Jerry Gollub – Lecture #3 Topics

(First 3 topics are contained in notes for Lecture #2)

- » Propulsion at low and high Re
- » Vorticity and vortices
- » Surface waves
- » Convection phenomena: Pattern formation
- » Instability of stretched polymeric fluids
- » Mixing - Introduction

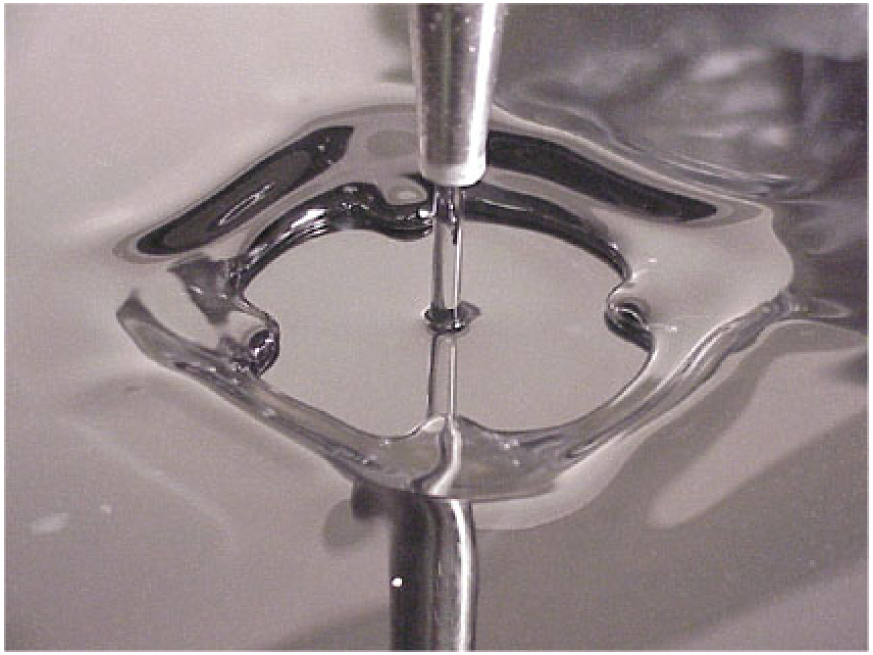
- Lecture 4 (next time)- Mixing of fluids; application to chemistry. Also: continuum mechanics in physics graduate and undergraduate curricula.

Pattern Formation

- Many flows become unstable when driven sufficiently hard. It can be difficult to predict what will happen.
- Movies
  - » Cylinder (vortex street)
  - » Soap_in phase
  - » Taylor-3 (vortices); Taylor

Hydraulic jump with symmetry-breaking

- J. Bush - MIT



Convective instability – an important example

- The most common induced flow thermally induced.
- Horizontal temperature gradients are a very powerful mechanism, and can be arbitrarily small; see VIDEO “Plume”. For 2 vertical plates separated by d and having a temperature difference Θ, the average velocity is

$$U = \alpha\Theta g d^2 / 24\nu$$

- Convective instability: If a layer is warmer at the bottom by ΔT , one potentially has instability, but only if the loss of buoyancy due to diffusion is sufficiently small relative to the drag on a rising blob. A calculation based on linear stability analysis shows that instability occurs if (d=layer thickness, α=thermal exp. coef., κ=thermal diffusivity)

$$Ra = g\alpha\Delta T d^3 / \kappa\upsilon > 1707$$

Equations for convection (M. Cross)

Equations for Convection (Boussinesq)

$$\sigma^{-1}(\partial_t + \mathbf{v}\cdot\nabla)\,\mathbf{v} = -\nabla p + RT\hat{\mathbf{z}} + \nabla^2\mathbf{v}$$

$$(\partial_t + \mathbf{v}\cdot\nabla)\,T = \nabla^2 T$$

$$\nabla\cdot\mathbf{v} = 0$$

Boundary conditions

$$\mathbf{v} = 0 \quad \text{at} \quad z = 0, 1$$

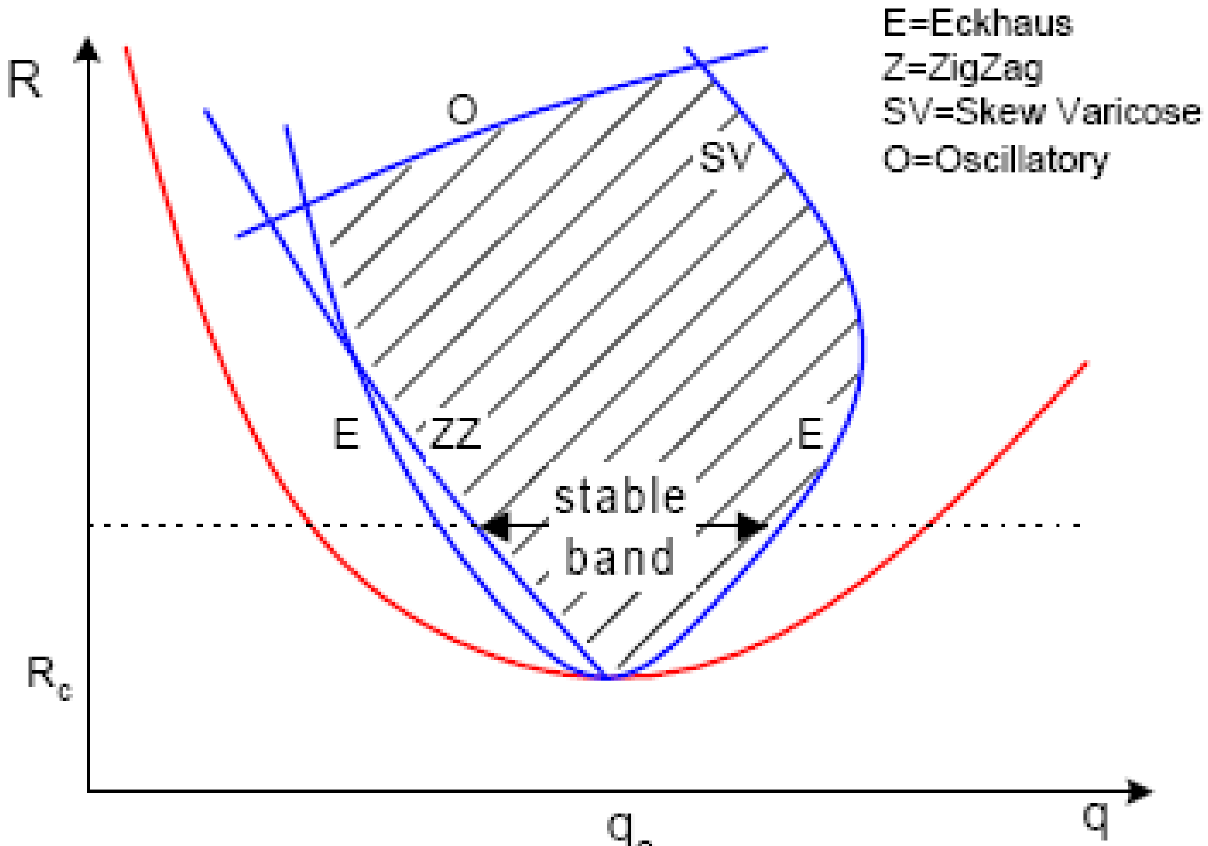
$$T = \begin{cases} 1 & \text{at} \quad z = 0 \\ 0 & \text{at} \quad z = 1 \end{cases}$$

Conducting solution: $\mathbf{v} = 0$, $T = 1 - z$

Convection Patterns

- Linear stability theory is not capable of determining the patterns that arise above onset of convection.
- A nonlinear analysis can be used to delineate the regimes of stability of different steady flow patterns: hexagons, stripes, etc., and to determine the secondary instabilities that occur outside that domain of stability.
- Regimes of "spatiotemporal chaos" occur that are still not quantitatively understood..

Stability for convection rolls

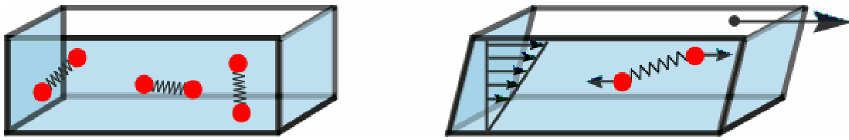


From M. Cross,
http://www.szfki.hu/~physbio/activities/lectures/Cross/lecture1

Polymer Hydrodynamics: Maxwell

- Treat elastic behavior as an ideal Hookean spring

$$\tau + \lambda \frac{\partial}{\partial t}\tau = \eta_0 \dot{\gamma}$$



- Stress is time dependent
- Fluid has an "elastic memory" of order λ

- Relaxation time;

$$\lambda = \frac{\eta_s R_g^3}{k_B T}$$

- Radius of gyration;

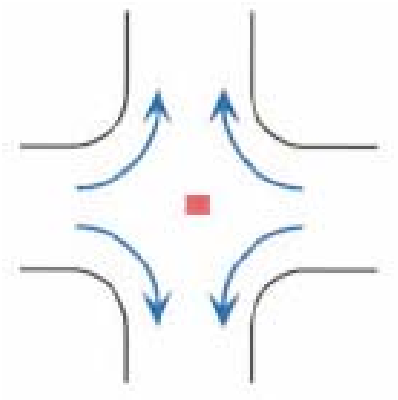
$$R_g = a * N^{3/2}$$

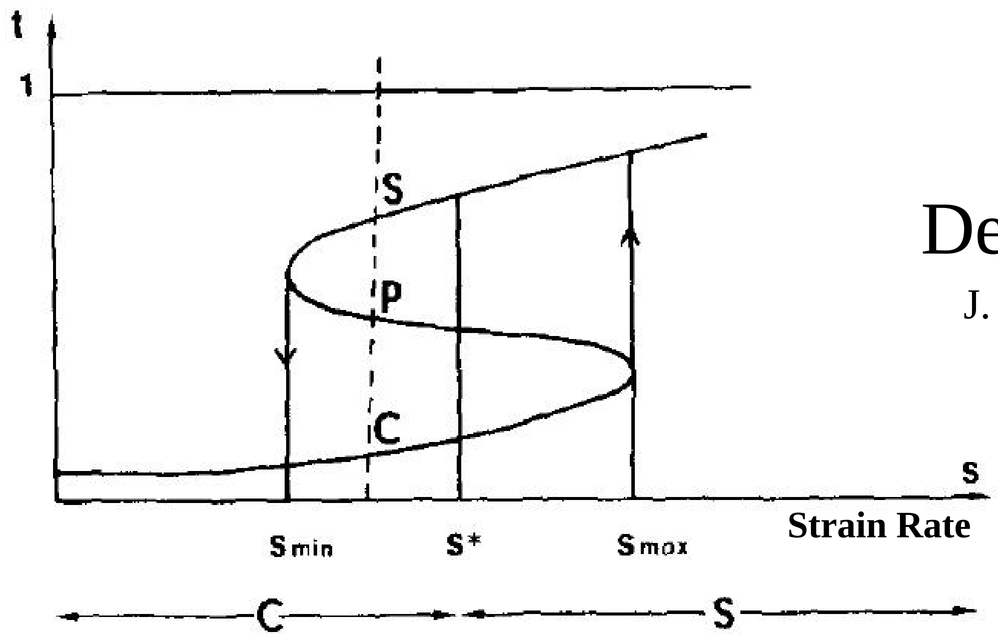
Modeling polymers as beads attached by freely jointed springs

- Deborah number;

$$De = \lambda \dot{\gamma} = \frac{elastic}{viscous}$$

‘Coil-Stretch’ Transition: Flexible Polymers in Extensional Flows





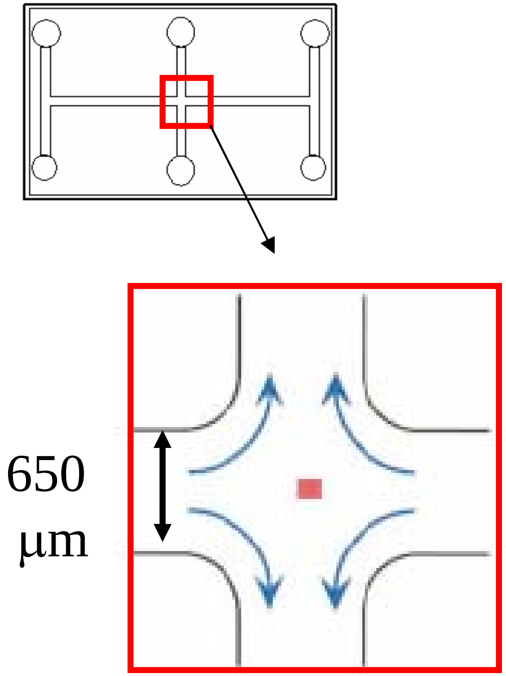
De Gennes, 1974

J. Chem. Phys., **60**, 1974

- For a given strain rate there are 3 possible states of the molecule: Coil (C), Unstable (P), Stretched (S)

- ‘Coil-Stretch’ transition at $\dot{\varepsilon} = 0.5\lambda^{-1}$

Stretching a polymer solution can cause instability



Square Microchannel

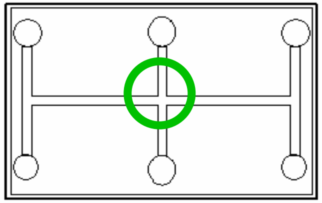
- Dye Experiments
  - Fluorescein
- Particle Tracking Velocimetry
  - 5 µm Polystyrene Particles

$$Re = \frac{\rho L v}{\mu} < 10^{-2}$$

$$De = \dot{\gamma}\lambda \qquad 0 < De < 26$$

$$0.2\ s^{-1} < \dot{\gamma} < 2.5\ s^{-1}$$

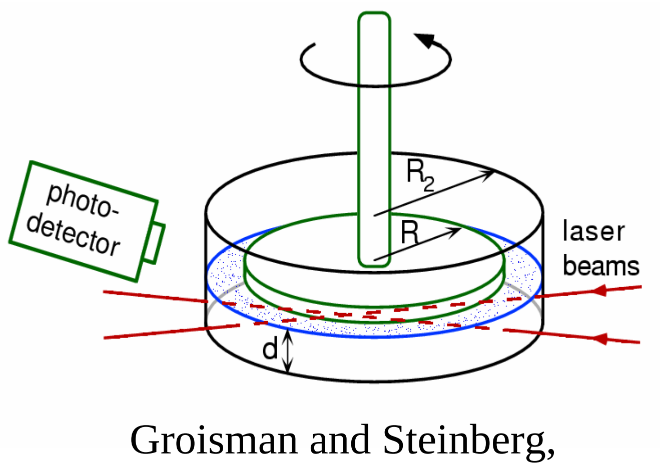
Velocity Fields:
Newtonian or Semi-Rigid vs Flexible



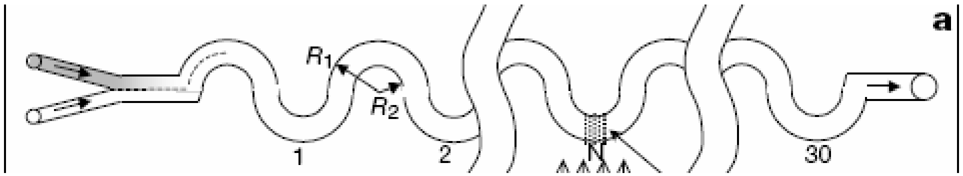
Flexible polymer velocity field deforms; pictures too large to include here; see PRL 2006, Arratia and Gollub. The flexible polymer velocity field is bistable when the molecules are stretched.

‘Turbulence’ with *no* inertia - Steinberg

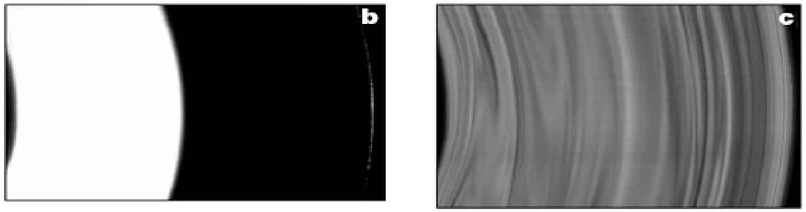
- Elastic Turbulence in *shear flows* at low Re – Victor Steinberg.
  - Power law decay in the velocity power spectra
  - higher viscosity → lower velocity



Groisman and Steinberg, Nature, 405, 2000



Newtonian **Polymer**



Groisman and Steinberg, Nature, 410, 2001

Other interesting topics

- Flow in porous media
- Turbulent boundary layers
- Etc.

END

Next time we will discuss the dynamics of mixing in fluids.