{E_f + U}}^{f^1 + e^- \leftrightarrow f^2} + \overbrace{\frac{(f^\dagger_{\sigma'} c_{k'\sigma'})(c^\dagger_{k\sigma} f_\sigma)}{-E_f}}^{f^1 \leftrightarrow f^0 + e^-} \right]$$

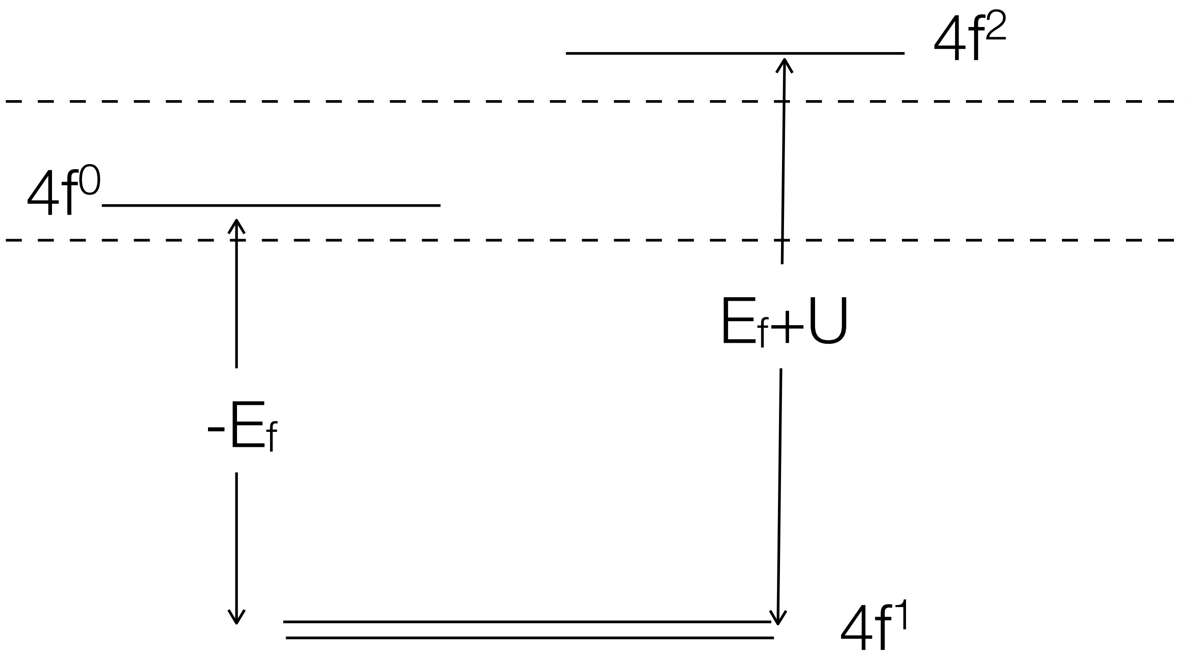
Schrieffer-Wolff Transformation.

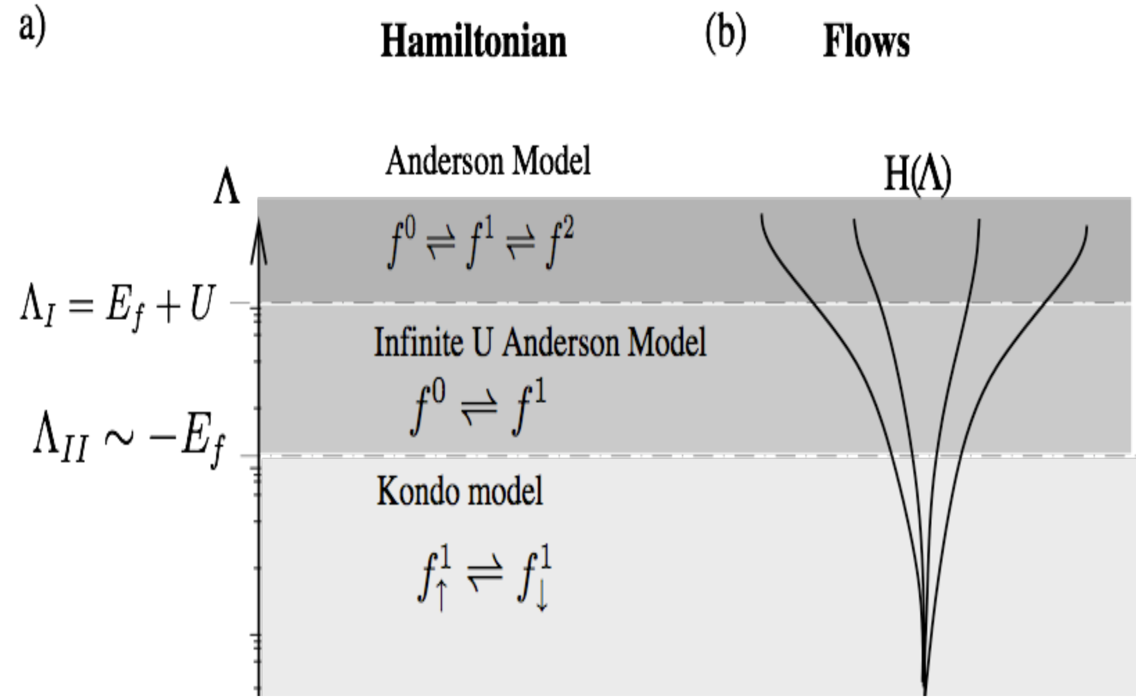


$$H_{int} = -\sum_{k\sigma,k'\sigma'} V^*_{k'} V_k \left[\overbrace{\frac{(c^\dagger_{k\sigma} f_\sigma)(f^\dagger_{\sigma'} c_{k'\sigma'})}{E_f + U}}^{f^1 + e^- \leftrightarrow d^2} + \overbrace{\frac{(f^\dagger_{\sigma'} c_{k'\sigma'})(c^\dagger_{k\sigma} f_\sigma)}{-E_f}}^{f^1 \leftrightarrow f^0 + e^-} \right]$$

$$\delta_{ab}\delta_{cd} + \vec{\sigma}_{ab} \cdot \vec{\sigma}_{cd} = 2\delta_{ad}\delta_{bc}$$

Schrieffer-Wolff Transformation.



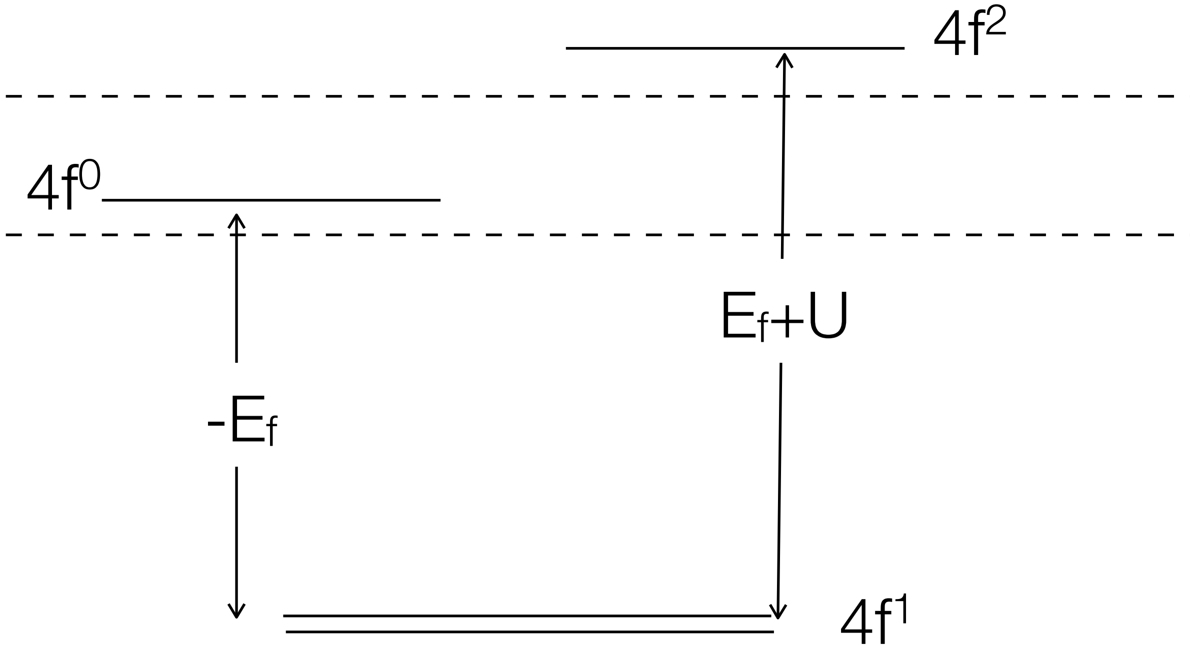


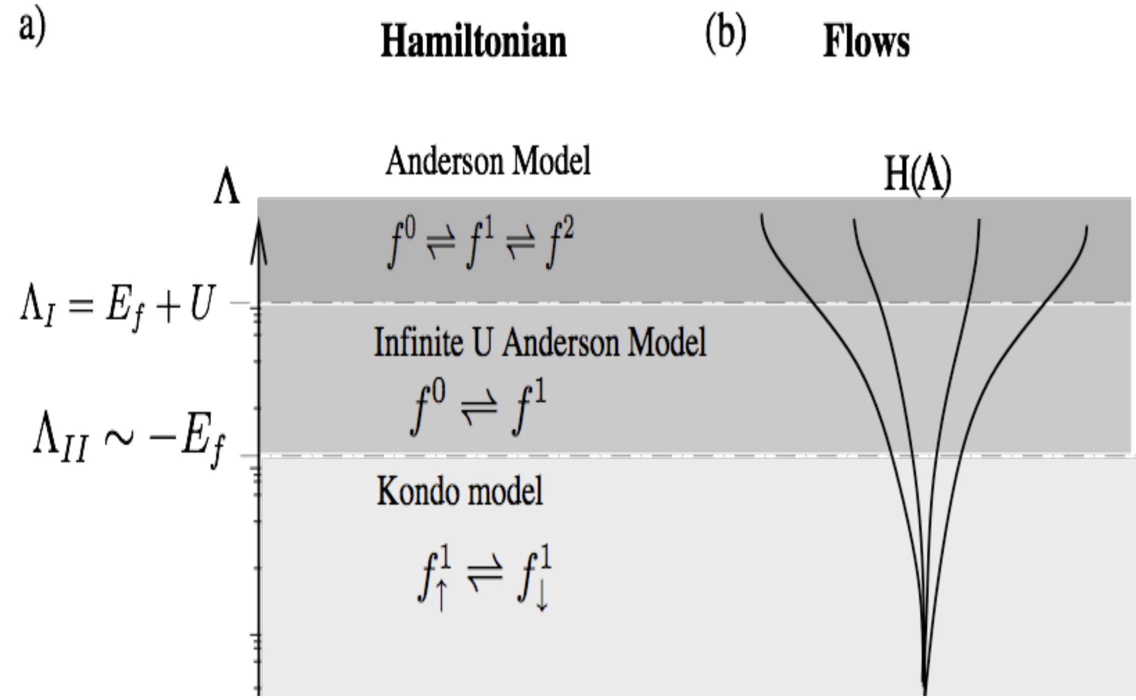
$$H_{int} = -\sum_{k\sigma,k'\sigma'} V^*_{k'}V_k \left[\overbrace{\frac{(c^\dagger_{k\sigma} f_\sigma)(f^\dagger_{\sigma'} c_{k'\sigma'})}{E_f + U}}^{f^1+e^- \leftrightarrow d^2} + \overbrace{\frac{(f^\dagger_{\sigma'} c_{k'\sigma'})(c^\dagger_{k\sigma} f_\sigma)}{-E_f}}^{f^1 \leftrightarrow f^0 + e^-} \right]$$

$$\delta_{ab}\delta_{cd} + \vec{\sigma}_{ab} \cdot \vec{\sigma}_{cd} = 2\delta_{ad}\delta_{bc}$$

$$H_{int} = \sum_{k\alpha,k'\beta} J_{k,k'} c^\dagger_{k\alpha} \vec{\sigma} c_{k'\beta} \cdot \vec{S}_f + H'$$

Schrieffer-Wolff Transformation.



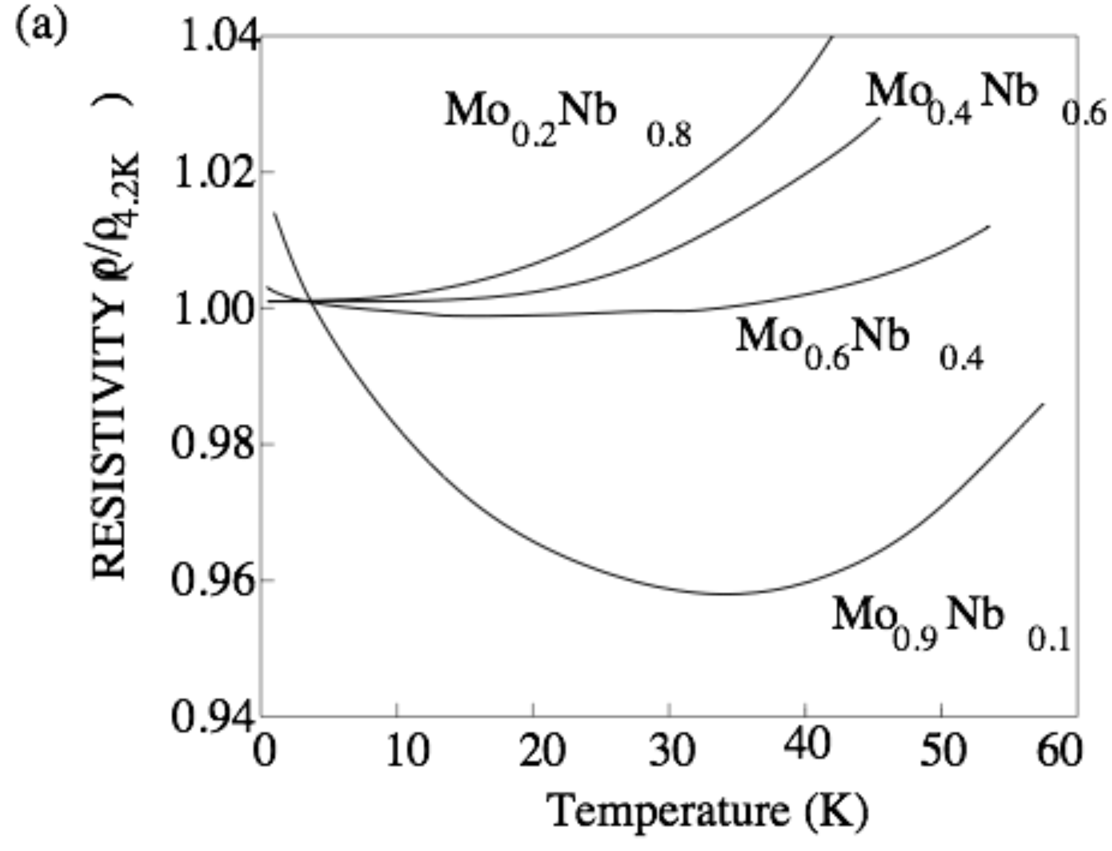


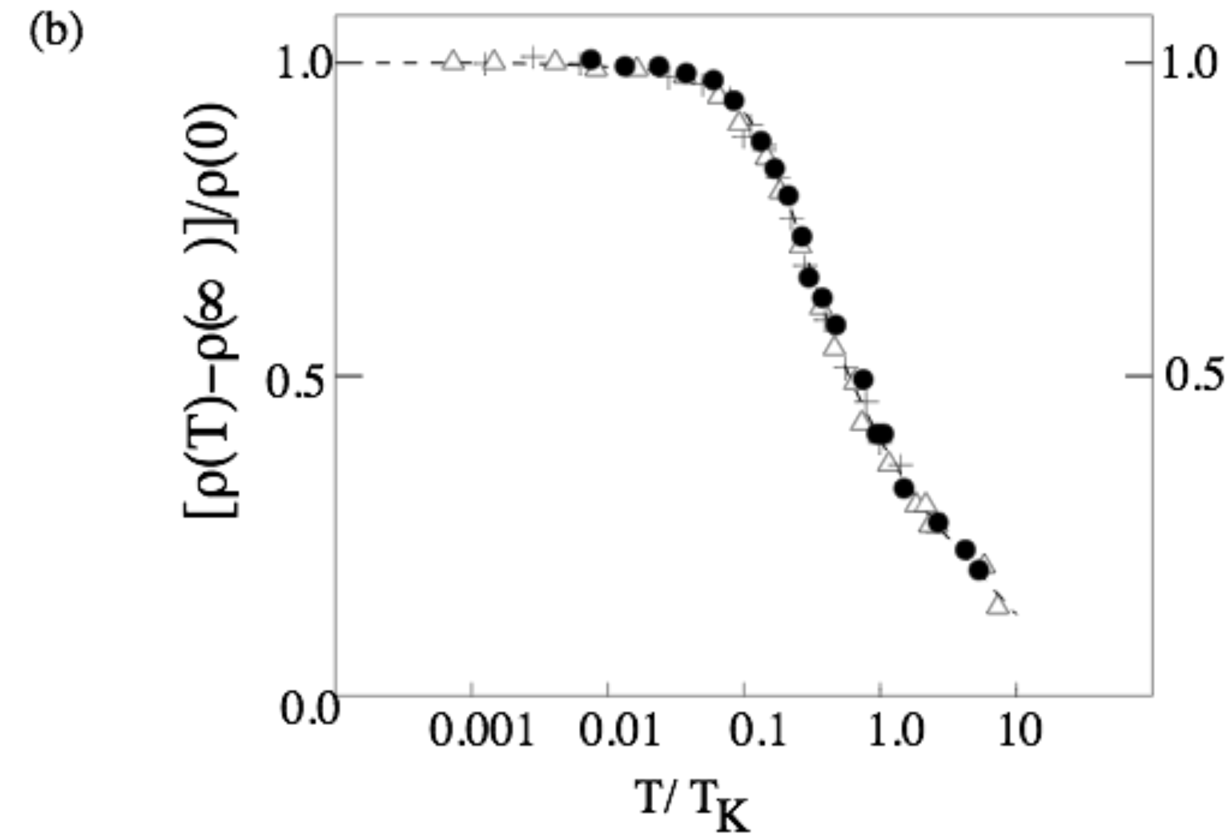
$$H_{int} = -\sum_{k\sigma,k'\sigma'} V^*_{k'} V_k \left[\overbrace{\frac{(c^\dagger_{k\sigma} f_\sigma)(f^\dagger_{\sigma'} c_{k'\sigma'})}{E_f + U}}^{f^1+e^- \leftrightarrow d^2} + \overbrace{\frac{(f^\dagger_{\sigma'} c_{k'\sigma'})(c^\dagger_{k\sigma} f_\sigma)}{-E_f}}^{f^1 \leftrightarrow f^0+e^-} \right]$$

$$\delta_{ab}\delta_{cd} + \vec{\sigma}_{ab} \cdot \vec{\sigma}_{cd} = 2\delta_{ad}\delta_{bc}$$

$$H_{int} = \sum_{k\alpha,k'\beta} J_{k,k'} c^\dagger_{k\alpha} \vec{\sigma} c_{k'\beta} \cdot \vec{S}_f + H'$$

$$J_{k,k'} = V^*_{k'} V_k \left[\overbrace{\frac{1}{E_f + U}}^{f^1+e^- \leftrightarrow f^2} + \overbrace{\frac{1}{-E_f}}^{f^1 \leftrightarrow f^0+e^-} \right]$$





Mathias and Clogston (62)
Sarachik et al, (1964)

White and Geballe, (1979)