Heavy Electron Physics:

Electrons on the edge of Magnetism

Piers Coleman, Rutgers CMT, NJ, USA

Boulder, Colorado
July 2008.

Lectures will be given on the blackboard
These viewgraphs are supplementary.

1. Landau Fermi liquids and Heavy Fermions.
2. Local moments and the Kondo effect.
3. Large N approach to the Kondo lattice.

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Supported by NSF DMR

Notes:

"Heavy Fermions: electrons at the edge of magnetism." PC. cond-mat/0612006.

"Local moment physics in heavy electron systems", PC, cond-mat/0206003

General reading:

Many Body Physics "unfinished frontier", PC, cond-mat/0307004.

The Theory of Quantum Liquids, Nozieres and Pines (Perseus 1999).

Quantum Criticality, P. Coleman and Andrew Schofield, Nature, 433, 226-229 (2005), cond-mat/0503002.

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Lecture I:

1. Overview of heavy fermion physics.

2. Landau Fermi liquid Theory.

3. Heavy Fermions from the Landau Perspective.

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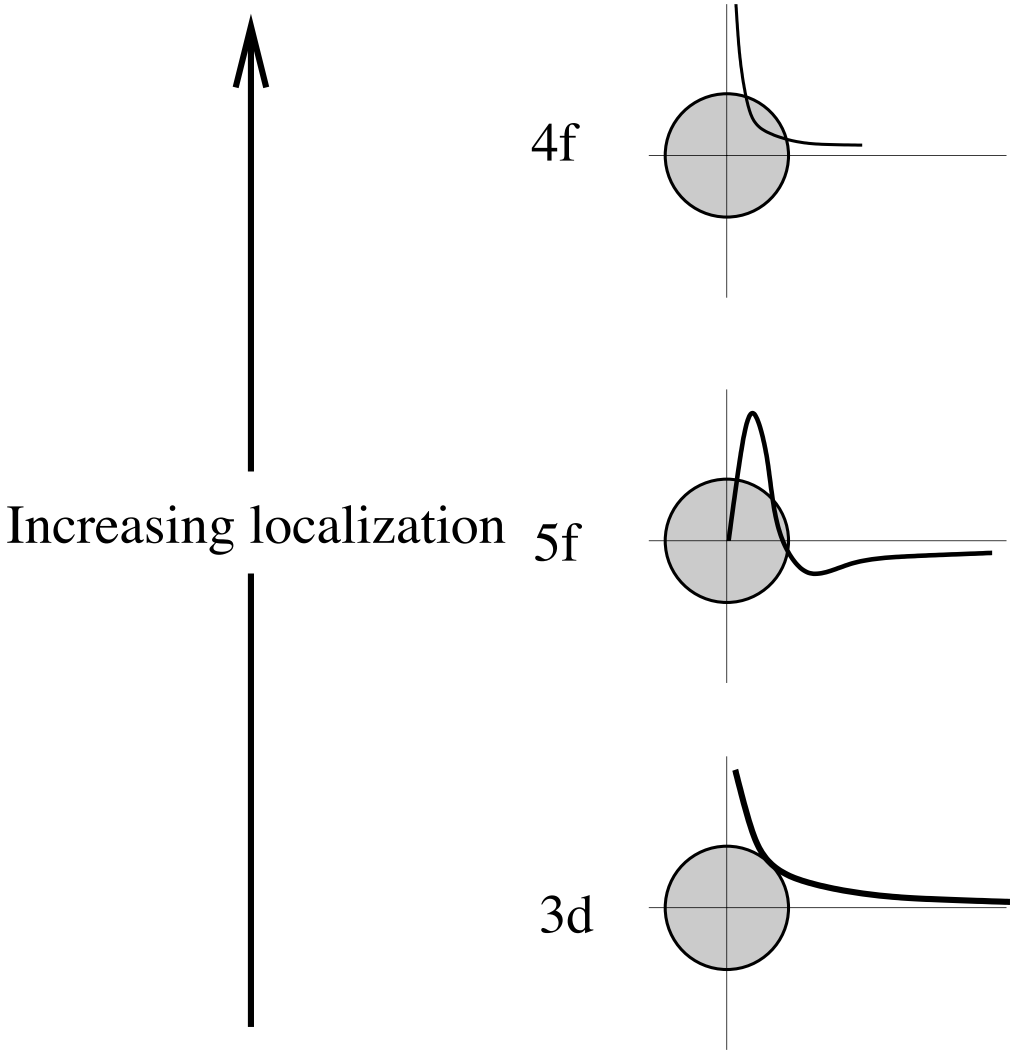
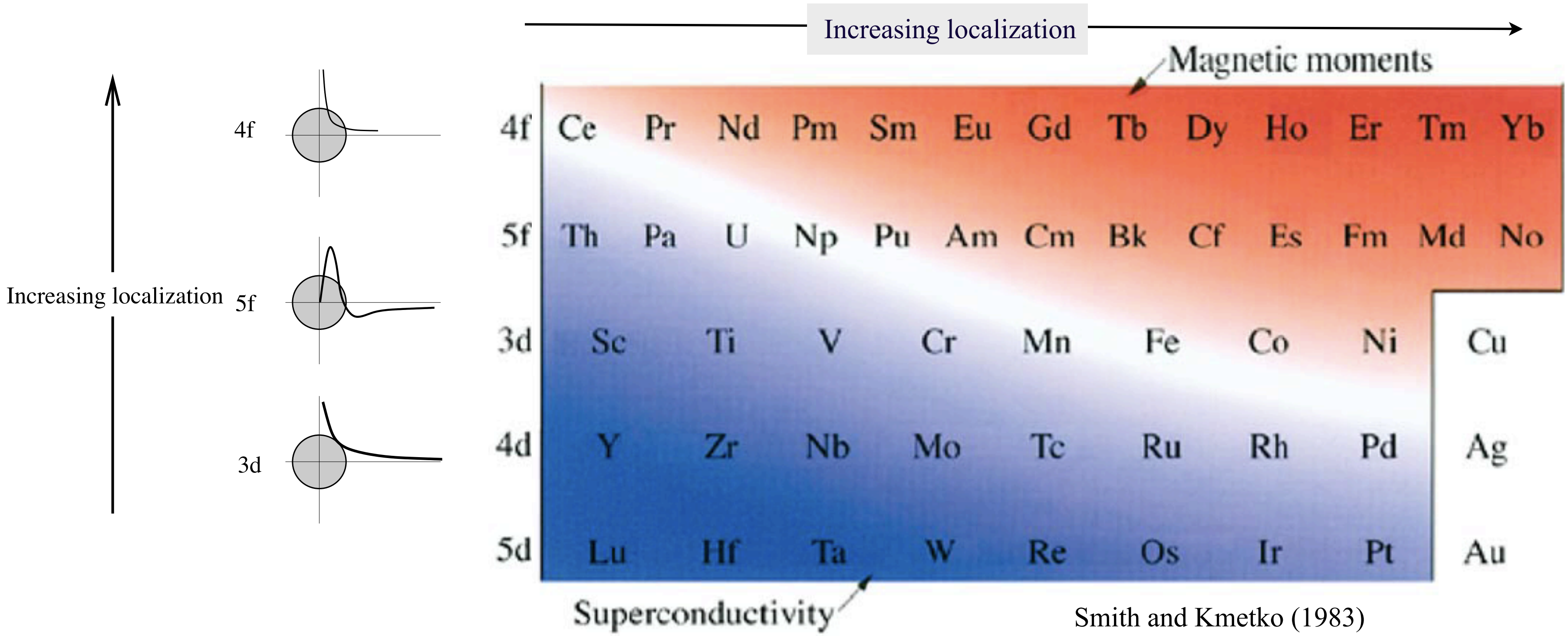
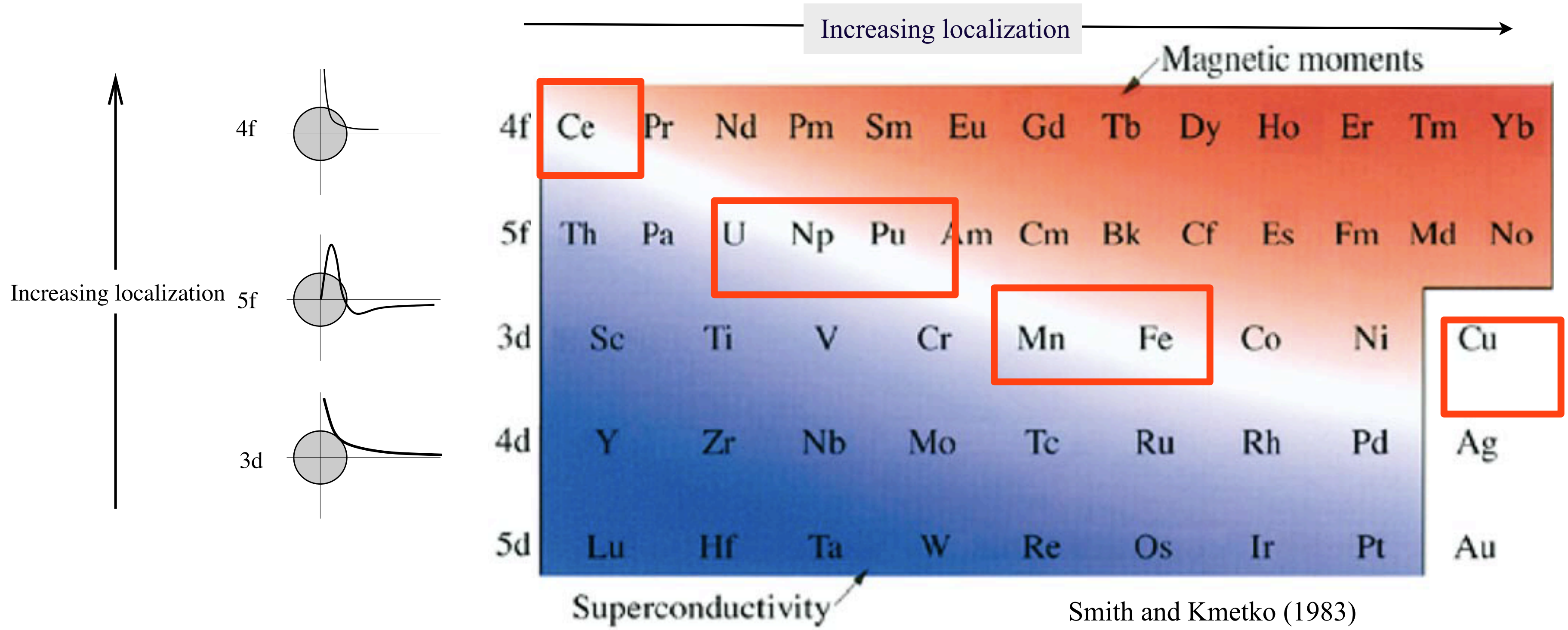
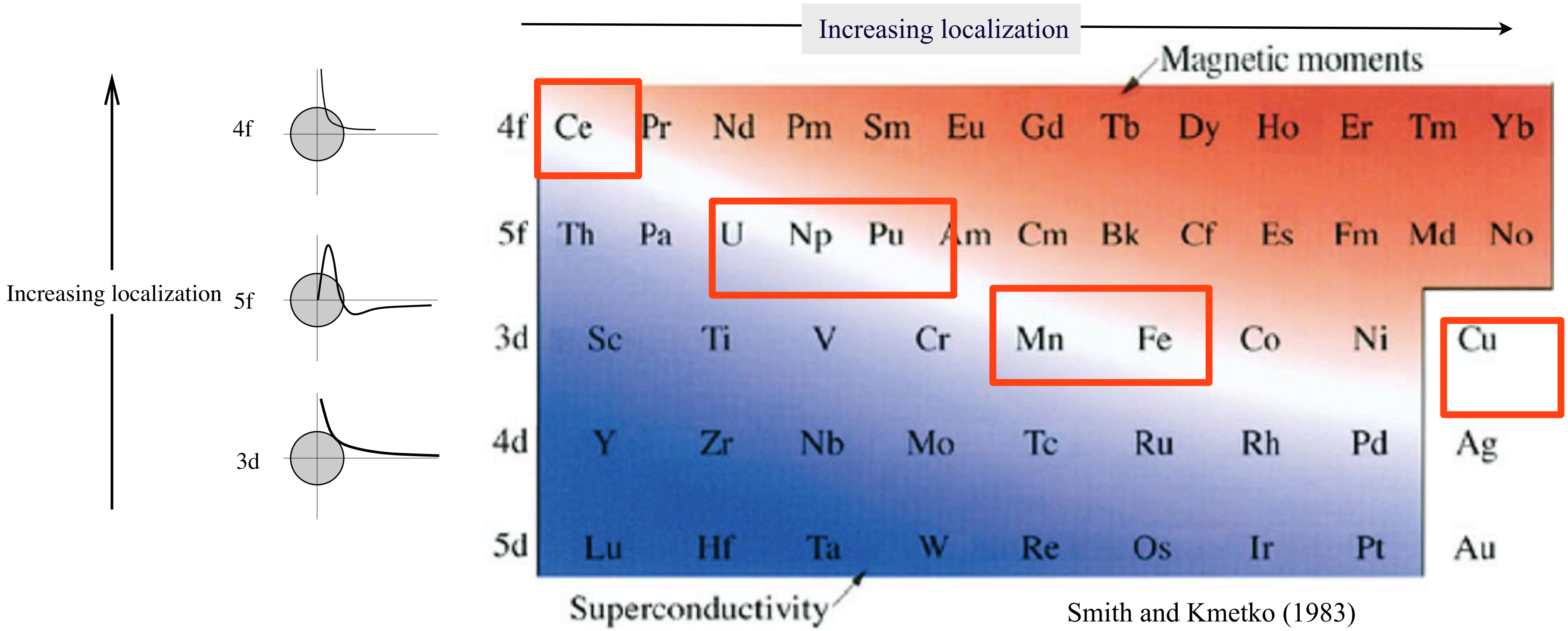

Increasing localization
4f
5f
3d


Increasing localization
Magnetic moments
Increasing localization
4f
5f
3d
4f Ce Pr Nd Pm Sm Eu Gd Tb Dy Ho Er Tm Yb
5f Th Pa U Np Pu Am Cm Bk Cf Es Fm Md No
3d Sc Ti V Cr Mn Fe Co Ni Cu
4d Y Zr Nb Mo Tc Ru Rh Pd Ag
5d Lu Hf Ta W Re Os Ir Pt Au
Superconductivity
Smith and Kmetko (1983)



A lot of action takes place on the brink of localization!

Heavy Fermions: f-spins are always localized, yet.....



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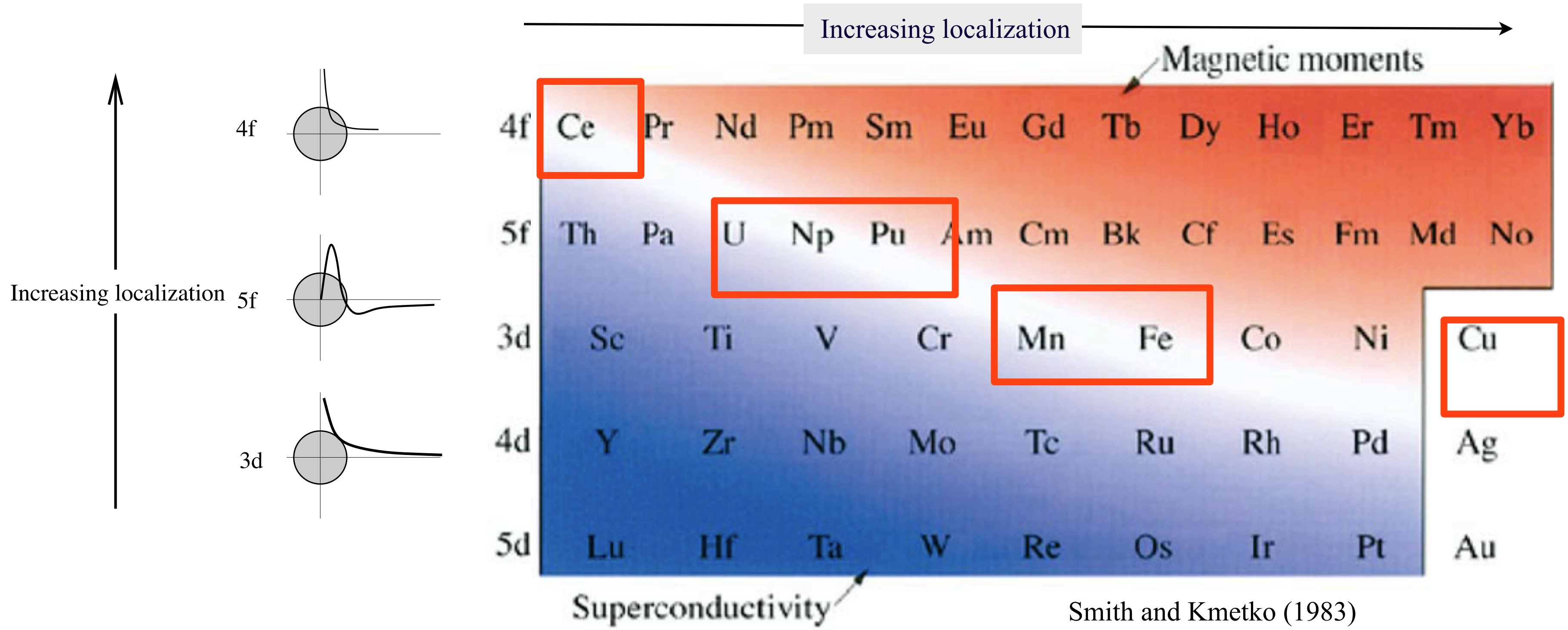
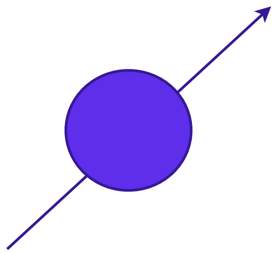
High Temperatures : local moment metals.



A lot of action takes place on the brink of localization!

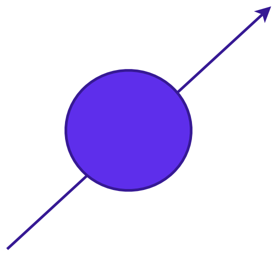
Heavy Fermions: f-spins are always localized, yet.....
High Temperatures : local moment metals.
Low Temperatures : Spins "quench" to form heavy fermions.

High Temperatures : local moments



Localized 4f or 5f Moment.

High Temperatures : local moments



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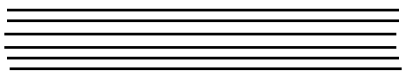
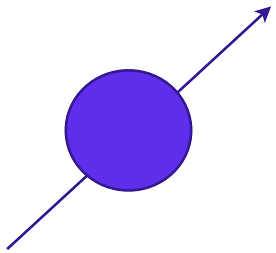


Low lying magnetic multiplet
N = 2j +1

High Temperatures : local moments



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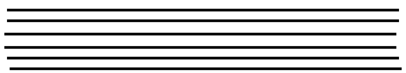

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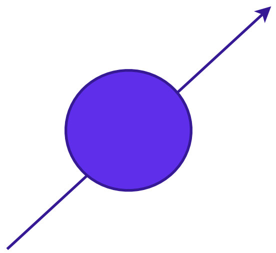
$N = 2j + 1$

e.g Ce^{3+} $| 4f^1 : j\ m \rangle$

$L=3$, $S = 1/2$, $j = L-S = 5/2$

$N = 6.$

High Temperatures : local moments



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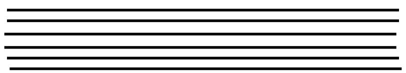
$$\chi = n_i \frac{M^2}{3T}$$

$$M^2 = g_J^2 \mu_B^2 J(J+1),$$



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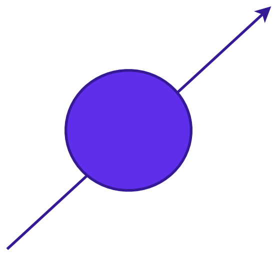

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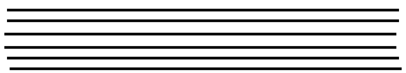
$$\chi = n_i \frac{M^2}{3T} \qquad M^2 = g_J^2 \mu_B^2 J(J+1),$$

$$S_Q = k_B \ln(2J+1) \qquad \text{spin entropy}$$



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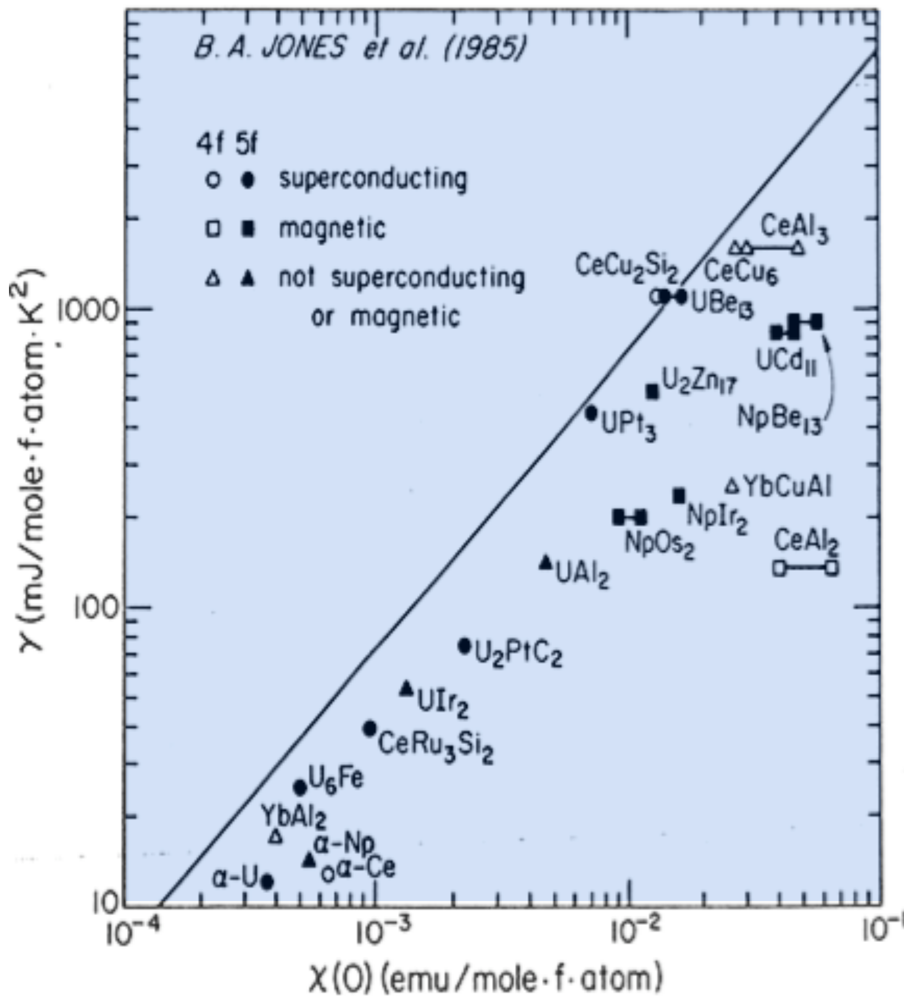

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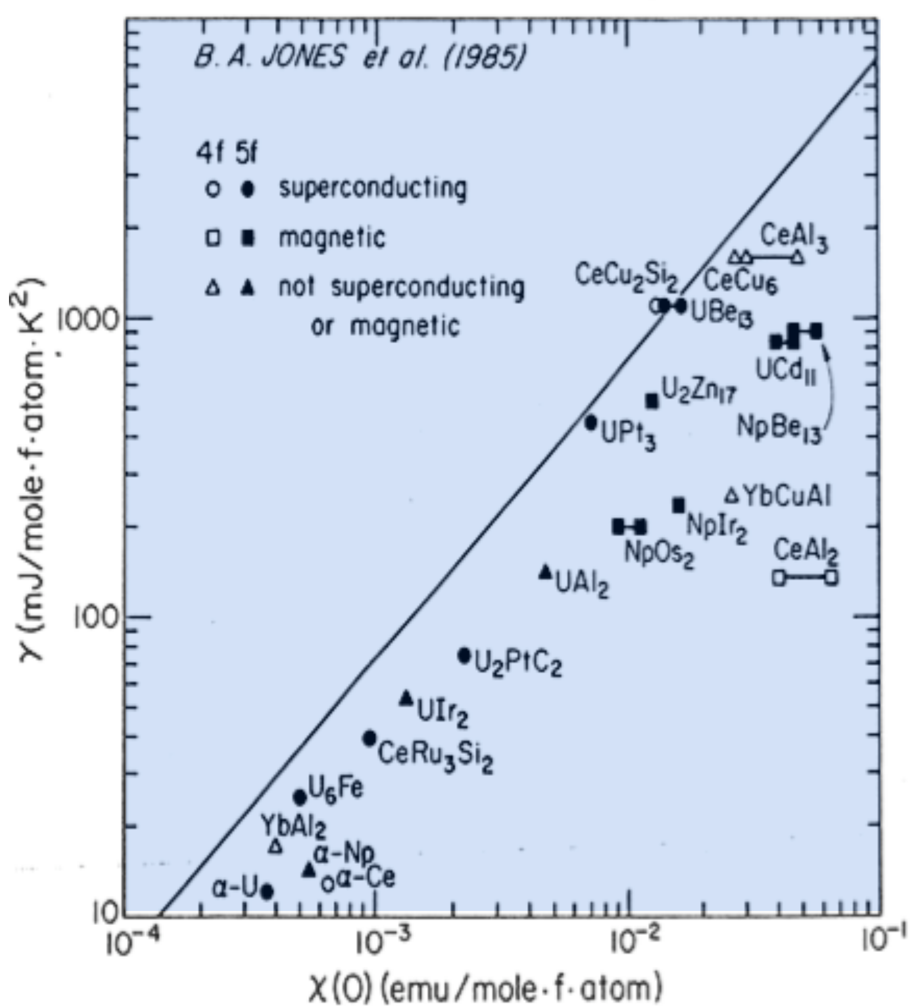

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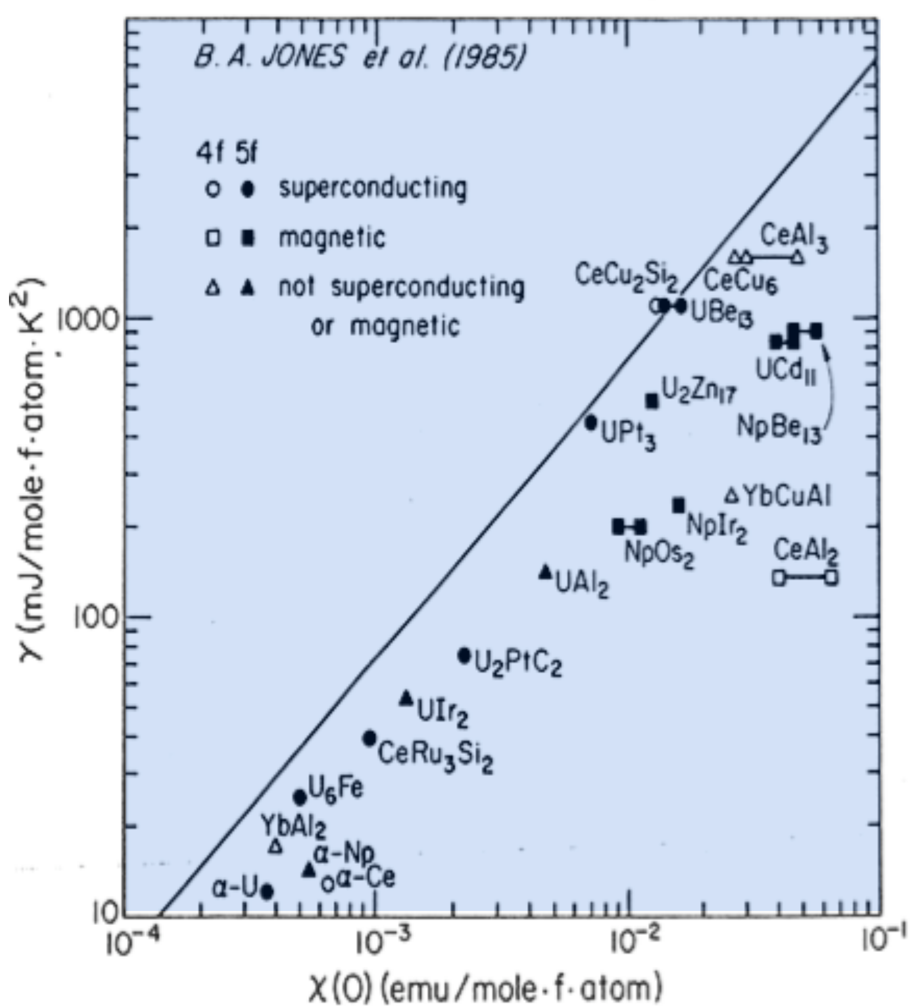
$$W = \frac{\chi}{\gamma} = 3\left(\frac{\mu_{\mathrm{B}}}{2\pi k_B}\right)^2 \frac{1}{1 + F_0^a}$$

"Wilson" or "Sommerfeld" ratio.



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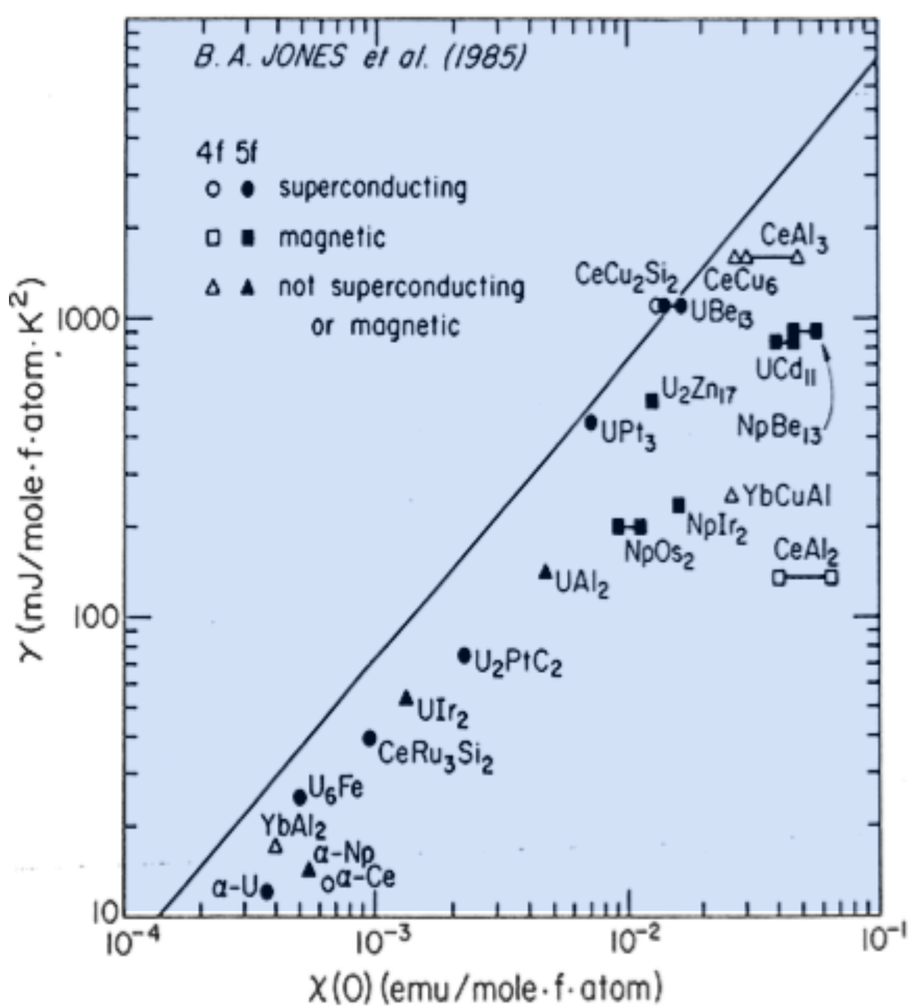

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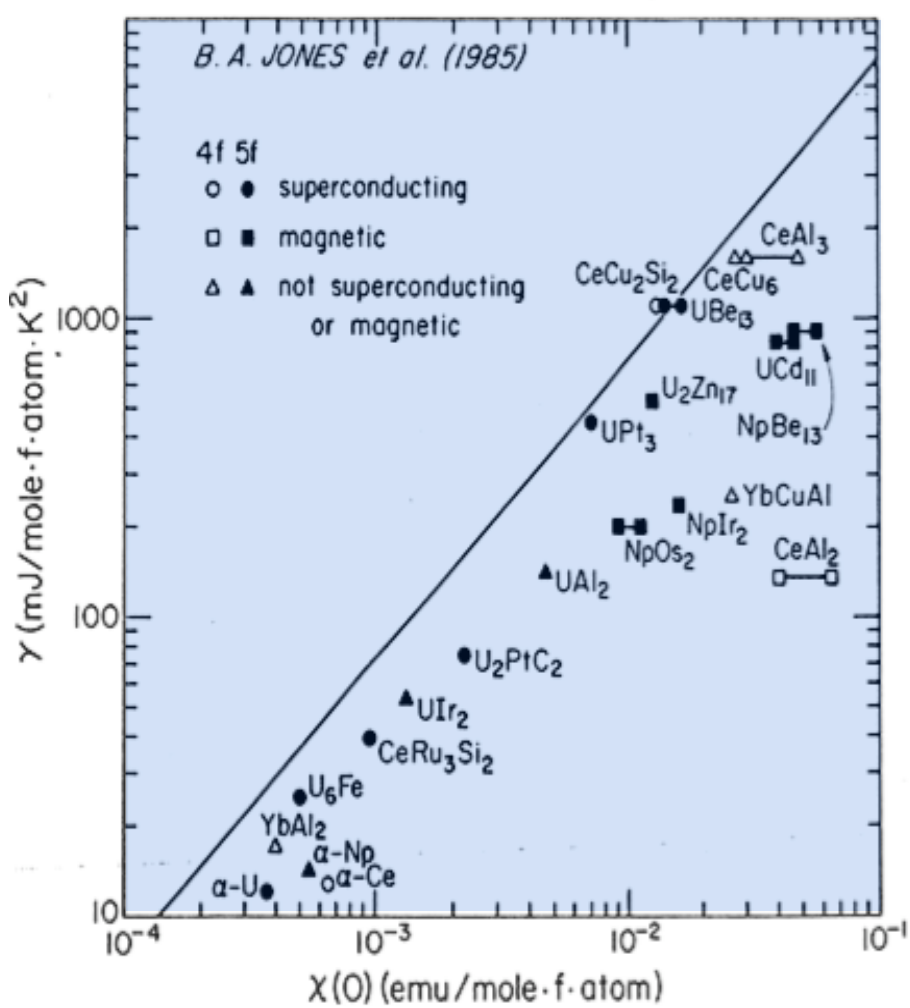

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γCu ~ 1 mJ/mol/K^2,

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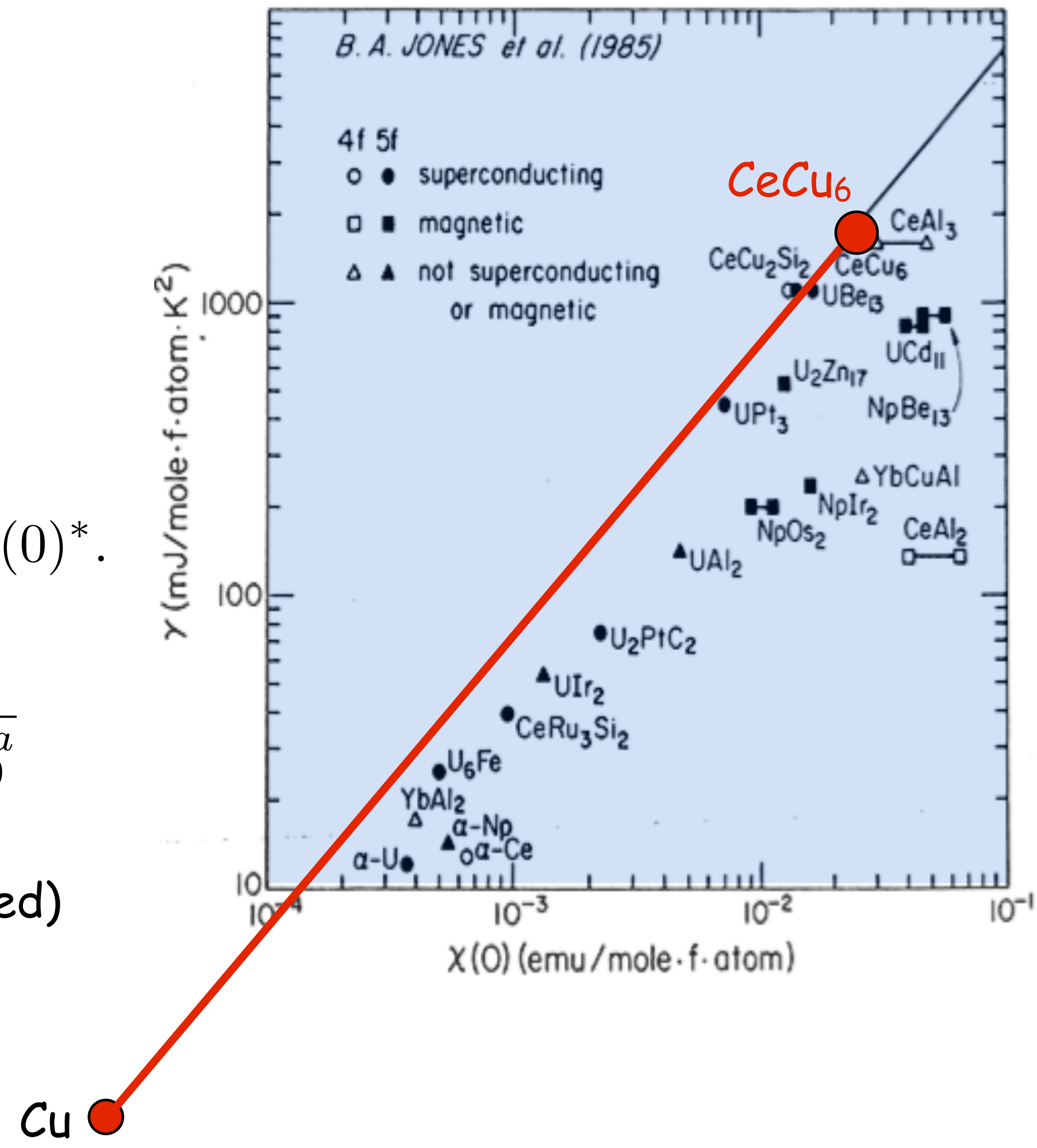
eg Cu vs $CeCu_6$ (copper, spin doped)

γCu ~ 1 mJ/mol/K^2,

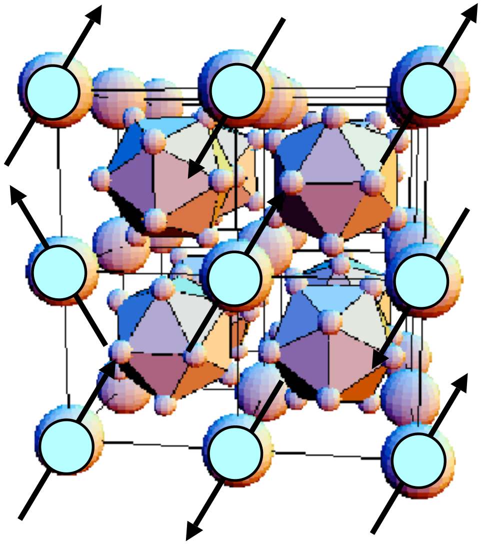
$\gamma[CeCu_6]$ ~ 1000 mJ/mol/K^2,

m^*/m_e ~ 1000

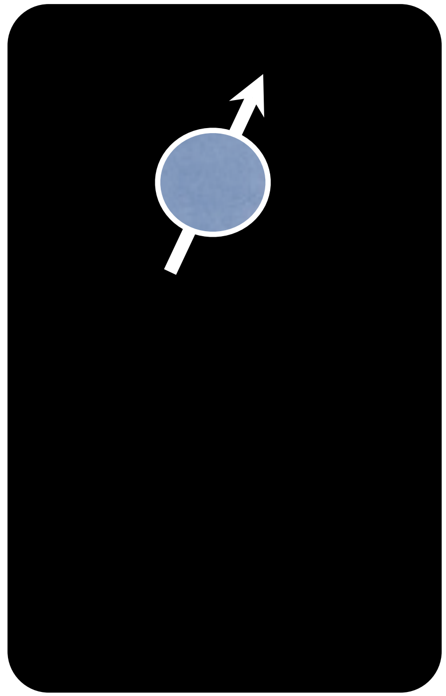


Heavy Fermion Metals

Review: cond-mat/0612006



UBe_{13}



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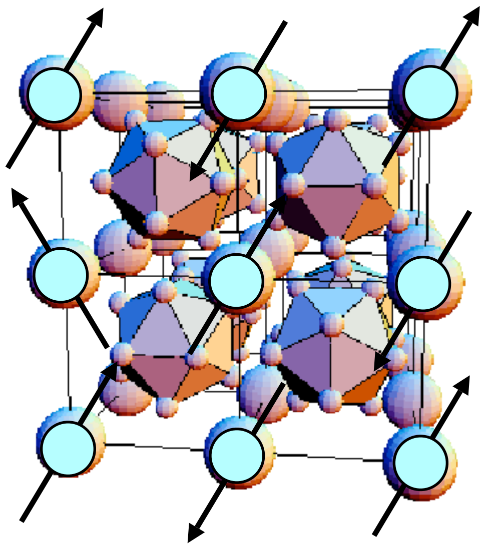

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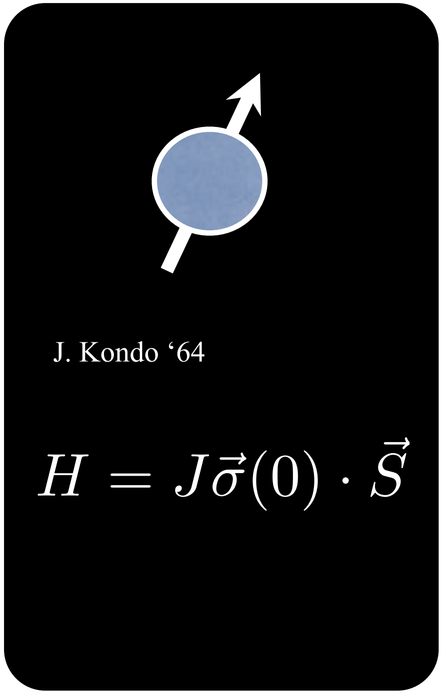

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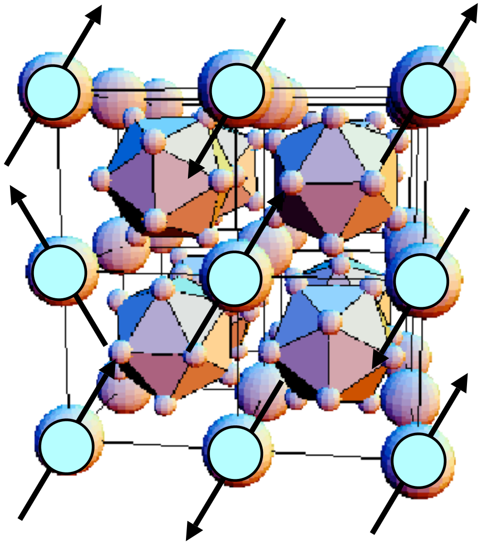

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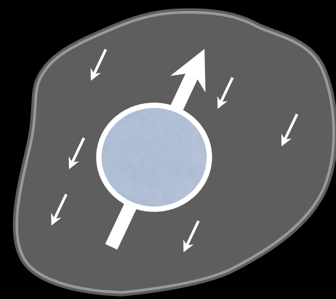

J. Kondo '64

$$H = J\vec{\sigma}(0)\cdot\vec{S}$$

$$T_K \sim \epsilon_F e^{-\frac{\epsilon_F}{J}}$$

"Kondo Effect"

Heavy Fermion Metals

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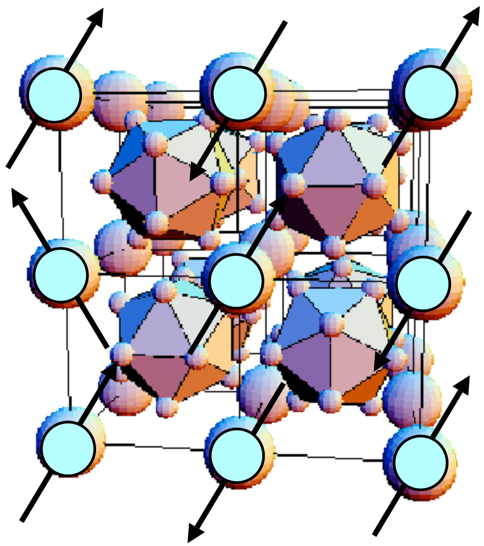

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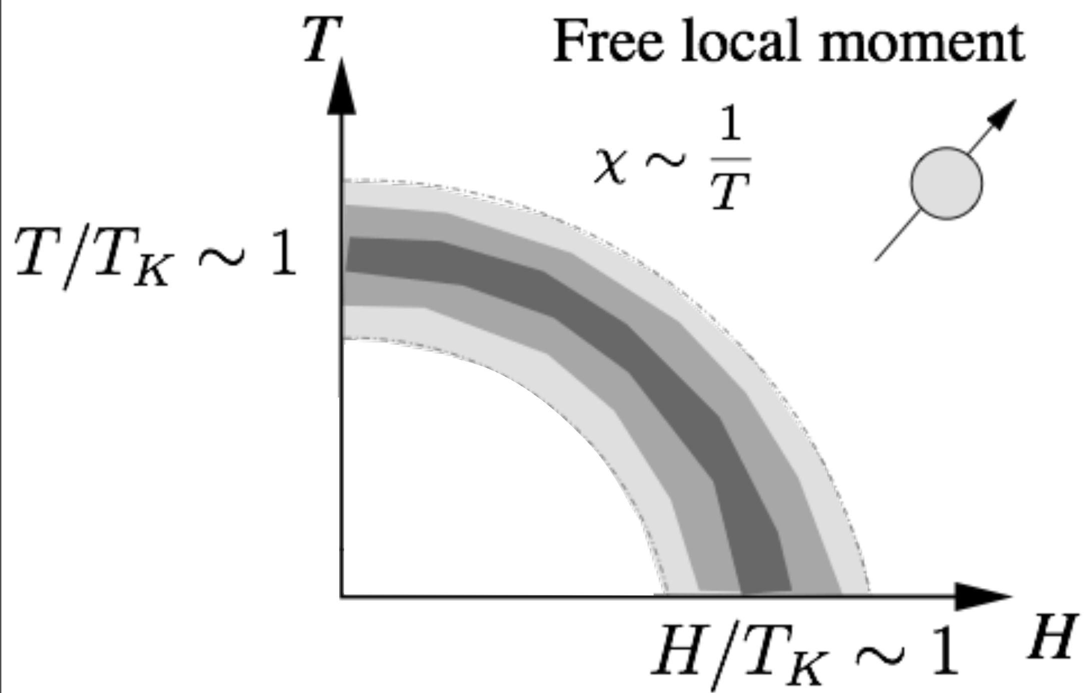

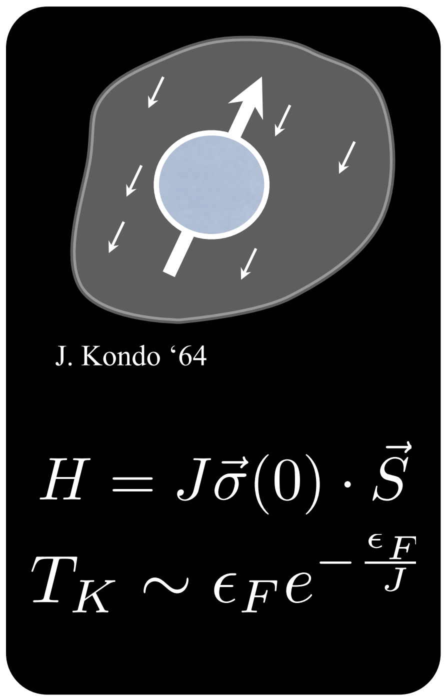

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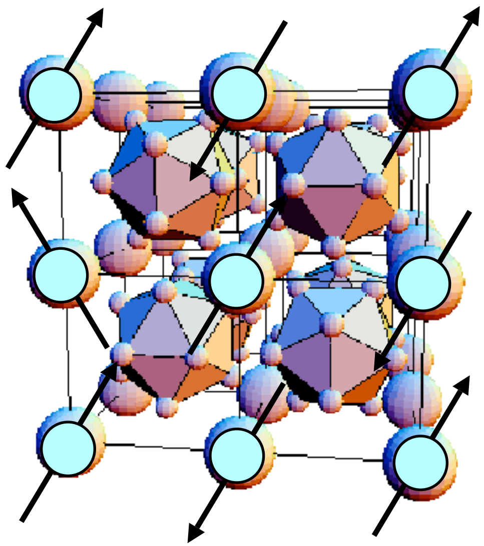

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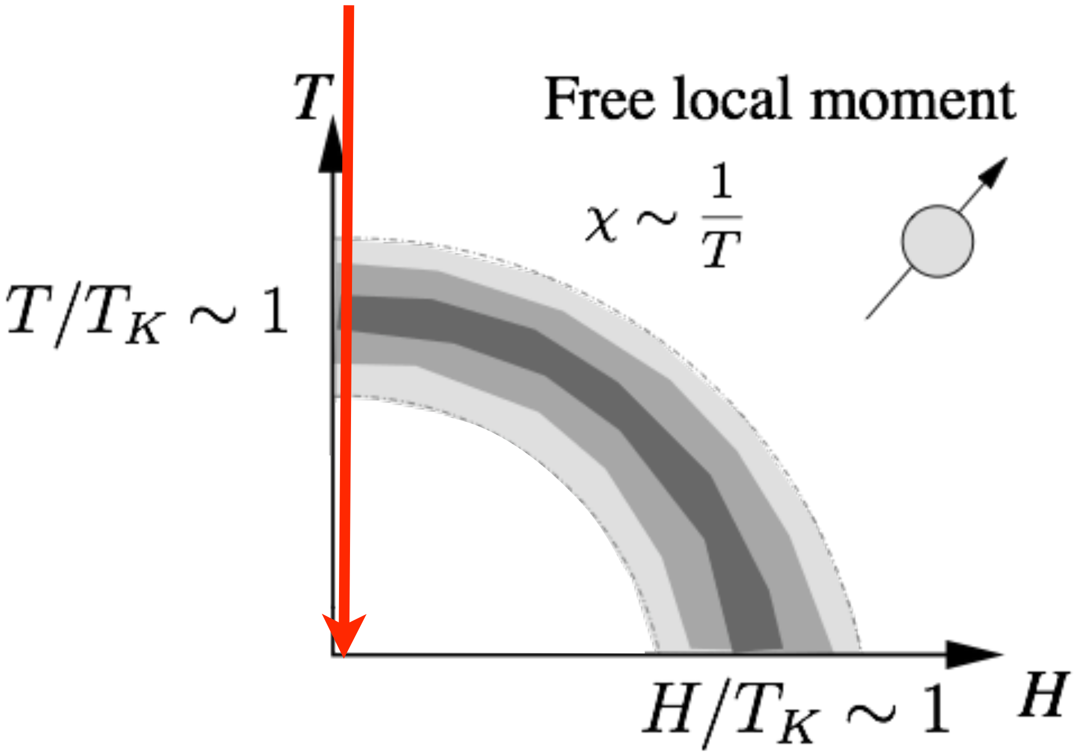

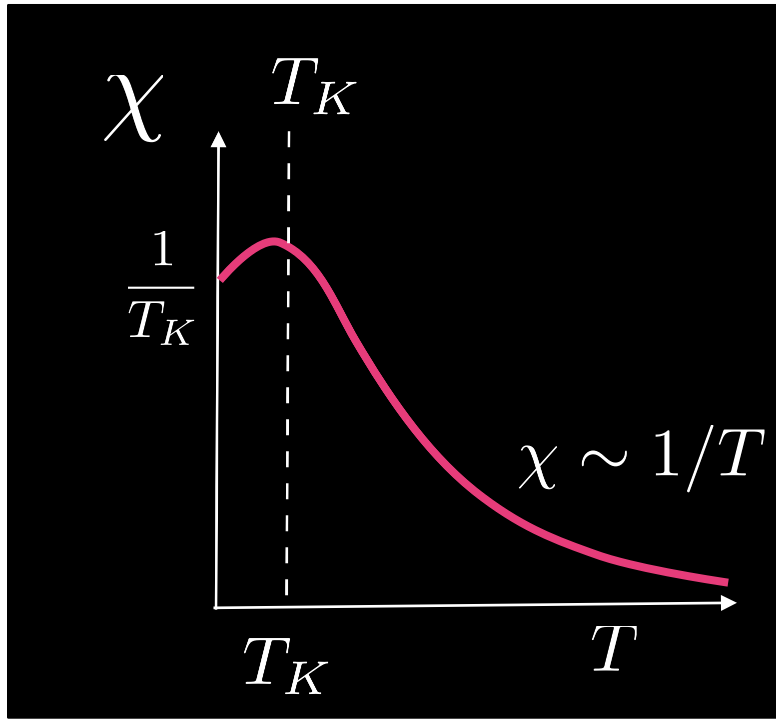

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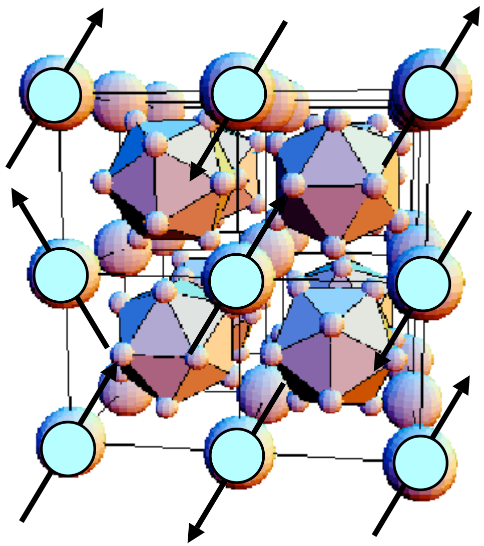

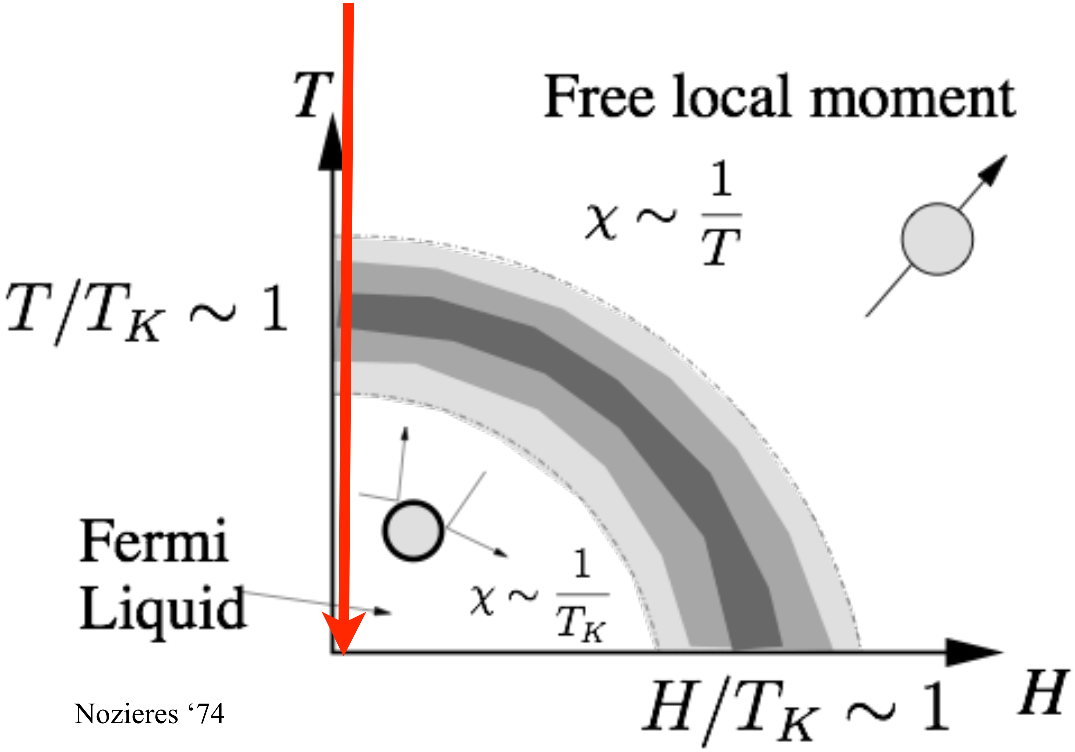

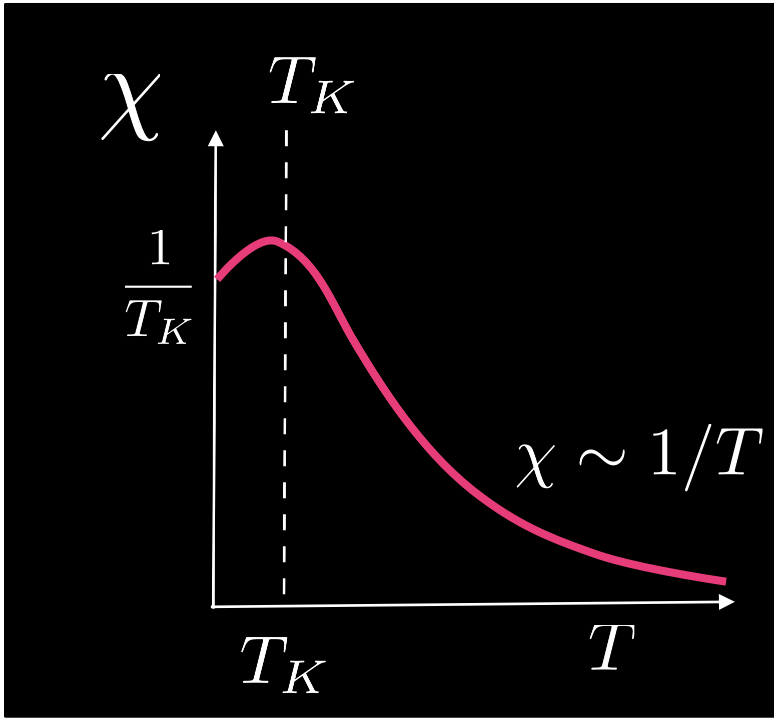

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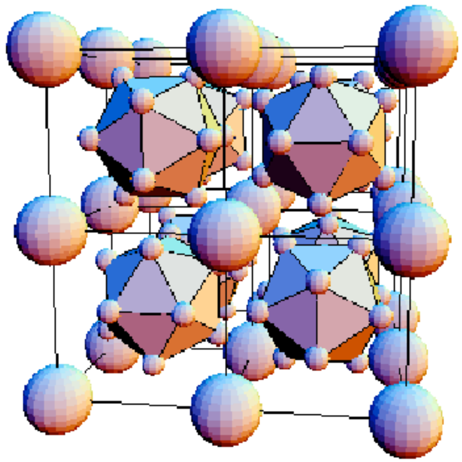

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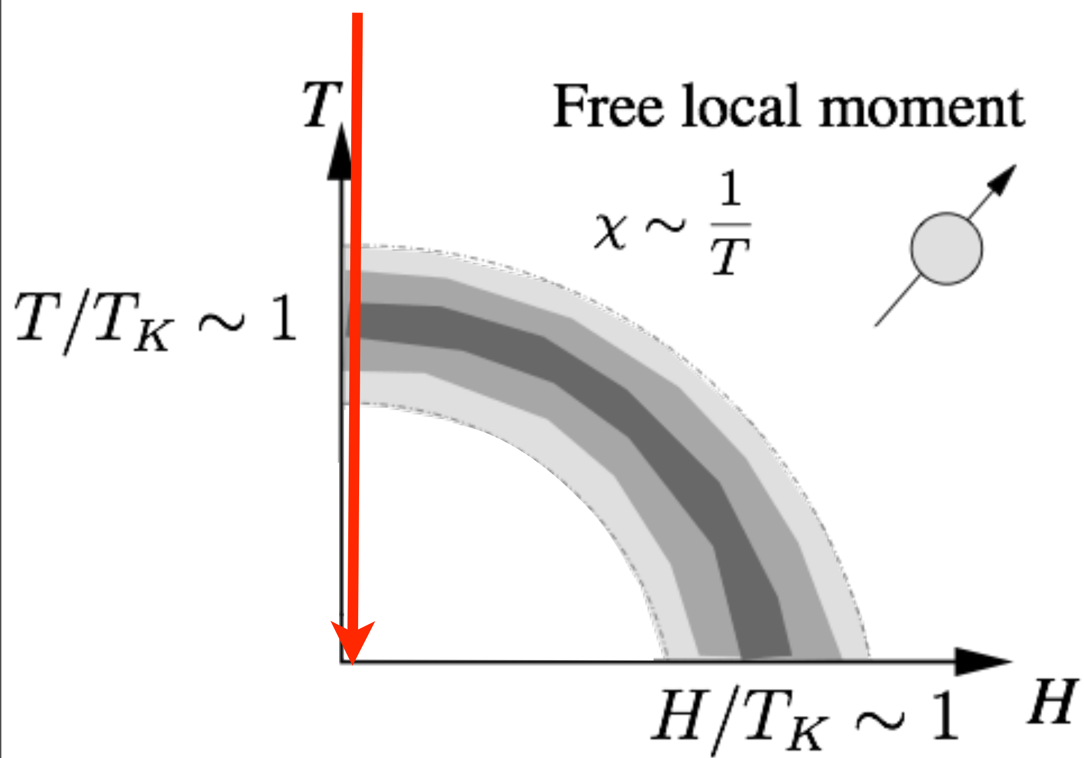

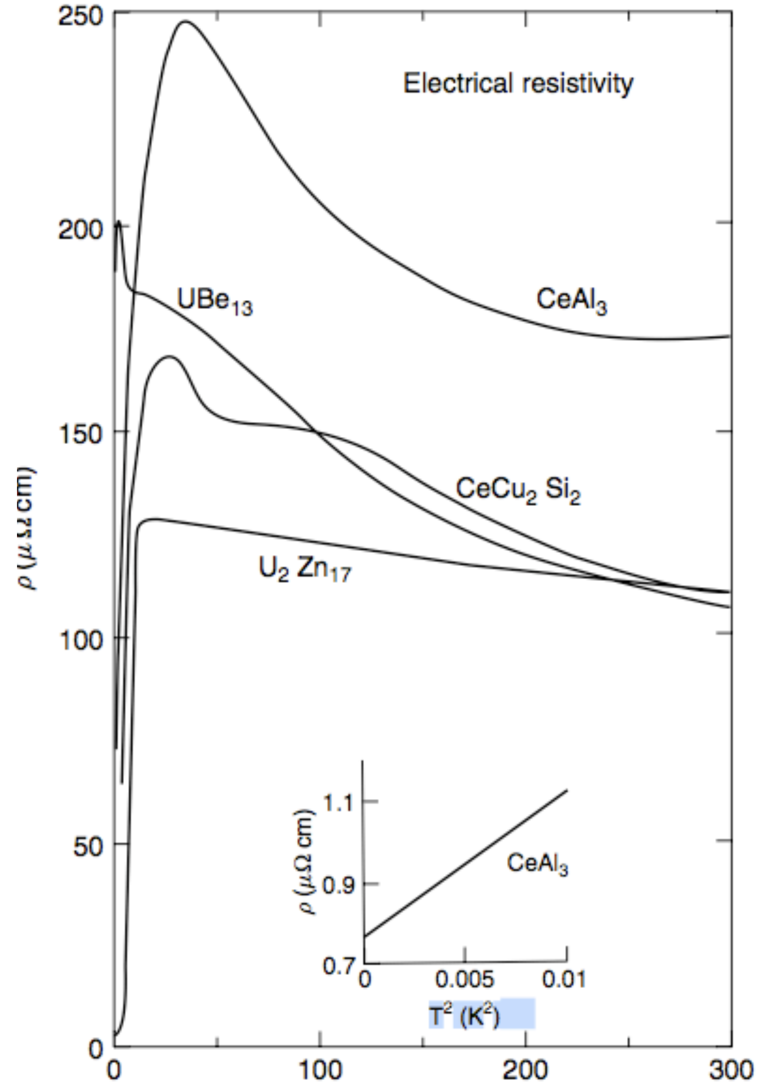

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Metal	$CeCu_6$	$10K$	-	Simple HF Metal	T^2	1600	[1]
Super-conductors	$CeCu_2Si_2$	$20K$	T_c=0.17K	First HFSC	T^2	800-1250	[2]
	UBe_{13}	2.5K	T_c=0.86K	Incoherent metal→HFSC	$\rho_c \sim 150\mu\Omega$cm	800	[3]
	$CeCoIn_5$	38K	T_c=2.3K	Quasi 2D HFSC	T	750	[4]
Kondo Insulators	$Ce_3Pt_4Bi_3$	$T_\chi \sim 80K$	-	Fully Gapped KI	$\sim e^{\Delta/T}$	-	[5]
	$CeNiSn$	$T_\chi \sim 20K$	-	Nodal KI	Poor Metal	-	[6]
Quantum Critical	$CeCu_{6-x}Au_x$	$T_0 \sim 10K$	$x_c = 0.1$	Chemically tuned QCP	T	$\sim \frac{1}{T_0} \ln\left(\frac{T_0}{T}\right)$	[7]
	$YbRh_2Si_2$	$T_0 \sim 24K$	$B_\perp$=0.06T $B_\parallel$=0.66T	Field-tuned QCP	T	$\sim \frac{1}{T_0} \ln\left(\frac{T_0}{T}\right)$	[8]
SC + other Order	UPd_2Al_3	110K	T_{AF}=14K, T_{sc}=2K	AFM + HFSC	T^2	210	[9]
	URu_2Si_2	75K	T_1=17.5K, T_{sc}=1.3K	Hidden Order & HFSC	T^2	120/65	[10]

Tour of Heavy Fermion Systems

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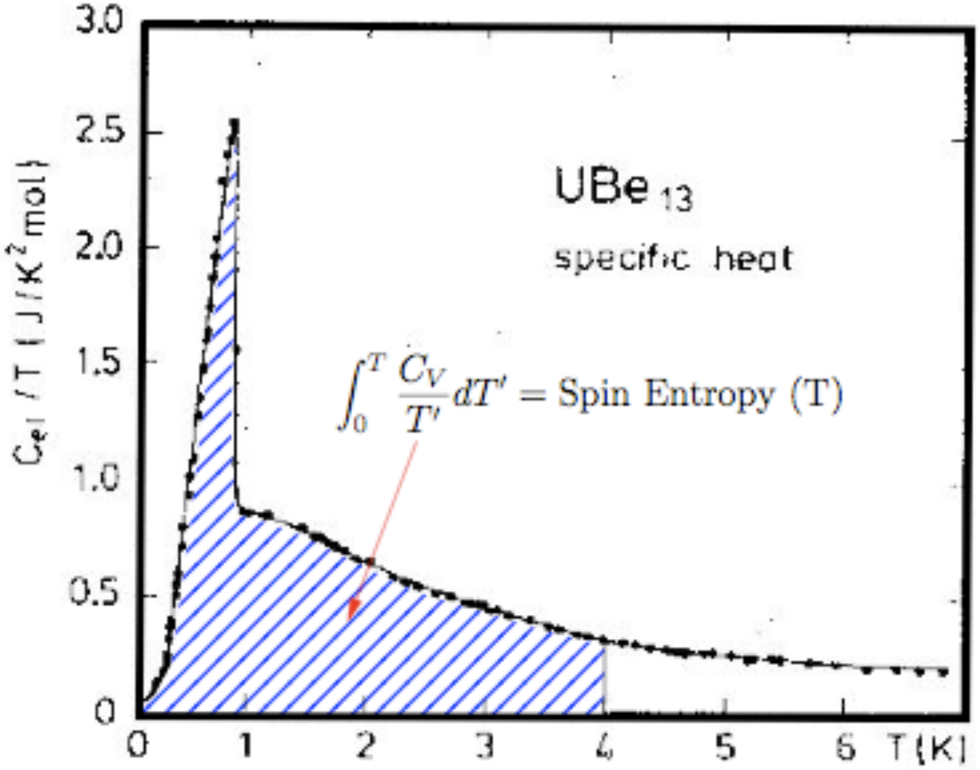
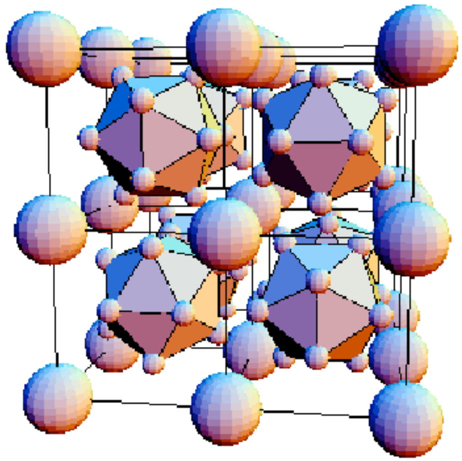


Ott et al, (1985)



Spin entropy contributes to the Superconducting and Fermi liquid thermodyanmics. Spins are forming the heavy fermions which are themselves pairing!

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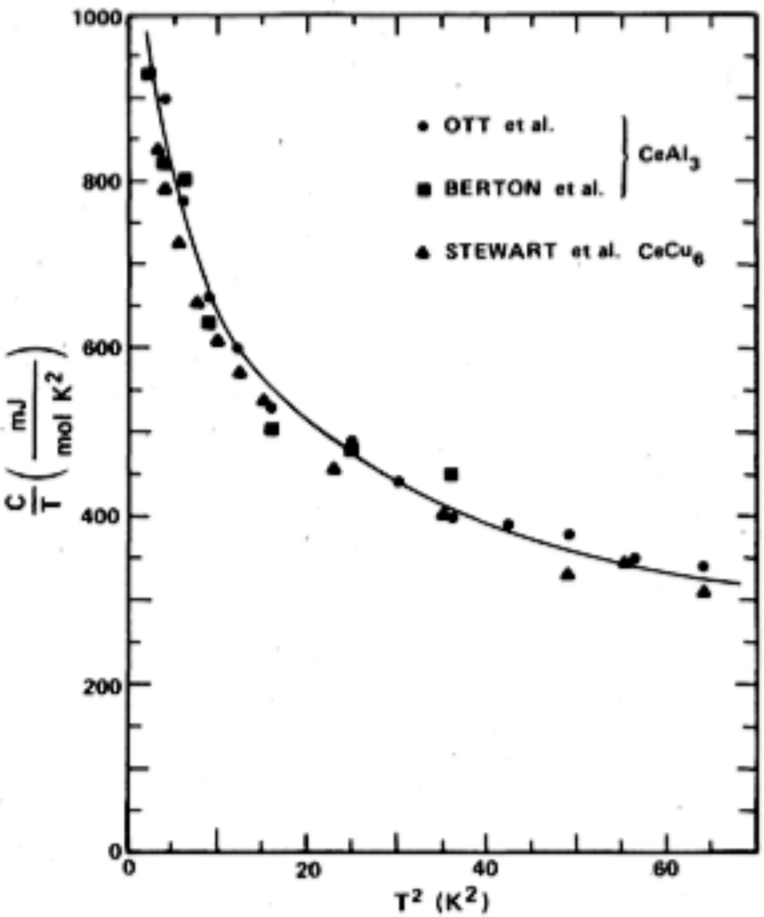
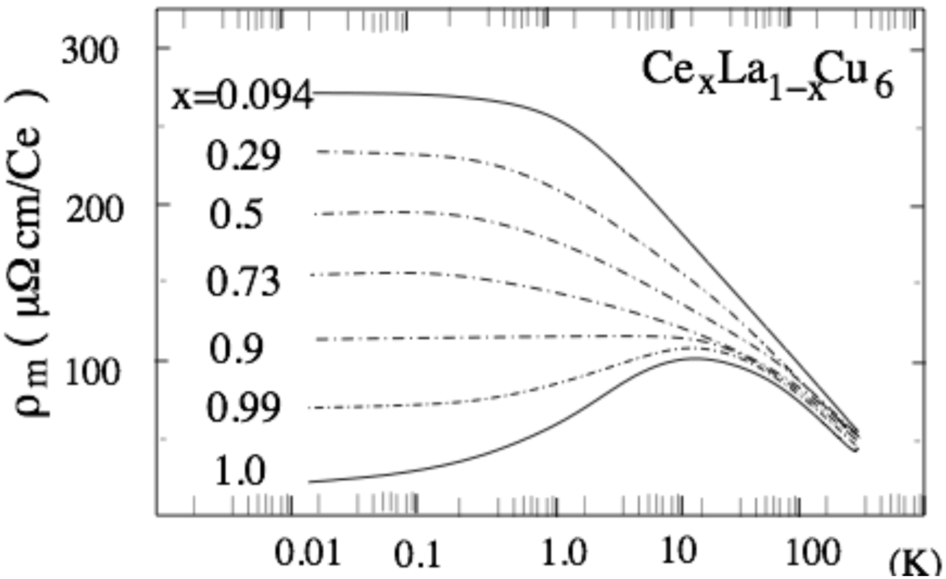
Center for Materials Theory

$CeCu_6$: From dilute impurity to dense "Kondo lattice", showing Development of coherence





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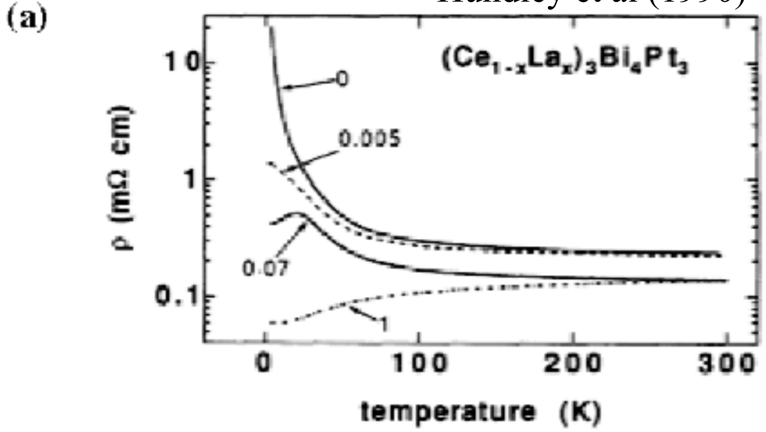
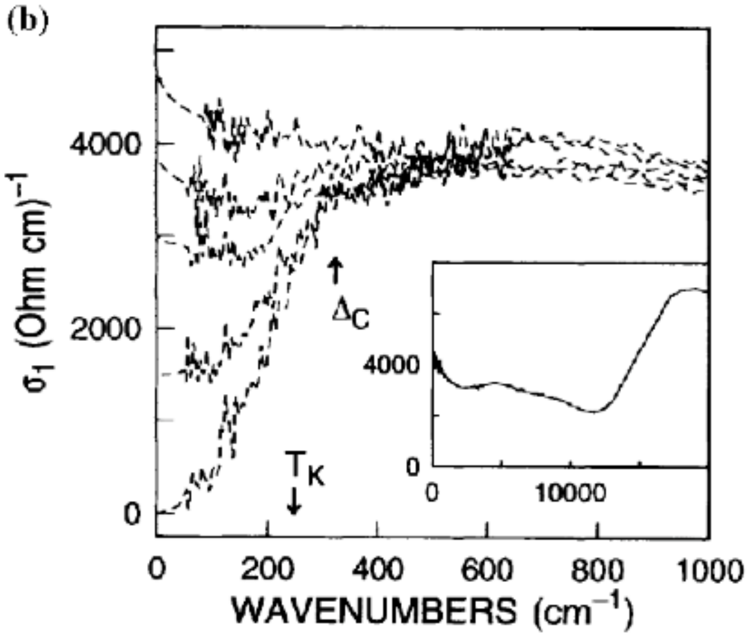
Metal	$CeCu_6$	$10K$	-	Simple HF Metal	T^2	1600	[1]
Super-conductors	$CeCu_2Si_2$	$20K$	T_c=0.17K	First HFSC	T^2	800-1250	[2]
	UBe_{13}	2.5K	T_c=0.86K	Incoherent metal→HFSC	$\rho_c \sim$ 150$\mu\Omega$cm	800	[3]
	$CeCoIn_5$	38K	T_c=2.3K	Quasi 2D HFSC	T	750	[4]
Kondo Insulators	$Ce_3Pt_4Bi_3$	$T_\chi \sim 80K$	-	Fully Gapped KI	$\sim e^{\Delta/T}$	-	[5]
	$CeNiSn$	$T_\chi \sim 20K$	-	Nodal KI	Poor Metal	-	[6]
Quantum Critical	$CeCu_{6-x}Au_x$	$T_0 \sim 10K$	$x_c = 0.1$	Chemically tuned QCP	T	$\sim \frac{1}{T_0}\ln\left(\frac{T_0}{T}\right)$	[7]
	$YbRh_2Si_2$	$T_0 \sim 24K$	$B_\perp$=0.06T $B_\parallel$=0.66T	Field-tuned QCP	T	$\sim \frac{1}{T_0}\ln\left(\frac{T_0}{T}\right)$	[8]
SC + other Order	UPd_2Al_3	110K	T_{AF}=14K, T_{sc}=2K	AFM + HFSC	T^2	210	[9]
	URu_2Si_2	75K	T_1=17.5K, T_{sc}=1.3K	Hidden Order & HFSC	T^2	120/65	[10]

$CeBi_4Pt_3$: Kondo Insulator.

Hundley et al (1990)





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115 Materials: layered heavy electron superconductors.

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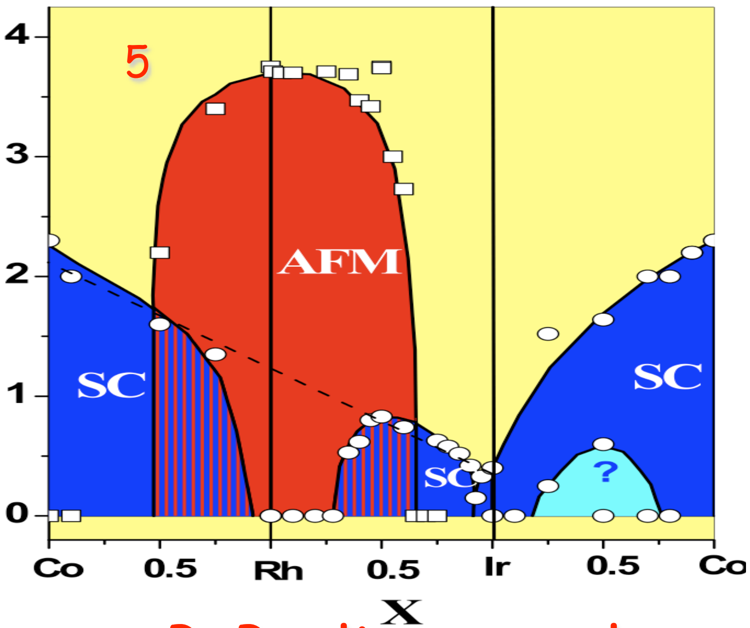

P. Pagliuso et al.

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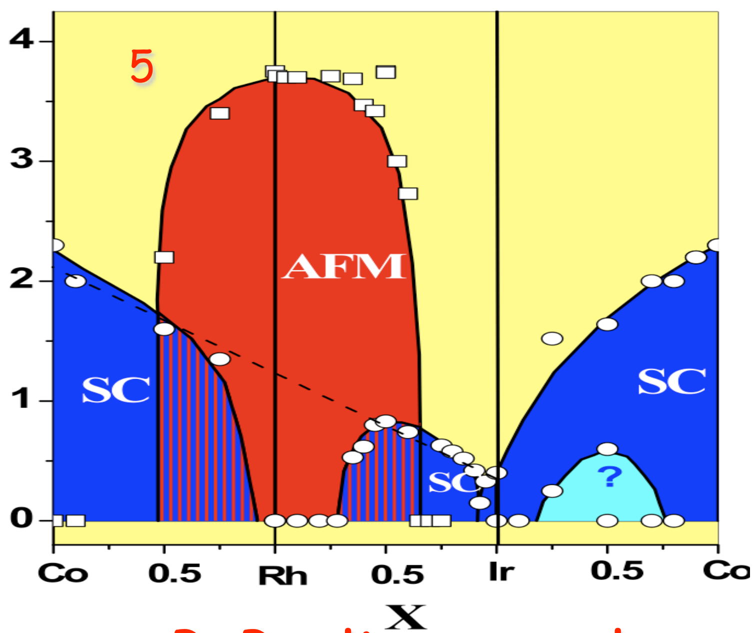
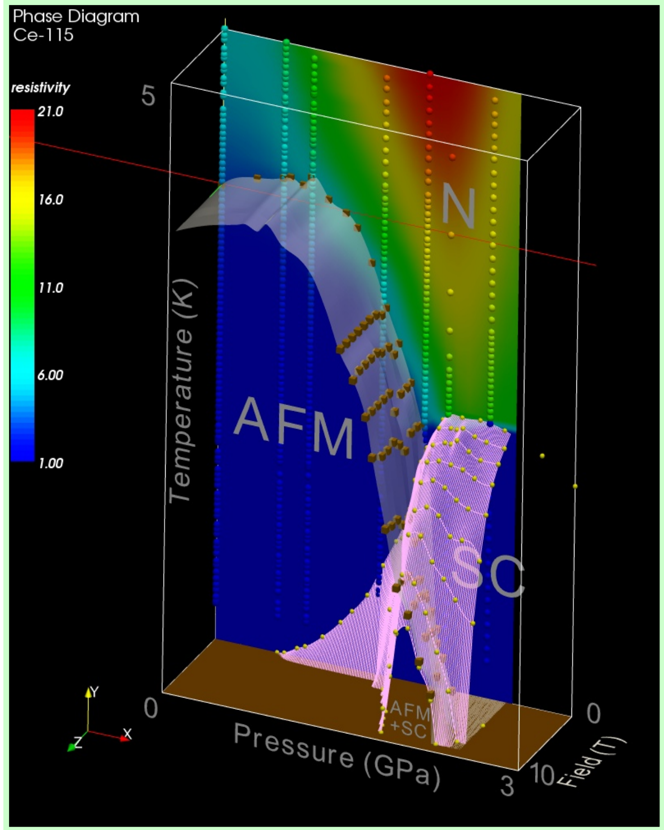


P. Pagliuso et al.

CeRhIn5



Data: Tuson Park
Figure rendition: Mathias Graf

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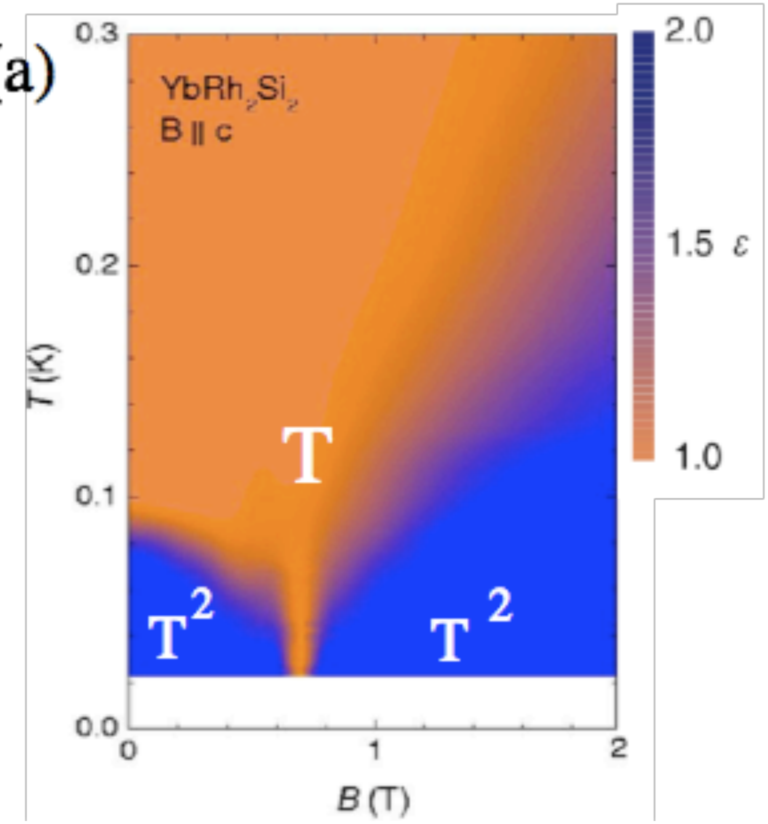
$YbRh_2Si_2$: Field tuned quantum criticality.

(a)



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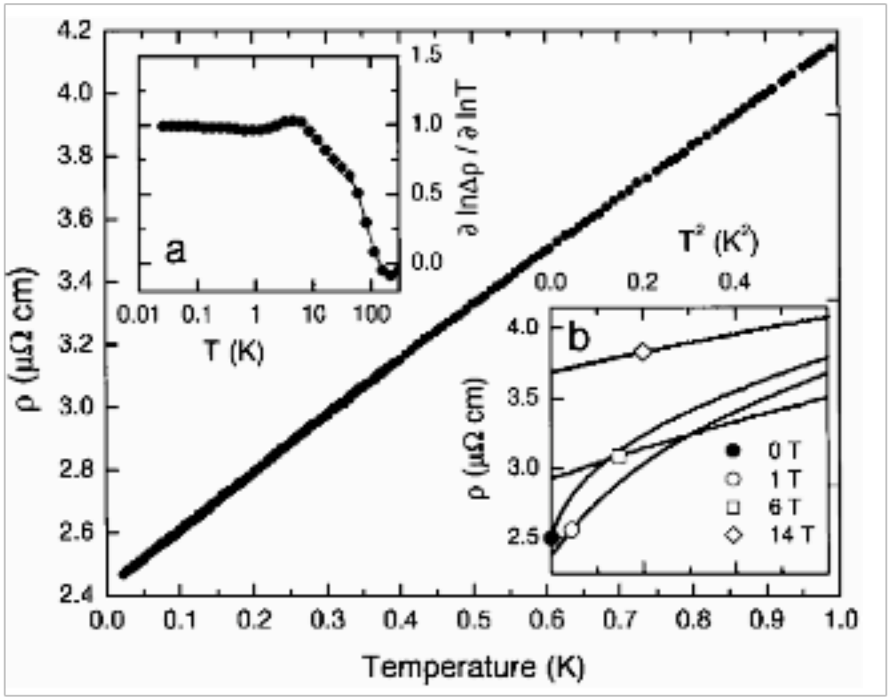

Custers et al, (2003)

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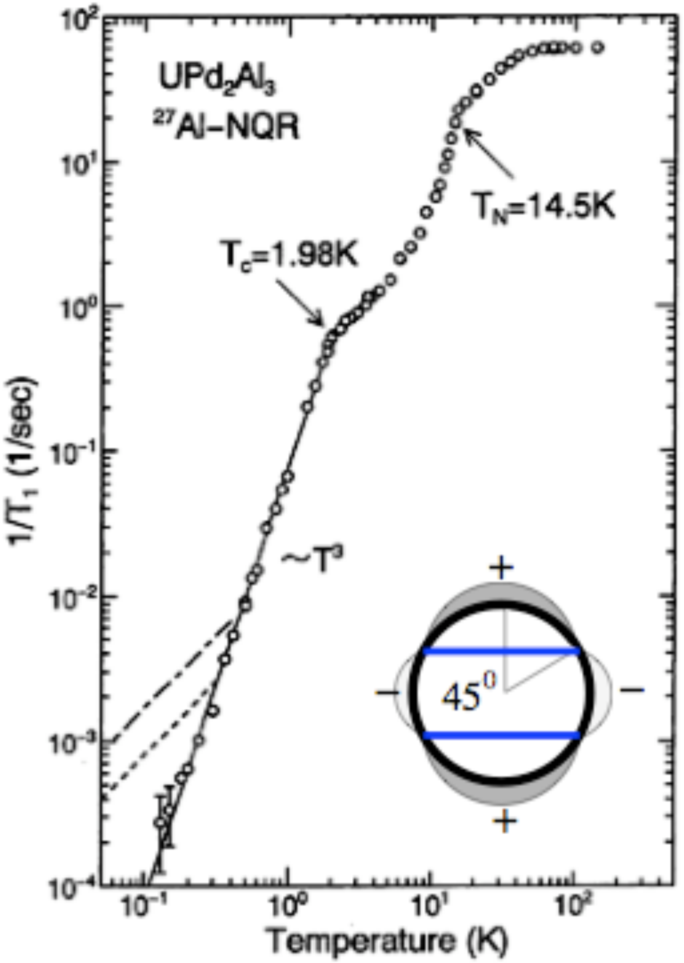
UPd_2Al_3 : Coexistent antiferromagnetism and nodal superconductivity.



Landau Fermi Liquid Theory

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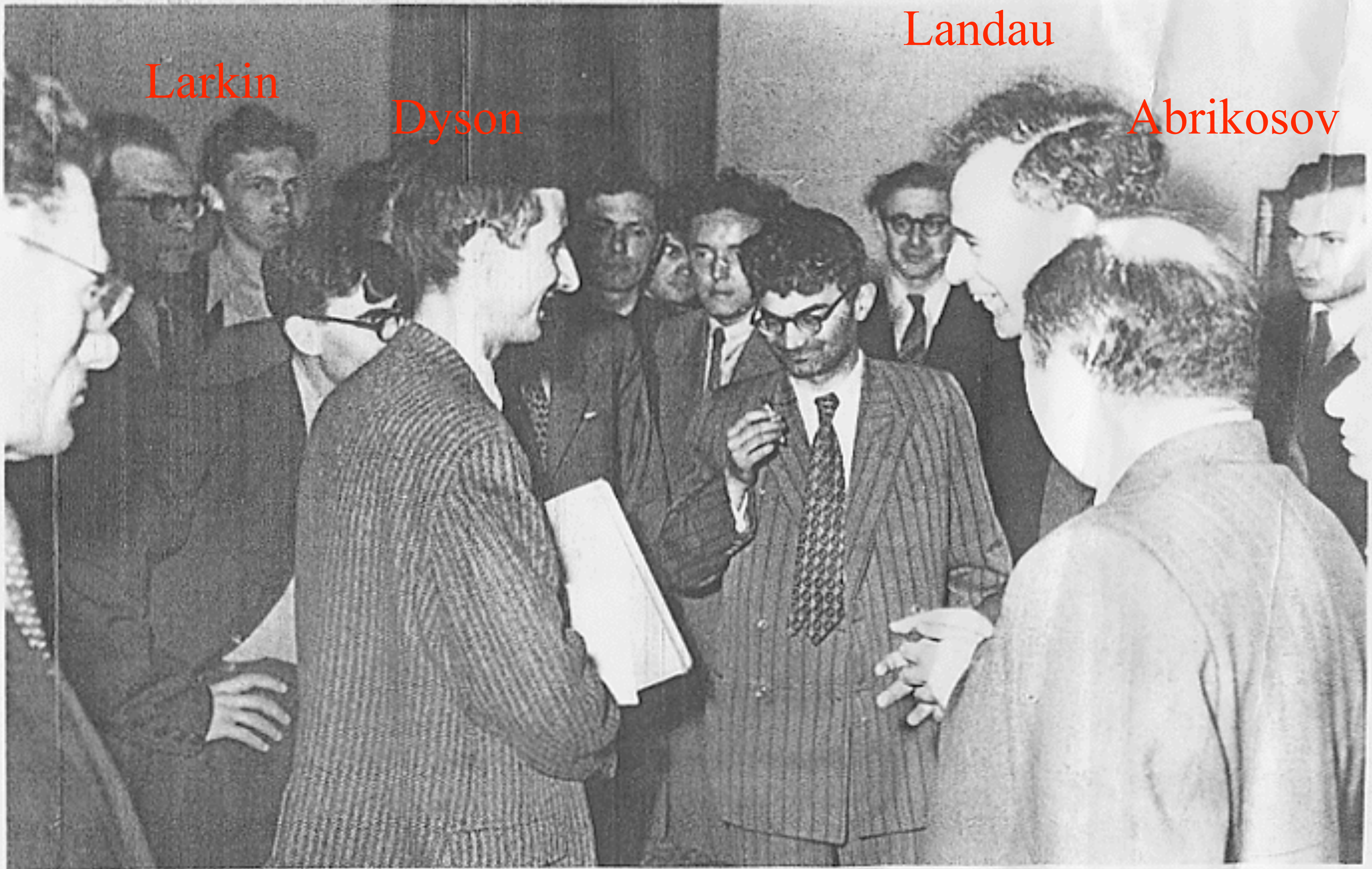

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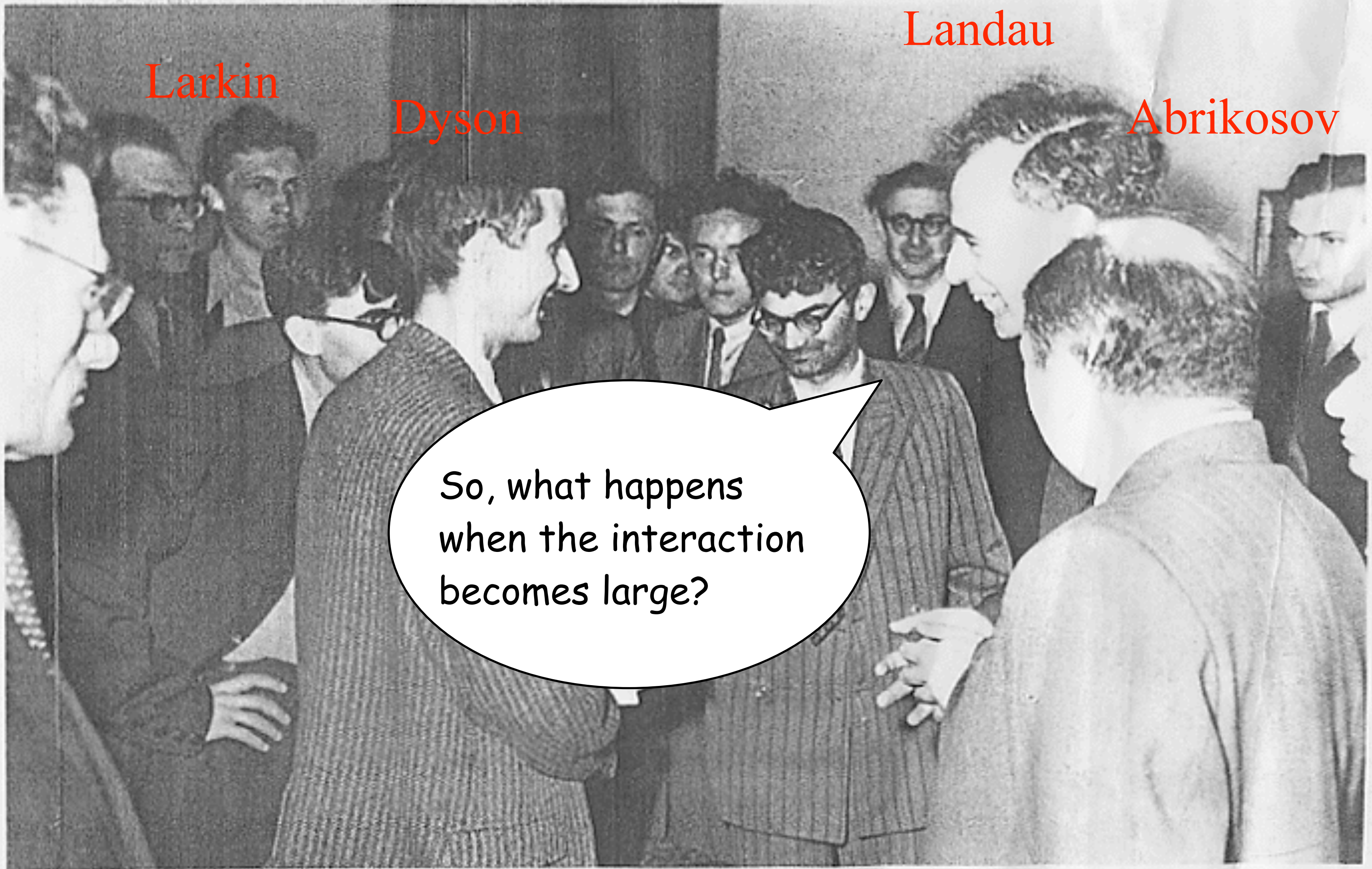

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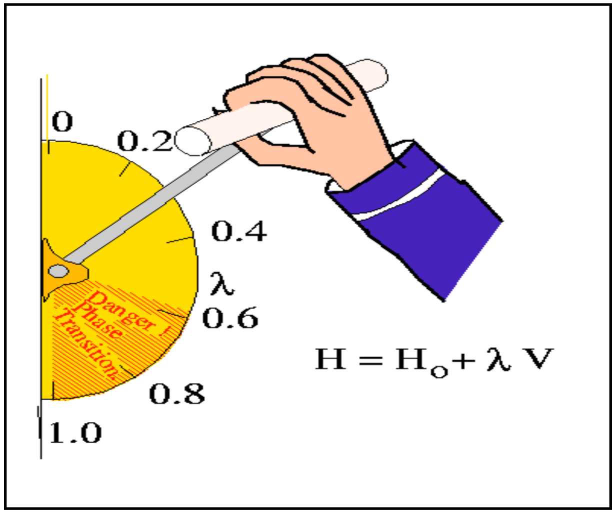

Interactions can be turned on adiabatically, preserving the excitation spectrum.

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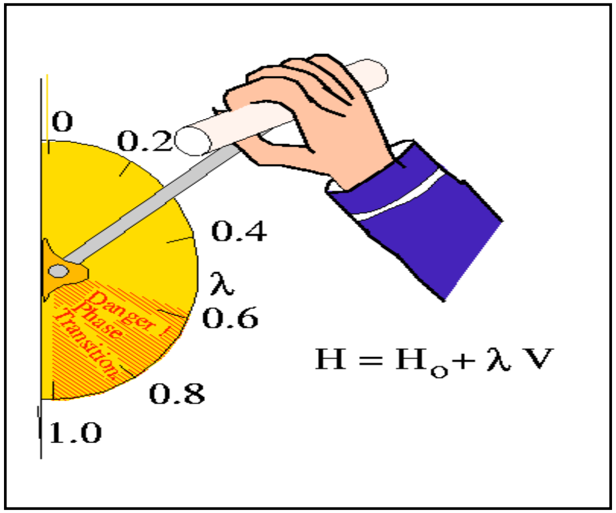

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States labelled by same quantum nos as non-interacting Fermi liquid

$$\Psi = \left| n_{\mathrm{p}_1\sigma_1},\ n_{\mathrm{p}_2\sigma_2}, \ldots \right\rangle$$

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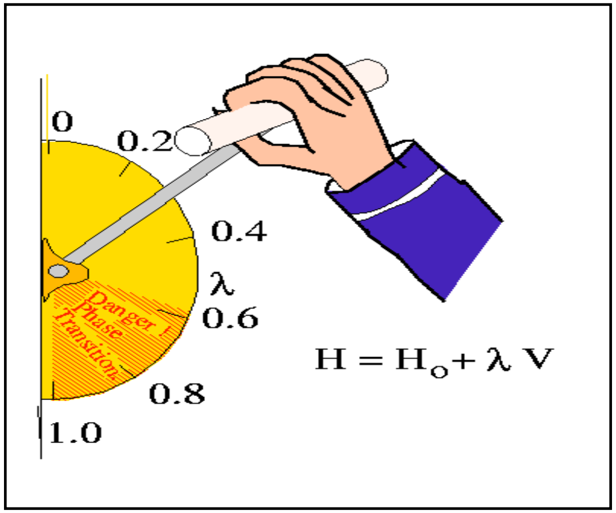

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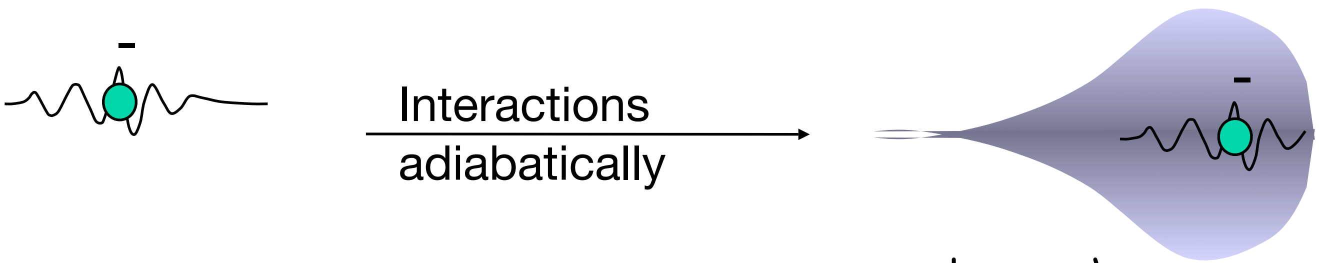
“Quasiparticle”



$$|e^-\rangle \qquad\qquad |qp^-\rangle$$

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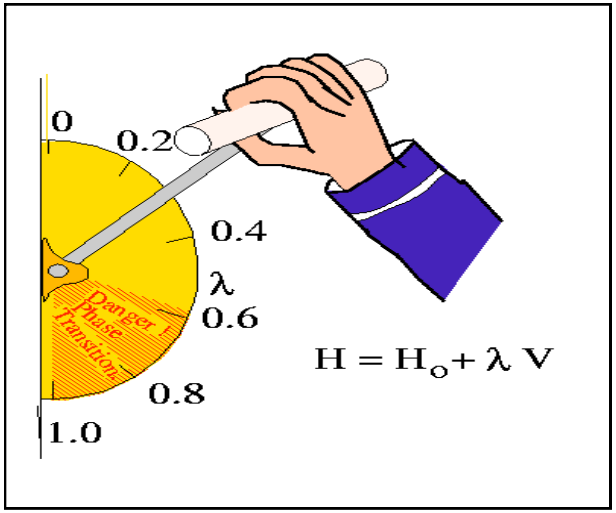

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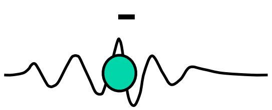
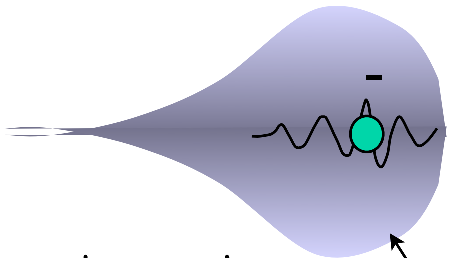


$\xrightarrow{\text{Interactions adiabatically}}$



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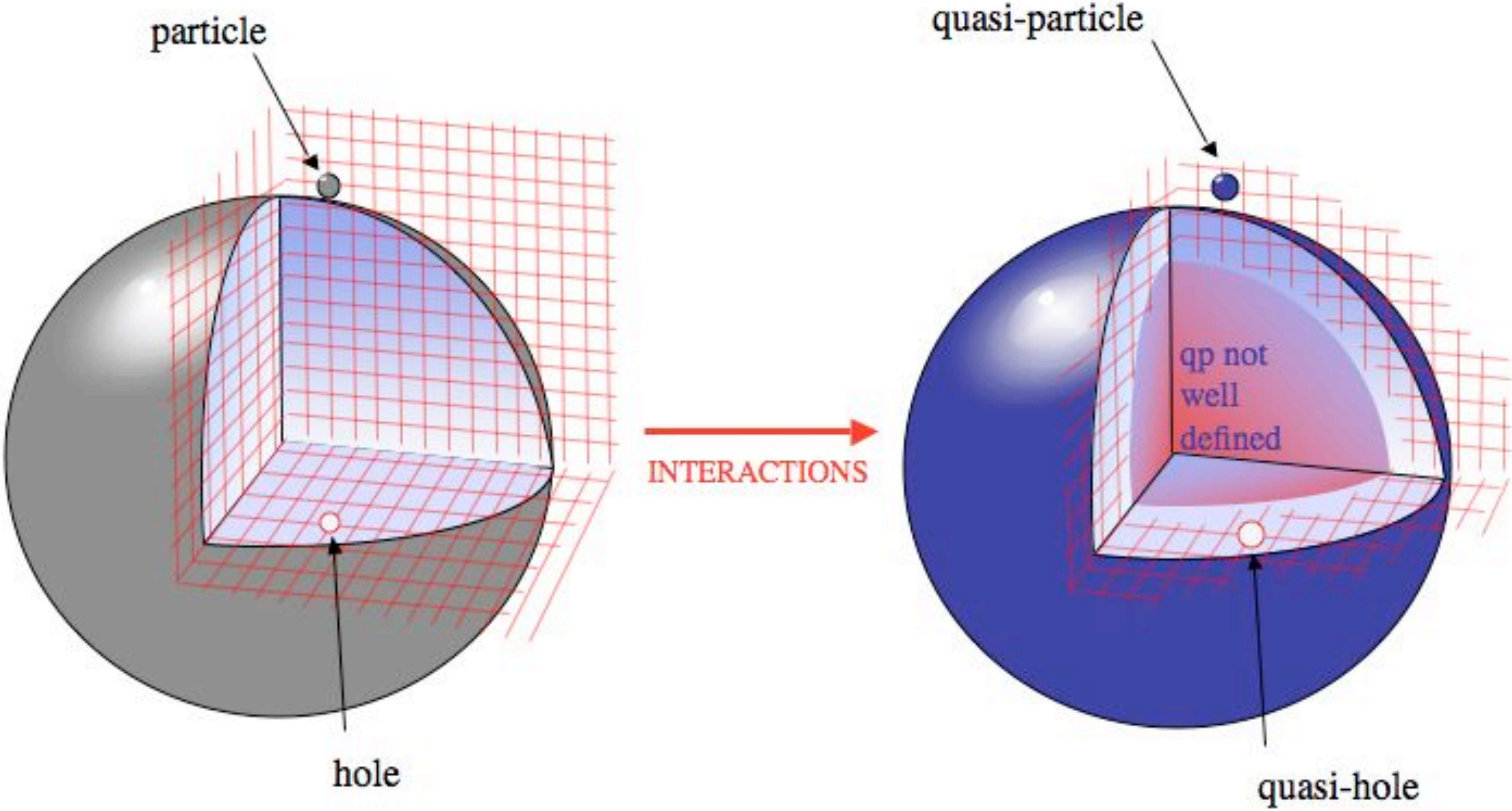
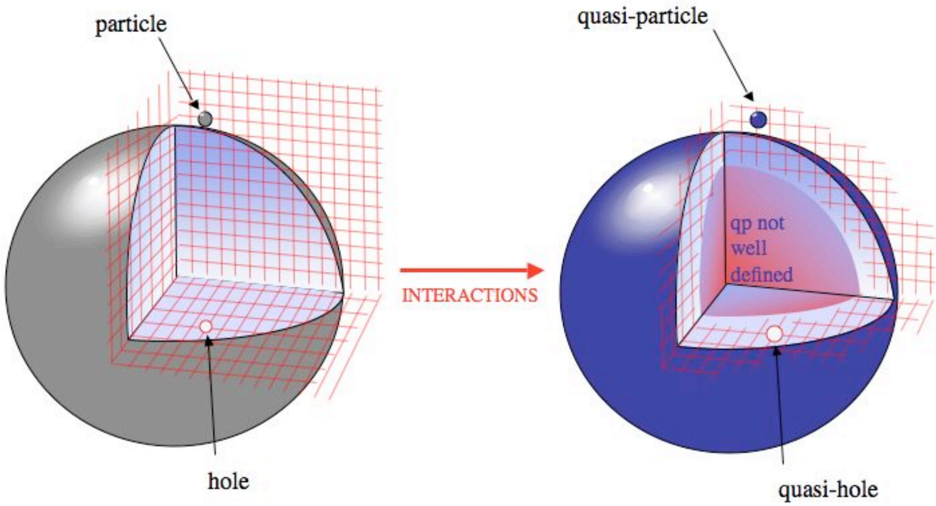
$$\frac{m^*}{m} = \frac{N(0)^*}{N(0)} = 1 + \frac{F_1^s}{3}$$



(a) Fermi Liquid

(b) Landau Fermi Liquid



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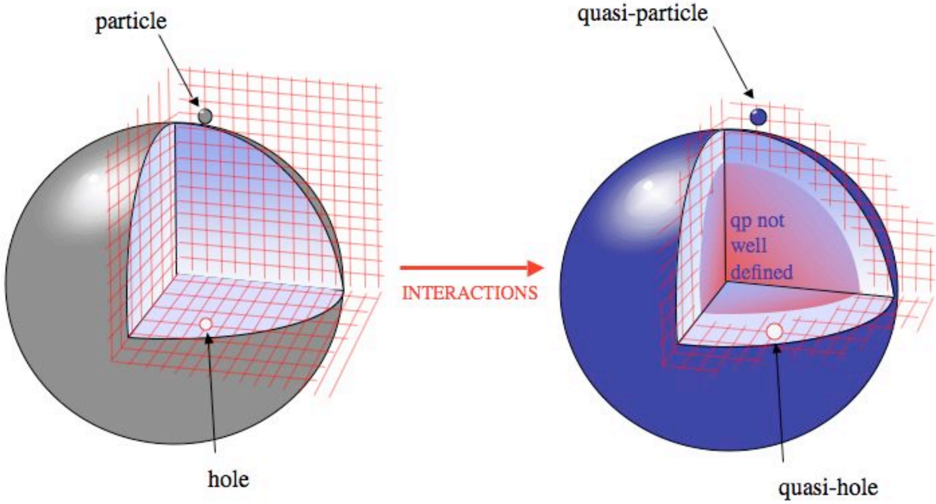

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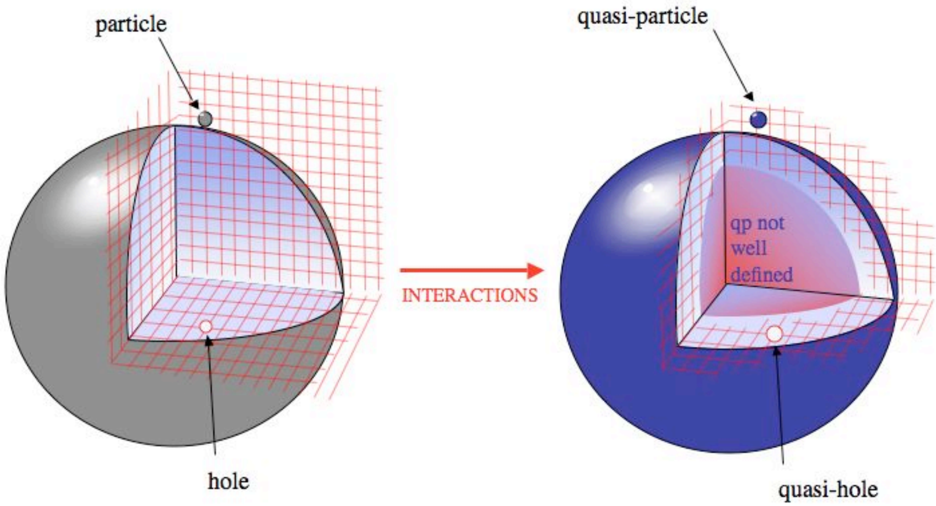

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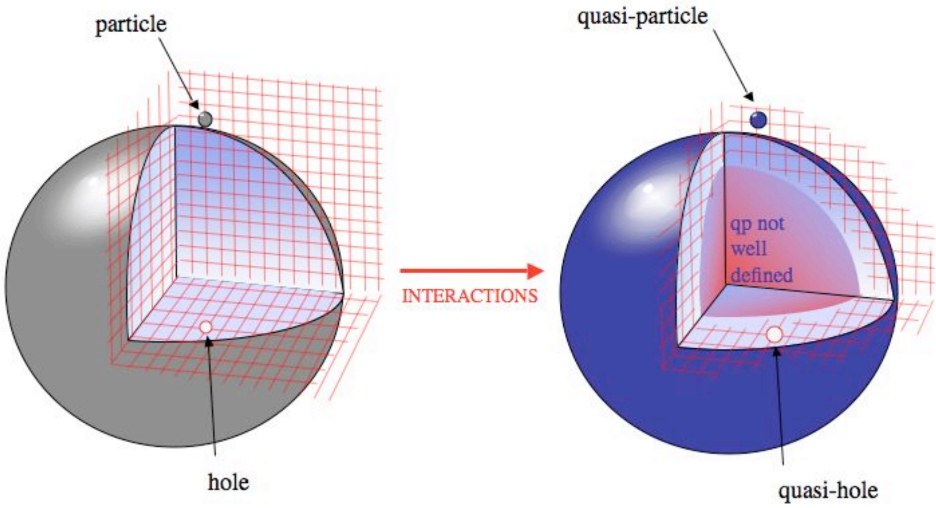

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$$\epsilon^{(0)}_{\mathrm{p_o}} = E^{(0)}_{\mathrm{p_o}} - \mu = \mathcal{E}(p_o) - \mathcal{E}_o$$ Quasiparticle excitation energy.

Key observation of Landau:

Pauli principle drives quasiparticle scattering rate to zero at the Fermi surface. Quasiparticles are well defined at the Fermi surface.

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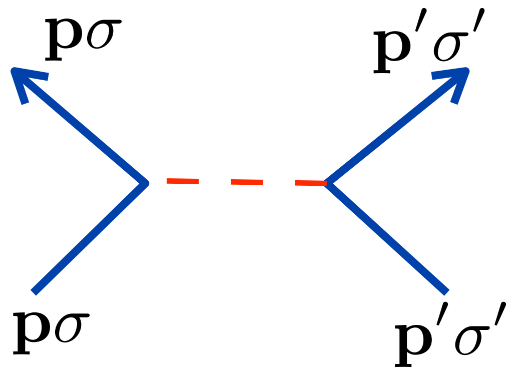

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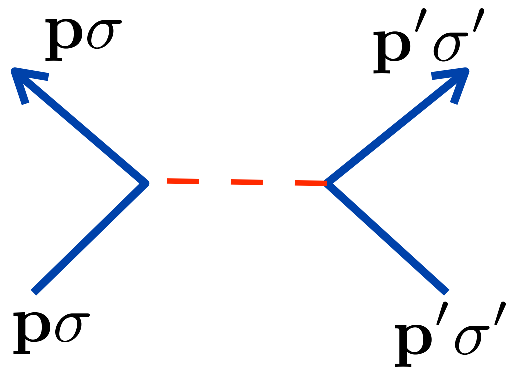

"Fixed point" Hamiltonian (Shankar, RMP 94)

$$H_{FP} = \sum_{\mathbf{p}\sigma} (E^{(0)}_{\mathbf{p}\sigma} - \mu)\hat{n}_{\mathbf{p}\sigma} + \frac{1}{2} \sum_{\mathbf{p},\sigma,\mathbf{p}'\sigma'} f_{\mathbf{p}\sigma,\mathbf{p}'\sigma'} \hat{n}_{\mathbf{p}\sigma} \hat{n}_{\mathbf{p}'\sigma'}$$

Interaction feedback on QP energy

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$$S = -k_B \sum_{\mathbf{p},\sigma} [n_{\mathbf{p}\sigma} \ln n_{\mathbf{p}\sigma} + (1 - n_{\mathbf{p}\sigma}) \ln(1 - n_{\mathbf{p}\sigma})]$$

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Note the strong feedback implicit in this statement.

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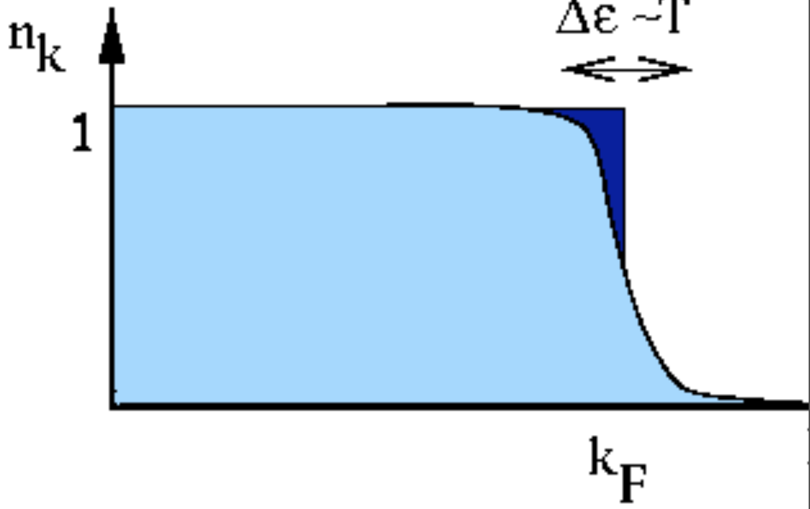

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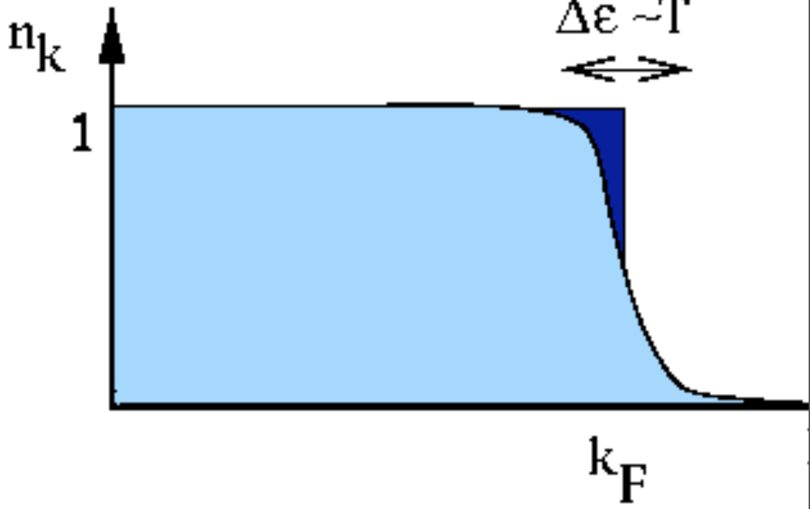

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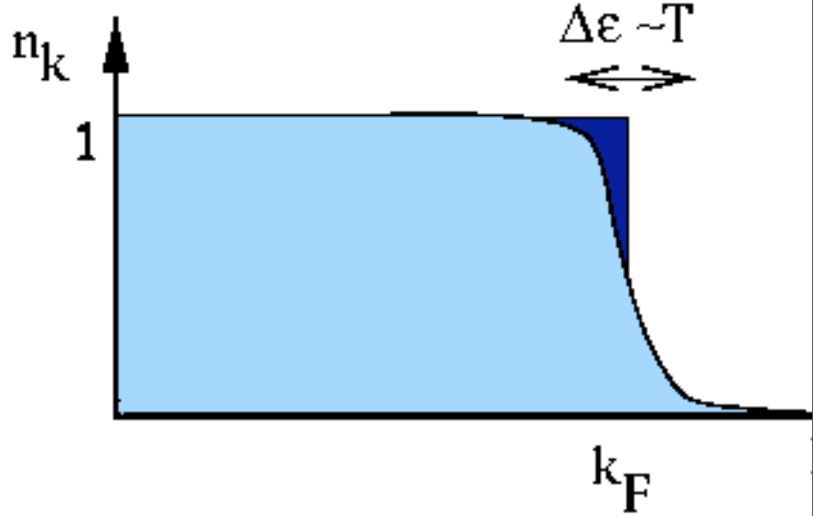

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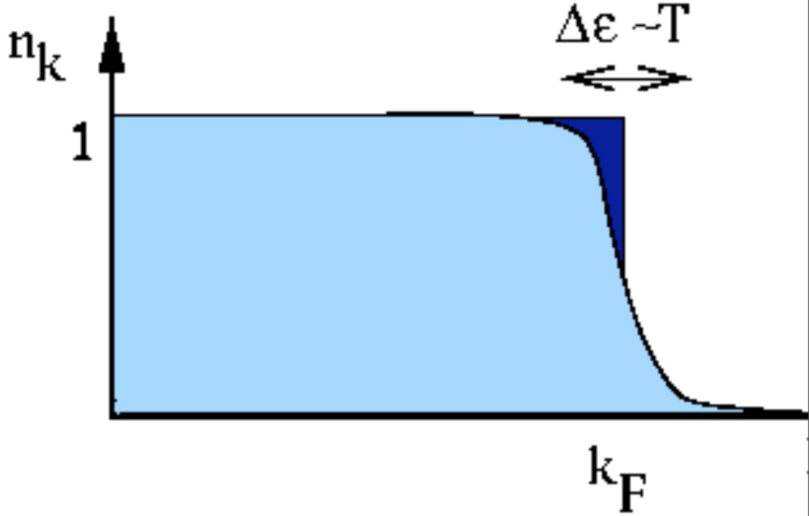

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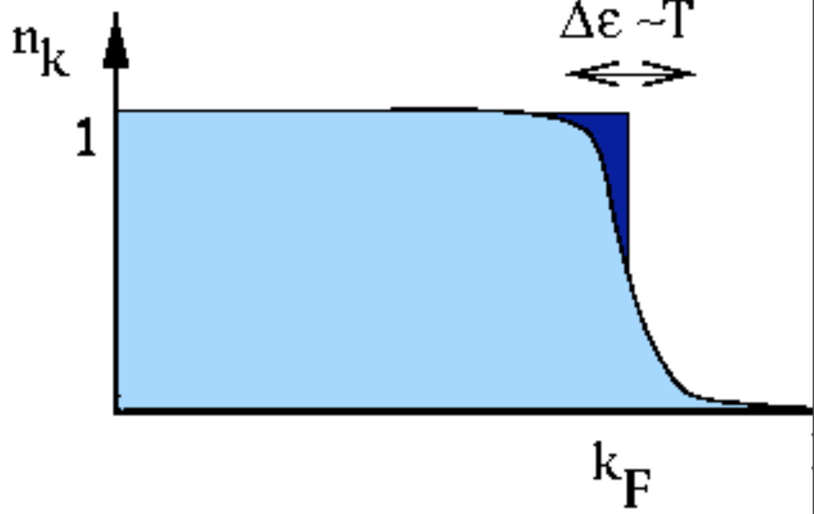

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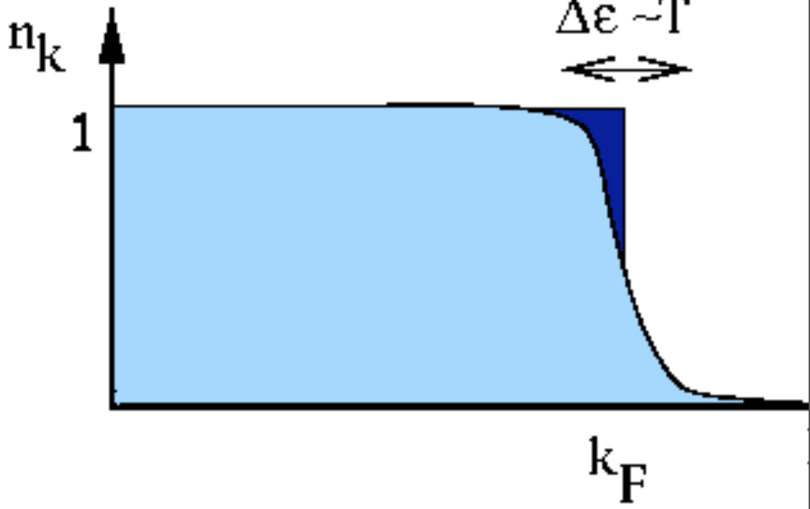

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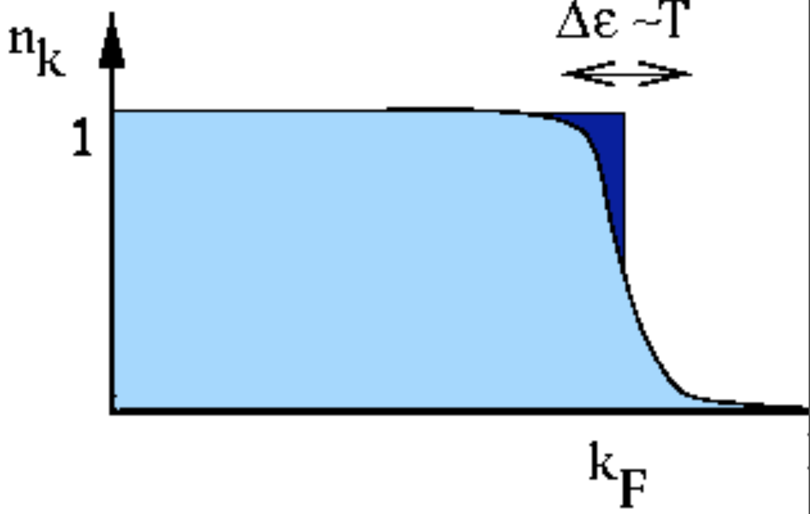

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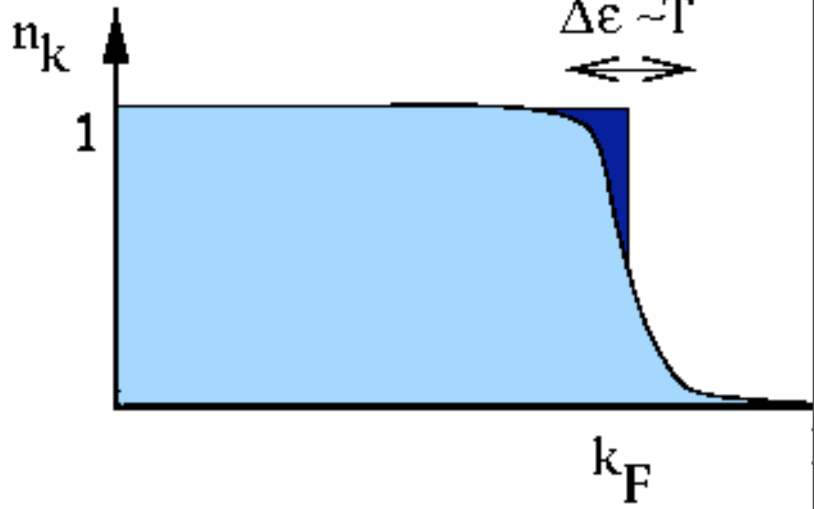

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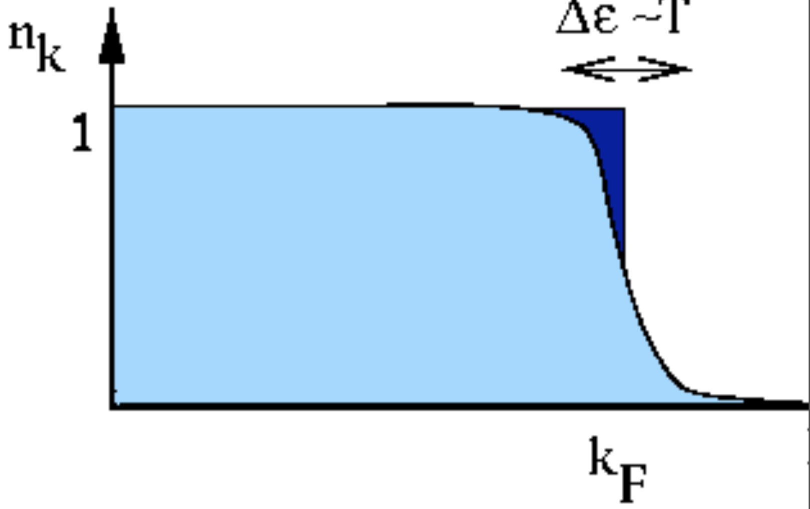

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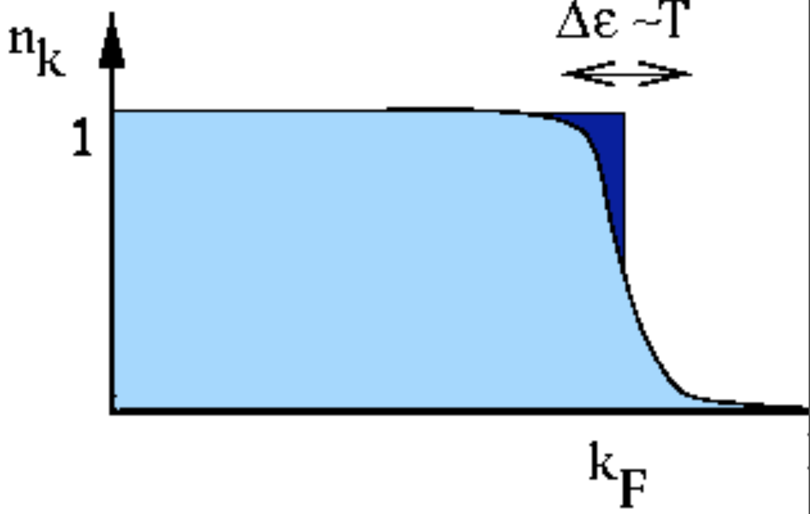

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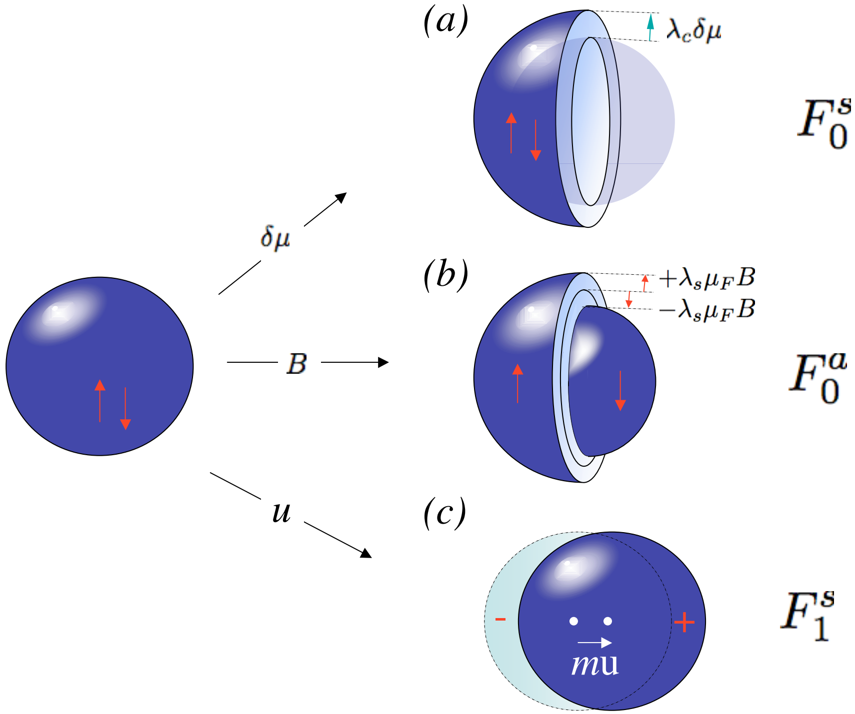
Landau Parameters

Each Landau Parameter captures the change in the quasiparticle energy corresponding to a given distortion of the Fermi surface.



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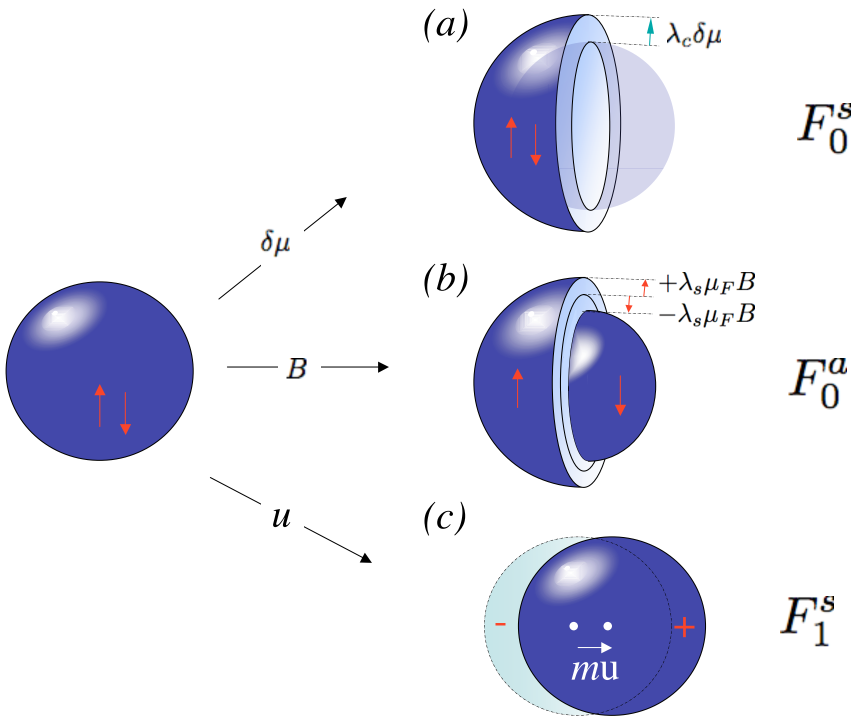

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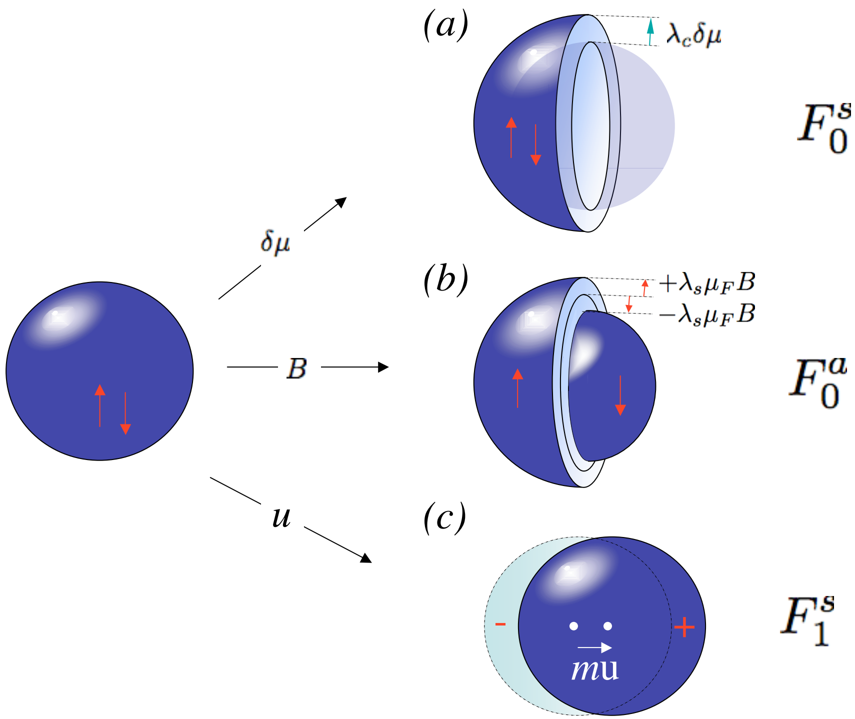

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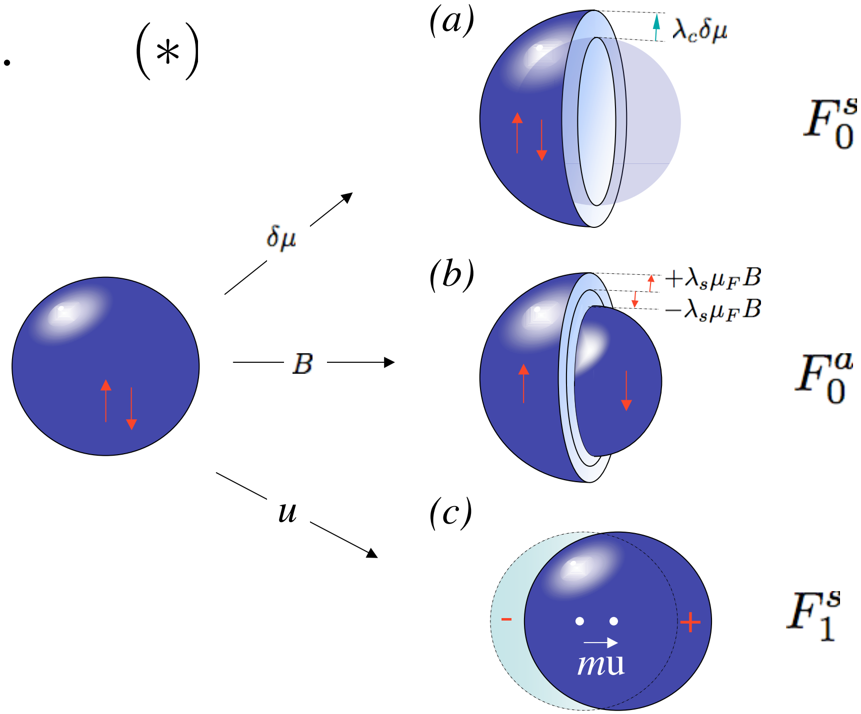

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$$a_l = b_l - F_l^s a_l$$

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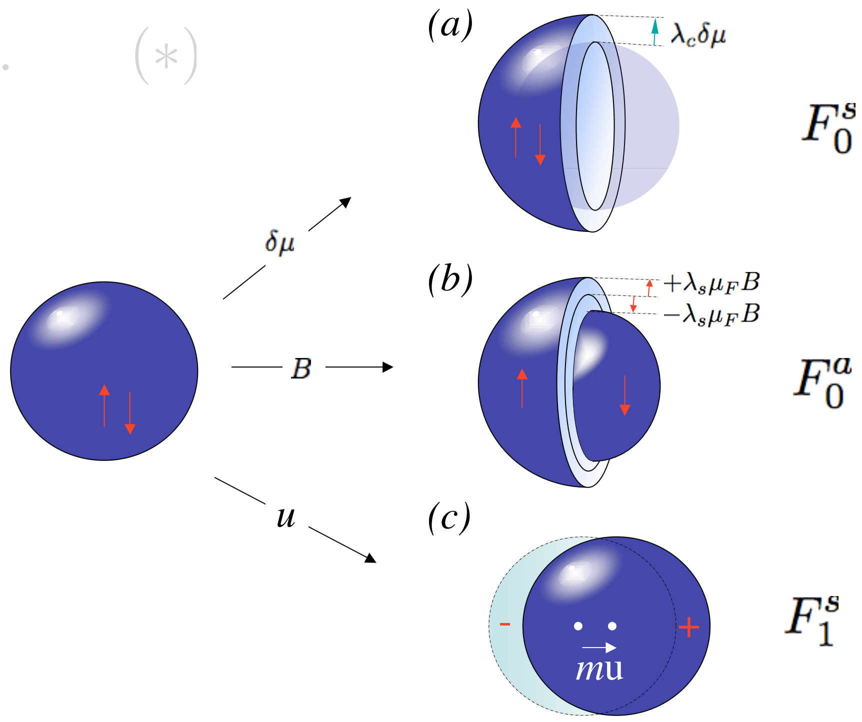

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Renormalization of Susceptibilties

The feedback effects renormalize the Fermi surface susceptibilities.

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where the quantities

$$A_0^s = \frac{F_0^s}{1+F_0^s} \qquad A_0^a = \frac{F_0^a}{1+F_0^a}$$

are interpreted as the t-matrix amplitudes for s-wave scattering

Table. 8.1 Key Properties of the Fermi Liquid .

PROPERTY	NON-INTERACTING	LANDAU FERMI LIQUID
Fermi momentum	p_F	unchanged
Density of particles	$2\frac{V_{FS}}{(2\pi)^3}$	unchanged
Density of states	$N(0) = \frac{m p_F}{\pi^2 \hbar^3}$	$N^*(0) = \frac{m^* p_F}{\pi^2 \hbar^3}$
Effective mass	m	$m^* = m(1 + F_1^s)$
Specific heat Coefficient $C_V = \gamma T$	$\gamma = \frac{\pi^2}{3} k_B^2 N(0)$	$\gamma = \frac{\pi^2}{3} k_B^2 N^*(0)$
Spin susceptibility	$\chi_s = \mu_F^2 N(0)$	$\chi_s = \mu_F^2 \frac{N^*(0)}{1+F_0^a}$
Charge Susceptibility	$\chi_C = N(0)$	$\chi_C = \frac{N^*(0)}{1+F_0^s}$
Collective modes	-	Sound ($\omega\tau << 1$) Zero sound ($\omega\tau >> 1$)

Heavy electrons: “Local Fermi Liquids”

Local Landau Fermi Liquid

- **Single scale** T^* The quasiparticle density of states $\rho^* \sim 1/T^*$ and scattering amplitudes $A_{\mathbf{k}\sigma,\mathbf{k}'\sigma'} \sim T^*$ scale approximately with a single scale T^*.
- **Almost incompressible.** Heavy electron fluids are "almost incompressible", in the sense that the charge susceptibility $\chi_c = dN_e/d\mu << \rho^*$ is unrenormalized and typically more than an order magnitude smaller than the quasiparticle density of states ρ^*. This is because the lattice of spins severely modifies the quasiparticle density of states, but leaves the charge density of the fluid $n_e(\mu)$, and its dependence on the chemical potential μ unchanged.
- **Local.** Quasiparticles scatter when in the vicinity of a local moment, giving rise to a small momentum dependence to the Landau scattering amplitudes. (Engelbrecht and Bedell, 1995; Yamada, 1975; Yoshida and Yamada, 1975) .

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Moreover, in local scattering the Pauli principle dictates that quasiparticles scattering at the same point can only scatter when in in opposite spin states, so that

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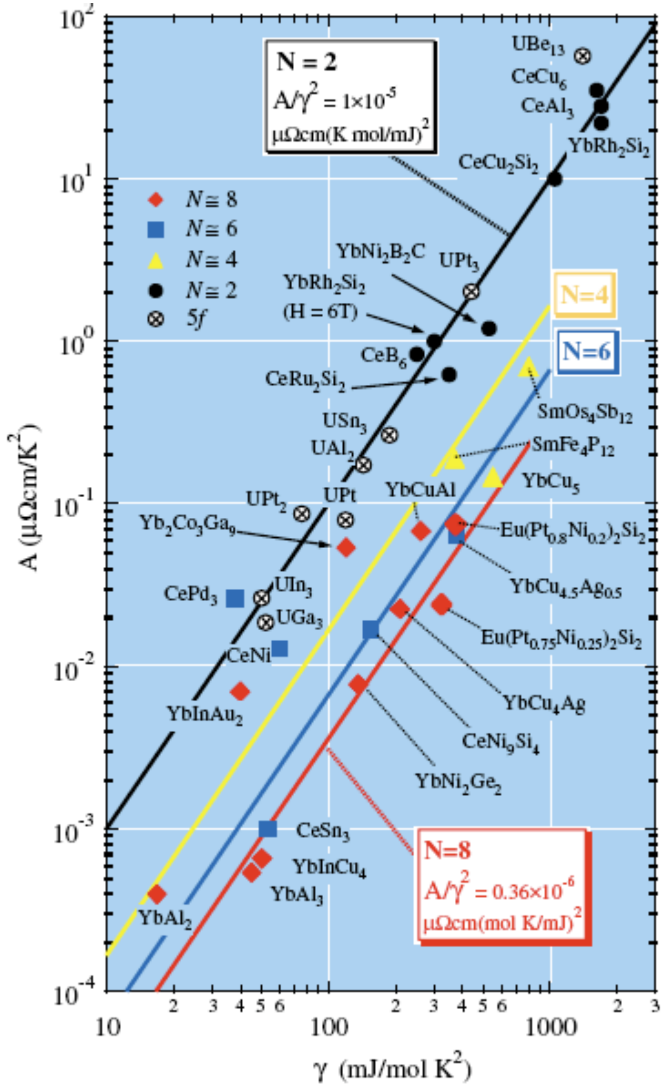
$$A^{(0)}_{\uparrow\uparrow} = A^0_s + A^0_a = 0$$

$$\chi_c \sim (1 - A_0^s) \sim 0$$

and hence $A^0_s = -A^0_a$. The additional assumption of incompressibility forces $A_0^s = 1$

so that now $A_0^s = -A_0^a \approx 1$ and all that remains is a single parameter ρ^*.



Tsuji et al (05)

Kadowaki Woods (1986)

$$
\begin{aligned}
\rho(T) &= \rho_0 + AT^2 \\
C_v(T) &= \gamma T
\end{aligned}
$$

$$
\alpha_{KW} = \frac{A}{\gamma^2} \approx (1 \times 10^{-5})\ \mu\Omega\mathrm{cm}(mol K^2/mJ)
$$