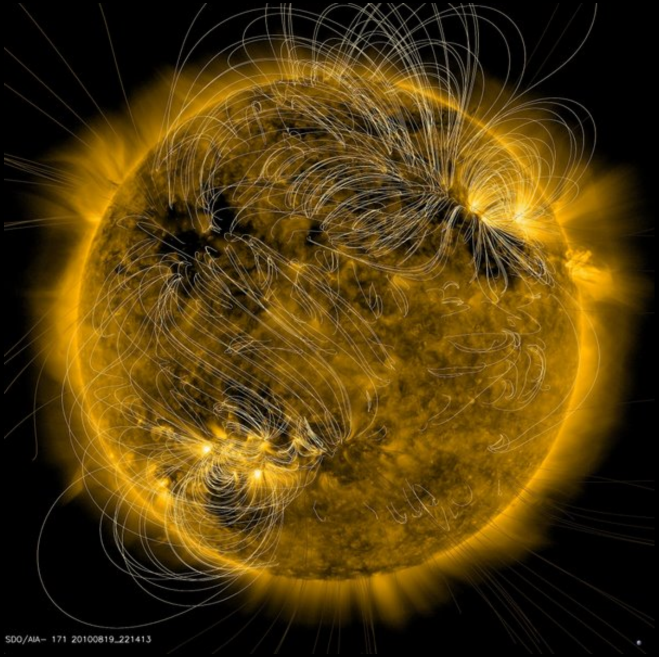
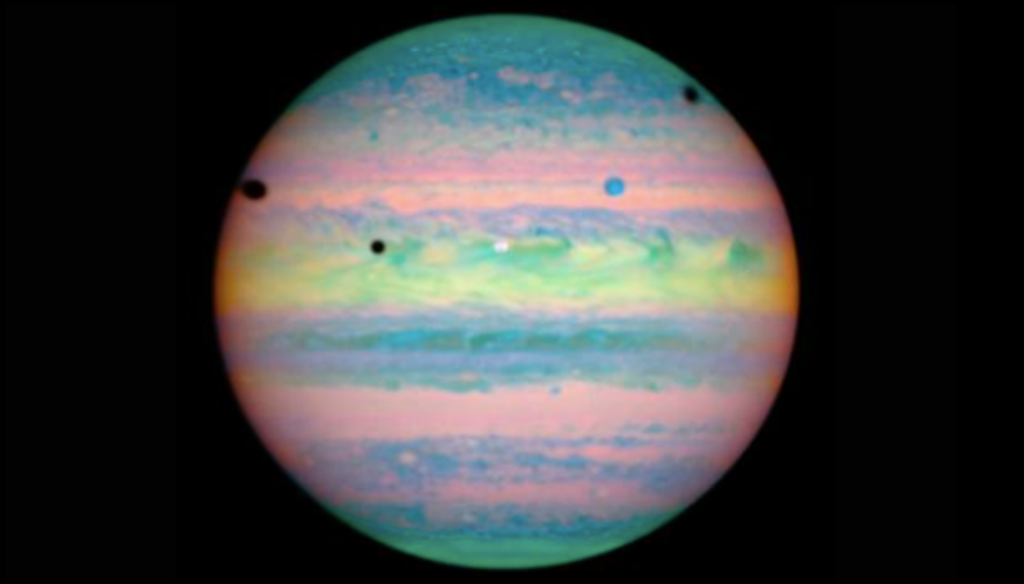
Direct Statistical Simulation: geophysical and astrophysical flows





Steve Tobias (University of Leeds)
(with thanks to Brad Marston)

Boulder Summer School: Lecture 1

Theme of talk: Can methods from non-equilibrium statistical mechanics help with problems in fluid dynamics even on very large-scale problems?

Collaborators: Brad Marston (Brown)

Katie Dagon, Wanming Qi (Brown), Greg Chini (New Hampshire)
Adam Child, Rainer Hollerbach (Leeds), Altan Allawala (Brown), Kuan Li, Girish Nivarti (Leeds), Rich Kerswell (Cambridge), Abby Plummer (Princeton)

S.M. Tobias
"The Turbulent Dynamo"
JFM Perspectives (2021)

J.B. Marston & S.M.Tobias
"Recent Developments in Theories of Inhomogeneous and Anisotropic Turbulence"
Ann Rev Fluid Mech. (2022)

Overall Outline: Lecture 1

- Motivation: Fluids and magnetic fields in Geophysics & Astrophysics (and plasma physics)

(cf Keith Julien's lecture 1)

  - The Sun and the Solar Cycle
  - Jupiter
  - Some flows on (and in!) the Earth
  - Fusion flows
- Modelling strategies for these fluid flows
- Dynamics vs Statistics

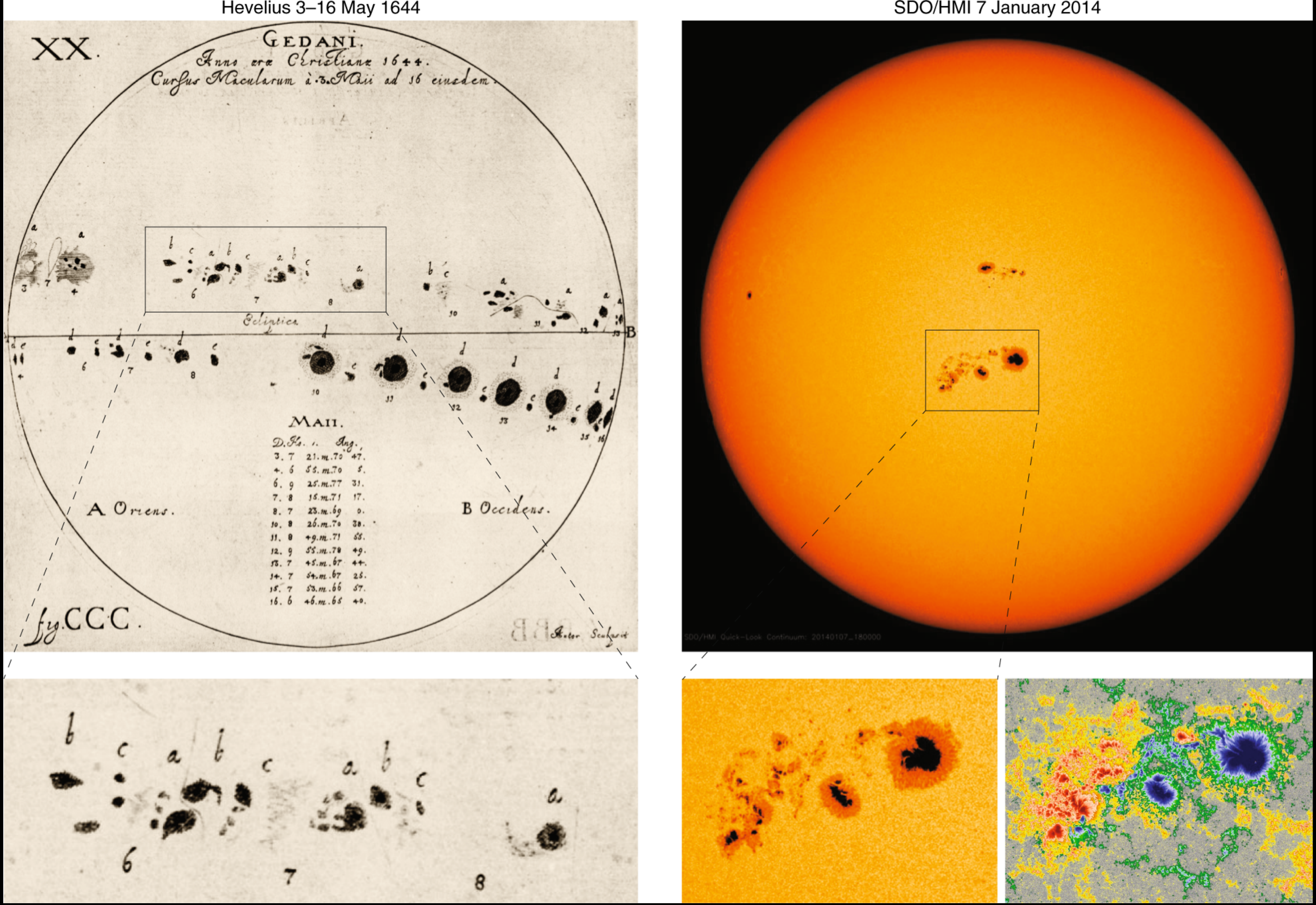
Overall Outline: Lecture 2

- The Quasilinear Approximation
  - Historical Perspective
  - Averaging Choices
  - The “Pain in the Neck Term”
  - Asymptotic Theories
  - The Kubo number
  - Infinite U(1) symmetry
- The Generalised Quasilinear Approximation
  - Interaction rules
  - Some examples
  - Why it works better…

Overall Outline: Lecture 3

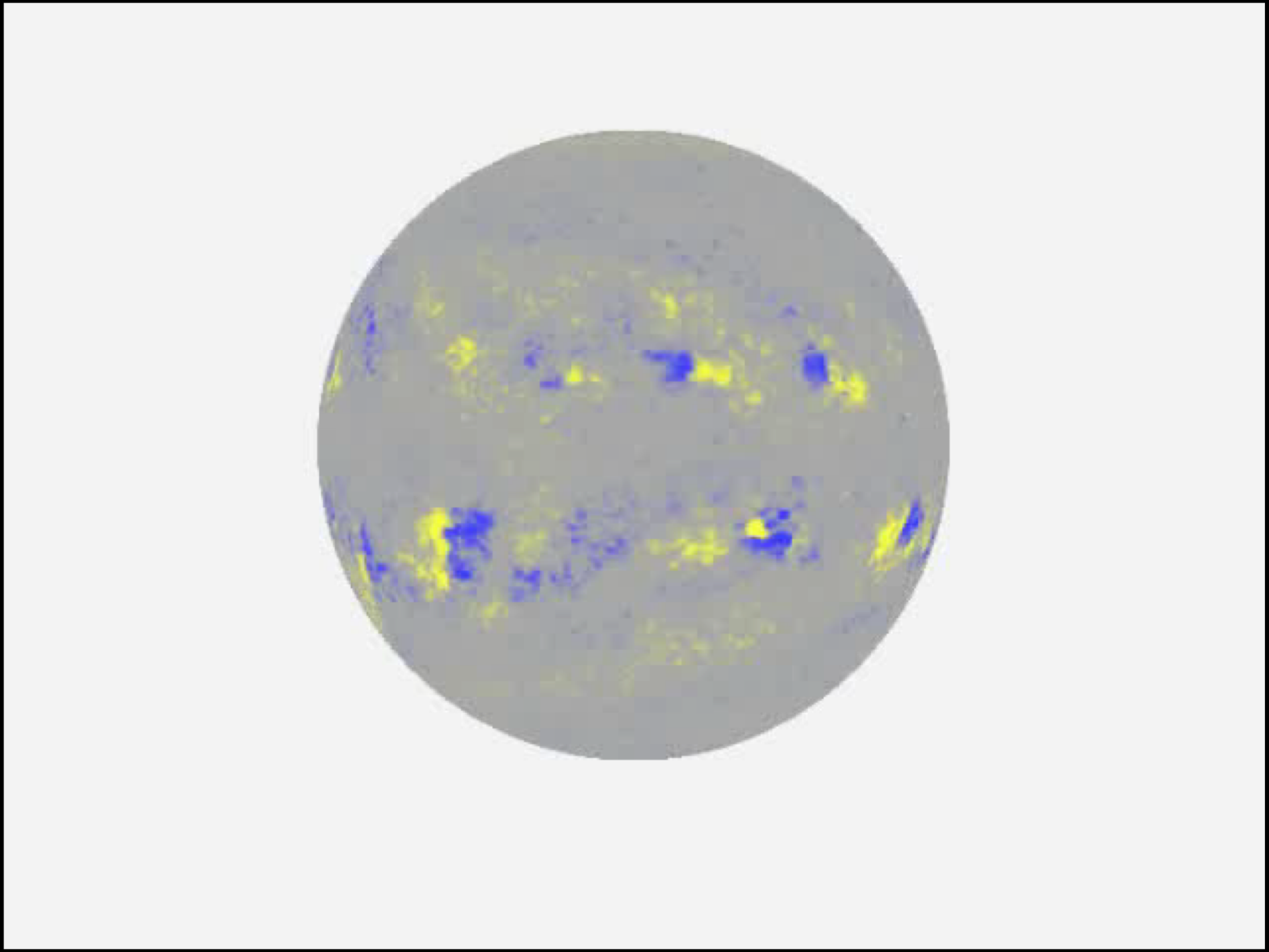
- Direct Statistical Simulation
  - Why simulate the statistics
  - What statistics could you look at
  - DSS via cumulant expansions
  - Combining DSS with DNS?

Motivation 1: The Solar Cycle

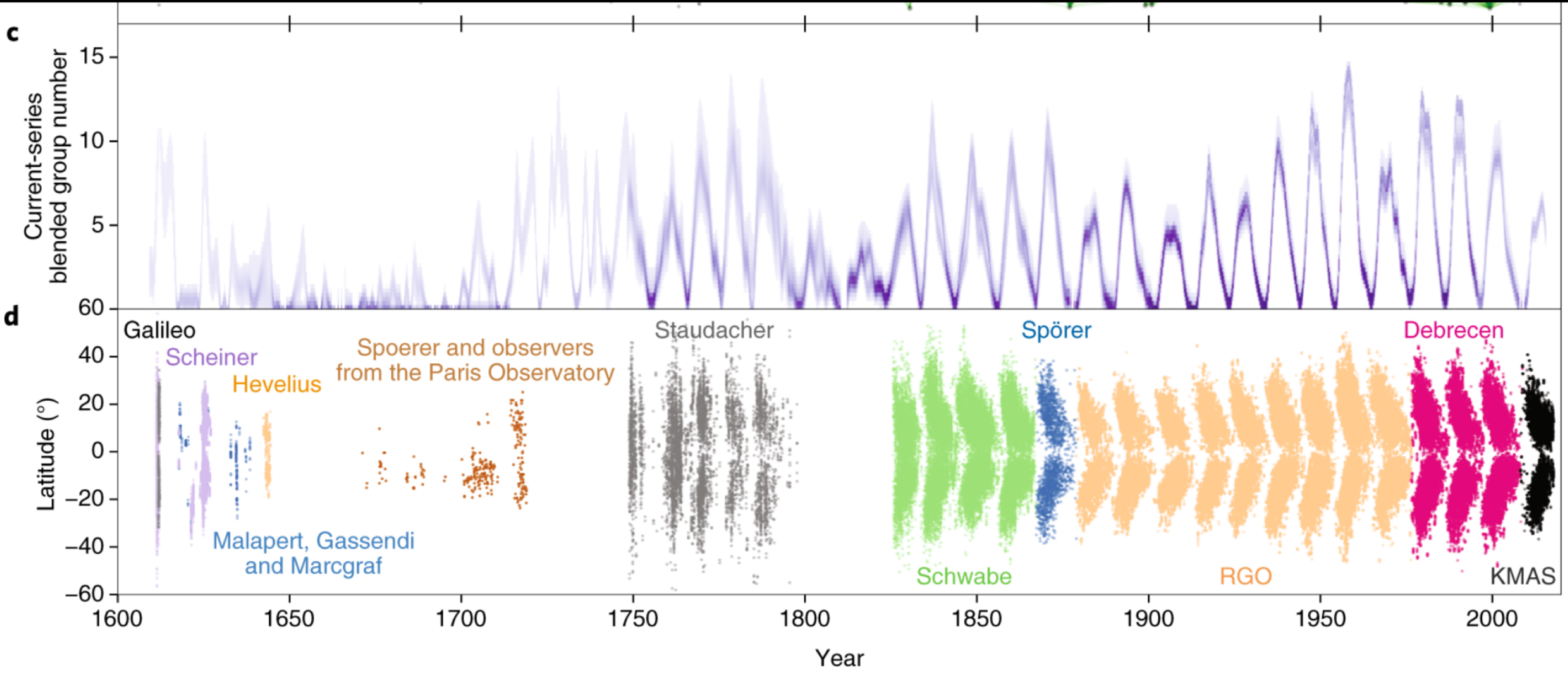


Sunspot Observations: Muñoz-Jaramillo & Vaquero
Nature Astronomy (2019)

Systematic Behaviour from Turbulence

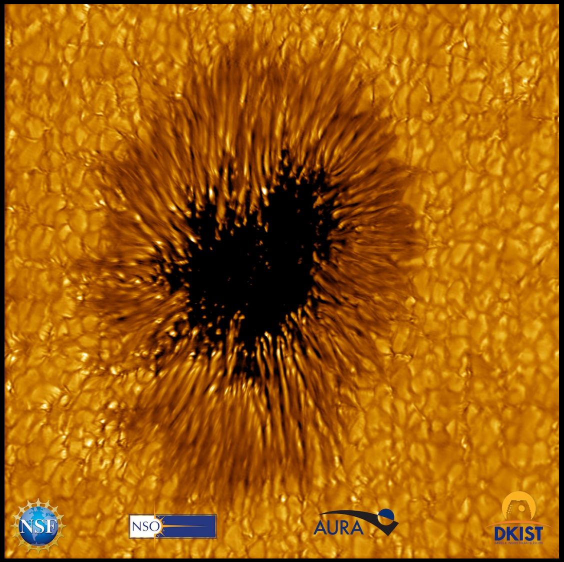


Courtesy
D. Hathaway



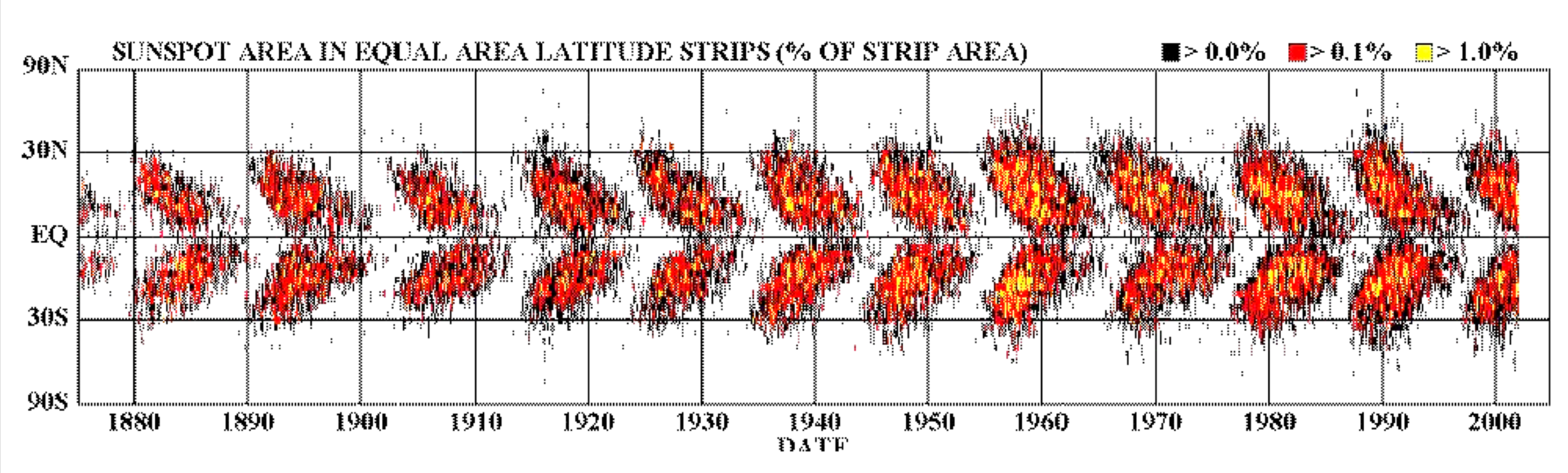
Solar Cycle

Muñoz-Jaramillo & Vaquero
Nature Astronomy (2019)



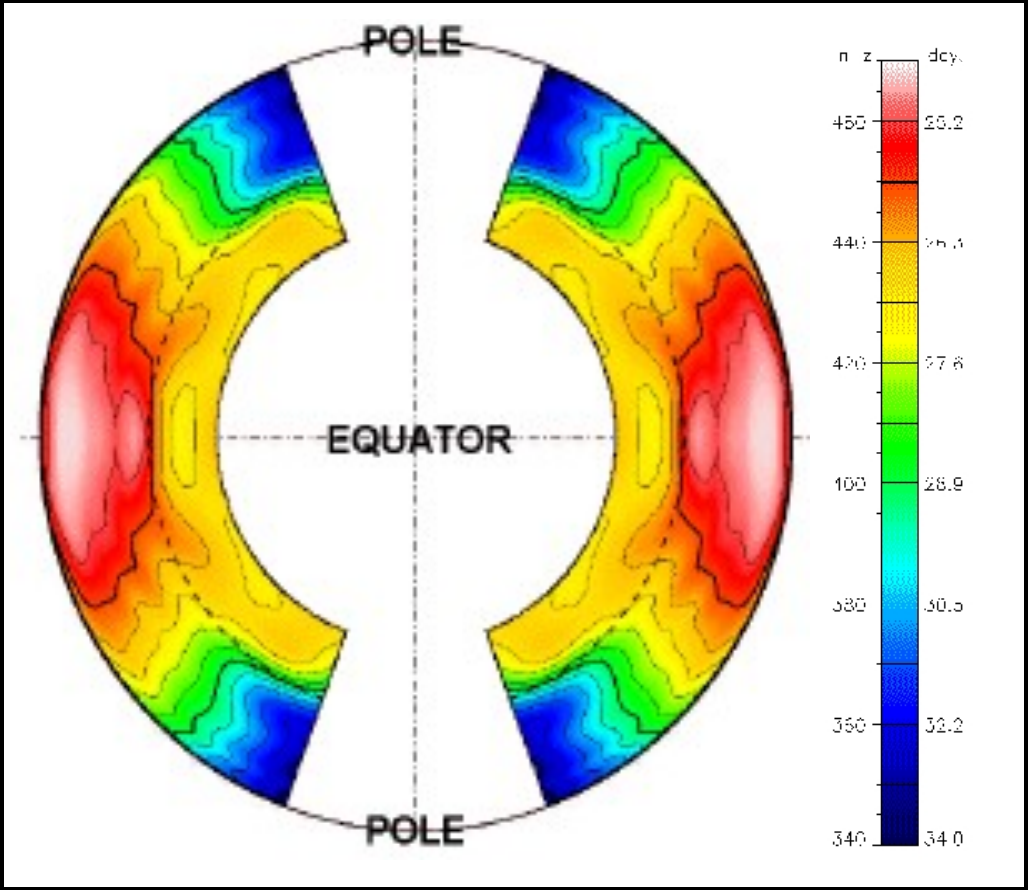
Sunspot: Interaction of magnetic field with convection

Courtesy Inouye Solar Telescope



- Solar cycle:
  - “Large-scale” in space
    - NOTE: ZONAL AVERAGE!
  - Systematic in time
    - Spatio-temporal ordering
    - Large-scale wave?
  - Very turbulent system → large-scale order

The solar interior

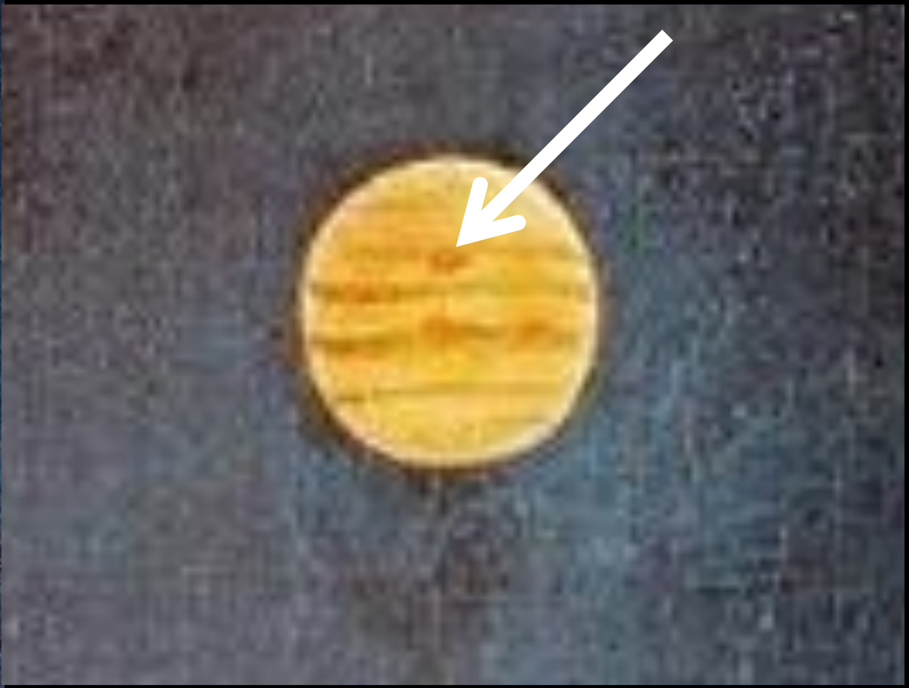


Schou et al (1998)

Solar interior:

- “Large-scale” differential rotation emerges in very turbulent environment
- Equator rotates faster than pole –large-scale jet stream
- Actually can be different for different stars depending on rotation rate “Anti-solar” differential rotn…

Motivation 2:
Jupiter (and other gas giants)



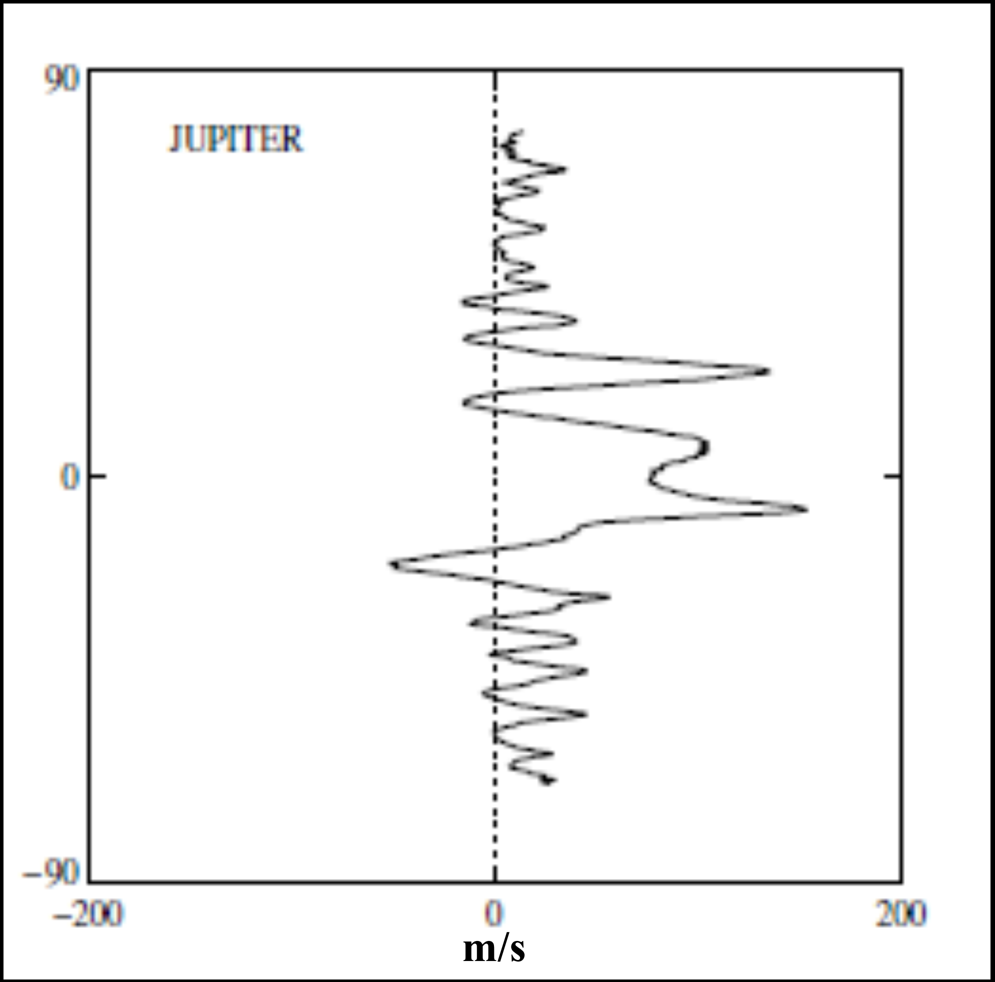
1711

Donato Creti

Jupiter Jets

Movie Courtesy NASA

Jupiter



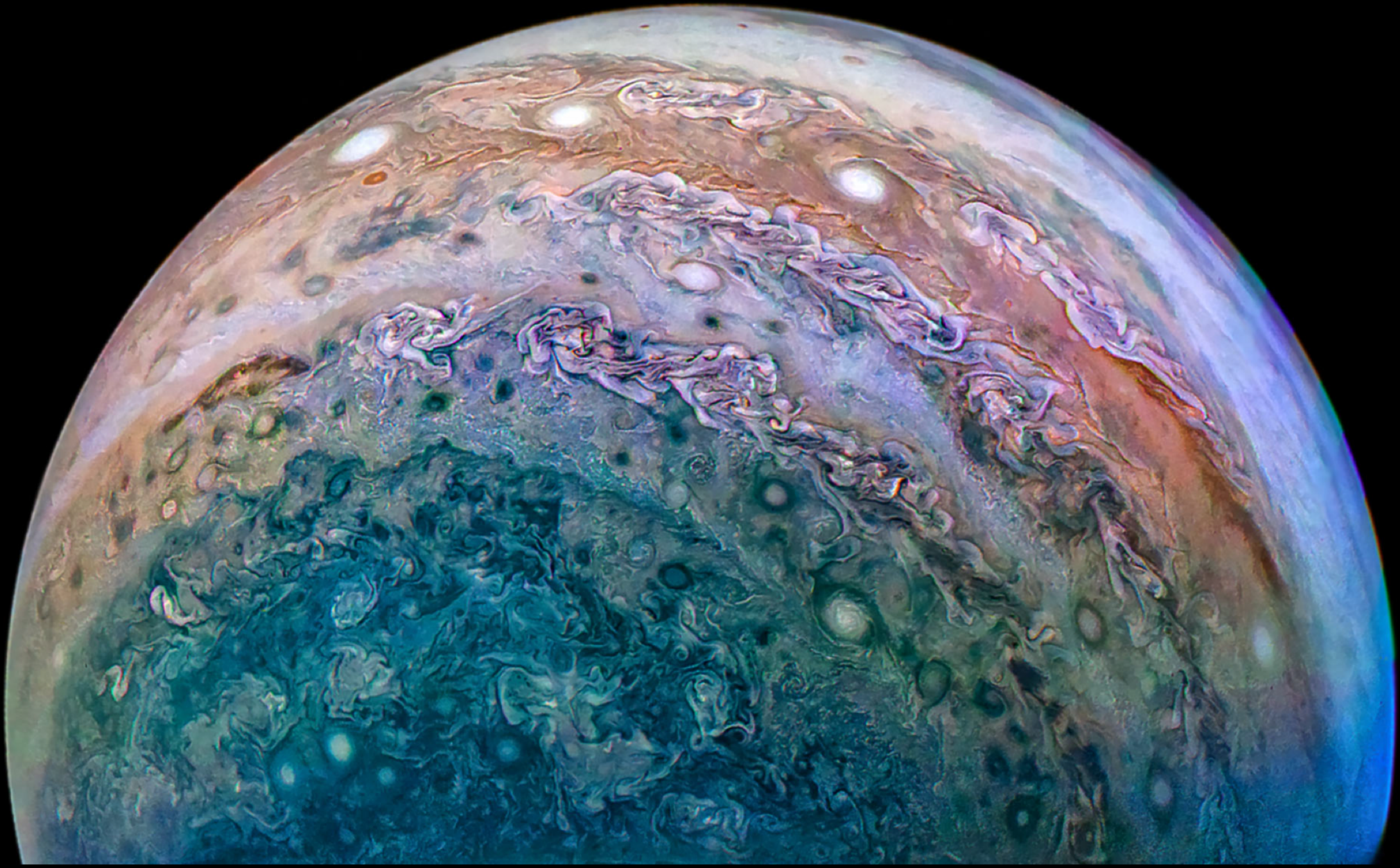
Systematic Behaviour from Turbulence: phase transition?

- equatorial jets:
  - . . . strongest
  - . . . super-rotating
- jet spacing increases with jet strength
- jets are stationary/very stable
- Two competing theories (Juno)

See later for model problem

Juno Mission

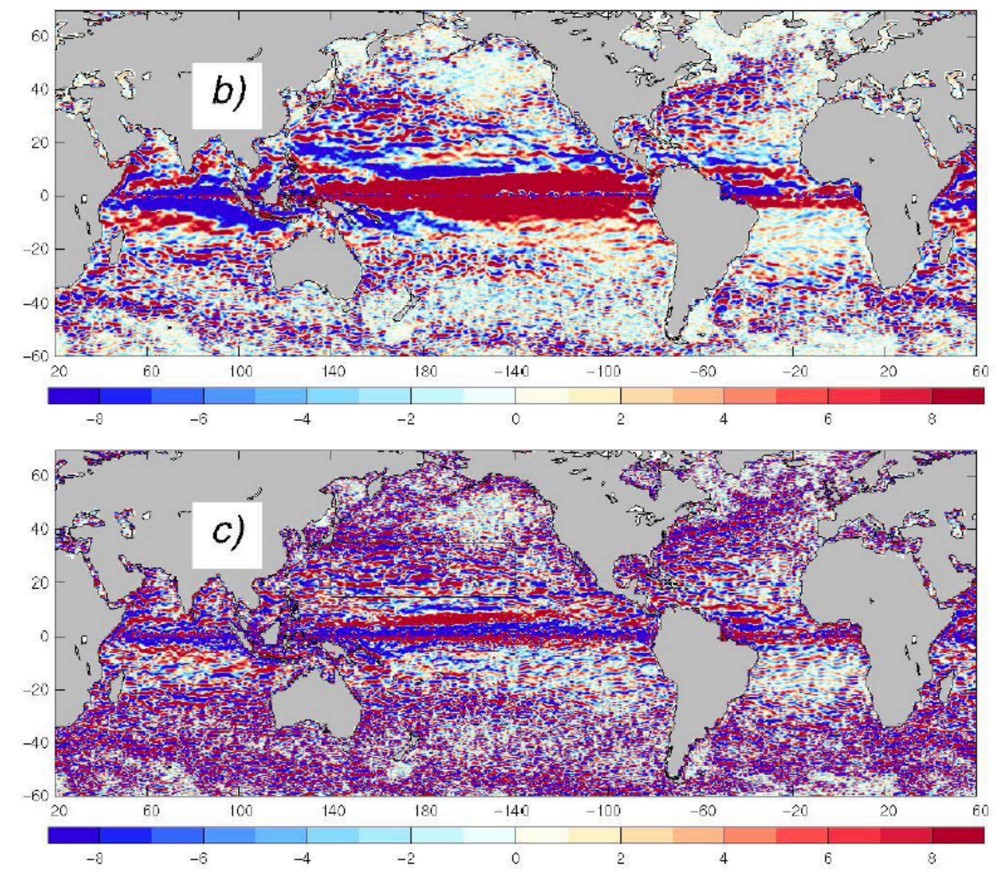
Juno Mission



Motivation 3:
Some jets closer to home…

Oceans...

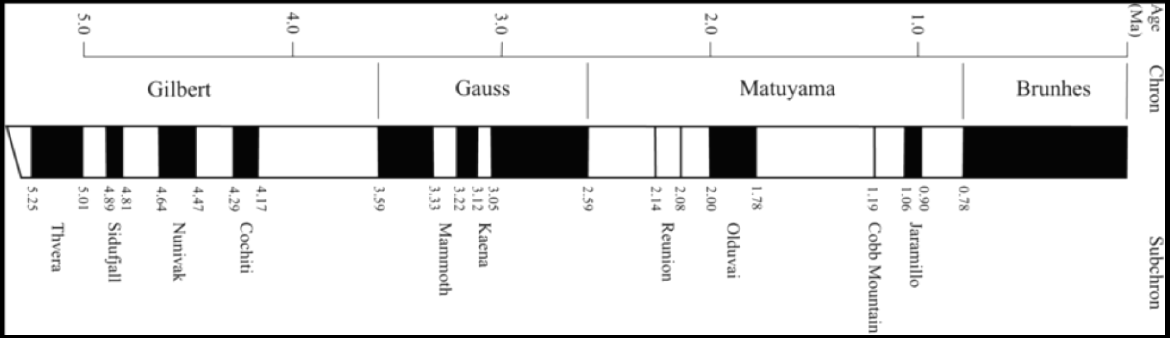
Maximenko et al. (2005)

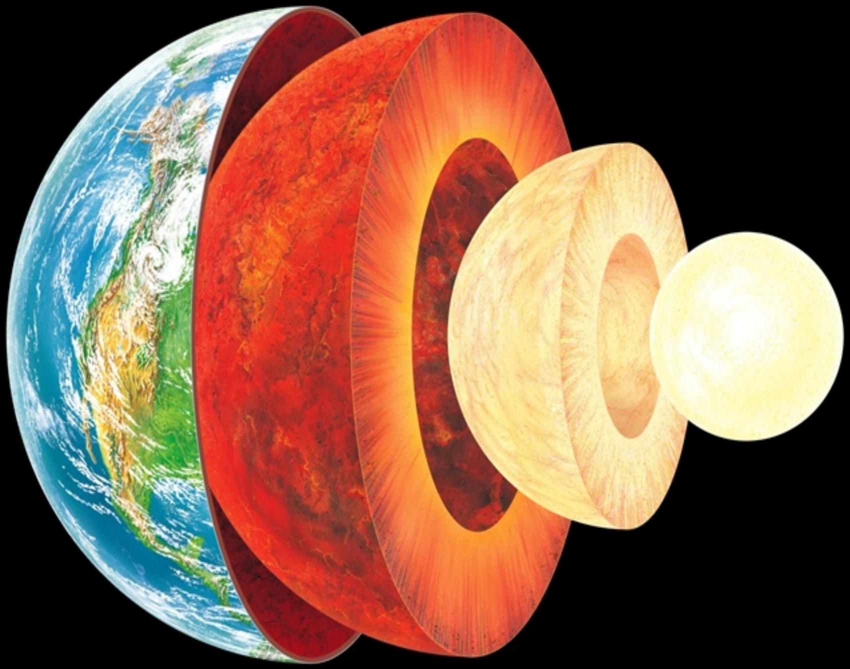


Jets time dependent
Emerge after averaging
(low Zonostrophy parameter)

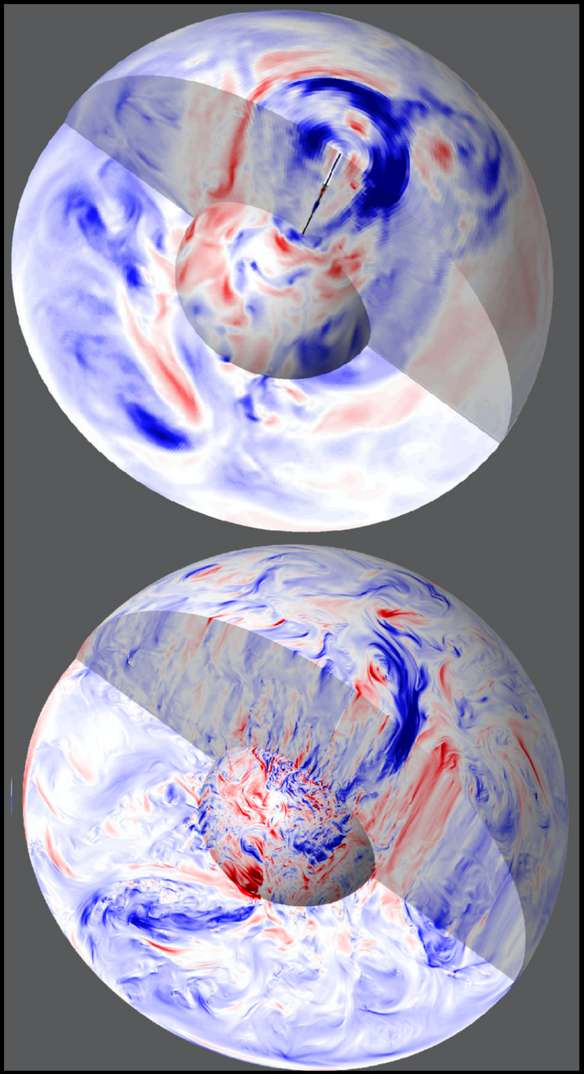
Formation of zonal flows is area of historic and current research
(e.g. Rhines, McIntyre, Vallis & Maltrud, Manfroi & Young, Sukoriansky & Galperin, Scott & Dritschel)

Earth’s interior



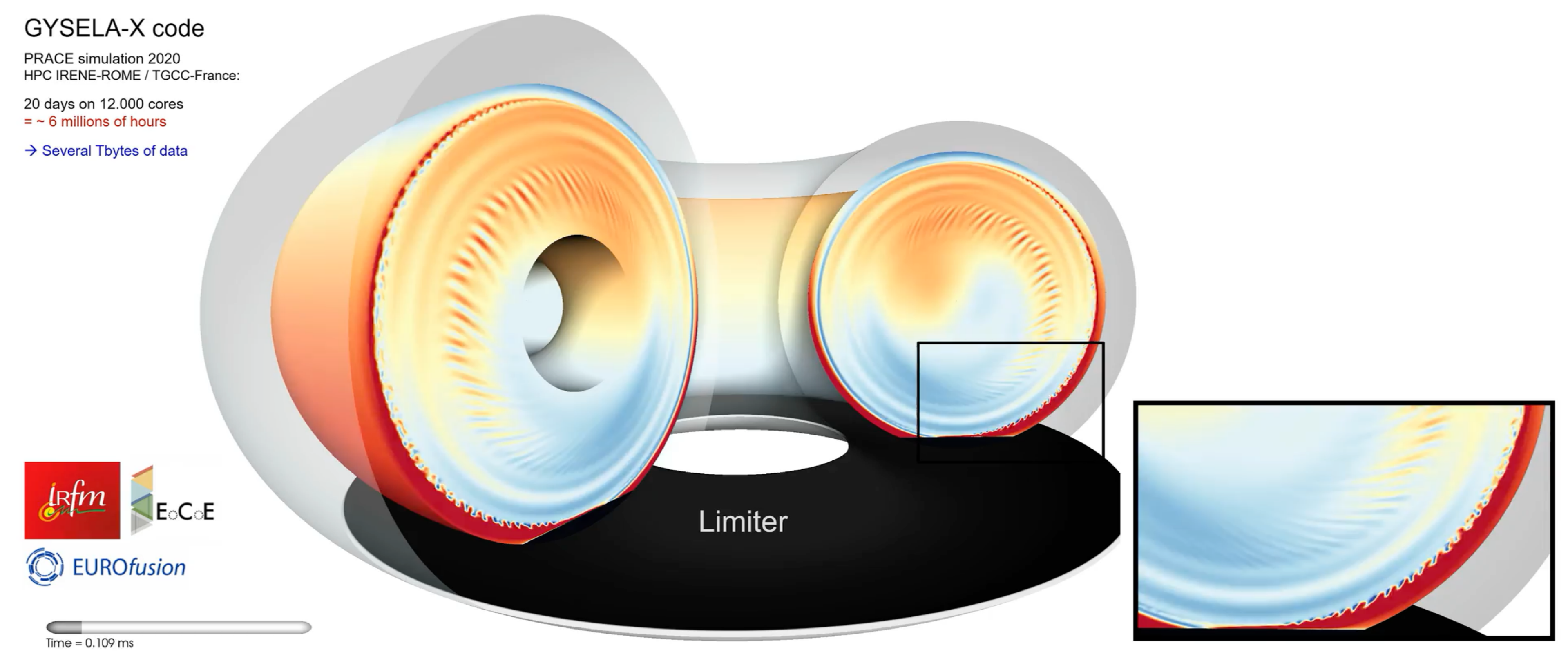


Courtesy J Aubert



Geodynamo: (massively) subcritical transition
(cf Nigel’s talk for transition)

Fusion flows: Electrostatic turbulence

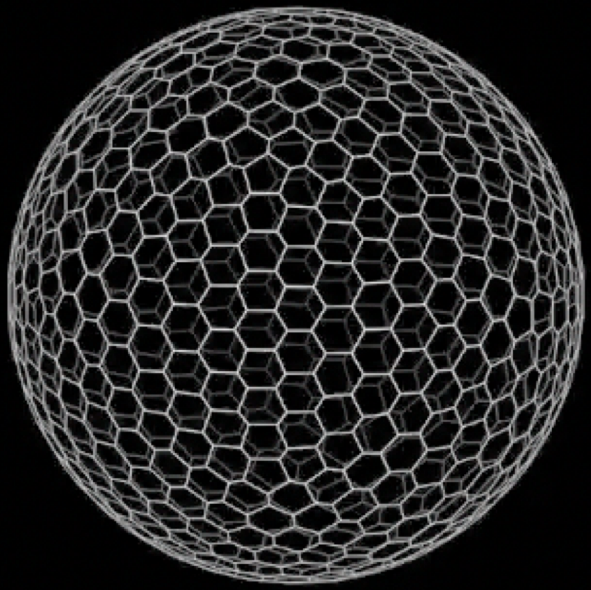


Flow inside a Tokamak
5D Gyrokinetic (3 space, 2 velocity) Semi-Lagrangian simulation
courtesy Guilhem Dif-Pradalier (IRFM (CEA), Marseille)

“Zonal flows in plasma – a review “
P H Diamond, S-I Itoh, K Itoh and T S Hahm
Plasma Physics and Controlled Fusion, Volume 47, Number 5

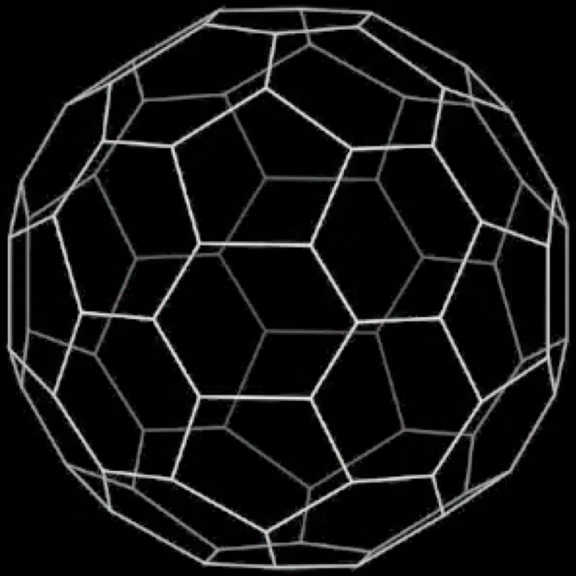
Chaos from Order in Turbulence

Plummer et al, 2018



- **Energy input via mean flows at large scales**
    - small scales emerge owing to instability of large scales.
    - these act back to modify large scales flows
- **KH instability**
- **Taylor-Couette**
- **Magnetorotational Instability**
- **Pipe Flow**
- **Rotating Couette Flow**

Order from Chaos in Turbulence



Allawala et al, JFM, 2020

- Energy input at small/moderate scales
  - large scales/mean flows emerge owing to correlations in turbulence (rotn/strat)
- Convective driving of mean flows in planets/stars
- Driving of zonal flows in plasma devices
- Driving of jets on giant planets
- Large-scale Dynamos

Why not just bung everything on a computer?

- Range of scales for astrophysical objects is huge
  - Large number of degrees of freedom
  - Non-dimensional parameters are in an extreme parameter regime .
- Even heroic calculations are the equivalent of climate models – modelling the largest scales (cf Baylor's talk)

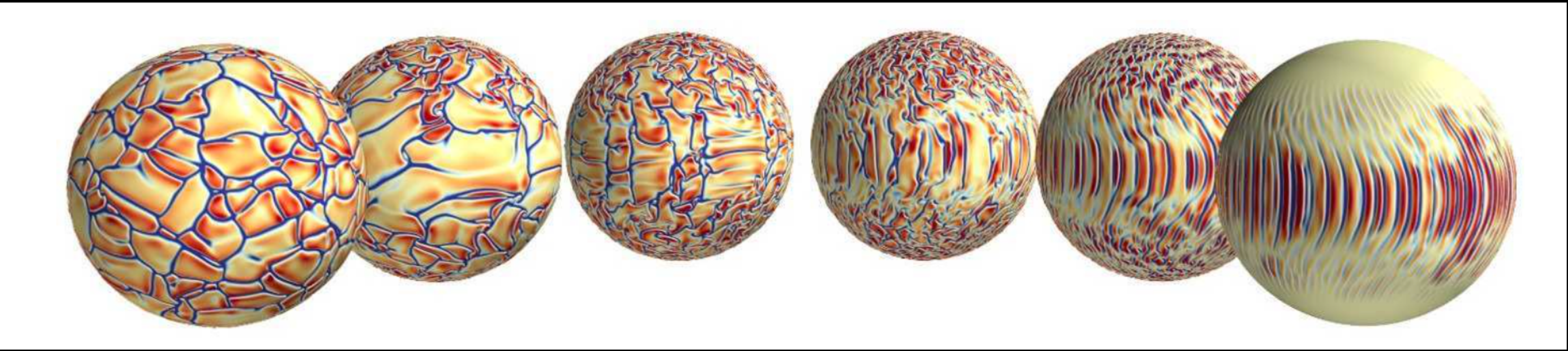
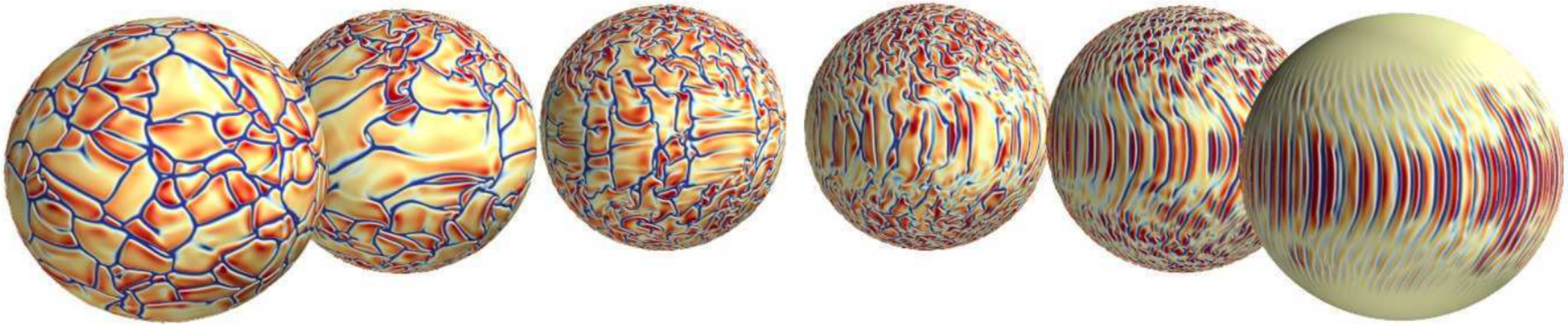


Image credit: Nick Featherstone

Why not just bung everything on a computer?



- Even if the computational resources were available the power required to simulate a star like the Sun is 10^{22} W.
  - This is equivalent to the luminosity of a M9V main sequence red dwarf.
- Important to bring mathematical models/understanding to bear on this problem
  - Reduced Models
  - Statistical Models
  - Statistical Models
  - Insight from data?

What’s the problem? We know the equations….

For example dynamics in the solar interior is governed by the following equations of MHD

INDUCTION

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \qquad (\nabla . \mathbf{B} = 0),$$

MOMENTUM

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla \mathbf{u}\right) = -\nabla p + \mathbf{j} \times \mathbf{B} + \rho \mathbf{g} + \mathbf{F}_{viscous} + \mathbf{F}_{other},$$

CONTINUITY

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{u}) = 0,$$

ENERGY

$$\frac{D(p\rho^{-\gamma})}{Dt} = \text{loss terms},$$

GAS LAW

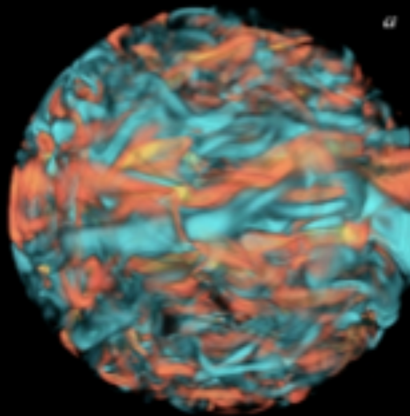
$$p = R\rho T.$$

What's the problem...?

	BASE OF CZ	**PHOTOSPHERE**
$Ra \equiv g\Delta\nabla d^4 / \nu\chi H_P$	10^{20}	10^{16}
$\mathrm{Re} \equiv UL / \nu$	10^{13}	10^{12}
$Rm \equiv UL / \eta$	10^{10}	10^{6}
$\mathrm{Pr} = \nu / \chi$	10^{-7}	10^{-7}
$\beta = 2\mu_0 p / B^2$	10^{5}	1
$Pm = \nu / \eta$	10^{-3}	10^{-6}
$M = U / c_s$	10^{-4}	1
$Ro = U / 2\Omega L$	0.1-1	10^{-3}-0.4

Ossendrijver (2003)

Modelling Strategies I



- **Direct Numerical Simulation (DNS)**
  - Discretise equations
  - Capture a range of spatial scales
    - *invoke eddy diffusivity (LES)*
    - *accept in **completely wrong** parameter regime*
    - *adopt a closure of small scales (subgrid modelling)*
  - Very good at describing the dynamics of the system.
  - Very expensive to calculate statistics.

Modelling Strategies II

- Asymptotic Theories (cf Keith Julien's talk)
  - Make use of small parameters in the problem to get reduced equation sets
  - For example in the Earth the Rossby number is small and we can expand there
  - Often these reduced equation sets need us to think about clever methods to solve them
  - The asymptotic expansion via a small parameter sometimes leads to QL-dynamics (certain nonlinearities become higher order)

Modelling Strategies III

- **Direct Statistical Simulation**
  - **Use our computer power to solve for the statistics of the flow instead of the dynamics**
  - **Low-order statistics are smoother in space than the instantaneous flow.**
  - **Statistics evolve slowly in time, or not at all, and hence may be described by a fixed point, or at least a slow manifold.**

Use ideas/techniques from non-equilibrium statistical mechanics

Modelling Strategies IV

- **Machine Learning**
  - **Make use of data (constrained by physics) to**
    - **Get reduced models of the very complicated interactions for these problems**
    - **Parameterise the effects of the small scales**
    - **Constrain the dependences in equation sets**
    - **Prediction of future dynamics or statistics (including transitions)**

- **Fourier Neural Operators**
- **Reservoir computing**
- **Sparse Identification of Nonlinear Dynamics (SINDy)**