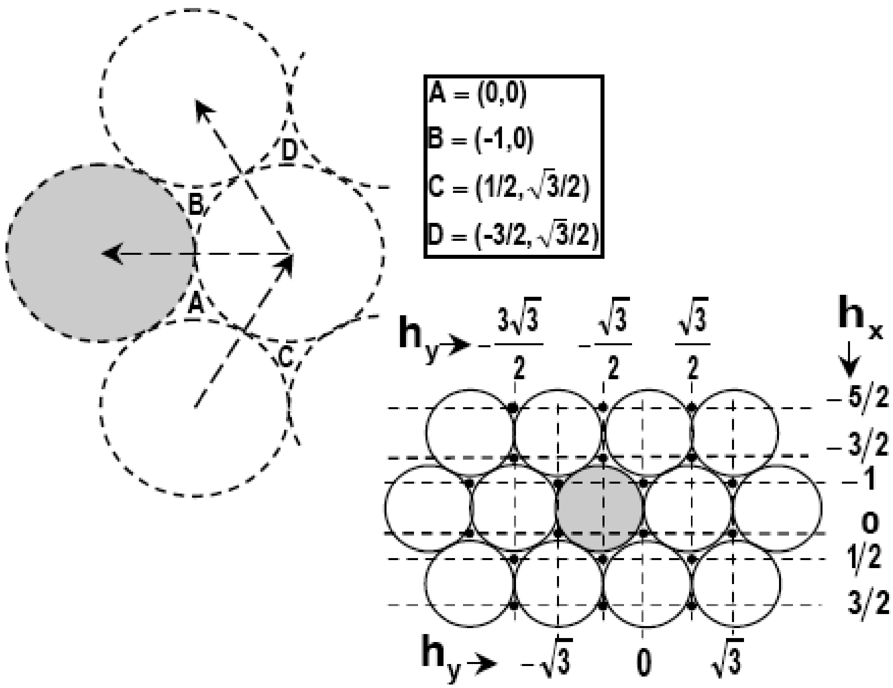
Granular packings

- Mapping of blocked configurations
- Conserved quantities
- Ensemble and relation to Edwards' ensemble

Force balance constraint



- Force Balance constraint ➔ a unique "height" field (loop forces defined by Ball and Blumenfeld)
- Frictionless constraint ==> height field is divergenceless.

$f_{ab} = h_2 - h_1$

$f_{ba} = h_1 - h_2$

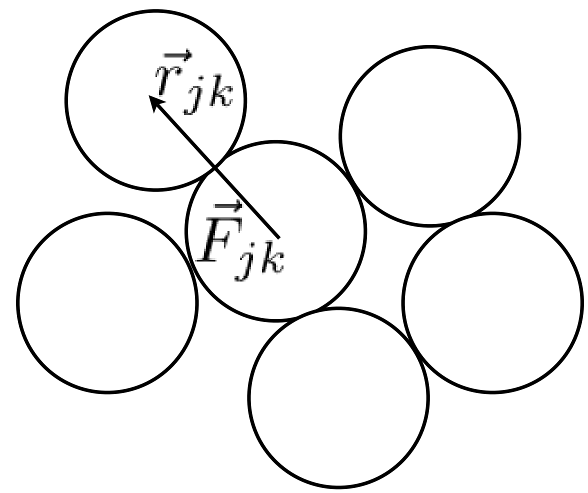
$$h_x = \partial_y \psi$$

$$h_y = -\partial_x \psi$$

The microscopic stress tensor (force moment) of grains is defined as

$$\hat{\sigma}_j = \sum_k \vec{r}_{jk} \vec{F}_{jk}$$

frictionless



Coarse-grained

$$\hat{\sigma}(\vec{r}) = (1/A) \sum_{j \subset A} \sum_k \vec{r}_{jk} \vec{F}_{jk}$$

$$\hat{\sigma} = \left[\begin{array}{cc} \partial_y^2 \psi & -\partial_x \partial_y \psi \\ -\partial_x \partial_y \psi & \partial_x^2 \psi \end{array} \right]$$

Conserved quantities

$p(x,y) = \nabla^2\psi = \vec{\nabla}\cdot\vec{\rho}(x,y),$
where $\vec{\rho}(x,y) = \vec{\nabla}\psi.$
In terms of our original "height" fields, $h_x = \rho_y$, $h_y = -\rho_x$

The global quantity:
$\Gamma = \int_0^L dx \int_0^L dy(\sigma_{xx} + \sigma_{yy}) = \int_0^L dx \int_0^L dy \vec{\nabla}\cdot\vec{\rho}(x,y)$
and can be written as; $\int \vec{\rho}\cdot d\hat{n}$ (using divergence theorem in 2D)

Taking our box to be a square,
$\Gamma = <\rho_x>_L - <\rho_x>_0 + <\rho_y>_L - <\rho_y>_0.$
$<\rho_x>_L \equiv \int_0^L \rho_x(L,y)dy$
$<\rho_y>_L \equiv \int_0^L \rho_y(x,L)dx$

Boundary terms

Ensemble

1. Any blocked state can be assigned a value of Γ. All states with the same value of Γ belong to the same topological sector. Different topological sectors are not connected by any local dynamics and, Γ is a conserved quantity. In terms of the components of the total force, $\Gamma = f_x + f_y$

2. in addition to Γ, which is the mean curvature of the surface $\psi(x, y)$, there is the integrated Gaussian curvature of the surface which is given by $\kappa = \int_0^L dx \int_0^L dy(\sigma_{xx}\sigma_{yy} - \sigma_{xy}^2)$. The integrated Gaussian curvature is, of course, a topological invariant for a closed surface. In the absence of a shear stress, the mean curvature and the Gaussian curvature are simply related. If there in average shear, though, the surface needs to be characterized by the two independent, integrated curvatures, Γ and κ. Each grain configuration is then labeled by Γ and κ not just Γ. A Boltzmann type entropy will then be $S(\Gamma, \kappa) = \ln \Omega(\Gamma, \kappa)$ where Ω counts the number of blocked states in the sector (Γ, κ)