Lecture II:
Liquid Crystal Elastomers

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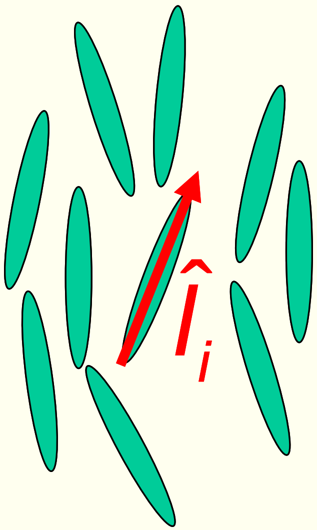
Outline

- review
- elastomers
- liquid crystal elastomers & gels
- photoactuation, energy generation
- modeling & simulations
- summary

What to remember:

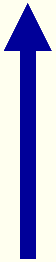
- order parameter



$$\mathbf{Q} = \left\langle \frac{1}{2}(3\hat{\mathbf{l}}\hat{\mathbf{l}} - \mathbf{I}) \right\rangle$$

$$\mathbf{Q} = S\frac{1}{2}(3\hat{\mathbf{n}}\hat{\mathbf{n}} - \mathbf{I})$$

$\hat{\mathbf{n}}$



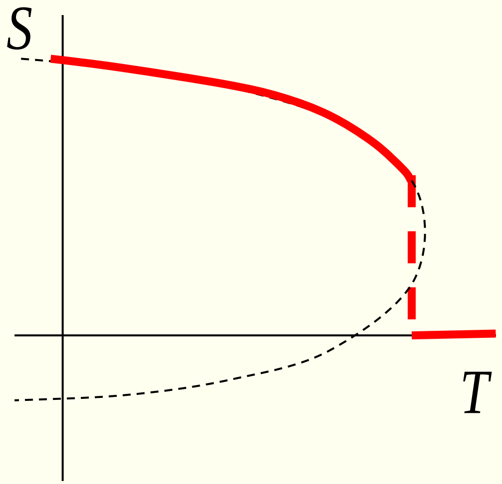
$$-\frac{1}{2} \le S \le 1$$

What to remember:

- free energy

$$\mathcal{F}_L = \frac{1}{2}A_o(\frac{T}{T_c}-1)\mathbf{Q}^2 - \frac{1}{3}B\mathbf{Q}^3 + \frac{1}{4}C\mathbf{Q}^4 + .. - \frac{1}{2}\Delta \qquad \textbf{QEE}$$

$$\mathcal{F}_L = \frac{1}{2}a_o(\frac{T}{T_c}-1)S^2 - \frac{1}{3}bS^3 + \frac{1}{4}cS^4 + .. - \frac{1}{2}\Delta\varepsilon(\hat{\mathbf{n}}\cdot\mathbf{E})^2$$



$$T_c \simeq \frac{\rho U}{5k} \qquad B \simeq \frac{kT_{NI}}{l_{mol}^{\;3}}$$

Liquid Crystal Elastomers

P.G. de Gennes, 1932-2007

- ANISOTROPIC RUBBERS
  with combined features of

LIQUID CRYSTALS
&
ELASTIC SOLIDS

H. Finkelmann

proposed by de Gennes

produced by Finkelmann

P.G. de Gennes, *C.R. Seances Acad.Sci.* **218**, 725 (1975)

H. Finkelmann, H.J. Kock, G. Rehage, *Makromol. Chem.*, Rapid Commun. **2**, 317 (1981)

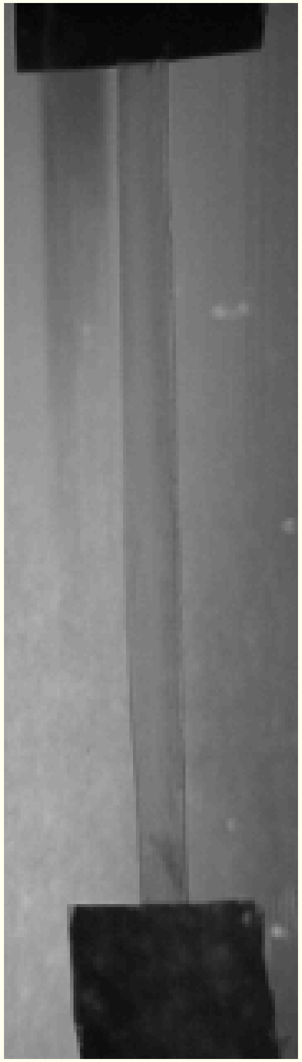
Why liquid crystals & elastic solids?

- both are soft
- liquid crystals:
  - respond to variety of stimuli
  - liquid (cannot support stress)

-complementary!

- elastic solids:
  - solid (supports stress)
  - respond to stress only

Liquid Crystal Elastomers: Behavior



- monodomain nematic LCE
- 5cm x 5mm x 0.3mm
- lifts 30g wt. on heating, lowers it on cooling
- large strain! (400%)

(H. Finkelmann)

Elastic solids:

- position of a material point before deformation: $\mathbf{r}$
- position of a material point after deformation: $\mathbf{r} + \mathbf{R}$
- displacement vector: $\mathbf{R}$
- strain tensor:

$$e_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial R_\alpha}{\partial x_\beta} + \frac{\partial R_\beta}{\partial x_\alpha} \right)$$

Elastic solids:

- for incompressible materials, the free energy density is

$$\mathfrak{F} = \frac{1}{2}\eta \mathbf{e}^2$$

- where η is Young's modulus

$$\eta \simeq \frac{3}{2}\rho_c kT$$

- and ρ_c is the cross-link density*.

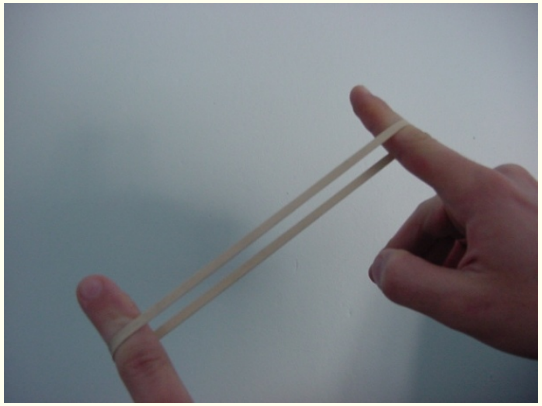
Elastic solids:

- for incompressible materials, the free energy density is

$$\mathcal{F} = \frac{1}{2}\eta \mathbf{e}^2 - \boldsymbol{\sigma}^{ext}\mathbf{e}$$

- where η is Young's modulus, and $\boldsymbol{\sigma}^{ext}$ is the external stress.

$\eta \simeq 10^6\,Pa$



Theory of Elasticity, L.D. Landau and E.M. Lifshitz

Liquid crystal elastomer

- polymer network incorporating mesogens

- polymer: anisotropic random walk
  - step length tensor:

$$\mathbf{L} = (\frac{l_{\parallel} + 2l_{\perp}}{3})\mathbf{I} + \frac{3}{2}(l_{\parallel} - l_{\perp})\mathbf{Q}$$

- liquid crystal: aligning effect of polymer chain

Liquid Crystal Elastomers

- free energy density*:

$$\mathcal{F} = \frac{1}{2} A\mathbf{Q}^2 + .. - \frac{1}{2}\Delta\varepsilon'\mathbf{QEE} - \gamma\mathbf{Qe} + \frac{1}{2}\eta\mathbf{e}^2 - \boldsymbol{\sigma}^{ext}\mathbf{e}$$

- sum of free energies of liquid crystal + elastic solid, *plus* new coupling term $\mathbf{Qe}$

- γ proportional to ρ_c and to step length anisotropy

- effect of strain on LC order is same as external field
- effect of LC order on strain is same as external stress

*P.G. de Gennes, *C.R. Seances Acad.Sci.* **218**, 725 (1975)

What happens:

$$\mathcal{F} = \frac{1}{2} A\mathbf{Q}^2 + .. - \frac{1}{2}\Delta\varepsilon'\mathbf{QEE} - \gamma\mathbf{Qe} + \frac{1}{2}\eta\mathbf{e}^2 - \boldsymbol{\sigma}^{ext}\mathbf{e}$$

- changing $\mathbf{Q}$ applies stress, causing shape change
  - heating causes contraction along director

- Note: $\gamma / A \approx \rho_m / \rho_c$

- mechanical strain has same effect as <u>strong</u> $\mathbf{E}$ -field*
  director $\hat{\mathbf{n}}$ reorients, S changes .

Liquid Crystal Elastomers

- key feature: coupling between orientational order and mechanical strain

order parameter tensor

$$f = f(Q_{\alpha\beta}, e_{\alpha\beta})$$

strain tensor

Free energy: another look

$$\mathcal{F} = \frac{1}{2}a\mathbf{Q}^2 - \frac{1}{3}b\mathbf{Q}^3 + \frac{1}{4}c\mathbf{Q}^4 \underbrace{- \gamma\mathbf{Q}\mathbf{e} + \frac{1}{2}\eta\mathbf{e}^2} - \boldsymbol{\sigma}^{ext}\mathbf{e} - U'\mathbf{Q}_o\mathbf{e}\ldots$$

- can also write

$$\mathcal{F} = \frac{1}{2}a'\mathbf{Q}^2 - \frac{1}{3}b\mathbf{Q}^3 + \frac{1}{4}c\mathbf{Q}^4 + \frac{1}{2}\eta(\mathbf{e} - \frac{\gamma}{\eta}\mathbf{Q})^2 + ..$$

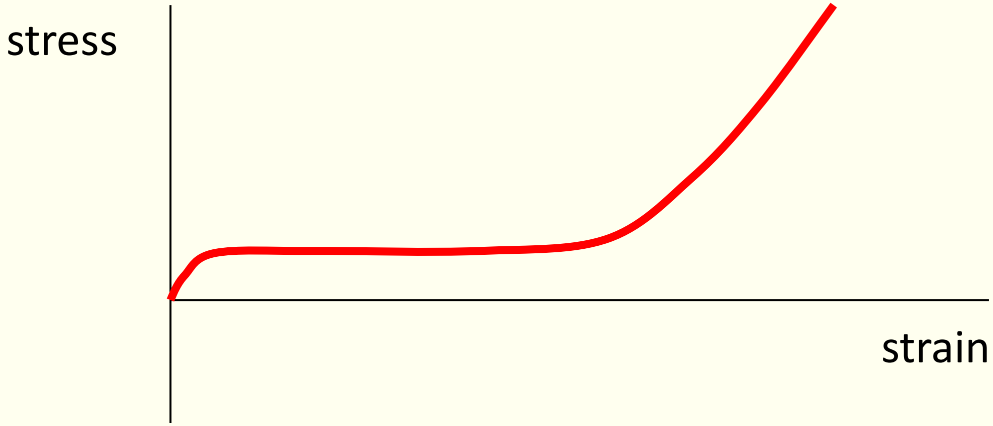
SOFT ELASTICITY!

Soft Elasticity

- if S is constant,

$$\mathcal{F} = \frac{1}{2}\eta(\mathbf{e} - \frac{\gamma}{\eta} S \frac{1}{2}(3\hat{\mathbf{n}}\hat{\mathbf{n}} - \mathbf{I}))^2$$

- changes in strain can be accommodated by director reorientation without energy cost

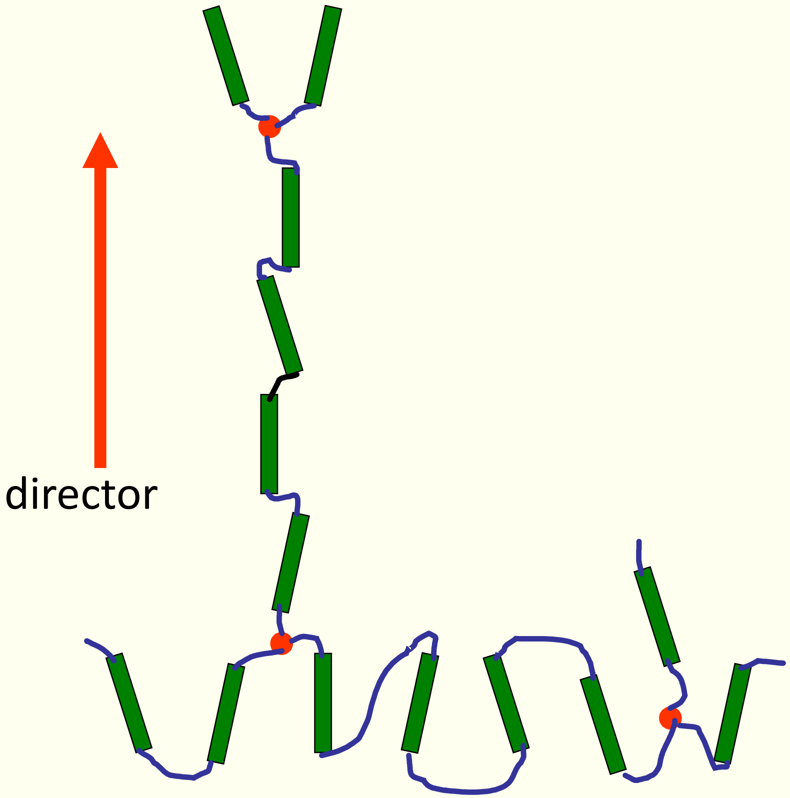


vibration damping

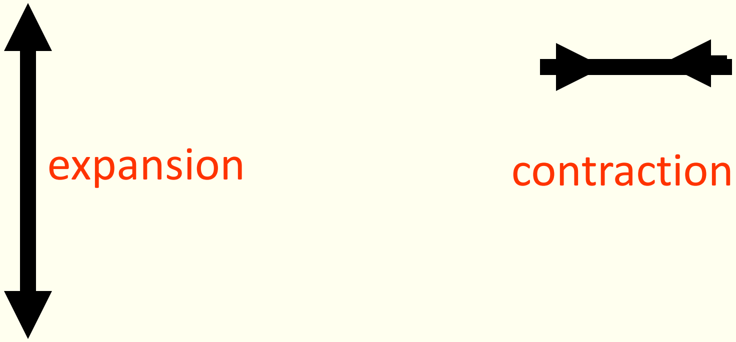
L. Golubovic and T.C. Lubensky, *Phys. Rev. Lett.* **63**, 1082 (1989)

LCE Samples

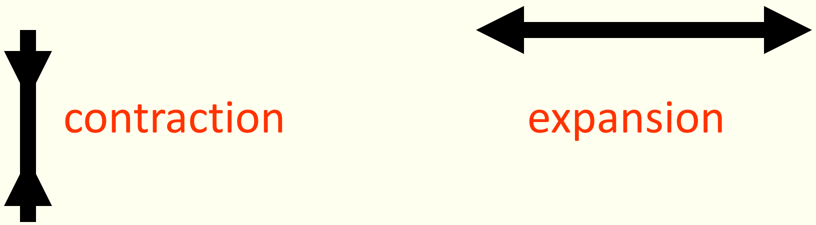
Structure



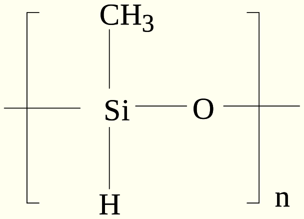
If orientational order increases,



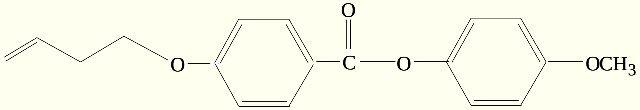
If orientational order decreases,



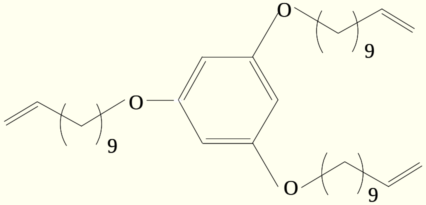
Composition of nematic LCE samples



- methylsiloxane monomer (main chain)



- mesogenic biphenyl (side group)

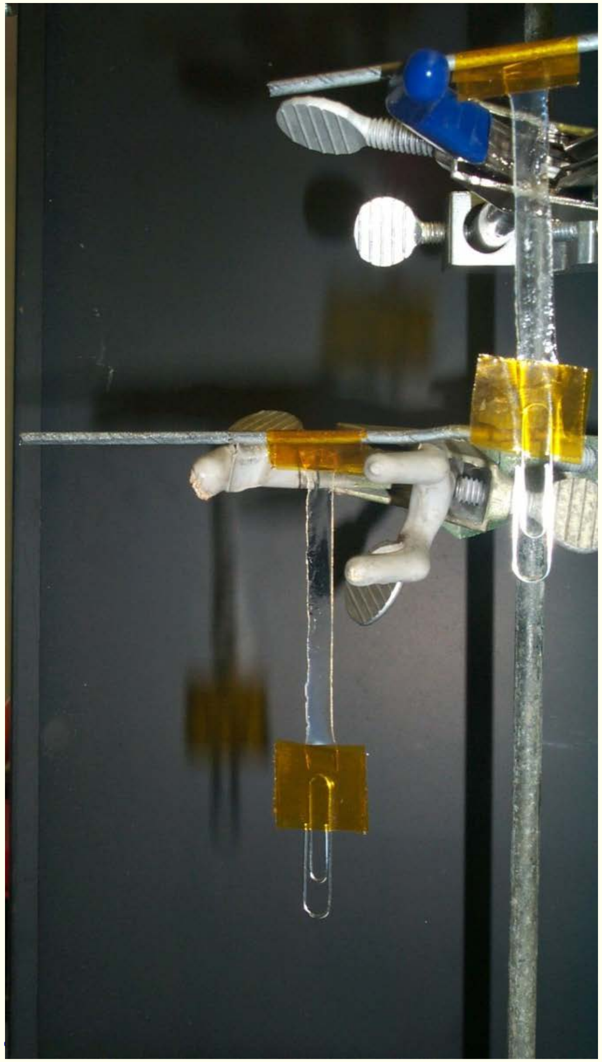


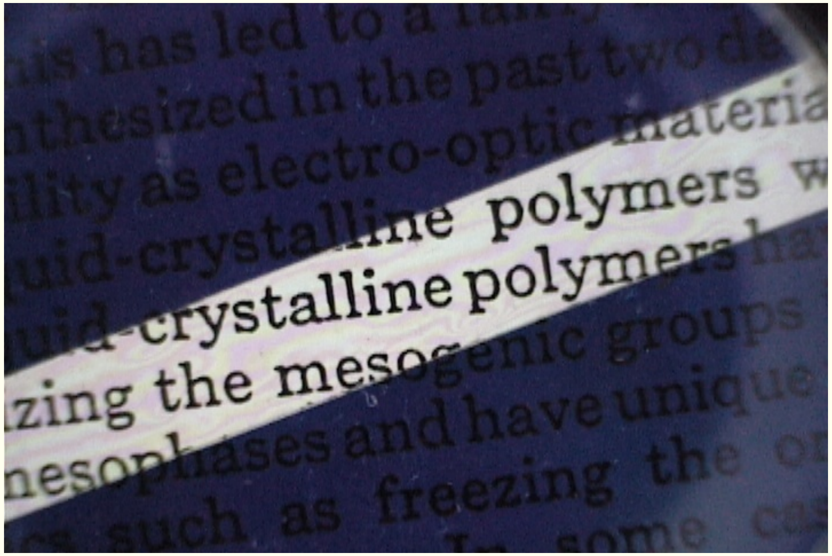
- trifunctional crosslinker

Sample preparation

- mix components in solvent
- centrifuge & evaporate solvent
- take out sample (partially polymerized)
- as solvent evaporates, polydomain nematic forms
- strain slightly to make sample monodomain

Appearance of nematic LCE samples





birefringent sample between crossed polarizers

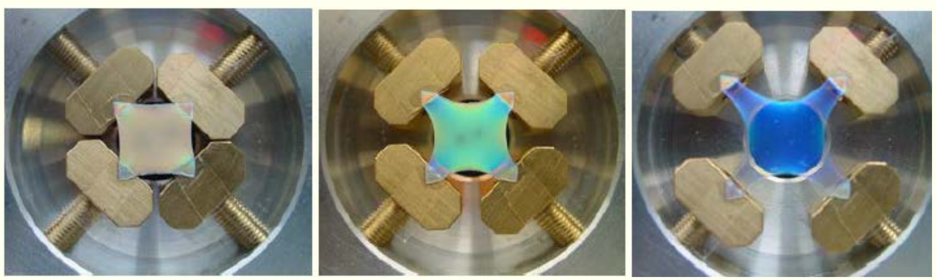
samples from:

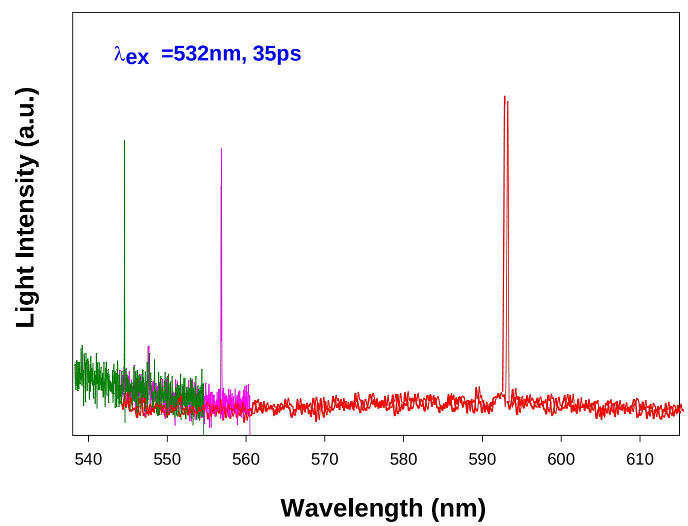
- H. Finkelmann, Freiburg
- now produced at the LCI

Strain changes optical properties

- changing sample shape changes Q and
  - magnetic permeability, dielectric permittivity
  - <u>optical properties</u>: cholesteric pitch

cholesteric elastomer





-mechanically tunable PBG material

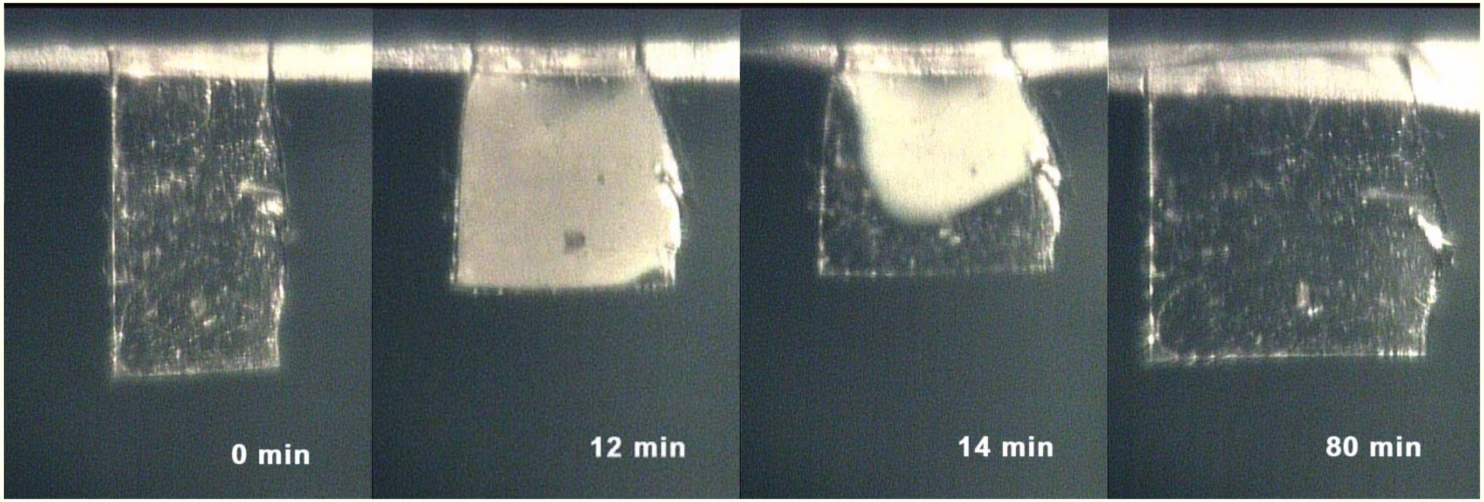
-mirrorless 'rubber' laser

H. Finkelmann, S-T. Kim, A. Munoz, P. Palffy-Muhoray and B. Taheri, *Adv. Mat.* **13**, 1069 (2001)

Effects of solvent vapors

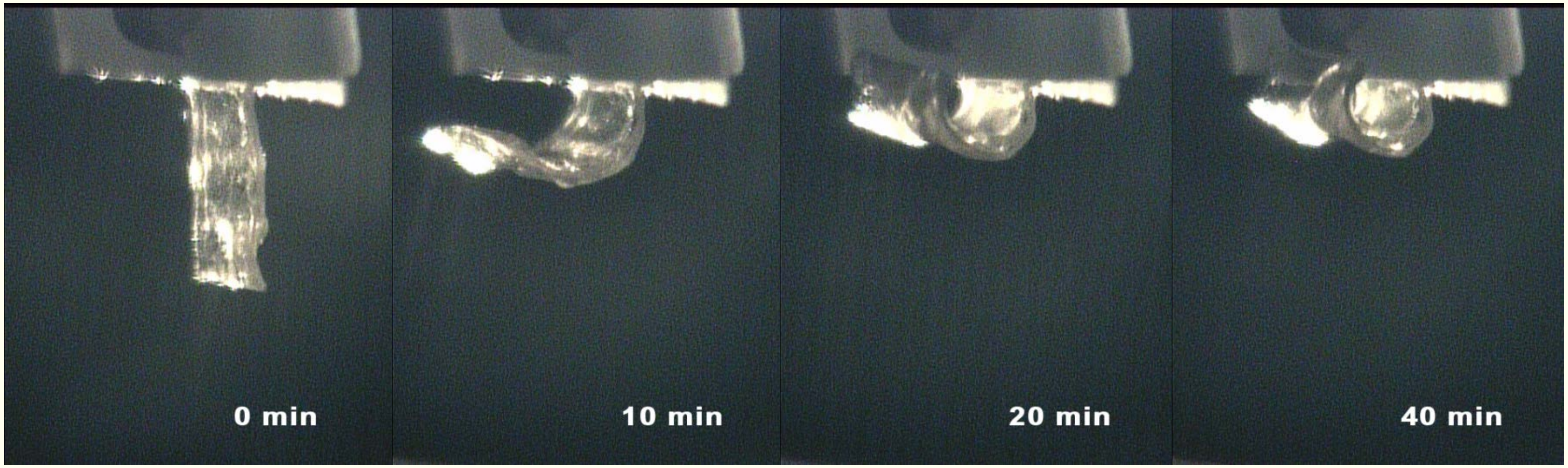
- siloxane based side chain nematic LCE
  - nematic single crystals
  - solvent: chloroform

- effect: vapor reduces mesogen density: $T_c = \frac{\rho U}{5k}$



Effects of solvent vapors

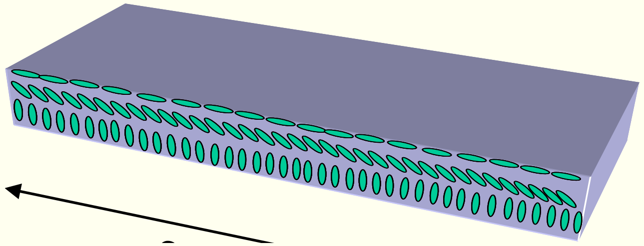
- Sidechain nematic + RTV silicone ‘bi-rubber’
  - RTV film cast on LCE
  - films have equal thickness
  - strong tendency to bend when exposed to solvent vapor
  - solvent: chloroform



T. Toth-Katona, P. Luchette, P. Palffy-Muhoray (unpublished)

Effects of solvent vapors

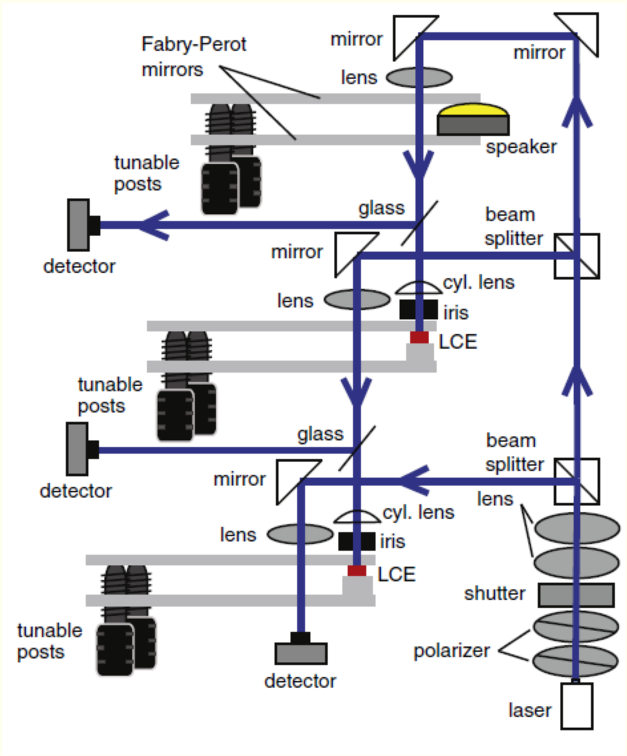
- hybrid aligned sample

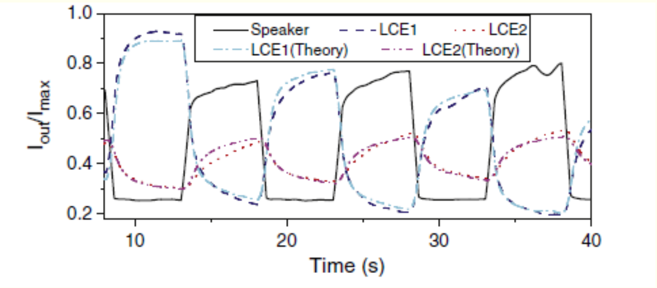


K. Harris, C. Bastiaansen, D. Broer, *J. MEM Syst.*, **16**, 480 (2007)

Light detection with LCEs

- order parameter depends on T
- ΔT causes strain
  - LCE is photomechanical transducer sensing light
  - strain in LCE tunes Fabry-Perot cavity
  - can be cascaded to increase sensitivity





N. Dawson, M. Kuzyk, J. Neal, P. Luchette, P. P-M., *Opt. Commun.* **284,** 991-993 (2011)

Liquid Crystal Gels

LC Gels

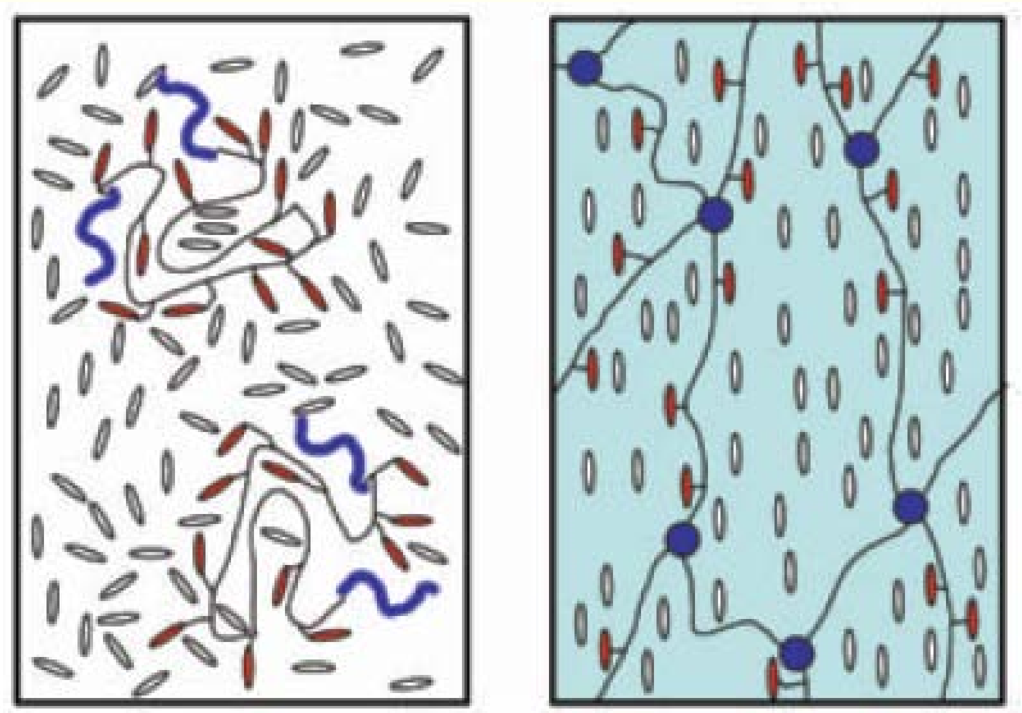
- here: gel = elastomer + solvent
- scenarios for LC gels:
  - LC elastomer + isotropic solvent
  - isotropic elastomer + LC solvent
  - LC elastomer + LC solvent

LC gels

- Kornfield group (Caltech)
  - triblock copolymer
    - long midblock with mesogenic sidechains
    - short polystyrene endblocks
  - dissolve in nematic LC (5CB)
  - in isotropic phase, have isotropic solution
  - in nematic phase, endblocks become insoluble, aggregate, forming physical crosslinks : <u>physical gel</u>

LC Gels

isotropic

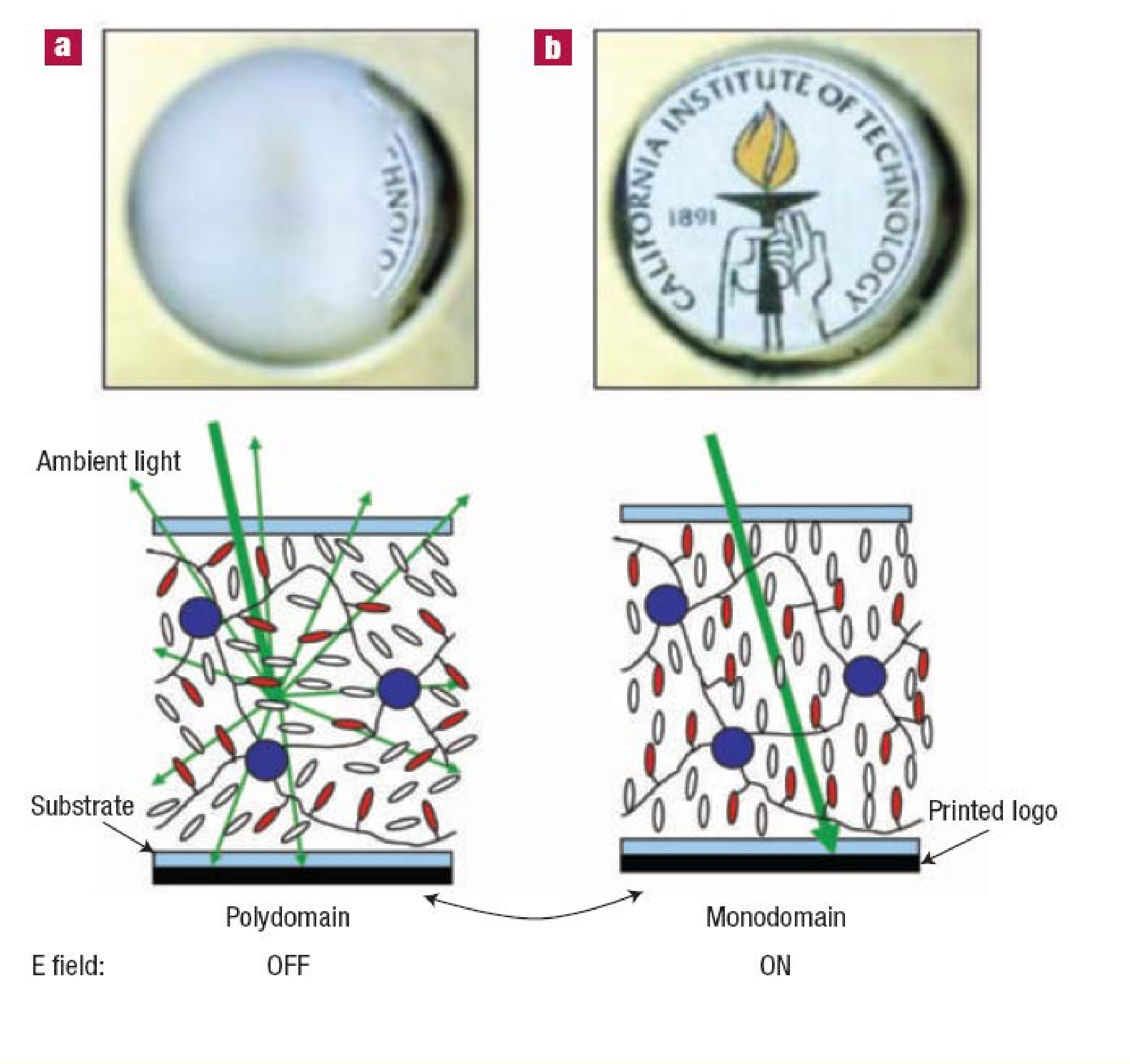


nematic

M.D. Kempe, N.R. Scruggs, R. Verduzco, J. Lal and J.A. Kornfield, “Self-assembled liquid-crystalline gels designed from the bottom up”, *Nature Mat.* **3**, 177-182 (2004)

P. Palffy-Muhoray, R.B. Meyer, “Liquid Crystal Gels: Bridging the experiment-theory gap”, *Nature Mat.*, **3**, 139-140 (2004).

LC Gels: electro –optic effect



LCE Photoactuators

Mechanisms of Optomechanical Effects in LCE

- optomechanical coupling:

light

↓

orientational order

↓

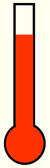
mechanical strain

↓

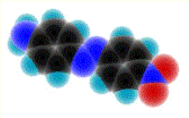
shape change

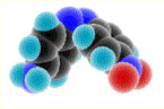
Effects of light on order parameter

- optical field changes order parameter via:
  - direct heating
    - absorption

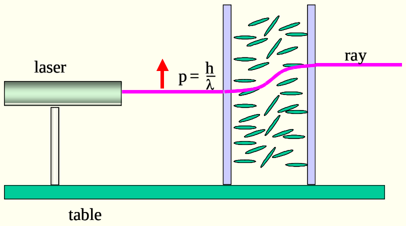


  - disruption of order
    - photoisomerization

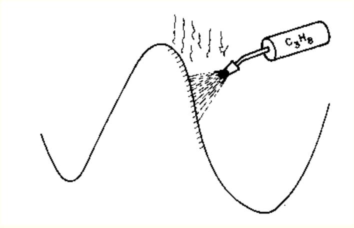




  - direct optical torque
    - angular momentum transfer from light



  - indirect optical torque
    - Landauer’s blowtorch
    - orientational Brownian ratchet
      - no angular momentum transfer from light;
      - light drives molecular motor



BOULDER SCHOOL

Experimental Results (Warner *et al.*)

- azo-dye incorporated in network

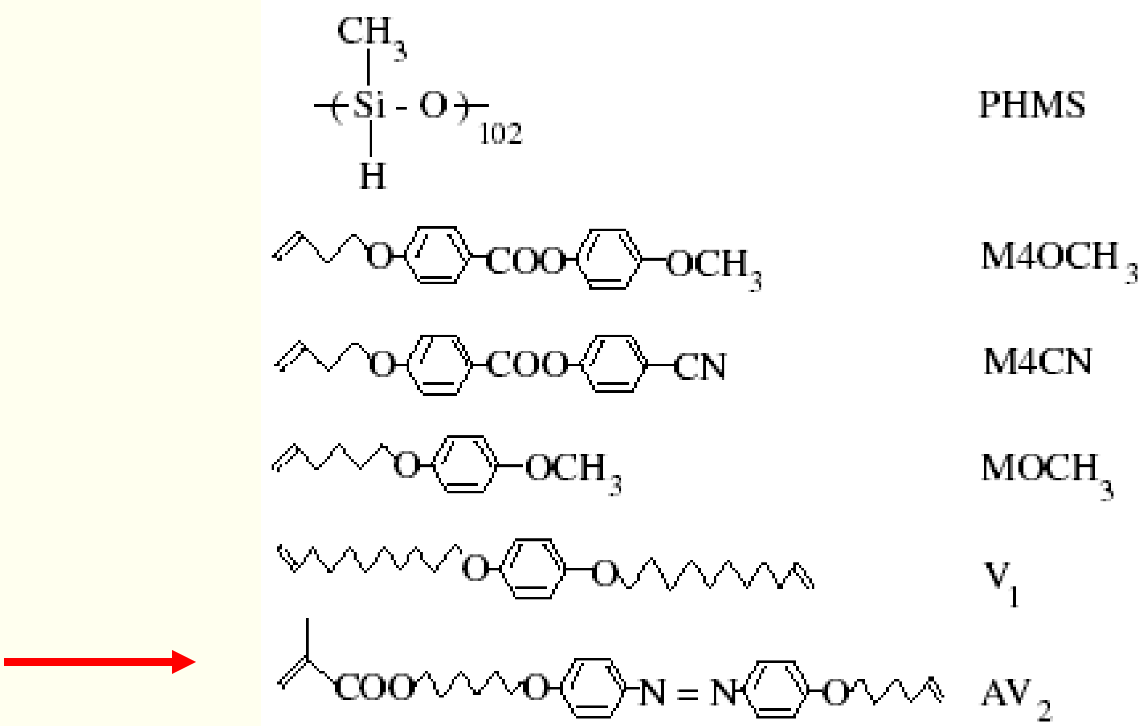


FIG. 1. Chemical structure of elastomer AE4.

H. Finkelmann, E. Nishikawa, G. G. Pereira and M. Warner, *Phys. Rev. Lett.* **87,** 015501 (2001)

Experimental Results (Warner *et al.*)

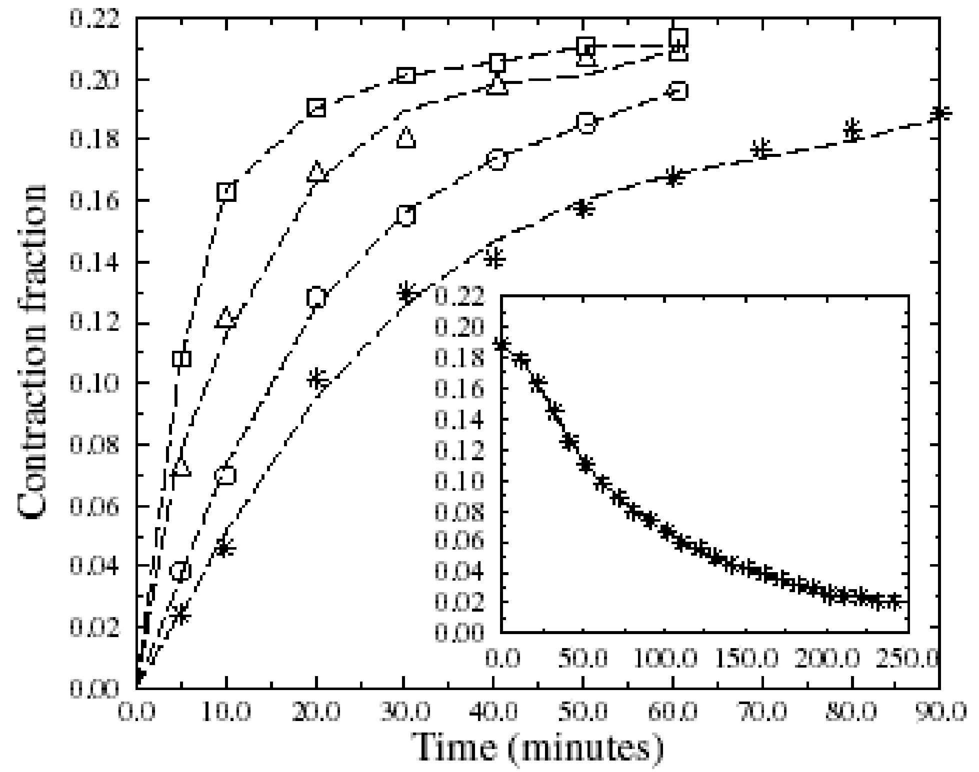
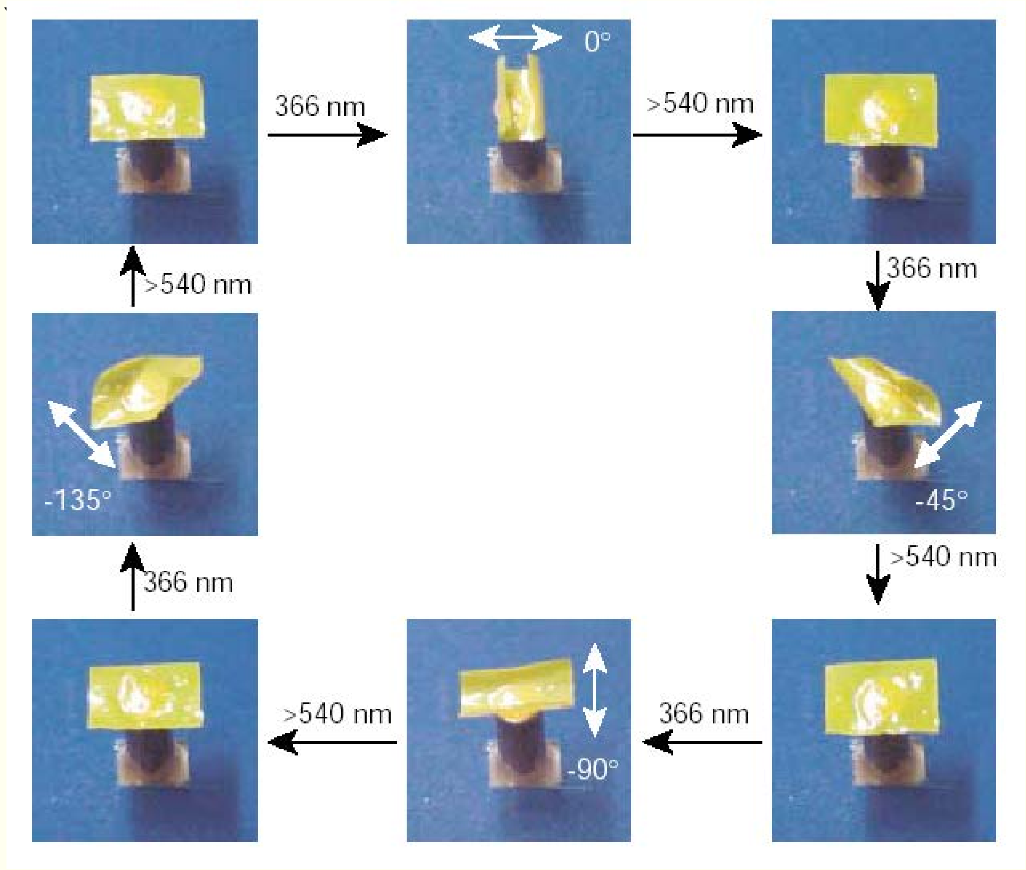


FIG. 3. Contraction fraction, $1 - e_{T_0}(t)$, versus time exposed to UV radiation for $T_0 = 298$ K ($\star$), 303 K ($\circ$), 308 K ($\triangle$), and 313 K ($\square$). Dashed lines are guides for the eye. Close to the origin curves may have an inflexion point. Inset: Recovery of the contraction for the 298 K elastomer after the 90 min of illumination.

Experimental Results (Ikeda et al.)



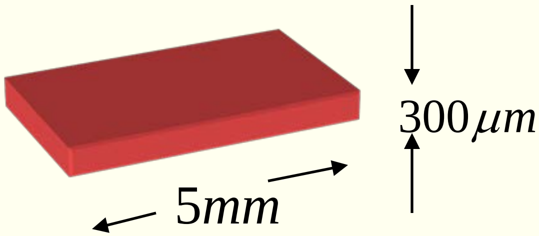
LC + diacrylate network
+ functionalized
azo-chromophore

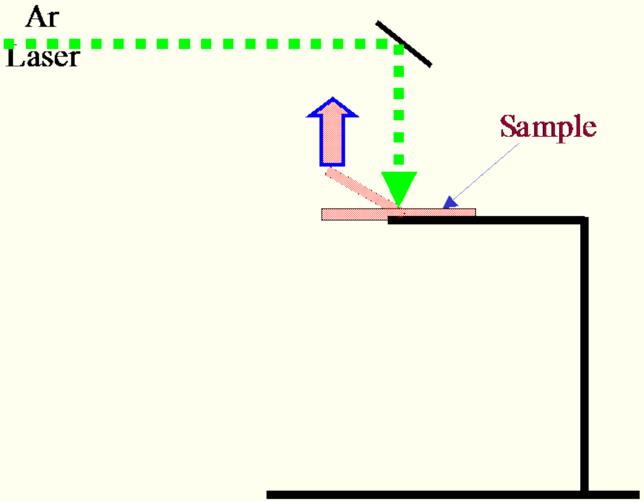
timescale: 10 s

Yanlei Yu,Makoto Nakano, Tomiki Ikeda, *Nature* **425**, 125 (2003)

Photoinduced Bending

sample: nematic elastomer EC4OCH3
+ 0.1% dissolved
Disperse Orange 1 azo dye

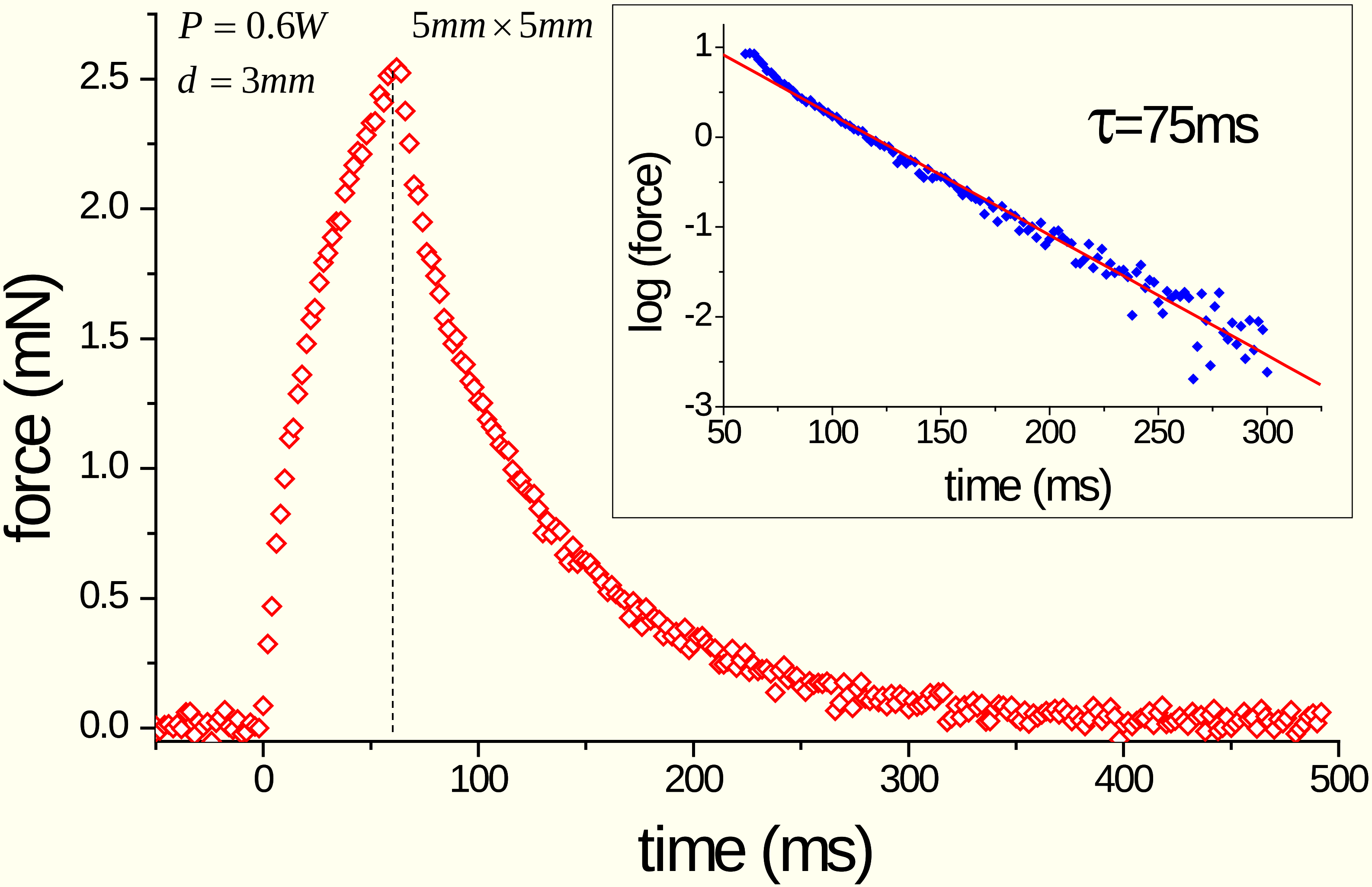




Response time: 70ms

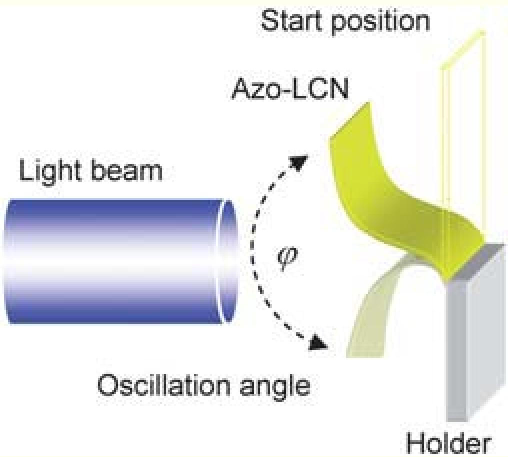
M. Chamacho-Lopez, H. Finkelmann, P. Palffy-Muhoray, M. Shelley, *Nature Mat.* **3**, 307, (2004)

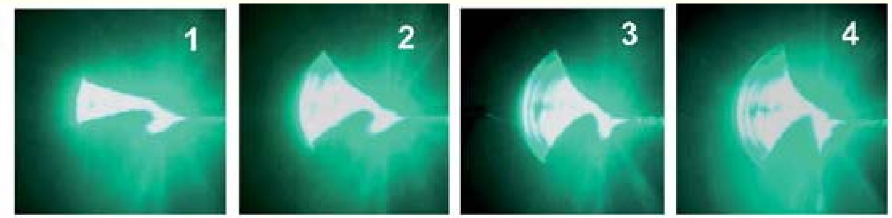
Dynamic Response

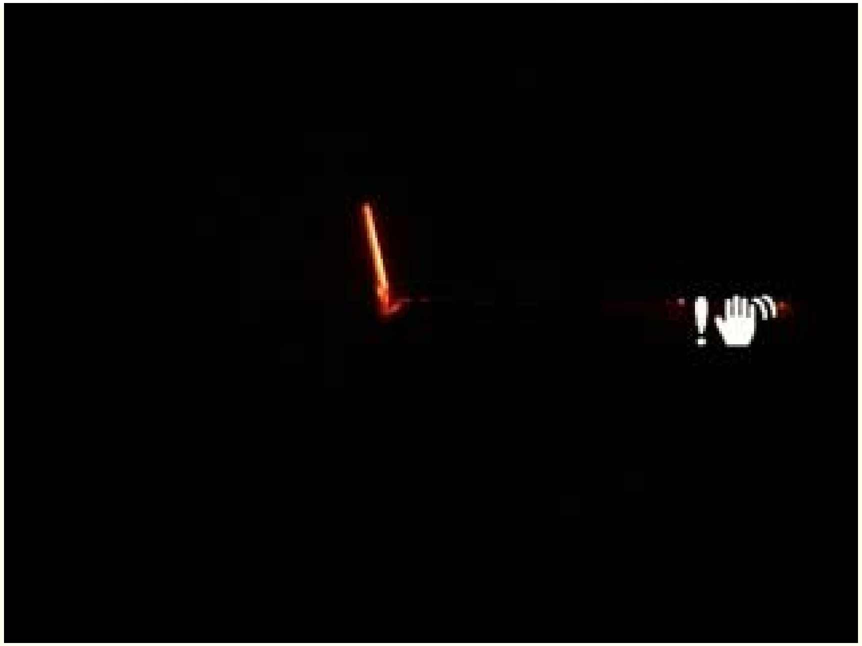


Photoinduced oscillations

- if sample bends $> 90^o$ both sides are illuminated, producing oscillations







nematic azo-elastomer

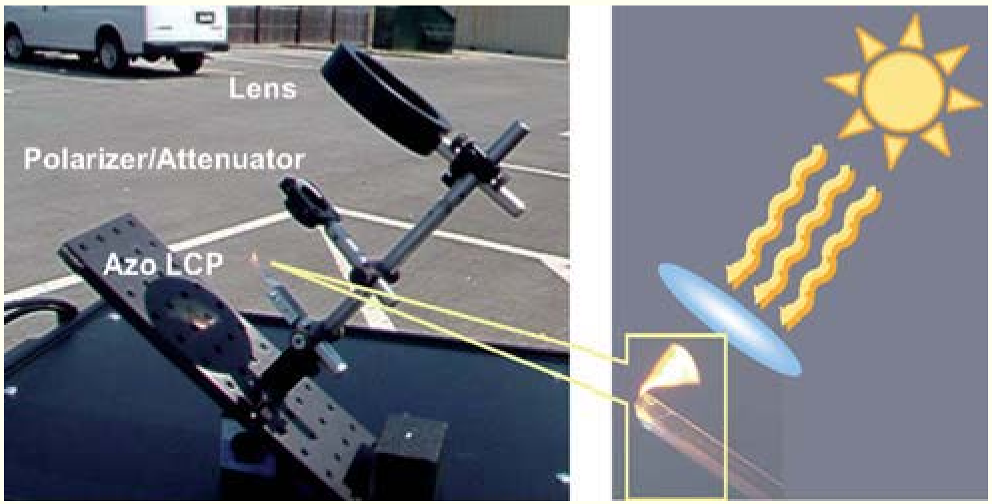
sample size: 5 mm 0.8 mm 0.05 mm

max. frequency = 270Hz

S. Serak, N. Tabiryan, R. Vergara, T. White, R. Vaia, T. Bunning, *Soft Matter* **6**, 779–783 (2010)

Photoinduced oscillations

- if sample bends $> 90^o$ both sides are illuminated, producing oscillations



sunlight operation is possible!

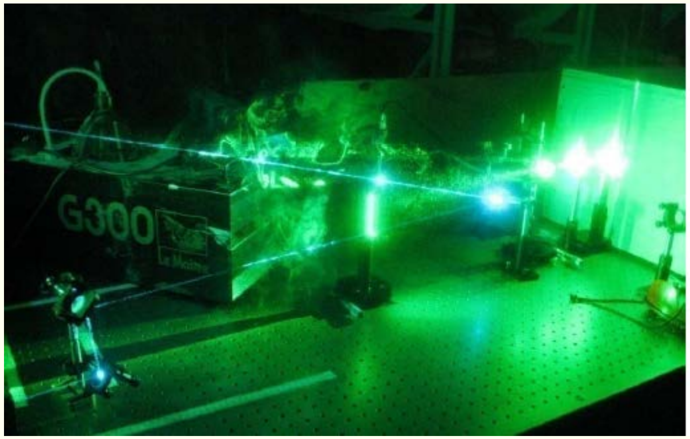
$\eta = 6 \times 10^8 Pa, \quad T_g = 65^o C, \quad \mathrm{T}_{NI} = 147^o C$

sample size: $5mm \times 0.8mm \times 0.05mm$

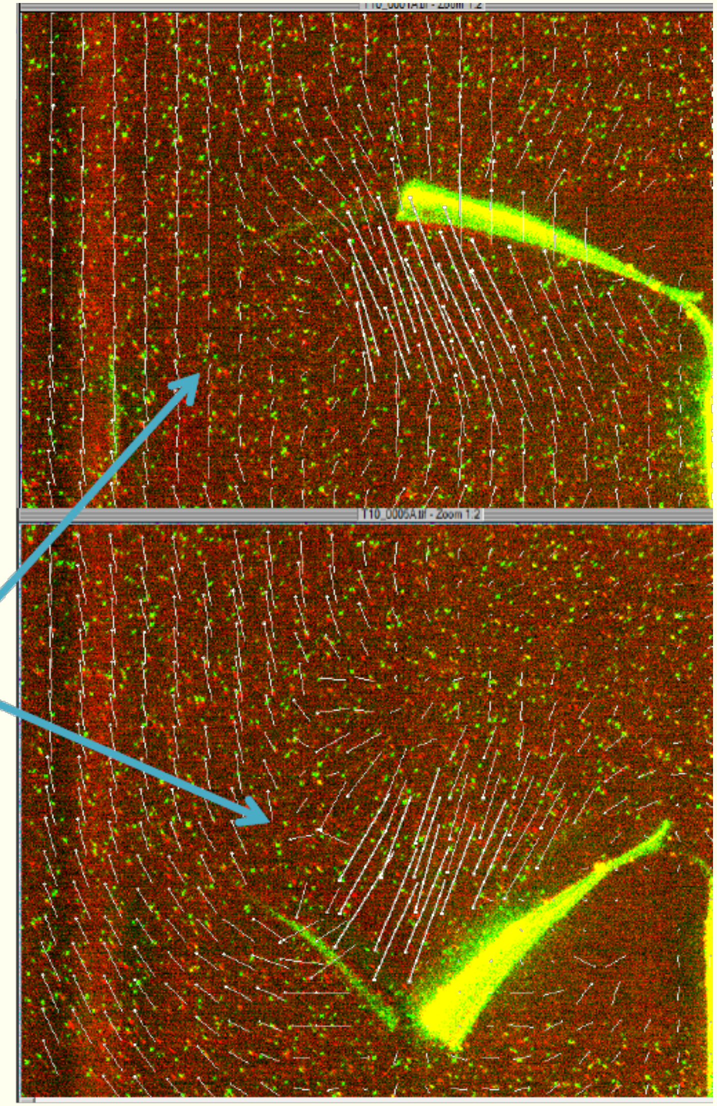
S. Serak, N. Tabiryan, R. Vergara, T. White, R. Vaia, T. Bunning, *Soft Matter* **6**, 779–783 (2010)

Photoinduced oscillations

- can it fly?



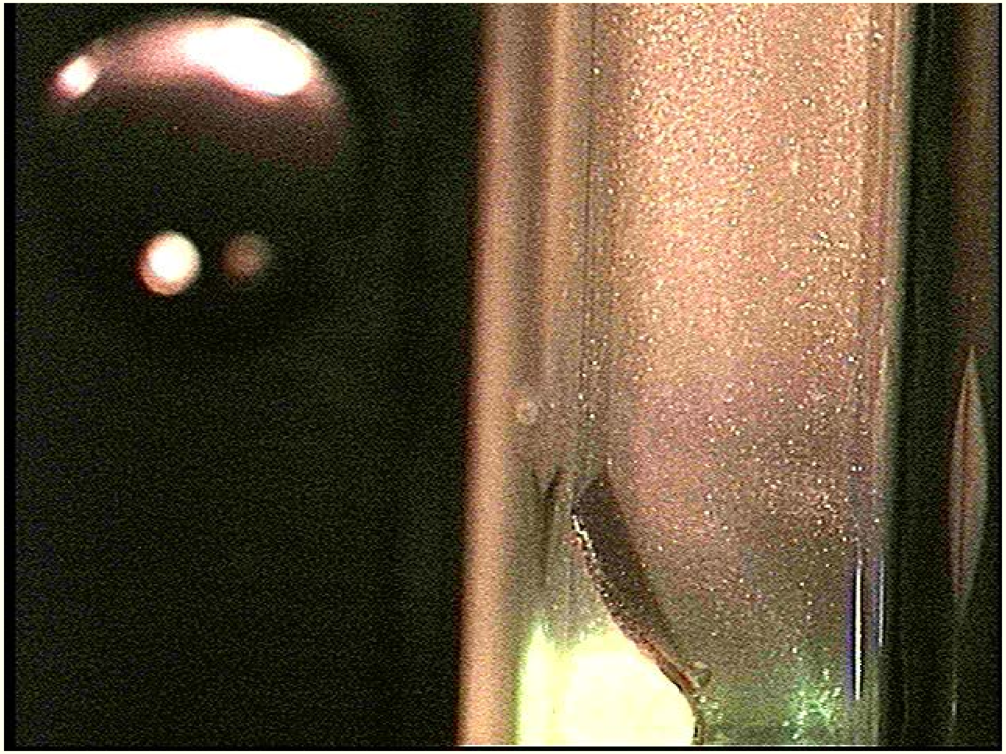
-particle velocimetry
to study lift of LCE wings



ongoing work:
T. White, AFOSR/WPAFB
A. Altman, U. Dayton

Photoinduced oscillations

- it <u>can</u> swim!



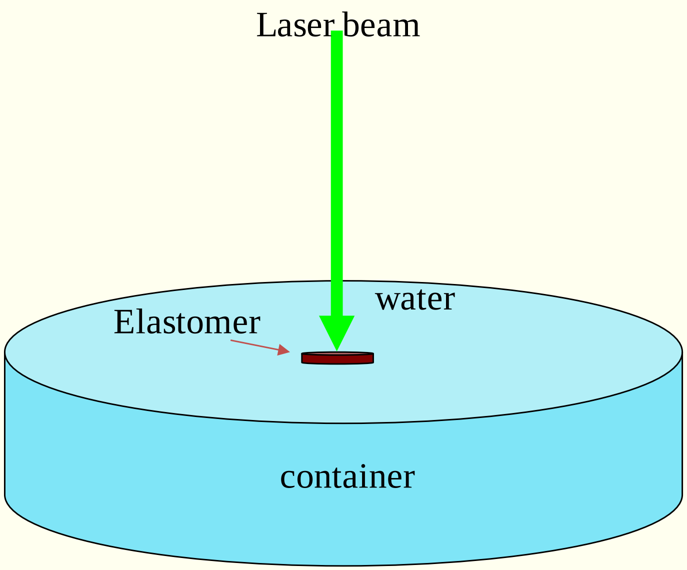
- azo-doped nematic
- LCE pumps water
- sample size:

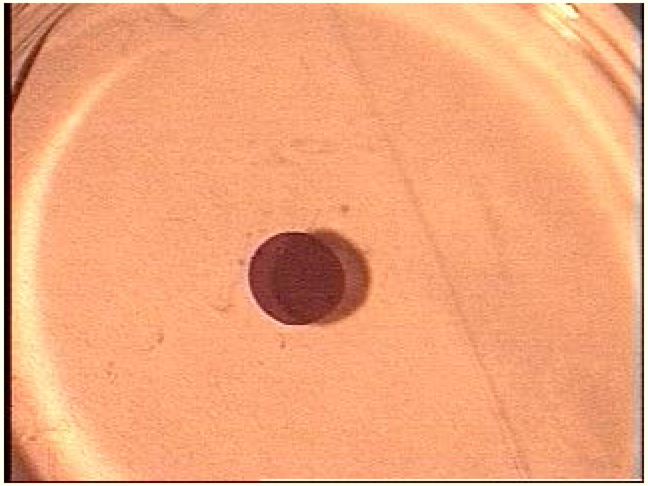
$$50mm \times 5mm \times 0.3mm$$

J. Neal, P, Palffy-Muhoray (unpublished)

Swimming away from the light

- floating nematic LCE sample illuminated from above

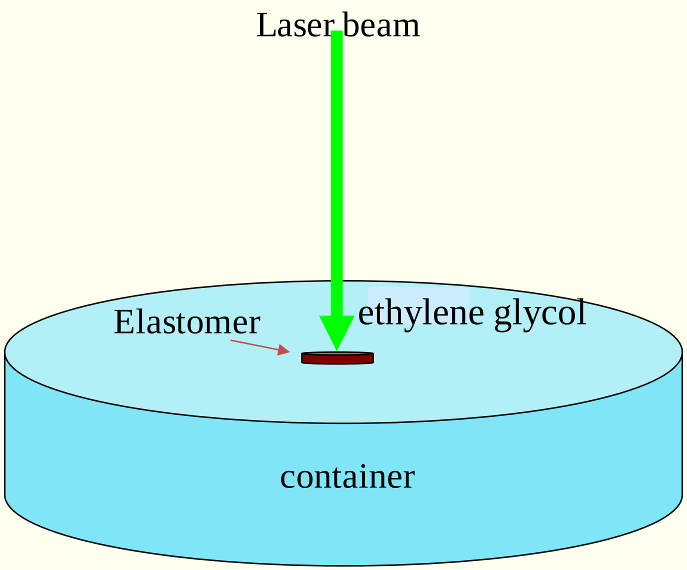


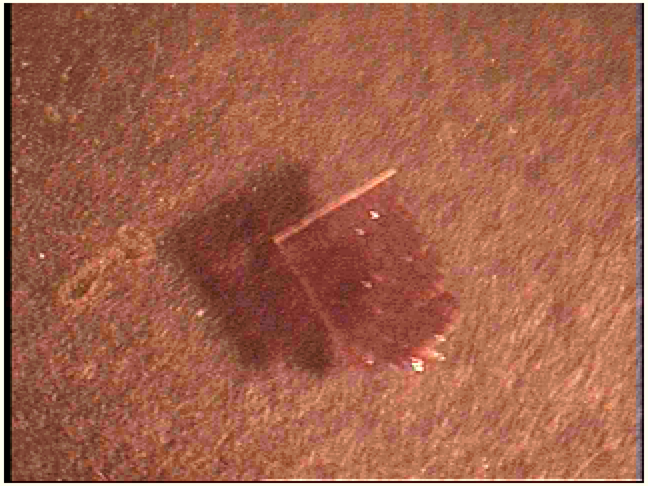


M. Chamacho-Lopez, H. Finkelmann, P. Palffy-Muhoray, M. Shelley, *Nature Mat.* **3**, 307, (2004)

Swimming away from the light

- floating nematic LCE sample illuminated from above





M. Chamacho-Lopez, H. Finkelmann, P. Palffy-Muhoray, M. Shelley, *Nature Mat.* **3**, 307, (2004)

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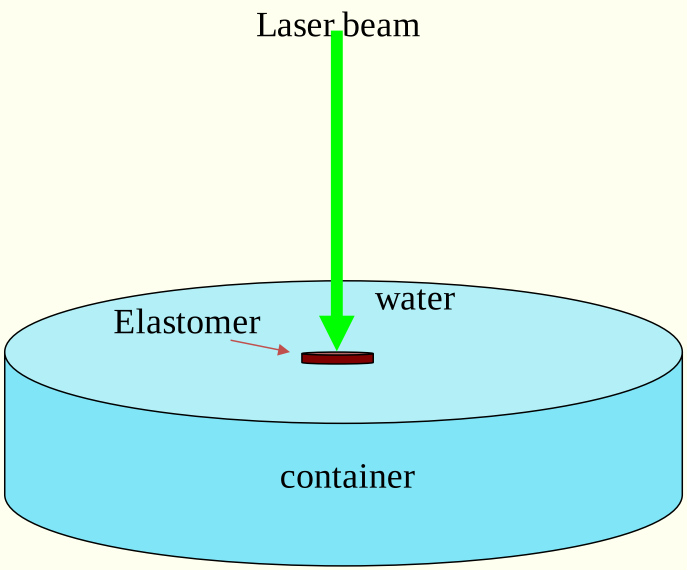
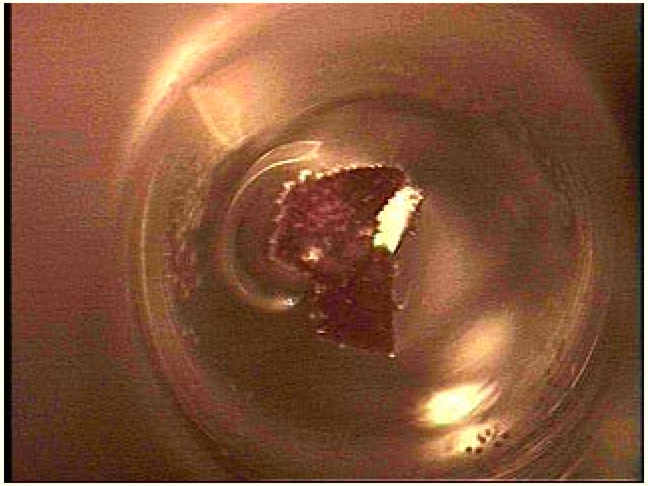




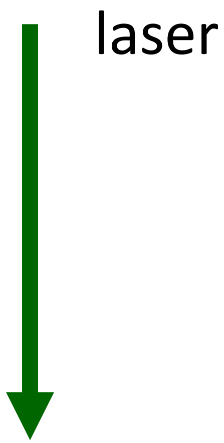
M. Chamacho-Lopez, H. Finkelmann, P. Palffy-Muhoray, M. Shelley, *Nature Mat.* **3**, 307, (2004)

the Puzzle:

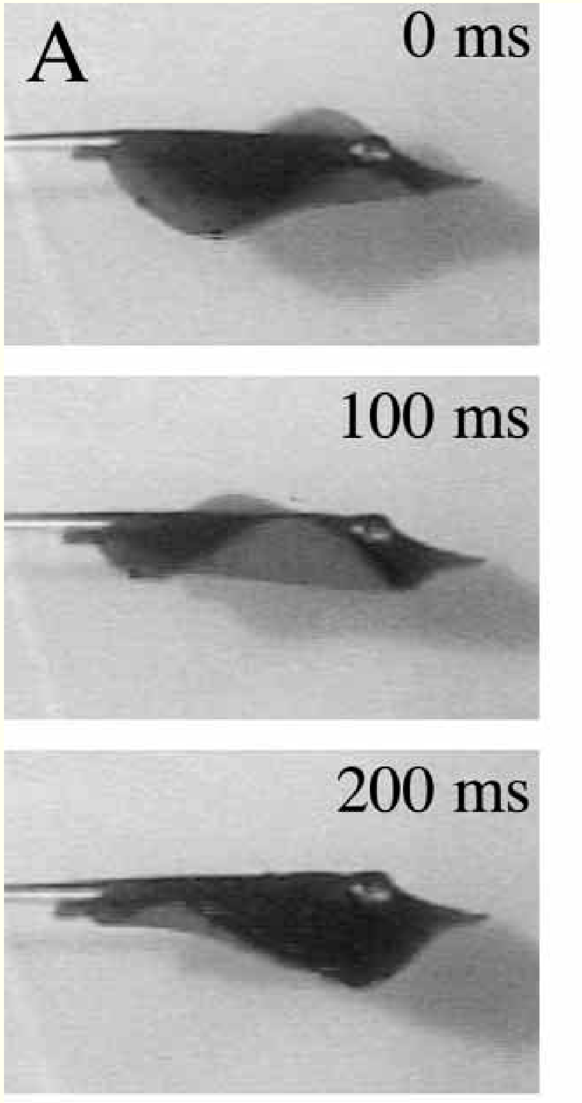
- how does it work??

- LCE is motor:
  - motion is due to transfer of energy only, not momentum!

what happens:

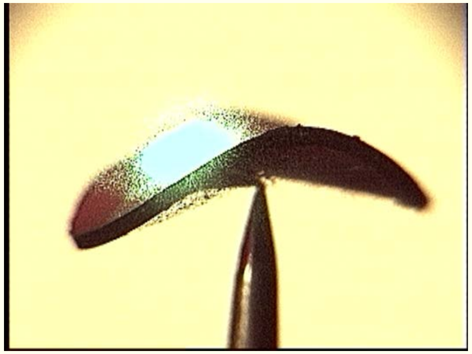


Locomotion in batoid fishes



Atlantic stingray.

Swims by propagating waves down the pectoral fins from anterior to posterior.



L.J. Rosenberger, *J. Exp. Biol.* **204**, 379-394 (2001).

Swimming dynamics

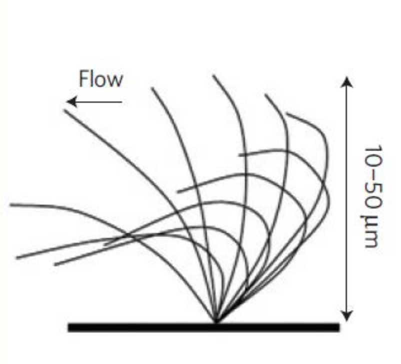
- elastomer
  - swims like a fish
  - intrinsic instability propagates wave-like deformation in elastomer

- system is a light-driven motor

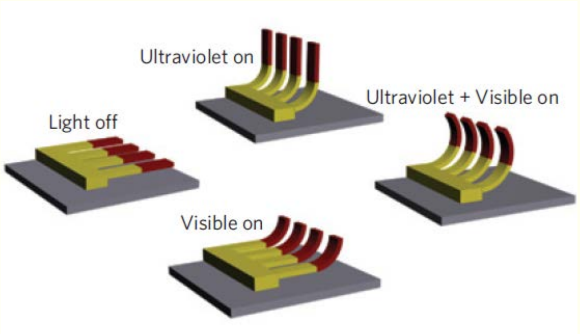
M. Chamacho-Lopez, H. Finkelmann, P. Palffy-Muhoray, M. Shelley, *Nature Mat.* **3**, 307, (2004)

Light-driven artificial cilia

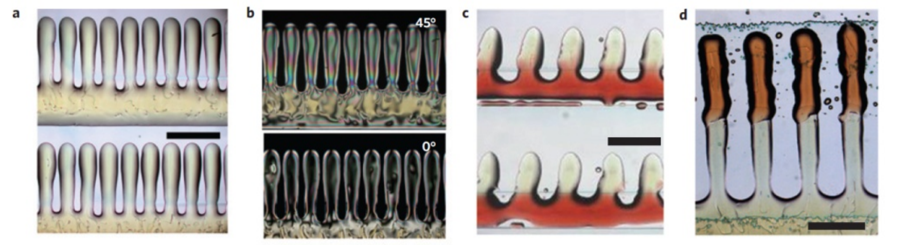
non-reciprocal motion can drive current:



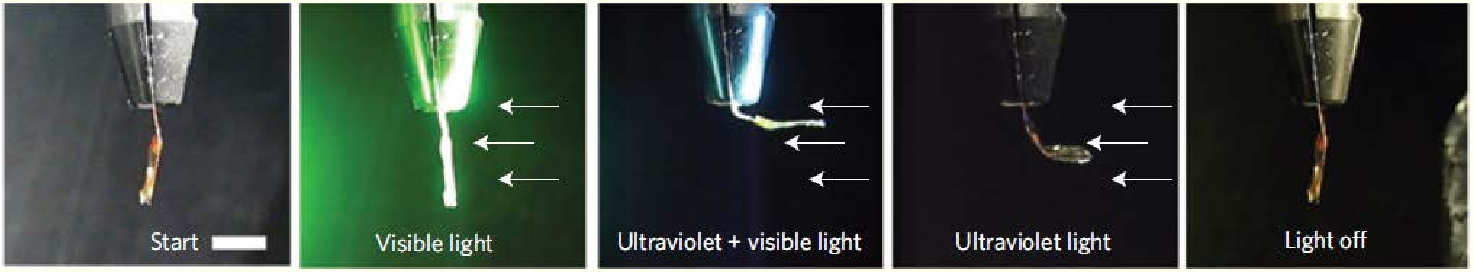
UV & vis. responsive LC network



can be printed via ink-jet technology (Fuji Dimatix)



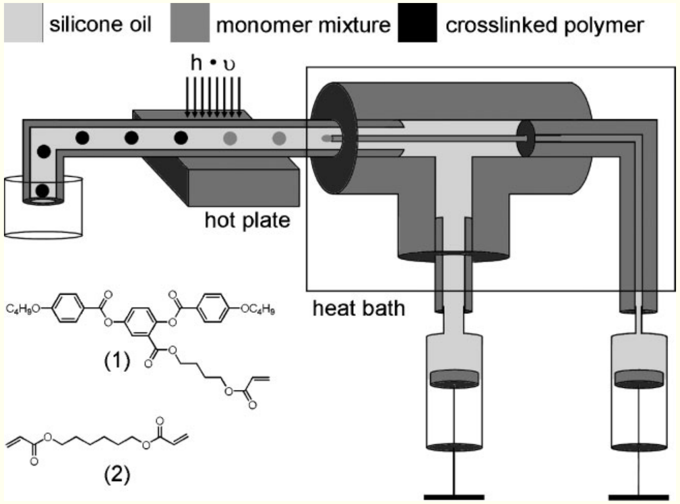
cilium in action:

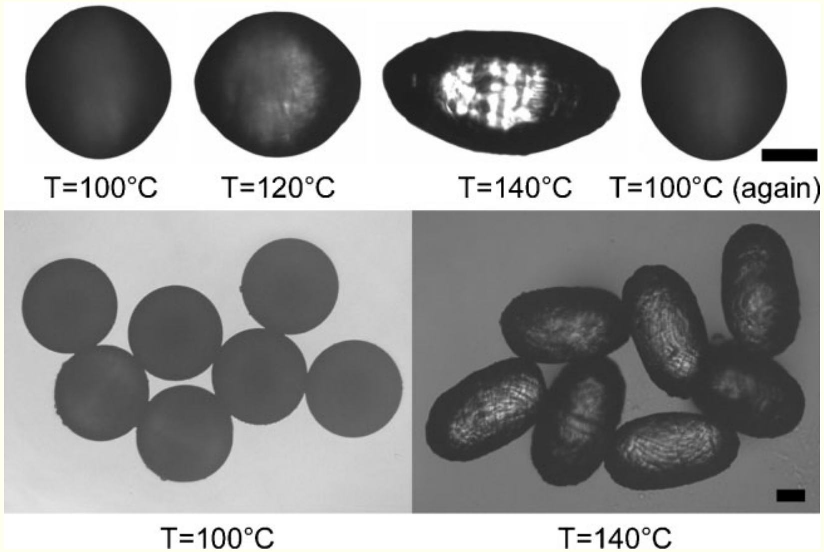


C. van Oosten, C. Bastiaansen and D. Broer, *Nat. Mater.* **8** 677-682 (2009)

Thermal actuation

- continuous production via microfluidics

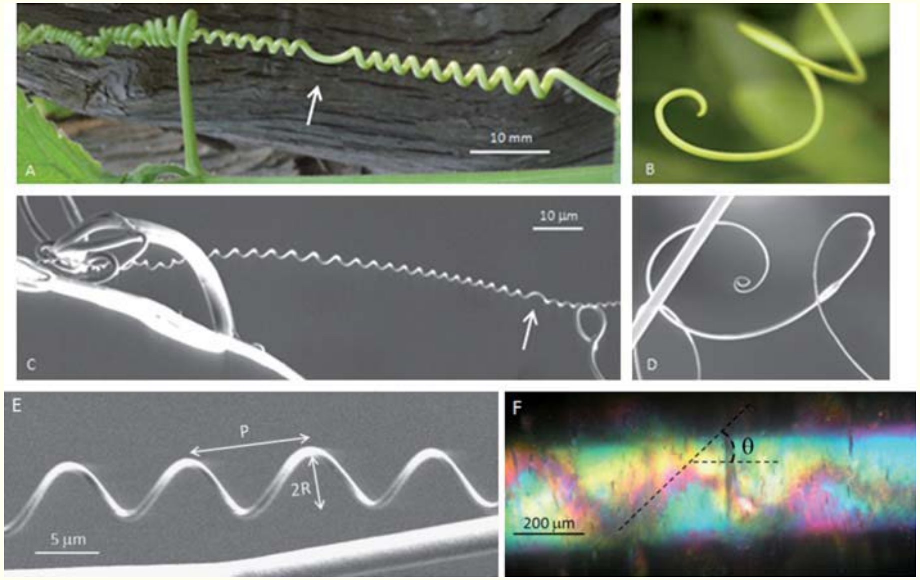




- uniform array of microactuators
  - can accurately control size & cross-link density

C. Ohm, C. Serra and R. Zentel, *Adv. Mater.* ***21,*** 4859–4862 (2009)

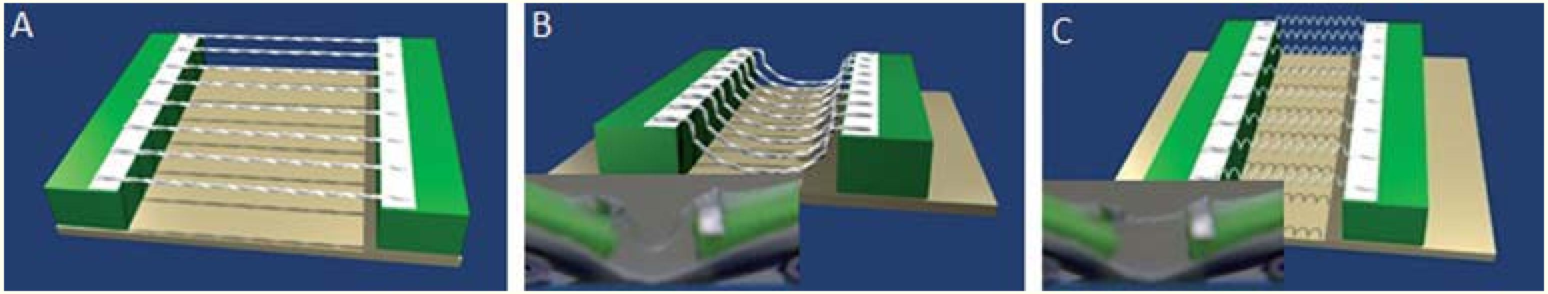
Responsive Helices



tendrils of *passiflora edulis*

liquid crystalline cellulosic fibers
via electrospinning
(note perversion)

form adaptive non-woven mats



H.M. Godinho, J. Canejo, G. Feioa and E. Terentjev, *Soft Matter*, **6,** 5965–5970 (2010)

Energy Conversion

Flexoelectricity

- the divergence of the dielectric tensor is a vector

$$\nabla \cdot \boldsymbol{\varepsilon} \sim \mathbf{P}$$

$\Rightarrow$ electric polarization

- if an isotropic rubber sample is deformed, $\nabla \cdot \boldsymbol{\varepsilon} \sim \mathbf{P} = 0$

Flexoelectricity

- the divergence of the dielectric tensor is a vector

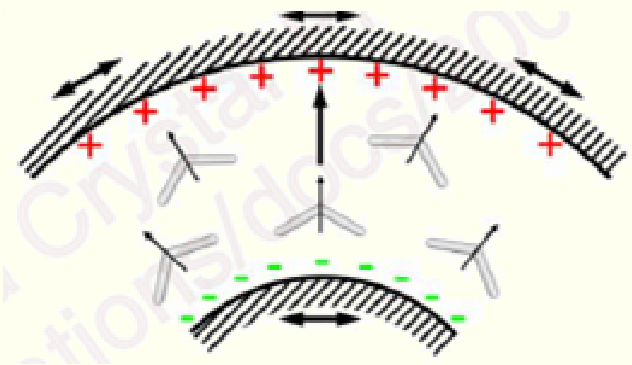
$$\nabla \cdot \boldsymbol{\varepsilon} \sim \mathbf{P}$$

$\Rightarrow$ electric polarization

- if an isotropic rubber sample is deformed, $\nabla \cdot \boldsymbol{\varepsilon} \sim \mathbf{P} = 0$

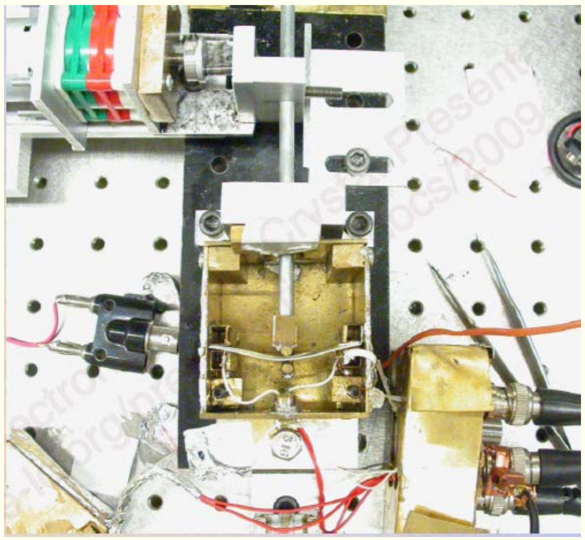
- if an LCE is deformed $\nabla \cdot \boldsymbol{\varepsilon} = \nabla \cdot (\varepsilon_i \mathbf{I} + \Delta\varepsilon \mathbf{Q}) \sim \mathbf{P} \neq 0$

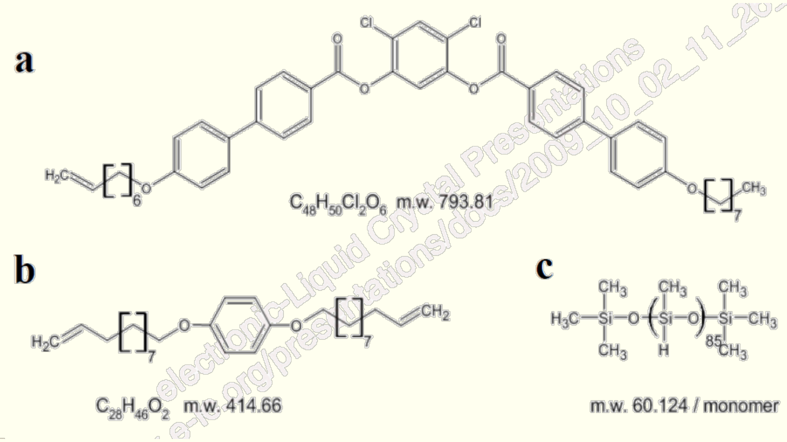
  – have 'giant' ferroelectricity!

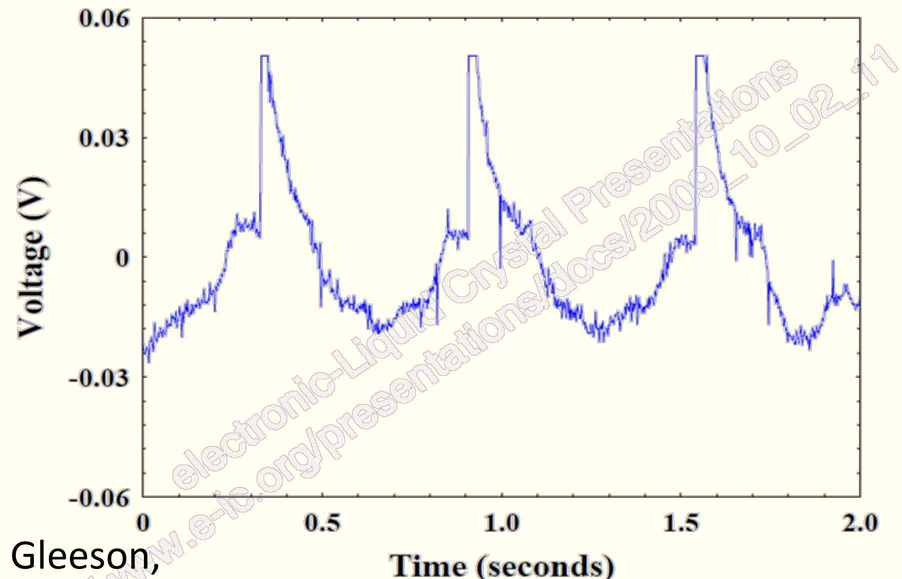


Giant flexoelectricity in 'banana' LCEs

- banana elastomers







$$P = e_3 k \qquad e_3 = 30 nC / m$$

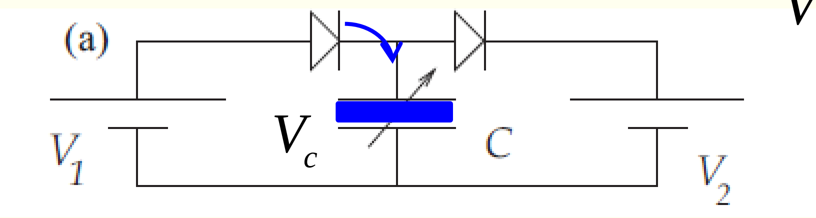
J. Harden, M. Chambers, R. Verduzco, P. Luchette, J. Gleeson, S. Sprunt, A. Jakli, *Appl. Phys. Lett.* **96**, 102907 (2010)

Solar to electrical energy conversion via LCEs

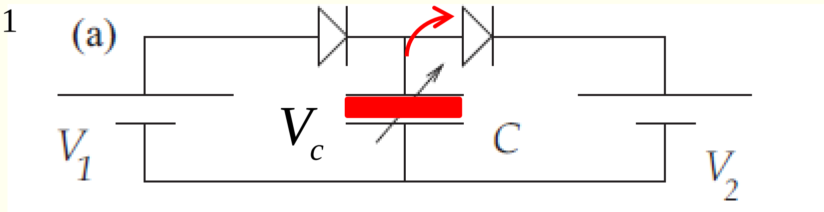
- dielectric tensor of LCs depends on orientational order

$$\boldsymbol{\varepsilon} = \varepsilon_i \mathbf{I} + \Delta\varepsilon \mathbf{Q}$$

- since the order parameter depends on temperature, capacitor with LCE* can act as a charge pump.



$$V_2 >> V_1$$



low temperature

high temperature

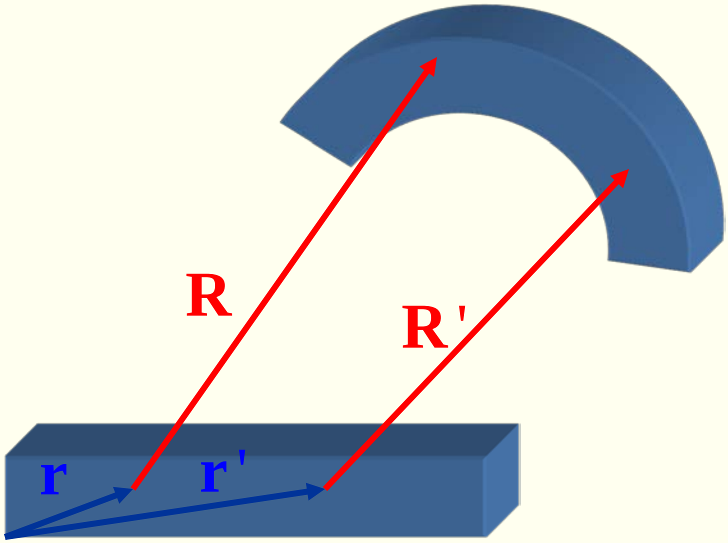
- as T increases, ε & C decrease, V_c increases.
- charge is pumped from V_1 to V_2; efficiency $\sim 5\%$!

* high dielectric breakdown

T. Hiscock, M. Warner and P Palffy-Muhoray, *J. Appl. Phys.* (to appear).

Modeling the dynamics of nematic LCEs

- Order parameters:
  - Displacement: $R_\alpha(\mathbf{r})$
  - Orientation: $Q_{\alpha\beta}(\mathbf{r})$



Strategy

- specify free energy density
  - nematic + elastomer

$$\mathfrak{F} = \mathfrak{F}(Q_{\alpha\beta}, R_{\alpha})$$

- specify dissipation
  - nematic + elastomer

$$\mathfrak{R} = \mathfrak{R}(\dot{Q}_{\alpha\beta}, \frac{\partial \dot{R}_{\alpha}}{\partial x_{\beta}})$$

Strategy

- obtain dynamics from

momentum conservation:

$$\int \left[\frac{d}{dt} \frac{\mathcal{E}_{KE}}{\partial R_\alpha} + \frac{\delta \mathcal{F}}{\delta R_\alpha} + \frac{\delta \mathcal{R}}{\delta \dot{R}_\alpha} \right] d^3\mathbf{r} = 0$$

non-conserved order parameter dynamics:

$$\int \left[\frac{\delta \mathcal{F}}{\delta Q_{\alpha\beta}} + \frac{\delta \mathcal{R}}{\delta \dot{Q}_{\alpha\beta}} \right] d^3\mathbf{r} = 0$$

Free energy:

$$\mathcal{F}(Q_{\alpha\beta}, R_{\alpha}) = ?$$

- mean field theory
- order parameters $Q_{\alpha\beta}$ and R_{α} vary in space
- use symmetry allowed squared gradient terms

squared gradient terms may be a problem!

Elastic free energy

– for isotropic polymers, distribution of separation of connected crosslinks is

$$P(\mathbf{R}_s) \sim \exp\left(-\frac{3}{2\mathcal{L}}\frac{R_s{}^2}{L}\right)$$

where $\mathcal{L}$ is the chain length, and L is the step length

– the free energy of the polymer chain between the crosslinks is

$$\mathcal{F} = -kT \ln P(\mathbf{R}_s)$$

Elastic free energy*

– for *anisotropic* polymers, distribution of separation of connected crosslinks is

$$P(\mathbf{R}_s) \sim \exp(-\frac{3}{2\mathcal{L}}\left[\mathbf{R}_s{}^T\mathbf{L}^{-1}\mathbf{R}_s\right])$$

– the matrix of effective step lengths $\mathbf{L}$ is anisotropic,

– for nematic LCEs

$$\mathbf{L} = l\mathbf{I} + \Delta l\mathbf{Q}$$

– while initially,

$$\mathbf{L}_o = l\mathbf{I} + \Delta l\mathbf{Q}_o$$

* *Liquid Crystal Elastomers,* M. Warner and E. Terentjev (Cambridge, 2003)

Non-local elastic free energy

$$\mathcal{F}_{el} = \frac{1}{2}\iint \rho_o^2 P_o(\mathbf{r},\mathbf{r}')g(\mathbf{r}+\mathbf{R},\mathbf{r}'+\mathbf{R})d^3\mathbf{r}d^3\mathbf{r}'$$

– $\mathbf{r}$ is Lagrangian coordinate, $\mathbf{R} = \mathbf{R}(\mathbf{r})$

$$g(\mathbf{r}+\mathbf{R},\mathbf{r}'+\mathbf{R}) = -kT\ln P(\mathbf{r}+\mathbf{R},\mathbf{r}'+\mathbf{R})$$

$$g = \frac{3kT}{2\mathcal{L}}(r_a^{'} + R_\alpha^{'} - r_a - R_\alpha)L_{\alpha\beta}^{-1}(r_\beta^{'} + R_\beta^{'} - r_\beta - R_\beta)$$

$$\mathcal{F}_{el} = \mathcal{F}_{el}(R_\alpha, Q_{\alpha\beta})$$

Non-local nematic free energy

$$\mathcal{F}_{LC} = \frac{1}{2}\iint [\frac{1}{2}aQ(\mathbf{r})^2 - \frac{1}{3}bQ(\mathbf{r})^3 + \frac{1}{4}cQ(\mathbf{r})^4 + ..$$

$$+ \rho U \frac{Q(\mathbf{r})(Q(\mathbf{r}) - Q(\mathbf{r}'))}{\underbrace{(\mathbf{r}' + \mathbf{R}' - \mathbf{r} - \mathbf{R})^6}_{\text{Eulerian separation}}}]d^3\mathbf{r}d^3\mathbf{r}'$$

$$\mathcal{F}_{LC} = \mathcal{F}_{LC}(R_\alpha, Q_{\alpha\beta})$$

Rayleigh dissipation function

$$2\mathfrak{R} = T\dot{S} = \nu^{(1)}_{\alpha\beta\gamma\delta}\dot{Q}_{\alpha\beta}\dot{Q}_{\gamma\delta} +$$

$$\nu^{(2)}_{\alpha\beta\gamma\delta}\left(\frac{\partial \dot{R}_\alpha}{\partial x_\beta}\dot{Q}_{\gamma\delta} + \dot{Q}_{\alpha\beta}\frac{\partial \dot{R}_\gamma}{\partial x_\delta}\right) +$$

$$\nu^{(3)}_{\alpha\beta\gamma\delta}\frac{\partial \dot{R}_\alpha}{\partial x_\beta}\frac{\partial \dot{R}_\gamma}{\partial x_\delta}$$

- viscosities $\nu_{\alpha\beta\gamma\delta}$ depend on $Q_{\alpha\beta}$.

$$\mathfrak{R} = \mathfrak{R}_{LC}(\dot{R}_\alpha, \dot{Q}_{\alpha\beta})$$

Dynamics

- for material points

$$\rho \ddot{R}_\alpha = -\frac{\delta \mathfrak{F}}{\delta R_\alpha} + \nu^{(2)}_{\alpha\beta\gamma\delta}\frac{\partial \dot{Q}_{\gamma\delta}}{\partial x_\beta} + \nu^{(3)}_{\alpha\beta\gamma\delta}\frac{\partial^2 \dot{R}_\gamma}{\partial x_\beta \partial x_\delta}$$

- for nematic order parameter

$$\nu^{(1)}_{\alpha\beta\gamma\delta}\dot{Q}_{\gamma\delta} = -\frac{\delta \mathfrak{F}}{\delta Q_{\alpha\beta}} - \nu^{(2)}_{\gamma\delta\alpha\beta}\frac{\partial \dot{R}_\gamma}{\partial x_\delta}$$

R. Ennis, L. Malacarne, P. Palffy-Muhoray, M. Shelley, http:www.e-lc.org/docs/2004_12_11_00_45_55

Dynamics

- for the surrounding fluid:

$$\rho \frac{D\mathbf{v}}{Dt} = \nabla \cdot (-p\mathbf{I} + 2\eta\mathbf{D})$$

$$\nabla \cdot \mathbf{v} = 0$$

- boundary conditions:

$$\boldsymbol{\sigma}_{\mathrm{fl}} \cdot \mathbf{n} = \boldsymbol{\sigma}_{\mathrm{el}} \cdot \mathbf{n}$$

$$\mathbf{V} = \mathbf{v}_{\mathrm{fl}} = \mathbf{v}_{\mathrm{el}}$$

on $\partial\Omega$

Simulations

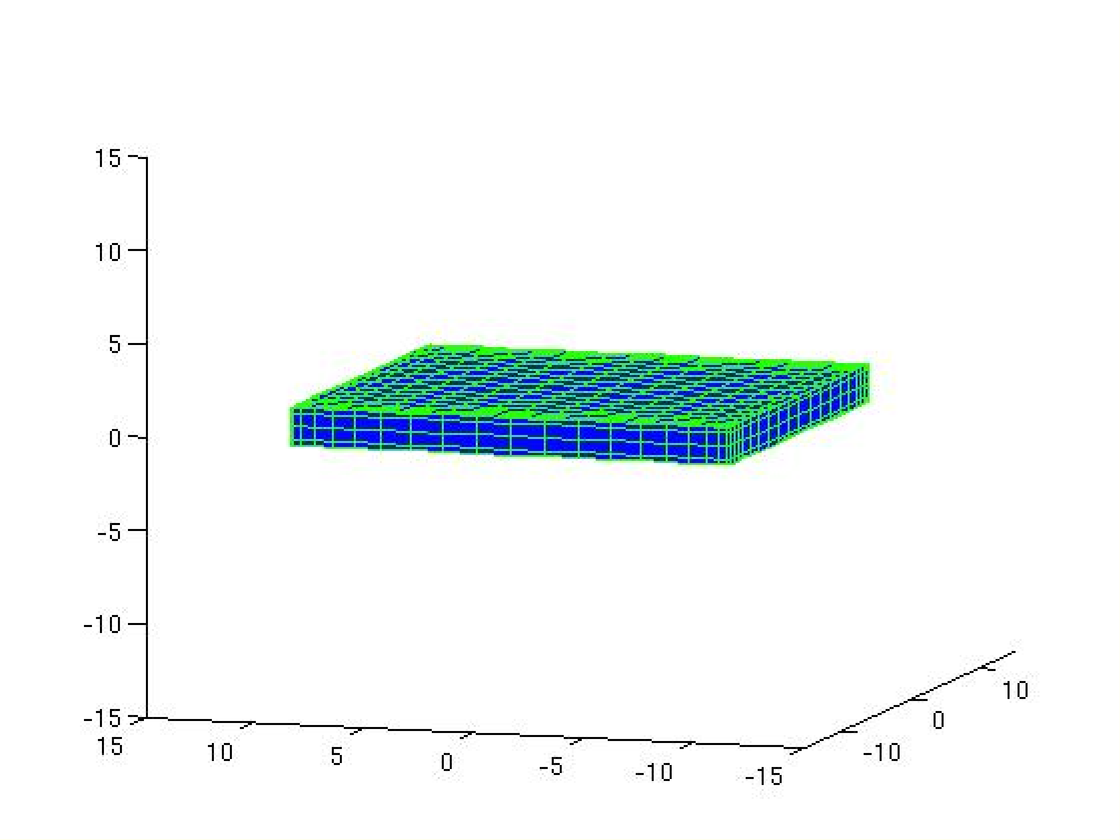
- Mike Shelley & Wei Zhu (Courant Institute, NYU)

- numerical solution of coupled PDEs:
  - spectral Chebyshev method

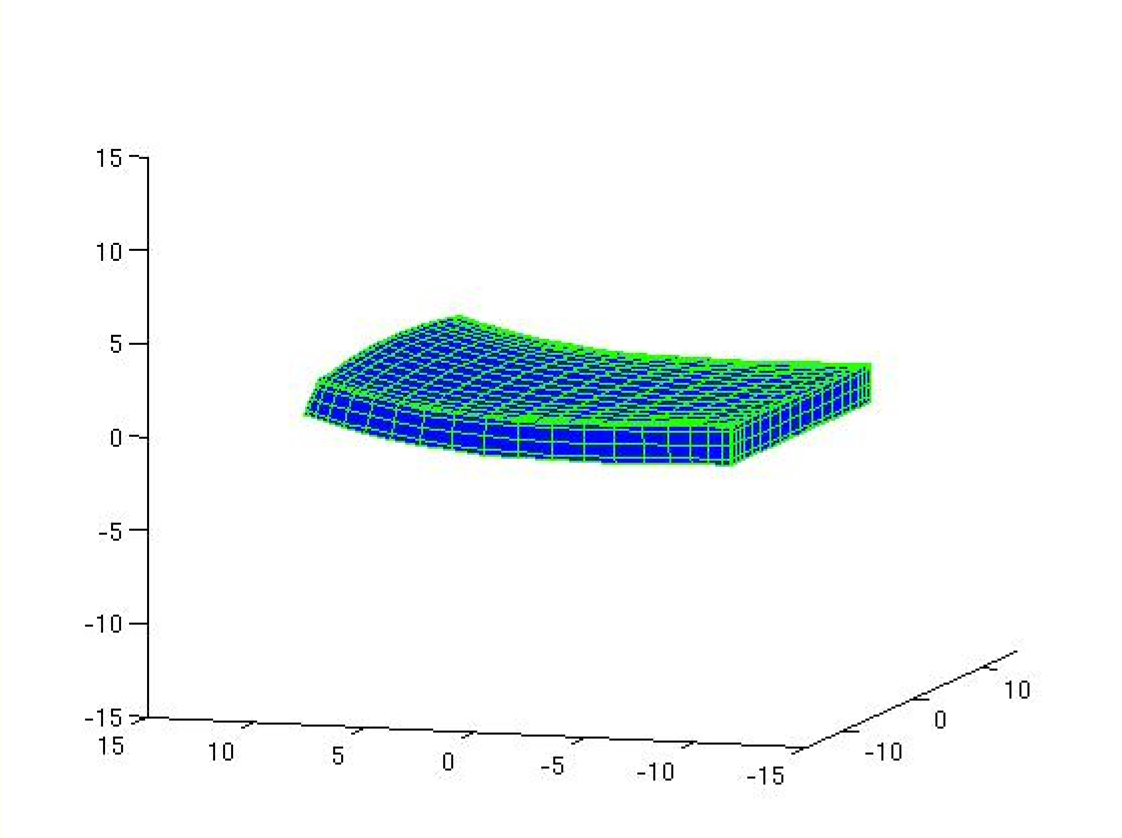
$$\rho \ddot{R}_\alpha = -\frac{\delta \mathcal{F}}{\delta R_\alpha} + \nu^{(2)}_{\alpha\beta\gamma\delta} \frac{\partial \dot{Q}_{\gamma\delta}}{\partial x_\beta} + \nu^{(4)}_{\alpha\beta\gamma\delta} \frac{\partial^2 \dot{R}_\gamma}{\partial x_\beta \partial x_\delta}$$

$$\nu^{(1)}_{\alpha\beta\gamma\delta} \dot{Q}_{\gamma\delta} = -\frac{\delta \mathcal{F}}{\delta Q_{\alpha\beta}} - \nu^{(2)}_{\gamma\delta\alpha\beta} \frac{\partial \dot{R}_\gamma}{\partial x_\delta}$$

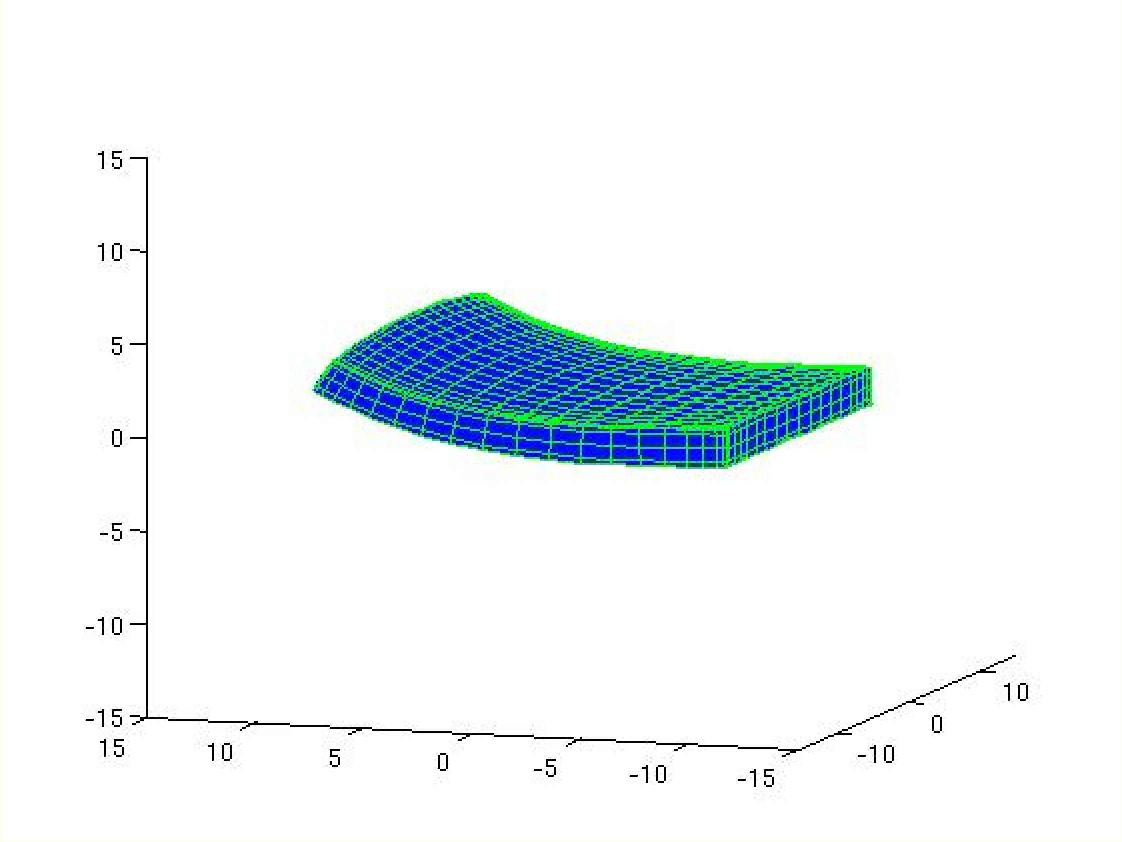
Bending



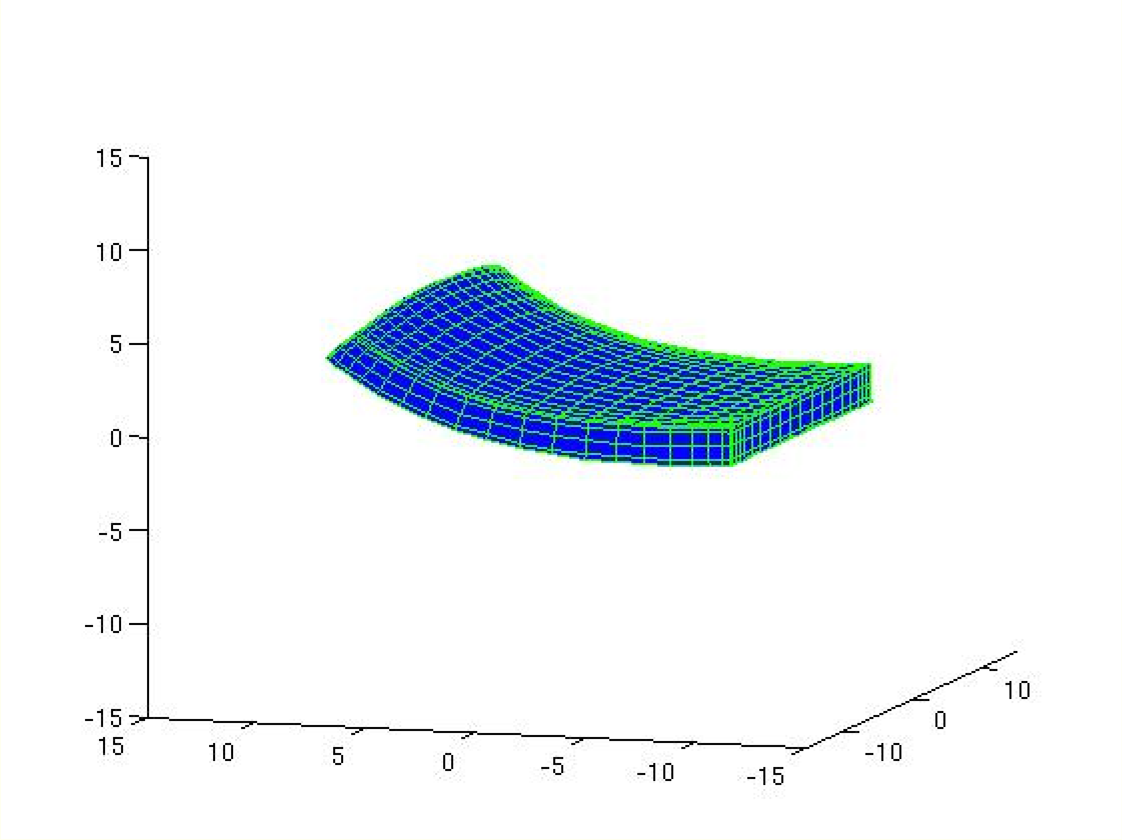
Bending



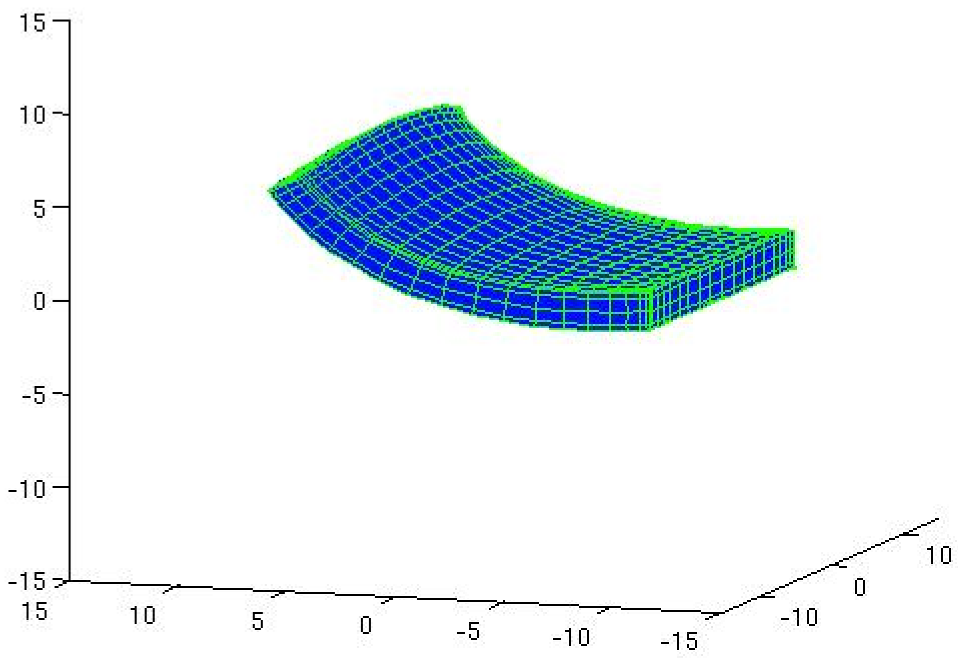
Bending



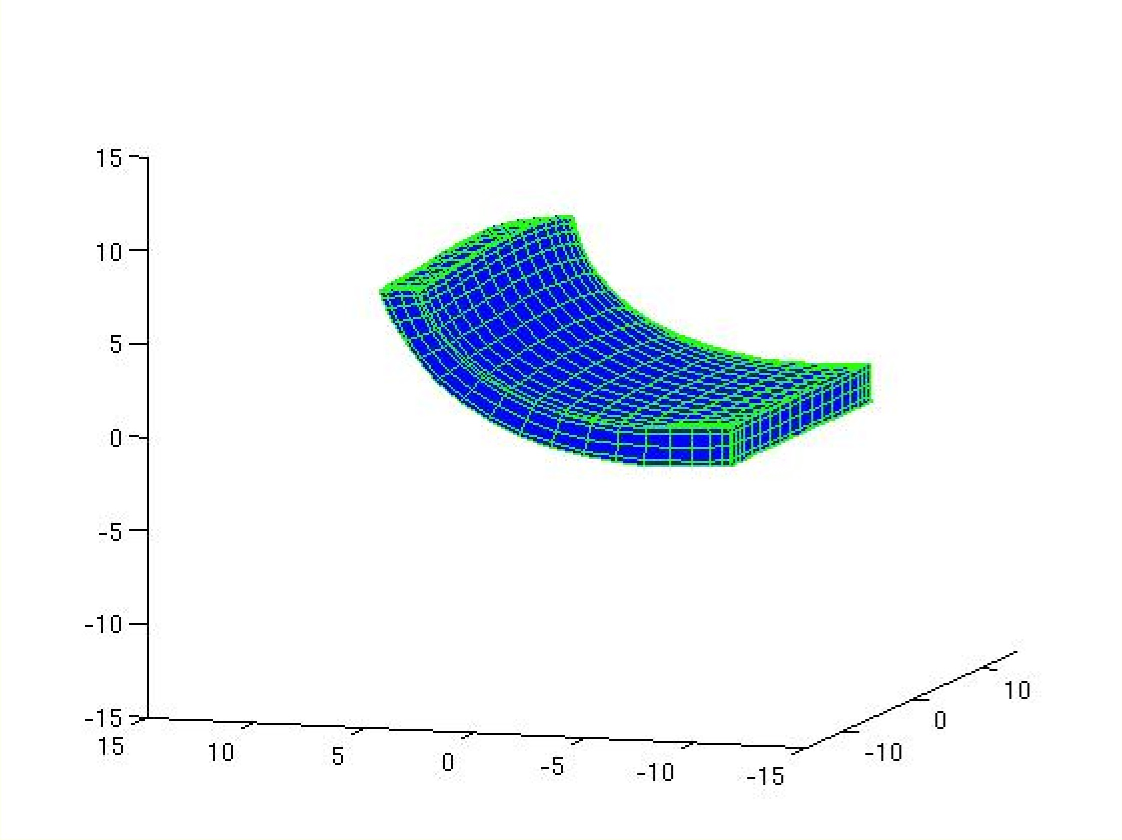
Bending



Bending

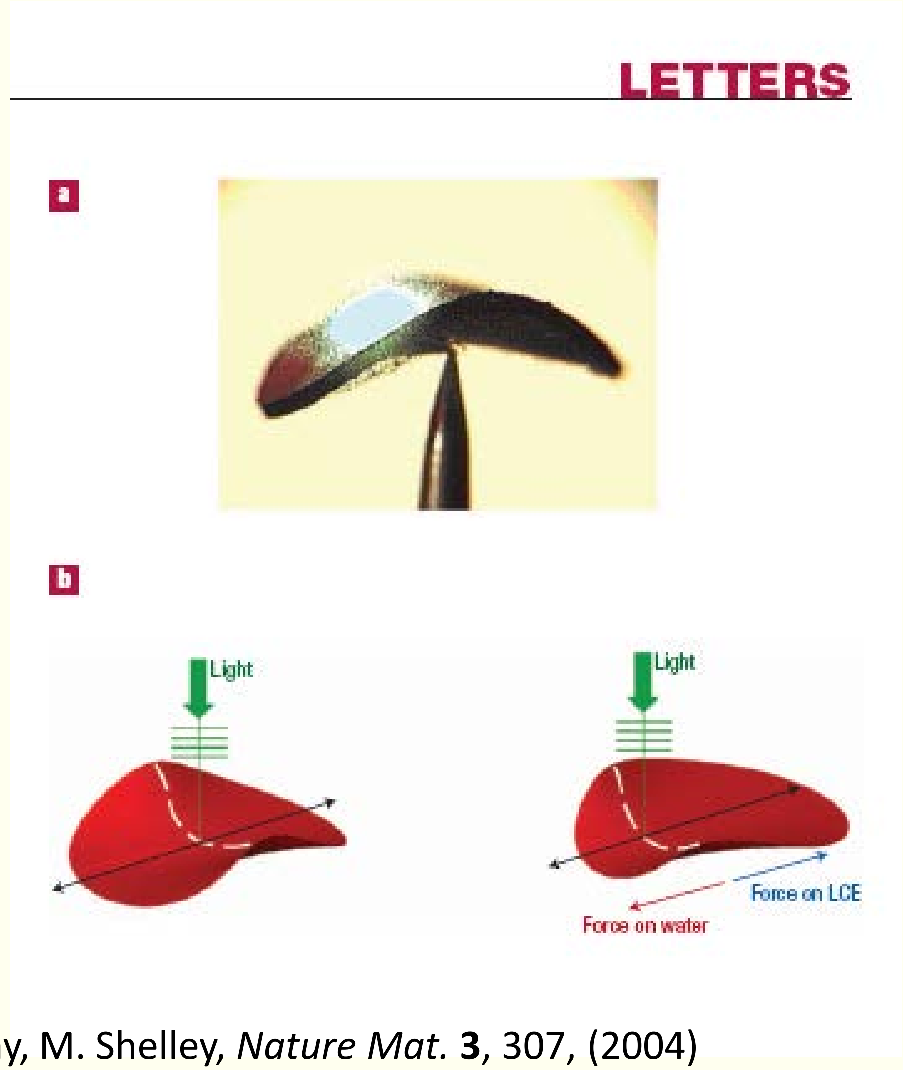


Bending



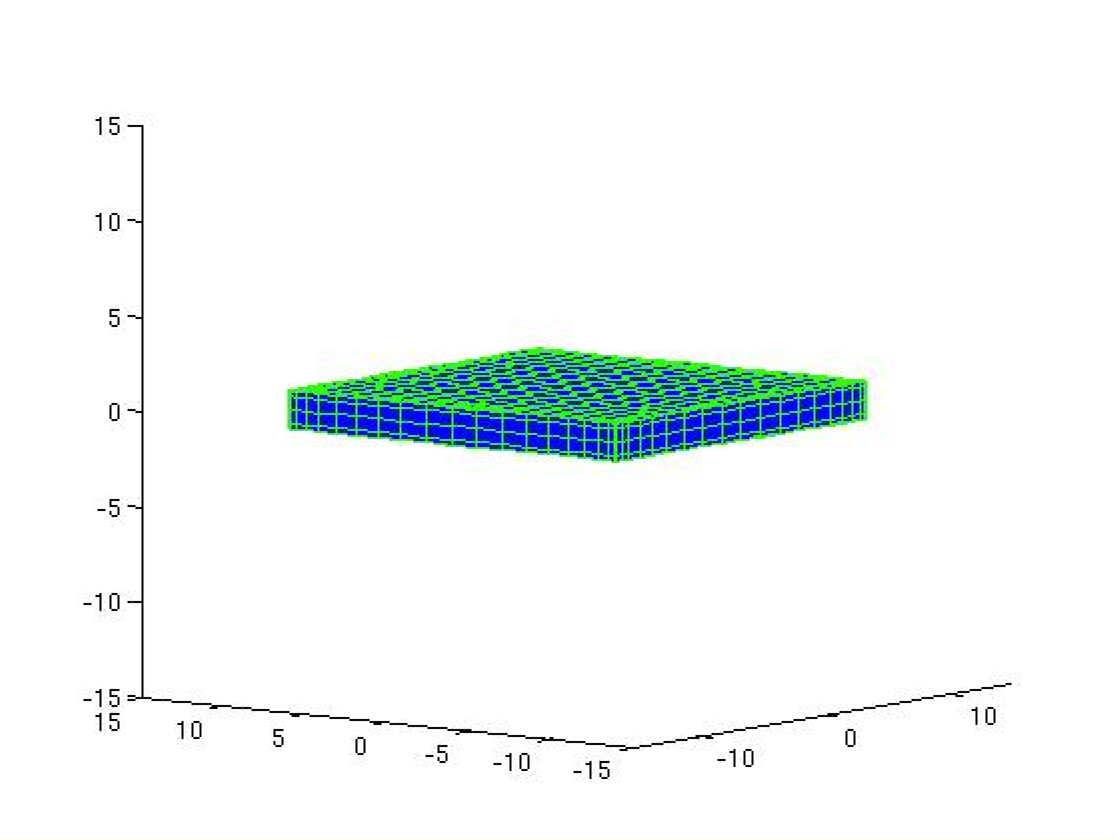
Saddle

- observed light induced saddle deformation
- want to simulate the dynamics
- simulation imposes temperature



M. Camacho-Lopez, H. Finkelmann, P. Palffy-Muhoray, M. Shelley, *Nature Mat.* **3**, 307, (2004)

SADDLE



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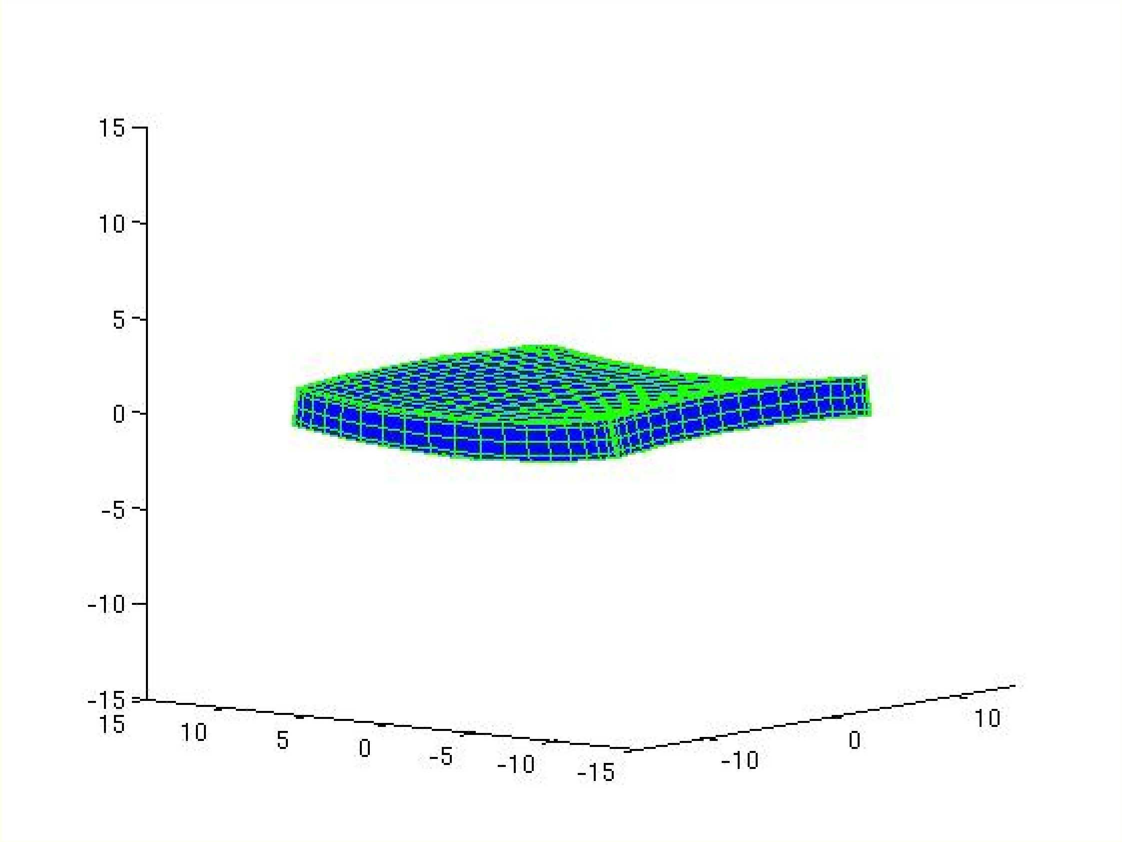

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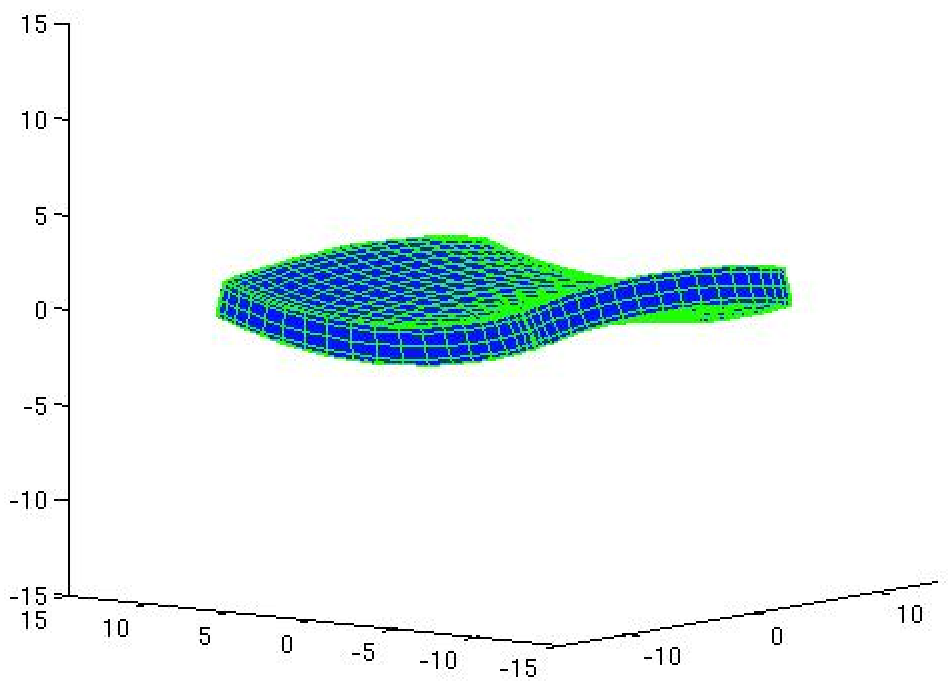

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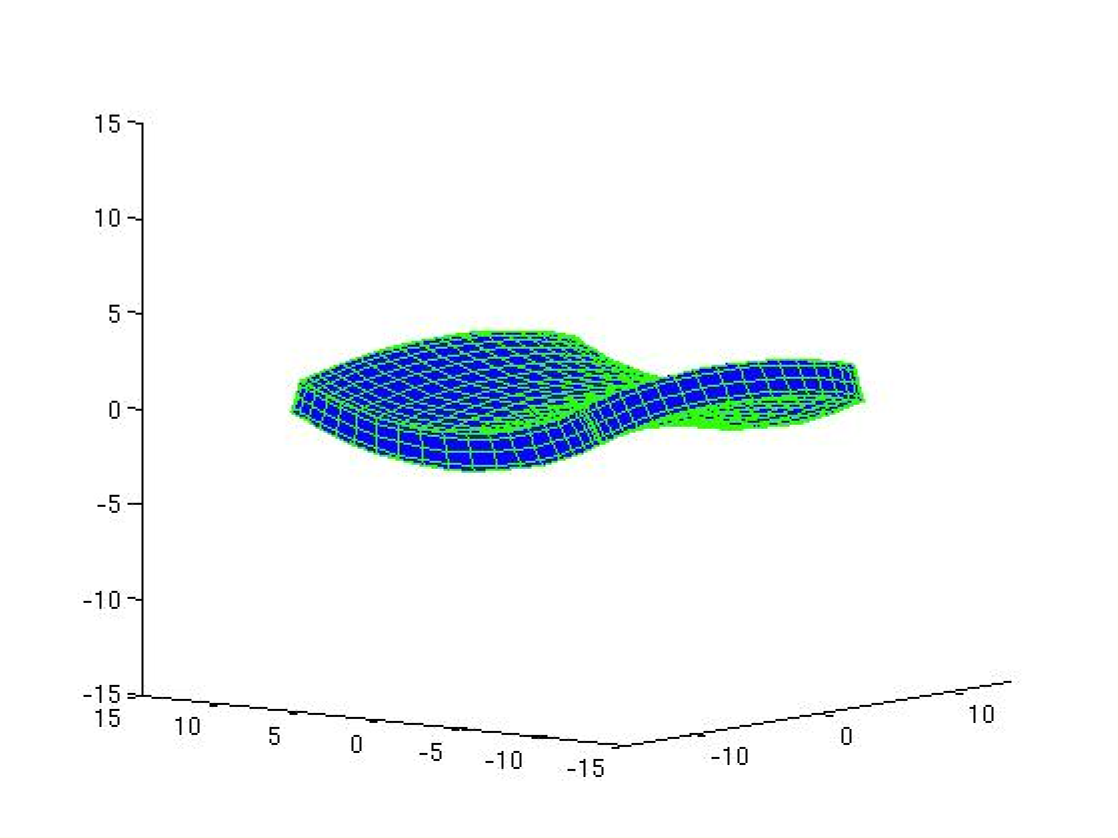

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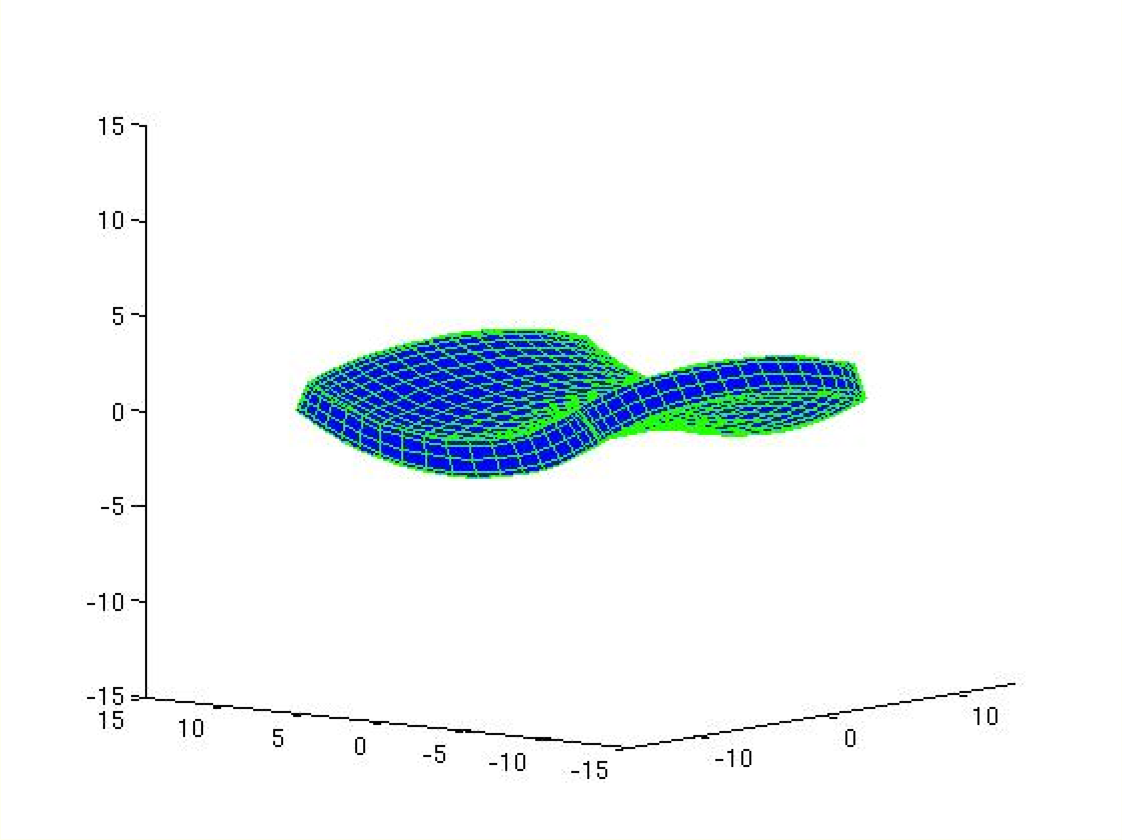

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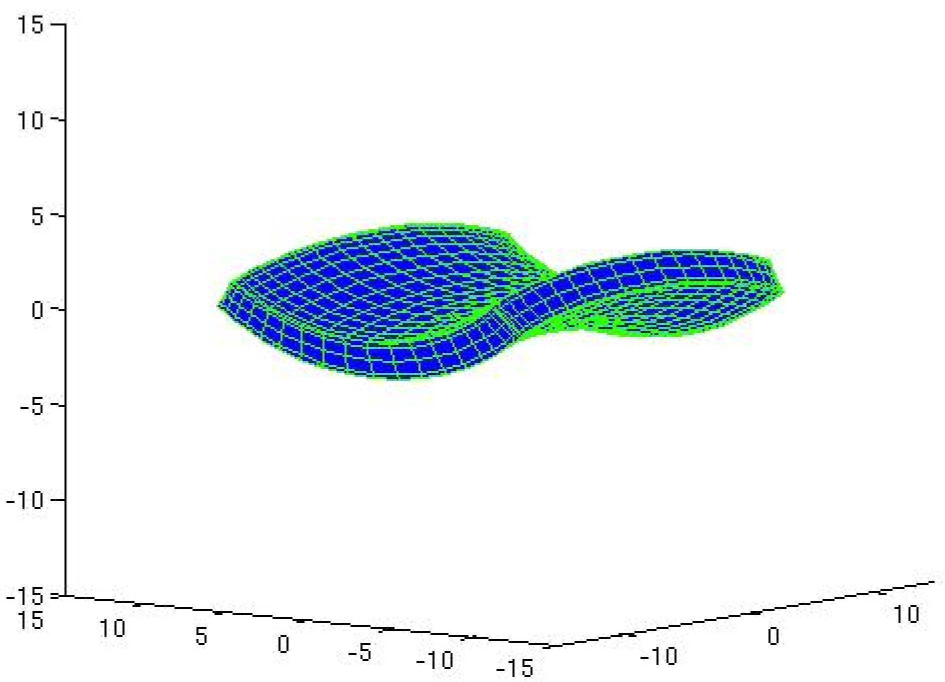

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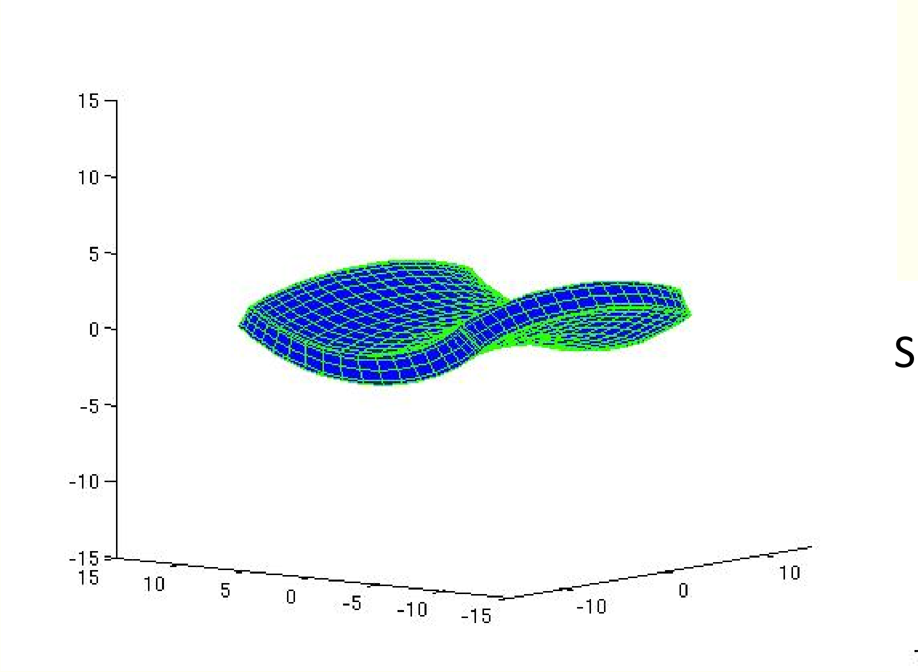
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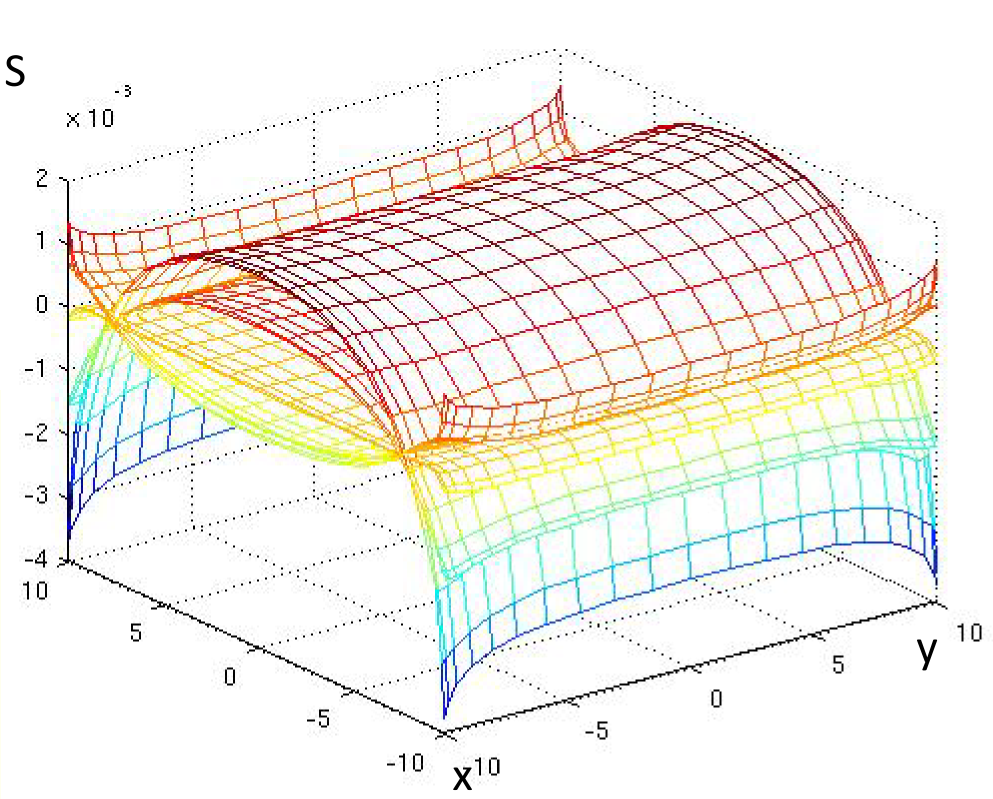
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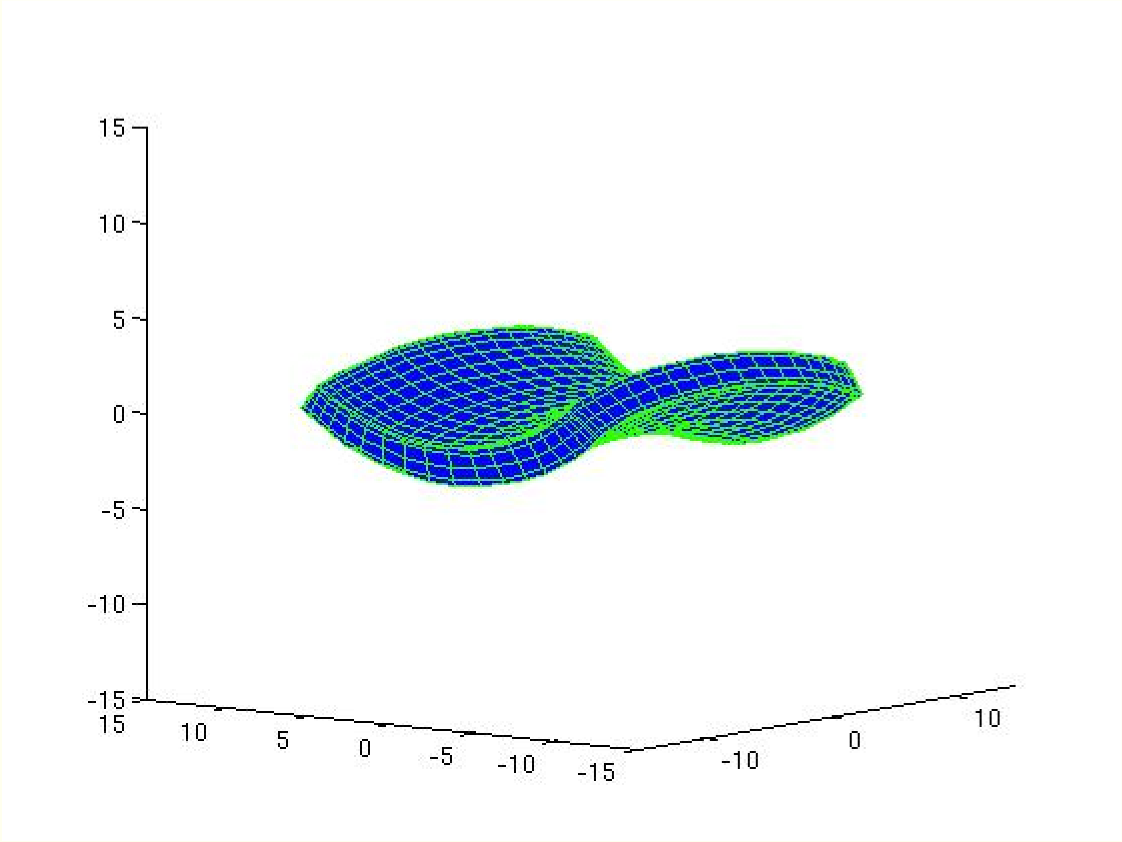
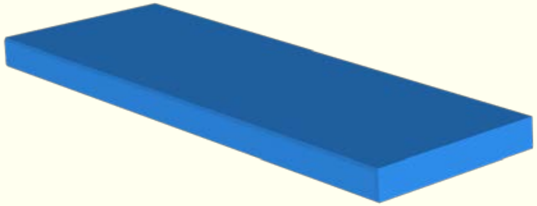
SADDLE



TWIST

- is it possible to induce spontaneous twist in a uniform achiral nematic sample with plane polarized light?



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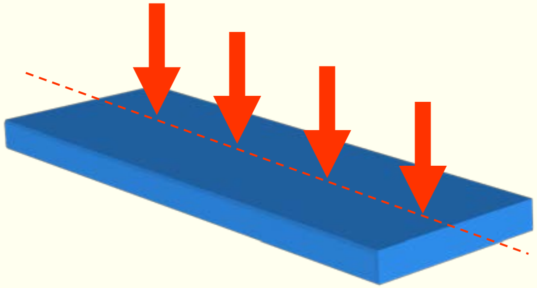
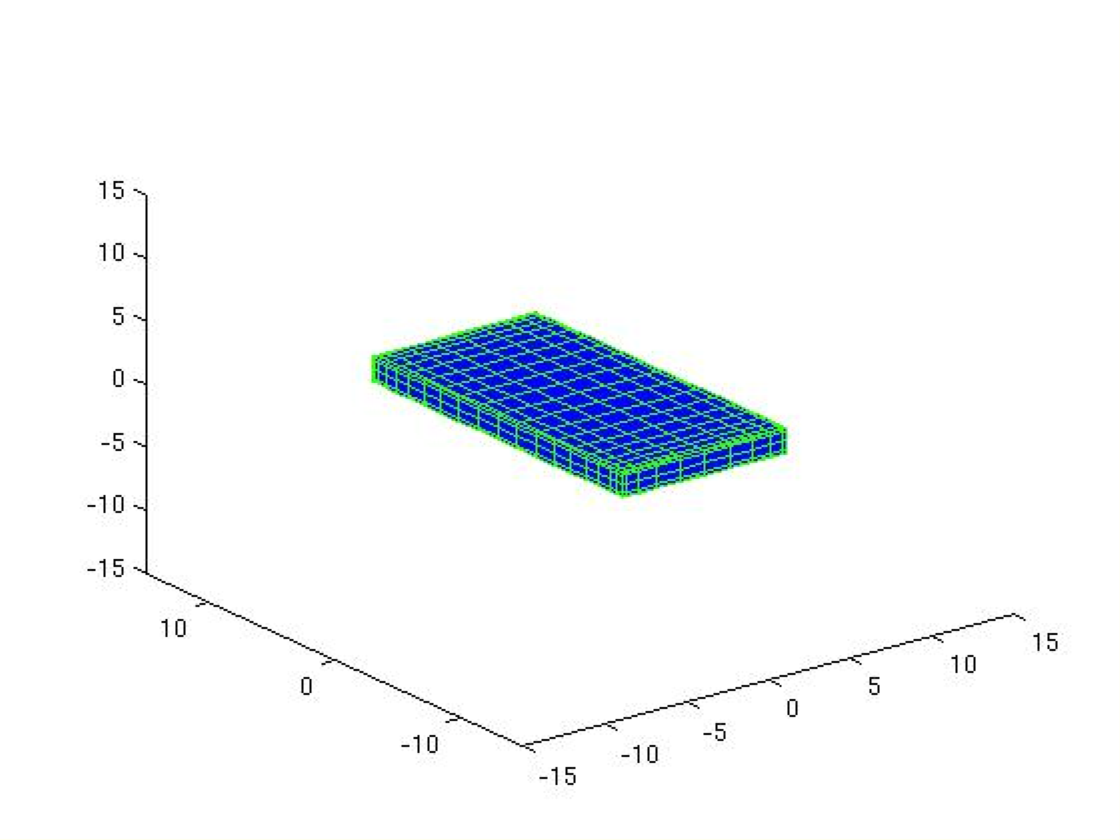
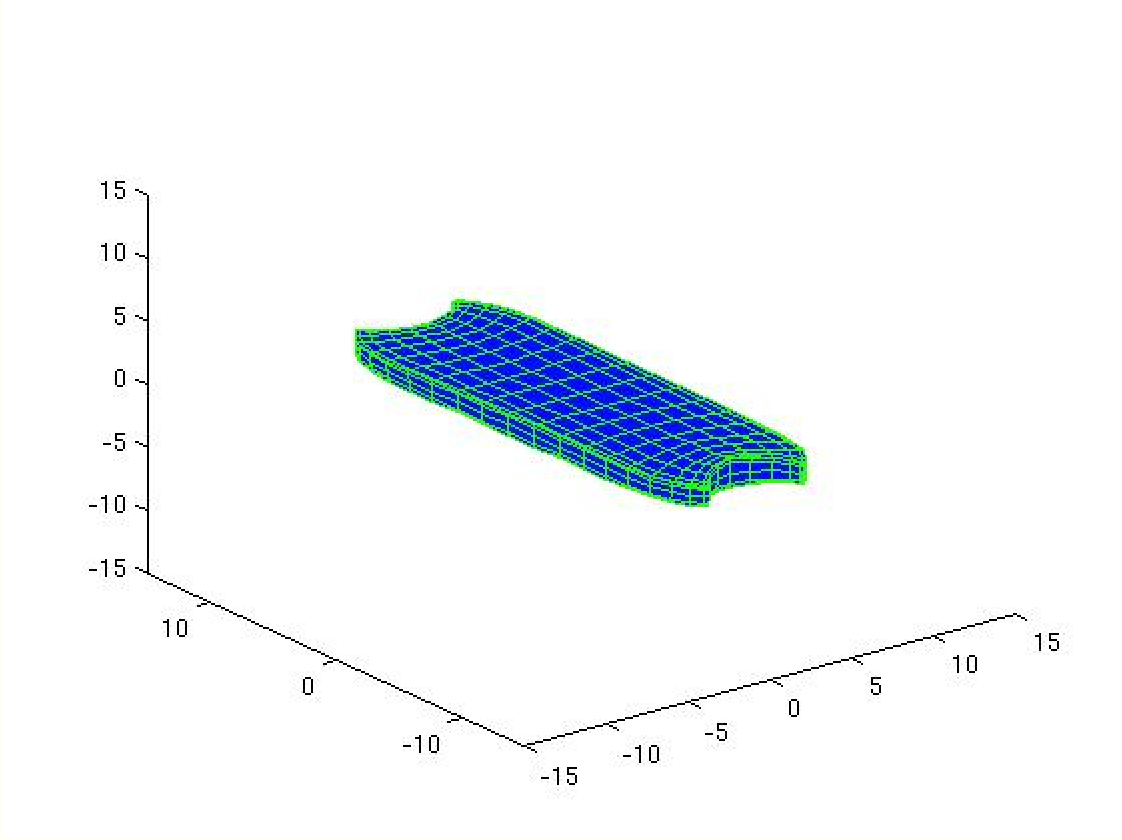


- illuminate (heat) along line in middle?

TWIST



TWIST



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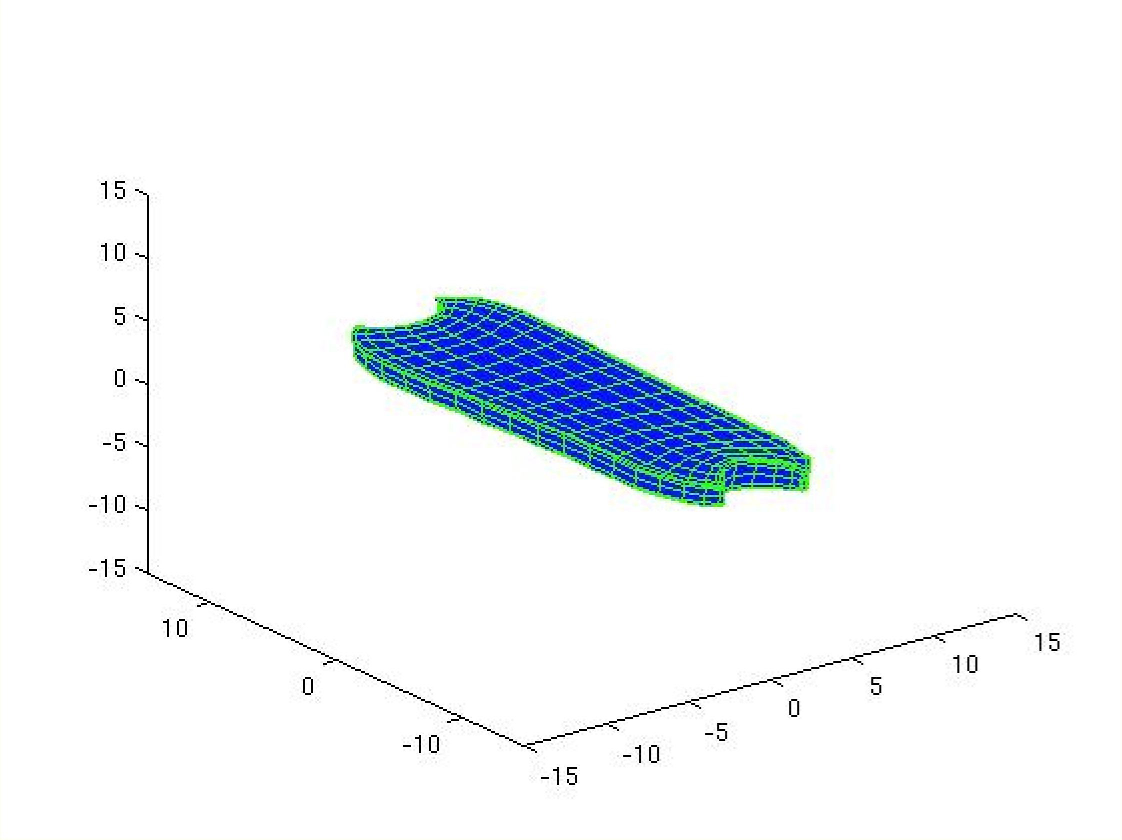
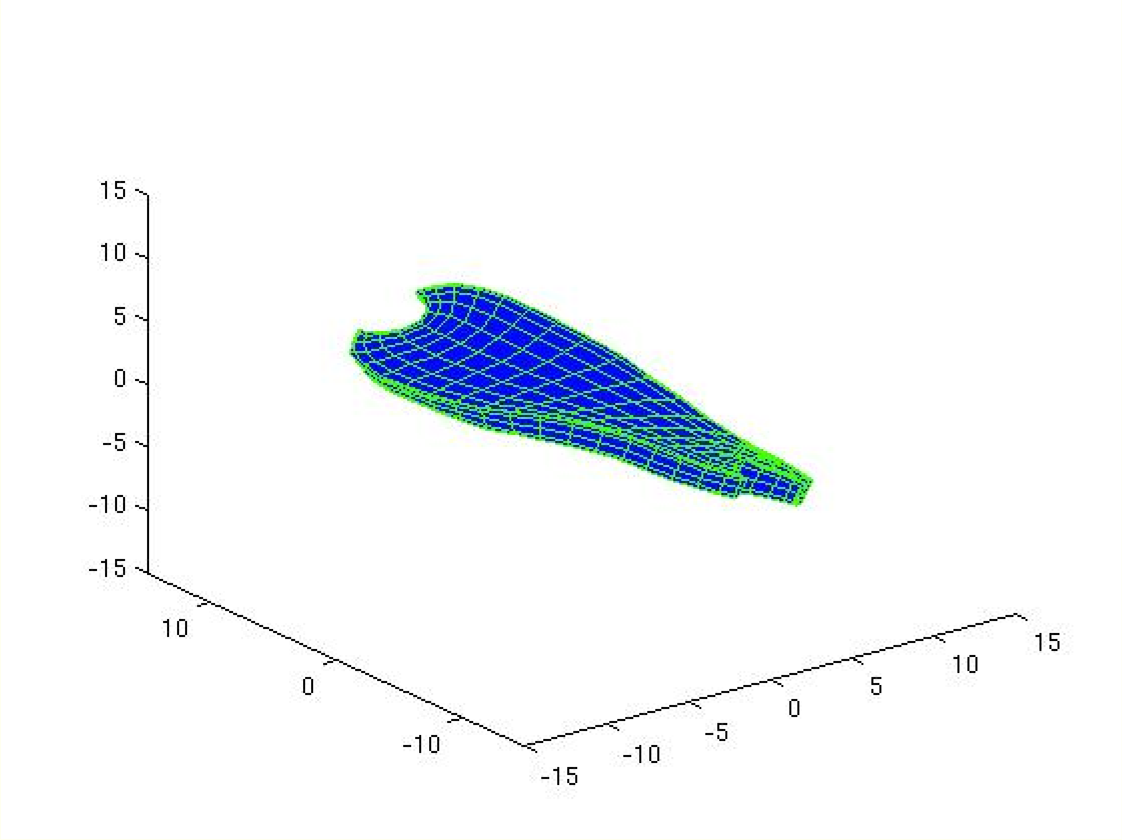


TWIST



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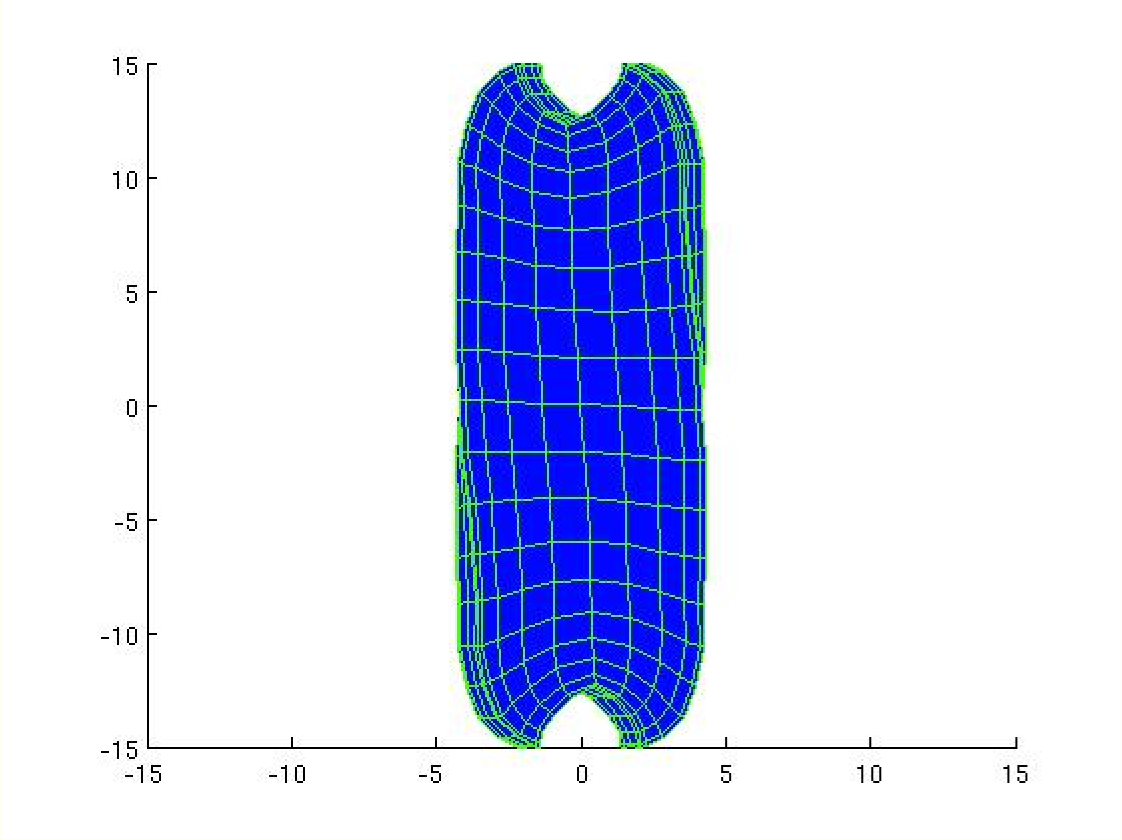
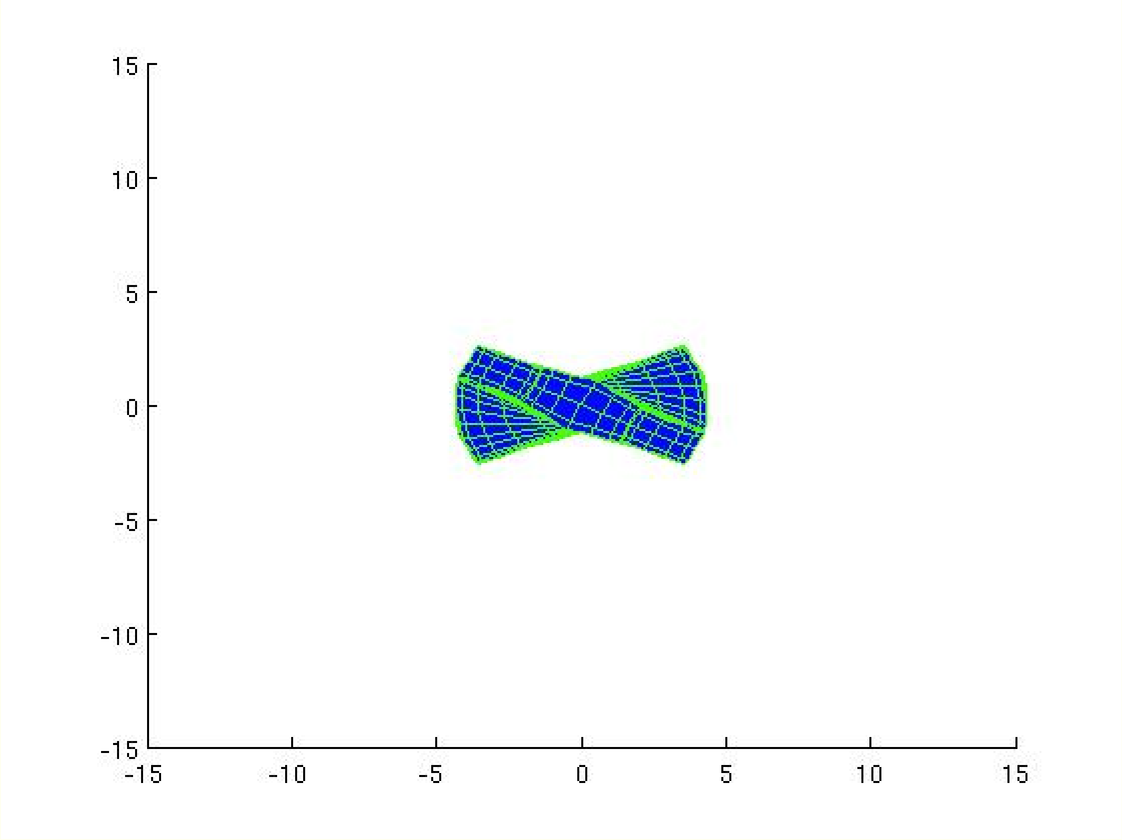
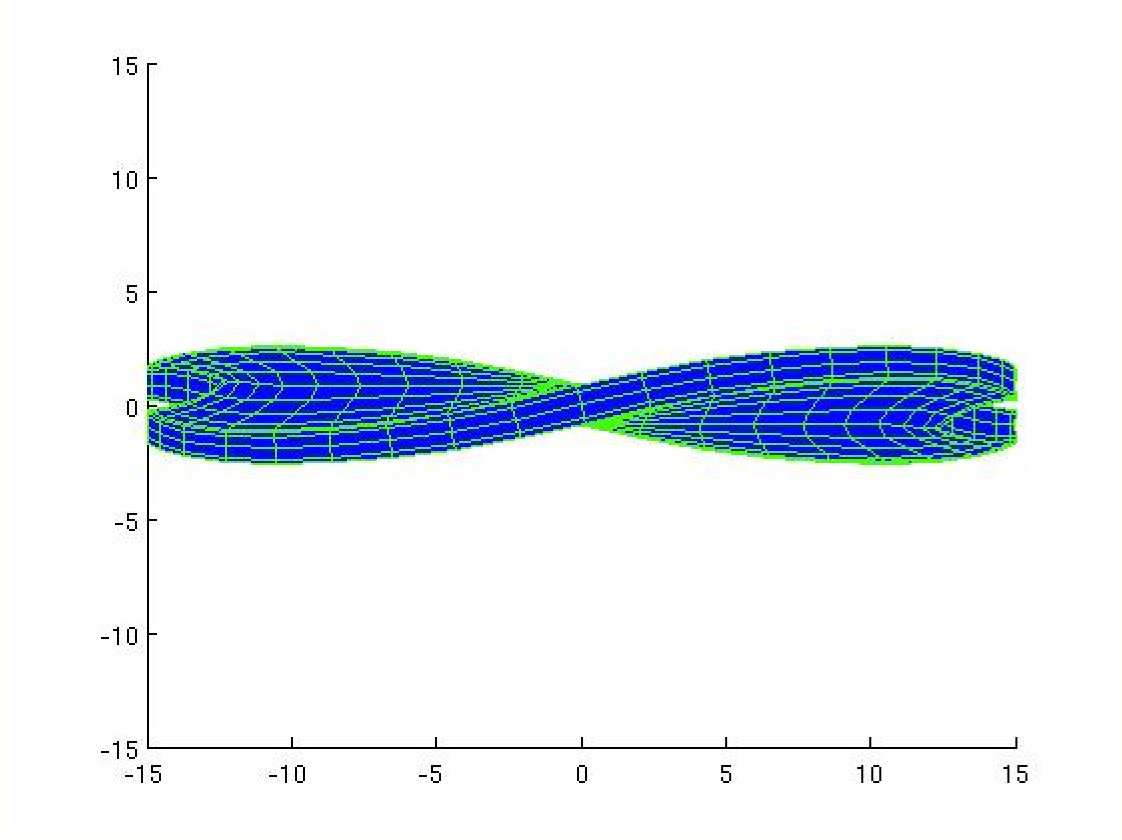


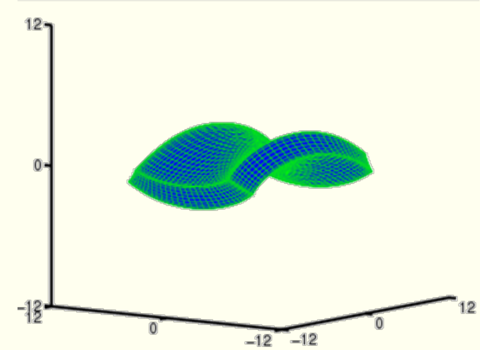
TWIST



TWIST



Modelling



- recent papers:

P. Cesana, A. DeSimone: Strain-order coupling in nematic elastomers: equilibrium configurations. *Math. Models Methods Appl. Sci.*, **19**, 601-630 (2009).

A. Fukunaga, K. Urayama, T. Takigawa, A. DeSimone, L. Teresi: Dynamics of electro-opto-mechanical eects in swollen nematic elastomers. *Macromol.* **41**, 9389-9396 (2008).

T.C. Lubensky, Fangfu Ye: Elastic response and Ward identities in stressed nematic elastomers. *Phys. Rev. E* **82**, 011704 (2010)

R. Selinger, B. Mbanga, J. Selinger, Modeling liquid crystal elastomers; actuators, pumps and robots. In *Emerging Liquid Crystal Technologies III, 69110A-5* SPIE (2008)

Wei Zhu, M. Shelley, P. Palffy-Muhoray, *Modeling and simulation on liquid crystal elastomers*, *Phys. Rev. E* **83**, 051703 (2011)

Summary

Summary

- LCEs combine features of LCs & elastomers
- key feature: coupling of orientational order & strain
- salient feature: responsivity
  - temperature, light, chemicals
- modeling underway