Superconductivity from repulsion

Lecture 2
RG treatment of superconductivity

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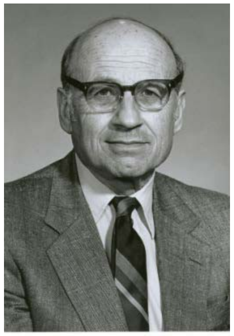
School on modern superconductivity, Boulder, CO, July 2014

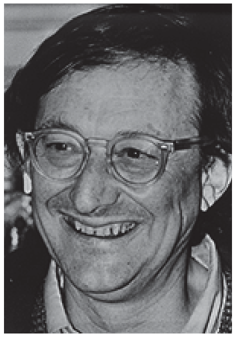
Yesterday's lecture:

- Generic conditions for SC
- Kohn-Luttinger mechanism
- p-wave pairing in isotropic systems



Walter
Kohn

Joaquin
Luttinger



For the rest of today's lecture I will explore KL idea that the effective pairing interaction is different from a bare repulsive U due to screening by other fermions, and may have attractive components in some channels

- cuprates
- doped graphene
- Fe-pnictides

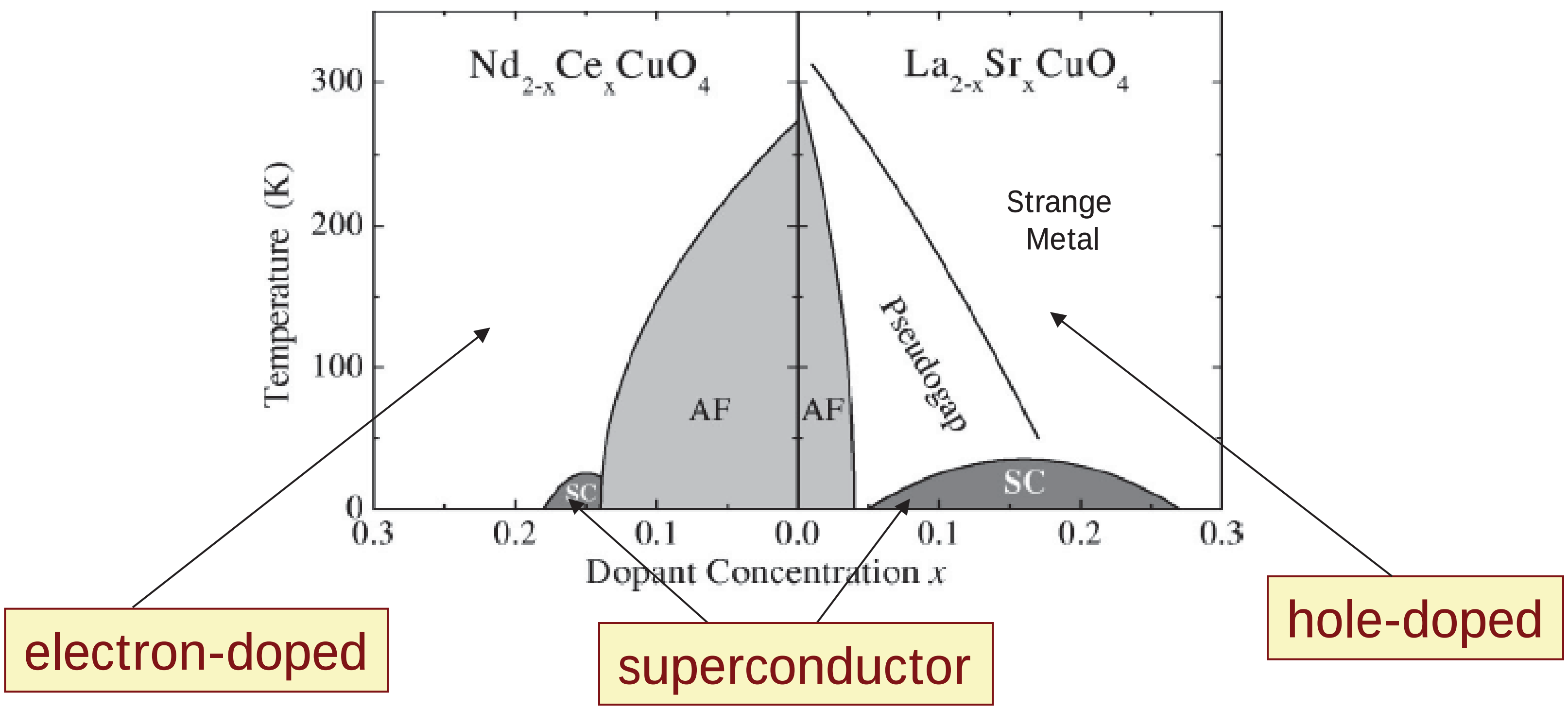
Each case will represent different lattice version of KL physics

Kohn-Luttinger story for Hubbard model:

At first order in Hubbard U,
no pairing interaction is non-s-wave channel

At second order in U, pairing interaction at large momentum transfer is enhanced more than at small momentum transfer, and the result is p-wave superconductivity

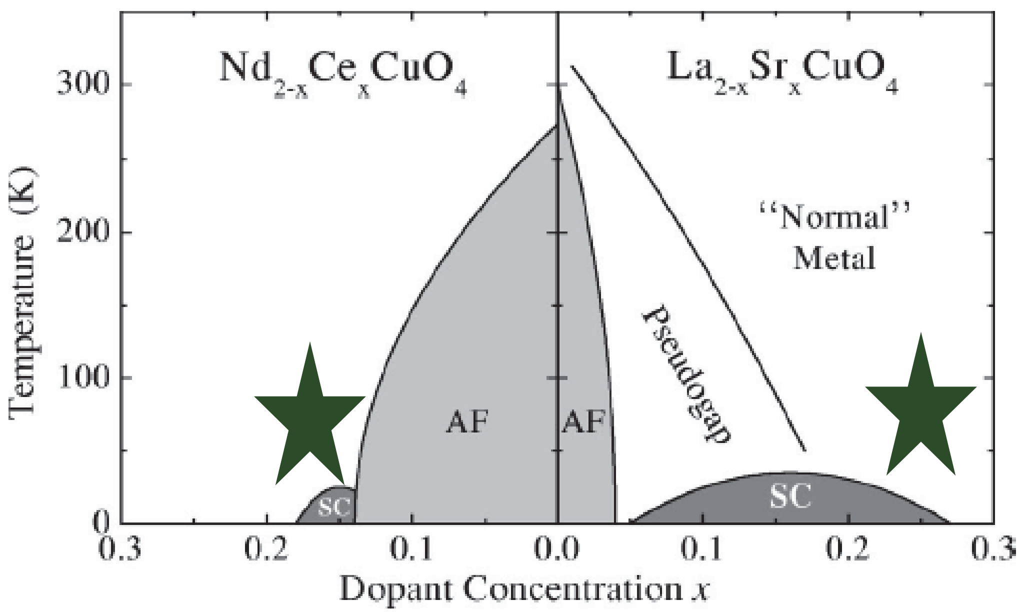
The cuprates (1986…)



Parent compounds are antiferromagnetic insulators

Superconductivity emerges upon either hole or electron doping

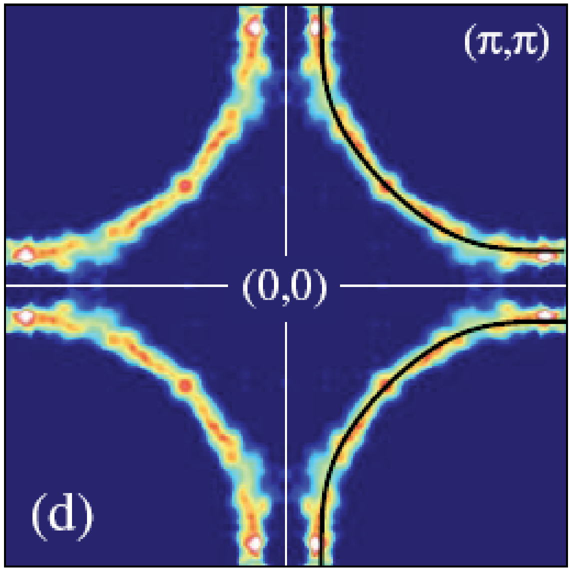
Overdoped compounds are metals and Fermi liquids



Overdoped compounds are metals and Fermi liquids

$Tl_2Ba_2CuO_{6+\delta}$

Photoemission



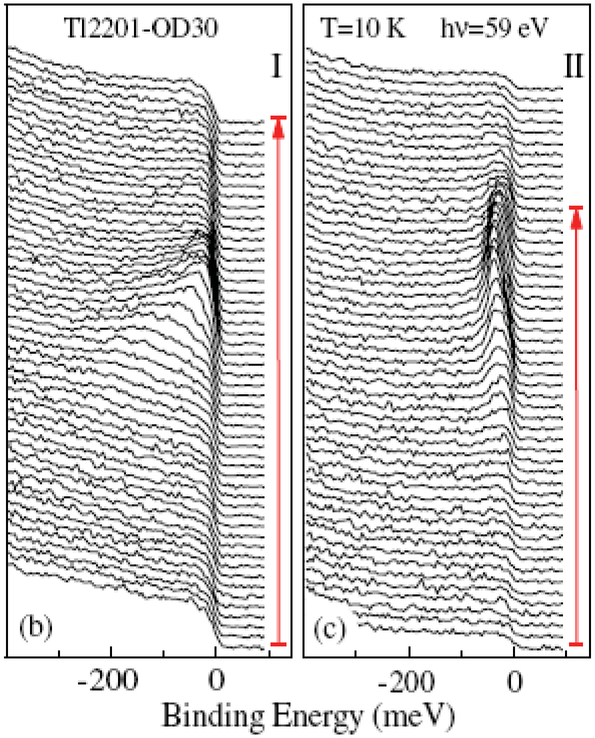
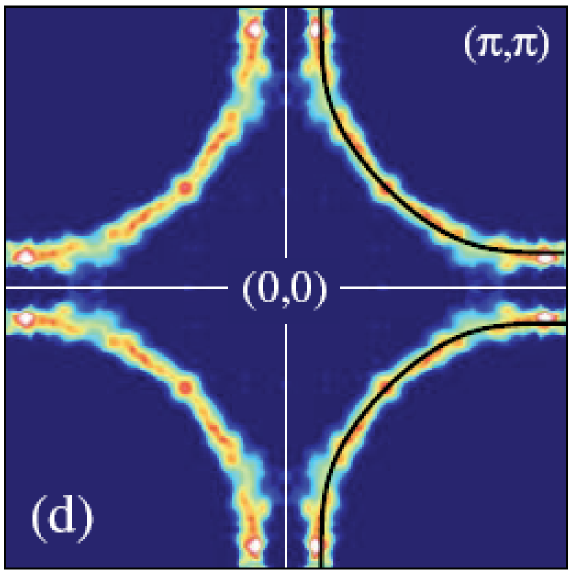


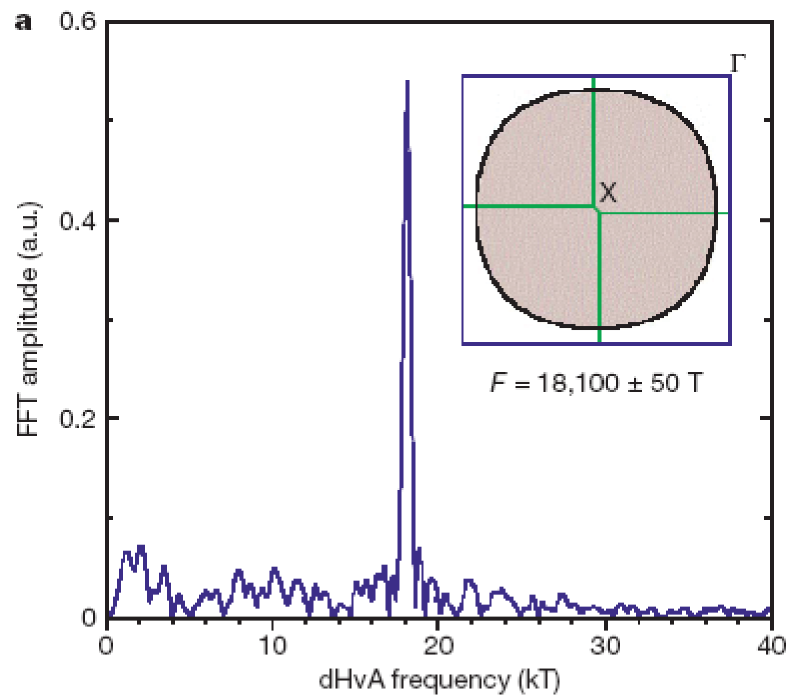
Plate et al

Overdoped compounds are metals and Fermi liquids

$Tl_2Ba_2CuO_{6+\delta}$



Oscillations of magnetoresistance

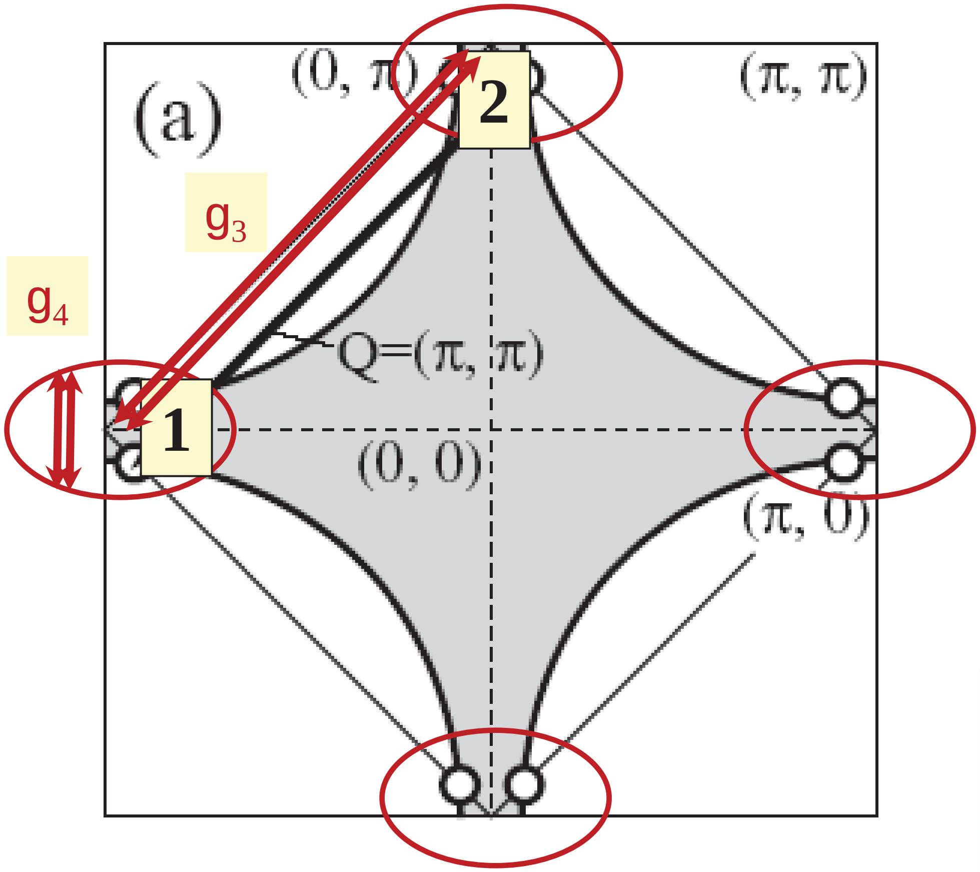


Vignolle et al

Area is consistent with Luttinger count for electrons in a Fermi liquid

Let's find lattice analog of expanding into harmonics

Let's look at regions with the highest density of states



We have repulsive interactions within a patch

$$U(1,1) = U(2,2) = g_4$$

and between patches

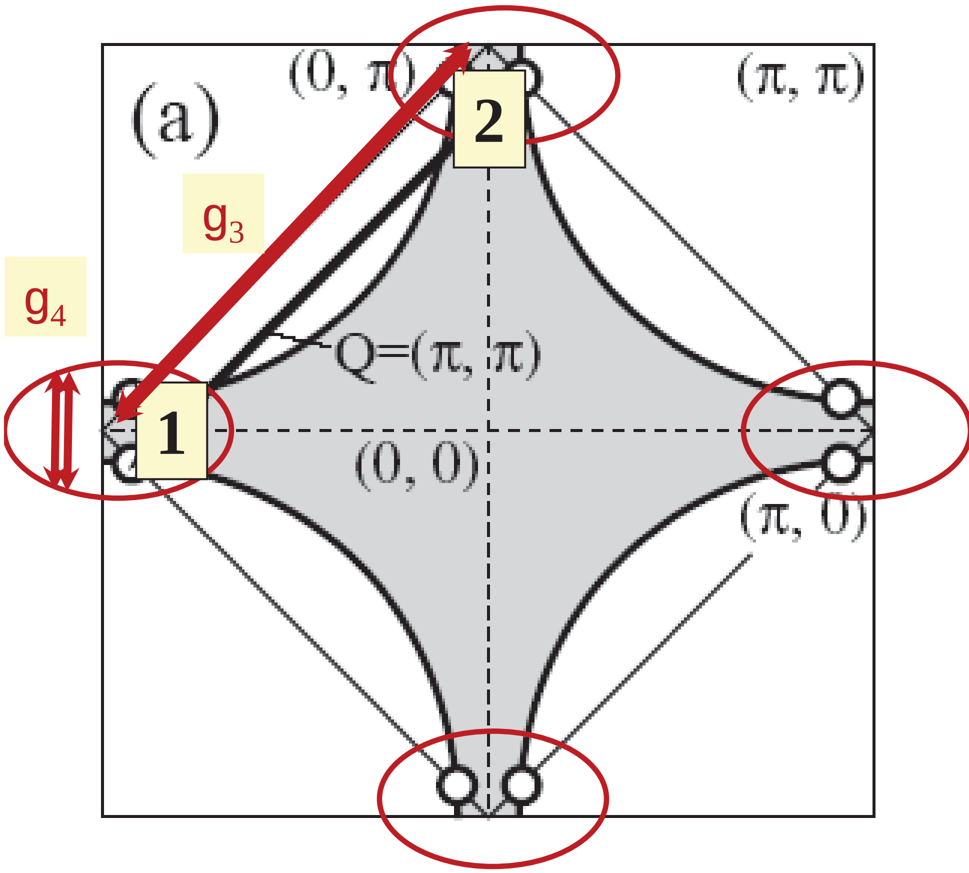
$$U(1,2) = g_3$$

Two pairing channels:

$$U_a = g_4 + g_3, \quad U_b = g_4 - g_3,$$

need $U < 0$ for pairing

$$U_a = g_4 + g_3, \quad U_b = g_4 - g_3,$$

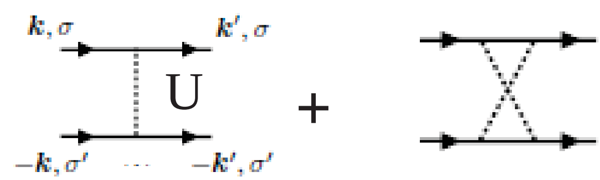


Do Kohn-Luttinger analysis for short-range repulsion U

To first order, we have a constant repulsive interaction – g_4=g_3=U, hence U_a >0, U_b =0

To order U^2

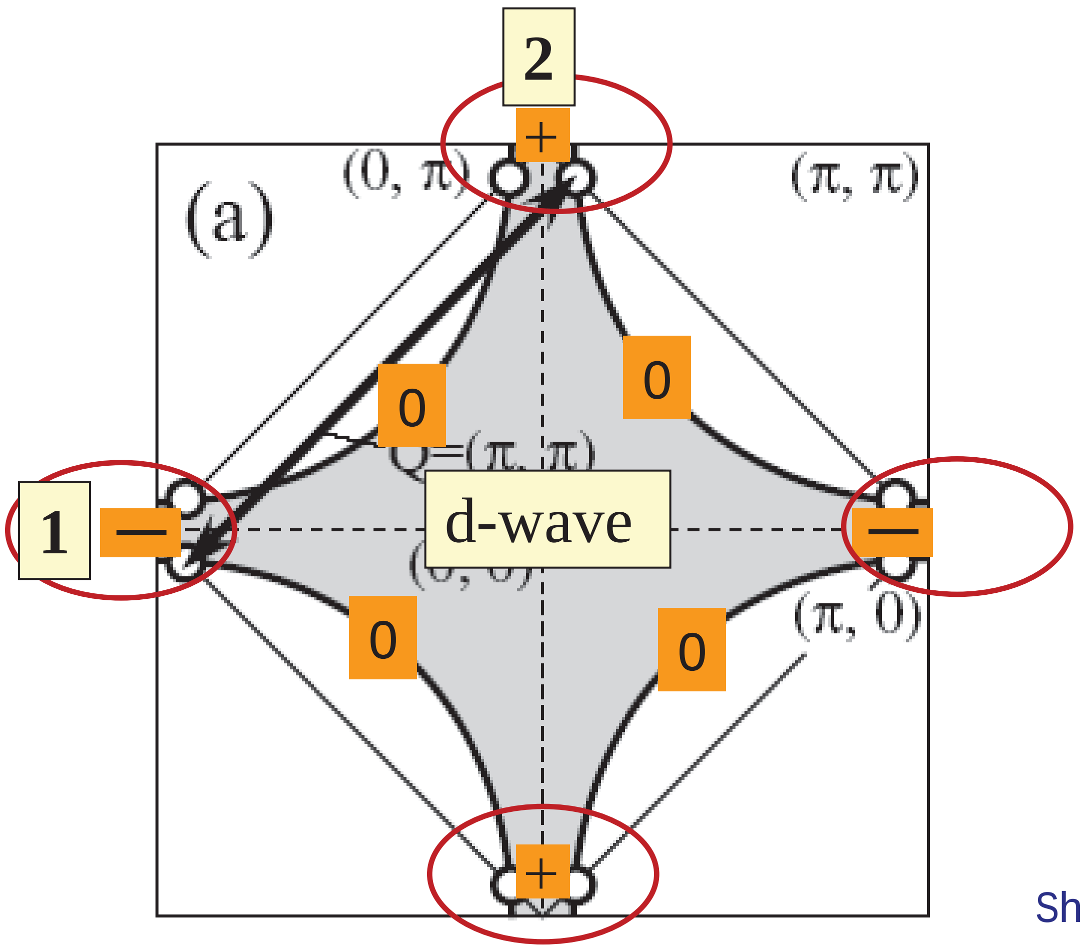
$g_{3,4} =$

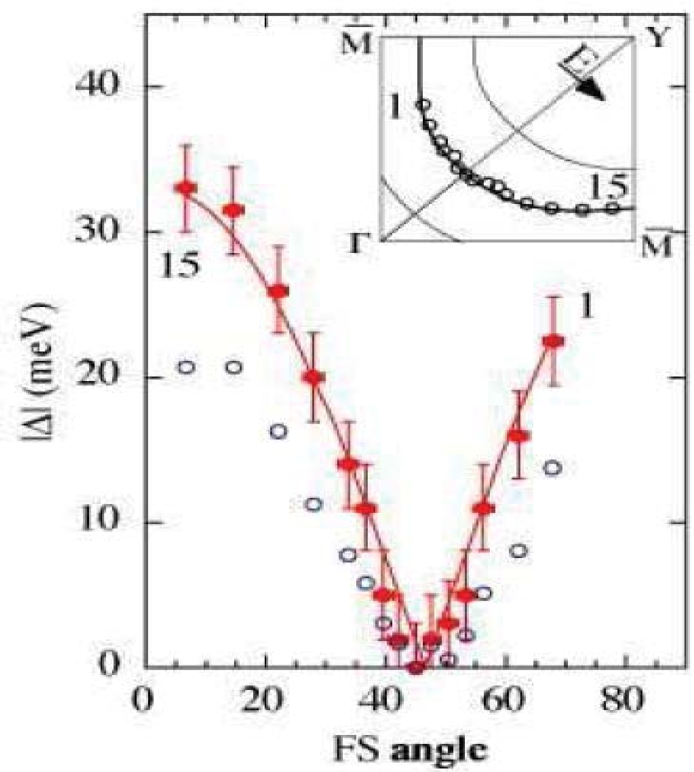


U^2 term, different for g_3 and g_4

Long story short: g_3> g_4, hence U_b <0

Eigenvector for $U_b = -g_4 + g_3 > 0$: superconducting order parameter Δ changes sign between patches





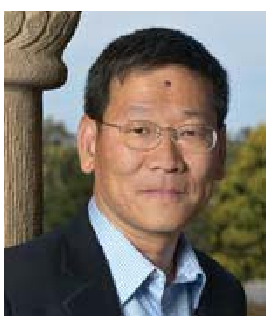
Shen, Dessau et al 93, Campuzano et al, 96

d-wave pairing is a well established phenomenon

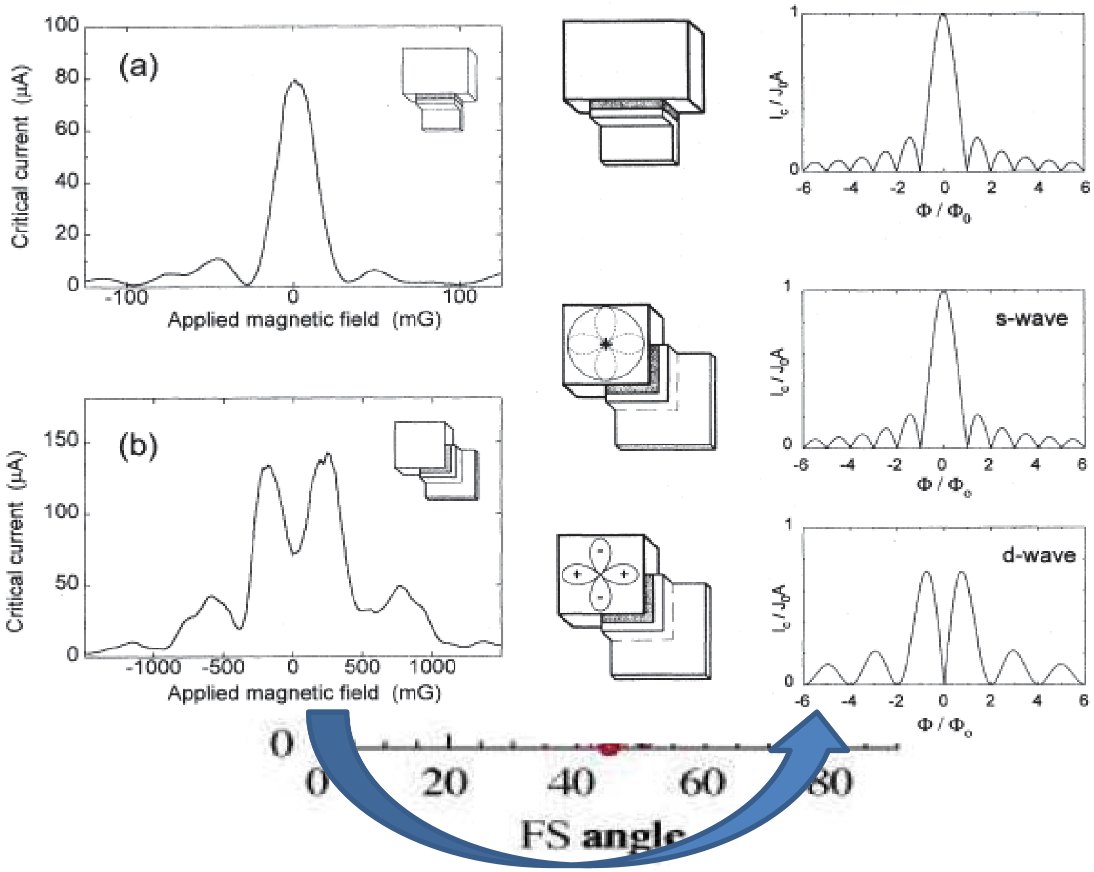
Oliver E. Buckley Condensed Matter Physics Prize

Campuzano

Johnson



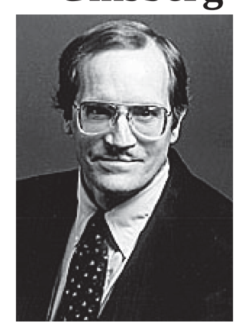
Z-X Shen



Tsuei

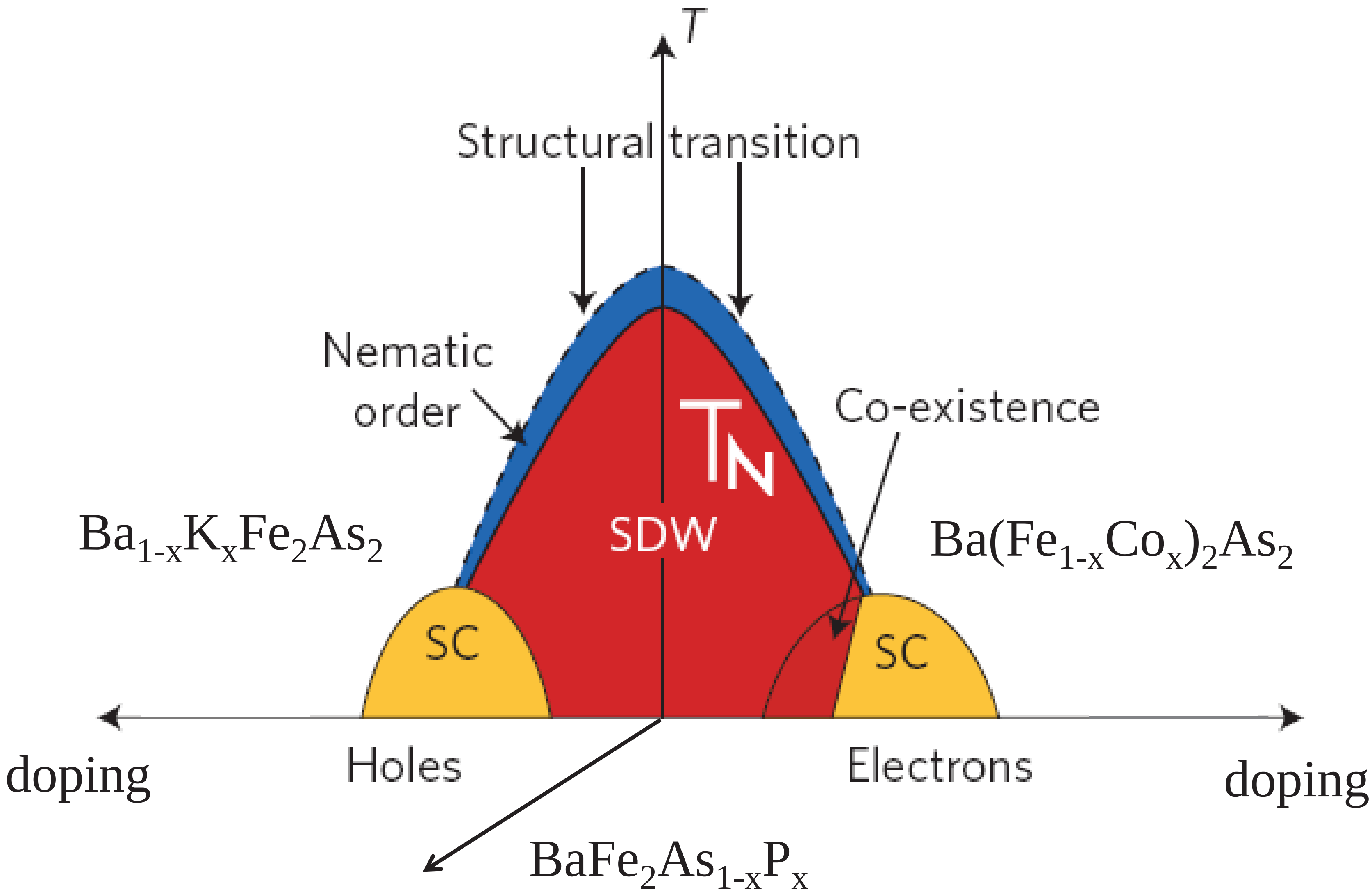
Van Harlingen

Ginsberg

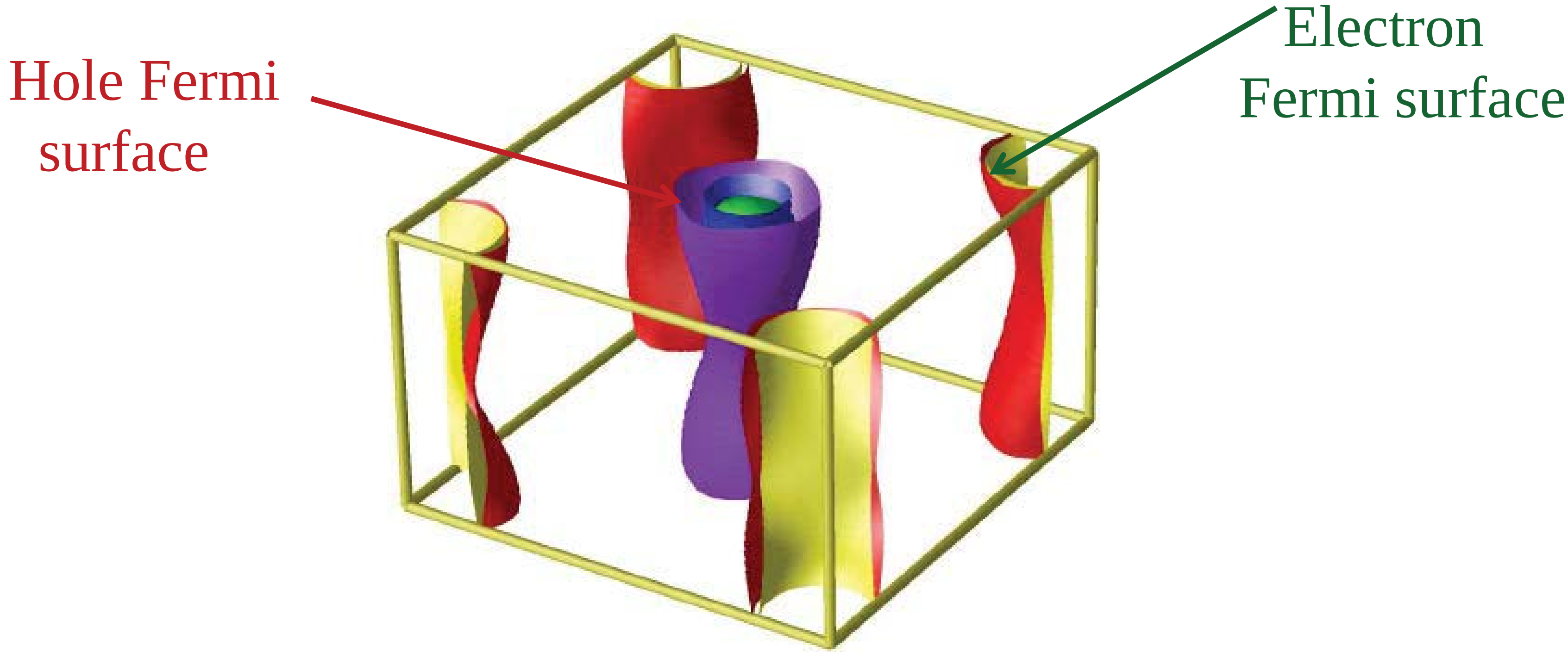


Kirtley

The pnictides (2008…)



These are multi-band systems



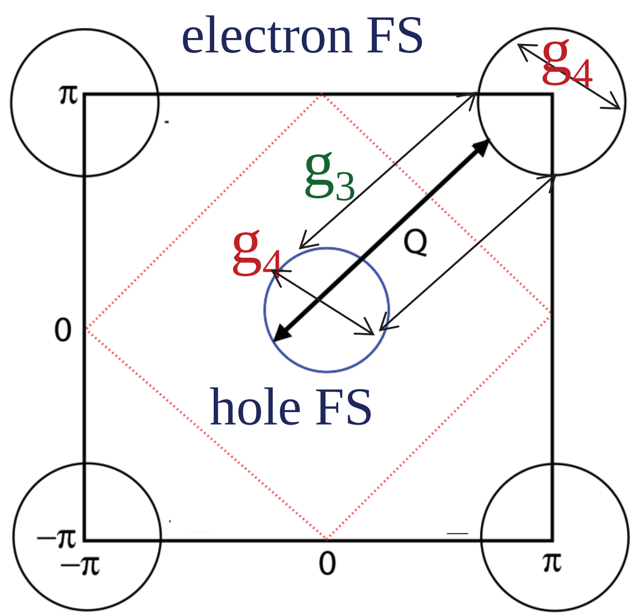
2-3 circular hole pockets around (0,0)

2 elliptical electron pockets around (π,π) (folded BZ), or $(0,\pi)$ and $(\pi,0)$ (unfolded BZ)

A toy model: one hole and one electron pocket

Inter-pocket repulsion g_4

Intra-pocket repulsion g_3



$U_a = g_3 + g_4,$

$U_b = g_4 - g_3,$

$U < 0$ is needed for SC

U_a is an ordinary s-wave

we will see what U_b is

$U_a = g_3 + g_4,$

$U_b = -g_3 + g_4,$

$U < 0$ is needed for SC

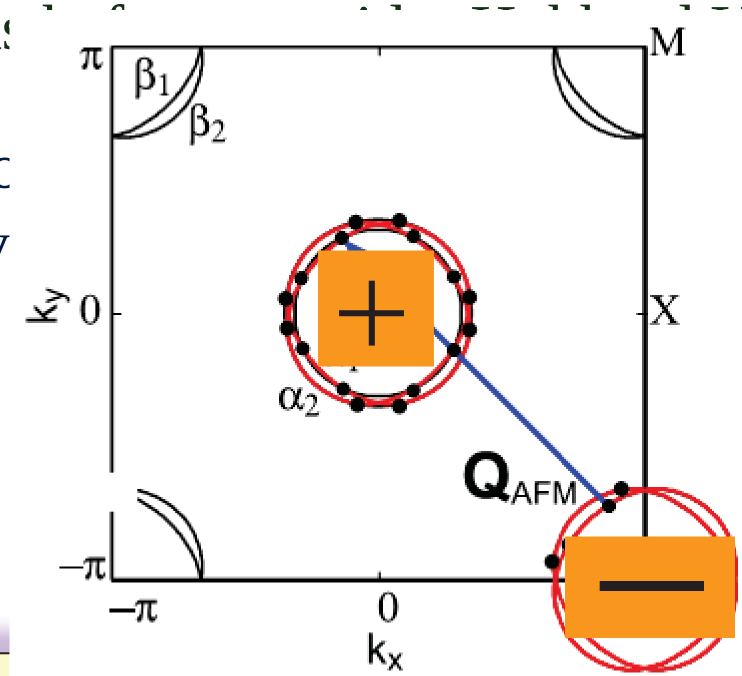
Do Kohn-Luttinger analysis:

As

To first c ... only have a

repulsiv ... U_b =0

To order U^2



Inter-pocket repulsion g_3 exceeds intra-pocker repulsion g_4, and U_b becomes negative, i.e., superconductivity develops

sign-changing s-wave gap s+-

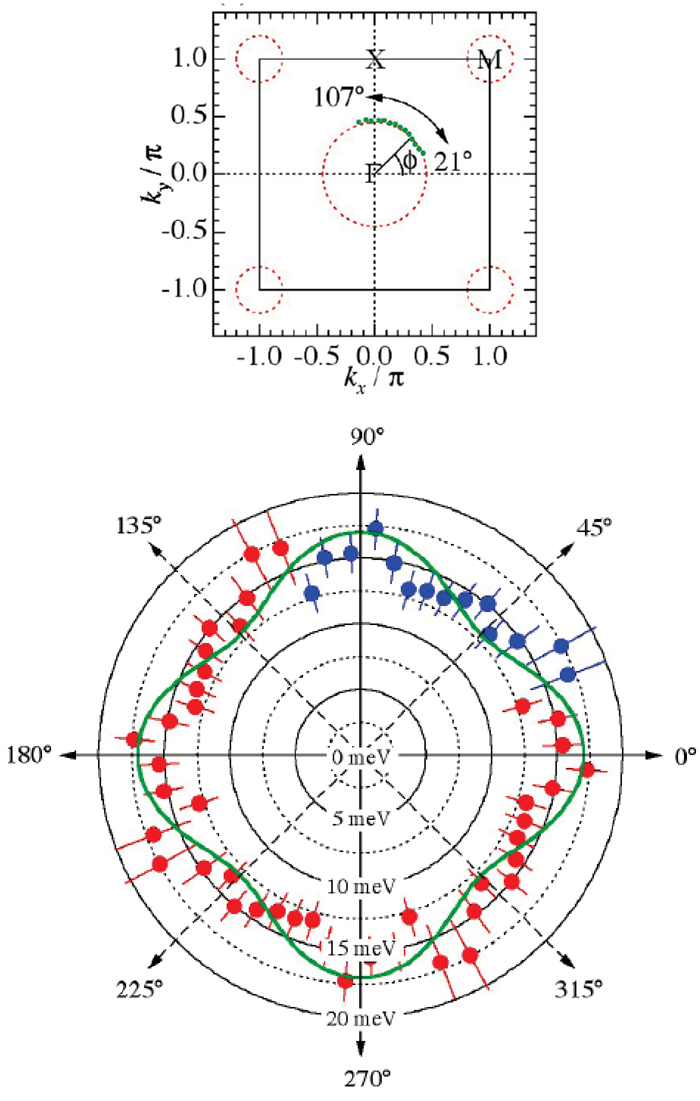
Agterberg, Barzykin, Gorkov,
Mazin, Kuroki, A.C, Tesanovic, D-H Lee..

S-wave

Photoemission in 1111 and 122 FeAs

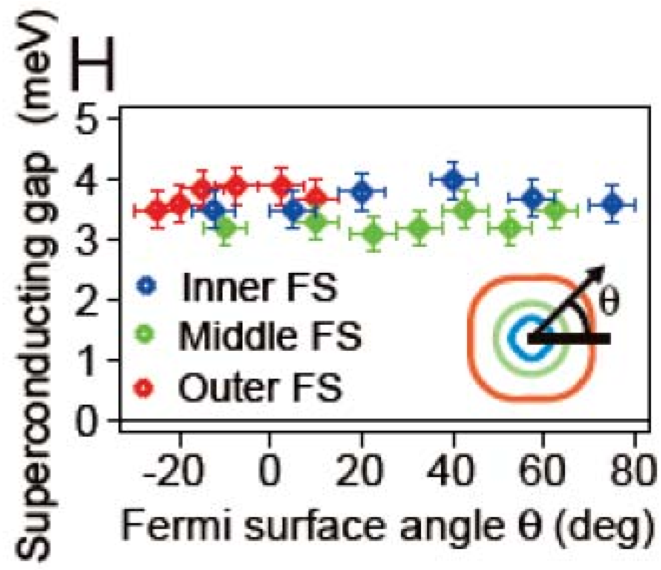
Data on the hole Fermi surfaces

$NdFeAsO_{1-x}F_x$



T. Kondo et al.

$BaFe_2(As_{1-x}P_x)_2$



T. Shimojima et al

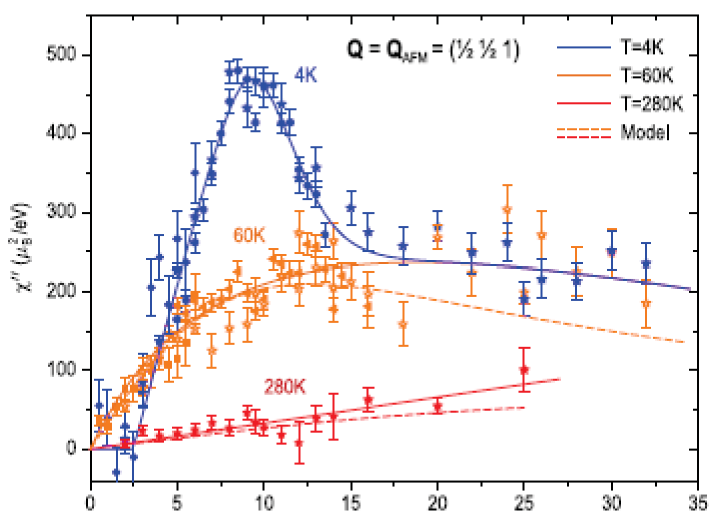
laser
ARPES

Almost angle-independent gap
(consistent with s-wave)

s+- gap

Neutron scattering – resonance peak below 2D

$BaFe_{1.85}Co_{0.15}As_2$ ($T_c = 25$ K)



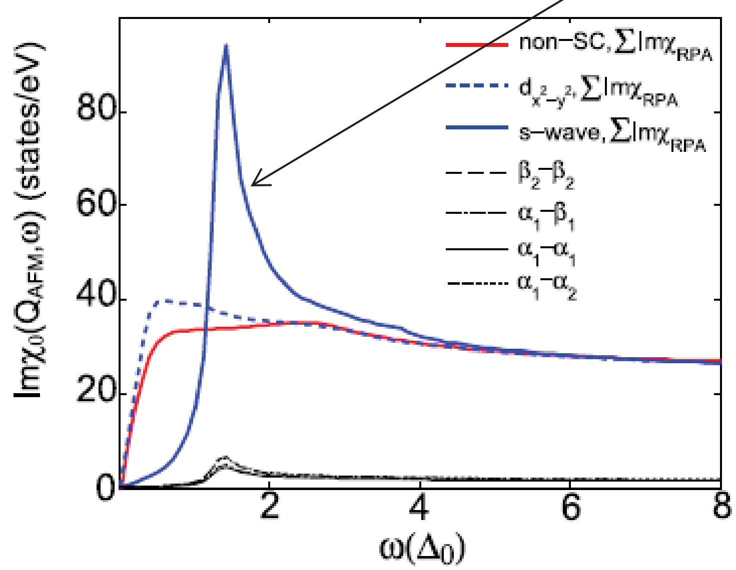
D. Inosov et al

s_{+-} gap

Theorists say :

one needs $\Delta_{k+\pi} = - \Delta_k$

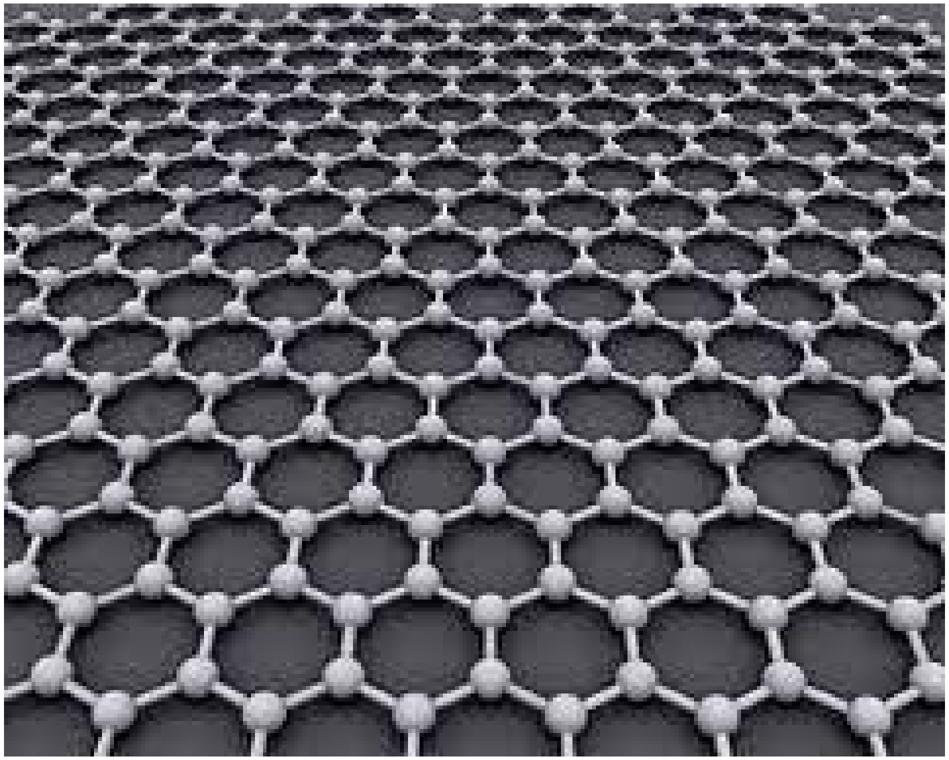
The “plus-minus” gap is the best candidate



Eremin & Korshunov Scalapino & Maier...

Doped graphene (2000…)

Graphene -- an atomic-scale honeycomb lattice
made of carbon atoms.

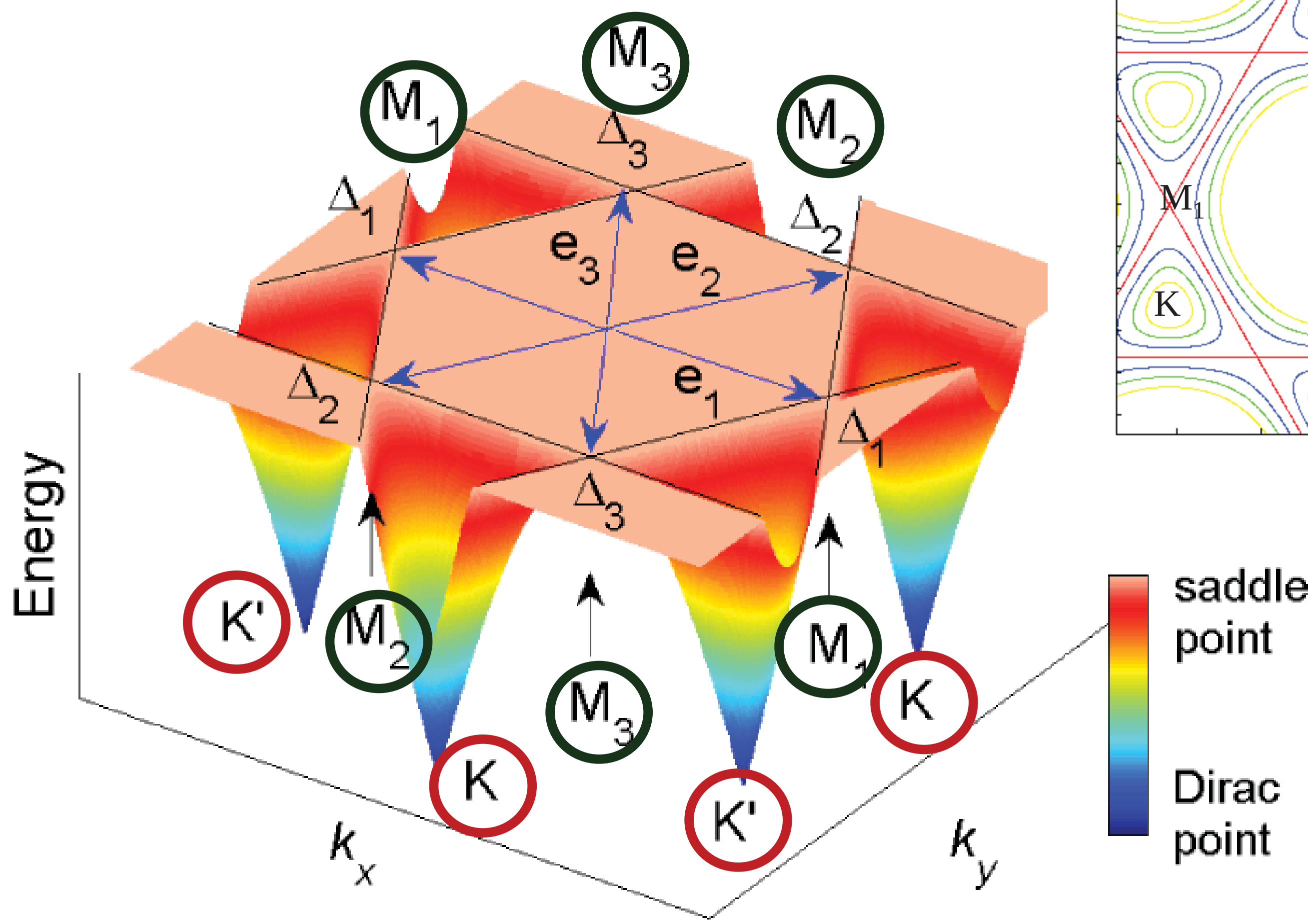


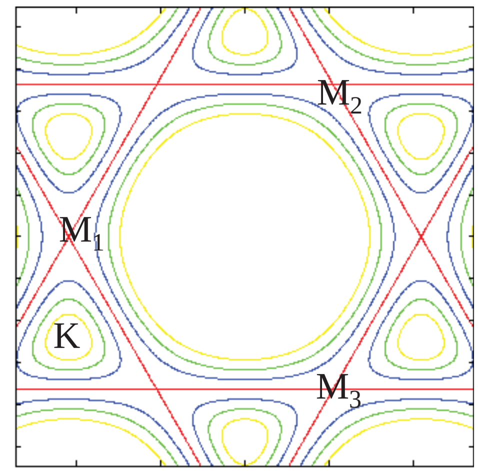
Nobel Prize 2010
Andre Geim, Konstantin Novoselov

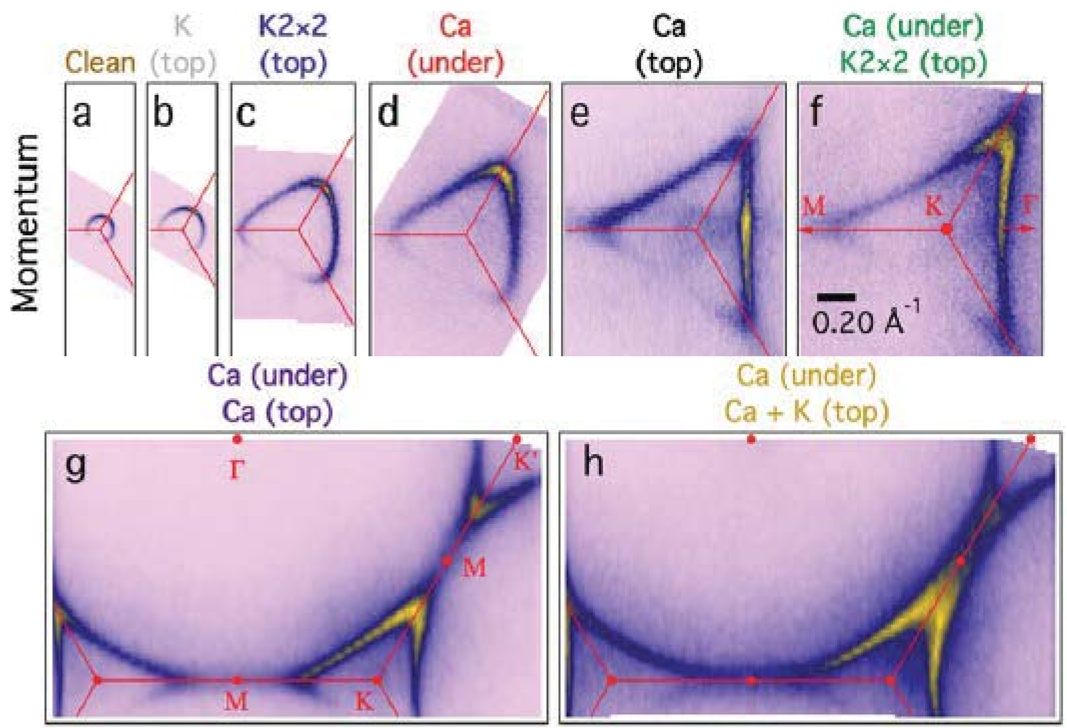
$$\varepsilon(\mathrm{k}) = \mathrm{t}_1\sqrt{1 + 4\cos\frac{\mathrm{k}_y\sqrt{3}}{2}\cos\frac{3\,\mathrm{k}_x}{2} + 4\cos^2\frac{\mathrm{k}_y\sqrt{3}}{2}} - \mu$$

$\mu = 0$, Dirac points

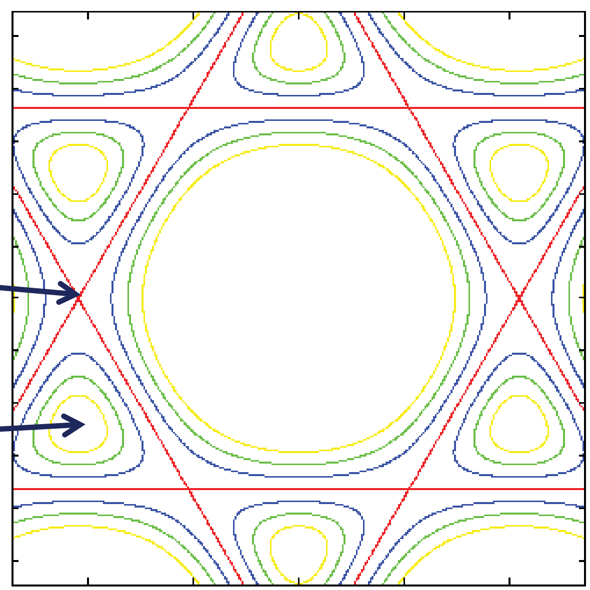
$\mu = \mathrm{t}_1$, van Hove points







E. Rotenberg et al
PRL 104, 136803 (2010)



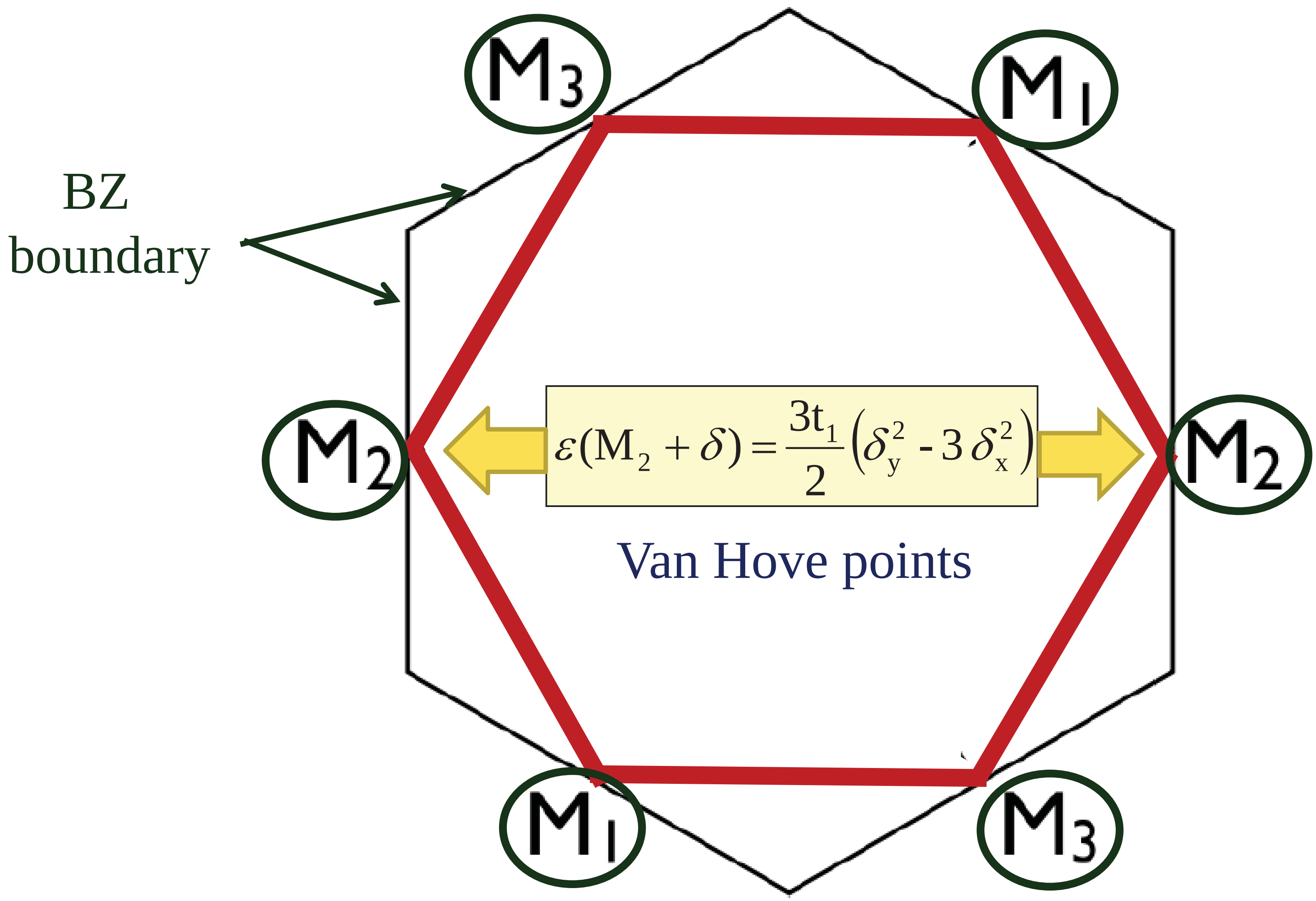
M

K

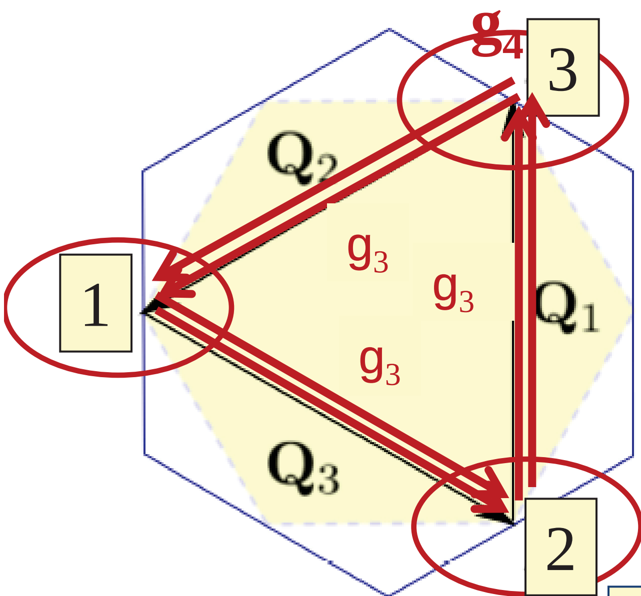
Dirac point

At van Hove doping

$$\varepsilon(\mathrm{k}) = t_1\sqrt{1+4\cos\frac{k_y\sqrt{3}}{2}\cos\frac{3k_x}{2}+4\cos^2\frac{k_y\sqrt{3}}{2}} - t_1$$



$$\varepsilon(M_2+\delta) = \frac{3t_1}{2}\left(\delta_y^2 - 3\,\delta_x^2\right)$$



$$U(1,1) = U(2,2) = U(3,3) = g_4$$
$$U(1,2) = U(2,3) = U(1,3) = g_3$$

Three pairing channels:

$$U_a = g_4 + 2g_3, \quad U_b = U_c = g_4 - g_3,$$

we need $U < 0$ for pairing

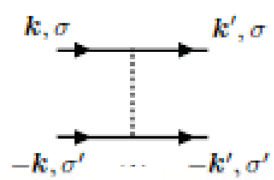
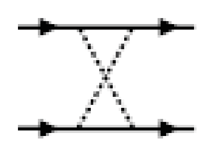
U_a is an ordinary s-wave

we will see what $U_{b,c}$ are

Do Kohn-Luttinger analysis for Hubbard U:

To first order in U, $g_4=g_3=U$, and we only have a repulsive s-wave component, $U_c > 0$, $U_{b,c} = 0$

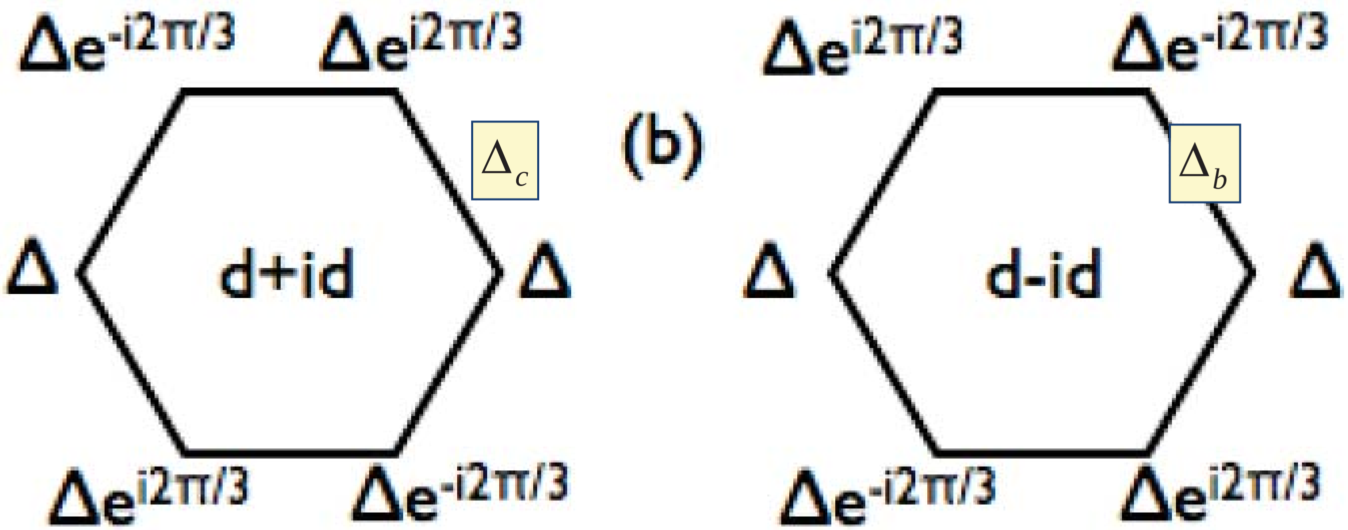
To order U^2

$g_{3,4} =$  + 

$g_3 > g_4$ and $U_{b,c} < 0$

Superconducting order para

$$\Delta_c = \frac{\Delta}{\sqrt{2}}(0, 1, -1) \qquad \Delta_b = \sqrt{\frac{2}{3}}\Delta\left(1, -\frac{1}{2}, -\frac{1}{2}\right)$$



The two d-wave solutions are degenerate by symmetry

Gonzales

Do they appear together?

$$F = \alpha(T - T_c)(|\Delta_a|^2 + |\Delta_b|^2) + K_1\left(|\Delta_a|^2 + |\Delta_b|^2\right)^2 + K_2|\Delta_a^2 + \Delta_b^2|^2 + O(\Delta^6)$$

Yes, and with relative factor i: d+id state

chiral superconductivity (phase winds up by 4 π)

Summary of Kohn-Luttinger physics for lattice systems:

At weak coupling, a fermionic system may undergo a superconducting instability, despite that the interaction is repulsive. The instability is never an ordinary s-wave

d-wave ($d_{x^2-y^2}$) pairing in the cuprates
s+- in Fe-pnictides
d+id ($d_{x^2-y^2} + d_{xy}$) in doped graphene

This story is a little bit too good to be true.

In all three cases, we assumed that bare interaction is a Hubbard U, in which case, in a relevant channel $\Gamma = 0$ to order U and becomes negative (attractive) to order U^2

In reality, to first order in interaction, $U_b = g_4 - g_3 = U_{small} - U_{large}$

small (large) is a momentum transfer

For any realistic interaction, $U_{small} > U_{large}$

Then bare U>0, and the second order term has to overcome it

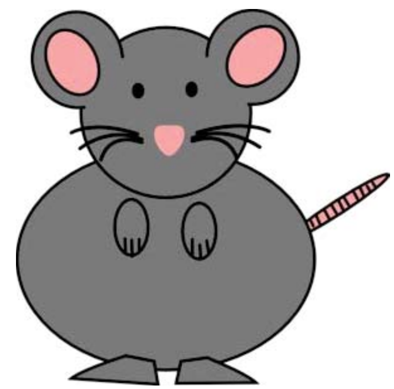
Physicists, we have a problem

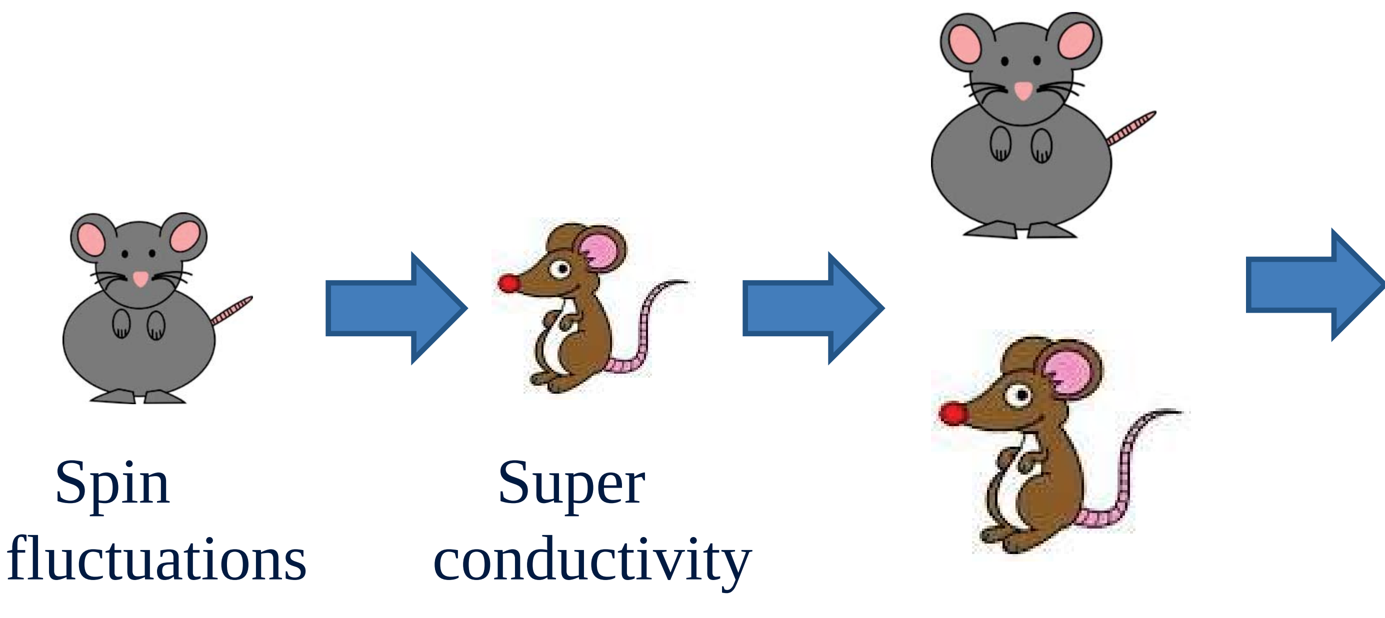
Two ways to resolve the problem:

Both assume that superconductivity is not the only instability in a given system, there is also a density-wave instability around.

One approach is to keep couplings weak, but see whether we can additionally enhance KL terms due to interplay with other potential instabilities, which develop along with SC.

Renormalization group (RG) approach





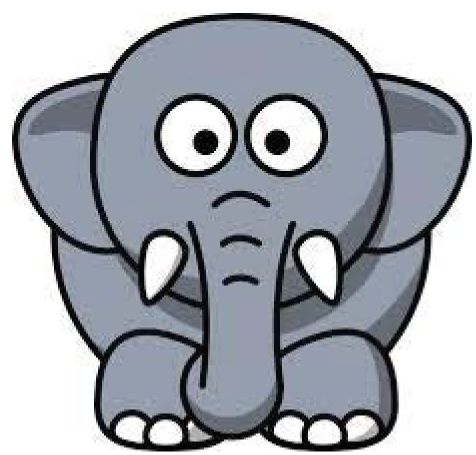
or

Two ways to resolve the problem:

Both assume that superconductivity is not the only instability in a given system, there is also a density-wave instability around.

Another approach is to abandon weak coupling and assume that density-wave instability (magnetism or charge order) comes from fermions at high energies, of order bandwidth. As an example, near antiferromagnetic instability, inter-pocket/inter-patch interaction g_3 is enhanced if we do full RPA summation in the particle-hole channel (or use any other method to account for contributions from high-energy fermions)

Super conductivity



Spin fluctuations

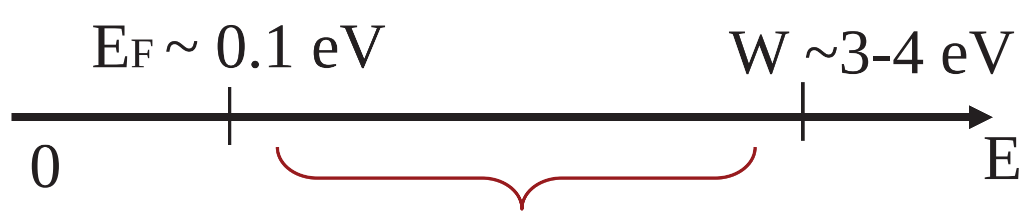
Let's start with RG

$$U_a = g_3 + g_4,$$
$$U_b = -g_3 + g_4,$$
$$U_{a,b} < 0 \text{ is needed for SC}$$

Consider Fe-pnictides as an example

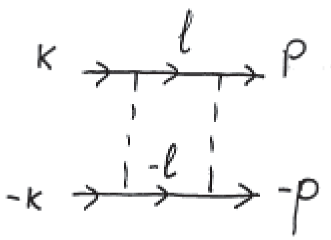
g_3 and g_4 are bare interactions, at energies of a bandwidth

For SC we need interactions at energies smaller than the Fermi energy



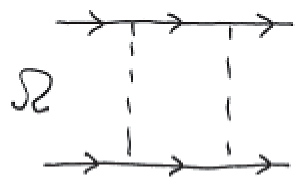
Couplings flow due to renormalizations in all channels (particle-particle AND particle-hole channels)

Recall: the pairing (particle-particle channel)
is logarithmically singular



$$\int \frac{d^d l \, d\omega_l}{(2\pi)^{d+1}} \, G_l \, G_{-l} = \frac{i N_F}{2} \int_{-\Lambda}^{\Lambda} \frac{d\varepsilon_l}{|\varepsilon_l|} = \log \text{ singular}$$

At a non-zero
total frequency
(or temperature)

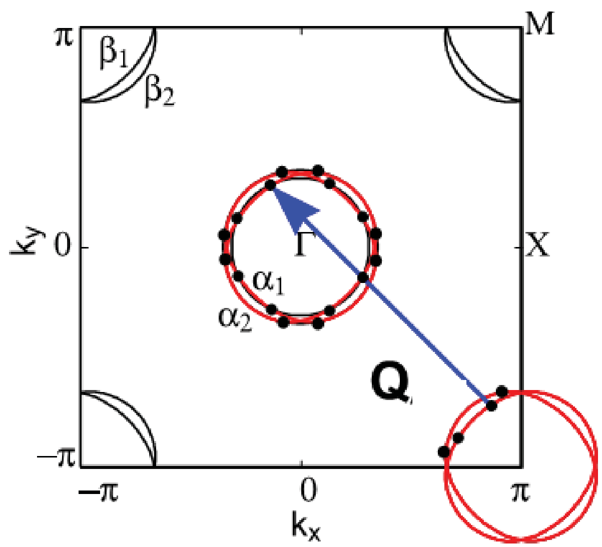


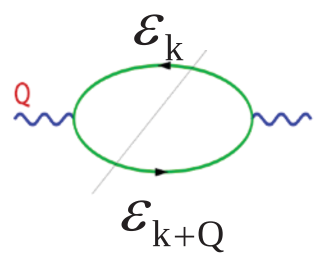
$$(\log \text{ singular}) \Rightarrow \log \frac{\Lambda}{|\Omega|} + i\,\pi/2$$

This is a Cooper logarithm

We cannot treat the pairing channel perturbatively because each time we add an extra power of (small) interaction, it gets multiplied by a large logarithm, and the product may be as large as we want.

Peculiarity of Fe-pnictides: because one pocket is electron-type and the other is hole-type, renormalizations in particle-hole channels are also logarithmically singular



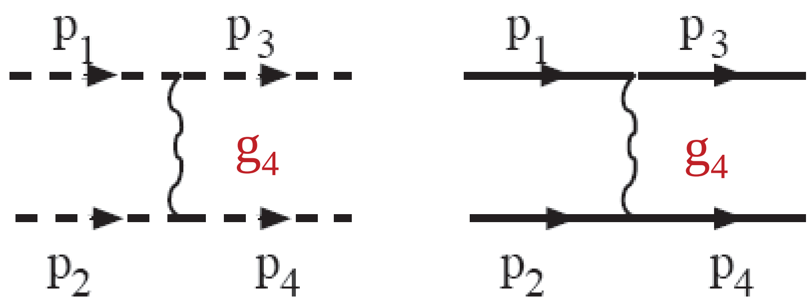


$$= \iint_T \frac{d\omega\, d\varepsilon_k}{\omega^2 + \varepsilon_k^2} = \log \frac{E_F}{T}$$

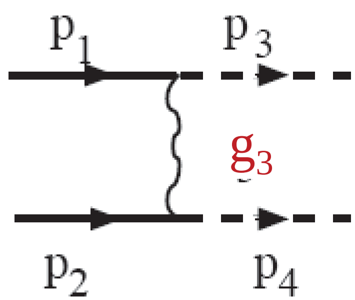
Then we have to treat particle-particle (SC) and particle-hole channels on equal footings

This is true EVEN if there is no nesting, as we consider renormalizations from fermions with $E_F < E < W$.

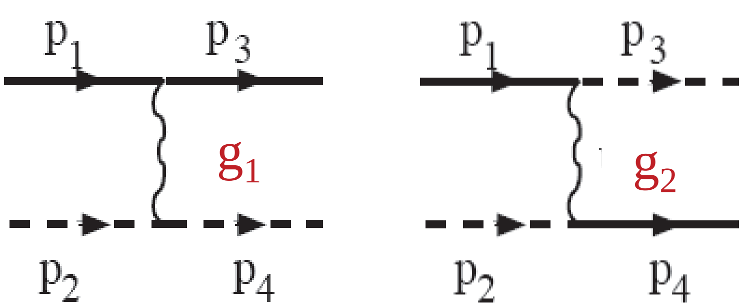
How to proceed:
introduce all relevant couplings between low-energy fermions



Intra-pocket repulsion



Inter-pocket repulsion



Inter-pocket forward and
backward scattering

What these other interactions g_1 and g_2 do?

a) They participate in KL renormalizations of g_3 and g_4

b) They lead to either spin-density-wave or charge-density-wave

Each of these orders obviously competes with superconductivity, but in the process of developing spin or charge order, fluctuations in the corresponding channels modify superconducting interactions, and modification is different for intra-pocket and inter-pocket interactions

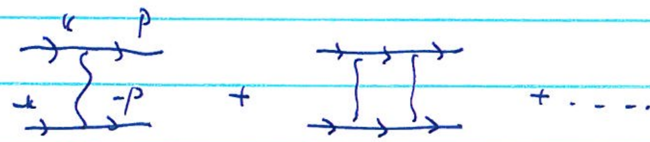
ABC of RG treatment of superconductivity
(let's use blackboard)

Addition 2

RG treatment of SC (follow-up of lectures by S. Kivelson)

*) consider a system with a constant attractive interaction U

We know: to get SC we need to sum up ladder diagrams



$$U_{eff} = \frac{U}{1 + U N(0) \log \frac{E_F}{T}} \qquad \text{need } U<0.$$

(**) Suppose we do this Wilsonian way, i.e, by progressively eliminating ~~...~~ fermions at energies $\Lambda < E < E_F$

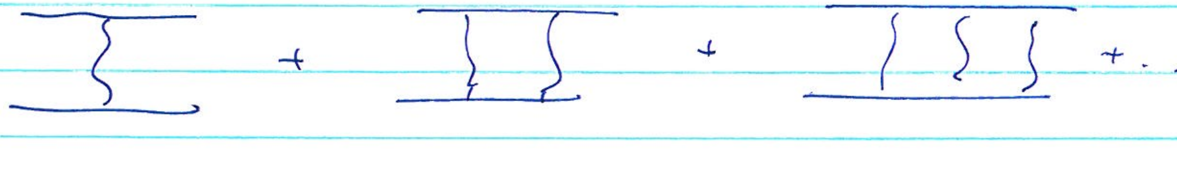
We can define $U(\Lambda)$: coupling at intermediate energies

$$U(\Lambda) = \frac{U}{1 + U N(0) \log \frac{E_F}{\Lambda}}$$ } from ladder diagrams

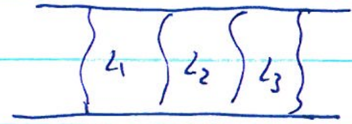
But we can also look at ladder series in a different way:

we need coupling at scale Λ, or at logarithmical scale $L = \log \frac{E_F}{\Lambda}$

Let's look at ladder series for $U(L)$



Let ~~me~~ take "some" diagram



$\int dL_1 \int dL_2 \int dL_3$

Larger energies $\equiv$ smaller L, at $\Lambda = E_F$, $L = \log \frac{E_F}{\Lambda} = 0$

Let me choose cross-section with the largest L_2 and integrate out in all other cross-sections over $0 < L_i < L_2$. (see p. 4 for explanations)

Then we can represent the result as

$$U(L) = -N_0 \int_{(\sim 1)}^{L} U^2(L_2)\, dL_2$$

or $\boxed{\dfrac{dU}{dL} = -U^2(L) N_0}$ $\qquad U(0) = U$

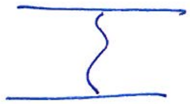
Solve this diff. eqn $\Rightarrow$ $U(*) = \dfrac{U}{1 + U N_0 \log \frac{E_F}{\Lambda}}$

Same eqn as in ladder summation.

So, SC can be equivalently viewed as a flow of the coupling towards larger L.

At some L, $U(L)$ diverges $\Rightarrow T_c$.

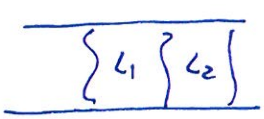
Explanations for how to obtain RG.

Look:  $= g$

Let's take external parameters (momentum, frequency)

at L $\Rightarrow$ $g = g(L)$

 $= g^2 \int_0^L dL_1$

 : $g^3 \int_0^L dL_1 \int_0^L dL_2 \equiv g^3 \left[\int_0^L dL_1 \int_0^{L_1} dL_2 + \int_0^L dL_2 \int_0^{L_2} dL_1 \right]$

Combine:

$$g(L) = g + g^2 \int_0^L dL_1 \left[1 + 2g \int_0^{L_1} dL_2\right] + \dots$$

$$1 + 2g \int_0^{L_1} dL_2 \Rightarrow \left(1 + g \int_0^{L_1} dL_2\right)^2$$

or $$g(L) = g + \int_0^L dL_1 \underbrace{\left[g\left(1 + g \int_0^{L_1} dL_2\right)\right]^2}_{g^2(L_1)} + \dots$$

$$\frac{dg}{dL} = g^2(L)$$

When more than one channel is involved,
one needs to combine logarithmic renormalizations
from particle-particle AND particle-hole channels

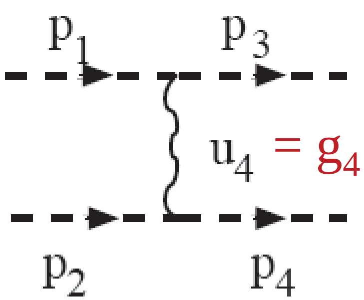
This leads to parquet RG equations – all coupling talk to each other and flow as we progressively integrate out contributions from fermions at energies larger than running E

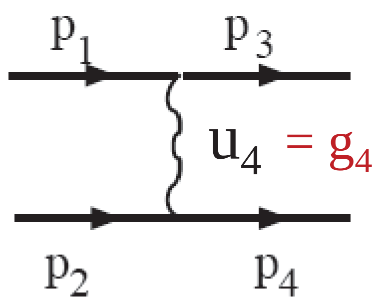
So, we need to introduce all relevant couplings between low-energy fermions

This is not too terrible – we only have 5 different couplings in the two-band model

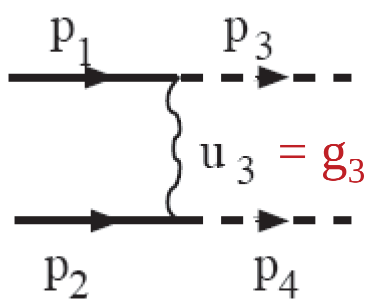
Introduce all relevant couplings between low-energy fermions

With apologies, I will from now label interactions as u_i instead of g_i

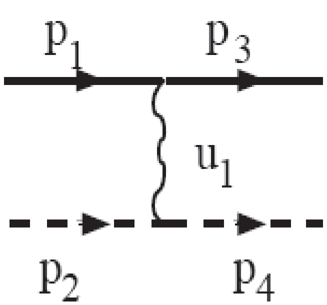


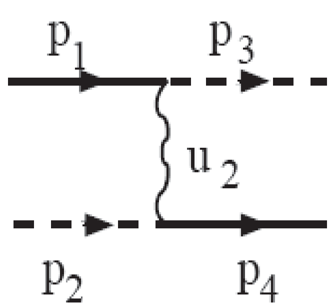


Intra-pocket repulsion



Inter-pocket repulsion

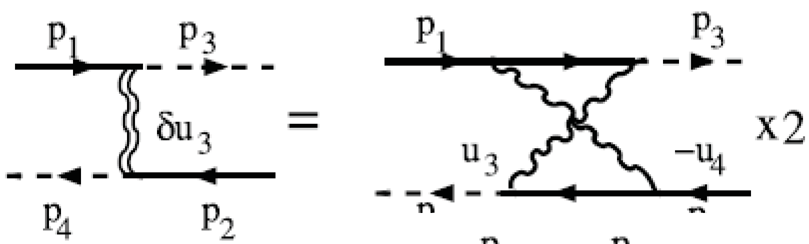




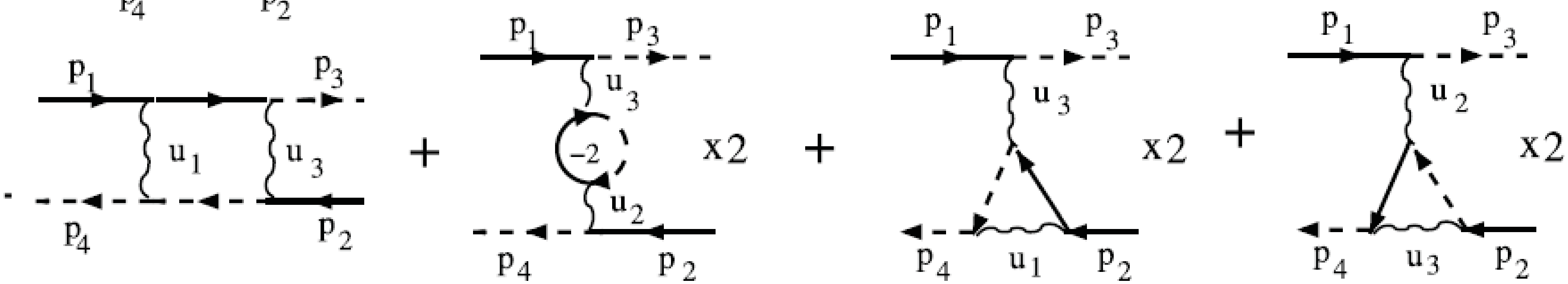
Inter-pocket forward and backward scattering

Recall: we need enhancement of u_3 relative to u_4 for superconductivity

Renormalization of u_3

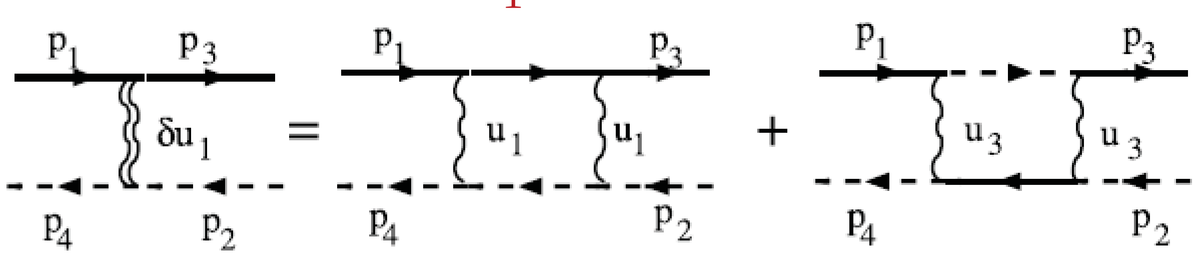


Particle-particle channel,
Cooper logarithm



Kohn-Luttinger diagrams, “nesting logarithms”

Renormalization of u_1



Also contains “nesting logaritms”

Combine all renormalizations into a set of RG requations

$$\frac{du_1}{dL} = u_1^2 + u_3^2,$$

$$\frac{du_2}{dL} = 2u_2\,(u_1 - u_2),$$

$$\frac{du_3}{dL} = u_3\,(4u_1 - 3u_2 - 2u_4),$$

$$\frac{du_4}{dL} = -u_3^2 - u_4^2,$$

$$L = \log\frac{W}{E}$$

Without coupling between particle-hole and particle-particle channels, we would have

$$\frac{d(u_3 + u_4)}{dL} = -(u_4 + u_3)^2$$

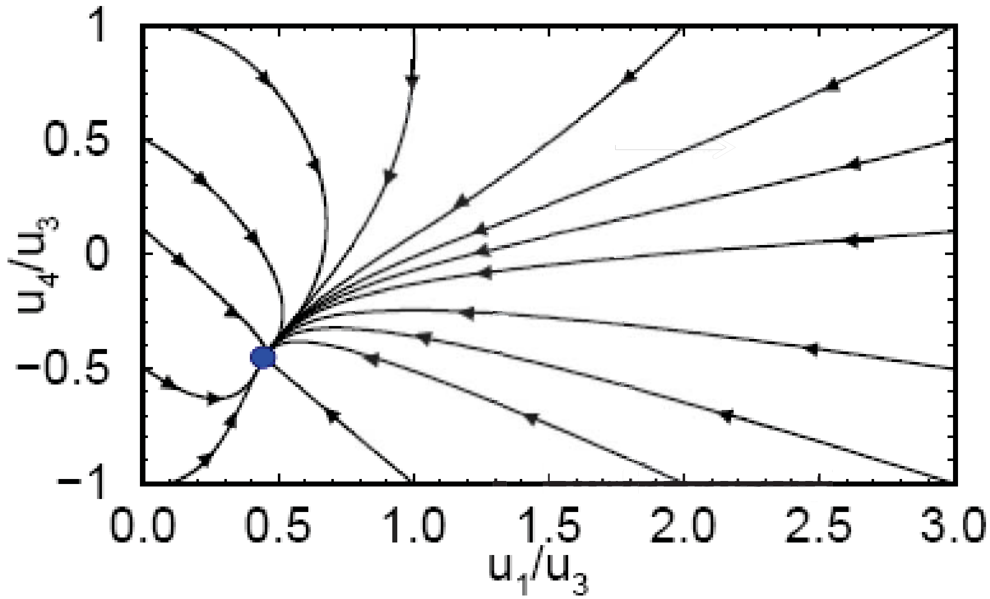
$$\frac{d(u_4 - u_3)}{dL} = -(u_4 - u_3)^2,$$

$$u_4 \Rightarrow \frac{a}{L},\ u_3 \Rightarrow \frac{b}{L^2}$$

both vanish at L >>1

With the coupling between particle-hole and particle-particle channels,

$$\dot{u}_1 = u_1^2 + u_3^2$$
$$\dot{u}_2 = 2u_2(u_1 - u_2)$$
$$\dot{u}_3 = 2u_3(2u_1 - u_2 - u_4)$$
$$\dot{u}_4 = -u_3^2 - u_4^2$$



The fixed point: the pair hopping term u_3 is the largest

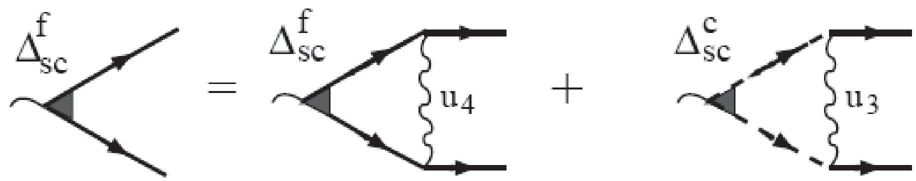
Physics: interaction g_1, which leads to SDW order at large momentum transfer, pushes up another interaction at large momentum transfer, which is g_3

$$u_1 = -u_4 = \frac{|u_3|}{\sqrt{5}}, u_2 \propto |u_3|^{1/3}$$

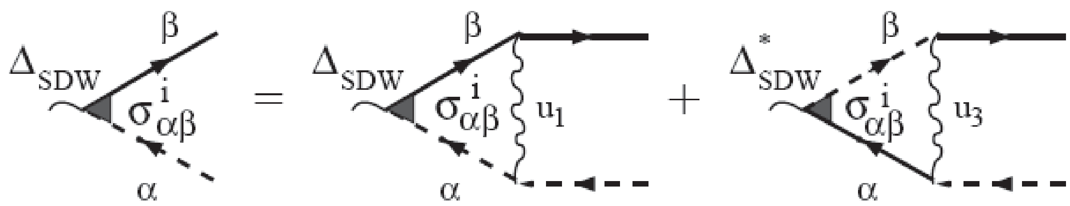
Over-screening: intraband interaction u_4 changes sign and becomes attractive below some scale.

We can re-write parquet RG equations as equations for density-wave and superconducting vertices

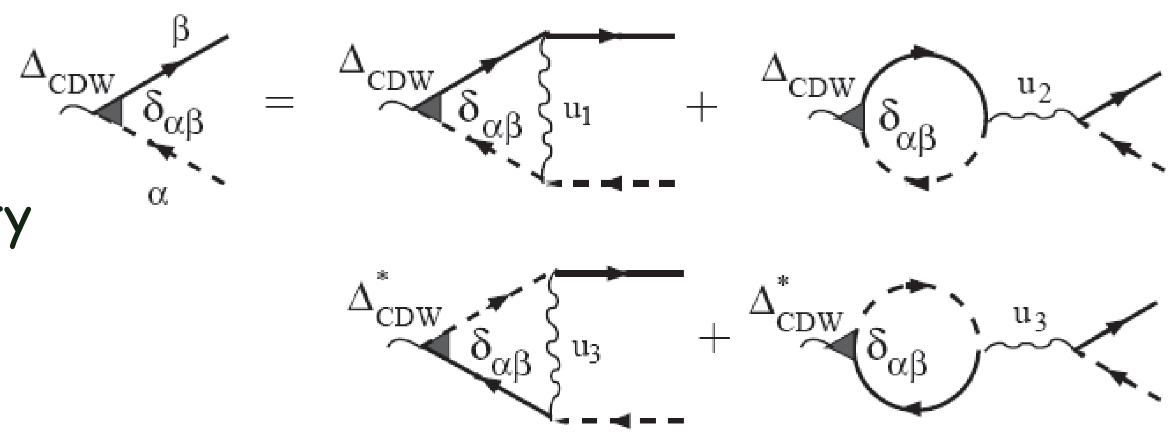
Super-conductivity



Spin-density wave

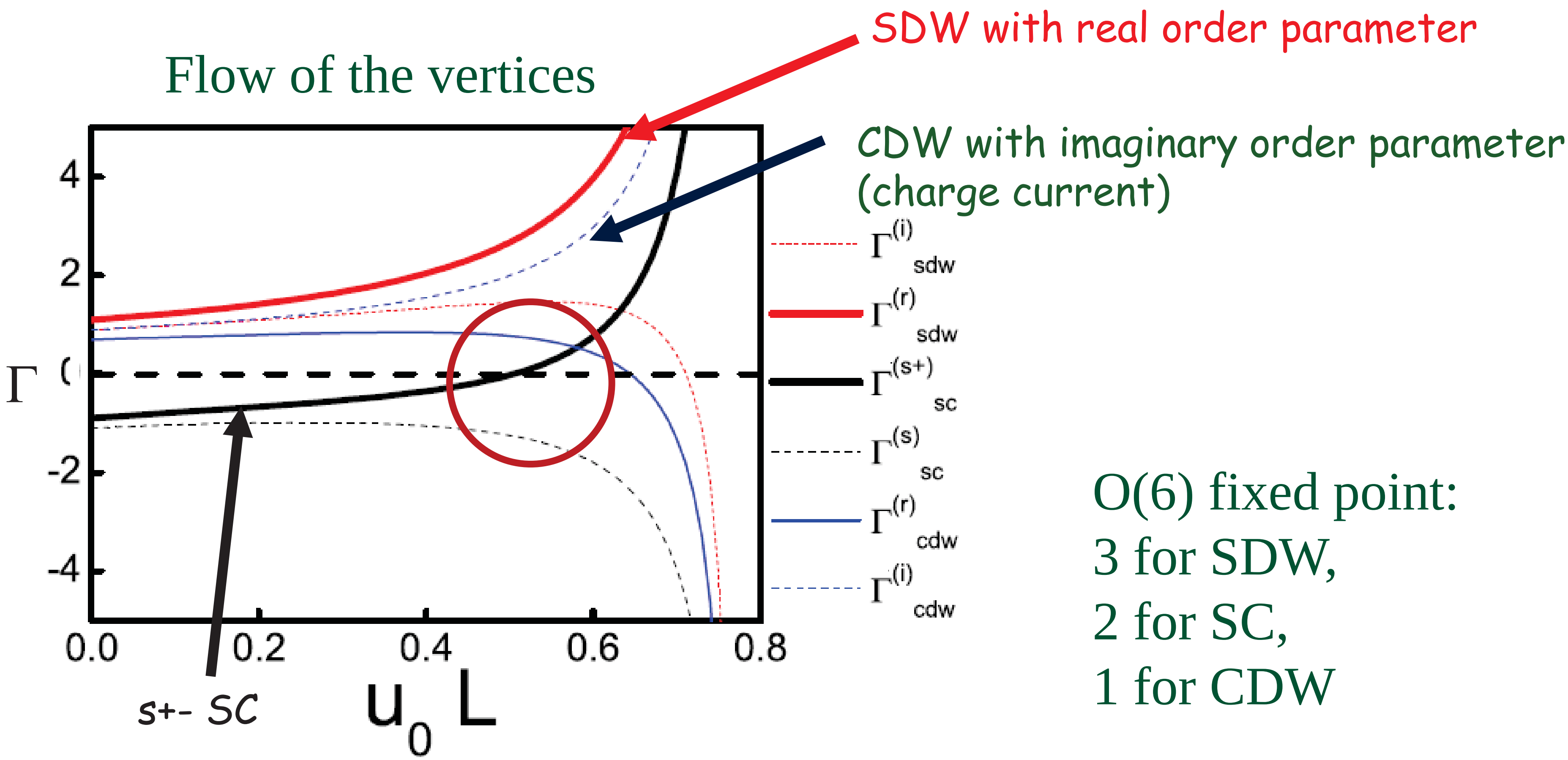


Charge-density wave



One-loop RG Flow – all channels

Flow of the vertices



O(6) fixed point:
3 for SDW,
2 for SC,
1 for CDW

At some scale, generated by the system, s+- SC vertex changes sign and becomes attractive

Lower boundary for parquet RG is the Fermi energy, E_F

Renormalization group equations

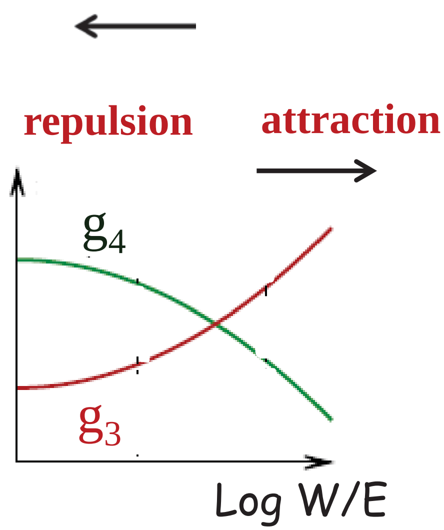
$$\frac{dg_1}{dL} = g_1^2 + g_3^2,$$

$$\frac{dg_2}{dL} = 2g_2(g_1 - g_2),$$

$$\frac{dg_3}{dL} = -g_3(4g_1 - 3g_2 - 2g_4),$$

$$\frac{dg_4}{dL} = -g_3^2 - g_4^2,$$

$$L = \log\frac{W}{E}$$

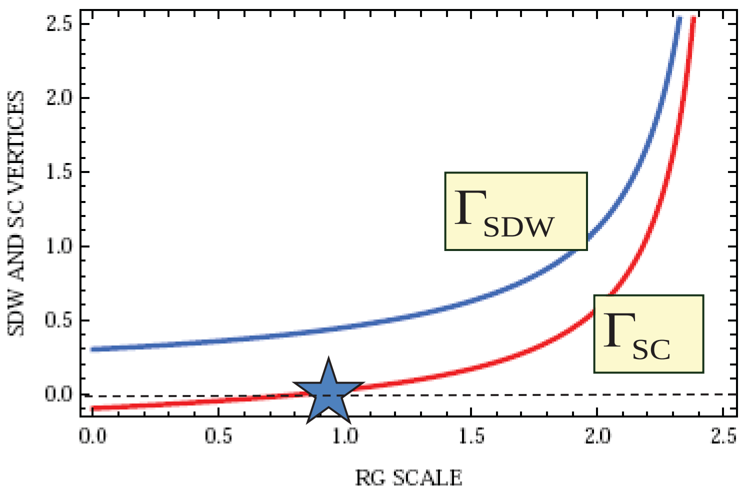


Physics: interaction g_1, which leads to SDW order at large momentum transfer, pushes up another interaction at large momentum transfer, which is g_3

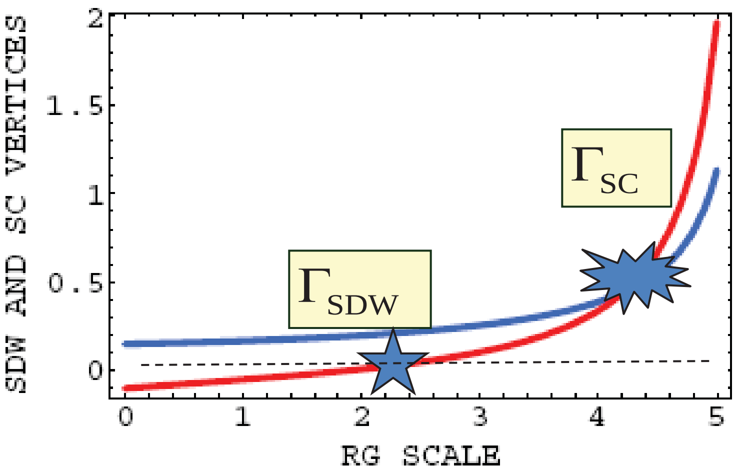
What happens after SC vertex becomes attractive depends on geometry and on doping

$\boxed{\Gamma_{SC} = (g_3 - g_4)}$ At zero doping $\boxed{\Gamma_{SDW} = (g_3 + g_1)}$

1 hole and 1 electron FSs

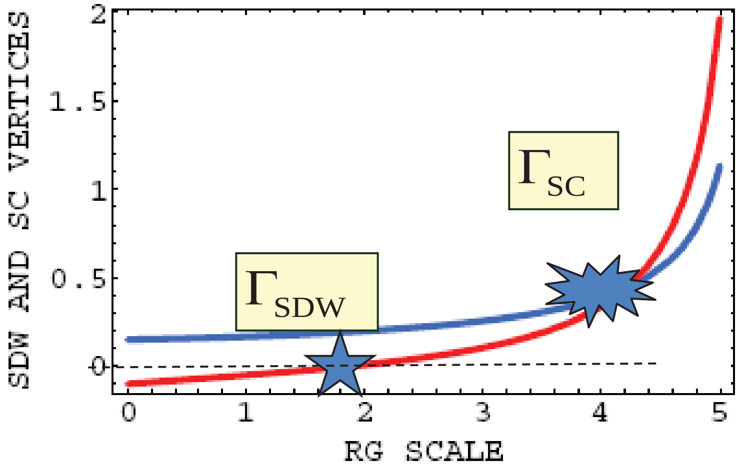


2 hole and 2 electron FSs

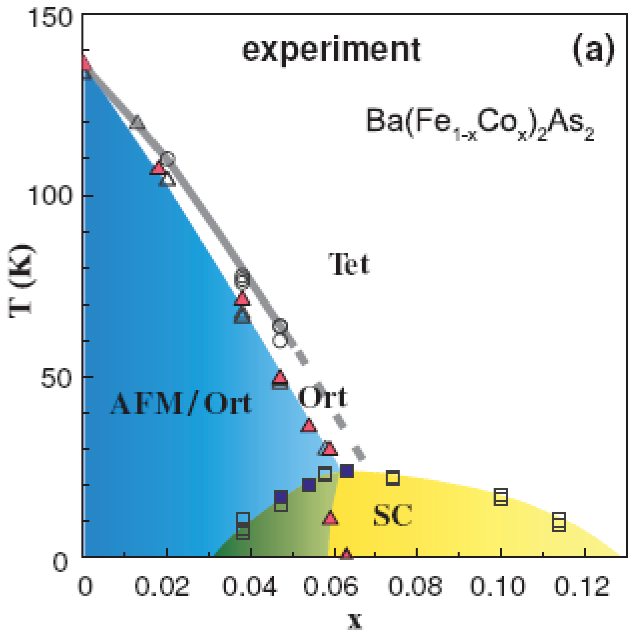


SC vertex can overshoot SDW vertex, in which case SC becomes the leading instability already at zero doping

At a finite doping



SC vertex always overshoots SDW vertex above some doping



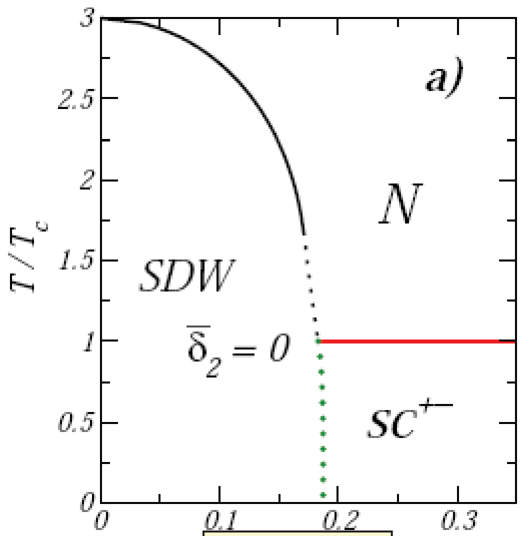
Zero doping –
SDW wins

A finite doping –
s^{+-} SC wins

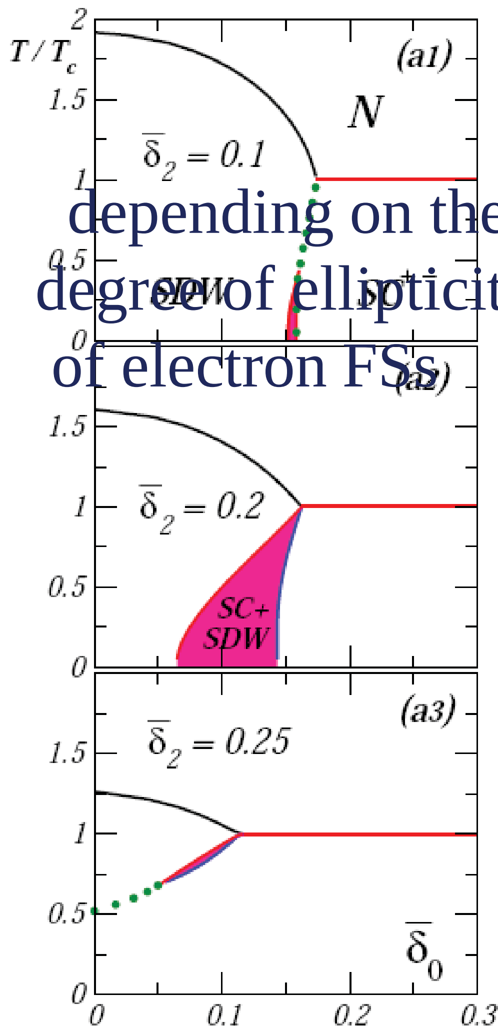
LiFeAs, LiFeP, LOFeP --
SC already at zero doping,
and no SDW order

Co-existence of SC and SDW:

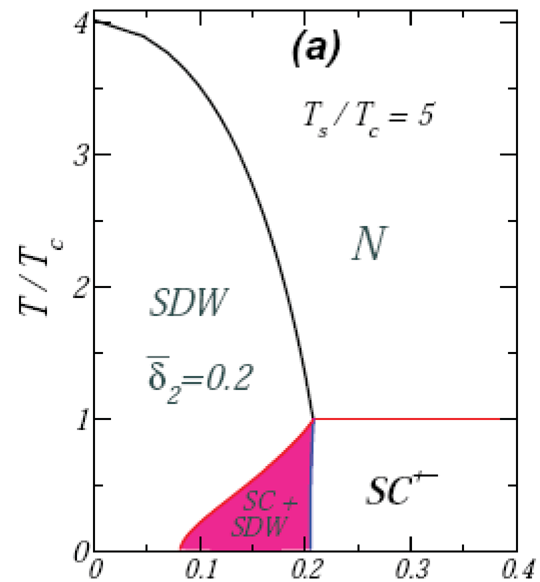
Suppose one order develops first, can the subleading one develop?



doping



depending on the degree of ellipticity of electron FSs



doping

Fernandes, Schmalian, et al,
Vorontsov, Vavilov, AC

Summary of RG

The essential aspect of the physics is the mutual support between superconducting and spin-density-wave fluctuations: magnetic fluctuations enhance tendency to superconductivity, and superconducting fluctuations enhance tendency to magnetism

However, once one order sets in, it fights against the appearance of the another one

Competition == good, monopoly == bad
(at least for the physics)

To continue