Course outline

1.

A reminder about concepts and an overview of experiments: how to entangle atoms and photons and realise quantum gates.

2.

Tests of complementarity and exploration of the quantum/classical boundary with coherent states of radiation

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Tests of complementarity and exploration of the quantum/classical boundary with coherent states of radiation

Entangle a qubit with a mesoscopic system: how to encode information in a large object

When is a coherent field “quantum” or “classical”?

How to prepare large Schrödinger cats with a resonant atom/field interaction?

Outline of lecture

2.1. **A complementarity experiment at the quantum/classical boundary**

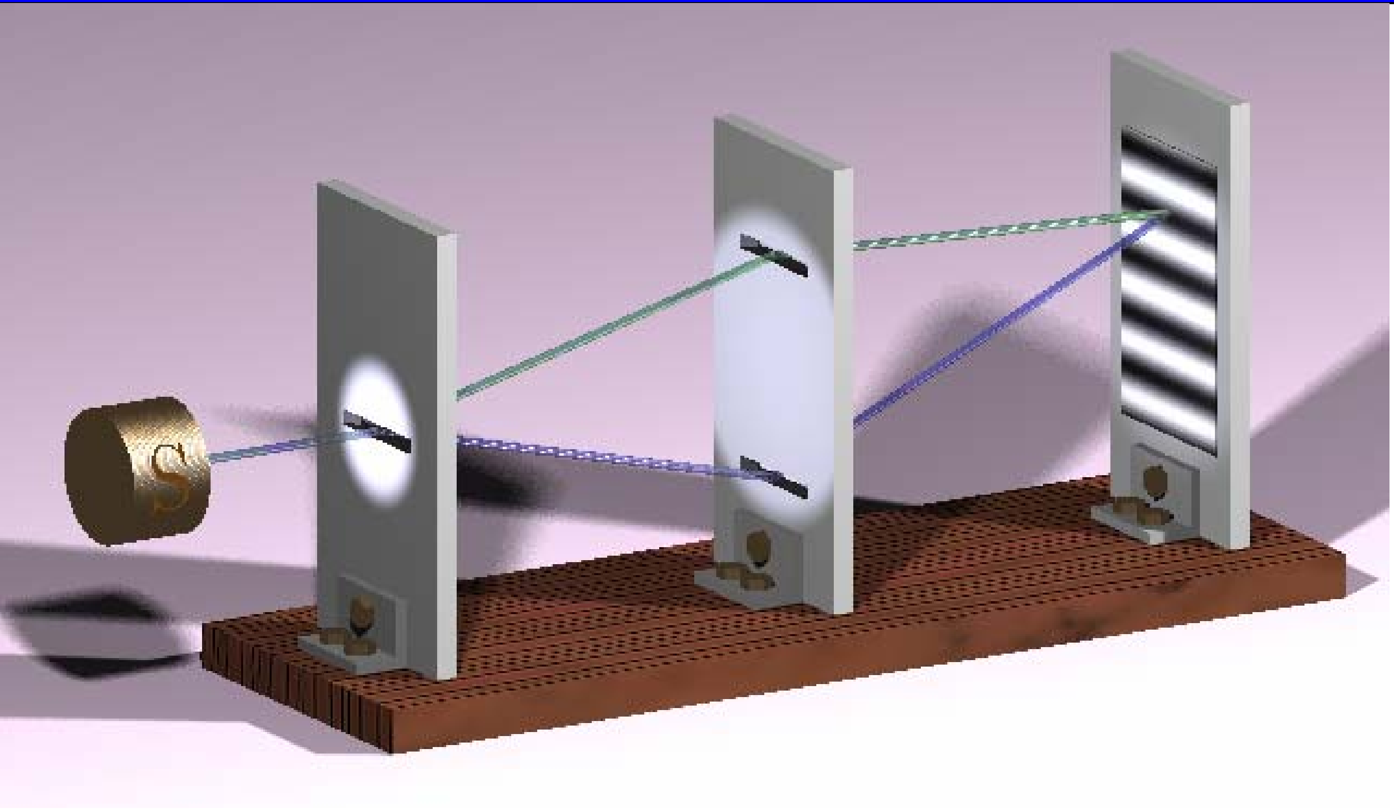
Realization of a thought experiment based on Rabi oscillation and Ramsey interferometry

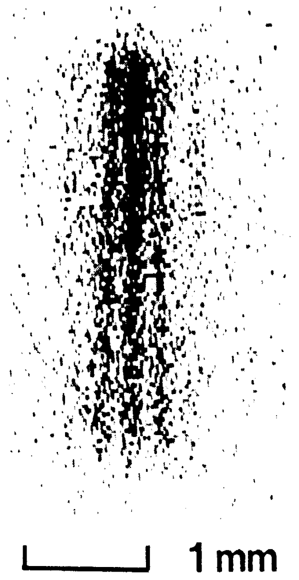
2.2. **Single atom/mesoscopic field entanglement: how a coherent field evolves from quantum to classical .**

- An unexpected aspect of Rab Oscillationi
- A new tool to prepare and study Schrödinger cats

2.1. A complementarity experiment at the Quantum/Classical boundary

The “strangeness” of the quantum





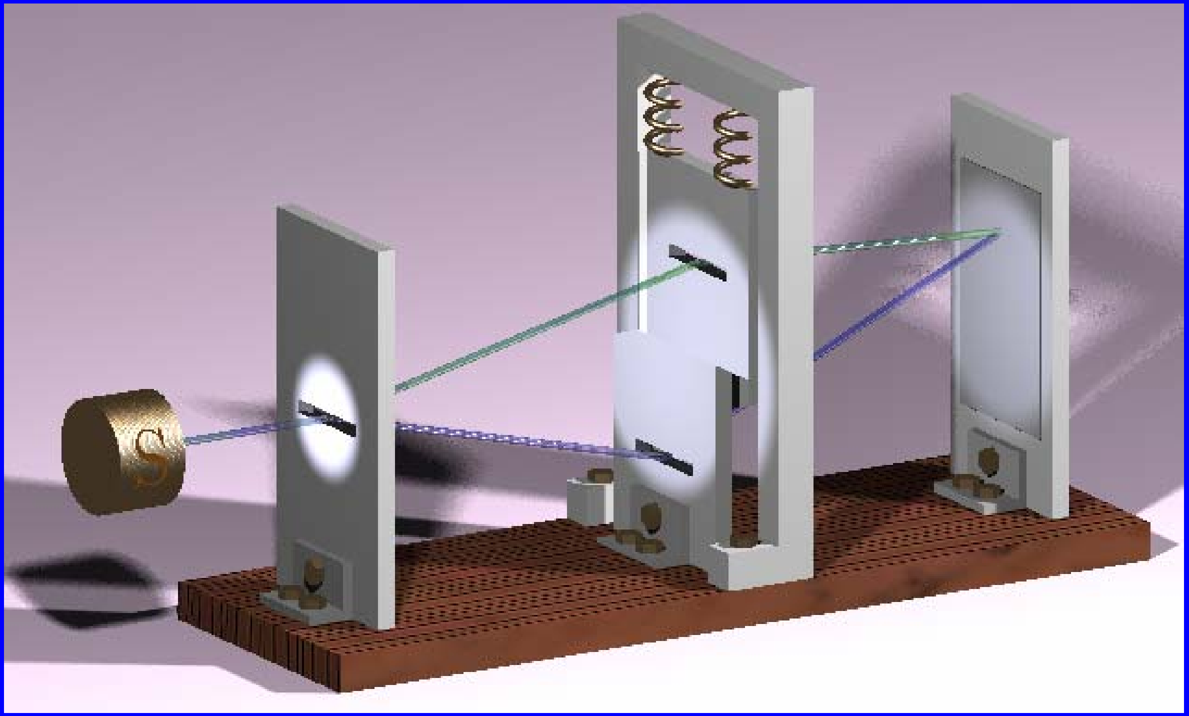
Shimizu et al 1992

- Feynman: Young’s slits experiment contains all the mysteries of the quantum

The “strangeness” of the quantum:

a thought experiment about complementarity

(Bohr-Einstein debate, Solvay 1927)

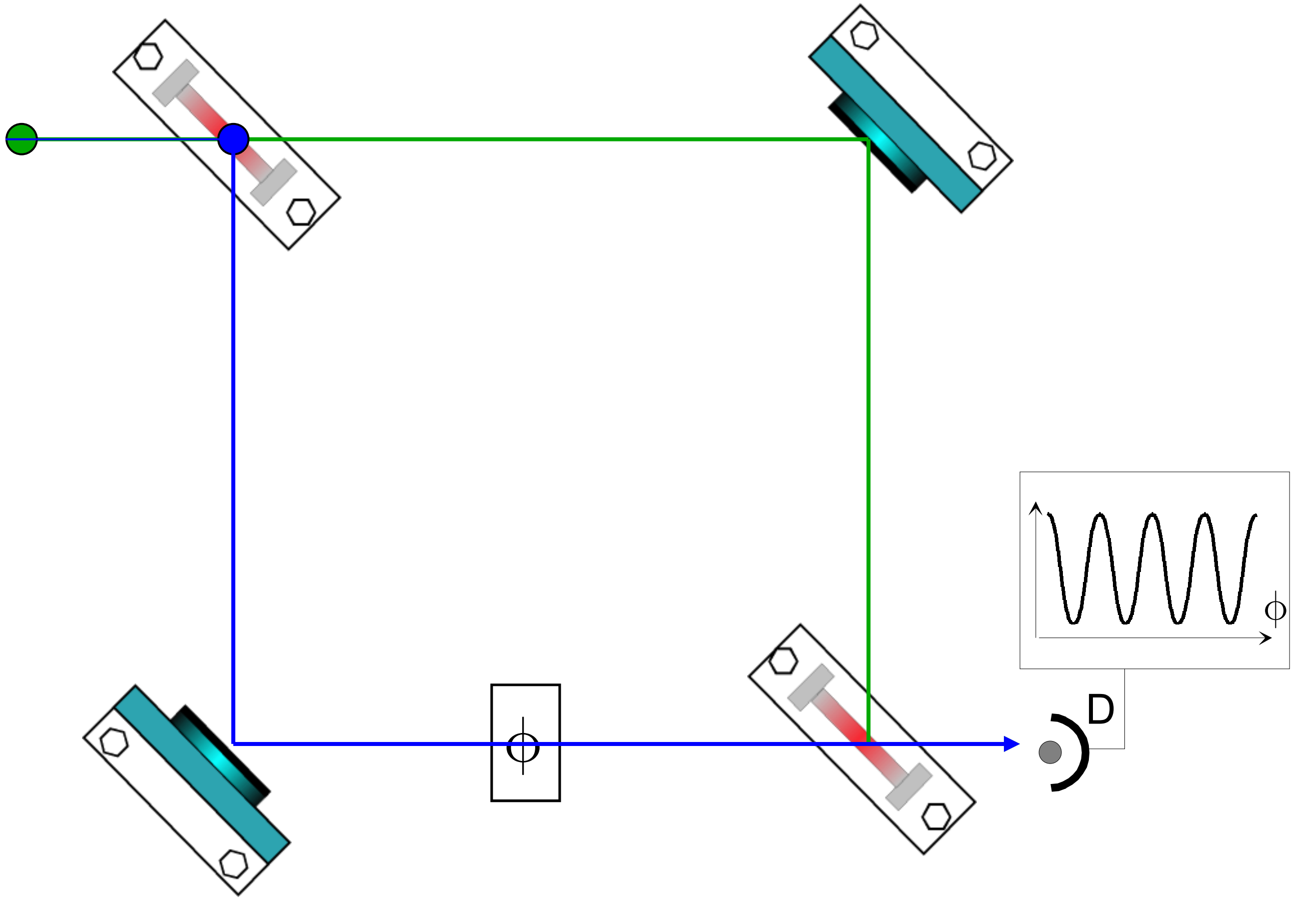


Particle/slit entanglement

- Microscopic slit: set in motion when deflecting particle. Which path information and no fringes
- Macroscopic slit: insensitive to interfering particle. No which path information: fringes are observed.
- Wave and particle are complementary aspects of the quantum object.

A “modern” version of Bohr’s proposal

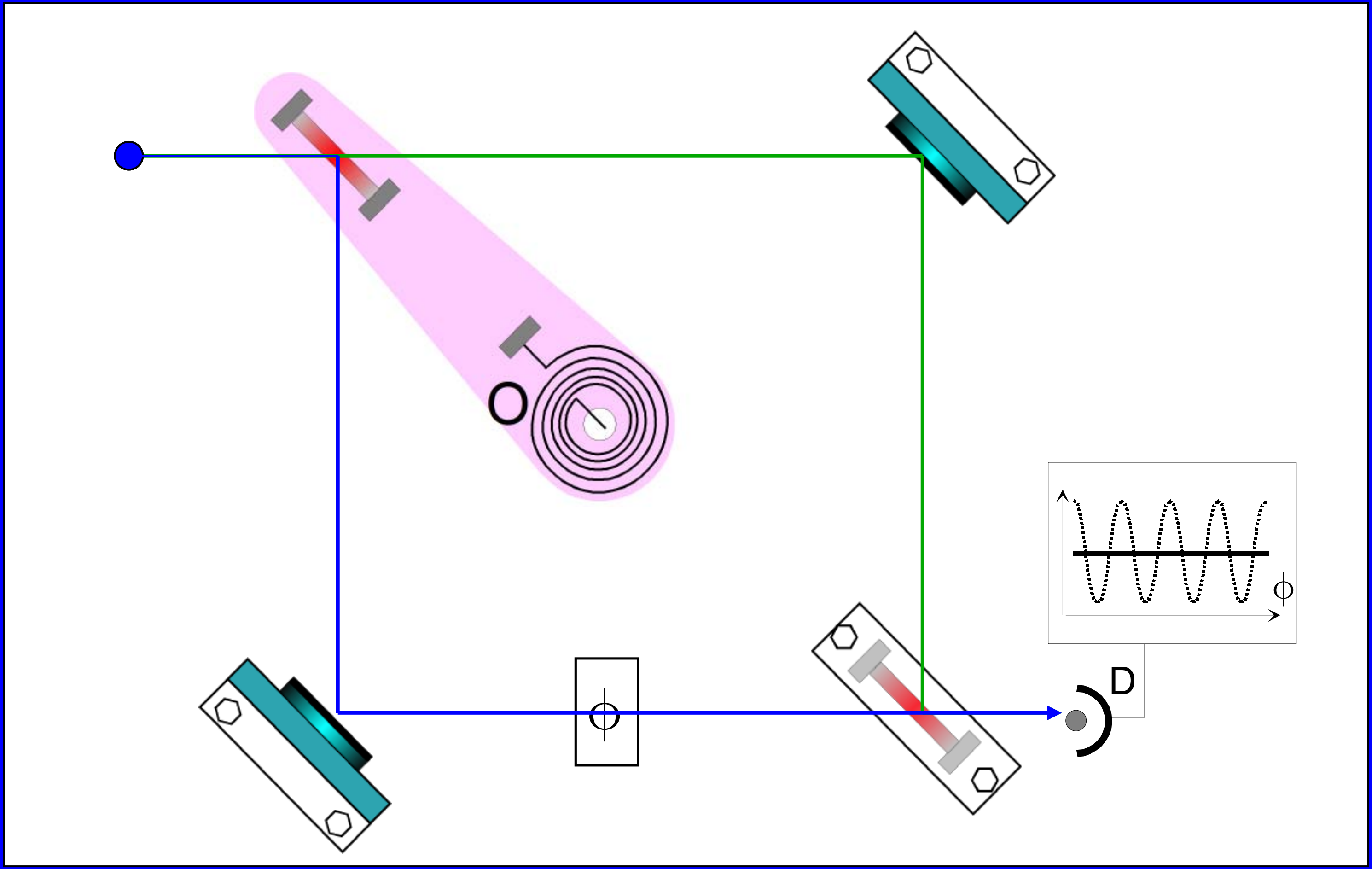
- Mach Zehnder interferometer



•Interference between two well-separated paths.

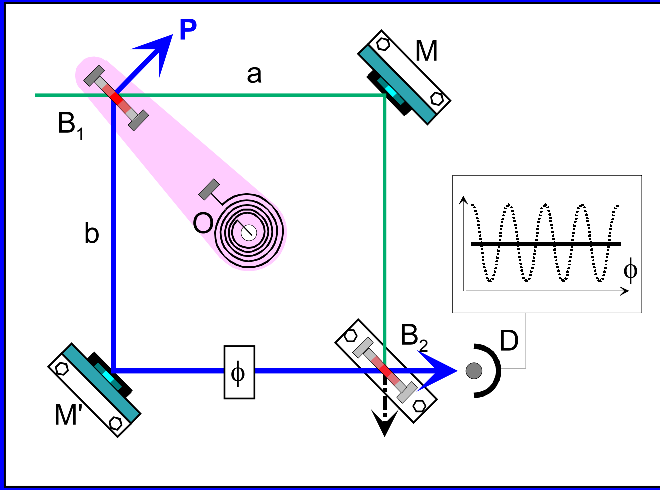
- Getting a which-path information?

A “modern” version of Bohr’s proposal



- Massive beam splitter: negligible motion, no which- path information, fringes
- Microscopic beam splitter: which path information and no fringes

Complementarity and entanglement



- A more general analyzis of Bohr’s experiment
  - Initial beam-splitter state $|0\rangle$
  - Final state for path b $|\alpha\rangle$
  - Particle/beam-splitter state $|\Psi\rangle = |\Psi_a\rangle|0\rangle + |\Psi_b\rangle|\alpha\rangle$
    - Particle/beam-splitter entanglement
    - (an EPR pair if states orthogonal)
  - **Final fringes signal** $\left|\langle\Psi_a|\Psi_b\rangle\langle 0|\alpha\rangle\right|$
    - Small mass, large kick

NO FRINGES $|\langle 0|\alpha\rangle| = 0$

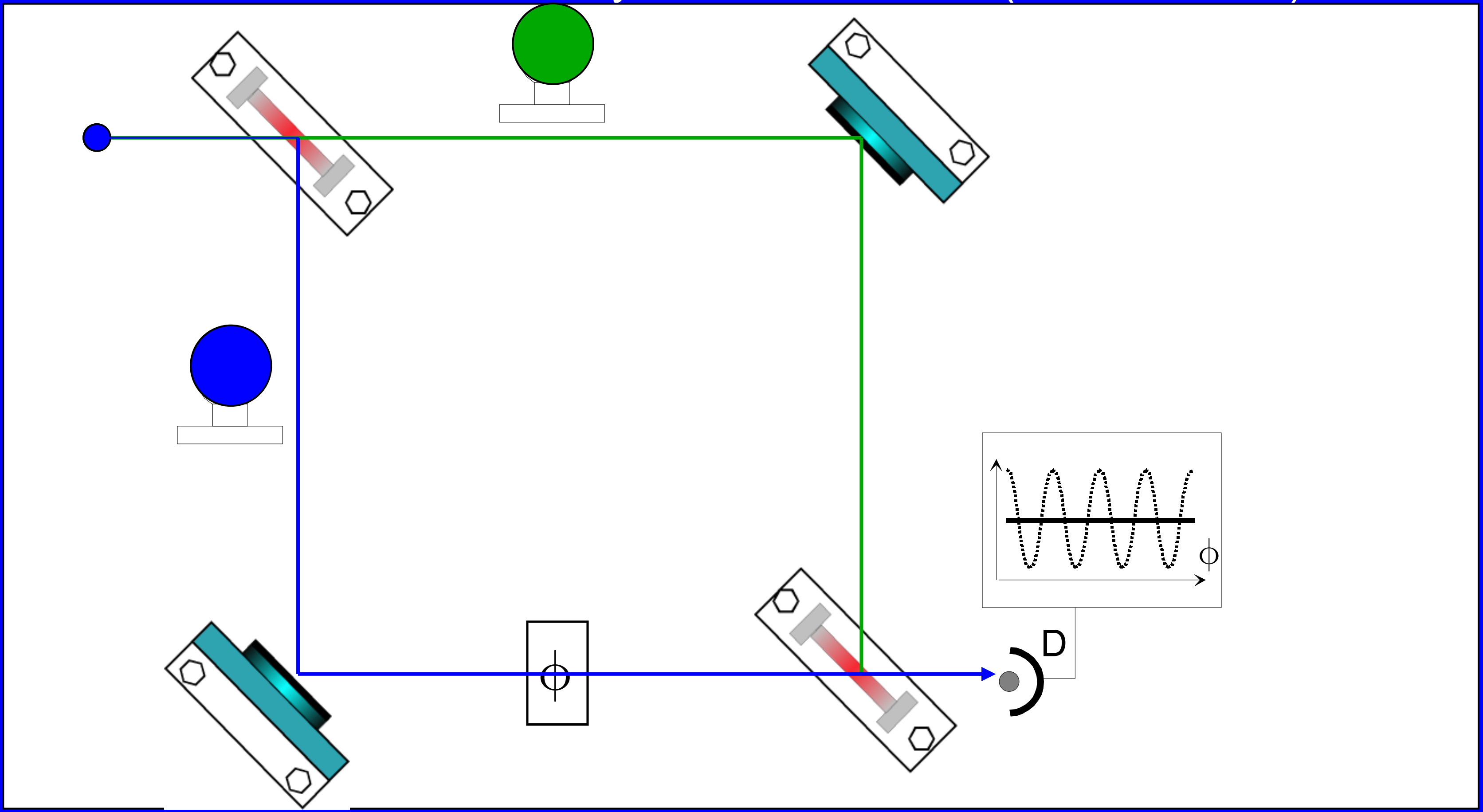
– **Large mass, small kick** $|\langle 0|\alpha\rangle| = 1$

FRINGES

Entanglement and complementarity

Entanglement with another system destroys interference

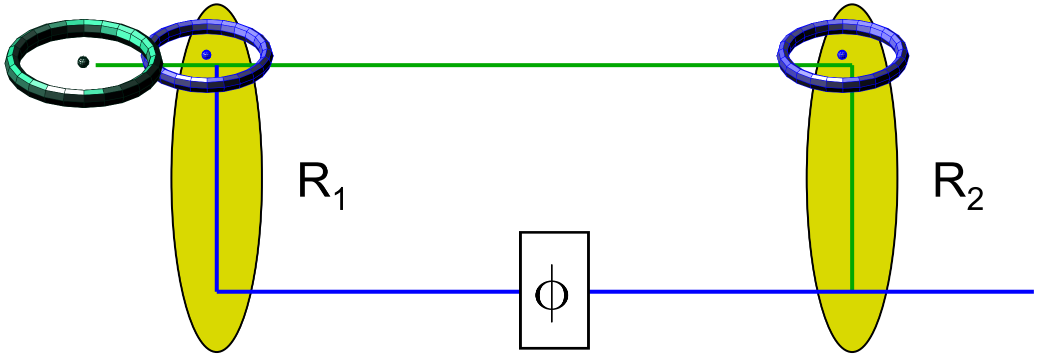
- explicit detector (beam-splitter/ external)
- uncontrolled measurement by the environment (decoherence)

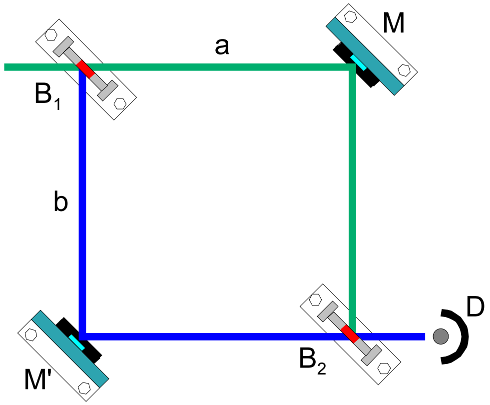


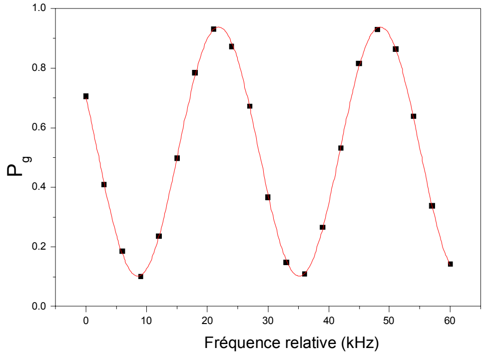
Complementarity, decoherence and entanglement intimately linked

A more realistic system: Ramsey interferometry

- Two resonant $\pi/2$ classical pulses on an atomic transition e/g







Which path information?

Atom emits one photon in R_1 or R_2

Ordinary macroscopic fields

(heavy beam-splitter)

Field state not appreciably affected. No "which path" information

FRINGES

Mesoscopic Ramsey field

(light beam-splitter)

Addition of one photon changes the field. "which path" info

NO FRINGES

Coherent states of the field: a system evolving from quantum to classical

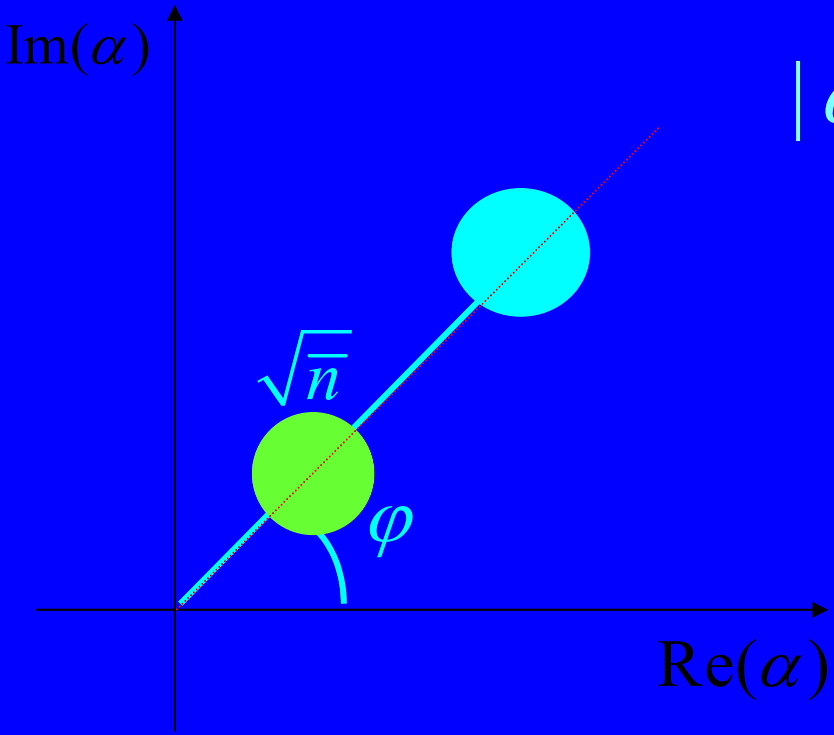
Field radiated by a classical source in the mode

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_n \frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

Poissonian distribution of the photon number

$$p(n) = e^{-|\alpha|^2}\frac{|\alpha|^n}{n!}$$

Representation in the complex plane



$$|\alpha| = \sqrt{\bar n} = \Delta n \Rightarrow \boxed{\frac{\Delta n}{\bar n} = \frac{1}{|\alpha|}}$$

$|\alpha| \approx 1$ "Quantum" field
Big fluctuations

$|\alpha| \gg 1$ "Classical" field
Small fluctuations

$|\alpha|$: a continuous parameter to explore the quantum classical boundary

Experimental requirements

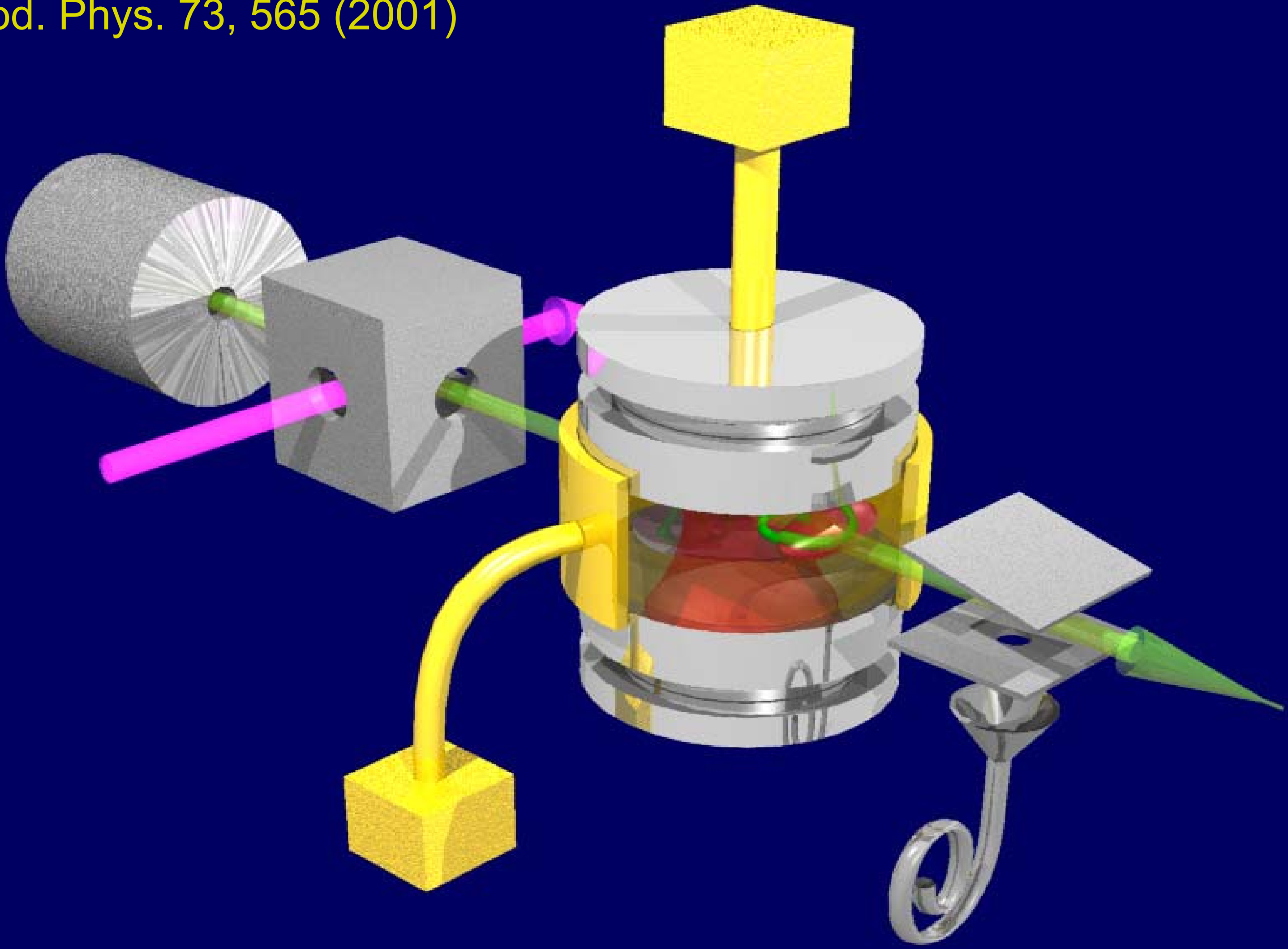
- Ramsey interferometry
  - Long atomic lifetimes
  - Millimeter-wave transitions
    - Circular Rydberg atoms

$\pi/2$ pulses in mesoscopic fields
  - Very strong atom-field coupling
    - Circular Rydberg atoms

- Field coherent over atom/field interaction
    - Superconducting millimeter-wave cavities

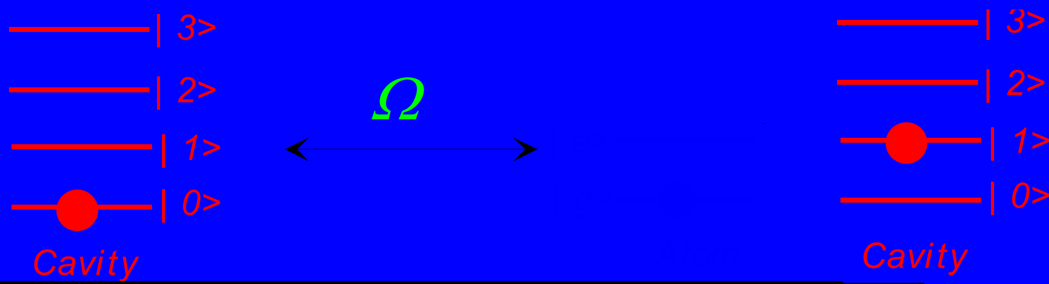
General scheme of the experiments

Rev. Mod. Phys. 73, 565 (2001)



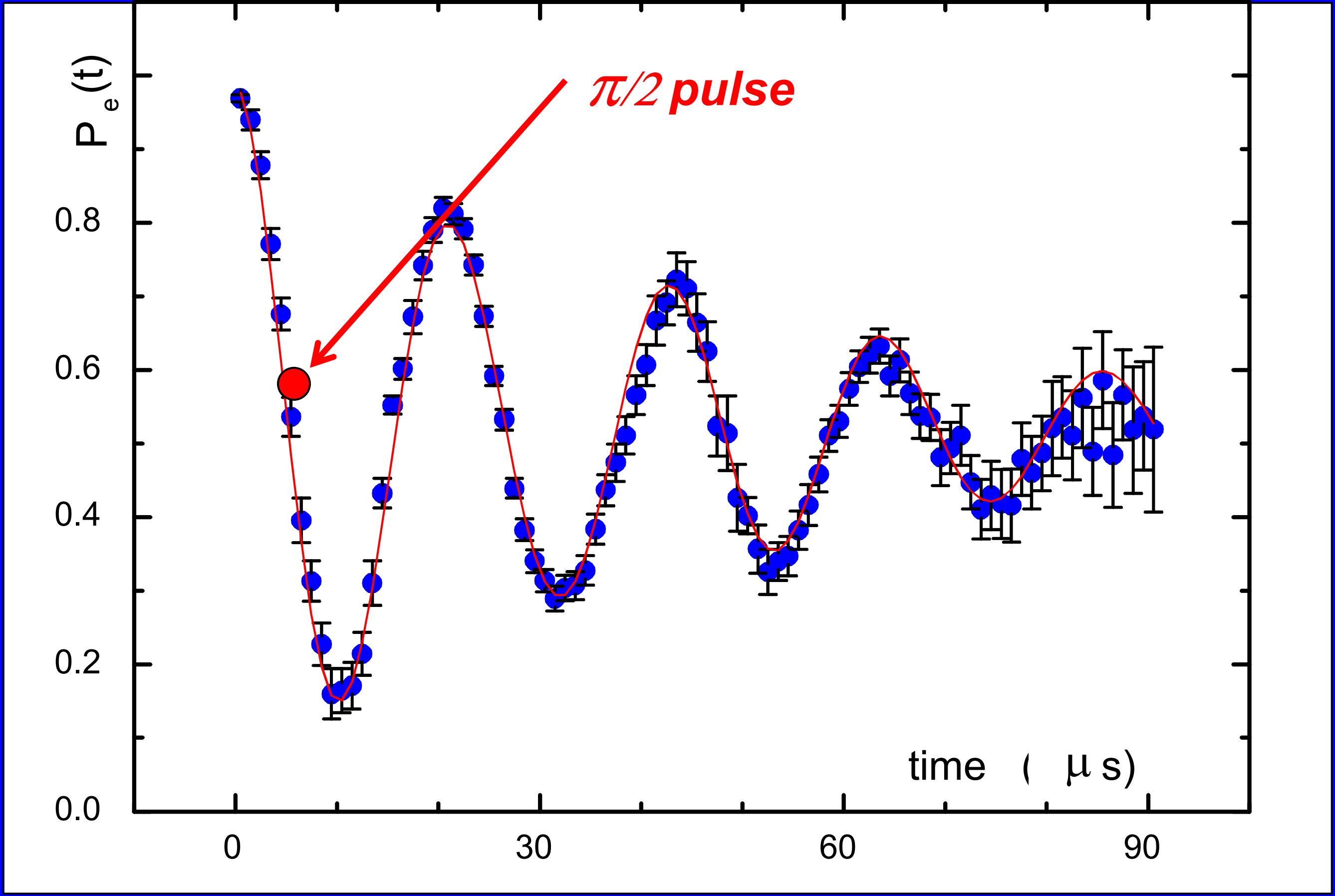
Resonant atom-cavity interaction: Rabi oscillation in vacuum

Initial state |e,0>



Vacuum Rabi frequency

$\Omega = 50$ kHz

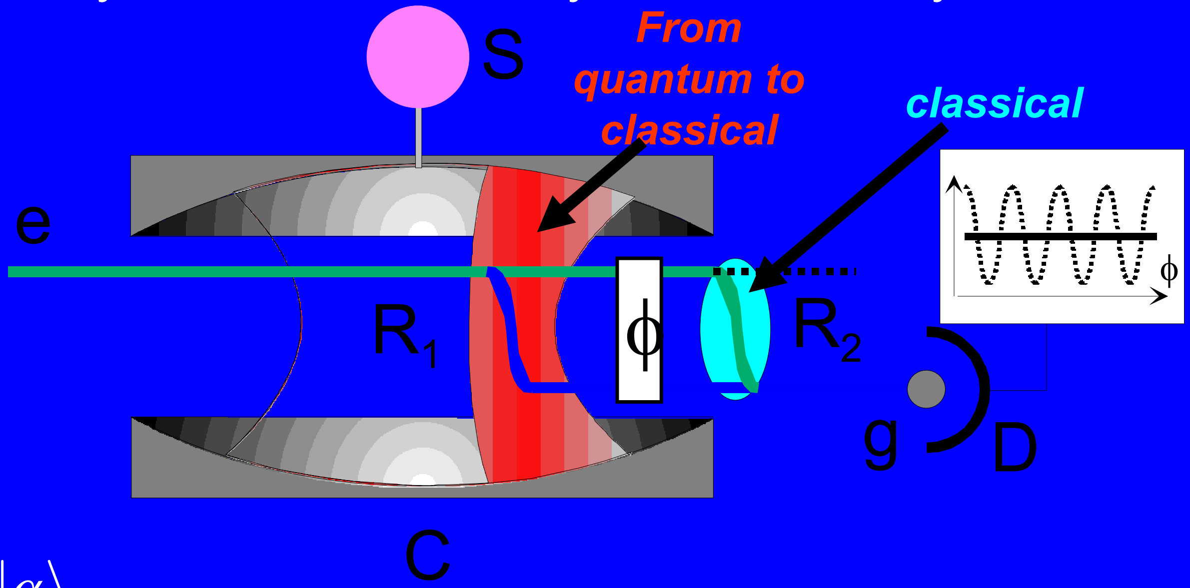


In a large coherent field, Rabi frequency becomes $\Omega\sqrt{n}$

Oscillatory Spontaneous emission and strong coupling regime.

Bohr's experiment with a Ramsey interferometer

- Illustrating complementarity: Store one Ramsey field in a cavity

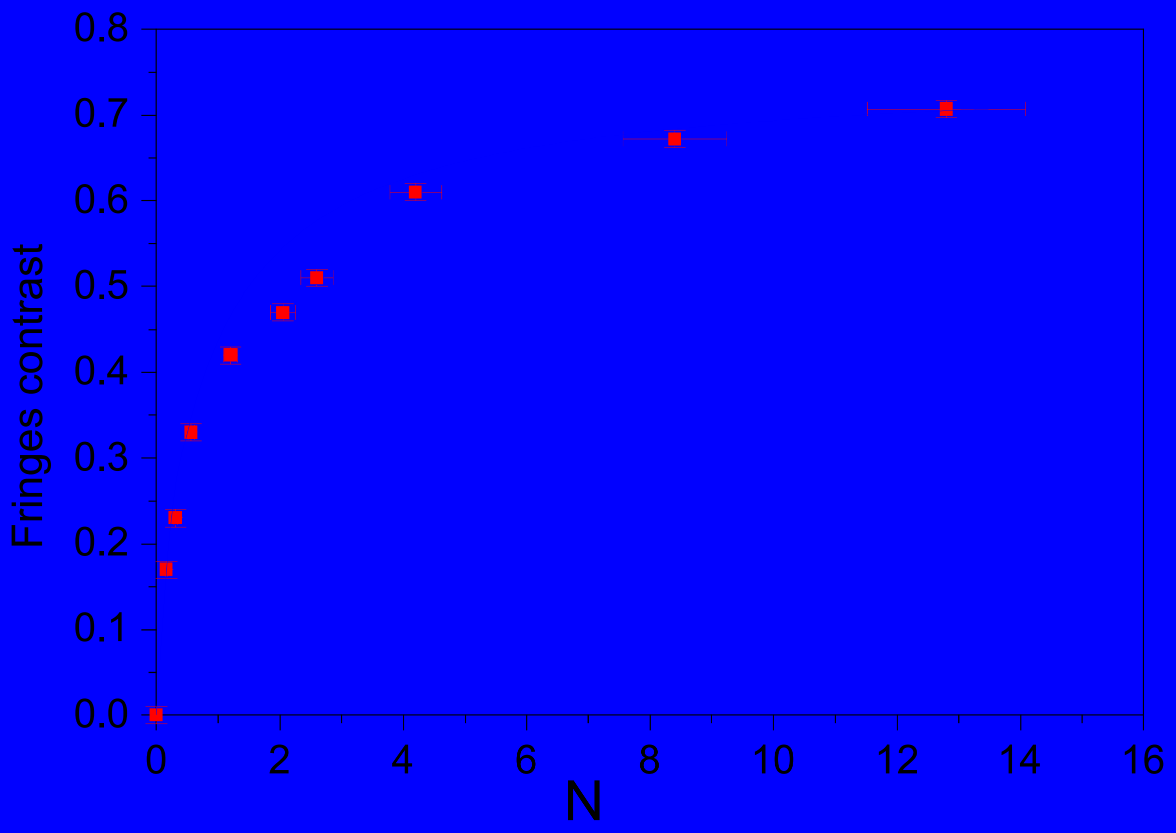


Atom-cavity interaction time
Tuned for π/2 pulse
Possible even if C empty

  - Initial cavity state $|\alpha\rangle$
  - Intermediate atom-cavity state $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|e,\alpha_e\rangle + |g,\alpha_g\rangle\right)$
    - Ramsey fringes contrast $|\langle\alpha_e|\alpha_g\rangle|$
  - Large field
    - $|\alpha_e\rangle \approx |\alpha_g\rangle \approx |\alpha\rangle$ FRINGES
  - Small field
    - $|\alpha_e\rangle = |0\rangle, |\alpha_g\rangle = |1\rangle$ NO FRINGE

Quantum/classical limit for an interferometer

Fringes contrast versus photon number N in first Ramsey field



Fringes vanish for quantum field

photon number plays the role of the beam-splitter's "mass"

Also an illustration of the $\Delta N \Delta \Phi$ uncertainty relation :

- Ramsey fringes reveal field pulses phase correlations.
- Small quantum field: large phase uncertainty and low fringe contrast

Nature, **411**, 166 (2001)

An elementary quantum eraser

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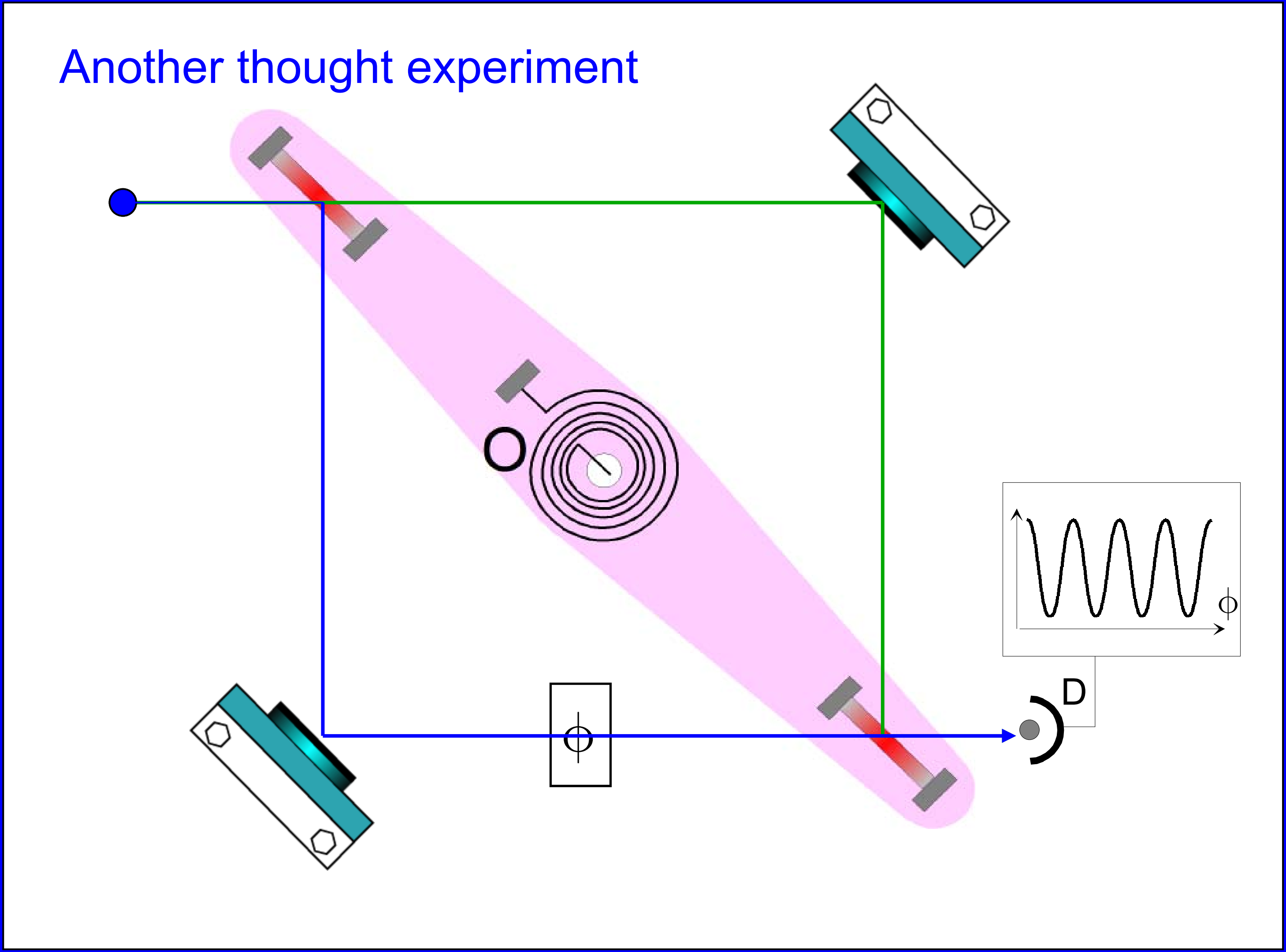
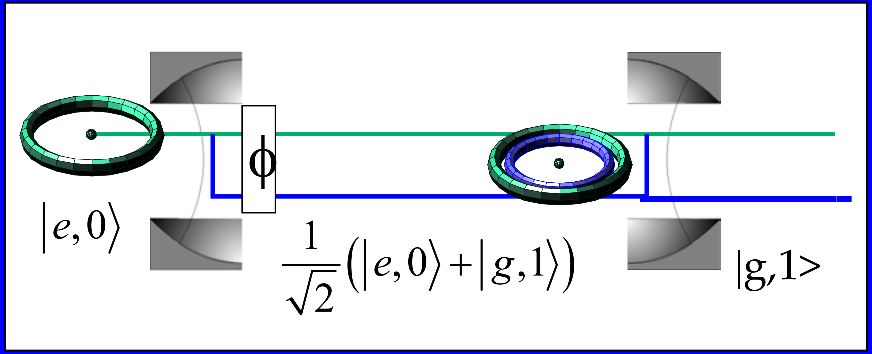


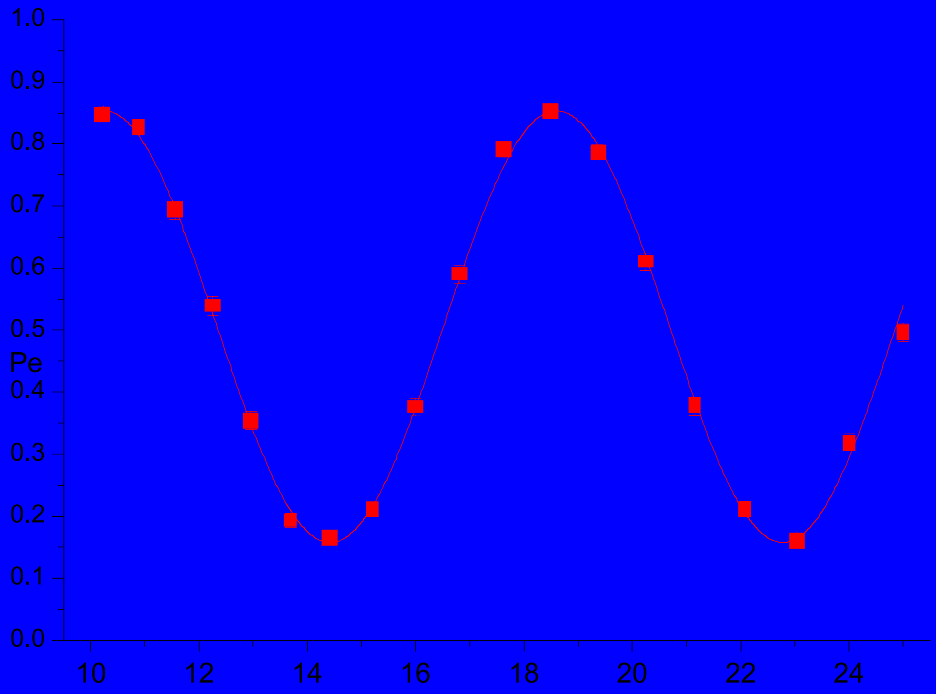
Two interactions with the same beamsplitter assembly erase the which path information and restore the interference fringes

Ramsey “quantum eraser”

- A second interaction with the mode erases the atom-cavity entanglement



Atom found in g: one photon in C
whatever the path:no info and fringes



- Ramsey fringes without fields !
  - Quantum interference fringes without external field
  - A good tool for quantum manipulations

Entanglement between a mesoscopic coherent field and a single atom

The Ramsey interference experiment shows that, ***during a $\pi/2$ pulse***, the atom and the field do not get entangled when ***$n \gg 1$***:

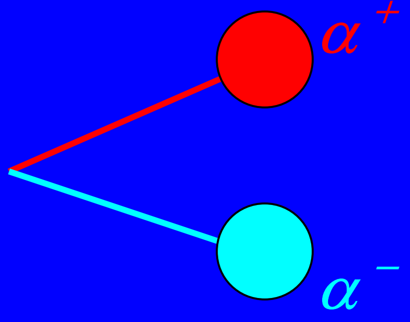
NO ENTANGLEMENT during time $t_{\pi/2} = \pi / 2 \Omega \sqrt{n}$

Atom and field get however ***ENTANGLED*** if they are coupled for a longer time, of the order of $2\pi/\Omega$:

Atom dipole states

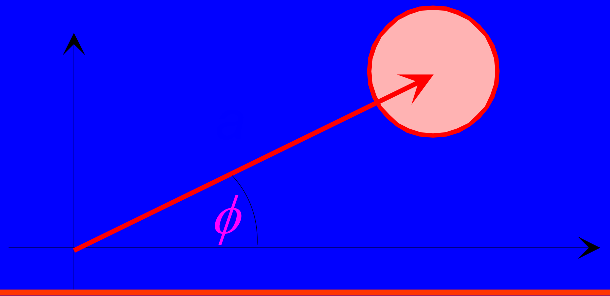
$$|e\rangle\,|\alpha\rangle \xrightarrow{t > 2\pi/\Omega} |\Psi^{+}_{\text{atom}}\rangle\,|\alpha^{+}\rangle + |\Psi^{-}_{\text{atom}}\rangle\,|\alpha^{-}\rangle$$

Coherent field split into two components: Mesoscopic superposition of coherent states with opposite phases



Rabi oscillation collapse and revivals revisited

To be classical a field in a cavity must be coherent and contain many photons on average.



Correspondance principle: a coherent field with many photons has small relative fluctuations and behaves asymptotically classically

The interaction with an atom, which can emit or absorb at most one photon, is expected to leave a « large » field practically « unperturbed » and the « atom + field system » unentangled:

$$| \alpha (0)> | \Psi_{atom}(0)> \rightarrow | \alpha (t)> | \Psi^{(\alpha)}{}_{atom}(t)>$$

How large must the photon number be for this classical limit to be valid?

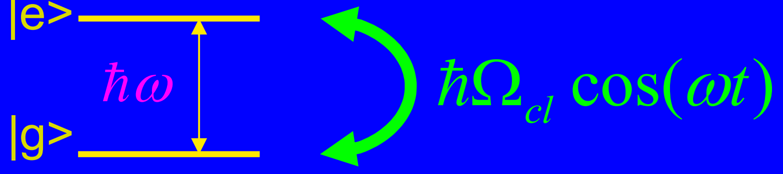
It depends on how long the interaction lasts…A large field exhibits quantum features if the interaction with the atom has enough time to create entanglement….**and if there is no decoherence**

Mesoscopic physics in Quantum Optics

2.2

Single atom/mesoscopic field entanglement:
how a coherent field evolves from quantum to classical

Classical Rabi oscillation

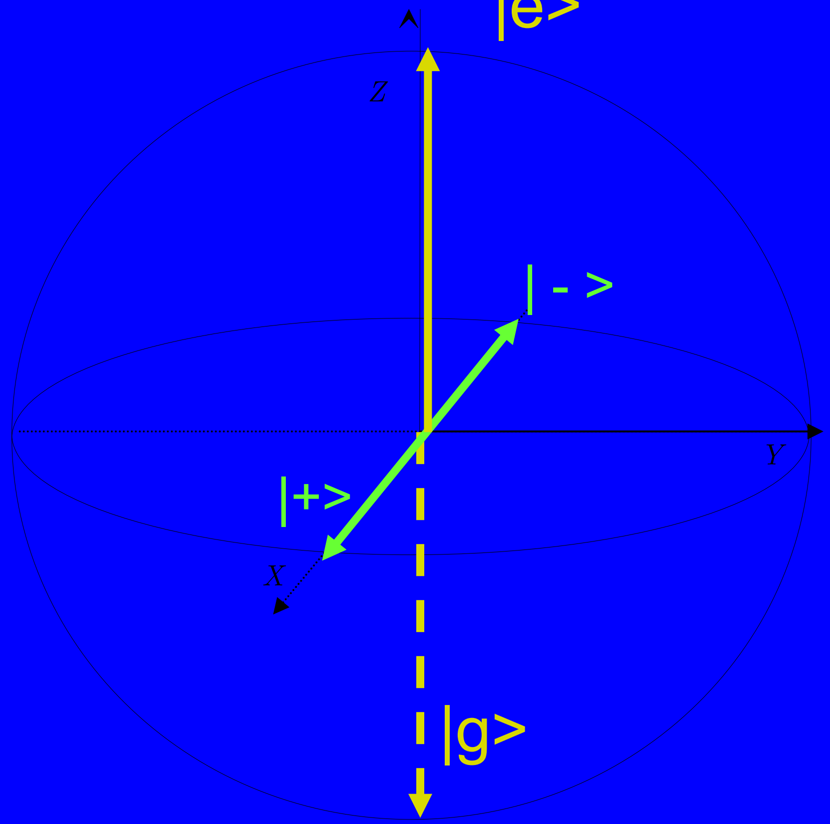


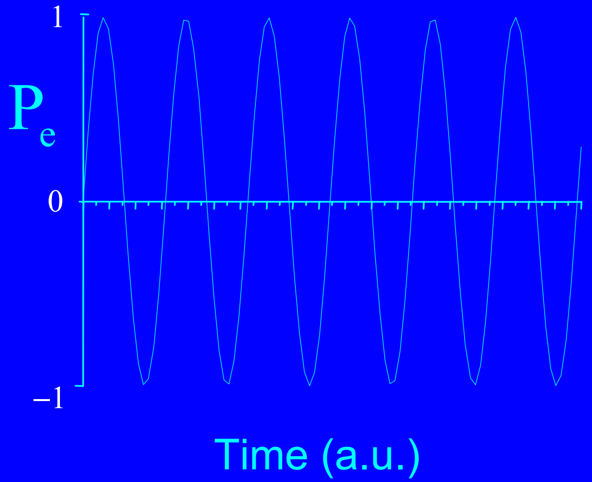
Two-level system {|e>;|g>}
interacting with a resonant field

Rotating frame
Rotating wave approximation

Eigenstates of the hamiltonian

$$\begin{cases} |+> = \dfrac{1}{\sqrt{2}}(|e>+|g>) \\ |-> = \dfrac{1}{\sqrt{2}}(-|e>+|g>) \end{cases}$$



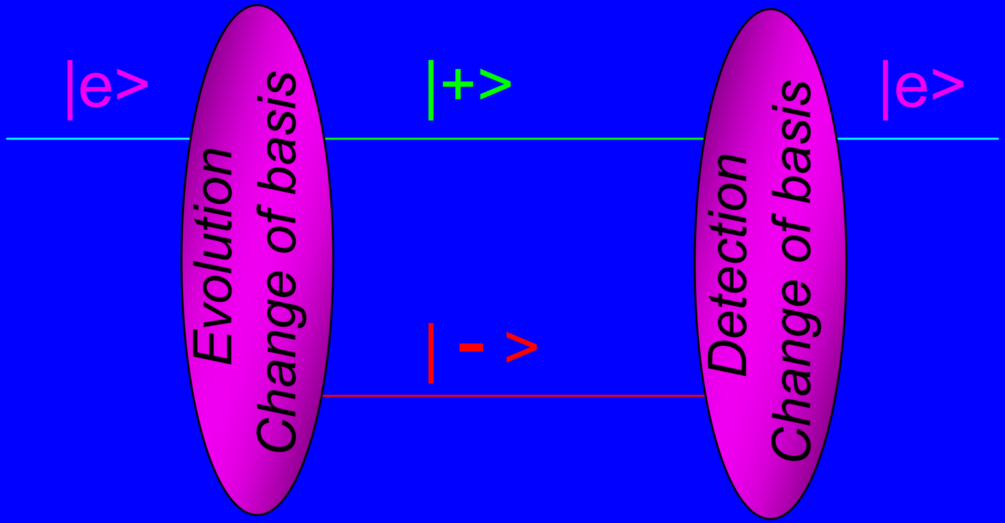


Rabi oscillation at frequency Ω_{cl}

Classical Rabi oscillation: an interference effect

Evolution $|e> \rightarrow |\psi(t)> = \frac{1}{\sqrt{2}}\left(e^{-i\Omega t/2}\,|+> + e^{i\Omega t/2}\,|->\right)$

Detection in {|e>,|g>} basis



|e>

Evolution
Change of basis

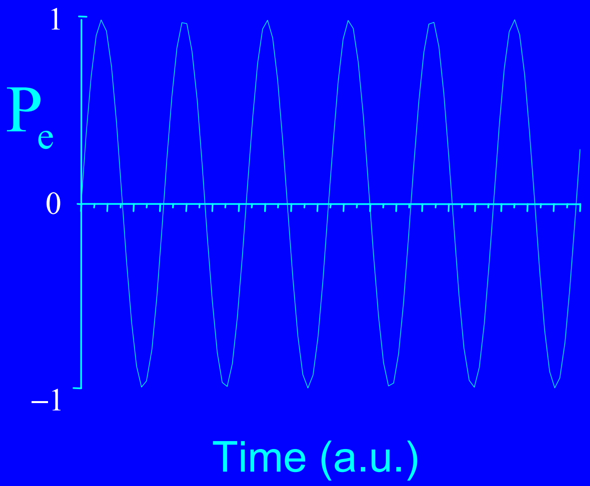
|+>

| - >

Detection
Change of basis

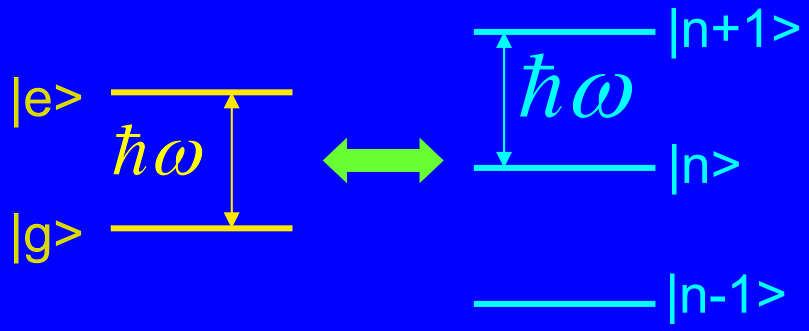
|e>

Classical Rabi oscillation:
a quantum beat between
two indistinguishable paths



Rabi oscillation in a quantized field

Two-level system {|e>;|g>} interacting with a resonant quantized field |n>



Jaynes-Cummings hamiltonian

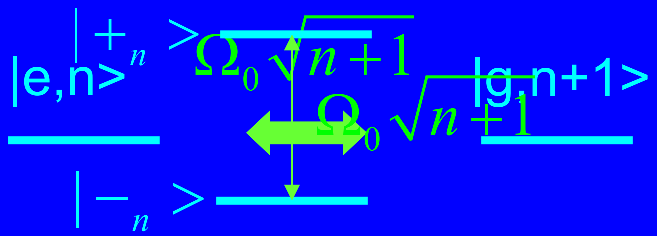
$$H_{JC} = \frac{\hbar\Omega_0}{2}\left(a^+ \,|\, g >< e \,|+ a \,|\, e >< g \,|)\right)$$

Exchange of a quantum of energy

|e,n>; |g,n+1> : two levels coupled and degenerated

Eigenstates of the hamiltonian: "Dressed states"

$$\begin{cases} |+_n> = \frac{1}{\sqrt{2}}(|e,n> + |g,n+1>) \\ |-_n> = \frac{1}{\sqrt{2}}(|e,n> - |g,n+1>) \end{cases}$$



Rabi oscillation between |e,n> and |g,n+1> at frequency

$$\Omega_0\sqrt{n+1}$$

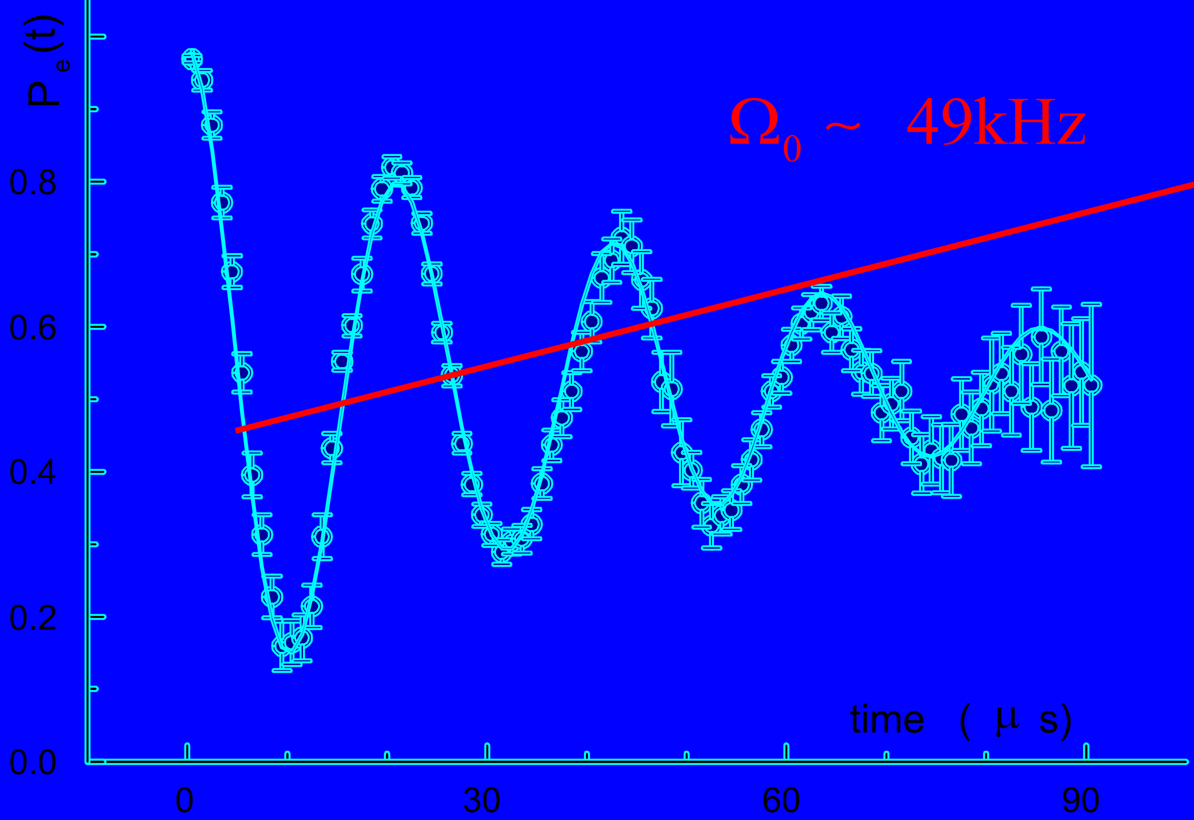
The vacuum Rabi oscillation

Initial state |e,0> → Rabi oscillation at Ω_0

Vacuum Rabi frequency

$$|e,0\rangle \rightarrow \cos\left(\frac{\Omega_0 t}{2}\right)|e,0\rangle - i\sin\left(\frac{\Omega_0 t}{2}\right)|g,1\rangle$$

Atom-field entangled state



$\Omega_0 \sim$ 49kHz

Maximal entanglement at

$$\Omega_0 t = \frac{\pi}{2}$$

Formation of an EPR-pair

Rabi oscillation in a quantum field

→ entanglement

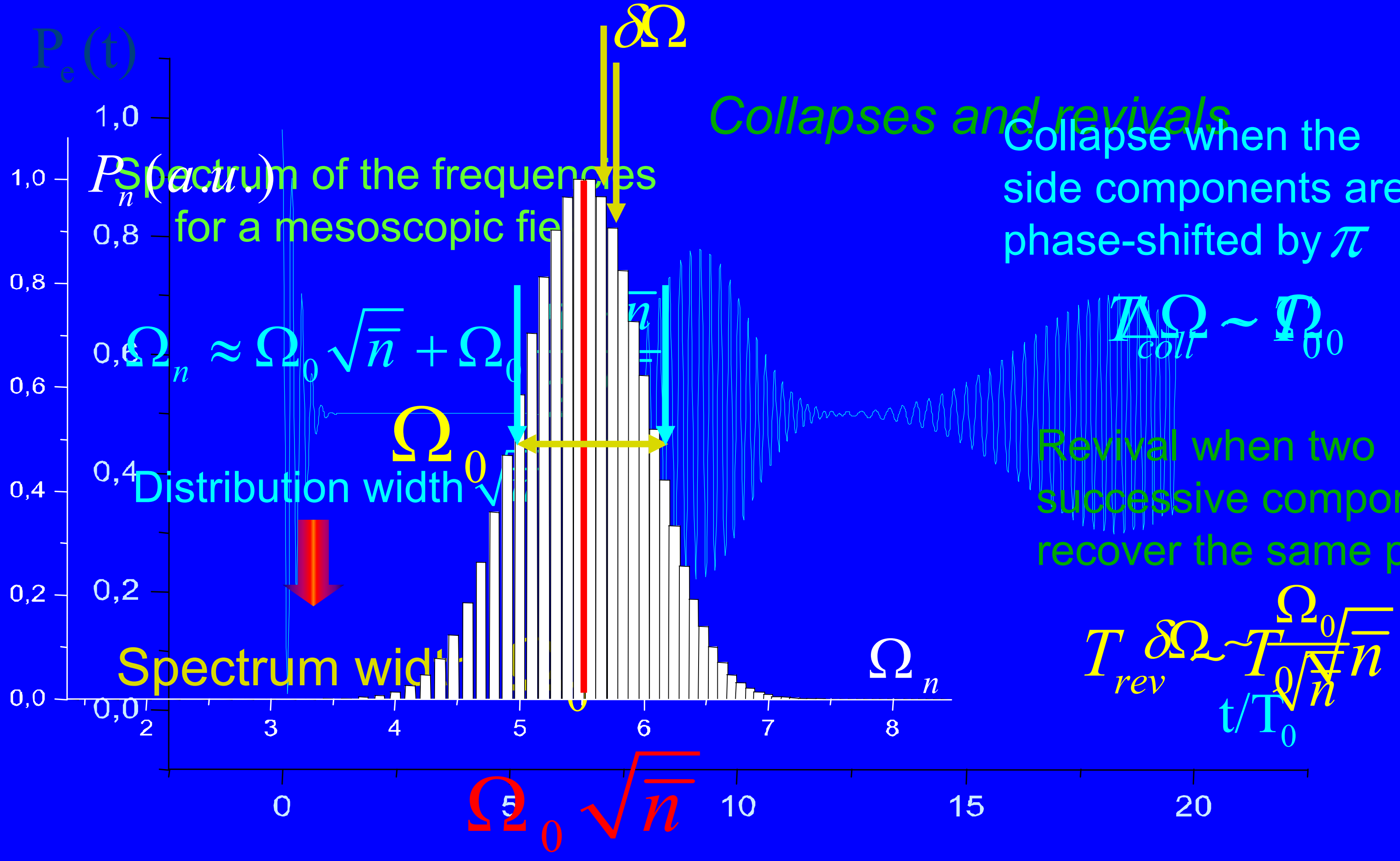
Rabi oscillation in a classical field

→ No entanglement

Continuous evolution?

Rabi oscillation in a mesoscopic coherent field

$$P_e(t)=\frac{1}{2}\sum_n p(n)\left(\cos(\Omega_n t)+1\right) \qquad \Omega_n=\Omega_0\sqrt{n+1}$$



Spectrum of the frequencies for a mesoscopic fie

$$\Omega_n \approx \Omega_0\sqrt{\bar n}+\Omega_0$$

Distribution width $\sqrt{}$

Ω_0

Spectrum widt

Collapses and revivals

Collapse when the side components are phase-shifted by π

$$\Delta\Omega\sim\Omega_0$$

$$T_{coll}\sim T_0$$

Revival when two successive components recover the same phase

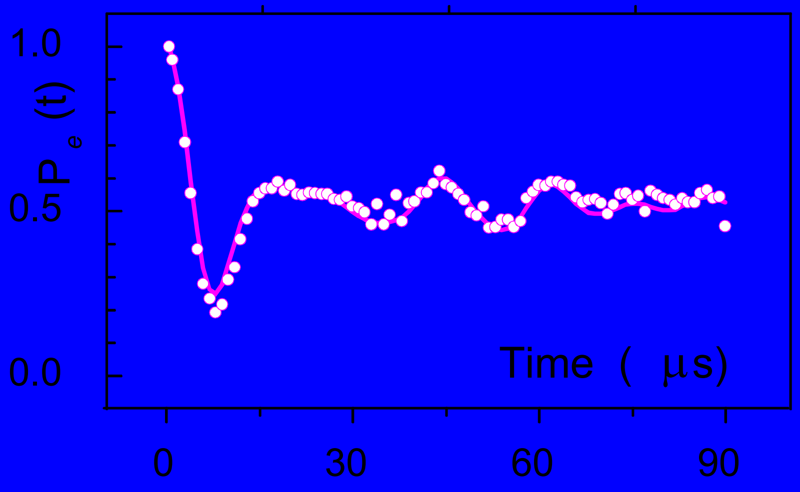
$$T_{rev}\,\delta\Omega\sim$$

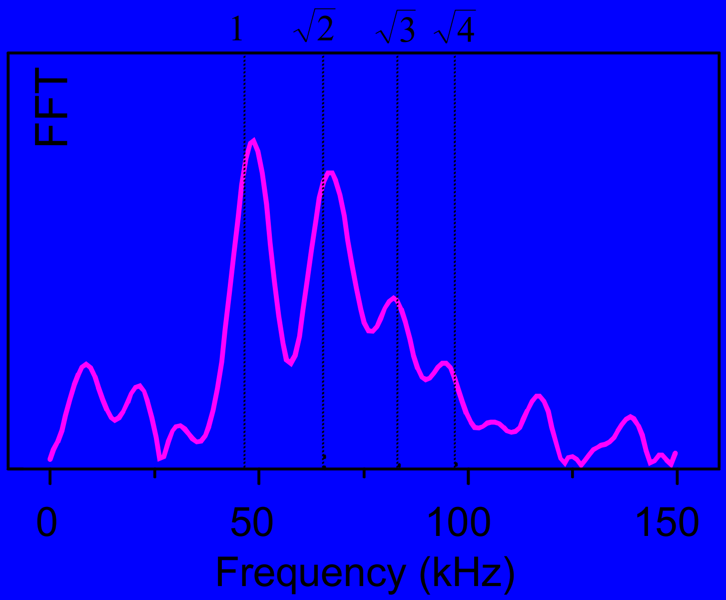
$$\frac{\Omega_0}{\sqrt{\bar n}}$$

$$T_{rev}\sim T_0\sqrt{\bar n}$$

Collapse and revival in cavity QED

(Brune et al PRL **76**, 1800)





Also observed for $\bar{n} \propto 1$ in

- closed cavities (Rempe, PRL **58**, 353)
- ion traps (Meekhof, PRL **76**, 1796)

Direct proof of field quantization (photon graininess)

What about larger n's?

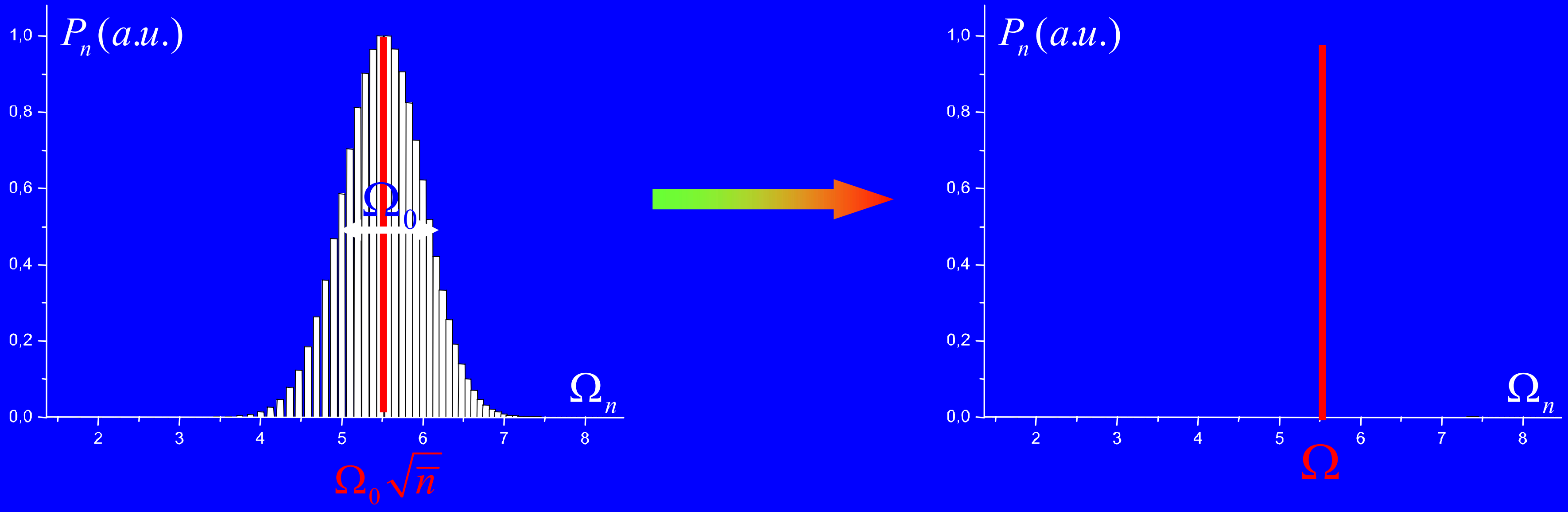
What about the field evolution in this complex Rabi oscillation process ?

Classical limit

Spectrum of the Rabi frequencies

In a coherent field

In the classical limit



« Classical limit »

$$\sqrt{\bar{n}} \to \infty$$

$$\Omega_0 \to 0$$

$$\Omega_0\sqrt{\bar{n}} \to \Omega$$

Atomic signal only depends on the energy of the field

Effect on the phase of the field?

Rabi oscillation in a mesoscopic field

Initial state: $|e,\alpha> = \sum_n C_n |e,n>$

$$|e,\alpha> = \frac{1}{\sqrt{2}}\left(\sum_n C_n |+_n> + \sum_n C_n |-_n>\right)$$

with $C_n = e^{-|\alpha|^2/2}\frac{\alpha^n}{\sqrt{n!}}$

$$|\psi_+(t)> \qquad |\psi_-(t)>$$

Mesoscopic field $|\psi_+(t)> = |\psi_+^{at}(t)>|\alpha_+(t)>$

$$\begin{cases} C_n \approx C_{n+1} \\ \sqrt{n} \approx \sqrt{\bar{n}} + \frac{n-\bar{n}}{2\sqrt{\bar{n}}} + o\left((n-\bar{n})^2\right) \end{cases}$$

$$\begin{cases} \bar{n} \gg 1 \\ \Omega_0 t \ll 16\sqrt{\bar{n}} \end{cases}$$

$$|\psi_+^{at}(t)> = \frac{1}{\sqrt{2}}\left(e^{-2i\bar{n}\varphi}e^{-i\varphi}|e> + |g>\right)$$

Atomic superposition of quantum phase $-\varphi(t)$

$$|\alpha_+(t)> = e^{i\bar{n}\varphi(t)}|\alpha e^{-i\varphi(t)}>$$

Coherent field of classical phase $-\varphi(t)$

$$\varphi(t) = \frac{\Omega_0 t}{4\sqrt{\bar{n}}}$$

→ Phase correlation

Rabi oscillation in a mesoscopic field

$$|e,\alpha> \rightarrow \frac{1}{\sqrt{2}}\Big(|\alpha_+(t)>|\psi_+^{at}(t)> + |\alpha_-(t)>|\psi_-^{at}(t)>\Big)$$

The atomic dipole and the field are phase-entangled

Generation of a Schrödinger-cat state

Classical limit

$$\sqrt{\bar{n}} \rightarrow \infty$$

$$\Omega_0 \rightarrow 0$$

$$\Omega_0\sqrt{\bar{n}} \rightarrow \Omega$$

$$\varphi = \frac{\Omega_0 t}{4\sqrt{\bar{n}}} \rightarrow 0$$

$$\bar{n}\varphi = \frac{\Omega_0\sqrt{\bar{n}}\,t}{4} \rightarrow \frac{\Omega t}{4}$$

$$|\alpha_+>, |\alpha_-> \rightarrow |\alpha>$$

$$|\psi_+> \rightarrow e^{-i\Omega t/2}|+>$$

$$|\psi_-> \rightarrow e^{i\Omega t/2}|->$$

$$|e,\alpha> \rightarrow \frac{1}{\sqrt{2}}\underbrace{\left(e^{-i\Omega t/2}|+> + e^{i\Omega t/2}|->\right)}_{\text{Classical Rabi oscillation}}\underbrace{|\alpha>}_{\text{Field unchanged}}$$

No atom-field entanglement

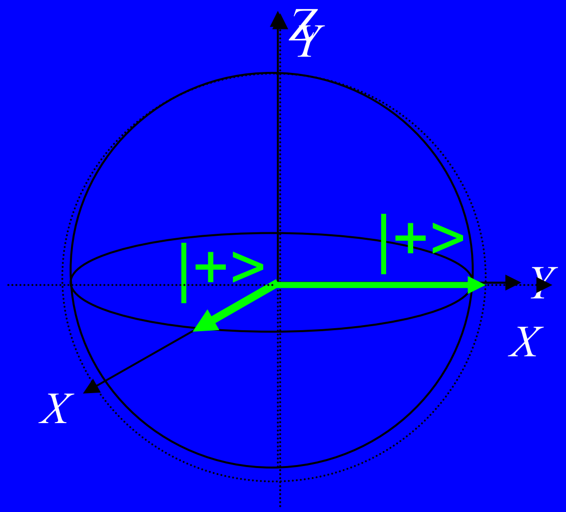
Field « classical »

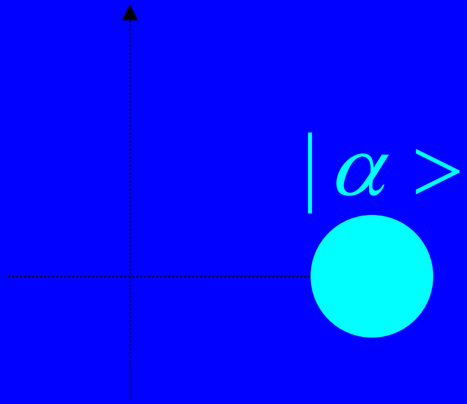
Geometrical representation

Atomic state in
the equatorial plane
of the Bloch sphere

Coherent field
in the Fresnel plane

Representation in the same plane





Equatorial plane
of the Bloch sphere

Phase correlation

⬍

Atomic dipole and field « aligned »

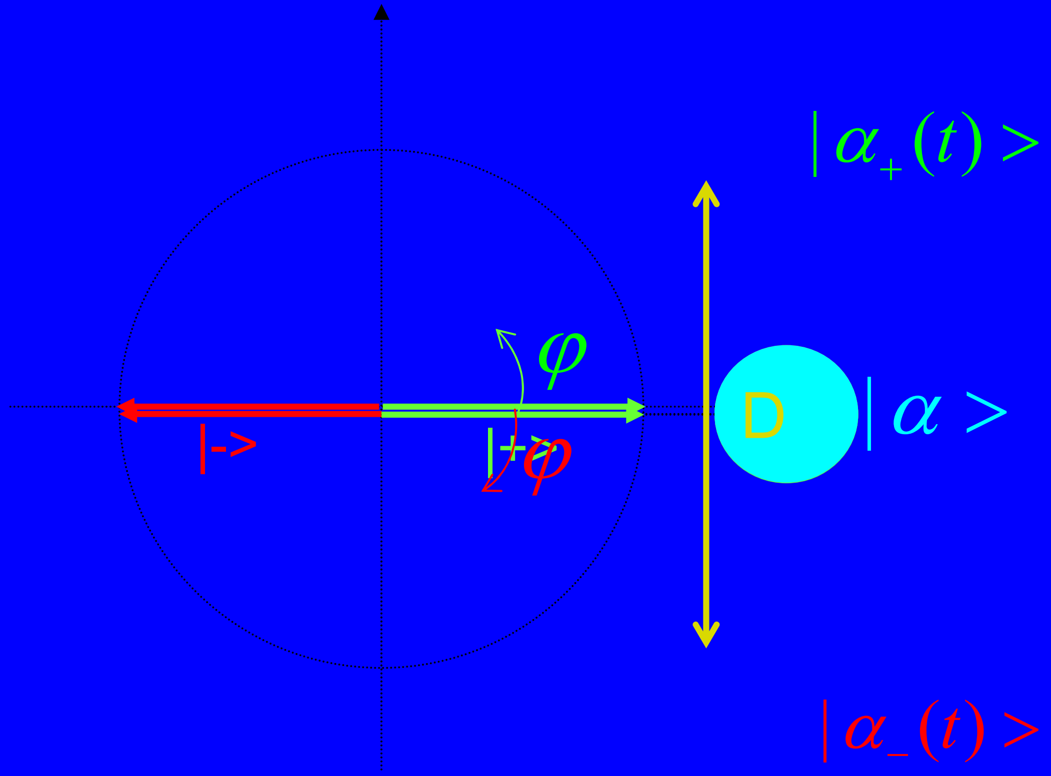
Evolution of the atom-field system

Initial state

$$|e,\alpha> \rightarrow \frac{1}{\sqrt{2}}\left(|\alpha_+(t)>|\psi_+^{at}(t)> + |\alpha_-(t)>|\psi_-^{at}(t)>\right)$$

$$|+,\alpha> \rightarrow |\psi_+^{at}(t),\alpha_+(t)>$$

$$|-,\alpha> \rightarrow |\psi_-^{at}(t),\alpha_-(t)>$$



A microscopic object
leaves its imprint
on a mesoscopic one

Schrödinger-cat situation

"Size" of the cat=D

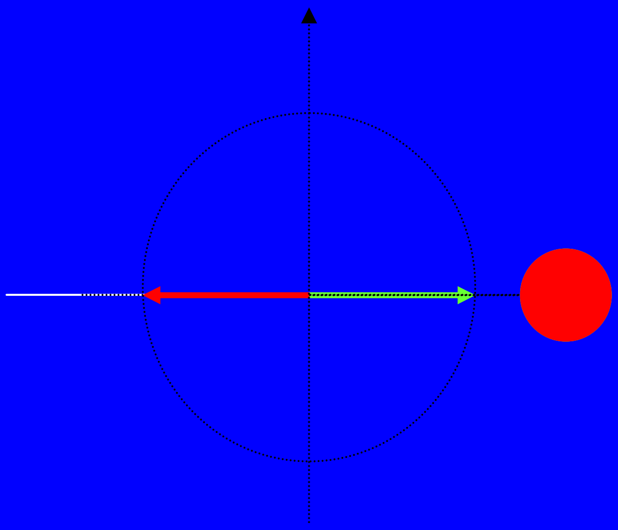
$$D = 2\sqrt{\overline{n}}\sin\left(\frac{\Omega_0 t}{4\sqrt{\overline{n}}}\right)$$

The field acts as a
Which-Path detector

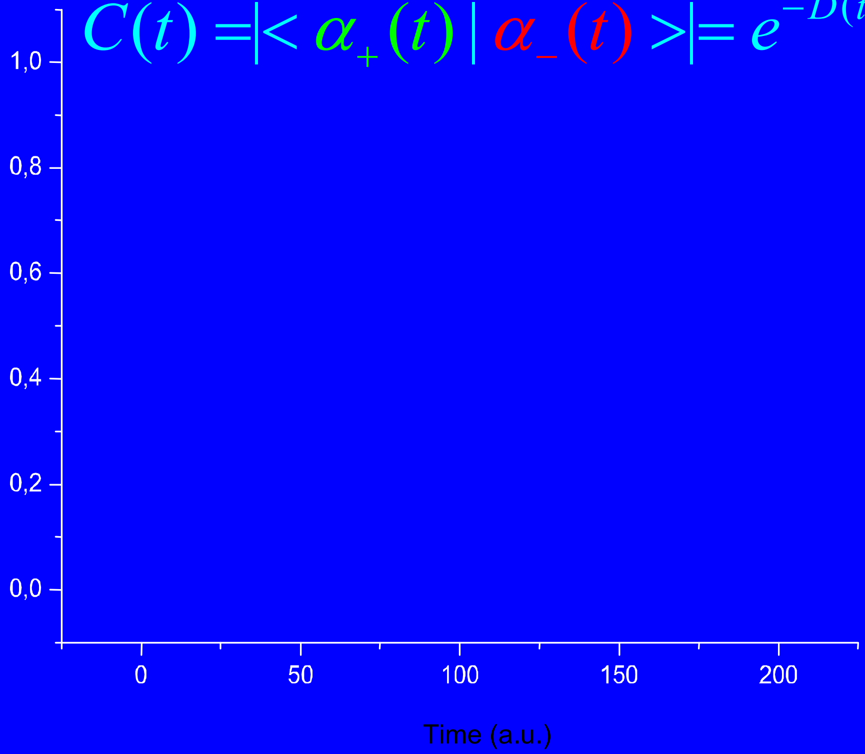
Contrast of the Rabi oscillation $C(t) = |<\alpha_+(t)|\alpha_-(t)>| = e^{-D^2(t)}$

New insights on collapse and revival

$$|e,\alpha> \rightarrow \frac{1}{\sqrt{2}}\left(|\alpha_+(t)>|\psi_+^{at}(t)> + |\alpha_-(t)>|\psi_-^{at}(t)>\right)$$



$$C(t) = |<\alpha_+(t)|\alpha_-(t)>| = e^{-D(t)^2}$$



Collapse as soon as the two components are well separated

Field states merge again

Factorization of a mesoscopic state

Revival of the Rabi oscillation

$$D \sim 2\frac{\Omega_0 t}{4\sqrt{\bar{n}}}\sqrt{\bar{n}}$$

$$|\psi_{field}> = \frac{1}{\sqrt{2}}\left(|i\alpha> + (-1)^{\bar{n}+1}|-i\alpha>\right)$$

$$\boxed{T_{coll} \sim \frac{2}{\Omega_0} \propto T_0}$$

$$\varphi = \frac{\Omega_0 T_R}{4\sqrt{\bar{n}}} \approx \pi \quad \Rightarrow \quad \boxed{T_R \approx 2T_0\sqrt{\bar{n}}}$$

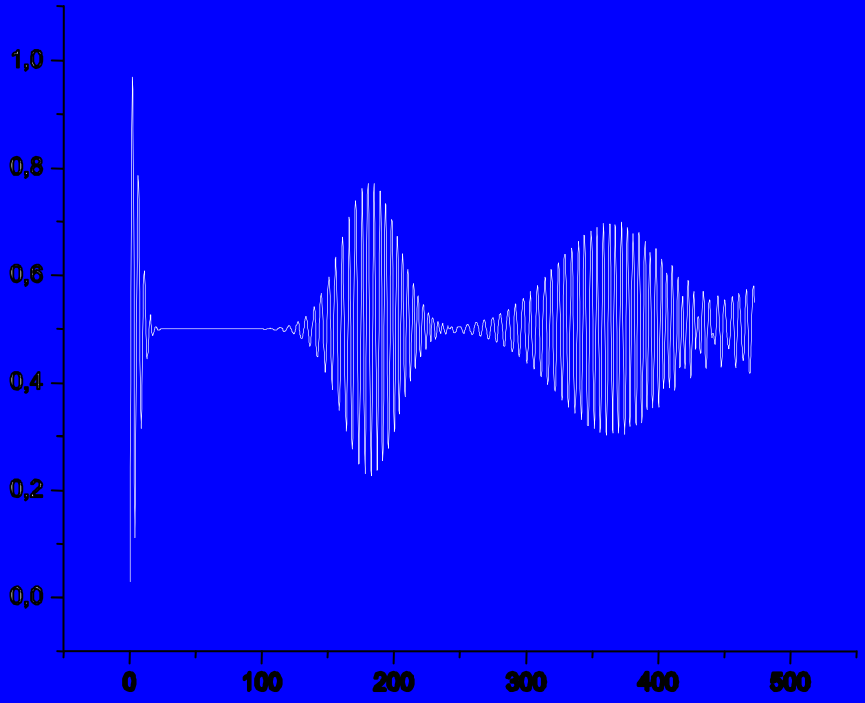
Maximal entanglement: "Schrödinger cat state"

"Size" of the cat=distance D

-Unconditional mesoscopic states superposition

-The field has a defined parity

An exact calculation



Rabi oscillation in 20 photons

Q function evolution in 20 photons
Atom initially in |g>

Field phase distribution measurement

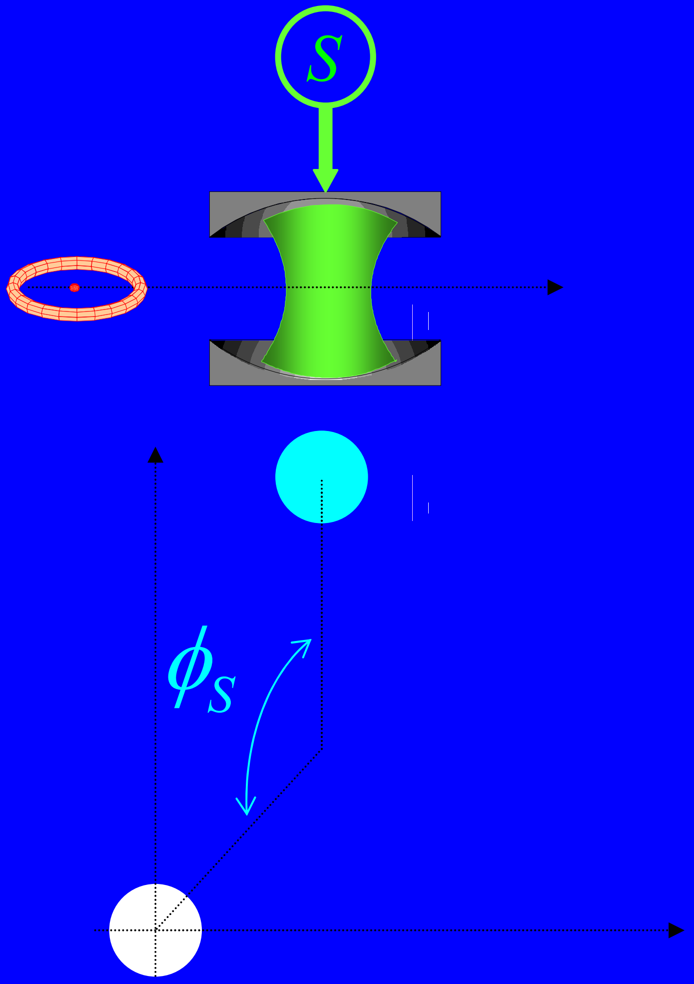
How to measure a coherent field phase-shift?

Homodyne method

Injection of a coherent field $|\alpha>$

Second injection $|-\alpha e^{i\phi_S}>$

Resulting field $|\alpha(1-e^{i\phi_S})>$



Back to the vacuum state ⟷ $\phi_S = 0$

A probe atom is sent in |g>

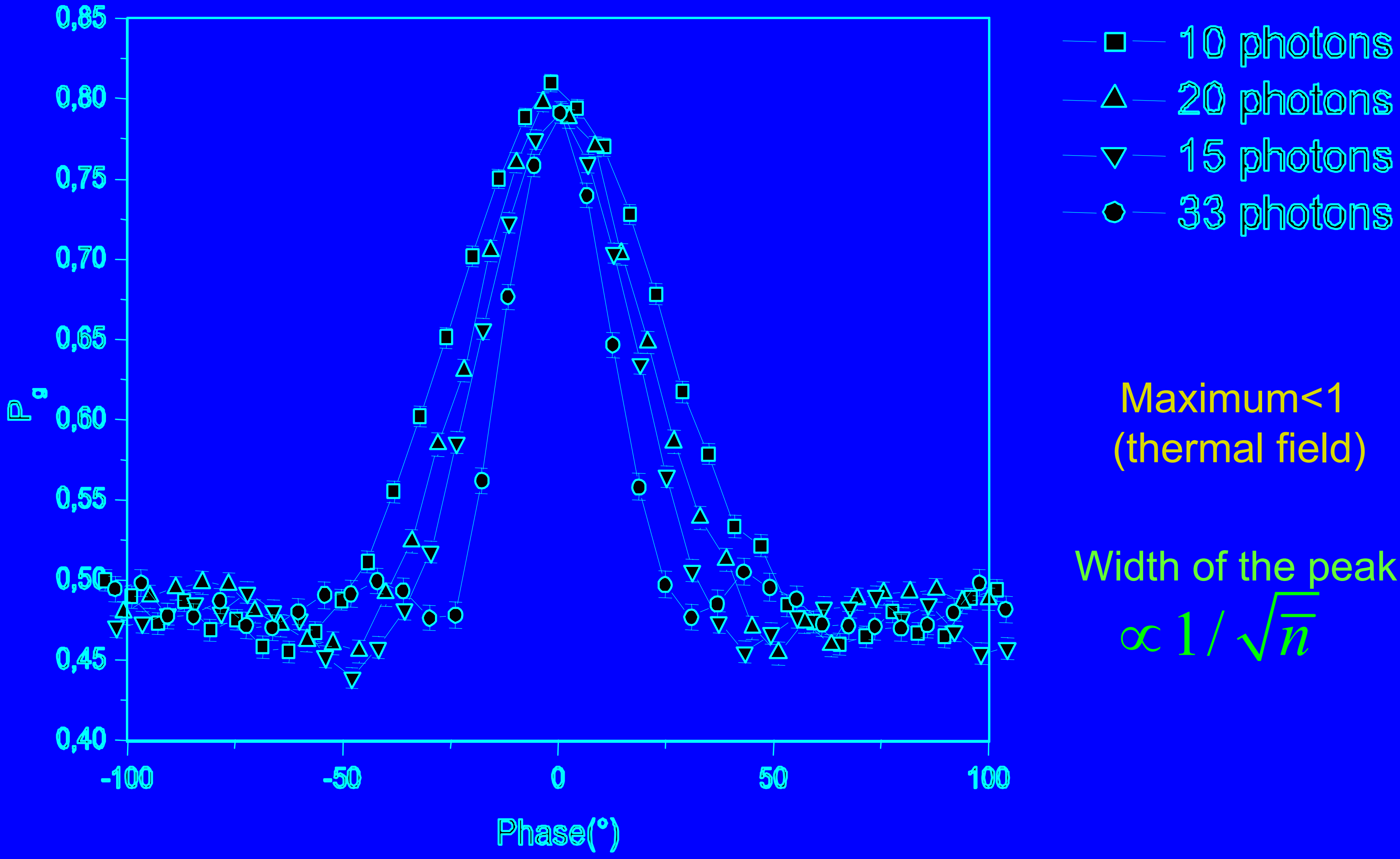
-Field in the vacuum state $P_g \approx 1$

-Field in an excited state $P_g \approx 1/2$

$P_g(\phi_S)$ = a signal to measure the field phase distribution

Field phase-shifted by $\Delta\phi$ ⟶ Maximum displaced by $\Delta\phi$

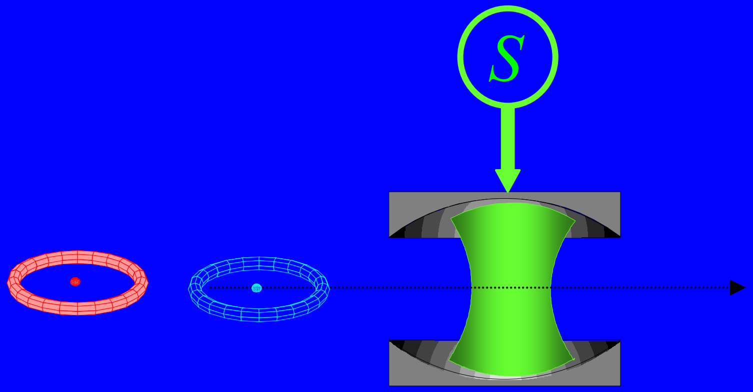
Experimental field phase distribution



Maximum<1
(thermal field)

Width of the peak
$\propto 1/\sqrt{\bar{n}}$

Phase splitting in quantum Rabi oscillation: timing of the experiment

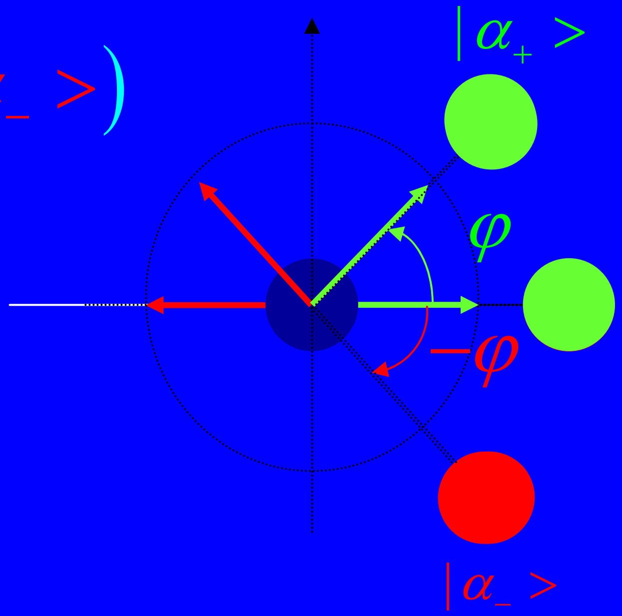


Injection of a coherent field $|\alpha>$

A first atom is sent and interacts resonantly with the field

Detection of the atom
Field projected on

$$|\psi_{field}>=\frac{1}{\sqrt{2}}\left(|\alpha_+>+|\alpha_->\right)$$



Injection of $|-\alpha e^{i\phi}>$

A probe atom is sent in |g>

$\phi_S=\varphi$

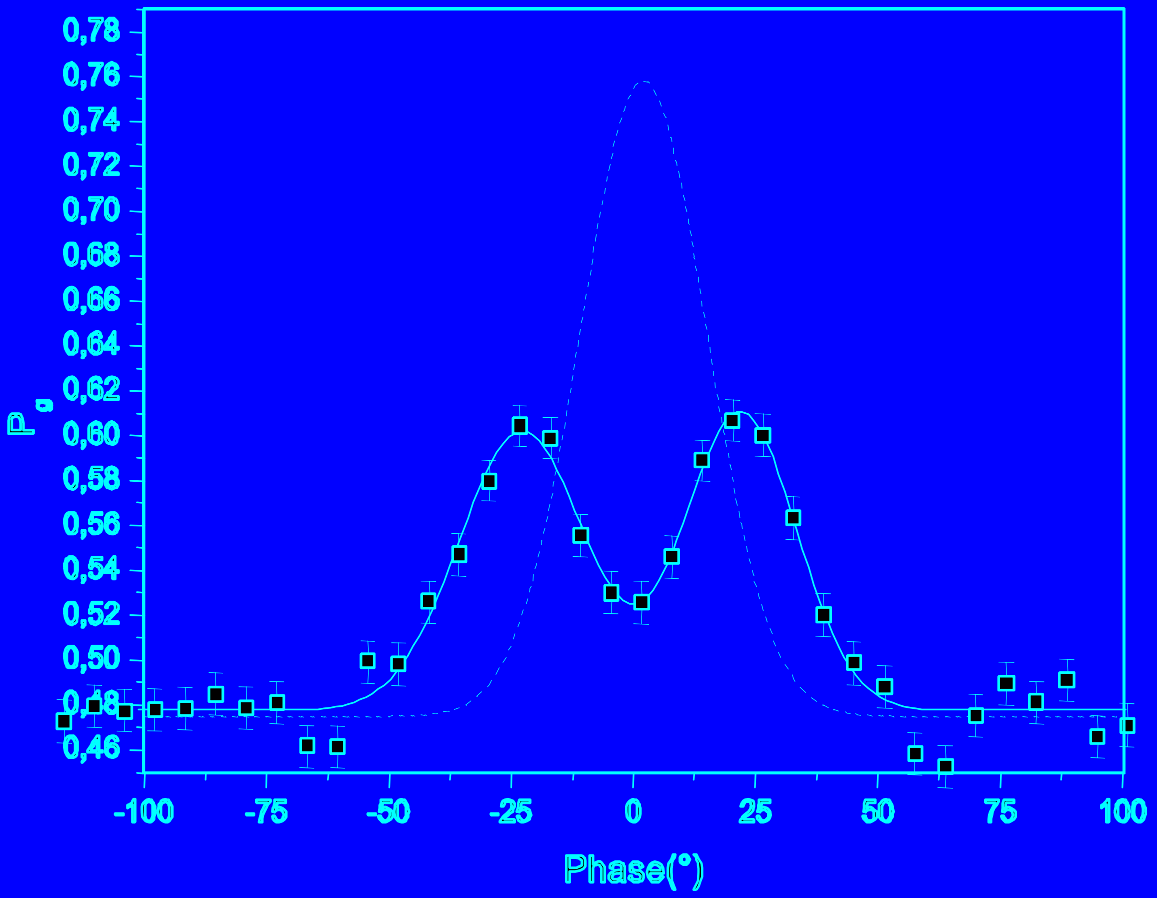
Vanishing of $|\alpha_+>$

$\phi_S=-\varphi$

Vanishing of $|\alpha_->$

$P_g(\phi)$: two peaks corresponding to the vanishing of each component

Evidence of the phase splitting



v=335m/s

$$T_{\text{int}} = 32\mu s \approx 1.5T_0$$

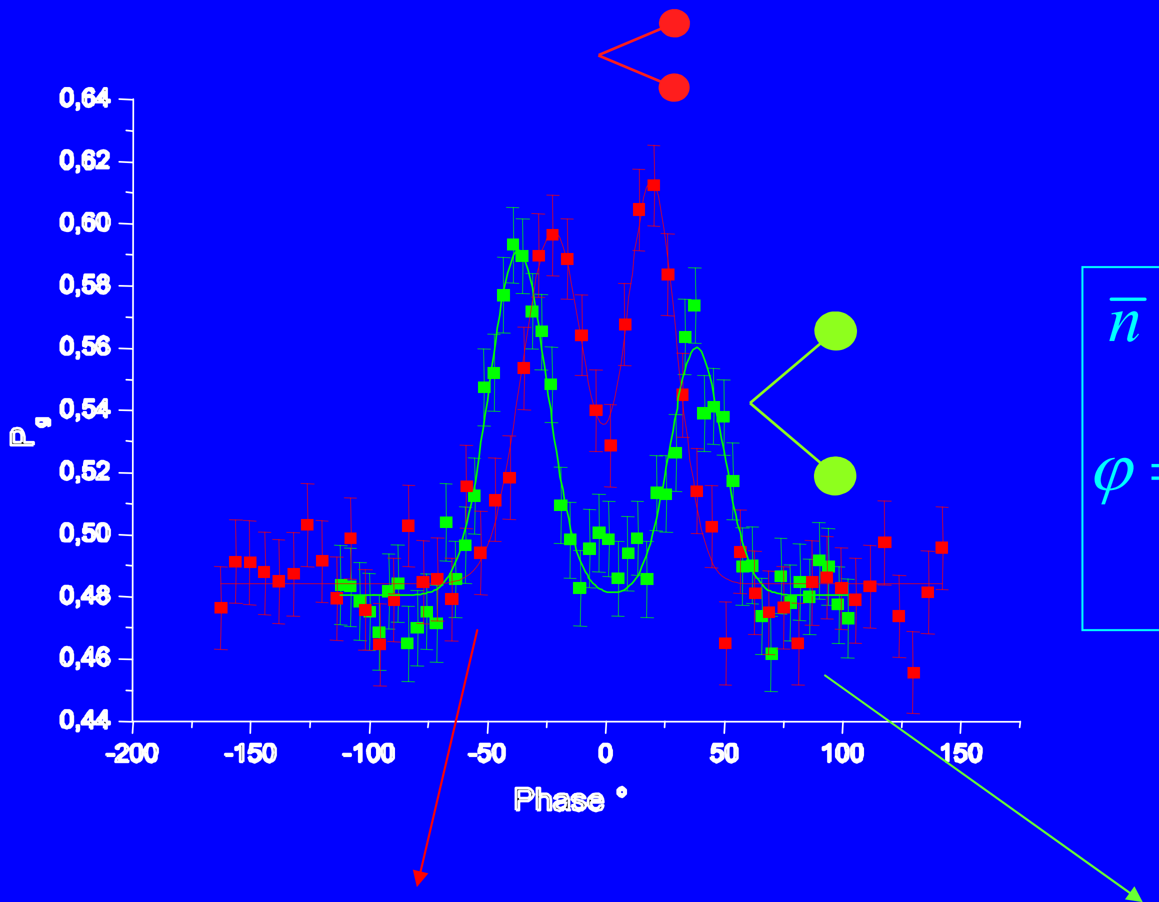
$$\bar{n} = 36$$

Measured phase $\varphi = 23°$

Expected value $\varphi = \dfrac{\Omega_0 t_{\text{int}}}{4\sqrt{\bar{n}}} = 23°$

Experiment and theory in very good agreement

Evolution of the phase distribution



$$\bar{n} = 30$$

$$\varphi = \frac{\Omega_0 t_{\text{int}}}{4\sqrt{\bar{n}}}$$

→ 2 velocities

→ Various number of photons

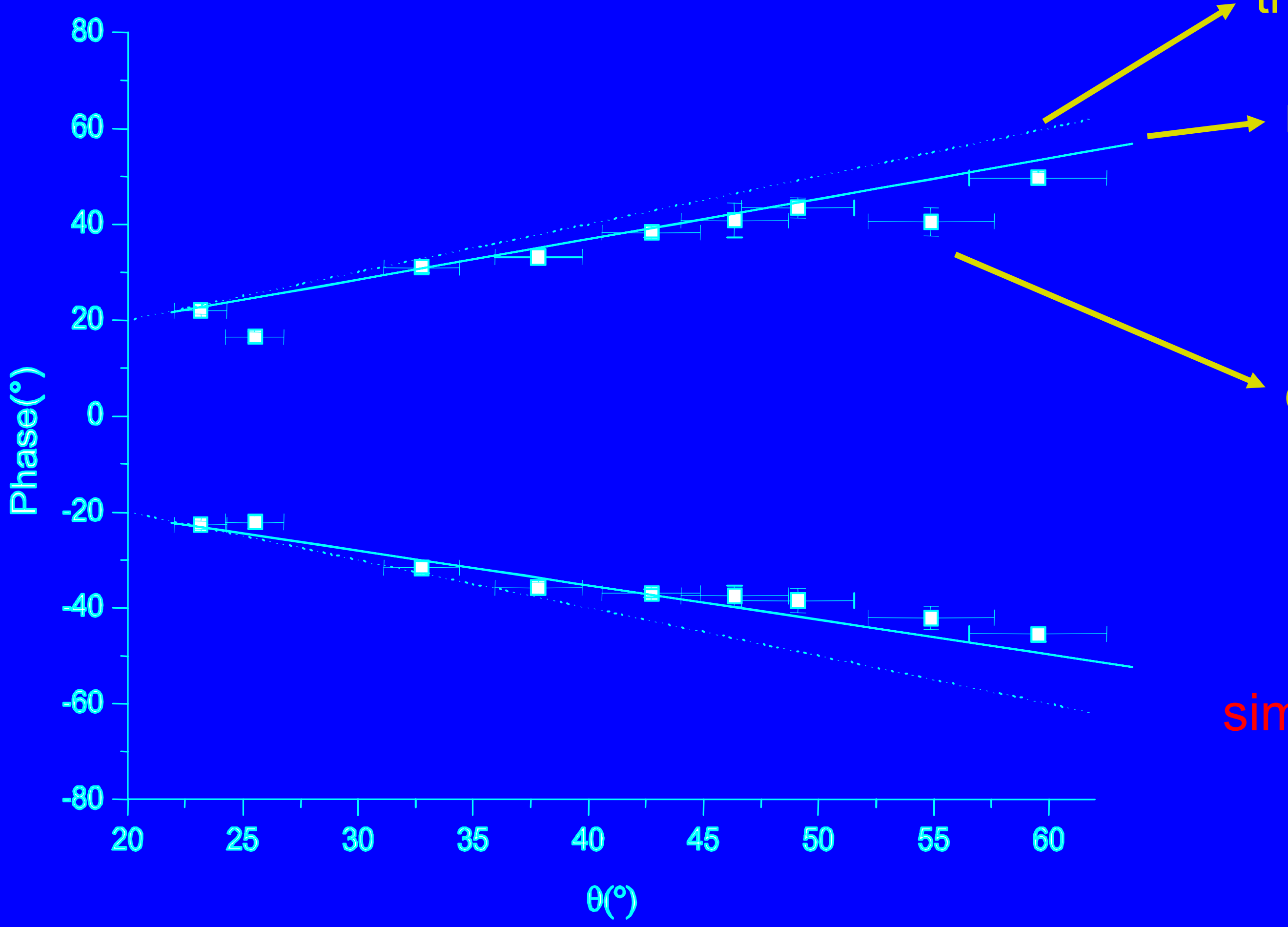
$$v_a = 335 m/s, \ \ t_{\text{int}} \approx 1.5 T_0$$

$$\varphi_{\exp} = 19°$$

$$v_b = 200 m/s, \ \ t_{\text{int}} \approx 2.5 T_0$$

$$\varphi_{\exp} = 37°$$

Experiment vs theory



theory (slope 1)

numerical simulations
-second mode
-thermal field
-relaxation

experimental points

Experiment and simulations in very good agreement

Measured phase vs theoretical phase $\theta = \dfrac{\Omega_0 t_{\text{int}}}{4\sqrt{\overline{n}}}$

Test of coherence: induced quantum revivals

Initial Rabi rotation

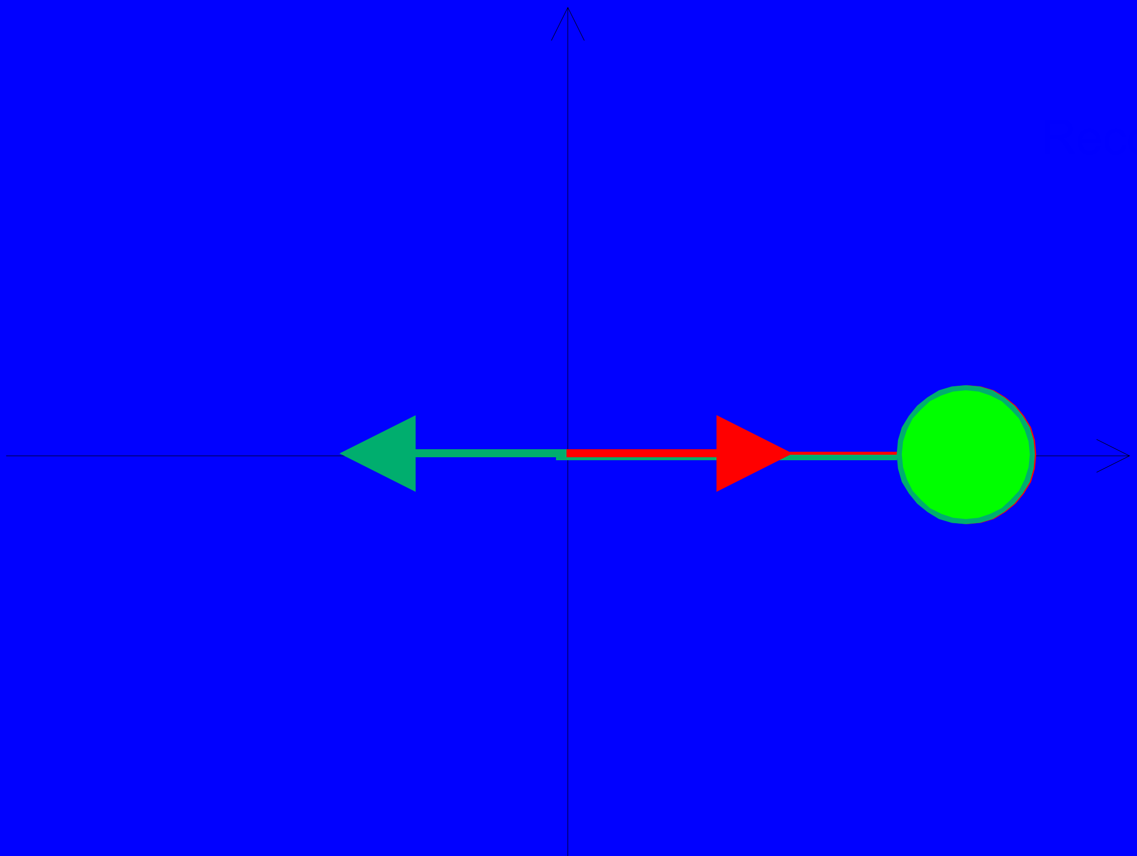
Collapse

And slow phase rotation

Stark pulse (duration short compared to phase rotation).

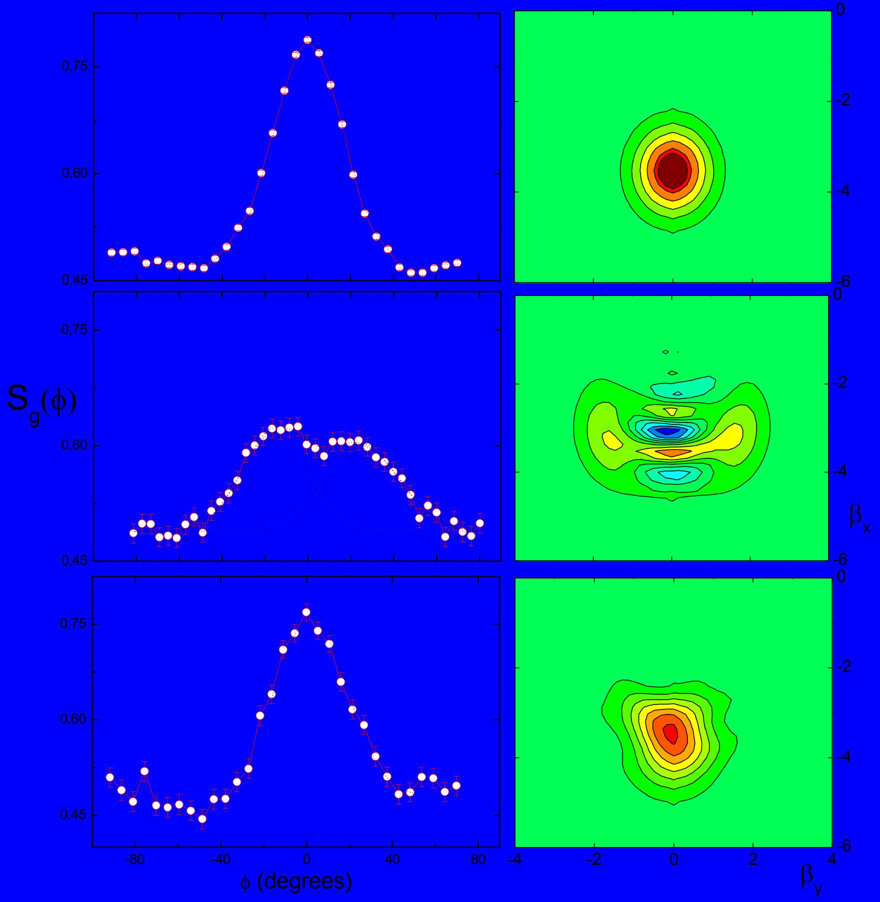
Equivalent to a Z rotation by π

Reverse phase rotation

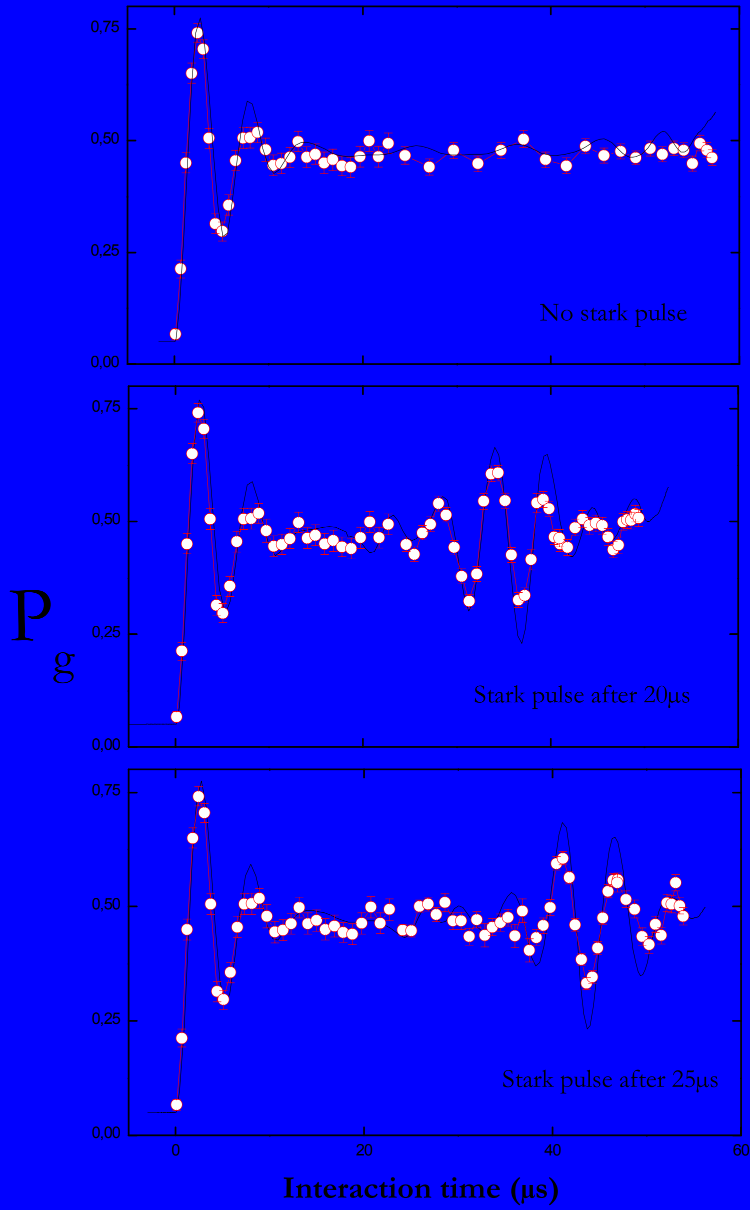


A spin echo experiment

Echo experiments in Cavity QED to study decoherence proposed by G.Morigi, E.Solano, B.G.Englert and H.Walther, Phys.Rev.A 65, 0401202(R),2002.



Separation and recombination of field components by Stark switching



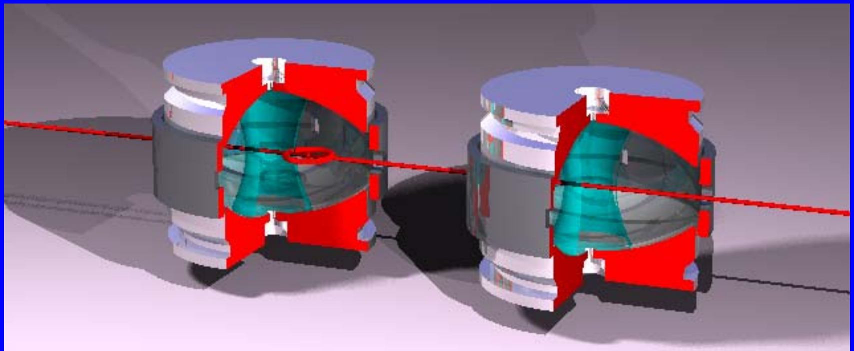
Rabi oscillation revivals

Conclusions and perspectives

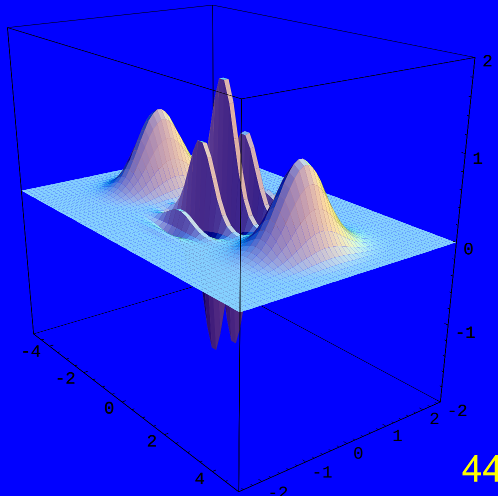
Larger and longer lived cats (n in the hundreds) with better cavities

Prepare and detect **$|\alpha, 0\rangle + |0, \alpha\rangle$**
(similar to $|n,0\rangle + |0,n\rangle$ « high noon states »)

Non local field states in two cavities



Wigner function measurements and decoherence studies of cat states

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