Superconductivity from repulsion

Lecture 3
Superconductivity near quantum criticality

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A quick reminder about yesterday’s lecture

Physicists, we have a problem

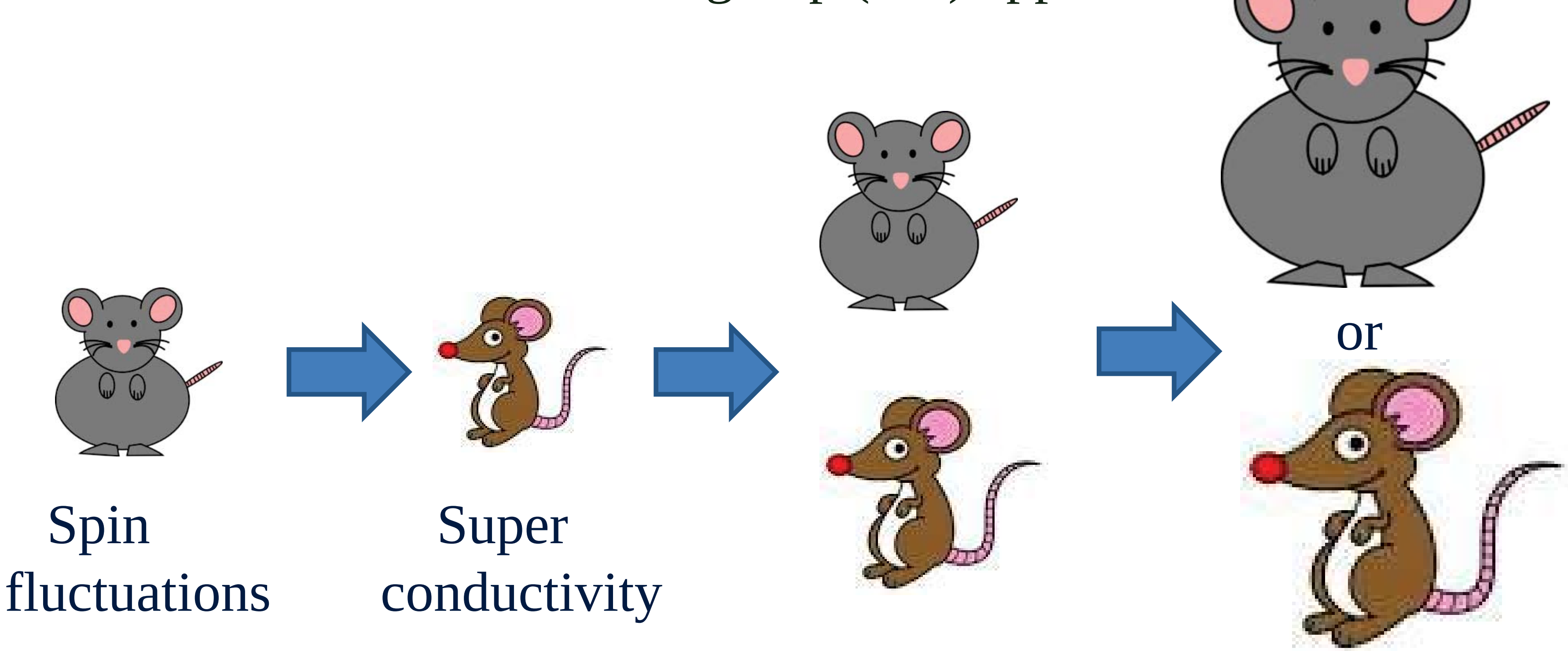
Bare interaction is generally repulsive in ALL channels,
Within perturbation theory, a simple Kohn-Luttinger
renormalization is not capable to overshoot the bare repulsion

Two ways to resolve the problem:

Both assume that superconductivity is not the only instability in a given system, there is also a density-wave instability around.

One approach is to keep couplings weak, but see whether we can additionally enhance KL terms due to interplay with other potential instabilities, which develop along with SC.

This is renormalization group (RG) approach

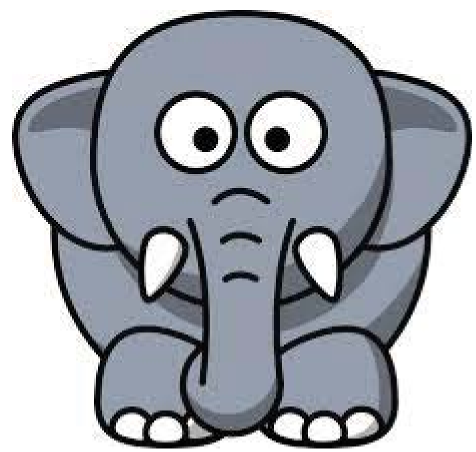


Two ways to resolve the problem:

Both assume that superconductivity is not the only instability in a given system, there is also a density-wave instability around.

Another approach is to abandon weak coupling and assume that density-wave instability (magnetism or charge order) comes from fermions at high energies, of order bandwidth. As an example, near antiferromagnetic instability, inter-pocket/inter-patch interaction g_3 is enhanced if we do full RPA summation in the particle-hole channel (or use any other method to account for contributions from high-energy fermions)

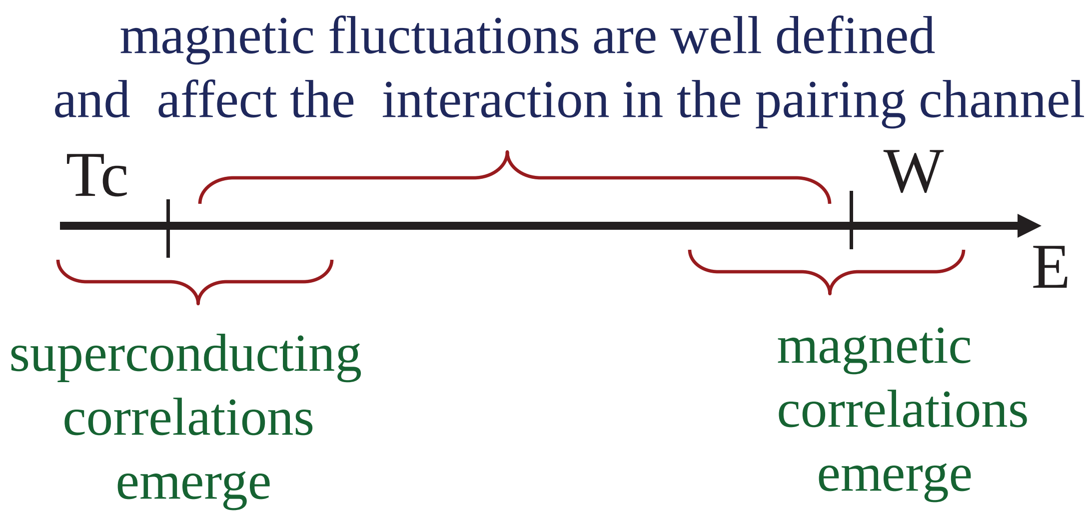
Super conductivity



Spin fluctuations

This lecture: spin-fermion model

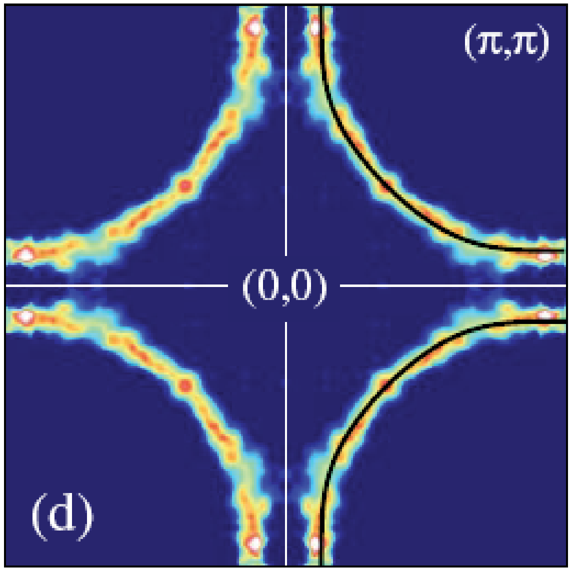
Let’s assume that magnetism emerges
already at scales comparable to the bandwidth, W



In this situation, one can introduce and explore the concept of spin-fluctuation-mediated pairing:
effective interaction between fermions is mediated by already well formed spin fluctuations

This is not a controlled theory: U/W ~1
(intermediate coupling)

The key assumption is that at U/W ~1 Mott physics does not yet develop, and the system remains a metal with a large Fermi surface

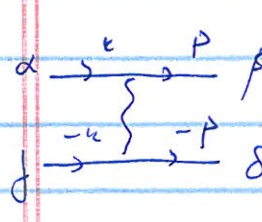


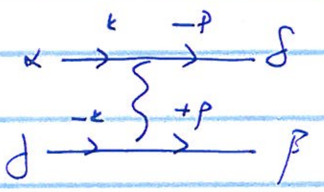
Problem I: how to re-write pairing interaction as the exchange of spin collective degrees of freedom?

(blackboard)

"Derivation" of spin-mediated pairing interaction

Look at anti-symmetrized interaction

$U_{eff} =$ 

$\ominus$ 

$$U(k-p)\,\delta_{\alpha\beta}\,\delta_{\gamma\delta} \;\ominus\; U(k+p)\,\delta_{\alpha\delta}\,\delta_{\gamma\beta}$$

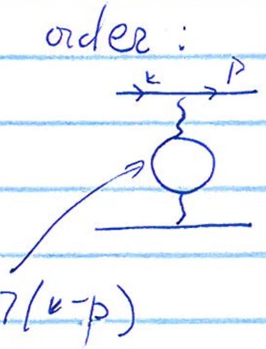
$$\equiv \left[U(k-p) - \tfrac{1}{2}U(k+p)\right]\delta_{\alpha\beta}\,\delta_{\gamma\delta} - \tfrac{1}{2}U(k+p)\,\vec{\sigma}_{\alpha\beta}\,\vec{\sigma}_{\gamma\delta}$$

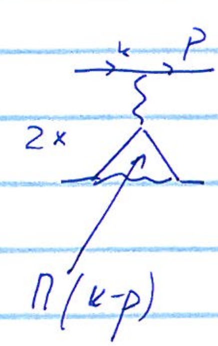
Let's go to second order and make two assumptions:

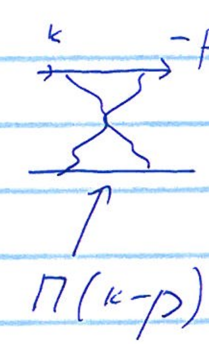
1) $U \equiv$ const

2) $\Pi(k-p)$ is peaked at $k-p=0$

Second order:

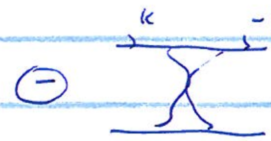
$U_{eff} \Rightarrow$ 

$+ \; 2\times$ 

$-$ 

$+$ (no other diagrams with $\Pi(k-p)$)

For a constant U, the first two diagrams cancel each other and we are left with second-order contribution

$\ominus$ 

$$\ominus\, U^2\Pi(k-p)\,\delta_{\alpha\delta}\,\delta_{\beta\gamma} =$$

from antisymmetrization $\Big\}$

$$= \ominus\, \frac{U^2\Pi(k-p)}{2}\left(\delta_{\alpha\beta}\delta_{\gamma\delta} + \vec{\sigma}_{\alpha\beta}\vec{\sigma}_{\gamma\delta}\right)$$

or

$$U_{eff} = \frac{U}{2}\left(1 - U\Pi(k-p)\right)\delta_{\alpha\beta}\delta_{\gamma\delta}$$

$$\ominus\, \frac{U}{2}\left(1 + U\Pi(k-p)\right)\vec{\sigma}_{\alpha\beta}\vec{\sigma}_{\gamma\delta}$$

<u>Homework:</u> do this at next order and check:

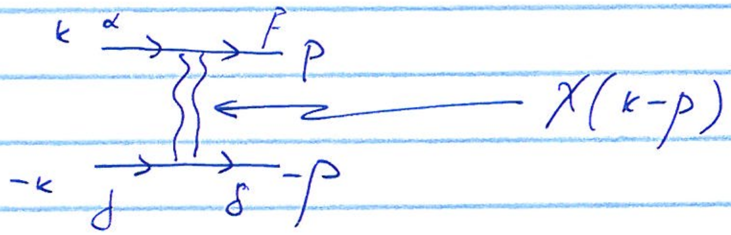
$$U_{eff} = \frac{U}{2}\left(1 - U\Pi(k-p) + U^2\Pi^2(k-p)\right)\delta_{\alpha\beta}\delta_{\gamma\delta}$$

$$- \frac{U}{2}\left(1 + U\Pi(k-p) + U^2\Pi^2(k-p)\right)\vec{\sigma}_{\alpha\beta}\vec{\sigma}_{\gamma\delta}$$

The extension to infinite order:

$$U_{eff} = \underbrace{\frac{U}{2}\,\frac{1}{1+U\Pi(k-p)}\,\delta_{\alpha\beta}\,\delta_{\gamma\delta}}_{\text{charge-fluctuation mediated pairing interaction}} \;-\; \underbrace{\frac{U}{2}\,\frac{1}{1-U\Pi(k-p)}\,\vec{\sigma}_{\alpha\beta}\,\vec{\sigma}_{\gamma\delta}}_{\text{spin-fluctuation mediated pairing interaction}}$$

$$\mathcal{H}_{eff} = -g\,\chi(k-p)\,\left(c_\alpha^{+}\,\vec{\sigma}_{\alpha\beta}\,c_\beta\right)\left(c_\gamma^{+}\,\vec{\sigma}_{\gamma\delta}\,c_\delta\right)$$



$\chi(k-p) \equiv$ static spin susceptibility

<u>N.B.</u> There is <u>no</u> rigorous justification of such procedure. But the result is quite appealing.

The outcome of this analysis is the effective Hamiltonian for instantaneous fermion-fermion interaction in the spin channel

Near a ferromagnetic instability

$$H^{eff} = -g\,(c^{+}_{\alpha}\,\vec{\sigma}_{\alpha\delta}\,c_{\delta})\,(c^{+}_{\gamma}\,\vec{\sigma}_{\gamma\beta}\,c_{\beta})\,\chi(q)$$

$$\chi(q) = \frac{1}{q^2 + \xi^{-2}}$$

Near an antiferromagnetic instability

$$H^{eff} = -g\,(c^{+}_{\alpha}\,\vec{\sigma}_{\alpha\delta}\,c_{\delta})\,(c^{+}_{\gamma}\,\vec{\sigma}_{\gamma\beta}\,c_{\beta})\,\chi(q+Q)$$

$$\chi(q+Q) = \frac{1}{q^2 + \xi^{-2}}$$

THIS IS THE SPIN-FERMION MODEL

It can also be introduced phenomenologically, as a minimal low-energy model for the interaction between fermions and collective modes of fermions in the spin channel

Check consistency with Kohn-Luttinger physics

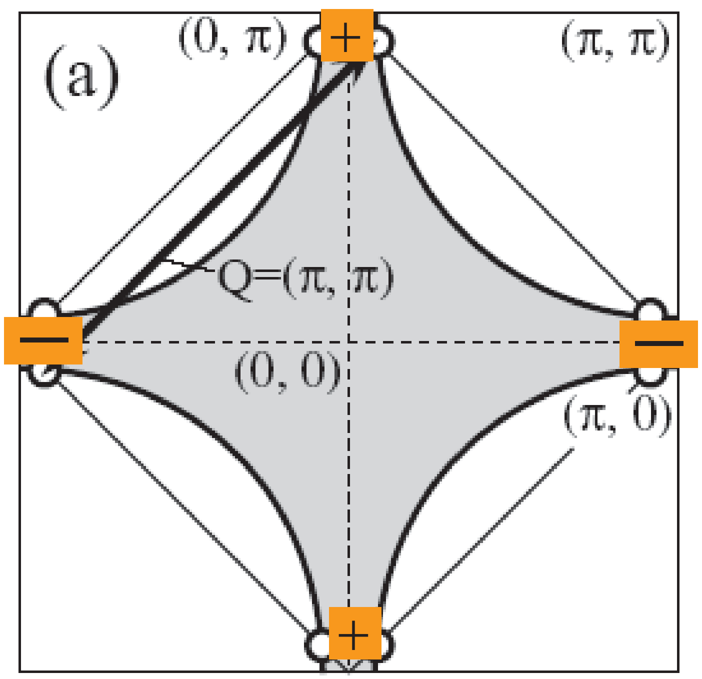
Antiferromagnetism for definitness

Eqn. for a sc gap

$$\Delta(k) = -g \int dq \frac{\Delta(q)}{\sqrt{\Delta^2(q) + E^2(q)}} \chi(k-q)$$

$q^2 + \xi^{-2}$

Effective interaction is repulsive, but is peaked at large momentum transfer



$d_{x^2-y^2}$ superconductivity

KL analysis assumes weak coupling
(static interaction, almost free fermions)

To properly solve for the pairing we need to know how fermions behave in the normal state

Energy scales:

- coupling g
- $v_F\ \xi^{-1}$
- bandwith W

Let's just assume for the next 30 min that g << W. Then high-energy and low-energy physics are decoupled, and we obtain a model with one energy scale g and one dimensional ratio $g/v_F\ \xi^{-1}$

$\lambda = \frac{g}{v_F \xi^{-1}}$ is the relevant parameter of the problem

$\lambda << 1$ truly weak coupling, KL pairing in a Fermi gas

$\lambda >> 1$, the system is still a metal, but with strong correlations

Problem II: how to construct normal state theory for $\lambda >> 1$

- fermions get dressed by the interaction with spin fluctuations

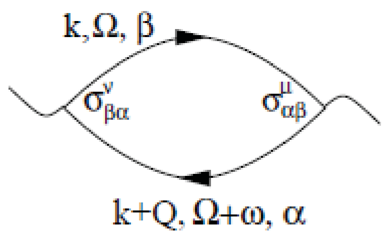
- spin fluctuations get dressed by the interaction with low-energy fermions

Bosonic and fermionic self-energies have to be computed self-consistently (see A. Millis talk)

Fermionic self-energy: mass renormalization & lifetime
Bosonic self-energy: Landau damping

At one loop level:

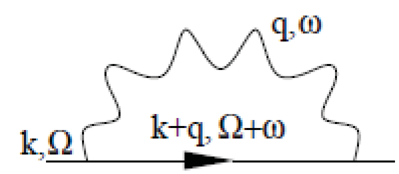
bosons (spin fluctuations) become Landau overdamped



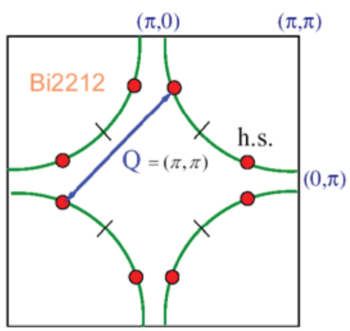
$$\chi(q,\omega) = \frac{1}{q^2 + \xi^{-2} - i\,\omega/\omega_{sf}}$$

$$\omega_{sf} \propto g/\lambda^2 \quad \left(= \frac{9}{64\pi}\frac{g}{\lambda^2} \right)$$

fermions acquire frequency dependent self-energy Σ(ω)



Hot spots



Fermi liquid — $\omega_{sf} \sim g/\lambda^2$ — Non-Fermi liquid — g — Fermi gas

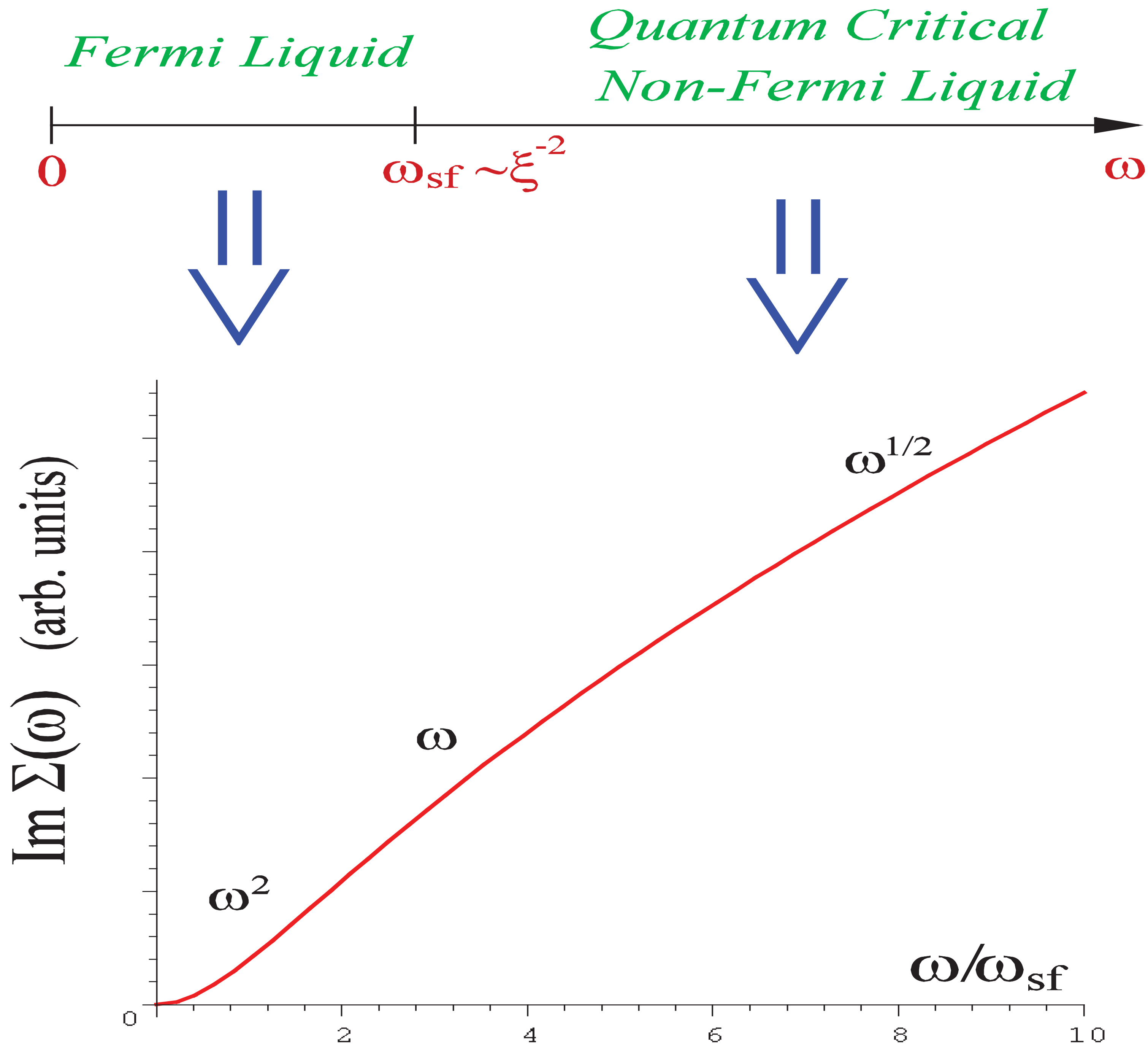
0 → E

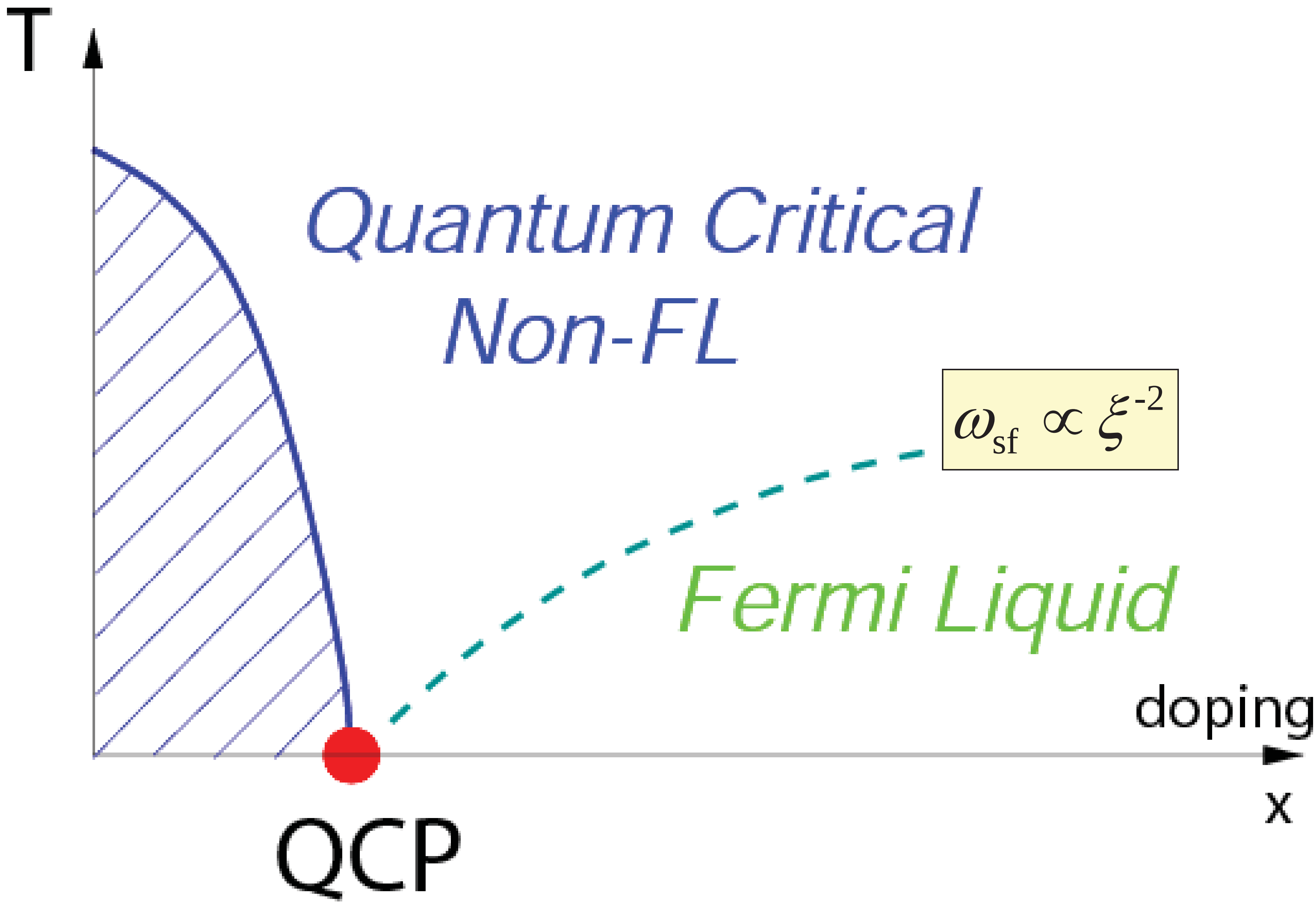
Fermi liquid:
$$\Sigma'(\omega) = \lambda\omega, \quad \Sigma''(\omega) \propto \omega^2$$

Non-Fermi liquid:
$$\Sigma'(\omega) \propto (\omega g)^{1/2} > \omega, \quad \Sigma''(\omega) \propto (\omega g)^{1/2} > \omega$$

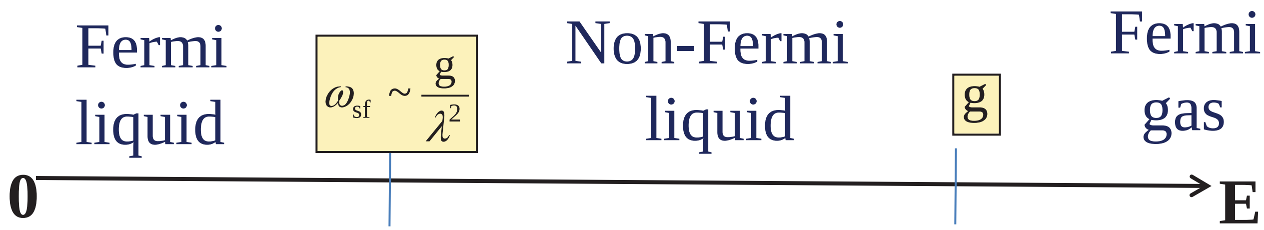
Fermi gas:
$$\Sigma(\omega) < \omega$$

At $\xi^{-1} = 0$, Fermi liquid region disappears at a hot spot


Fermi Liquid
Quantum Critical
Non-Fermi Liquid
0
$\omega_{sf} \sim \xi^{-2}$
ω
$\omega^{1/2}$
ω
ω^2
ω/ω_{sf}
Im $\Sigma(\omega)$ (arb. units)
0
2
4
6
8
10


T
Quantum Critical
Non-FL
$\omega_{sf} \propto \xi^{-2}$
Fermi Liquid
doping
x
QCP

Problem III: pairing at $\lambda >> 1$



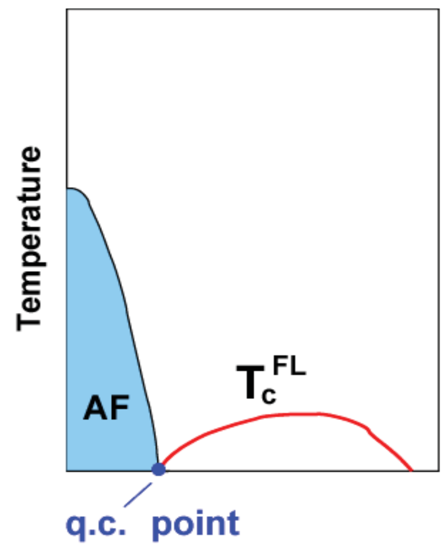
Pairing in the Fermi liquid regime is KL physics

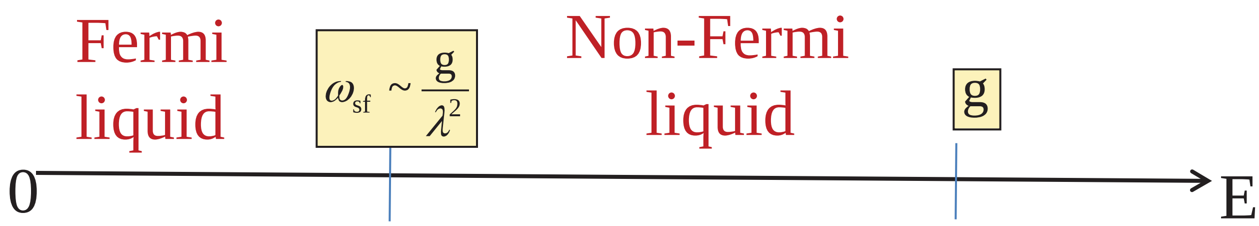
McMillan formula for phonons   by analogy

$$T_c \sim \omega_D \exp[-(1+\lambda)/\lambda]$$

$$Tc \propto \xi^{-2} \exp(-\frac{\xi+\xi_0}{\xi})$$

If only Fermi liquid region would contribute to d-wave pairing, Tc would be zero at a QCP





Pairing in non-Fermi liquid regime is a new phenomenon

Pairing vertex Φ becomes frequency dependent $\Phi(\Omega)$

Gap equation has non-BCS form

$$\Phi(\Omega) = \frac{\pi}{2} T \sum_{\omega > \omega_{sf}} \frac{\Phi(\omega)}{|\omega|^{1/2} |\Omega - \omega|^{1/2}} \frac{1}{1 + (|\omega|/g)^{1/2}}$$

$\Sigma(\omega) \propto \omega^{1/2}$

$\int dq \chi(q, \omega) \propto \omega^{-1/2}$

$1 + \omega/\Sigma(\omega)$, soft cutoff

Compare BCS and QC pairings

Quantum-critical pairing

$$\Phi(\Omega) = \frac{\pi}{2} T \sum_{\omega > \omega_{sf}} \frac{\Phi(\omega)}{|\omega|^{1/2} |\Omega - \omega|^{1/2}} \frac{1}{1 + (|\omega|/g)^{1/2}}$$

BSC pairing

$$\Phi(\Omega) = \frac{\lambda}{1+\lambda} \pi T \sum_{\omega} \frac{\Phi(\omega)}{|\omega|} \frac{1}{(1 + |\omega|/\omega_{sf})^{1/2}}$$

- pairing problem in the QC case is universal (no overall coupling)

- pairing kernel is $|\omega|^{-1}$, like in BCS theory, only a half of $|\omega|$ comes from self-energy, another from interaction

Is the quantum-critical problem like BCS?

Let's check: Pairing kernel $|\omega|^{-1}$ → logarithms!

BCS:

$$\Phi(\Omega) = \bar{\lambda} T \sum^{\omega_{sf}} \frac{\Phi(\omega)}{|\omega|} + \Phi_0, \qquad \bar{\lambda} = \frac{\lambda}{1+\lambda}$$

sum up logarithms

$$\Phi = \Phi_0 \left(1 + \bar{\lambda} \log \frac{\omega_{sf}}{T} + \bar{\lambda}^2 \log^2 + \ldots\right) = \frac{\Phi_0}{\bar{\lambda} \log \frac{T}{T_c}}$$

pairing instability at any coupling

QC case:

$$\Phi(\Omega) = \frac{\pi T}{2} \sum^{g} \frac{\Phi(\omega)}{|\omega|^{1/2} |\Omega - \omega|^{1/2}} + \Phi_0$$

sum up logarithms

$$\Phi(\Omega = 0, T) = \Phi_0 \left(1 + \frac{1}{2} \log \frac{g}{T} + \frac{1}{2} \left(\frac{1}{2} \log \frac{g}{T}\right)^2 + \ldots\right) = \Phi_0 \left(\frac{g}{T}\right)^{1/2}$$

no divergence at a finite T

Let's now look at the solution of the equation without Φ_0

$$\Phi(\Omega) = \frac{\pi}{2} T \sum \frac{\Phi(\omega)}{|\omega|^{1/2} |\Omega - \omega|^{1/2}} \frac{1}{1 + (|\omega|/g)^{1/2}}$$

Search for power-law solution at $T < \Omega < g$,

$$\Phi(\Omega) = \Omega^{-(1/4 - 2\beta)}$$

Substitute: no solutions for real β

Strange. We summed up logarithms five minutes ago and did obtain power- law solution

$$\Phi(\Omega = 0, T) = \Phi_0 \left(1 + \frac{1}{2}\log\frac{g}{T} + \frac{1}{2}\left(\frac{1}{2}\log\frac{g}{T}\right)^2 + \ldots\right) = \Phi_0 \left(\frac{g}{T}\right)^{1/2}$$

But we do remember that we sum up logs ONLY when a coupling is small

In the case we are looking at, the coupling = 1/2

Let's artificially add a small ε to compare with summing up logs

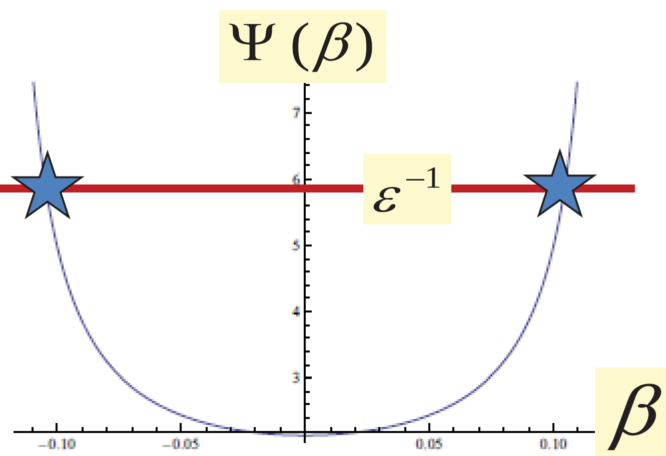
$$\Phi(\Omega) = \varepsilon \frac{\pi}{2} T \sum \frac{\Phi(\omega)}{|\omega|^{1/2} |\Omega - \omega|^{1/2}} \frac{1}{1 + (|\omega|/g)^{1/2}}$$

Search again for power-law solution at $T < \Omega < g$,

$$\Phi(\Omega) = \Omega^{-(1/4 - 2\beta)}$$

Substitute, we get

$$\frac{1}{\varepsilon} = \Psi(\beta)$$



Yes, at small ε, we do have a power-law solution with a real β

$$\Phi(\Omega) = A\Omega^{-(1/4 - 2\beta)} + B\,\Omega^{-(1/4 + 2\beta)}$$

We now need to see whether this solution satisfies boundary conditions

The linearized gap equation, which we just solved,
has two boundaries: an upper one at g and a lower one at T

$$\Phi(\Omega) = \frac{\varepsilon}{4}\int_{T}^{g} \frac{\Phi(\omega)}{|\omega|^{1/2}} \left(\frac{1}{|\Omega - \omega|^{1/2}} + \frac{1}{|\Omega + \omega|^{1/2}} \right)$$

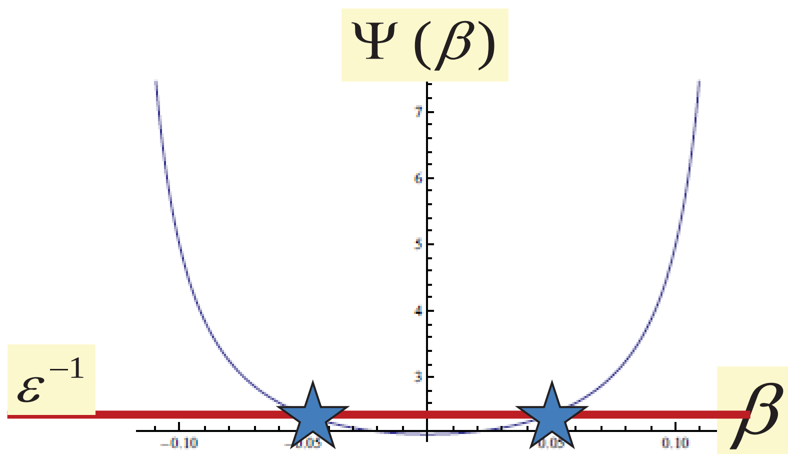
With $\Phi(\Omega) = A\Omega^{-(1/4-2\beta)} + B\Omega^{-(1/4+2\beta)}$ one can satisfy one boundary condition, say, at g, but not both

No QC superconductivity at small ε.

$$\Phi(\Omega) = \frac{\varepsilon}{4}\int_{T}^{g} \frac{\Phi(\omega)}{|\omega|^{1/2}}\left(\frac{1}{|\Omega - \omega|^{1/2}} + \frac{1}{|\Omega + \omega|^{1/2}}\right)$$

$$\Phi(\Omega) = A\Omega^{-(1/4-2\beta)} + B\,\Omega^{-(1/4+2\beta)}$$

On a more careful look, we find that perturbation theory works only up to $\varepsilon_{max} < 1$



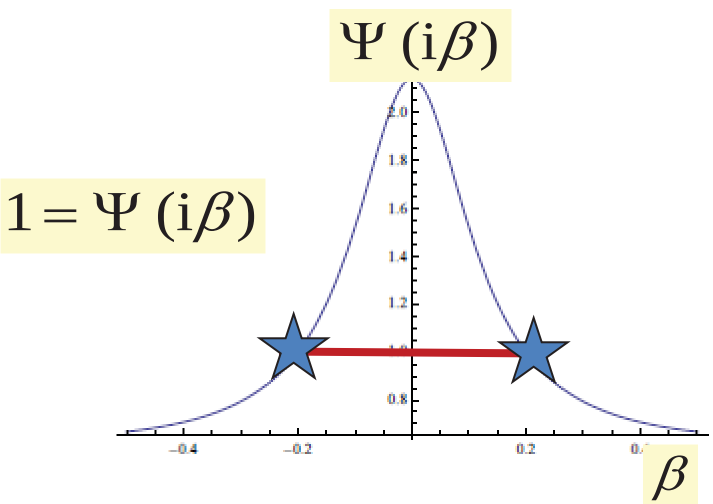
$$\Psi(0) = 2.13 \Rightarrow \varepsilon_{max} = 0.47$$

Set ε=1

$$\Phi(\Omega) = \frac{1}{4}\int \frac{\Phi(\omega)}{|\omega|^{1/2}}\left(\frac{1}{|\Omega-\omega|^{1/2}} + \frac{1}{|\Omega+\omega|^{1/2}}\right)$$

Still search for a a power-law solution $\Phi(\Omega) = \Omega^{-(1/4-2\beta)}$

But now take β to be imaginary, i β $1 = \Psi(i\beta)$



Combine solutions with β and –β into a real function Φ(Ω)

$$\Phi(\Omega) = C\left(\frac{1}{\Omega}\right)^{1/4} \cos(2\beta\log(\Omega) + \phi_0)$$

This is an oscillating function of frequency – multiple zeros!

Actual equation:

$$\Phi(\Omega) = \frac{1}{4}\int_{T}^{g} \frac{\Phi(\omega)}{|\omega|^{1/2}} \left(\frac{1}{|\Omega - \omega|^{1/2}} + \frac{1}{|\Omega + \omega|^{1/2}} \right)$$

Two boundaries: one at g, another at T_c

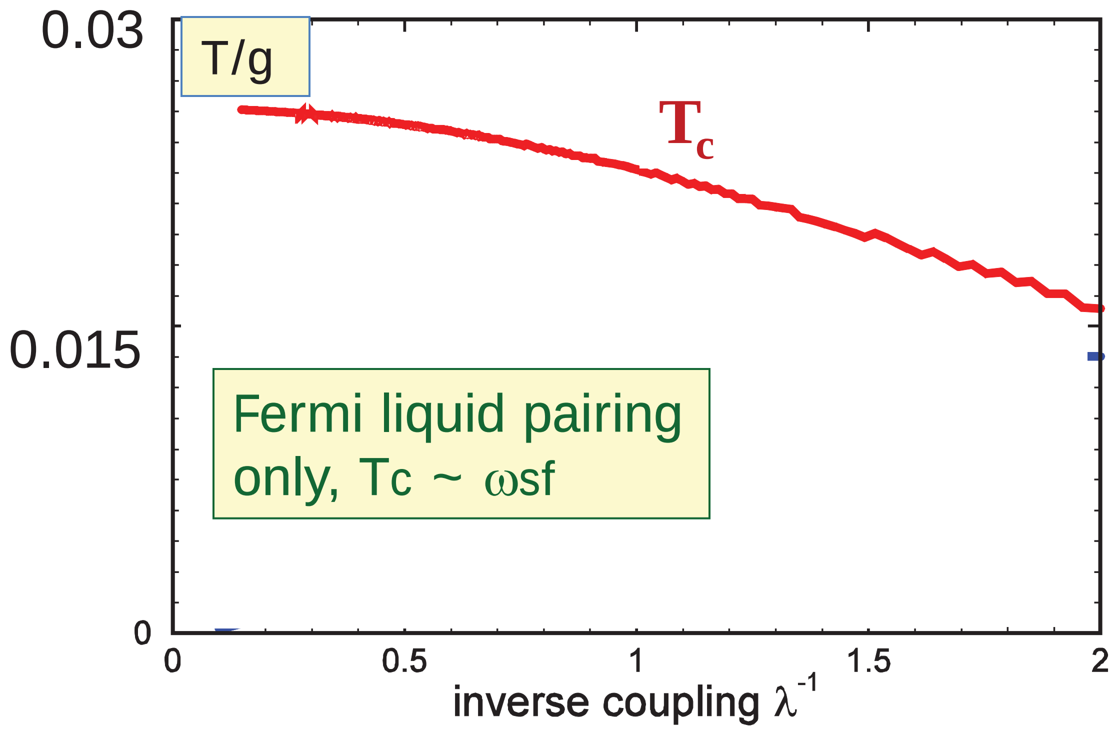
Roughly, $\Phi(\Omega)$ should vanish at both boundaries

$$\Phi(\Omega) = C \left(\frac{1}{\Omega} \right)^{1/4} \cos\left(2\beta \log(\Omega) + \phi_0\right)$$

at $\Omega = g$ $\quad \cos\left(2\beta \log(g) + \phi_0\right) = 0, \quad \cos\left(2\beta \log(T_c) + \phi_0\right) = 0 \quad$ at $\Omega = T_c$

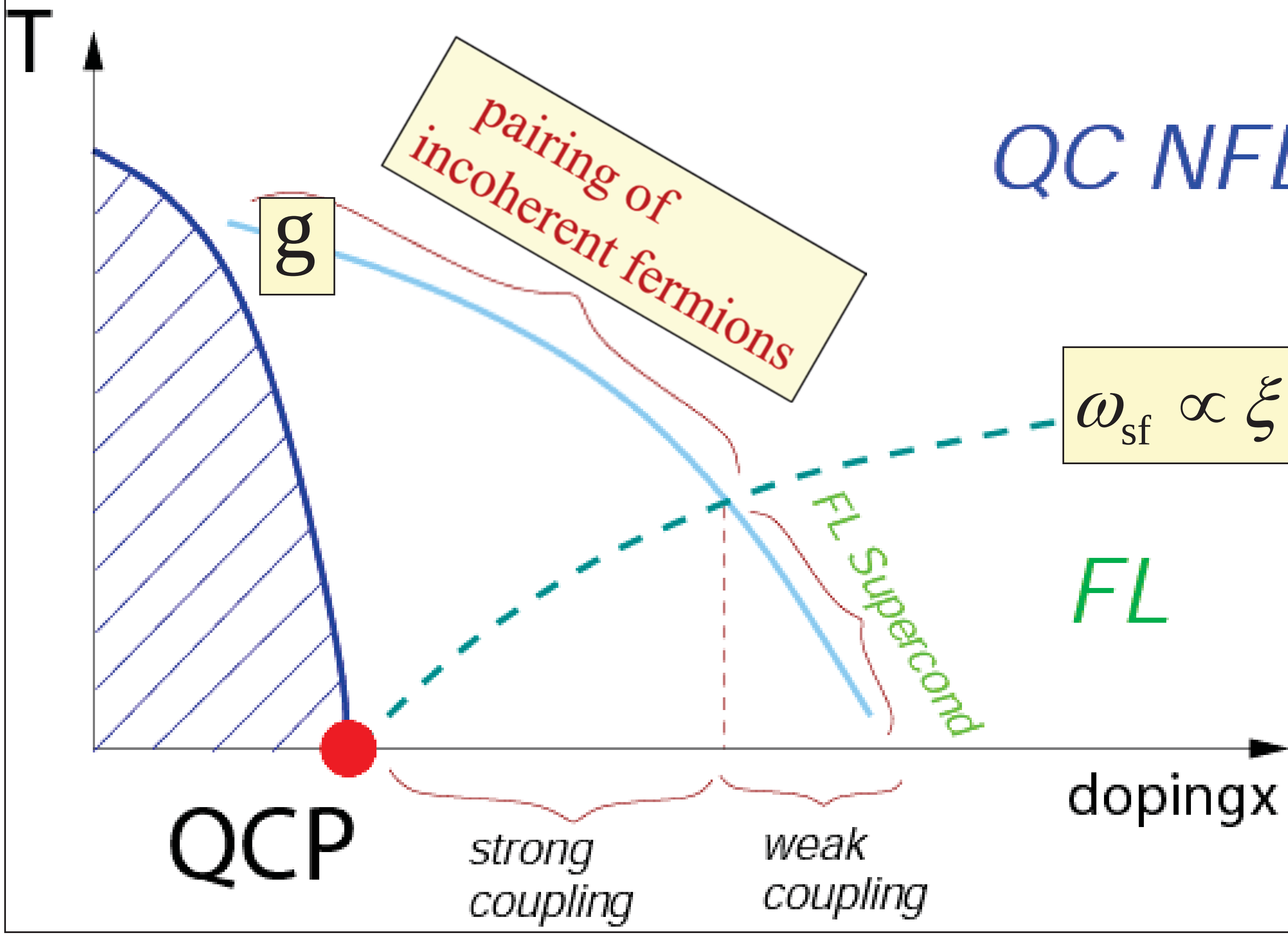
$$T_c \sim g\, e^{-\frac{1}{2\beta}}$$

The result: a finite Tc right at the quantum-critical point



$T_c = 0.025\,g$ at QCP

Dome of a pairing instability above QCP



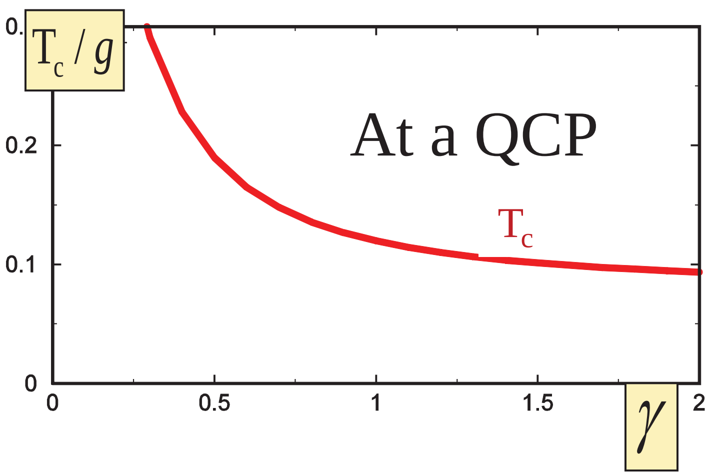
This problem is quite generic and goes beyond the cuprates

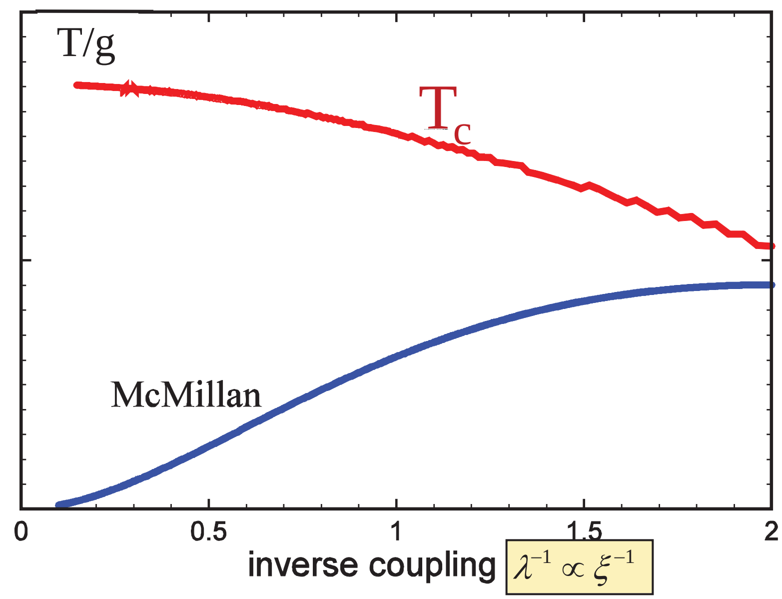
$$\Phi(\Omega) = \frac{1-\gamma}{2}\int_0^g d\omega \frac{\Phi(\omega)}{|\omega|^{1-\gamma}}\left(\frac{1}{|\Omega-\omega|^{\gamma}} + \frac{1}{|\Omega+\omega|^{\gamma}}\right)$$

Abanov et al, Moon, She, Zaanen

$\gamma = 1/2$	Antiferromagnetic QCP	Abanov et al, Metlitski, Sachdev
$\gamma = 1/3$	FM QCP, nematic, composite fermions, $\Omega^{2/3}$ problem	Bonesteel, McDonald, Nayak, Haslinger et al, Millis et al, Bedel et al…
$\gamma = +0 \ (\log \omega)$	3D QCP, Color superconductivity	Son, Schmalian, A.C, Metlitski, Sachdev
$\gamma = 1$	Z=1 pairing problem	Schmalian, A.C….
$\gamma = +0 \rightarrow \gamma = 1$	pairing in the presence of SDW	Moon, Sachdev
$\gamma \approx 0.7$	fermions with Dirac cone dispersion	Metzner et al
$\gamma = 2$	Pairing by near-gapless phonons	
	$T_c^{ad} = 0.1827\, g$	Allen, Dynes, Carbotte, Marsiglio, Scalapino, Combescot, Maksimov, Bulaevskii, Dolgov, …..

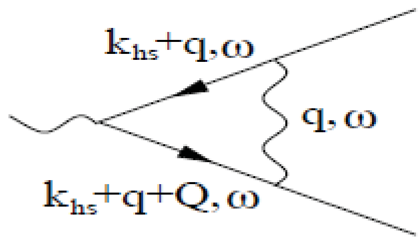
It turns out that for all γ, the coupling (1 - γ)/2 is larger than the threshold





Accuracy: corrections are O(1), the leading ones can be accounted for in the 1/N expansion

Leading vertex corrections are log divergent



$$\frac{\Delta g}{g} = \frac{Q(v)}{N} \log \lambda$$

$$Q(v) = \frac{4}{\pi} \tan^{-1} \frac{v_x}{v_y}$$

To order O(1/N):

$$\chi(\mathrm{q}, \omega) \propto \frac{1}{(\mathrm{q}^2 + |\omega|)^\eta}$$

$$\eta = 1 - \frac{1}{2\mathrm{N}}$$

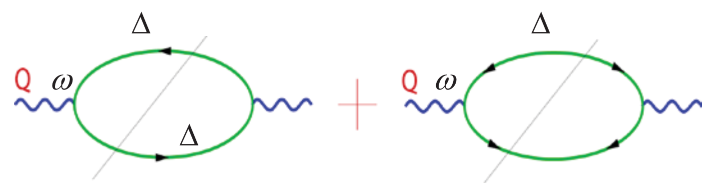
The only change is

$$\gamma \rightarrow \frac{1}{2} - \frac{1}{2\mathrm{N}}$$

The superconducting phase

Spin dynamics changes because of
d-wave pairing -- the resonance peak appears

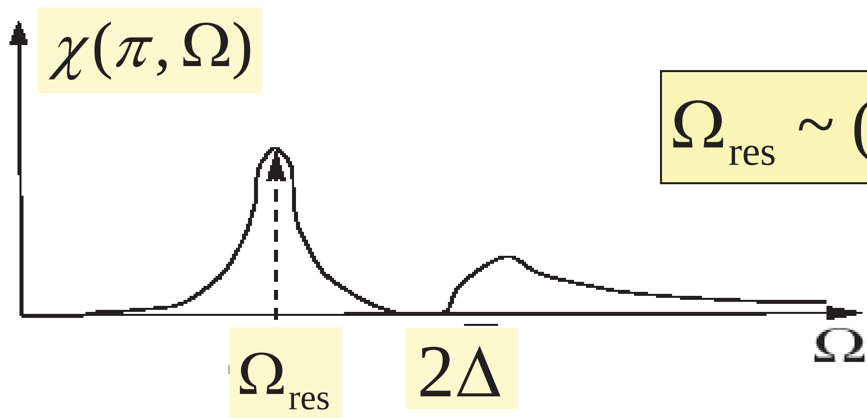
- no low-energy decay below 2Δ due to fermionic gap



- residual interaction is “attractive” for d-wave pairing

Collective spin fluctuation mode
at the energy well below 2Δ

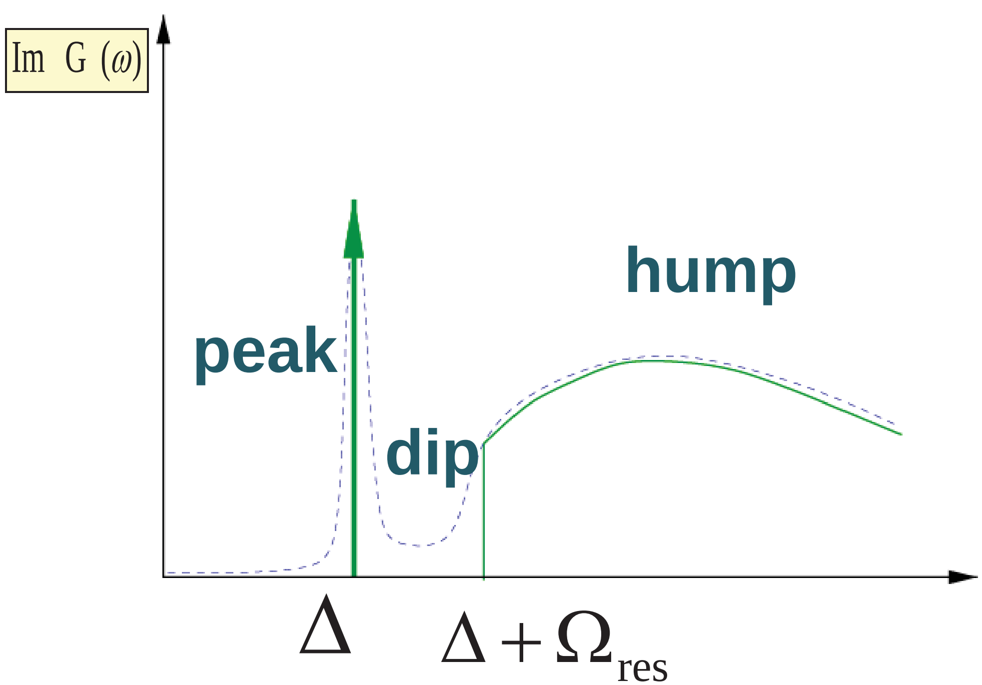
$$\chi(\Omega) \sim \frac{1}{\Omega^2 - \Omega_{res}^2}$$

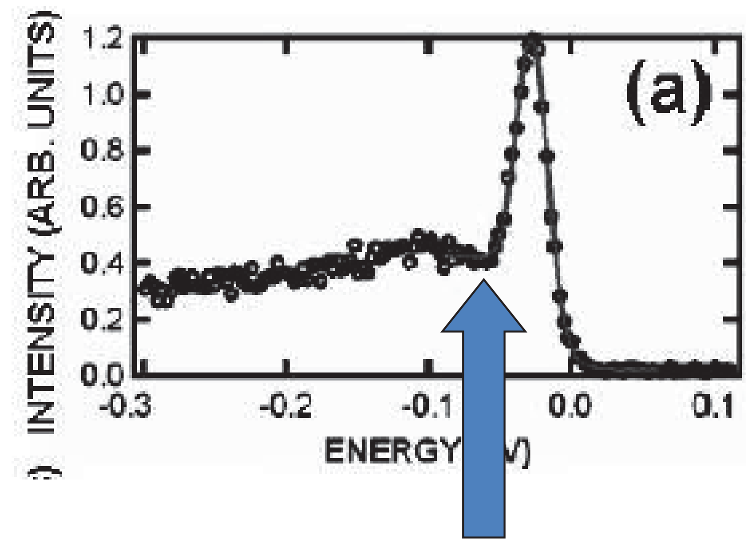


$$\Omega_{res} \sim (g\,\omega_{sf})^{1/2} \sim \xi^{-1}$$

By itself, the resonance is NOT a fingerprint of spin-mediated pairing,
nor it is a glue to a superconductivity

A fingerprint is the observation how the resonance peak affects the electronic behavior, if the spin-fermion interaction is the dominant one



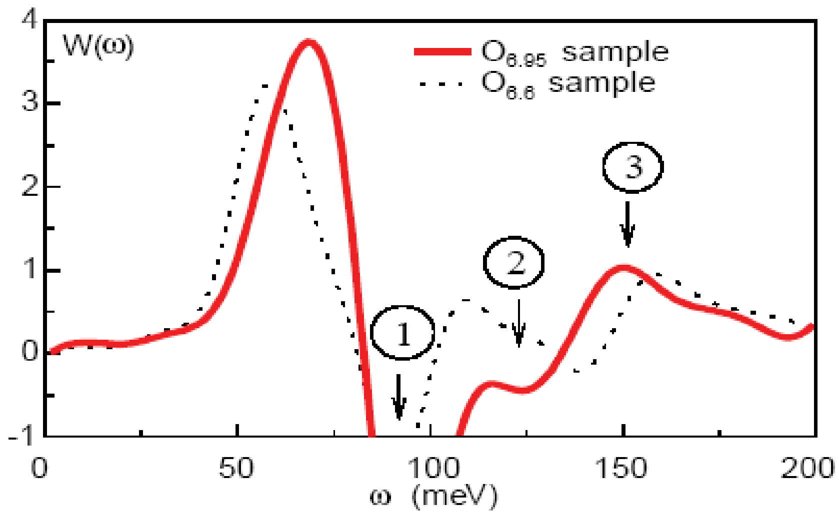


$\Omega_{mode} \sim 38 - 40$ meV

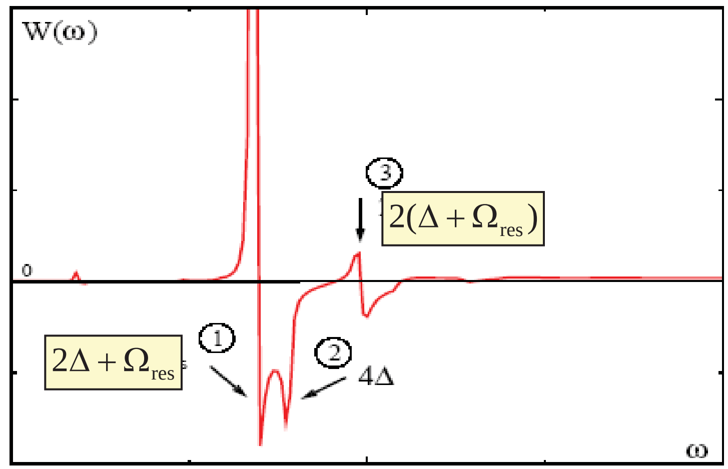
The resonance mode also affects optical conductivity

$$W(\omega) = \frac{d^2}{d\omega^2}\left[\omega \, \mathrm{Re}\, \frac{1}{\sigma(\omega)}\right]$$

$YBCO_{6.95}$



Theory



$$\Delta \approx 30\ \mathrm{meV}, \quad \Omega_{res} \approx 40\ \mathrm{meV}$$

Basov et al,
Timusk et al,
J. Tu et al…..

Abanov et al
Carbotte et al

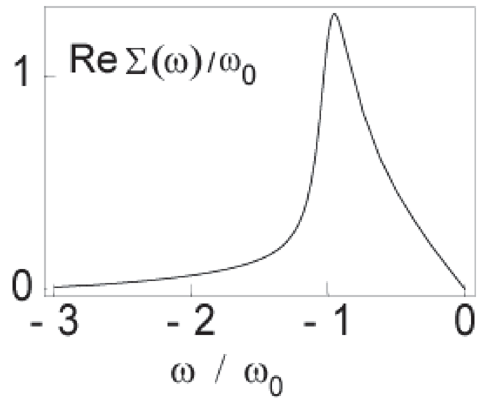
Dispersion anomalies along the Fermi surface

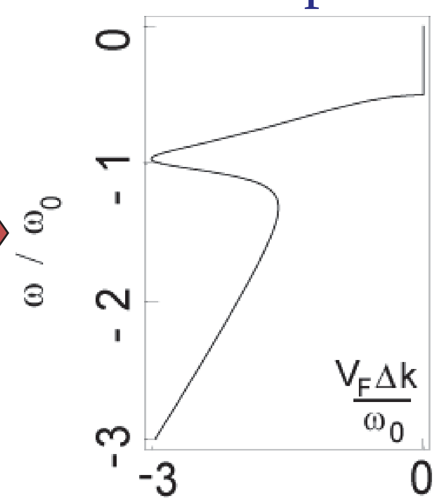
Norman, AC

The self-energy

The dispersion

Antinodal direction

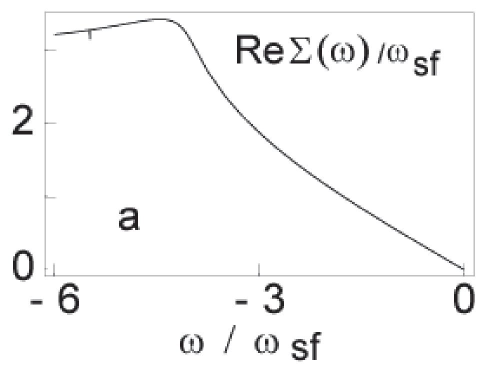


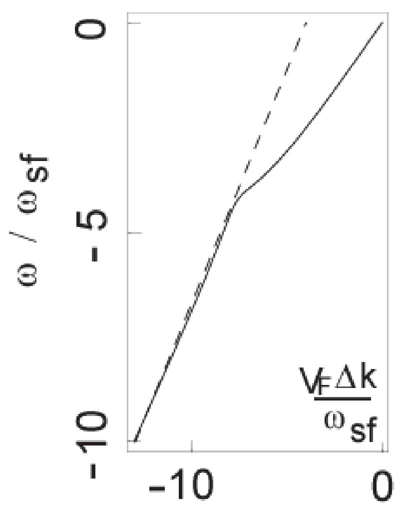


the S-shape dispersion

$$\omega_0 = \Delta + \Omega_{res}$$

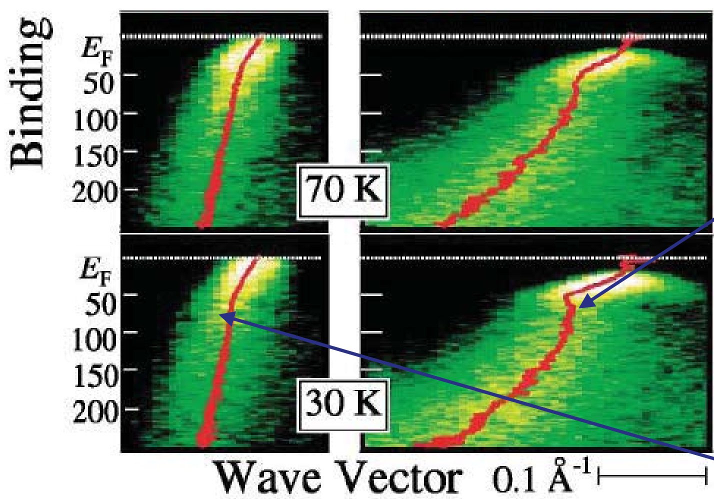
Nodal direction

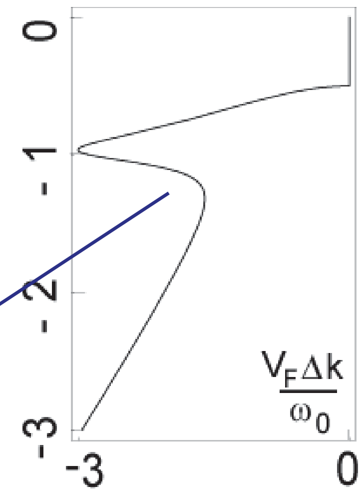




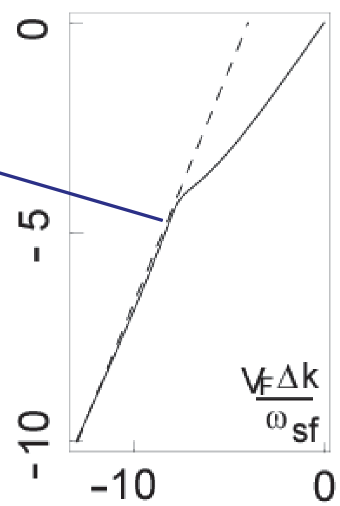
The kink

Nodal Antinodal





Antinodal



Nodal
(diagonal)

The S-shape disappears at Tc

Summary of spin-fermion model

Spin-fermion model: the minimal model which describes fermion-fermion interaction, mediated by spin collective degrees of freedom

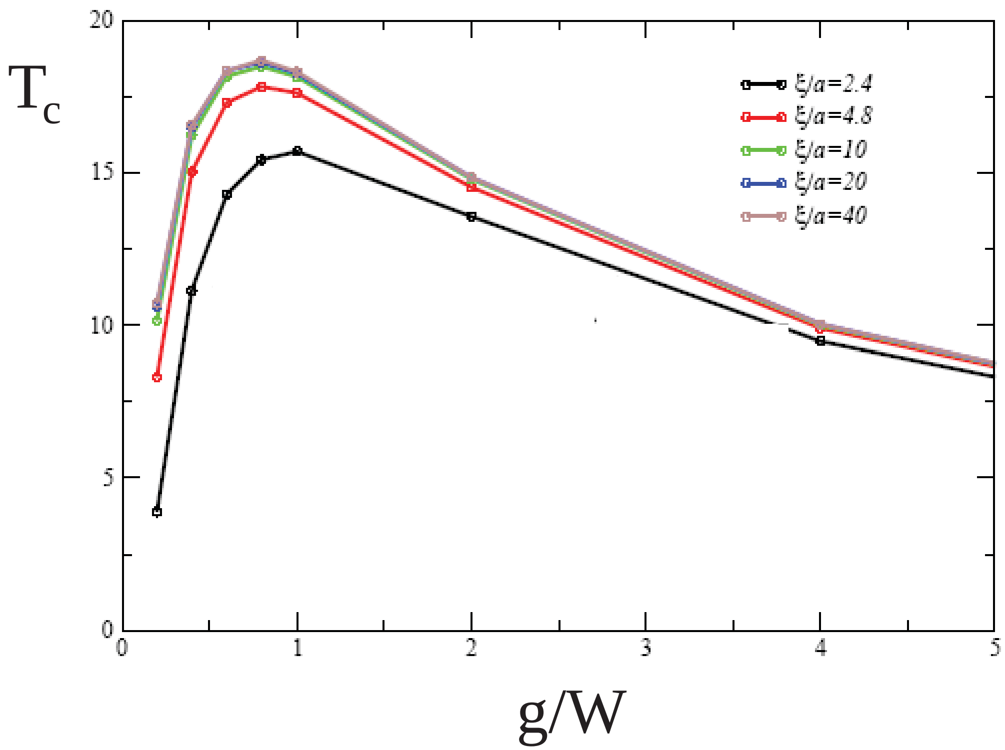
Some phenomenology is unavoidable (or RPA)

Once we selected the model, $\Sigma(\omega)$ in the normal state and superconducting Tc are obtained explicitly.

- Non-Fermi liquid in the normal state, in hot regions
- d-wave superconductivity near a QCP
- universal pairing scale
- feedbacks from SC on electronic properties

THANK YOU

The calculation of Tc can be extended to larger g



Universal pairing scale

$$T_{c,\,max} \sim 0.02\,\frac{v_F}{a}$$

The gap

$$\Delta(k) \approx \Delta\,(\cos k_x - \cos k_y)$$

Low-energy collective mode

$$\Omega_{res} \sim \Delta/\lambda < \Delta$$