Boulder Summer School

Flow in the Sun

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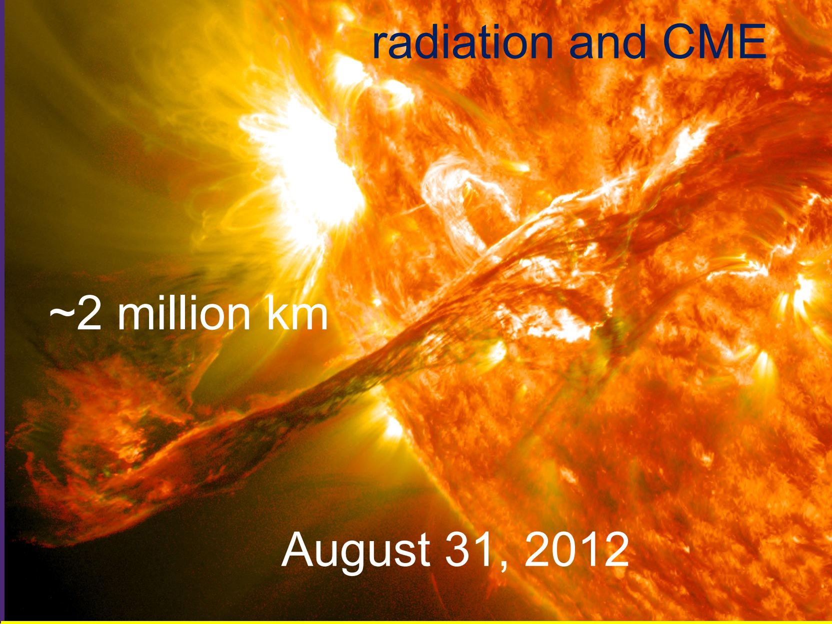
New York University

July 5, 2022

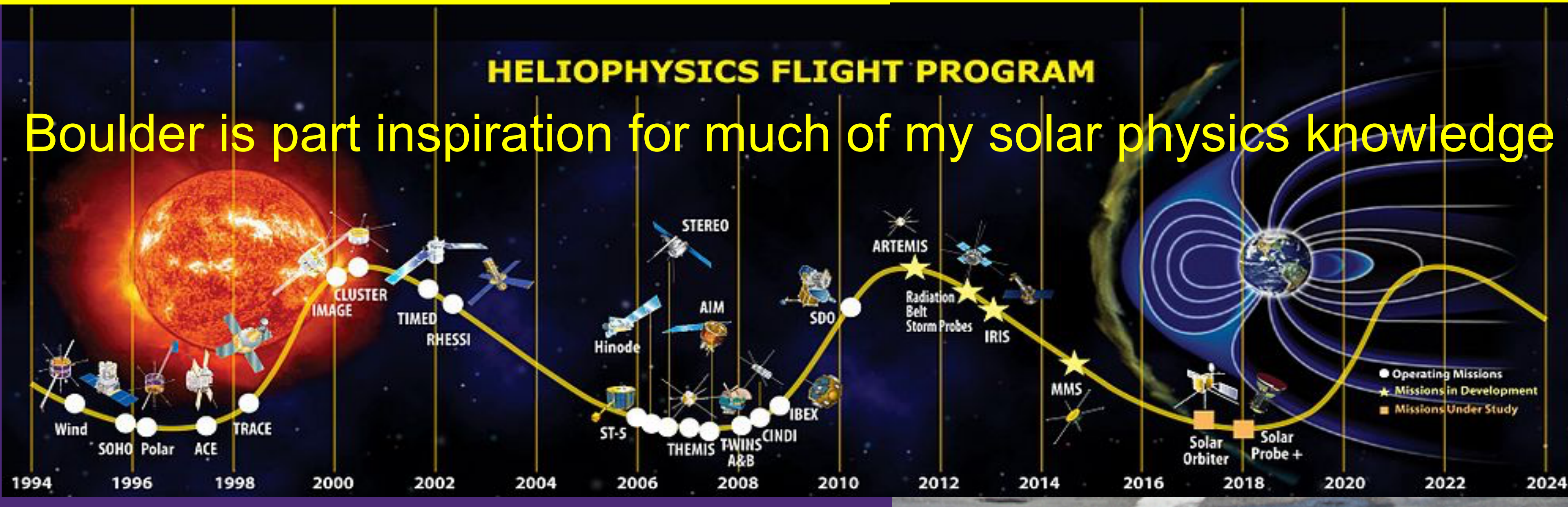

radiation and CME

~2 million km

August 31, 2012

Much is known about the Sun as a main sequence star: its mass, size, age, nuclear reactions, surface temperature, the power it generates, etc. But much less certainty exists about the Sun as a dynamical object.

NAS Report estimates $2 trillion damage in 2008 dollars (= 20 Katrinas). Knowledge is being accumulated rapidly because of heliophysics programs---satellites as well as ground based.



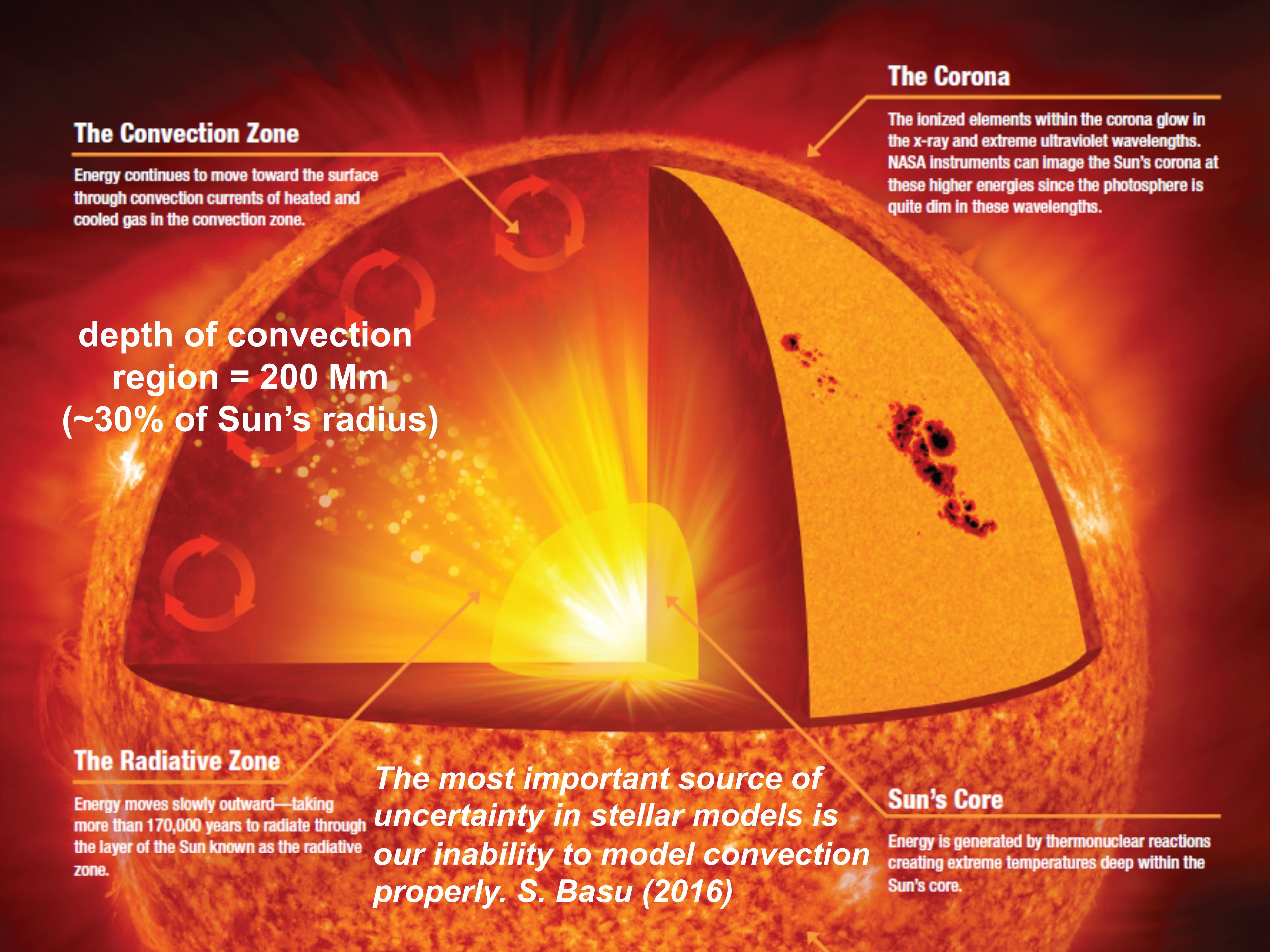
Boulder is part inspiration for much of my solar physics knowledge

Reigning Paradigm of Solar Physics Today

All surface phenomena of the Sun are driven by internal dynamics (not the result of external forcing)

Sun’s angular momentum: Only 0.3%

Jupiter 62%; period is equal to periodicity of sunspots



The Convection Zone

Energy continues to move toward the surface through convection currents of heated and cooled gas in the convection zone.

The Corona

The ionized elements within the corona glow in the x-ray and extreme ultraviolet wavelengths. NASA instruments can image the Sun's corona at these higher energies since the photosphere is quite dim in these wavelengths.

depth of convection region = 200 Mm (~30% of Sun's radius)

The Radiative Zone

Energy moves slowly outward—taking more than 170,000 years to radiate through the layer of the Sun known as the radiative zone.

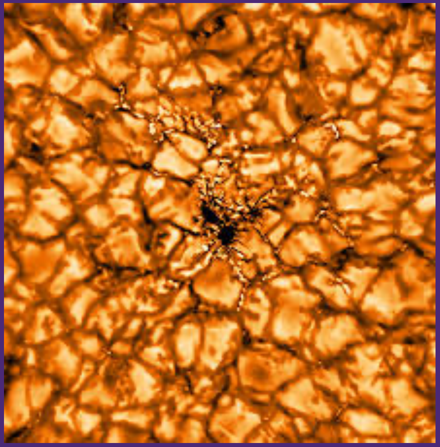
The most important source of uncertainty in stellar models is our inability to model convection properly. S. Basu (2016)

Sun's Core

Energy is generated by thermonuclear reactions creating extreme temperatures deep within the Sun's core.

Some naïve estimates for the convective flow in the Sun

- * Ra ~ 10^{24} (instability Ra = $O(10^6)$)
- * Pr ~ 10^{-6} (water ~ 5, air ~1)
- * Re ~ 10^{14}, scale separation $O(10^9)$; high level of turbulent activity



granular structures on the surface of the sun, of the order of 1000 km, disintegrate in short times of the order of 10 min, which would require a strong eddy viscosity in the range of $10^8 - 10^9$ m^2/s (only the surface value; see later when we discuss mixing length theory, and also Rossby waves)

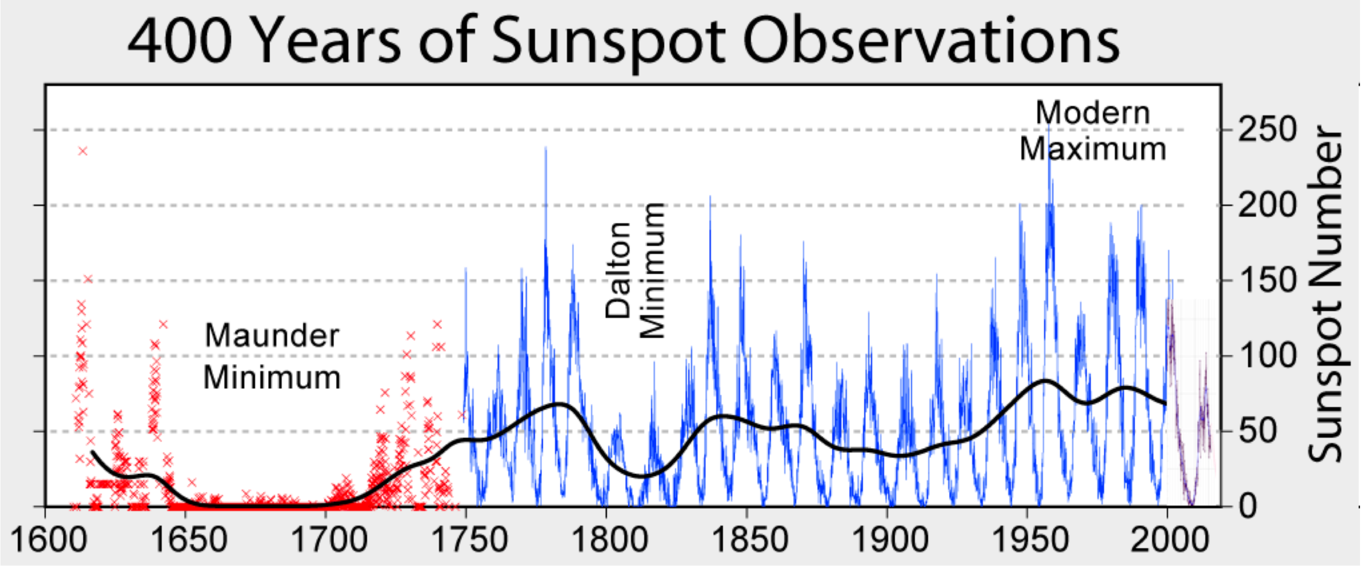
- * Free fall velocity ~100 m/s
- * Subsonic (speed of sound ~ 2×10^5 m/s)
- * Convection layer thickness ~ 2×10^8 m (outer 3/10ths of solar radius)
- * $\nu \sim 10^{-3} m^2/s$, $\kappa \sim 10^3 m^2/s$ (assuming that the fluid is fully ionized)
- * Nusselt number by some extrapolation is $O(10^6)$, amply satisfying bounds

The fluid dynamical question

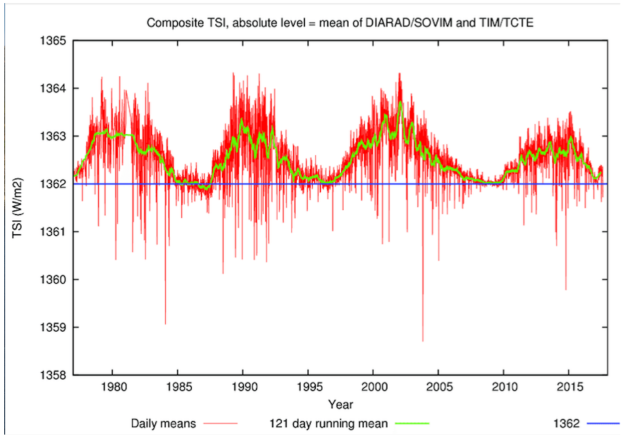
(as it occurred to me ~30 years ago)

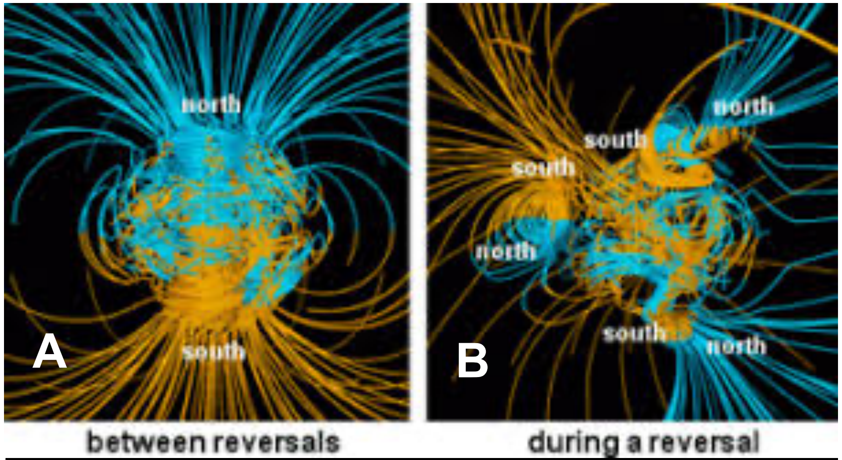
- If the Reynolds and Rayleigh numbers are so high and the turbulence is correspondingly very strong, how do several highly coherent activities survive?
- Obviously the combined effects of rotation*, stratification, magnetic fields, etc., cooperate to produce these activities. How precisely?

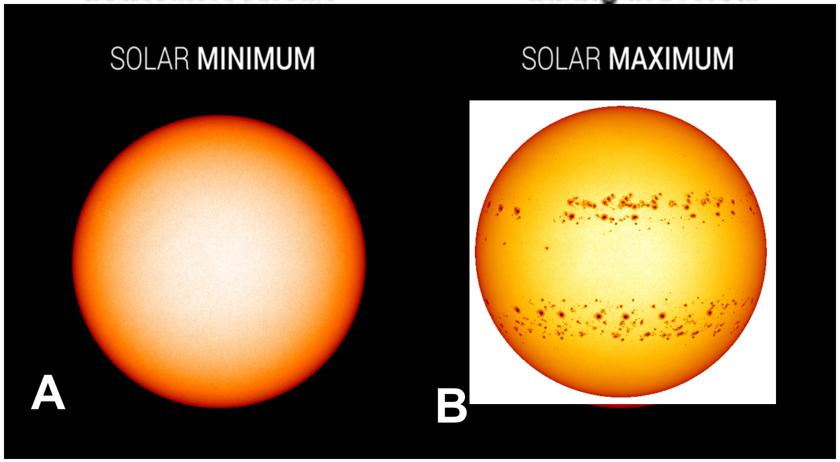
*Rossby number for the sun as a whole is > O(1)

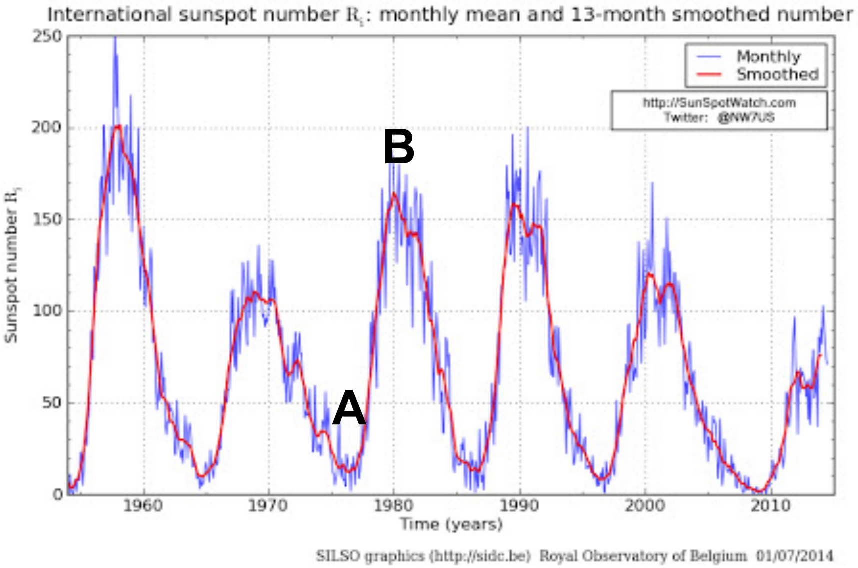


The observed year-to-year variation in the sunspot number spanning the period from the earliest use of the telescope through 2007.







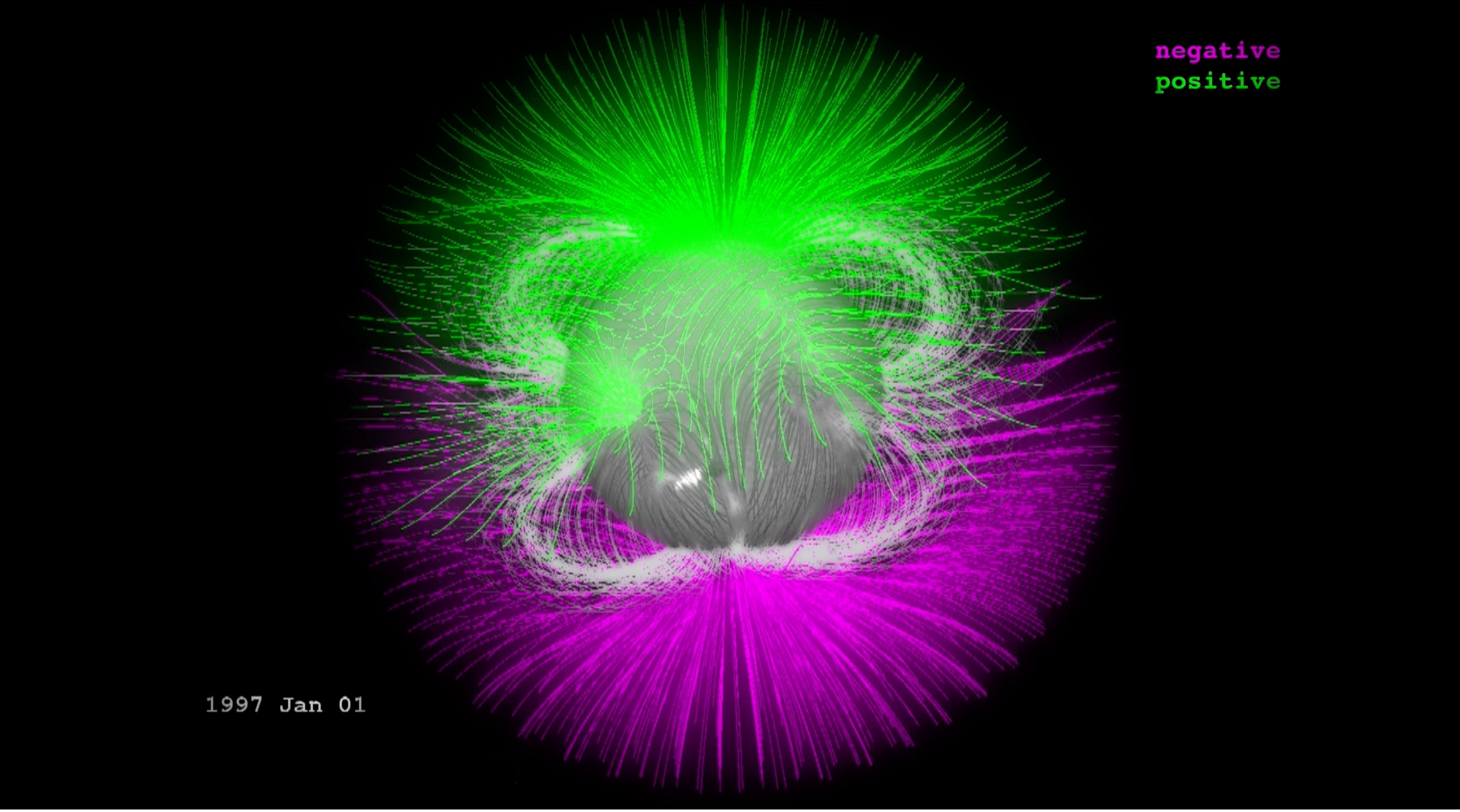


SILSO graphics (http://sidc.be) Royal Observatory of Belgium 01/07/2014

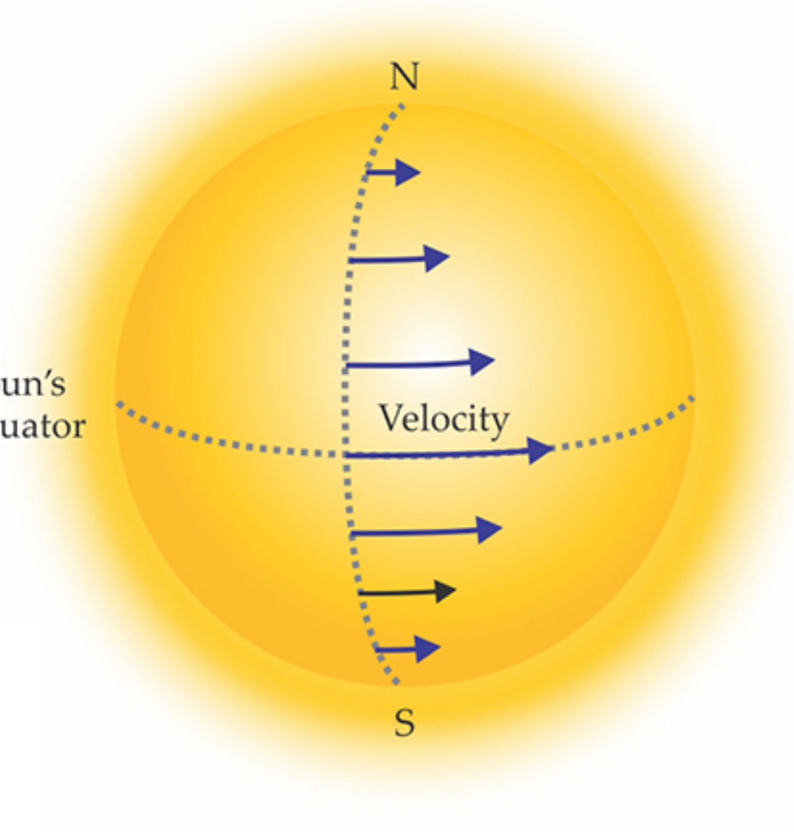
When the Sun has one or the other polarity (as shown in the top left), or the magnetic activity is not widespread, it is the solar minimum, **A** in all the figures,

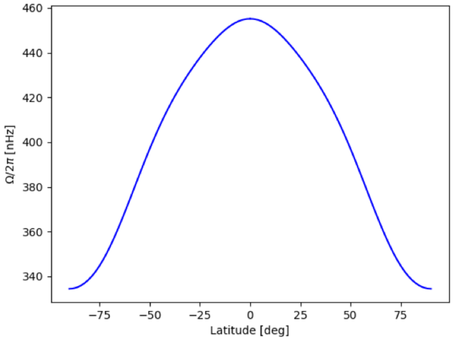
During maximum activity, **B** in all the figures, the polarity begins to reverse itself, but the process is complex.

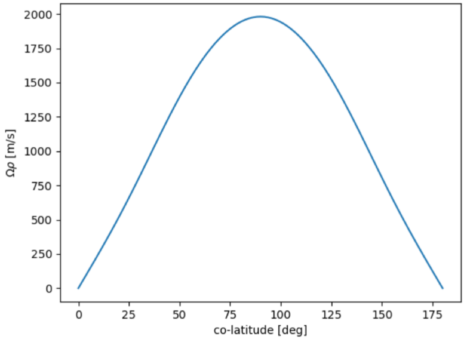
The period between maxima or minima is roughly 11 years.



16 years of Sun’s magnetic field activity compressed to 25 secs; time on bottom left; green is positive; magenta is negative

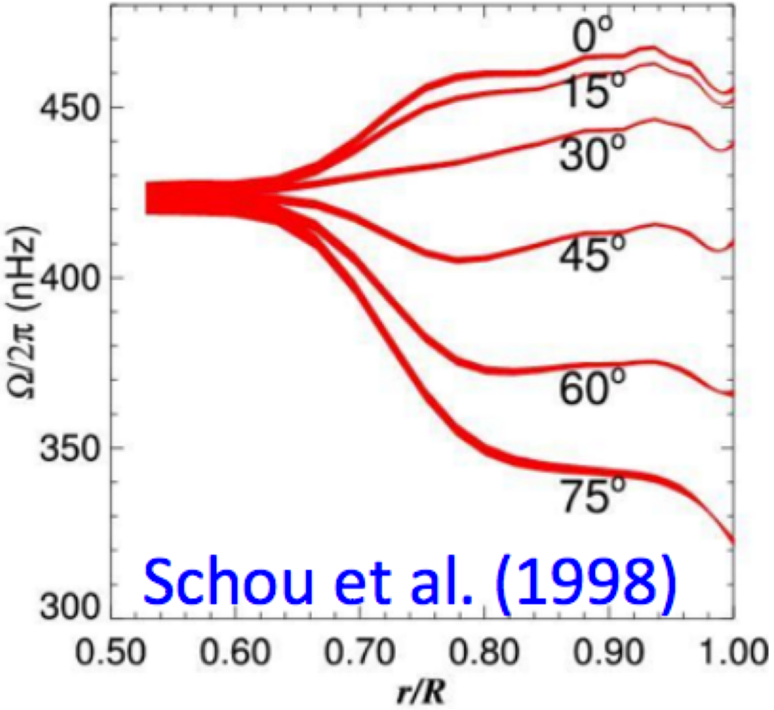




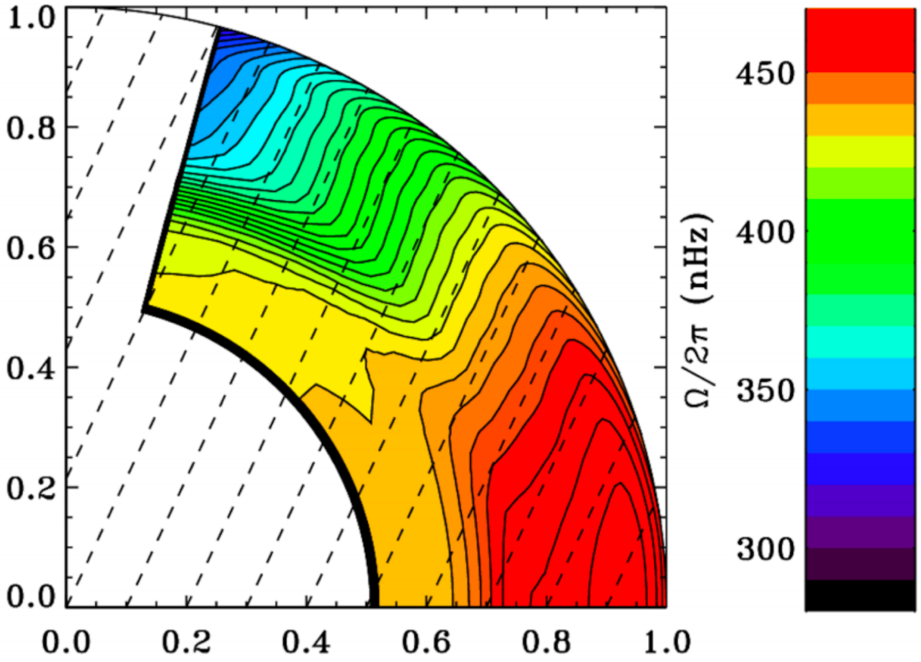


Differential rotation: The Sun takes 25 days at the equator for one revolution and about 34 days near the pole.

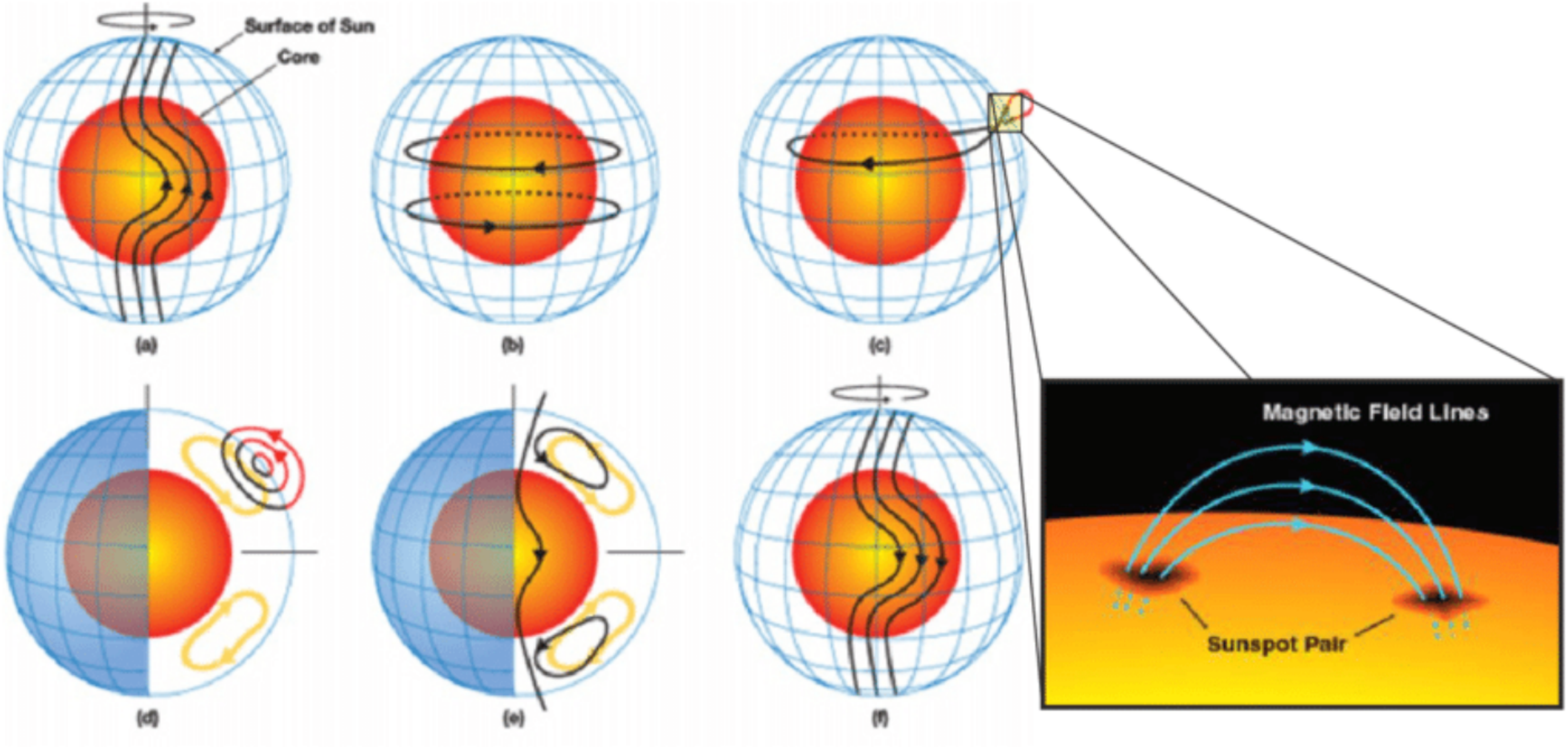
Differential rotation



Schou et al. (1998)



- Monotonic decrease of angular velocity with latitude, with a contrast of about 25% between the equator and 60°
- Angular velocity contours at mid-latitudes tilted at about 25° to the pole
- Narrow layers of strong shear near the top and bottom of the convection zone
- Periodic and non-periodic temporal variations.



Panel (a) represents the deformation of poloidal magnetic field lines (black line) by differential rotation in the bottom of the solar convection zone. After a few turns (b), the field lines are wound around the Sun into a toroidal configuration forming structures called flux tubes (many intertwined field lines). This happens in tachocline between the radiative and convective zones where the radial shearing action of rotation is higher. Due to magnetic buoyancy instabilities (c), the flux tubes rise towards the surface. In the place where these field loops pierce the photosphere (what we loosely call solar surface) we have the appearance of a sunspot pair. These sunspots will eventually decay (d) and rearrange the magnetic field into the poloidal direction again. This surface field is then carried by meridional circulation (yellow lines) toward the poles (e) contributing to the increase of the global toroidal component of the field (f). Note the change of direction in the magnetic field between panels (a) and (f); it denotes a polarity change of the observed large scale solar dipolar field. Adapted from: NASA

- Theory (turbulence, instability modes)
- Numerical simulations
- Observations (satellites and ground based)

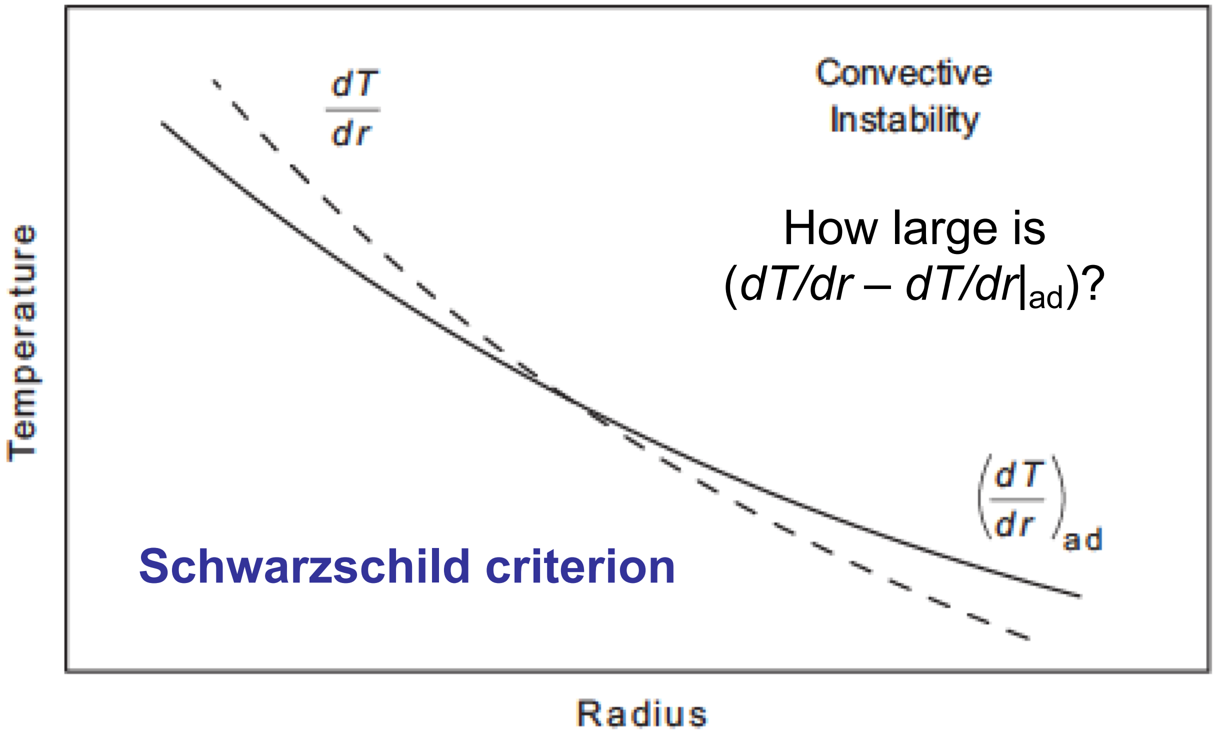
“What appliance can pierce through the outer layers of a star and test conditions within?”

Arthur Eddington (1926) in *The Internal Constitution of the Stars*

- The answer is modern helioseismology, which employs:
  - (a) Sound waves naturally generated at Sun’s surface, mostly by convective parcels of fluid arriving from inside and hitting the outer surface with its large density contrast. Imagine little pebbles of sand hitting the surface of the a bell. The sound waves thus generated travel through the Sun’s interior and give information about internal motion;
  - (b) Actual velocity field itself, as a result of many years of observational data now available;
  - (c) Sophisticated data analysis.

shortcomings of inverse methods

Normally, convection might be expected if *dT/dr* < 0. But hydrostatics sets up a huge temperature gradient in the Sun, so convection is possible only if the actual gradient exceeds the hydrostatics gradient.

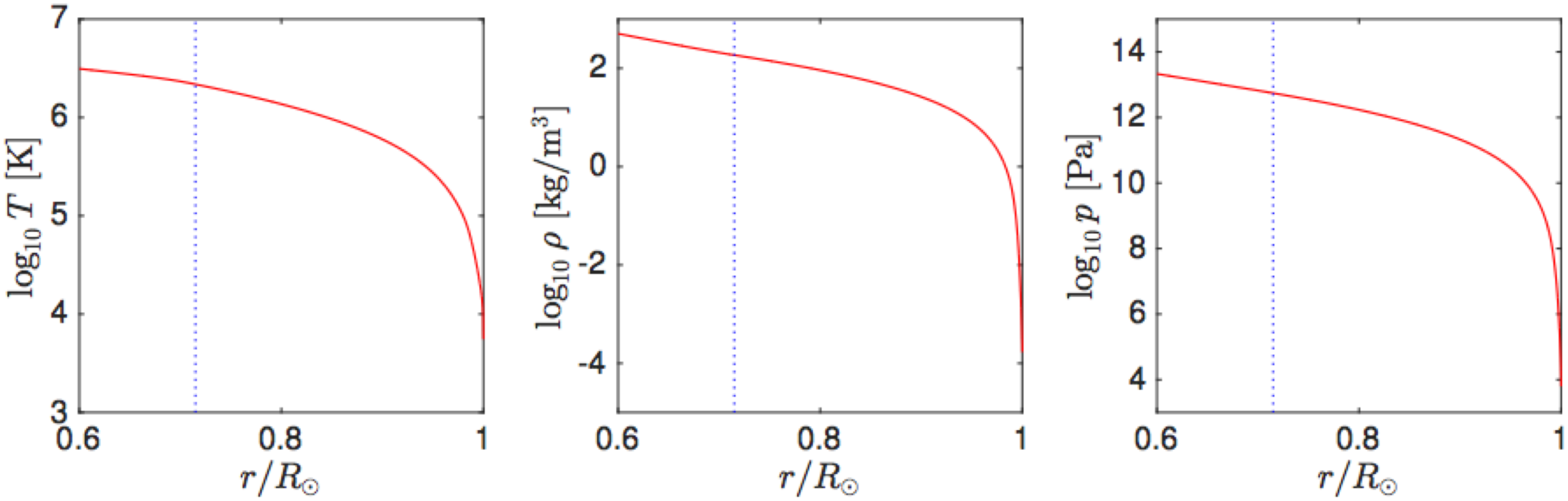


How large is $(dT/dr - dT/dr|_{ad})$?

Schwarzschild criterion

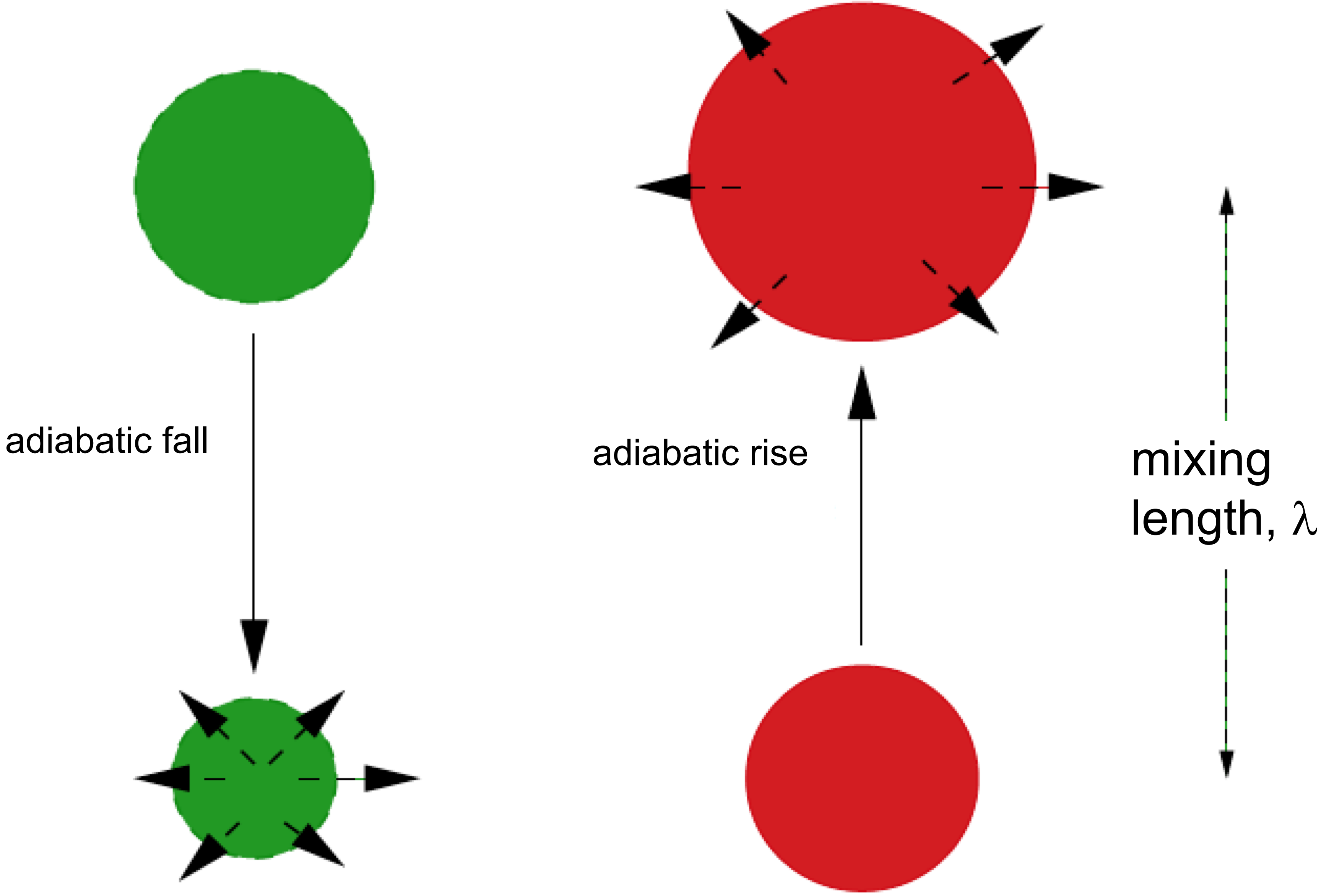
Figure 6.11: Schematic illustration (solid line) of the critical temperature gradient for convection. In this example the actual temperature gradient (dashed line) is steeper than the adiabatic gradient, so the region is convectively unstable.

From J. Schumacher & KRS, *Rev. Mod. Phys.* (2020)



Surface convection: outermost 1% or so
Deep convection: Everything between tachocline and the outer skin.

Mixing length theory: L. Prandtl (1925)



Erika Böhm-Vitense
1923 -- 2017

Mixing length theory

Böhm-Vitense ('53, '58)

Spruit ('74, '77)

Chan & Sofia ('89)

Robinson et al. (2003)

Heat flux = $\rho V C_p \Delta T$,
where ΔT is the temperature difference across λ.

$\Delta T = (dT/dr - dT/dr|_{ad})\lambda$

Take V from free-fall velocity ($\sim\Delta T^{1/2}$)
If the total heat flux = radiation L_r from the Sun,

$$(dT/dr - dT/dr|_{ad})^{3/2} = \frac{L_r/(4\pi r^2)}{C_p\rho(GM/Tr^2)^{1/2}\lambda^2}$$

Taking $\lambda =$ scale height, we get

$(dT/dr - dT/dr|_{ad}) = 10^{-7}$ K/m (what does it imply?)

Thus, dT/dr is almost $dT/dr|_{ad}$, but the difference is crucial, also may show that there is high-level of mixing. Does it?

Since $dT/dr|_{ad} = g/C_p$ is $O(10^{-2}$ K/m), we get

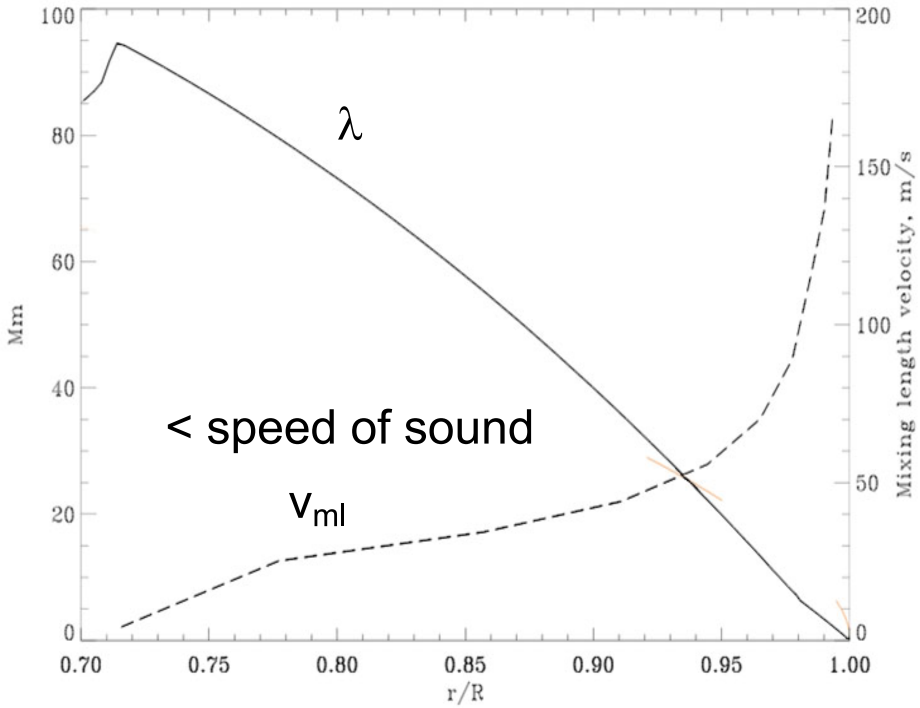
$$\frac{dT/dr - dT/dr|_{ad}}{dT/dr|_{ad}} = 10^{-5}$$

density scale height, λ

mixing length velocity, v_{ml}

$v_{ml}\lambda/\nu$ varies from
5×10^{11} (r/R = 0.75)
to 10^{12} (r/R = 0.95)

eddy viscosity (max): $O(10^9\ m^2/s)$



< speed of sound

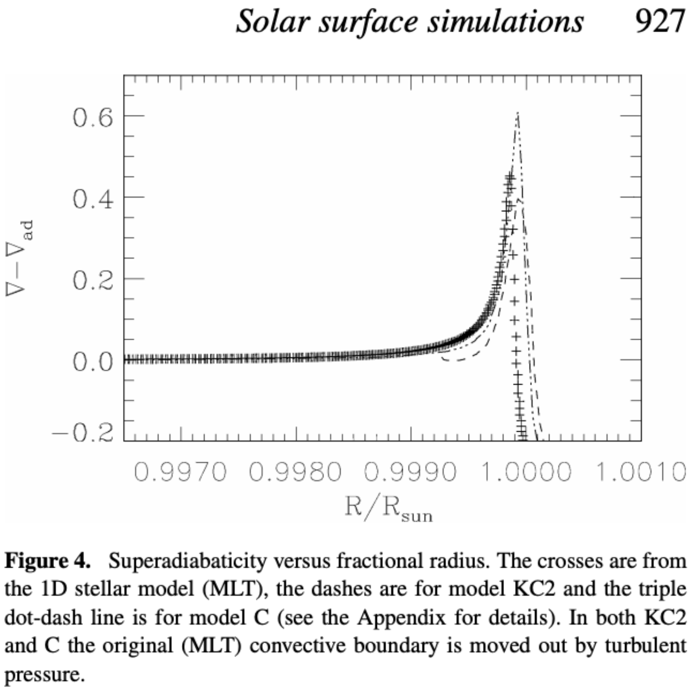
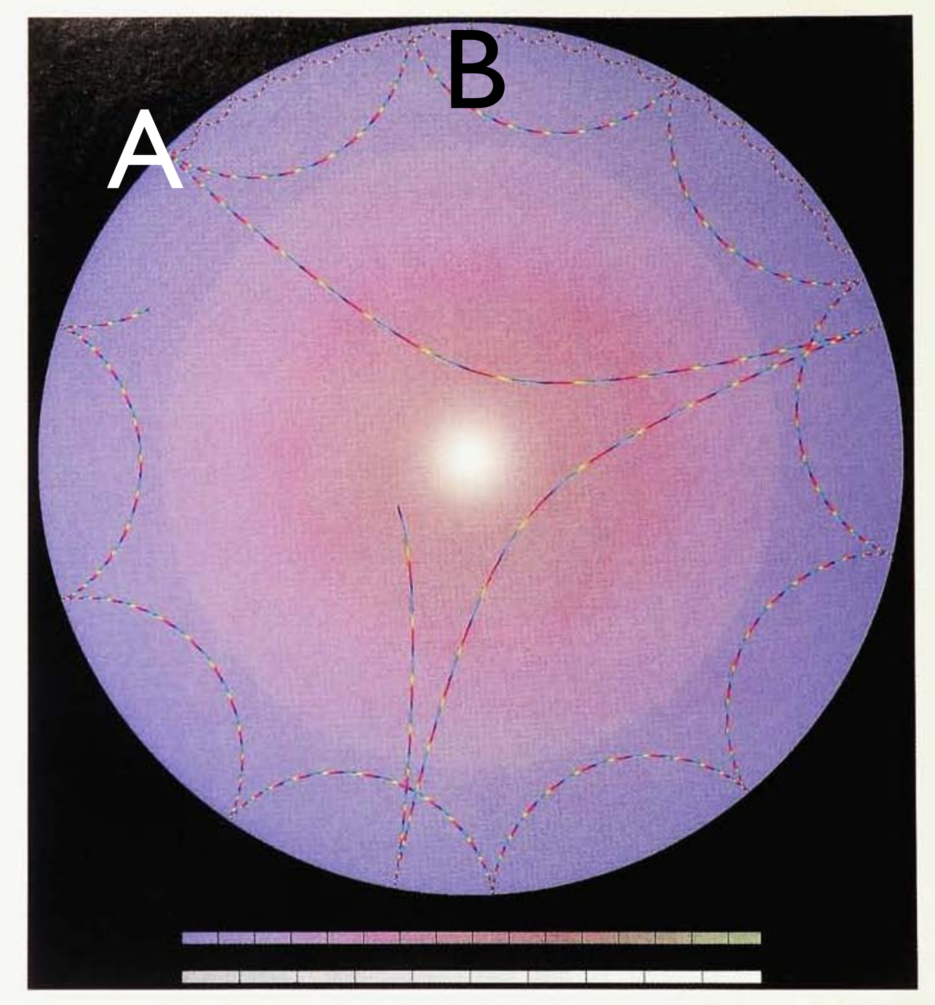

Solar surface simulations 927

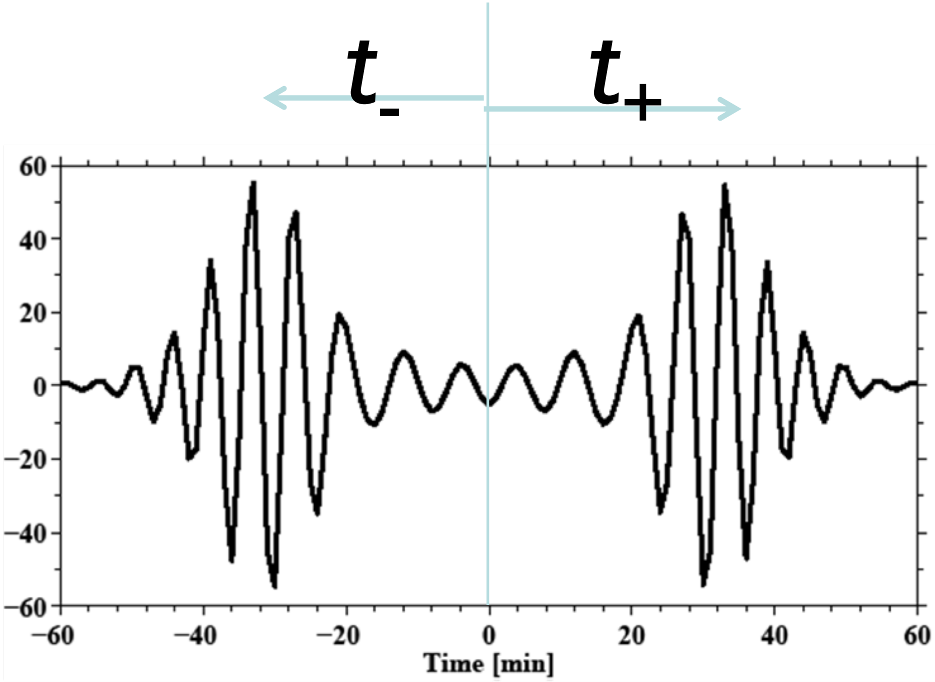
Figure 4. Superadiabaticity versus fractional radius. The crosses are from the 1D stellar model (MLT), the dashes are for model KC2 and the triple dot-dash line is for model C (see the Appendix for details). In both KC2 and C the original (MLT) convective boundary is moved out by turbulent pressure.


S.M. Hanasoge


L. Gizon

Normal modes of the Sun





$$\delta\tau = t_+ - t_-$$

cross-correlations!

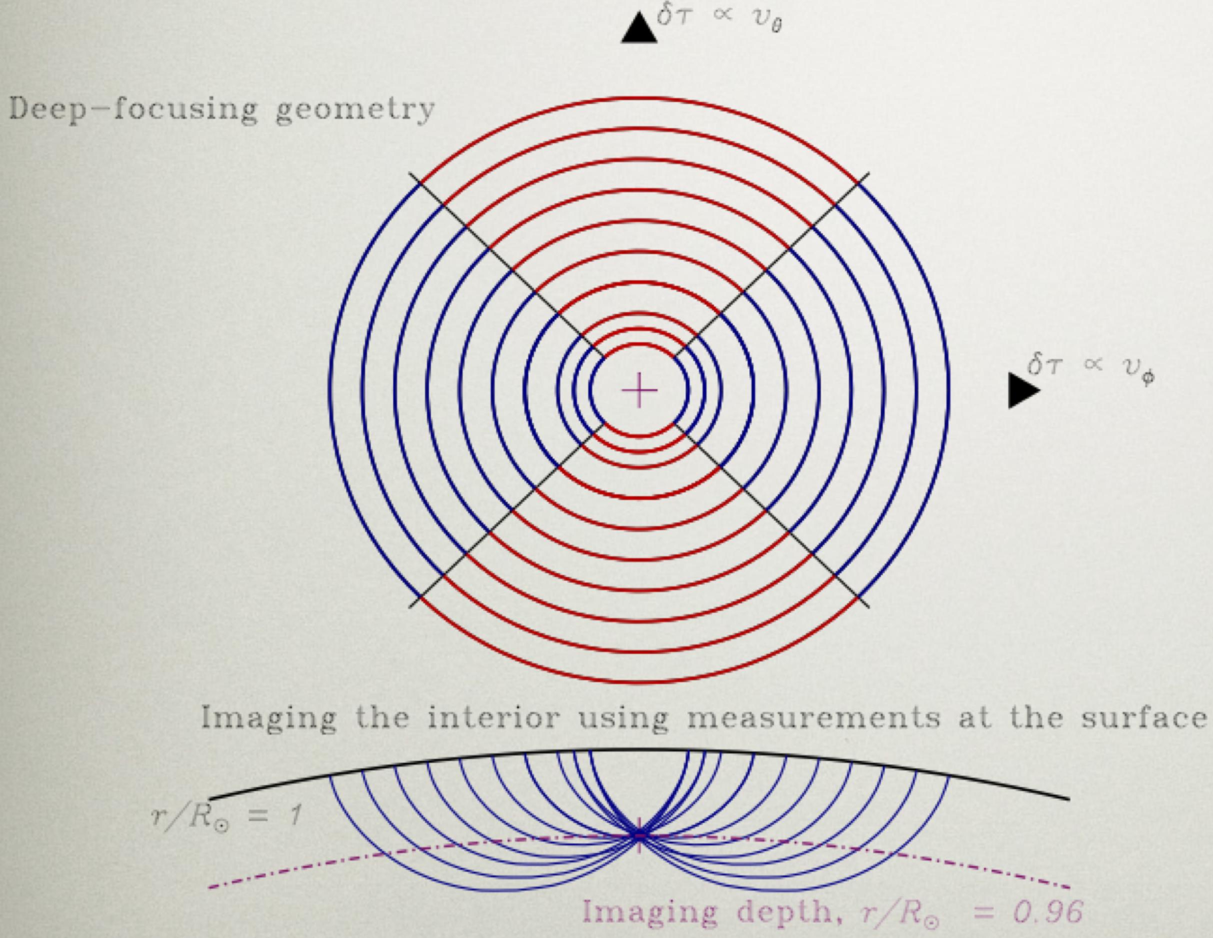
$$\delta\tau \sim v_\theta \text{ or } v_\phi$$

Duvall et al. *Nature* **362**, 430 (1993)

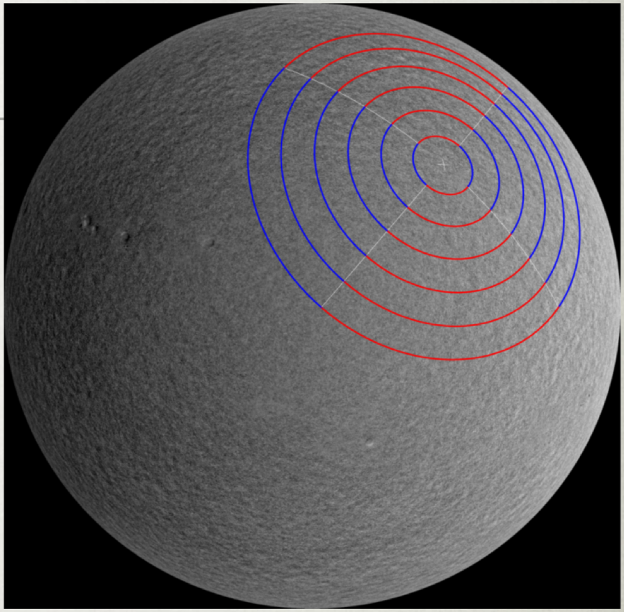
Hanasoge, Duvall & KRS, *PNAS* **109**, 1928 (2012)

Any flow breaks symmetry

SOME DETAILS



Hanasoge, Duvall & S, PNAS 2012



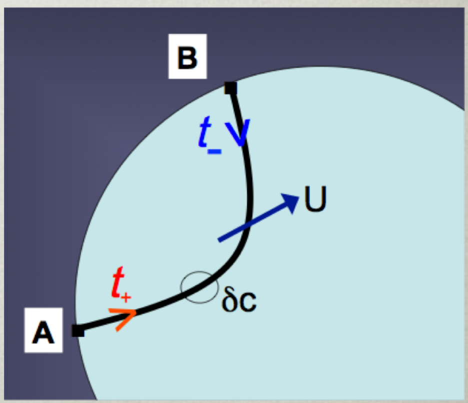


Figure courtesy of L. Gizon

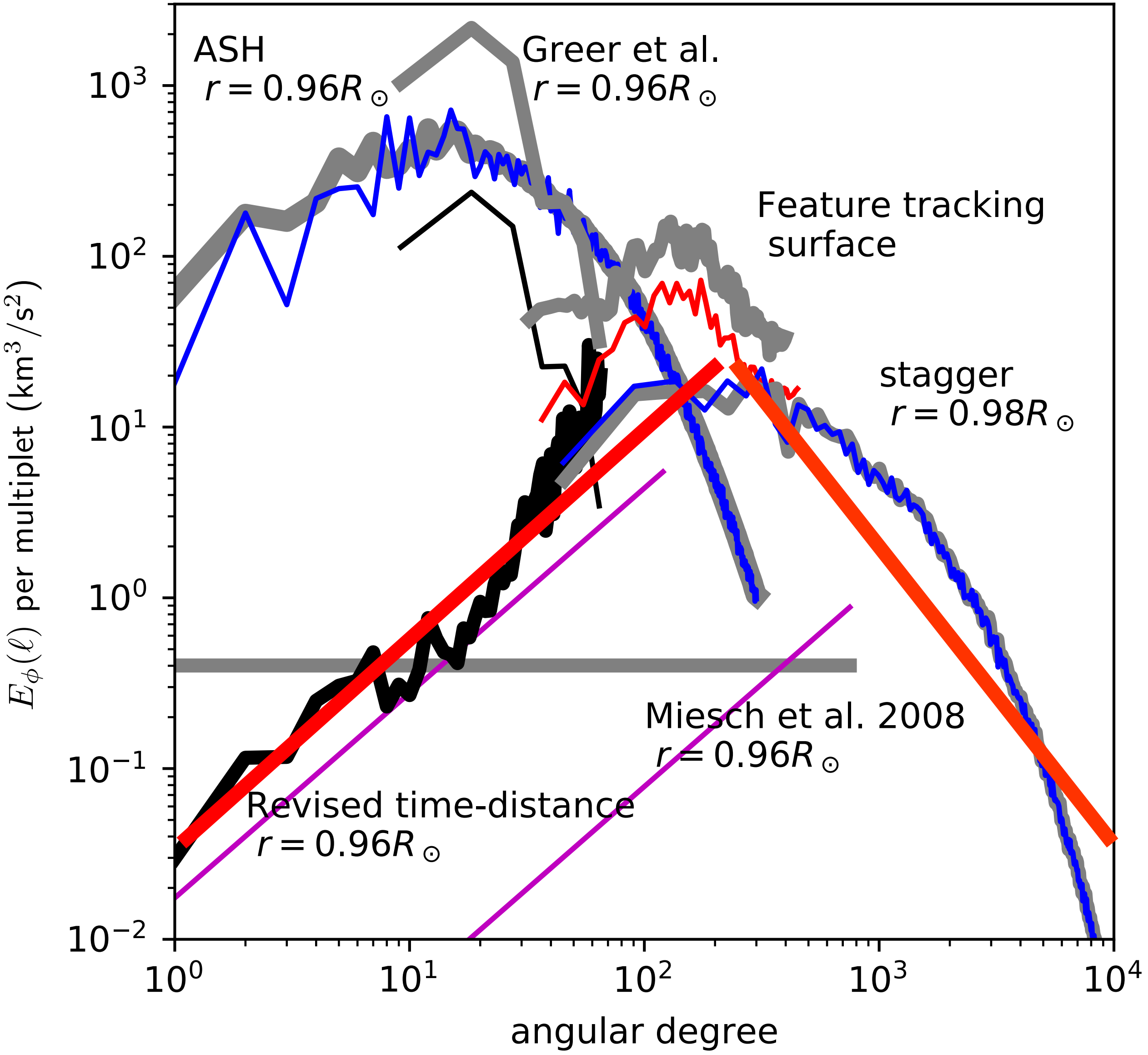
ASH SIMULATIONS

E.G., MIESCH, BRUN, DEROSA & TOOMRE (2008)

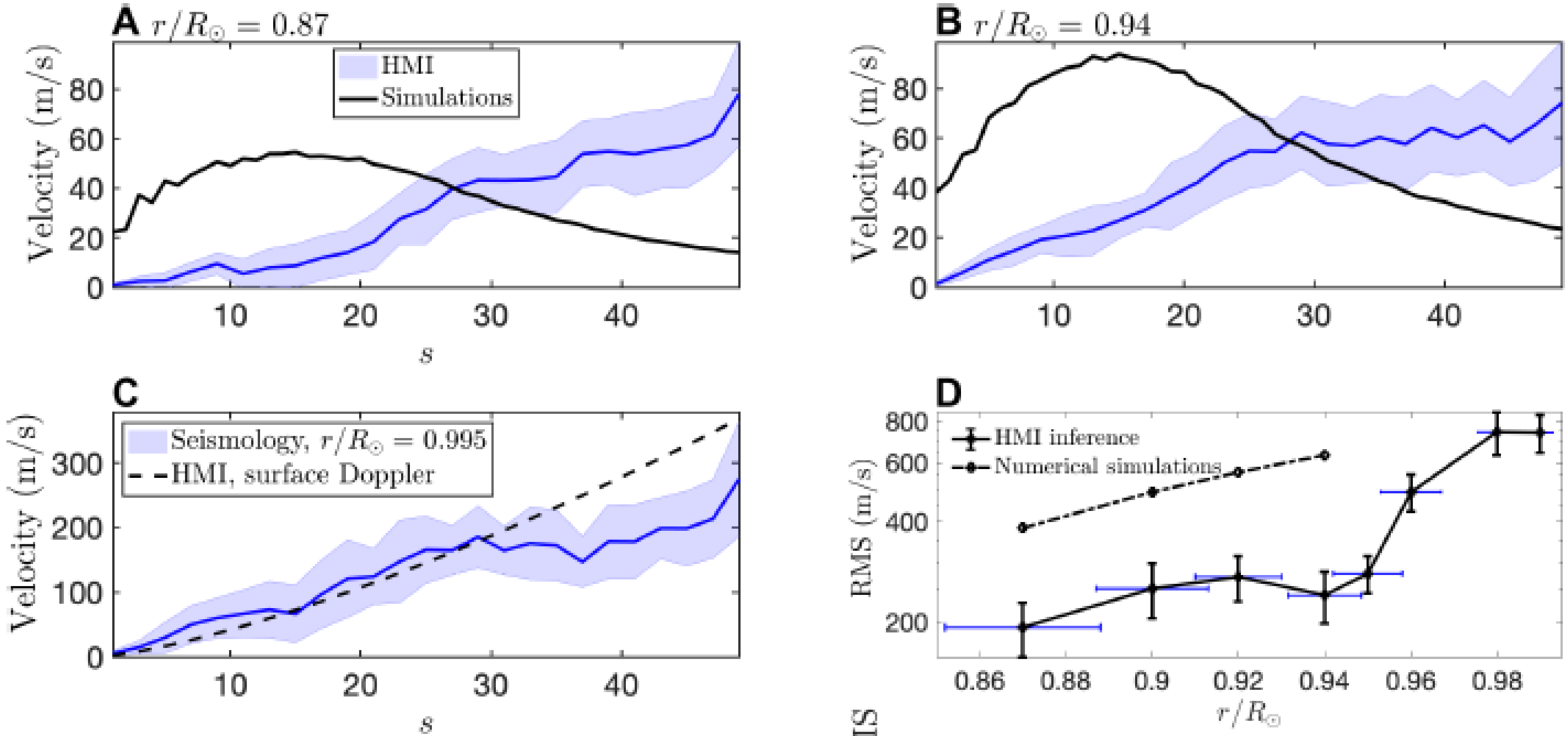
ASH simulations have provided considerable insights into the intricate convective patterns that are likely to exist in the deep solar convection zone.

- Use anelastic approximation of the equations
- The domain is from $r/R_\odot = 0.71$ to 0.98
- Boundaries are taken as impermeable and free of tangential stresses; entropy flux is prescribed at the lowest boundary and constant heat flux at the upper boundary
- Typical fluid parameters assumed are: $\nu = 1.2 \times 10^{12}$ cm^2/s (some 15 orders of magnitude larger), $\kappa = 4.8 \times 10^{12}$ cm^2/s (some 12 orders of magnitude larger), Pr = 0.25 (some 3 orders of magnitude larger)
- Shows large variations ~ 300% with time (not in observations)

Hanasoge, Duvall, KRS, PNAS, 109, 11928 (2012)



A. Birch … KRS, In Catalyzing Solar Connections, 2018



Turbulence in the Sun is suppressed on large scales and confined to equatorial regions, S.M. Hanasoge, H. Hotta and KRS, *Science Advances* **6**, eaba9639, 2020

- Very low Prandtl number
- Non-Boussinesq effects
- Rotation

J. Schumacher

A. Pandey

Non-Boussinesq low-Prandtl number convection with a temperature-dependent thermal diffusivity, A. Pandey, J. Schumacher, K. R. Sreenivasan, The Astrophysical Journal 907 (1), 56 (2020)

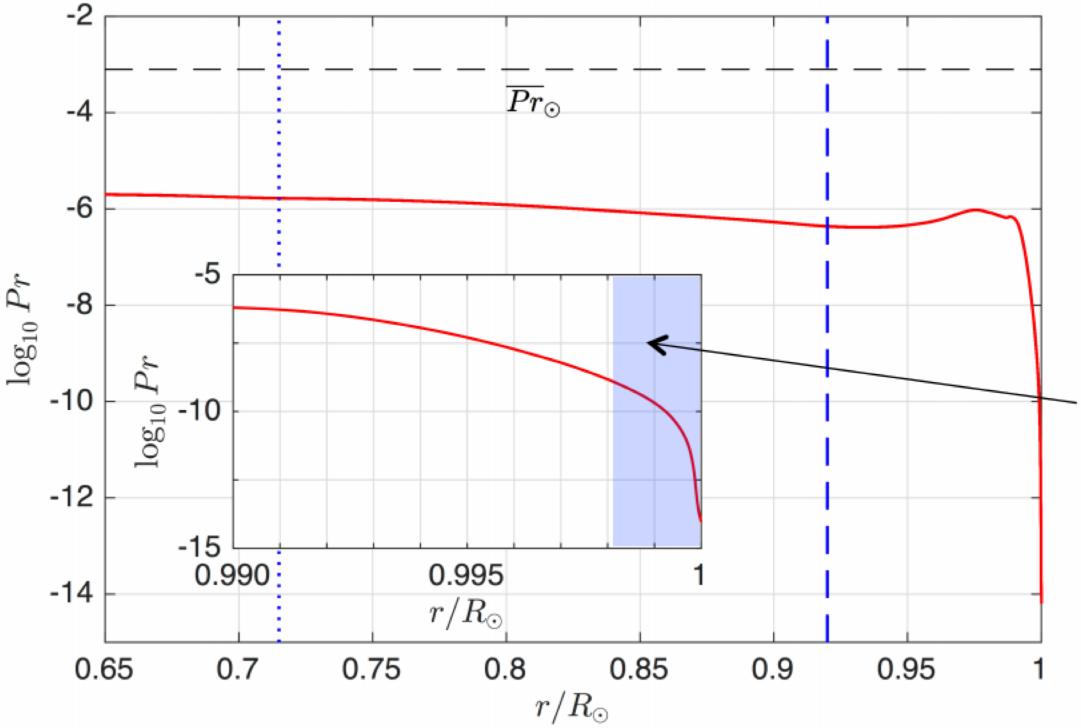
Non-Boussinesq convection at low Prandtl numbers relevant to the Sun, A Pandey, J Schumacher, KR Sreenivasan, Physical Review Fluids 6 (10), 100503 (2021)

Thermal boundary layer structure in low-Prandtl-number turbulent convection, A Pandey, JFM 910 (2021)

Convective mesoscale turbulence at very low Prandtl numbers, A Pandey, D Krasnov, KR Sreenivasan, J Schumacher, arXiv:2202.09208 (likely to appear in J. Fluid Mech.)

From J. Schumacher & KRS, *Rev. Mod. Phys.* (2020)

Prandtl number in solar convection



Partial ionization of H below T=1.5×10^4 K

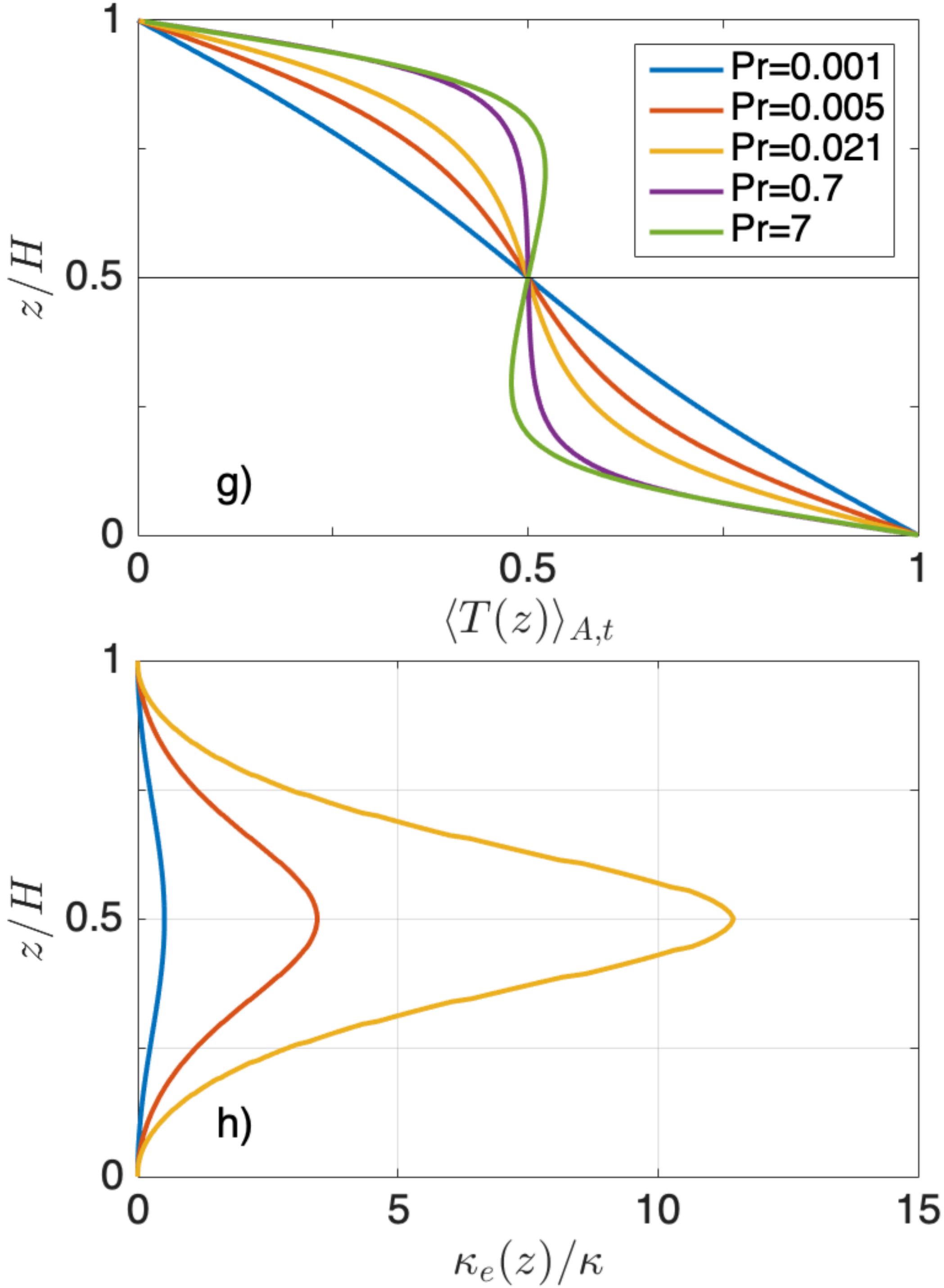
$$Pr_\odot(r) = \frac{\nu(r)}{\kappa(r)} = \frac{c_p \eta(T(r))}{k(T(r))}$$

Momentum transfer by electrons

$$\eta_q = \frac{1}{3}\langle v_q\rangle\langle\lambda_q\rangle n_q m_q$$

Heat transfer by photons

$$k_q = \frac{1}{3}\langle v_q\rangle\langle\lambda_q\rangle C_{v,q}$$

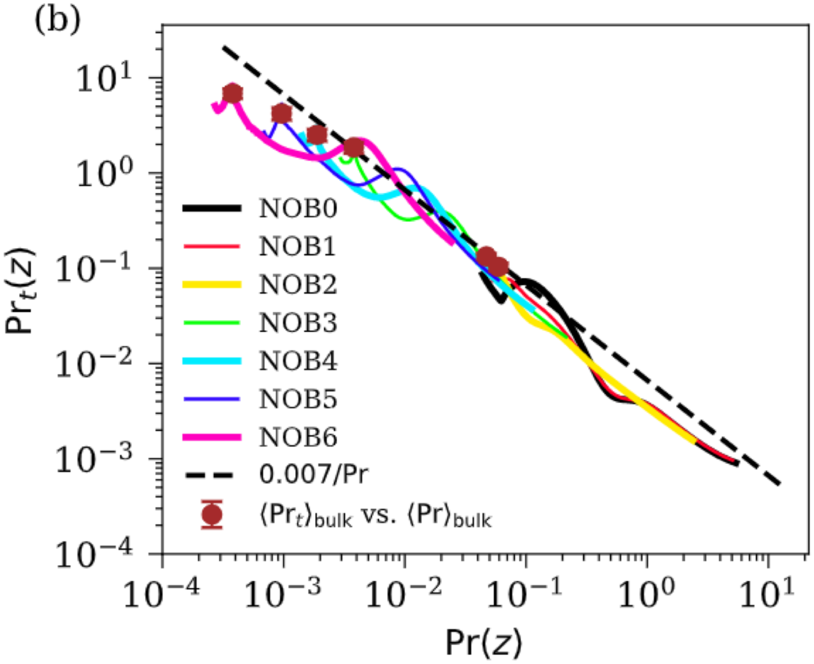


J. Schumacher & KRS,
Rev. Mod. Phys. (2020)

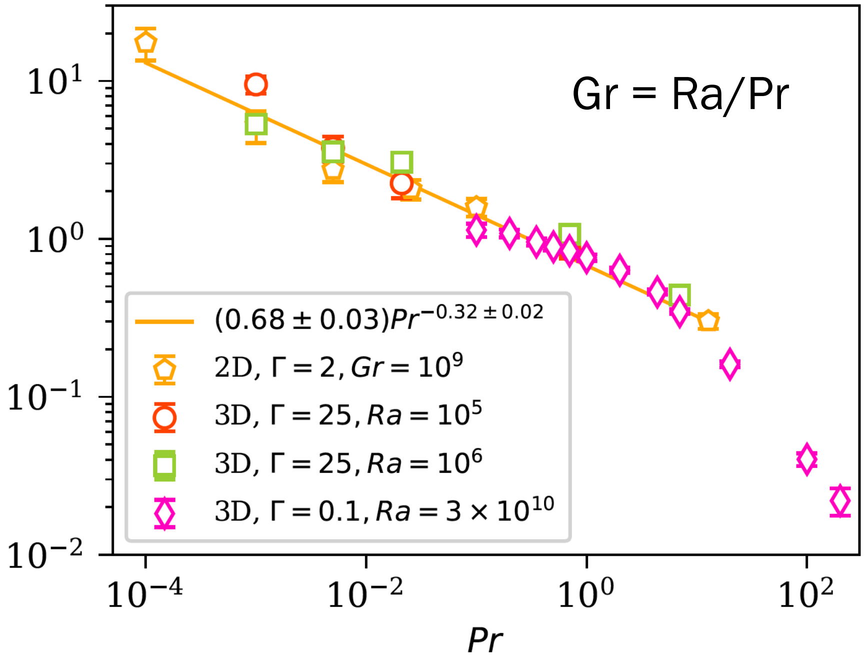
A wild conjecture

For any Rayleigh number, however high, there exists a low enough Prandtl number below which heat transport is accomplished mostly microscopically.

Turbulent vs. molecular Prandtl number



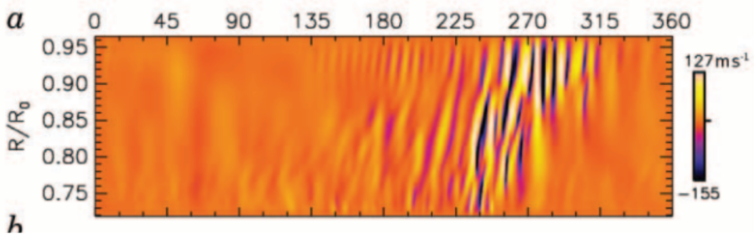
Six orders of magnitude in Pr is covered



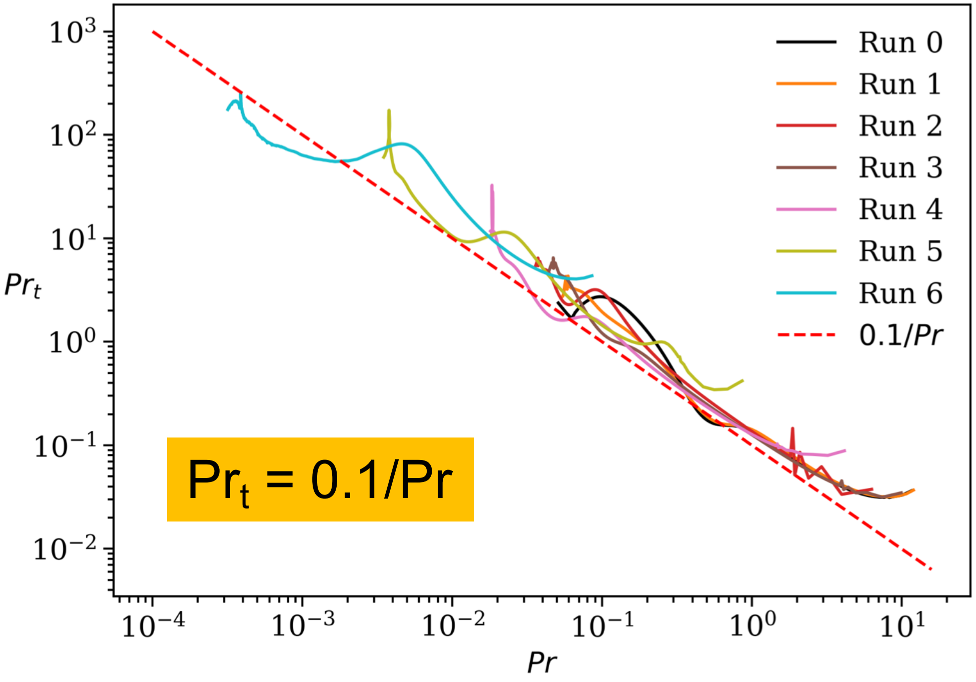
Gr = Ra/Pr

$Pr_t = \nu_t/\kappa_t$

M. Rempel, M. Miesch, others

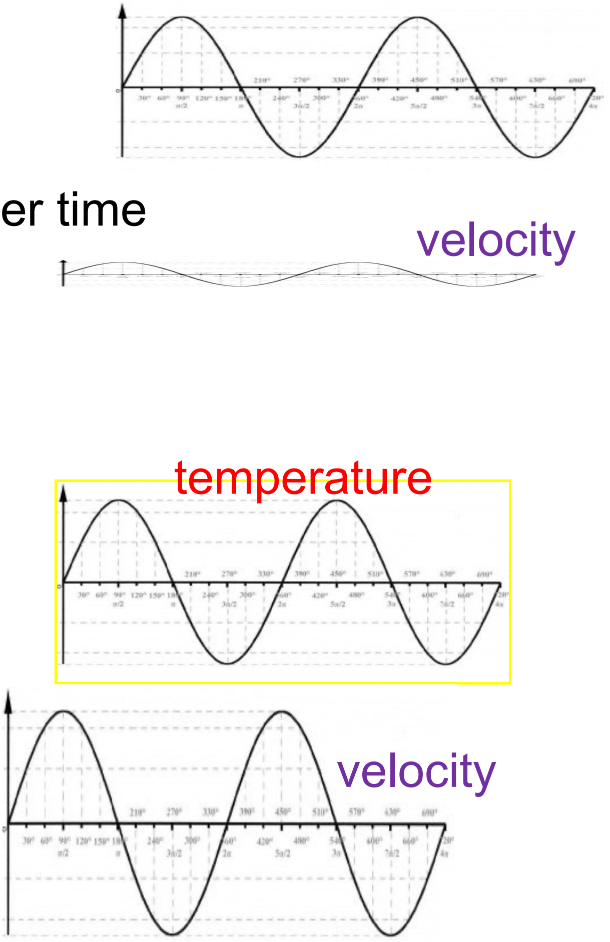


In reality, ν_t increases slightly with decreasing Pr, whereas κ_t decreases considerably.



$Pr_t = 0.1/Pr$

A. Pandey, J. Schumacher & KRS,
Phys. Rev. Fluids (2021)



temperature

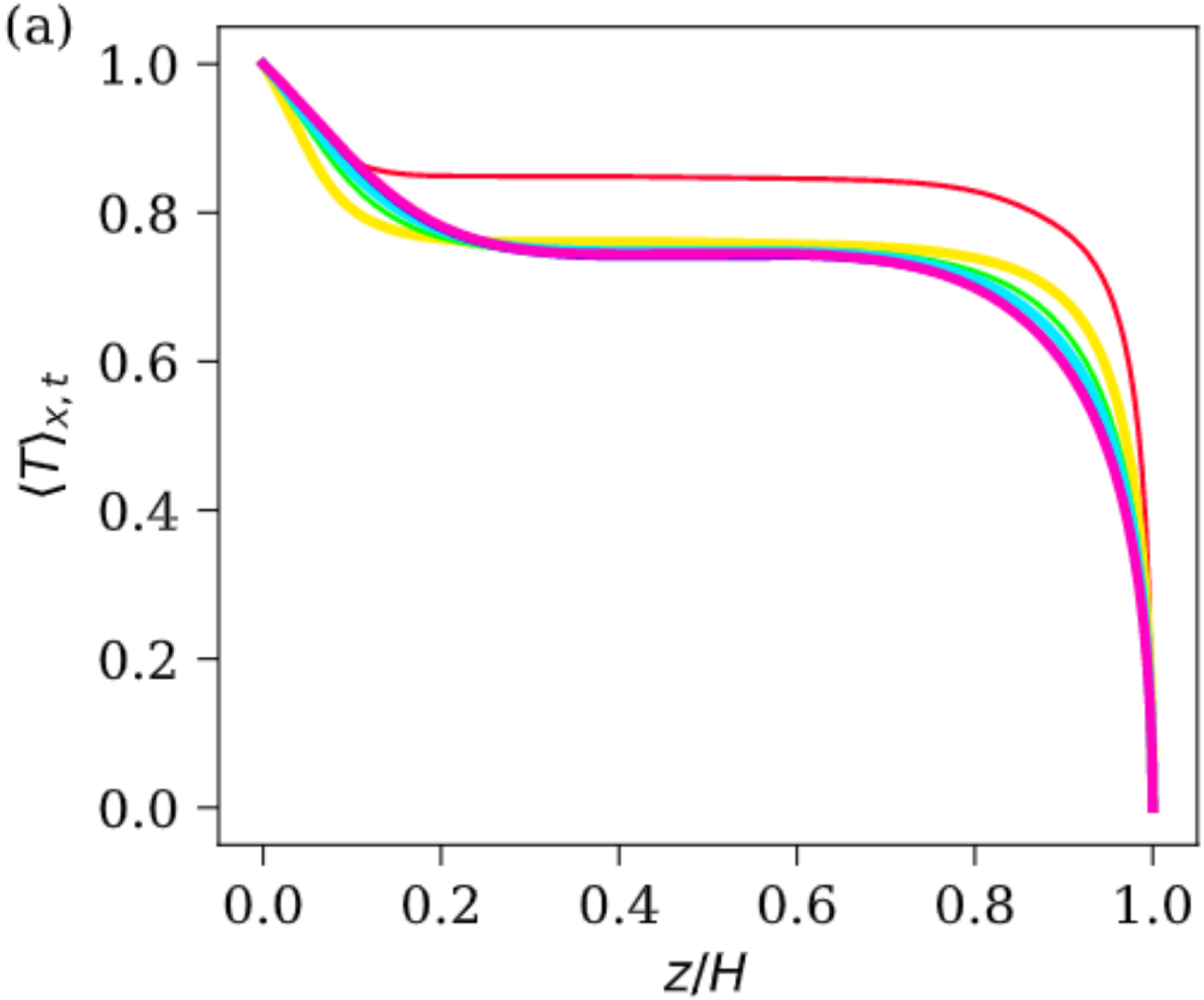
at a later time

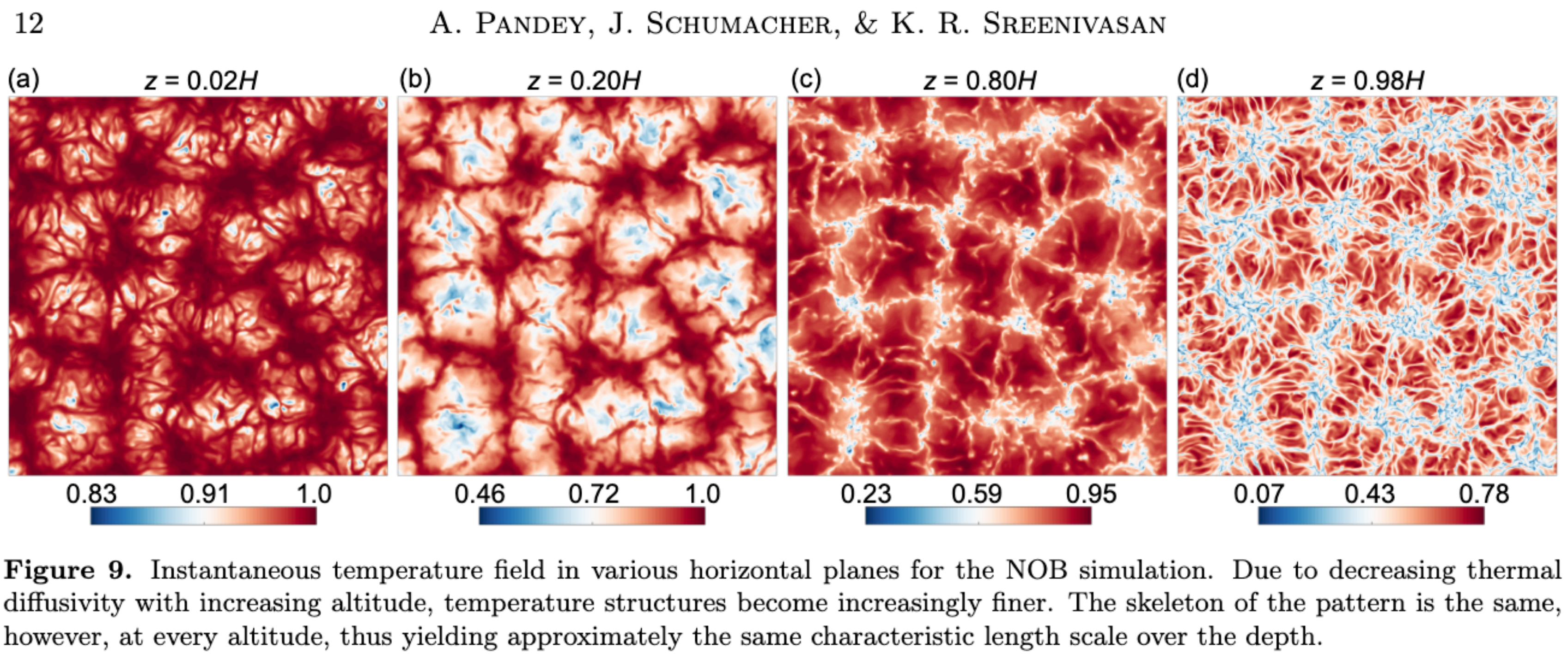
velocity

temperature

t = 0

velocity


(a)
1.0
0.8
0.6
0.4
0.2
0.0
$\langle T \rangle_{x,t}$
0.0
0.2
0.4
0.6
0.8
1.0
z/H



12 A. PANDEY, J. SCHUMACHER, & K. R. SREENIVASAN

Figure 9. Instantaneous temperature field in various horizontal planes for the NOB simulation. Due to decreasing thermal diffusivity with increasing altitude, temperature structures become increasingly finer. The skeleton of the pattern is the same, however, at every altitude, thus yielding approximately the same characteristic length scale over the depth.

Structure set by the wall with higher diffusivity

Non-Boussinesq convection: summary

- The layer with higher thermal diffusivity controls the structure of convection
- Heat transport goes down because the temperature and velocity fields become increasingly uncorrelated. Temperature field becomes smoother and velocity field becomes more and more intermittent.
- Energy moves to smaller scales (spectral result)
- As for the Sun, the convection region does not begin probably until 8/10 Sun's radius

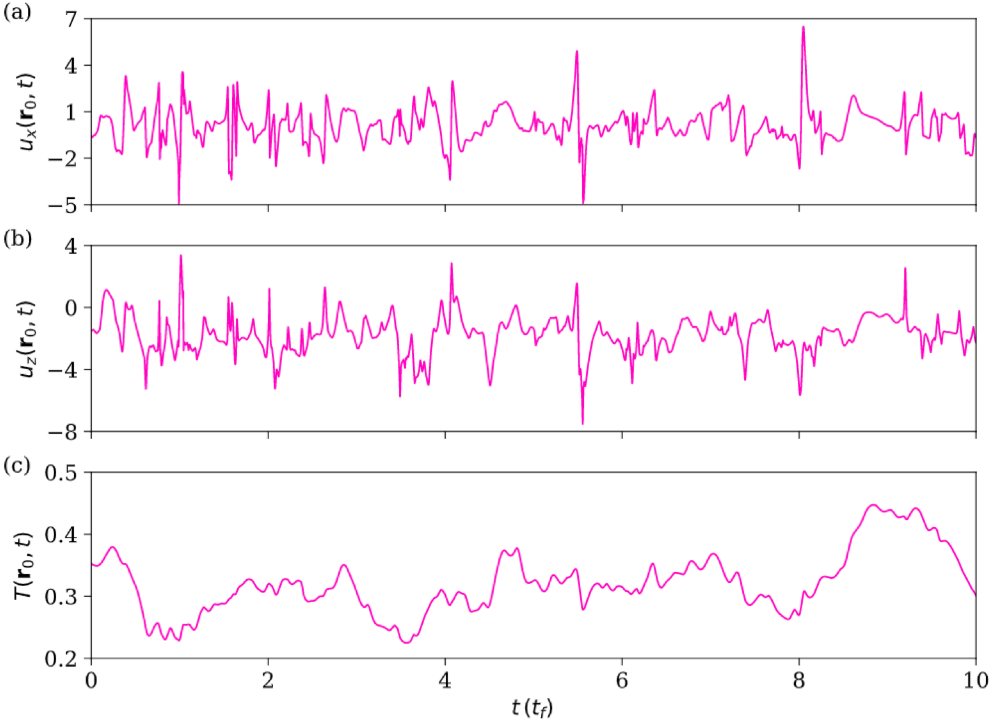
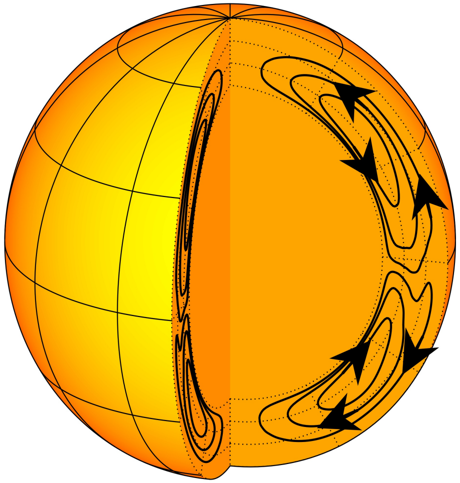
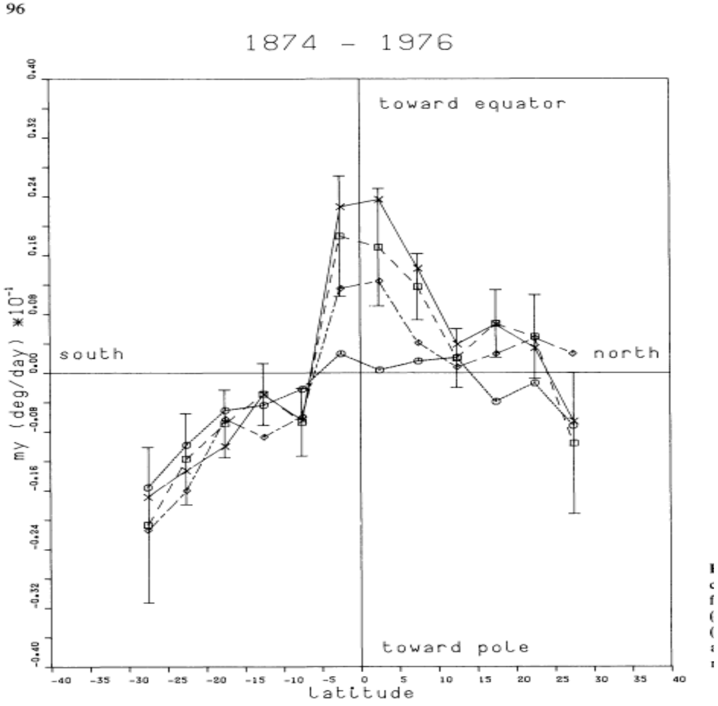


FIG. 4. Time traces of (a) horizontal velocity, (b) vertical velocity, and (c) temperature taken at the center at $\mathbf{r}_0 = (L/2, H/2)$ for run OB7 with $Pr = 10^{-4}$ and $Ra = 10^5$. A shorter segment of the entire time trace is shown only to highlight the irregular and stochastic nature of all fields. The signals indicate that the flow is turbulent despite the moderate Rayleigh number, but that the temperature field is coarse due to high diffusivity.

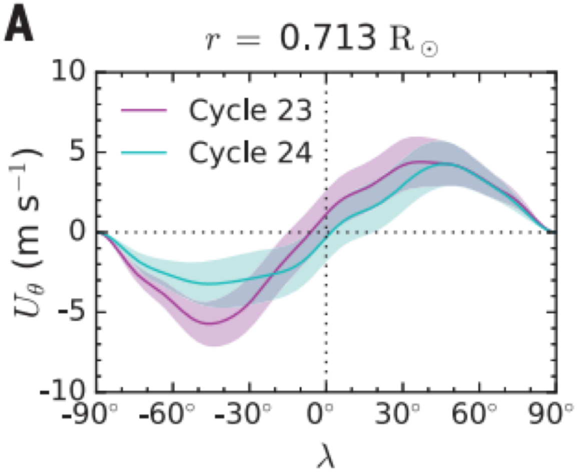


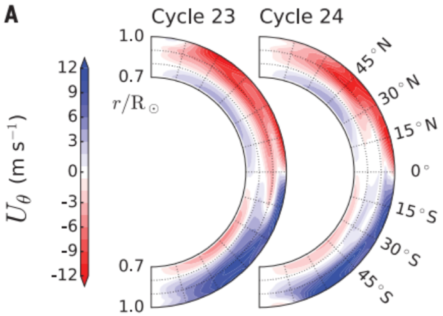
Single-celled meridional flow in each hemisphere O(10m/s)



Enormous amount of averaging is possible

1996-2019



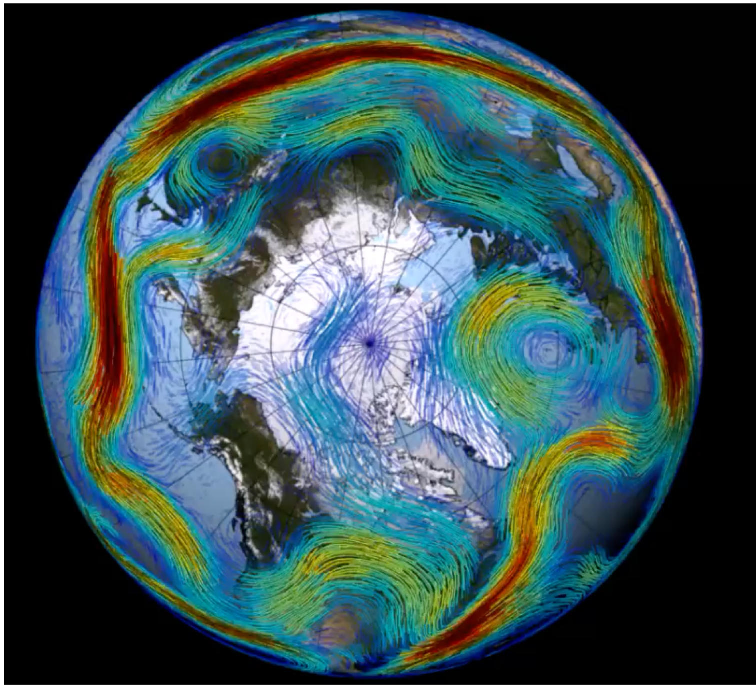


Gizon et al., Science 368, 1469-1472 (2020)

Solar inertial modes

McIntosh et al., Nature (2017); Löptien et al. Nature Astron. 2, 568 (2018); Hanasoge & Mandal, ApJL (2019); Dikpati & McIntosh, Space Weather (2020); Gizon et al. A&A (2021)

When rotation is included, other modes become possible, e.g., Rossby waves



Result of conservation of potential vorticity and changing Coriolis force

In thin 2D shells, they are retrograde waves relative to the rotating frame with specified dispersion relationship.

Theoretically, prograde Rossby waves are possible but the conditions on the Sun may not be conducive.

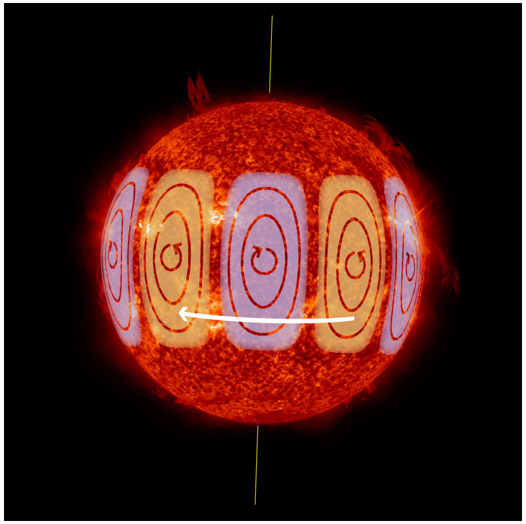
Requires observations over many solar rotations; first observed helioseismically by H. Löptien et al.

From Gizon et al. (2022): Superadiabaticity $< 2 \times 10^{-7}$;

ν_t at the bottom of the convection layer < 100 km^2/s

Competing ideas for redistribution of momentum

- Latitudinal entropy gradients (thermal wind balance); see M.S. Miesch, “Large-Scale Dynamics of the Convection Zone and Tachocline”, *Living Rev. Solar Phys.* **2**, 1 (2005).
- Rossby waves
- Giant cells, supergranules, etc



Results from H. Löptien et al. *Nature Astron*. **2**, 568 (2018)

- In rotating frame, phase velocity

  $$\omega/m = -2\Omega/[\ell(\ell+1)]$$
- ℓ is spherical harmonic degree and m is the azimuthal order
- For sectorial modes $\ell = m$, these authors found waves with
- retrograde phase velocity

  $$\omega/m = -2\Omega/[m(m+1)].$$
- The group velocity is prograde, given by $2\Omega/[(m+1)^2]$.

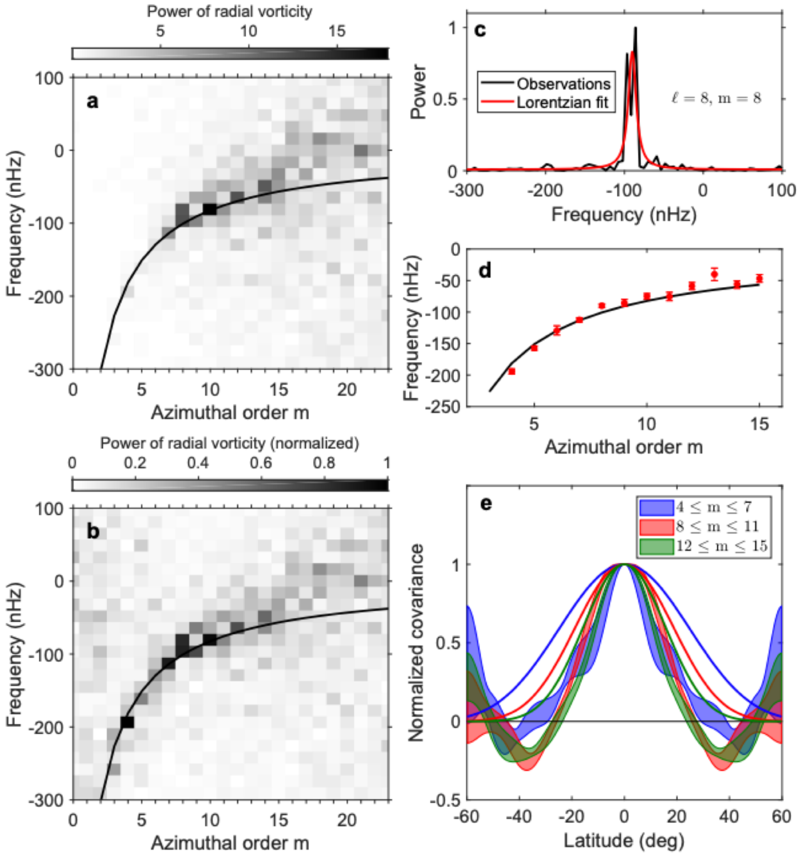


Figure 3: **Dispersion relation and horizontal eigenfunctions of equatorial Rossby waves.** (**a**) Power spectrum of surface radial vorticity from granulation tracking for $m = \ell$ using a spherical harmonic decomposition. The frequency spacing is 16 nHz. The vorticity is measured in a frame rotating at the surface equatorial angular velocity Ω_{ref}. The black curve shows the theoretical dispersion relation for sectoral Rossby waves in the rotating frame. (**b**) Same as panel (a) except that the power at each m is normalized by its mean over frequencies between −310 and 110 nHz. (**c**) Power spectrum of surface radial vorticity at $\ell = m = 8$ (black curve, without smoothing or binning). The red curve shows a Lorentzian fit to the data, used to measure the peak frequency and the linewidth. (**d**) Measured peak frequencies in the surface power spectrum (red points with one-sigma errors) and theoretical mode frequencies of classical Rossby waves (black curve). (**e**) Estimate of the latitudinal eigenfunctions of the Rossby waves derived from the covariance of the surface vorticity at the equator and other latitudes. Averages over three sets of azimuthal orders (see inset) are plotted as shaded areas ($\pm 1\sigma$ around the mean). The thin lines show averages of the corresponding Legendre polynomials.

Various forms of inertial modes are being explored currently, and many have been found (and not understood). See, e.g., Hanson, Hanasoge & KRS, Nature Astronomy 6, 708 (2022)

C. Hanson

For possible applications in determining and predicting space weather via Rossby waves, see Dikpati & McIntosh, Space Weather (2020)

M. Dikpati

For possible generation mechanisms of Rossby waves in the Sun, Dikpati et al., Ap. J. (2022)

A few closing remarks

- The Sun has been the subject of systematic scientific observations for several hundred years, and some fluid flow aspects were known already by the 1830's.
- The turbulent nature of these flows were known by the 1930's (a little after mixing length ideas appeared in the turbulence literature)
- Advances in turbulence theory such as LES, VLES, added diffusivities, and intermittency have all had some role to play.
- All this work has produced a wealth of ideas, for some of which are amenable to testing from careful observations of local helioseismology. Progress in the last 20 years has been exciting for this reason.
- Not only sound waves but direct measurement of surface velocities are measurable; compatible instability modes have been proposed; also as predictive tool for space weather.
- The new missions such as the Solar Orbiter, along with new methods of analysis (for example, from machine learning), will accelerate progress.
- The result of all this work suggests that models of solar convection needs serious rethinking; and the next decade promises to be as exciting as the last.