Quantum Optics and Information

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Outline for the three lectures

- interacting photons in Rydberg media

- dynamics of quantum systems with long-range interactions
Interacting photons in Rydberg media

References:

Quantum optics:
• lecture notes for Misha Lukin’s class Modern Atomic and Optical Physics II, compiled by Lily Childress: https://lukin.physics.harvard.edu/teaching
• Meystre and Sargent, “Elements of Quantum Optics”
• Loudon, “Quantum Theory of Light”

Waveguide QED:

Interacting photons in Rydberg media:

Thanks to colleagues whose slides I am borrowing: Misha Lukin, Bill Phillips, Thomas Pohl,…
Outline

- motivation and basic idea
- E&M field quantization
- propagation of light through atomic ensembles; electromagnetically induced transparency (EIT)
- Rydberg atoms
- basic idea revisited
- photon interacting with stationary excitation
  - on resonance: single-photon switch, subtractor
  - off resonance: two-photon quantum gate
- dynamics of multiple photons
  - on resonance: source of single photons
  - off resonance: two-photon gate, bound states, many-body physics
- more applications
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Photons interact only in Sci-Fi
But light CAN act on light under the right circumstances
One of the first scientific achievements following the first laser...

...was the first demonstration of non-linear optics
The first non-linear optics needed the first laser
Electrons vs. Photons

\[ F = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} \]  (large)

transistor

small electrical signals control huge currents

photons hardly interact

nonlinear crystal

even huge optical intensities only control tiny optical signals
Electronics vs. Photonics

- **Electrons** process information.
- **Photons** transmit information.

- On-chip optical routing.
Electronics vs. Photonics

- **Electrons**
  - Process information

- **Photons**
  - Transmit information

Converting photons to electrical signals is “expensive”
Electronics vs. Photonics

Electrons
process information

Photons
transmit information

Can we make photons process without conversion?
• photon-photon interactions too weak for processing information
Photon-photon interactions

Typical approach to achieving interactions between optical photons:

• nonlinearity induced by individual atoms (or artificial atoms)

Kimble @ Caltech

Vuckovic @ Stanford

Hard!
Photon-photon interactions

Typical approach to achieving interactions between optical photons:

- nonlinearity induced by individual atoms (or artificial atoms)

This talk: Map strong atom-atom interactions onto strong photon-photon interactions
Medium where photons interact strongly

Map strong atom-atom interactions onto strong photon-photon interactions

EIT = electromagnetically induced transparency

Experiments: Adams, Kuzmich, Lukin & Vuletic, Pfau & Löw, Grangier, Weidemüller, Hofferberth, Dürr & Rempe, Simon, Firstenberg, Ourjoumtsev, H. de Riedmatten, etc…

Theory: Kurizki, Fleischhauer, Petrosyan, Mølmer, Pohl, Lesanovsky, Kennedy, Brion, Büchler, Sørensen, most experimental groups above, etc…
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E&M field quantization

- Lukin/Childress lecture notes
- Meystre and Sargent, "Elements of Quantum Optics"
- consider free field (no sources)
- Maxwell’s equations $\Rightarrow$ wave equation
  \[
  \nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0
  \]
- knowing $\mathbf{E}$, find $\mathbf{B}$ via $\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$  (SI units used)
- large cavity of length $L$ & volume $V$, with $\mathbf{E} = 0$ on mirrors
- eigenmodes = standing waves
  \[
  \sin(k_j z) \quad k_j L = \pi j
  \]
  $j = 1, 2, 3, \ldots$
consider \( \hat{x} \)-polarized field

\[
E_x(z, t) = \sum_j \sqrt{\frac{2\nu_j^2}{\varepsilon_0 V}} q_j(t) \sin k_j z \quad \nu_j = ck_j
\]

\[A_j \text{ amplitude}\]

\[
\Rightarrow B_y(z, t) = \frac{1}{c^2} \sum_j \frac{\dot{q}_j(t)}{k_j} A_j \cos k_j z
\]
• classical energy:

\[ H = \frac{1}{2} \int dV \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) \]

\[ = \sum_j \frac{\nu_j^2 q_j^2}{2} + \frac{\dot{q}_j^2}{2} \to \frac{1}{2} \sum_j (\nu_j^2 \dot{q}_j^2 + \dot{p}_j^2) = \sum_j \hbar \nu_j \left( \hat{a}_j^\dagger \hat{a}_j + \frac{1}{2} \right) \]

• independent harmonic oscillators with frequency \( \nu_j \), unit mass, position \( q_j \)

• Quantization: \( q_j \to \hat{q}_j \quad \dot{q}_j \to \hat{p}_j \quad [\hat{q}_j, \hat{p}_j] = i \hbar \delta_{j,j'} \)

• creation/annihilation:

\[ \hat{a}_j = \frac{1}{\sqrt{2\hbar \nu_j}} \left( \nu_j \hat{q}_j + i \hat{p}_j \right) \]

\[ \hat{q}_j = \sqrt{\frac{\hbar}{2\nu_j}} \left( \hat{a}_j + \hat{a}_j^\dagger \right) \]

\[ \hat{a}_j^\dagger = \frac{1}{\sqrt{2\hbar \nu_j}} \left( \nu_j \hat{q}_j - i \hat{p}_j \right) \]

\[ \hat{p}_j = -i \sqrt{\frac{\hbar \nu_j}{2}} \left( \hat{a}_j - \hat{a}_j^\dagger \right) \]
• $\hat{x}$ component of electric field operator

$$\hat{E}_x(z) = \sum_j A_j \sqrt{\frac{\hbar}{2\nu_j}} (\hat{a}_j + \hat{a}^\dagger_j) \sin k_j z$$

$$\sqrt{\frac{\hbar \nu_j}{\epsilon_0 V}} = \text{electric field per photon}$$

(makes sense: $\hbar \nu_j \sim \text{energy} \sim \epsilon_0 E^2 V$)

• for running waves, including all polarizations & directions

$$\hat{E}(r) = \hat{\mathcal{E}}(r) + \hat{\mathcal{E}}^\dagger(r)$$

$$\hat{\mathcal{E}}(r) = \sum_{k,\alpha} \epsilon_\alpha \sqrt{\frac{\hbar \nu_j}{2\epsilon_0 V}} \hat{a}_{k,\alpha} e^{ik \cdot r}$$

transverse polarization
Atom-field interactions

• starting point: dipole Hamiltonian for a 2-level atom

\[ \hat{V}_{af} = -\hat{E} \cdot \hat{d} \]

\[ = - (\hat{E} + \hat{E}^\dagger) \cdot (\langle 2 | \hat{d} | 1 \rangle | 2 \rangle \langle 1 | + \langle 1 | \hat{d} | 2 \rangle | 1 \rangle \langle 2 |) \]

• 4 types of terms:

\[ \hat{a} | 2 \rangle \langle 1 | \quad \hat{a} | 1 \rangle \langle 2 | \quad \hat{a}^\dagger | 2 \rangle \langle 1 | \quad \hat{a}^\dagger | 1 \rangle \langle 2 | \]

(Heisenberg evolution under \( \hat{H} = \hbar \nu_j \hat{a}^\dagger_j \hat{a}_j \) : \( \hat{a}_j(t) = \hat{a}_j(0)e^{-i\nu_j t} \))

• RWA \approx \text{energy conservation}

• with RWA:

\[ \hat{V}_{af} = - \sum_{k,\alpha} \hbar g_{k,\alpha} | 2 \rangle \langle 1 | \hat{a}_{j,\alpha} + \hbar g_{k,\alpha}^* | 1 \rangle \langle 2 | \hat{a}_{k,\alpha}^\dagger \]

• single-photon Rabi frequency:

\[ g_{k,\alpha} = \frac{\mu_\alpha}{\hbar} \sqrt{\frac{\hbar \nu_j}{2\varepsilon_0 V}} e^{i\mathbf{k} \cdot \mathbf{r}} \]

(• if standing wave mode, \( |g| \propto \sin(kz) \))

\[ \mu_\alpha = \langle 2 | d_\alpha | 1 \rangle \]
Remarks

• no sources ($\nabla \cdot \mathbf{E} = 0$), wave equation
  ⇒ didn’t need $\mathbf{A}$; used $\hat{V}_{af} = -\hat{\mathbf{E}} \cdot \hat{d}$
  ⇒ didn’t need to choose gauge

• with sources ($\nabla \cdot \mathbf{E} \neq 0$), no wave equation
  ⇒ need $\mathbf{A}$
  ⇒ choose Coulomb gauge $\nabla \cdot \mathbf{A} = 0$
  ⇒ quantized similarly to this lecture
    [see Cohen-Tannoudji et al., “Photons and Atoms”]
Medium where photons interact strongly

EIT = electromagnetically induced transparency
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Three-level medium

\[ |r\rangle \uparrow \quad \hat{E}_2 \quad \uparrow \quad |e\rangle \quad \Delta \quad \downarrow \quad \hat{E}_1 \quad \downarrow \quad |g\rangle \]

\[ \omega_1 = \omega_{eg} + \Delta \quad \quad \omega_2 = \omega_{re} - \Delta \]

\[ \hat{E}_1(z) = e_1 \left( \frac{\hbar \omega_1}{4\pi c \varepsilon_0 A} \right)^{1/2} \int d\omega \hat{a}_\omega e^{i\omega z/c} + h.c. \]

\[ \left[ \hat{a}_\omega, \hat{a}_{\omega'}^\dagger \right] = \delta(\omega - \omega') \]

\[ E_2(t) = \epsilon_2 \mathcal{G}_2(t) \cos(\omega_2 t) \]

A = cross section of beam and of ensemble

Fleischhauer, Lukin, PRA 65, 022314 (2002)
AVG, Adre, Lukin, Sorensen, PRA 76, 033805 (2007)

Loudon, “Quantum Theory of Light”
Three-level medium

\[ |r\rangle \quad |e\rangle \quad |g\rangle \]

\[ \hat{E}_1(z) = \epsilon_1 \left( \frac{\hbar \omega_1}{4 \pi c \epsilon_0 A} \right)^{1/2} \int d\omega \hat{a}_\omega e^{i\omega z/c} + h.c. \]

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\[ E_2(t) = \epsilon_2 \mathcal{G}_2(t) \cos(\omega_2 t) \]

\[ A = \text{cross section of beam and of ensemble} \]

\[ \omega_1 = \omega_{eg} + \Delta \quad \omega_2 = \omega_{re} - \Delta \]

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Loudon, “Quantum Theory of Light”

\[ \hbar \omega_1 \]

\[ \epsilon_1 \]

\[ \epsilon_2 \]
Three-level medium

$$\hat{H} = \hat{H}_0 + \hat{V}$$

$$\hat{H}_0 = \int d\omega \hbar \omega \hat{a}^\dagger_\omega \hat{a}_\omega + \sum_{i=1}^{N} \left( \hbar \omega_{rg} \hat{\sigma}_{rr}^i + \hbar \omega_{eg} \hat{\sigma}_{ee}^i \right)$$

$$\hat{V} = -\sum_{i=1}^{N} \hat{d}_i \cdot \left[ E_2(t) + \hat{E}_1(z_i) \right]$$

$$\hat{V} = -\hbar \sum_{i=1}^{N} \left( \Omega(t) \hat{\sigma}_{re}^i e^{-i\omega_2 t} + g \sqrt{\frac{1}{2\pi c}} \int d\omega \hat{a}_\omega e^{i\omega z_i/c} \hat{\sigma}_{eg}^i + h.c. \right)$$

$$\hat{\sigma}_{\mu\nu} = |\mu\rangle_{ii} \langle \nu|$$

$$\Omega(t) = \langle r | (\hat{d} \cdot \epsilon_2) | e \rangle \mathcal{E}_2(t)/(2\hbar) = \text{Rabi frequency}$$

$$g = \langle e | (\hat{d} \cdot \epsilon_1) | g \rangle \sqrt{\frac{\omega_1}{2\hbar \epsilon_0 A}}$$

- $\pi$ pulse takes time $\pi/(2\Omega)$
- I will often set $\hbar = 1$
Three-level medium

define slowly varying operators

\[ |r\rangle \]

\[ |e\rangle \]

\[ |g\rangle \]

\[ \Omega \]

\[ \Delta \]

\[ n \approx 1 \] atoms

\[ n = \text{atom density} \]

\[ \hat{P}^\dagger(z, t) = \sqrt{n} \frac{1}{N_z} \sum_{i=1}^{N_z} \hat{\sigma}_{eg}^i(t) e^{-i\omega_1(t-z_i/c)} \]

\[ \left[ \hat{P}(z, t), \hat{P}^\dagger(z', t) \right] = \delta(z-z') \]

creates \( |e\rangle \) excitation at \( z \)

\[ \hat{S}^\dagger(z, t) = \sqrt{n} \frac{1}{N_z} \sum_{i=1}^{N_z} \hat{\sigma}_{rg}^i(t) e^{-i\omega_1(t-z_i/c)} - i\omega_2 t \]

\[ \left[ \hat{S}(z, t), \hat{S}^\dagger(z', t) \right] = \delta(z-z') \]

creates \( |r\rangle \) excitation at \( z \)

\[ \hat{\mathcal{E}}^\dagger(z, t) = \sqrt{\frac{1}{2\pi c}} e^{-i\omega_1(t-z/c)} \int d\omega \hat{\mathcal{E}}^\dagger_\omega(t) e^{-i\omega z/c} \]

\[ \left[ \hat{\mathcal{E}}(z, t), \hat{\mathcal{E}}^\dagger(z', t) \right] = \delta(z-z') \]

creates photon at \( z \)

\( \bullet \) thin enough that fields continuous

\( \bullet \) assume almost all atoms in ground state at all times

homework exercise

\[ \hat{\mathcal{E}}(z, t), \hat{\mathcal{E}}^\dagger(z', t) \]

\[ \delta(z-z') \]

\[ e^{-i\omega_1(t-z_i/c)} \]

\[ e^{-i\omega_1(t-z_i/c)} - i\omega_2 t \]

\[ e^{-i\omega z/c} \]

\[ \delta(z-z') \]
Three-level medium

\[
\frac{\hat{H}}{\hbar} = -ic \int dz \hat{\varepsilon}(z) \frac{\partial}{\partial z} \hat{\varepsilon}(z)
\]

collective enhancement

Heisenberg evolution:

\[
(\partial_t + c \partial_z) \hat{\varepsilon} = ig \sqrt{n} \hat{P}
\]

\[
\partial_t \hat{P} = i \Delta \hat{P} + ig \sqrt{n} \hat{\varepsilon} + i \Omega \hat{S} - \gamma \hat{P} + \sqrt{2 \gamma} \hat{F}_P
\]

\[
\partial_t \hat{S} = i \Omega^* \hat{P} - \gamma_s \hat{S} + \sqrt{2 \gamma_s} \hat{F}_S
\]

Langevin noise

only nonzero noise correlations are:

\[
\langle \hat{F}_P(z, t) \hat{F}^*_P(z', t') \rangle = \delta(z - z') \delta(t - t')
\]

\[
\langle \hat{F} \rangle = \langle \hat{F} \hat{F} \rangle = \langle \hat{F}^\dagger \hat{F} \rangle = 0
\]

\[
\langle \hat{F}_S(z, t) \hat{F}^*_S(z', t') \rangle = \delta(z - z') \delta(t - t')
\]
Three-level medium

\[
(\partial_t + c \partial_z) \hat{E} = ig \sqrt{n} \hat{P}
\]
\[
\partial_t \hat{P} = - (\gamma - i\Delta) \hat{P} + ig \sqrt{n} \hat{E} + i\Omega \hat{S} + \sqrt{2\gamma} \hat{F}_P
\]
\[
\partial_t \hat{S} = - \gamma_s \hat{S} + i\Omega^* \hat{P} + \sqrt{2\gamma_s} \hat{F}_S
\]

• assume all atoms initially in ground state, i.e. no P or S excitations

• assume 1 incoming photon

\[
|\psi(t)\rangle = \int dz E(z, t) \hat{E}^\dagger(z) |0\rangle + \int dz P(z, t) \hat{P}^\dagger(z) |0\rangle + \int dz S(z, t) \hat{S}^\dagger(z) |0\rangle
\]

\[
(\partial_t + c \partial_z)E = ig \sqrt{n}P
\]

\[
E(z, t = 0) = P(z, t = 0) = S(z, t = 0) = 0
\]

\[
\partial_t P = - (\gamma - i\Delta)P + ig \sqrt{n}E + i\Omega S
\]

\[
\partial_t S = - \gamma_s S + i\Omega^* P
\]

• same as equations for coherent input
\[(\partial_t + c\partial_z)E = ig\sqrt{n}P\]

\[\partial_t P = - (\gamma - i\Delta)P + ig\sqrt{n}E + i\Omega S\]

\[\partial_t S = i\Omega^* P\]

sanity check: no atoms

\[g\sqrt{n} = 0\]

\[(\partial_t + c\partial_z)E = 0\]

\[E(z, t) = E(0, t - z/c)\]

undistorted propagation at \(C\)
Two-level medium

| $r$ \rangle \quad \Omega \quad \Delta \quad | e \rangle \quad \gamma \quad | g \rangle \\

\begin{align*}
(\partial_t + c\partial_z)E &= ig\sqrt{n}P \\
\partial_t P &= -(\gamma - i\Delta)P + ig\sqrt{n}E + i\Omega S \\
\partial_t S &= i\Omega^*P
\end{align*}

assume: resonant incoming photon, no control

$\Delta = 0 \quad \Omega = 0$
Two-level medium

\[(\partial_t + c\partial_z)E = ig\sqrt{n}P\]

\[\partial_t P = - (\gamma - i\Delta)P + ig\sqrt{n}E + i\Omega S\]

\[\partial_t S = i\Omega^* P\]

\[\Delta = 0\quad \Omega = 0\]

assume: resonant incoming photon, no control

\[E(z, t) = \int d\omega \tilde{E}(\omega, t)e^{-i\omega t}\]

\[P(z, t) = \int d\omega \tilde{P}(\omega, t)e^{-i\omega t}\]
Two-level medium

\[(\partial_t + c\partial_z)E = ig\sqrt{n}P\]
\[\partial_t P = - (\gamma - i\Delta)P + ig\sqrt{n}E + i\Omega S\]
\[\partial_t S = i\Omega^* P\]

assume: resonant incoming photon, no control
\[\Delta = 0\]
\[\Omega = 0\]

\[(\partial_t + c\partial_z)E = ig\sqrt{n}P\]
\[\partial_t P = - \gamma P + ig\sqrt{n}\tilde{E}\]

\[E(z, t) = \int d\omega \tilde{E}(\omega, t)e^{-i\omega t}\]
\[P(z, t) = \int d\omega \tilde{P}(\omega, t)e^{-i\omega t}\]

\[(-i\omega + c\partial_z)\tilde{E} = ig\sqrt{n}\tilde{P}\]
\[-i\omega \tilde{P} = - \gamma \tilde{P} + ig\sqrt{n}\tilde{E}\]
Two-level medium

\[\begin{align*}
(-i\omega + c\partial_z)\tilde{E} &= ig\sqrt{n}\tilde{P} \\
-i\omega\tilde{P} &= -\gamma\tilde{P} + ig\sqrt{n}\tilde{E} \\
\tilde{P} &= \frac{ig\sqrt{n}}{\gamma - i\omega}
\end{align*}\]

\[c\partial_z\tilde{E} = \left(i\omega - \frac{g^2n}{\gamma - i\omega}\right)\tilde{E}\]

\[\tilde{E}(L, \omega) = \tilde{E}(0,\omega) \exp\left[i\omega\frac{L}{c} - \frac{g^2nL/c}{\gamma - i\omega}\right]\]

\[\tilde{E}(L, \omega) = \tilde{E}(0,\omega) \exp\left[i\omega\frac{L}{c} - \frac{d\gamma}{\gamma - i\omega}\right]\]

\[d = \frac{g^2nL}{\gamma c}\]

\[|\tilde{E}(L, \omega)|^2 = |\tilde{E}(0,\omega)|^2 \exp\left[-\frac{2d}{1 - (\omega/\gamma)^2}\right]\]
Two-level medium

\[ |\tilde{E}(L, \omega)|^2 = |\tilde{E}(0, \omega)|^2 \exp \left[ -\frac{2d}{1 - (\omega/\gamma)^2} \right] \]

- on resonance: \( I_{out} = I_{in} e^{-2d} \)
- \( 2d = \) optical depth
- assume \( d \gg 1 \)

absorption line

\begin{align*}
\text{transmission} & \quad 1.0 \quad 0.8 \quad 0.6 \quad 0.4 \quad 0.2 \quad 0.0 \\
\omega & \quad -10 \quad -5 \quad 0 \quad 5 \quad 10
\end{align*}

\[ \sim \gamma \sqrt{d} \]
Electromagnetically induced transparency (EIT)

\[ (\partial_t + c \partial_z)E = ig\sqrt{n}P \]
\[ \partial_t P = -\gamma P + ig\sqrt{n}E + i\Omega S \]
\[ \partial_t S = i\Omega^*P \]

- assume \( \Omega \) real

\[ (-i\omega + c \partial_z)\tilde{E} = ig\sqrt{n}\tilde{P} \]
\[ -i\omega \tilde{P} = -\gamma \tilde{P} + ig\sqrt{n}\tilde{E} + i\Omega \tilde{S} \]
\[ -i\omega \tilde{S} = i\Omega \tilde{P} \]

- at \( \omega = 0 \):
  \[ \tilde{P} = 0 \]
  \[ \tilde{S} = -\frac{g\sqrt{n}}{\Omega} \tilde{E} \]
  \[ \partial_z \tilde{E} = 0 \quad \text{perfect transmission, i.e. no scattering} \]

\[
E(z, t) = \int d\omega \tilde{E}(\omega, t)e^{-i\omega t}
\]
\[
P(z, t) = \int d\omega \tilde{P}(\omega, t)e^{-i\omega t}
\]
\[
S(z, t) = \int d\omega \tilde{S}(\omega, t)e^{-i\omega t}
\]

- Dark-state polariton: coupled atom-photon excitation
  [Fleishhauer & Lukin, 2000, 2002]
  - destructive interference
Electromagnetically induced transparency (EIT)

\[ (-i\omega + c\partial_z)\tilde{E} = ig\sqrt{n}\tilde{P} \]
\[ -i\omega\tilde{P} = -\gamma\tilde{P} + ig\sqrt{n}\tilde{E} + i\Omega\tilde{S} \]
\[ -i\omega\tilde{S} = i\Omega\tilde{P} \]

- near \( \omega = 0 \)

\[ |\tilde{E}(L, \omega)|^2 \approx |\tilde{E}(0, \omega)|^2 \exp \left[ -\frac{2d\gamma^2\omega^2}{\Omega^4} \right] \]

- EIT transparency window of bandwidth \( \omega_{\text{EIT}} \sim \frac{\Omega^2}{\gamma\sqrt{d}} \)

homework exercise
Electromagnetically induced transparency (EIT)

\[ (-i \omega + c \partial_z) \tilde{E} = ig \sqrt{n} \tilde{P} \]
\[ -i \omega \tilde{P} = -\gamma \tilde{P} + ig \sqrt{n} \tilde{E} + i \Omega \tilde{S} \]
\[ -i \omega \tilde{S} = i \Omega \tilde{P} \]

- near \( \omega = 0 \)

\[ \partial_z \tilde{E} \approx i \frac{\omega}{v_g} \tilde{E} \]
\[ v_g \approx \frac{\Omega^2}{g^2 n} c \ll c \]

\[ (\partial_t + v_g \partial_z) E = 0 \]

reduced group velocity

“slow light”

- pulse compression

homework exercise
Electromagnetically induced transparency (EIT)

$$\begin{align*}
\partial_z \tilde{E} & \approx i \frac{\omega}{v_g} \tilde{E} \\
v_g & \approx \frac{\Omega^2}{g^2n} c \ll c
\end{align*}$$

$$\begin{align*}
(\partial_t + v_g \partial_z)E & = 0 \\
\text{reduced group velocity} \\
\text{“slow light”}
\end{align*}$$

- pulse compression
Photon storage and retrieval

- dark state polariton

\[ |ψ⟩ \sim \int dz f(z - v_g t) \left( \Omega \hat{ℰ}(z) - g\sqrt{n}\hat{S}^+(z) \right) |0⟩ \]

- while pulse is inside medium, turn \( \Omega \) off

\[ |ψ⟩ \sim \int dz f(z)\hat{S}^+(z) |0⟩ \quad \text{photon stored in “spinwave”} \]

- when turn \( \Omega \) back on, photon is retrieved

\[ v_g \approx \frac{\Omega^2}{g^2n} c \quad \text{reduced group velocity} \]

- pulse compression
Off-resonant two-level medium

\[(\partial_t + c \partial_z)E = ig\sqrt{n}P\]
\[\partial_t P = -(\gamma - i\Delta)P + ig\sqrt{n}\hat{E}\]

\[
\text{large } \Delta
\]

- Fourier transform in time & drop \(\gamma\):
\[
(-i\omega + c \partial_z)\tilde{E} = ig\sqrt{n}\tilde{P}
\]
\[-i\omega\tilde{P} \approx i\Delta\tilde{P} + ig\sqrt{n}\tilde{E}\]
Off-resonant two-level medium

\[(\partial_t + c \partial_z)E = ig\sqrt{n}P\]
\[\partial_t P = - (\gamma - i\Delta)P + ig\sqrt{n}\hat{E}\]

- Fourier transform in time & drop \(\gamma\):

\[(-i\omega + c \partial_z)\tilde{E} = ig\sqrt{n}\tilde{P}\]
\[-i\omega\tilde{P} \approx i\Delta\tilde{P} + ig\sqrt{n}\tilde{E}\]

- near \(\omega = 0\):

\[c\partial_z\tilde{E} \approx ig\sqrt{n}\tilde{P}\]
\[0 \approx i\Delta\tilde{P} + ig\sqrt{n}\tilde{E}\]
\[\tilde{P} \approx -\frac{g\sqrt{n}}{\Delta}\tilde{E}\]
\[\partial_z\tilde{E} \approx -i\frac{g^2n}{c\Delta}\tilde{E}\]

\[\tilde{E}(z = L, \omega \approx 0) \approx \tilde{E}(z = 0, \omega \approx 0) \exp\left[-i\frac{d\gamma}{\Delta}\right]\]

- atoms imprint a phase on photon
EIT with large single-photon detuning

\[ (-i\omega + c\partial_z)\tilde{E} = ig\sqrt{n}\tilde{P} \]
\[ -i\omega\tilde{P} = -(\gamma - i\Delta)\tilde{P} + ig\sqrt{n}\tilde{E} + i\Omega\tilde{S} \]
\[ -i\omega\tilde{S} = i\Omega\tilde{P} \]

• at \( \omega = 0 \):

same as for \( \Delta = 0 \)

\[ \tilde{P} = 0 \quad \tilde{S} = -\frac{g\sqrt{n}}{\Omega}\tilde{E} \]

\[ \partial_z\tilde{E} = 0 \quad \text{perfect transmission due to EIT} \]

Dark-state polariton: coupled atom-photon excitation
[Fleishhauer & Lukin, 2000, 2002]

\[ (\partial_t + v_g\partial_z)E = 0 \quad \text{reduced group velocity} \]

“slow light”
Medium where photons interact strongly

EIT = electromagnetically induced transparency

Summer school lectures by Browaeys, Hazzard, and possibly Kaufman, Bakr, etc…
Outline

• motivation and basic idea
• E&M field quantization
• propagation of light through atomic ensembles; electromagnetically induced transparency (EIT)
• Rydberg atoms
• basic idea revisited
• photon interacting with stationary excitation
  - on resonance: single-photon switch, subtractor
  - off resonance: two-photon quantum gate
• dynamics of multiple photons
  - on resonance: source of single photons
  - off resonance: two-photon gate, bound states, many-body physics
• more applications
electronic levels in atom:

$n = 1$
electronic levels in atom:

\[ n = 2 \]
\[ n = 1 \]
electronic levels in atom:

\[ n = 1 \]
\[ n = 2 \]
\[ n = 3 \]
electronic levels in atom:

n = 4
n = 2
n = 1
• large size: $r \sim n^2$

electronic levels in atom:

Rydberg states e.g. $n = 100$

$r \sim 1\mu m$

$n = 5$

$n = 2$

$n = 1$

$r \sim 0.1\ nm$
\[ n = 1 \]
Rydberg states

- huge size: $r \sim n^2$
- huge electric dipole moment
- strong, distant interactions

map on strong, distant photon-photon interactions
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Medium where photons interact strongly

Map strong atom-atom interactions onto strong photon-photon interactions

EIT = electromagnetically induced transparency
Basic idea

- one photon (polariton) drags along a Rydberg excitation
Basic idea

- one photon (polariton) drags along a Rydberg excitation
- another photon drags along a Rydberg excitation
- Rydberg excitations feel strong, distant interactions

⇒ strong, distant photon-photon interactions
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