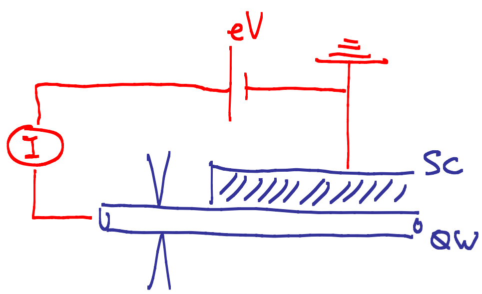
Majorana in condensed matter systems – PART III

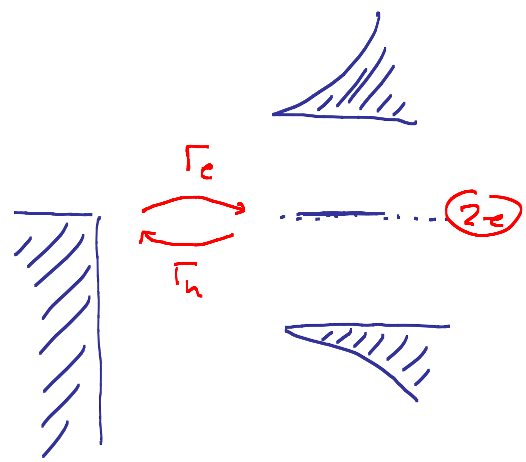
IX) Signatures of Majoranas

* Zero-bias conductance peak:



SC : fixed μ

charge fluctuates



Andreev reflection.

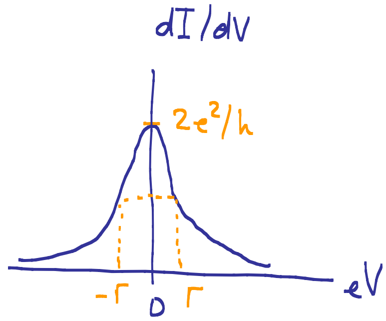
Majorana mode : resonant tunneling from electron in lead to Majorana and as hole back to lead

$\rightsquigarrow$ transfer of two Cooper pair into SC (charge 2e)

$$\frac{dI}{dV} = \frac{2e^2}{h} \frac{\Gamma_e \Gamma_h}{(eV)^2 + \left(\frac{\Gamma_e}{2} + \frac{\Gamma_h}{2}\right)^2}$$

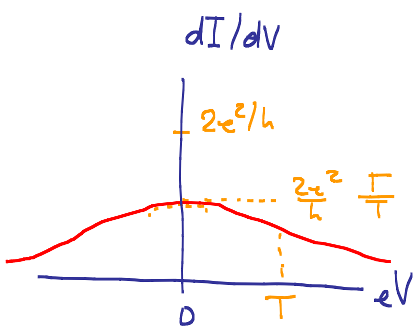
ph symmetry at $E=0$: $\Gamma_e = \Gamma_h$

$$\boxed{\frac{dI}{dV} = \frac{2e^2}{h} \frac{\Gamma^2}{(eV)^2 + \Gamma^2}} \qquad T=0 .$$

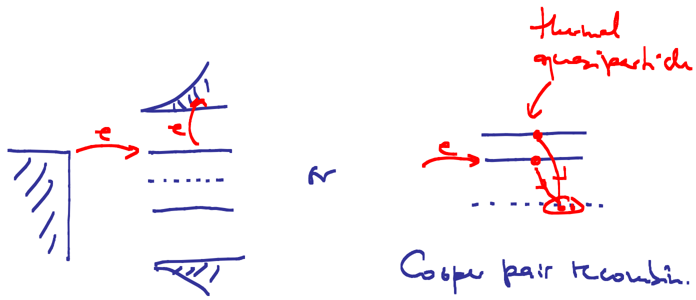


Remarks:

- finite temperature: broadens peak & reduces peak height



- and introduces inelastic processes → single-electron tunneling



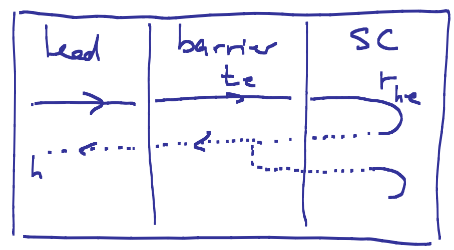
  - additional current channel
  - $\Gamma \to \Gamma + \Gamma_{relax}$ in Andreev contribution (suppression)
  - particularly relevant for STM (often at ~ 1K) and/or soft gap

- Superconducting tip (Peng et al. PRL 2015)
  - suppression of thermal broadening in tip
  - Majorana peaks at $eV = \pm\Delta_{tip}$ (NOT zero bias!)
  - symmetric peak heights at $\pm\Delta_{tip}$
  - Andreev: $\left.\frac{dI}{dV}\right|_{eV=\pm\Delta_{tip}} = (4-\pi)\frac{2e^2}{h}$.
  - peak width $\propto \Gamma^{2/3}$ (i.e., larger than Γ for normal state)

Alternative derivation using scattering theory:

$$A_{he} = t_h \left[1 + r_{he} r_e r_{eh} r_h + (r_{he} r_e r_{eh} r_h)^2 + \dots \right] r_{he} t_e$$

$$= \frac{t_h r_{he} t_e}{1 - r_{he} r_e r_{eh} r_h}$$



denominator vanishes at $r_e = r_h = 1$ and using $|r_{eh}|^2 = 1$ characteristic of topological SC (see below)

$\rightsquigarrow$ reflection of formation of Majorana bound state

reflection from SC:

$$r = \begin{pmatrix} r_{ee} & r_{eh} \\ r_{he} & r_{hh} \end{pmatrix}$$

- unitary for subgap energies (no transmission through SC)
- ph symmetry $\tau_x r(-E) \tau_x = r^*(E)$

$E = 0$: $r_{ee} = r_{hh}^*$; $r_{eh} = r_{he}^*$ $\rightsquigarrow$ $\det r(E=0) \in \mathbb{R}$

$$\boxed{\det r(E=0) = \begin{cases} +1 & \text{reflection from trivial SC}, \ |r_{ee}|=1, \ |r_{eh}|=0 \\ -1 & \text{'' TSC}, \ |r_{ee}|=0, \ |r_{eh}|=1 \end{cases}}$$

Remember that for spinless fermions, trivial phase is essentially vacuum ($\mu<0$)
→ only normal reflection.

More complete calculation:

- multiply $|A_{he}|^2$ by $\underbrace{n_F(\omega - eV)}_{\text{incoming electrons}} \, [1 - \underbrace{n_F(\omega + eV)}_{\text{outgoing holes}}]$
- integrate over all energies
- add contribution of incoming holes and outgoing electrons and divide by 2 to avoid double counting

$$I = \frac{1}{2} \, 2e \int \frac{d\omega}{2\pi\hbar} \, |A_{he}|^2 \left[n_F(\omega - eV) - n_F(\omega + eV) \right]$$

Andreev reflection:

$$r_{he} = \exp\left\{ -i \arccos \frac{\omega}{\Delta} \right\} \qquad (\Delta = \Delta' p_F)$$

$$r_{he} = \exp\left\{ i \arccos \frac{\omega}{\Delta} \right\}$$

- near Fermi level (ω small): $\quad r_{he} r_{eh} \simeq 1 + 2i\omega/\Delta$
- weak tunneling: $\quad r_{e/h} \simeq 1 - \frac{1}{2} t_{e/h}^2 \quad$ (assuming $r_{e/h} \in \mathbb{R}$)

$$\rightarrow \quad |A_{he}|^2 = \frac{t_h^2 t_e^2}{4\omega^2/\Delta^2 + \frac{1}{4}(t_e^2 + t_h^2)}$$

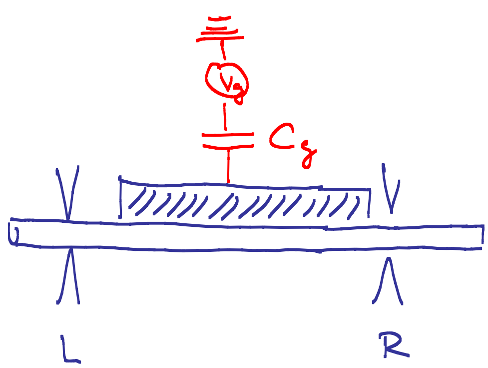
and w/ $\Gamma = \Gamma_{e/h} = \frac{1}{2} \Delta t_{e/h}^2$:

$$\boxed{\frac{dI}{dV} = \frac{2e^2}{h} \frac{\Gamma^2}{(eV)^2 + \Gamma^2}} \qquad T = 0.$$

* charging physics & Majoranas

- new effects: Majorana teleportation (Fu PRL 2010)
- manipulation & readout (Aasen et al. arxiv 2015)

now consider: floating SC



fixed N (closed contacts L, R)

→ charging energy

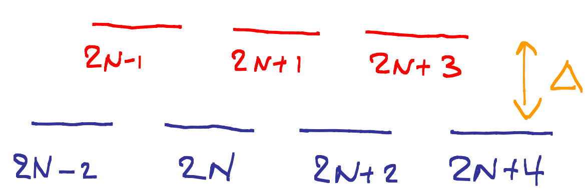
$$E = E_c (n - Q_0/e)^2$$

w/ gate charge $Q_0 = C_g V_g$

charging energy $E_c = \frac{e^2}{C_\Sigma}$

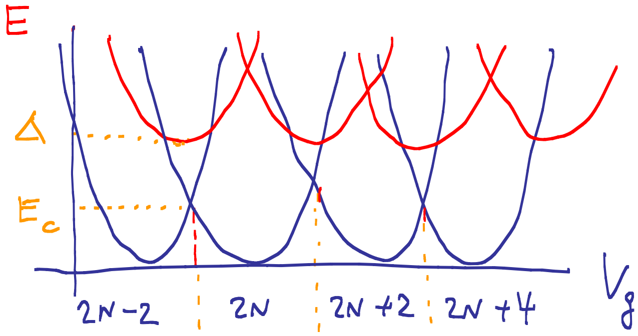
Conventional SC:

- zero E_c

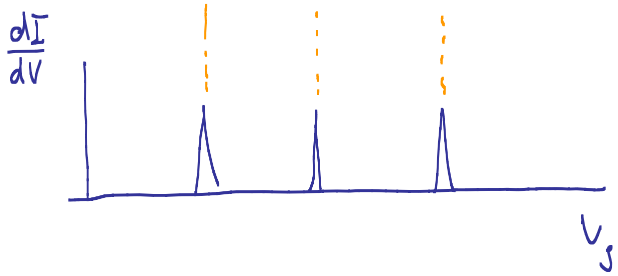


- finite E_c

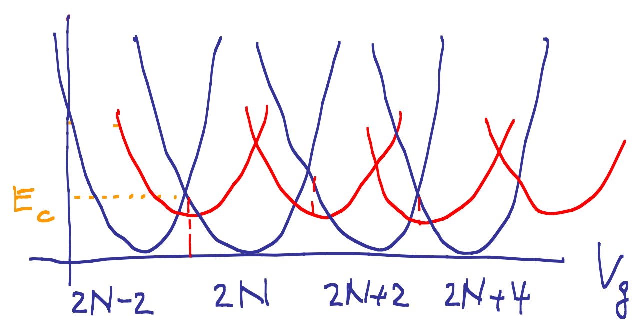
$\boxed{\Delta > E_c}$



- addition of electron pairs



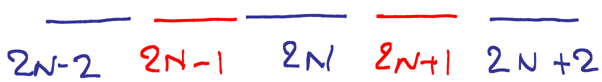
$\boxed{\Delta < E_c}$

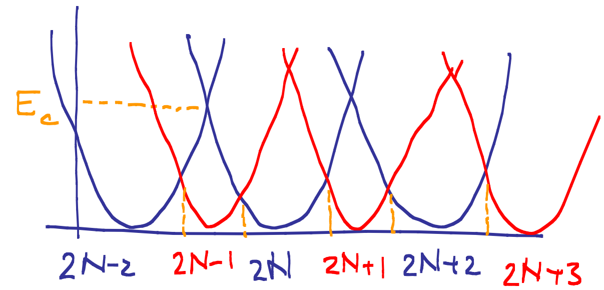


- addition of single electrons
- even-odd effect

topological SC w/ pair of Majorana zero modes γ_1 and γ_2:

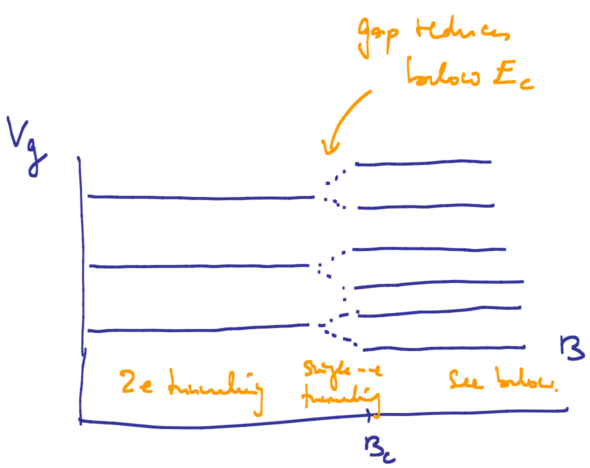
- zero E_c
- finite $E_c < \Delta$

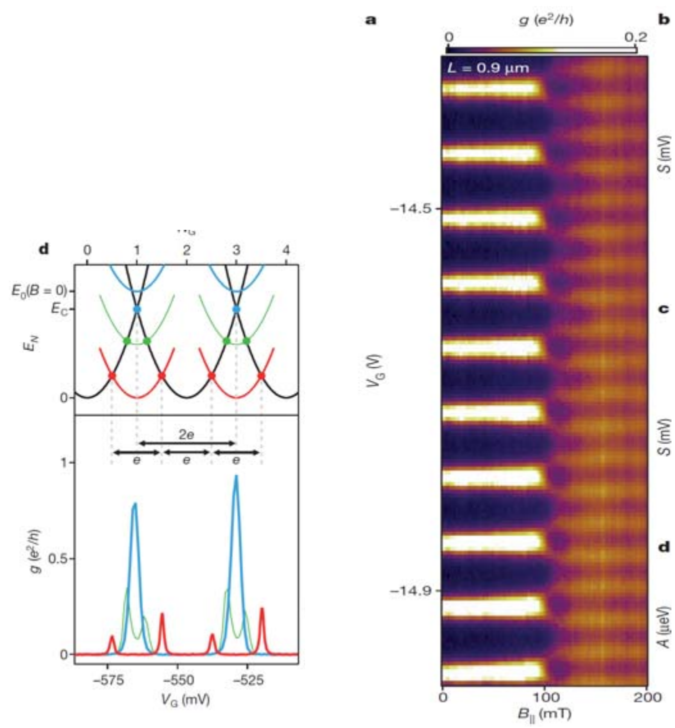




even & odd parity ground states:

- addition of individual electrons
- period halving upon transition to topological phase (e.g., by increasing B in semiconductor QW setting)





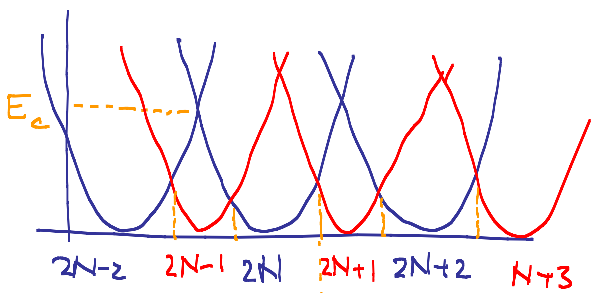
Albrecht et al, Nature 2016

Heck, Lutchyn, Glazman PRB 2016

also: small even-odd variation in Coulomb blockade plateau width when Majoranas hybridize and acquire finite energy

Albrecht et al. : Exponential dependence of splitting on length.

Nature of current flow in topological regime:



focus on vicinity of this point

⇝ finite conductance

- system changes between states w/ 2N & 2N+1 electrons
- same # of Cooper pairs, but different fermion parity.
- define $d = \frac{1}{2}(\gamma_1 - i\gamma_2)$ such that

$d\,|\text{even}\rangle = 0 \; ; \quad |\text{odd}\rangle = d^\dagger |\text{even}\rangle$

$$\boxed{H_{\text{eff}} \simeq H_L + H_R + \frac{\delta}{2} d^\dagger d + \sum_k \left\{ \left[\lambda_L c_{Lk}^\dagger d - i \lambda_R c_{RL}^\dagger d \right] + \text{h.c.} \right\}}$$

This is Hamiltonian for resonant tunneling through single, nondegenerate level:

$$\boxed{\left.\frac{dI}{dV}\right|_{V=0} = \frac{e^2}{h} \frac{\Gamma_L \Gamma_R}{\delta^2 + \left(\frac{\Gamma_L}{2} + \frac{\Gamma_R}{2}\right)^2}}$$

w/ peak conductance at most e^2/h

- resonant tunneling is coherent process → Majoranas enable coherent process over arbitrarily large distance

→ electron teleportation (Fu PRL 2010)

(but remember that this effect requires $eV \ll E_c$)

- reflects long-range nature of Green function

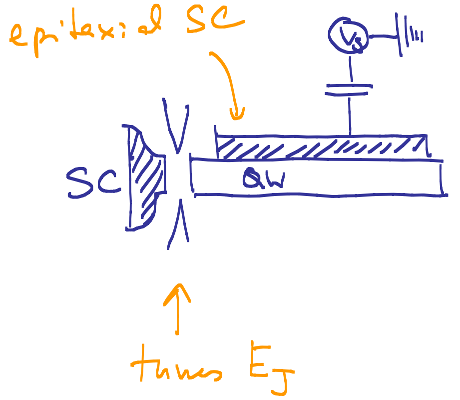
$$G^{e,o}(r_L, t \to \infty; r_R, 0) = \langle c(r_L t \to \infty)\, c^\dagger(r_R 0)\rangle_{e,o}$$

low-energy limit

$$\sim \mp i\, \xi_R^*(r_R)\, \xi_L(r_L)$$

Majorana wavefunction

* Interplay of charging & Josephson



$$\boxed{H = E_c\left(2N + n_d - \frac{Q_0}{e}\right)^2 + E_J \cos\phi}$$

junction w/ trivial SC → only Cooper pair tunneling

Analogy to 1d band-structure problem:

$$H = \frac{1}{2m}(p-k)^2 + V(x)$$

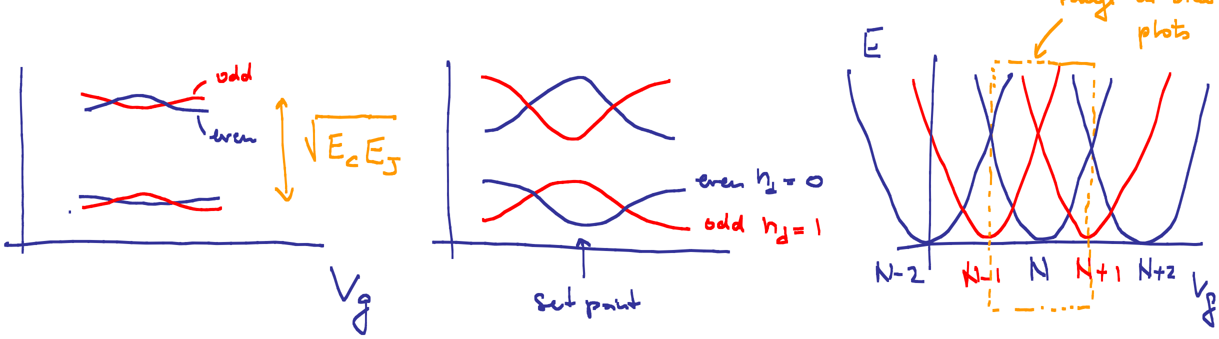
$$[N, \phi] = i$$

"momentum" "position"

Q_0: Bloch momentum

- $e^{\pm i\phi}$ add/remove Cooper pair to/from SC (translation operator for N: $N \to N \pm 1$) → H conserves fermion parity

$E_J \gg E_c$	$E_J \simeq E_c$	$E_J \ll E_c$
tight-binding limit of band-structure problem	intermediate limit	perturbative periodic potential



Level spacing for $E_J \gg E_c$:

$$H \simeq E_c (2N + n_d - Q_0)^2 + \frac{E_J}{2} \phi^2$$

$$\simeq (E_c E_J)^{1/2} \left\{ \left(\frac{E_c}{E_J}\right)^{1/2} (2N + n_d - Q_0)^2 + \frac{1}{2} \left(\frac{E_J}{E_c}\right)^{1/2} \phi^2 \right\}$$

canonical transformation : $\tilde{N} = \left(\frac{E_c}{E_J}\right)^{1/4} N$, $\tilde{\phi} = \left(\frac{E_J}{E_c}\right)^{1/4} \phi$

w/ $[\tilde{\phi}, \tilde{N}] = 1$

$$= (E_c E_J)^{1/2} \left\{ (2\tilde{N} + \tilde{n}_d - \tilde{Q}_0)^2 + \frac{1}{2} \tilde{\phi}^2 \right\}$$

$$\rightarrow \Delta E \sim (E_c E_J)^{1/2}.$$

fermion parity readout: parity-to-charge readout

- charge sensing: even and odd states have different charge for any non-integer gate charge
- dispersive readout : set point for $E_c \simeq E_J$ and integer (say even) gate charge $Q_0 = Ne$
  - both even and odd state have same charge Ne

- even state: middle of Coulomb blockade plateau

  → weak variation of charge w/ gate voltage

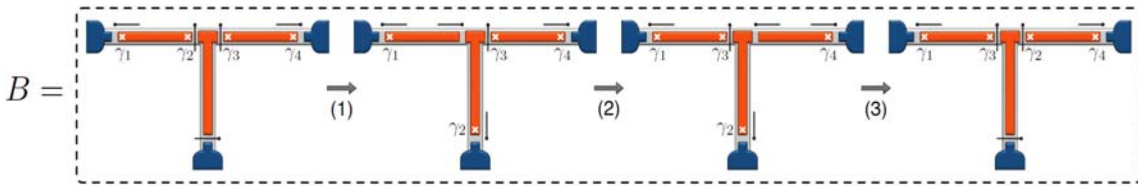
- odd state: anticrossing between N-1 and N+1 electron state

  → charge varies rapidly w/ gate voltage

→ Quantum capacitance $\delta C = \frac{d(en)}{dV_g}$ differentiates between even and odd states.

Braiding: (see Aasen et al. arXiv: 1511.05153)

(a) Basic braiding operation



4 Majoranas $\rightarrow$ topological qubit at fixed fermion parity

e.g., $|0\rangle = |0_{12}, 0_{34}\rangle \qquad |1\rangle = |1_{12}, 1_{34}\rangle$ for even parity

occupation of $f_{12} = \frac{1}{2}(\gamma_1 - i\gamma_2)$

occupation of $f_{34} = \frac{1}{2}(\gamma_3 - i\gamma_4)$

$\rightarrow$ braid γ_2 and γ_3: $\quad U_{34} = \exp\left\{\frac{\pi}{4}\gamma_2\gamma_3\right\} = \exp\left\{\frac{i\pi}{4}\sigma_x\right\}$

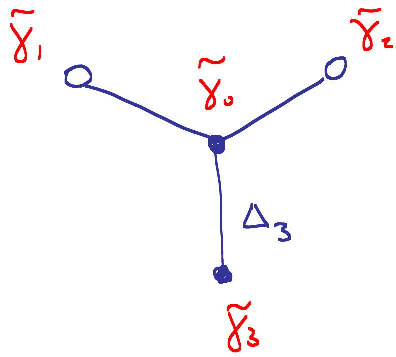
in logical qubit basis $|0\rangle, |1\rangle$

$$-i\gamma_2\gamma_3 = -i\left(i f_{12} - i f_{12}^{\dagger}\right)\left(f_{34} + f_{34}^{\dagger}\right)$$

$$= f_{12} f_{34} - f_{12}^{\dagger} f_{34}^{\dagger} + \text{terms which act in odd subspace}$$

$$= f_{12} f_{34} + \text{h.c.} \longrightarrow \sigma_x \text{ in logical qubit space}$$

relation to braiding protocol in 1^{st} lecture:



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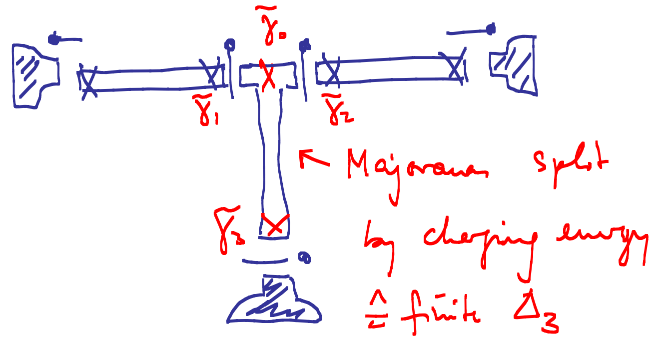
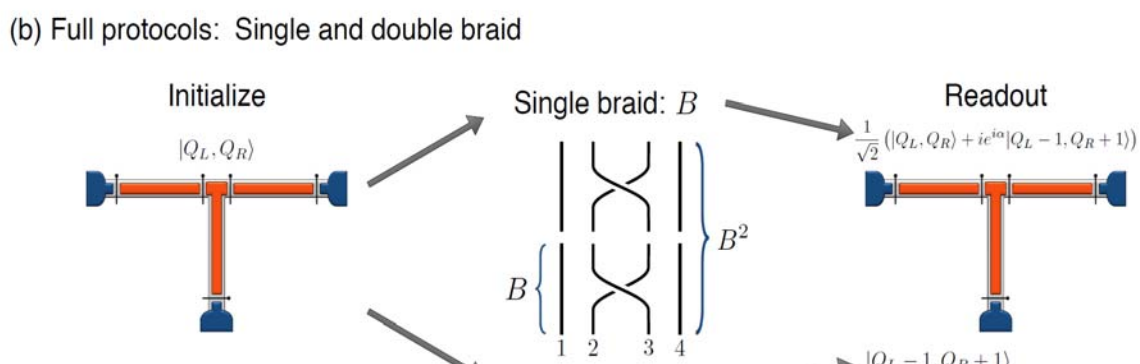


Couplings Δ_1 and Δ_2 are realized by direct overlap w/ Majorana $\tilde{\gamma}_0$

Experimental protocol:



relax to ground state;

then open outer valves

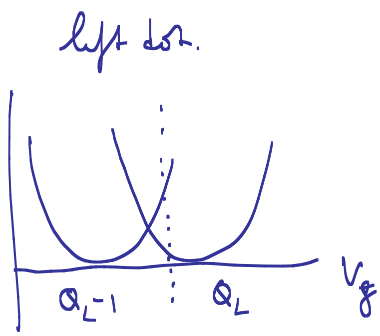
$\rightarrow$ initialized in $|0\rangle = |0_{12}, 0_{34}\rangle$

parity to charge conversion

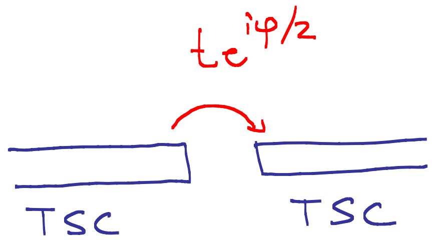
single braid: $|0\rangle \longrightarrow U_{34}|0\rangle = \frac{1}{\sqrt{2}}(1 + i\underbrace{\gamma_3\gamma_4}_{\sigma_x})|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

double braid: $|0\rangle \longrightarrow U_{34}^2|0\rangle = \gamma_3\gamma_4|0\rangle = i\sigma_x|0\rangle = i|1\rangle$

w/o mutual capacitance.



* fractional Josephson effect



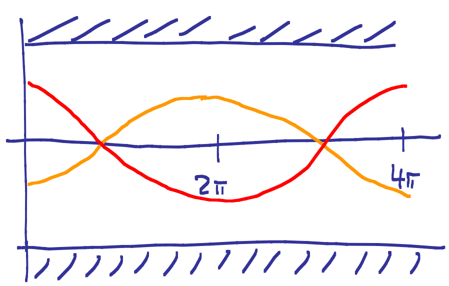
$$H_T = t e^{i\varphi/2} \psi_R^\dagger \psi_L + h.c.$$

$$\psi_L \simeq u_L \gamma_L$$

$$\psi_R \simeq i u_R \gamma_R$$

$u_{L/R}$ real Majorana wavefunctions

$$\boxed{H_T = 2 t u_L u_R \cos\frac{\varphi}{2} \; i\gamma_R\gamma_L}$$



- overall spectrum 2π periodic
- fixed fermion parity : 4π periodic
  → fractional Josephson effect
- level crossing protected by fermion parity

$$I = 2e \frac{dI}{d\varphi} = \pm 2 e t u_L u_R \sin\frac{\varphi}{2}$$ + (2π)-periodic non-Majorana contributions.

- quasiparticle poisoning: switching between branches
  → Josephson reverts to 2π periodicity
- swift variation of phase difference: ac Josephson w/ applied bias;
  ac current at half the usual Josephson frequency
- possibly faster than quasiparticle poisoning, but diabatic transitions
  to continuum
- still signature in noise (Badiane et al. PRL 2011)
- analogous: Shapiro steps
- switching current measurements w/ current pulses (Peng et al. 2016)