Quantum Computing with Trapped Ions

Boulder Summer School

July 2023

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Plan for lectures this week

1. Ion trapping 101
    1. How to trap an ion
    2. Ion qubit examples
    3. Light-single ion interaction
2. Trapped ion 2 qubit gates
    1. Many ions in one trap
    2. Molmer-Sorensen gate (XX gate)
3. Architecture and Magic

References for this lecture

- Roos thesis (https://www.quantumoptics.at/en/publications/ph-d-theses.html)
- Monroe slides (http://iontrap.umd.edu/publications/presentations/)
- Lots of great ion resources here: https://iontrap.duke.edu/resources/useful-references/

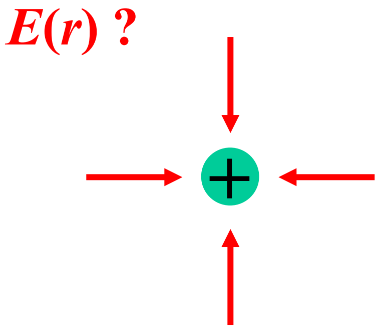
Why talk about hardware?

- Error mechanisms matter
- Co-design is faster
- We are far from fault tolerance
- Knowledge —> Power —> FUN

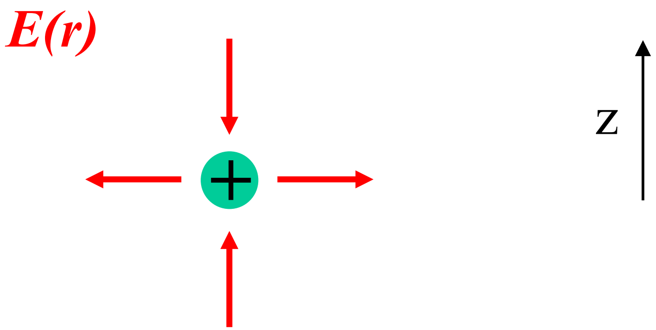
How to trap an ion

Penning and Paul

How to trap an ion



NO! $\nabla\cdot\mathbf{E} = 0$



quadrupole: $\mathbf{E}(\boldsymbol{r}) = \varepsilon(\boldsymbol{x} + \boldsymbol{y} - 2\boldsymbol{z})$

Ion Trap Tricks to "get around" $\nabla\cdot\mathbf{E} = 0$:

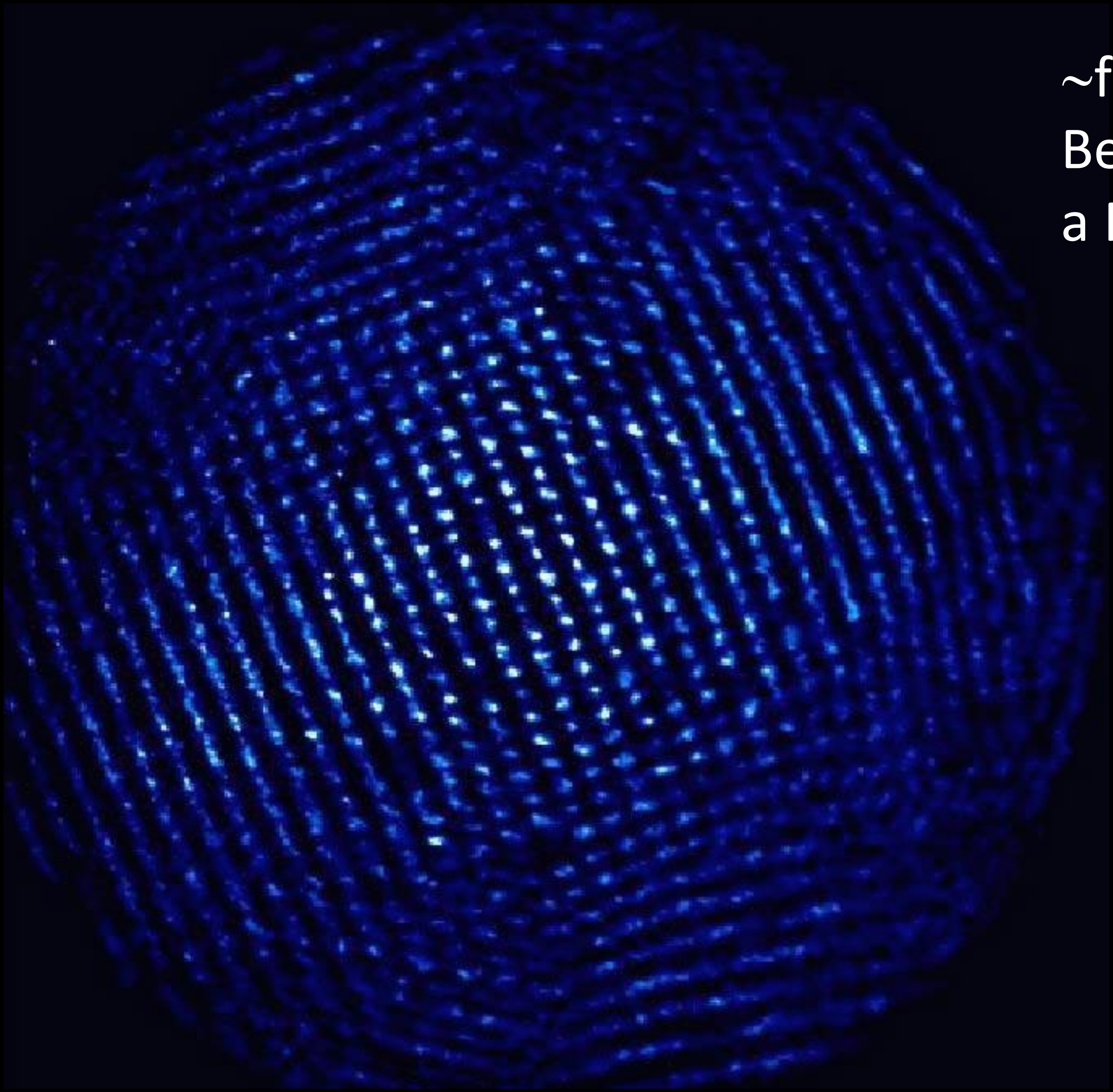
(1) Apply magnetic field along z; e**v**×**B** Lorentz force confines in xy plane

PENNING TRAP

- large capacity (1-10^8)
- ions rotate around z
- confinement frequency limited by $\omega_c = \dfrac{eB}{mc}$

$m = 9$ amu
$B = 1$ T

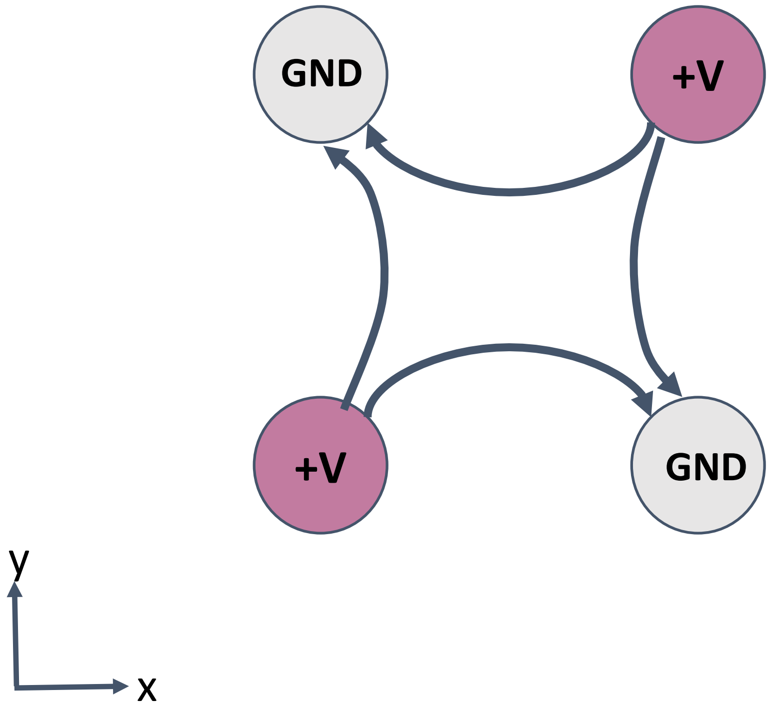
$\boldsymbol{\omega_c = 2\pi(1\ \mathrm{MHz})}$

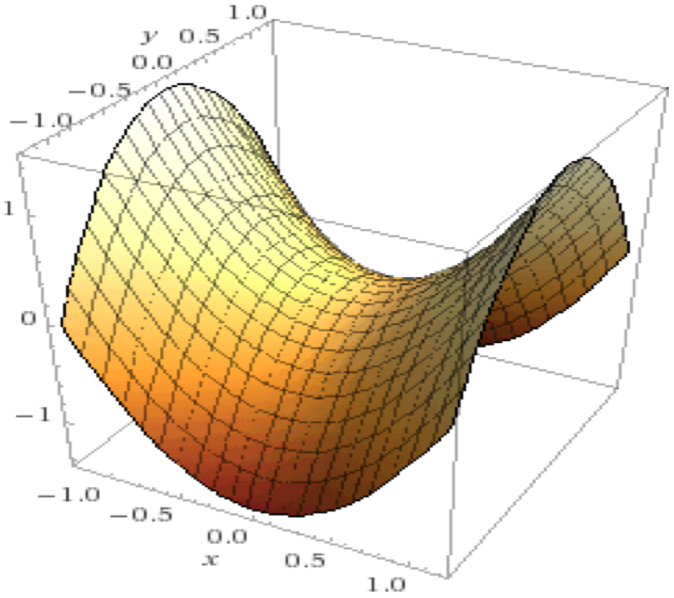


~few 1000
Be$^+$ ions in
a Penning Trap

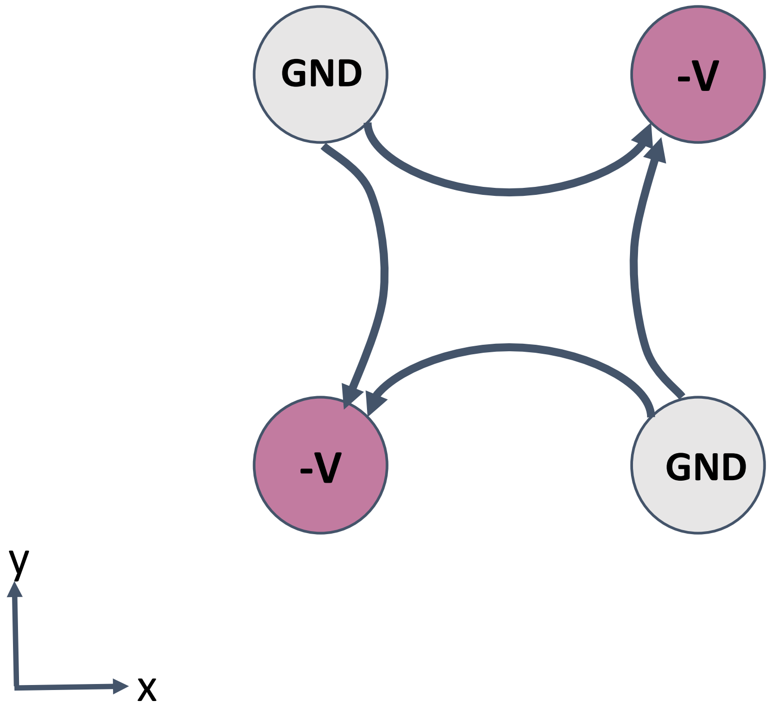
J. Bollinger, NIST
A. M. Rey, JILA

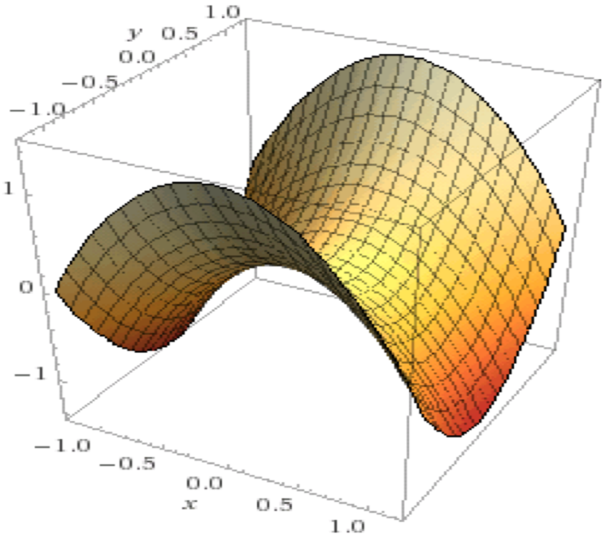
Paul Trap



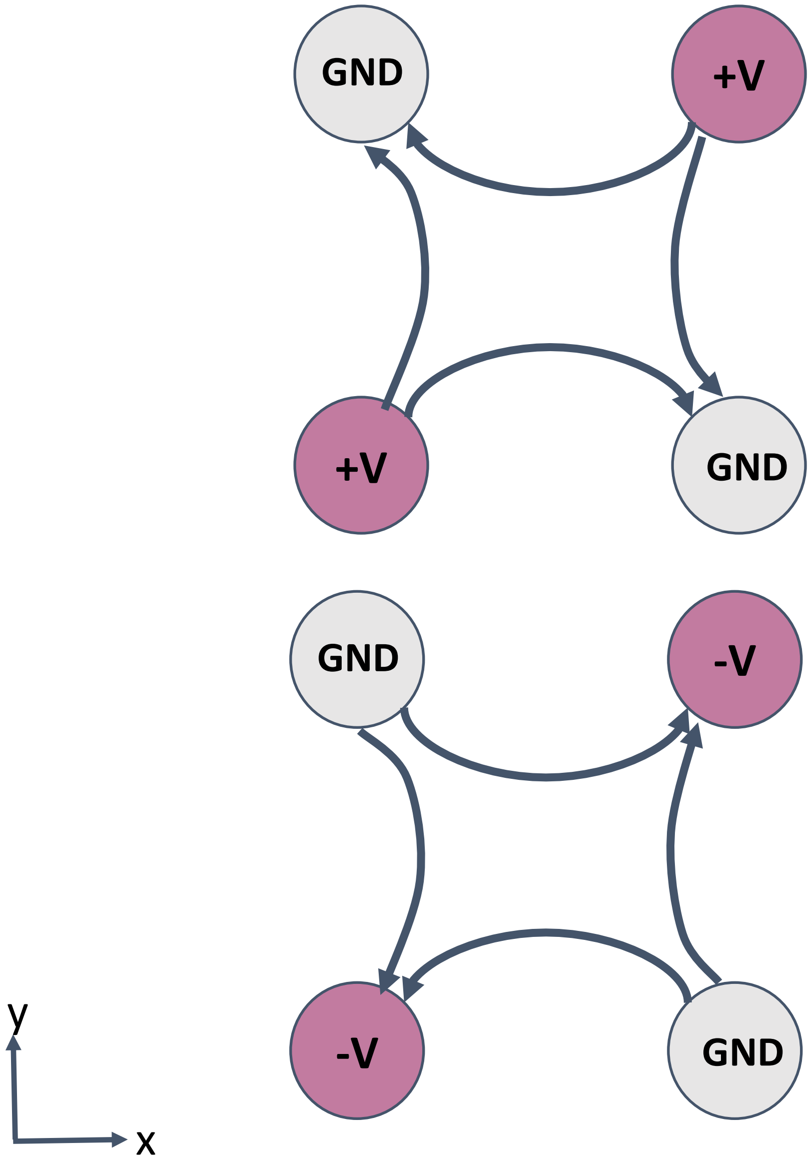


Paul Trap

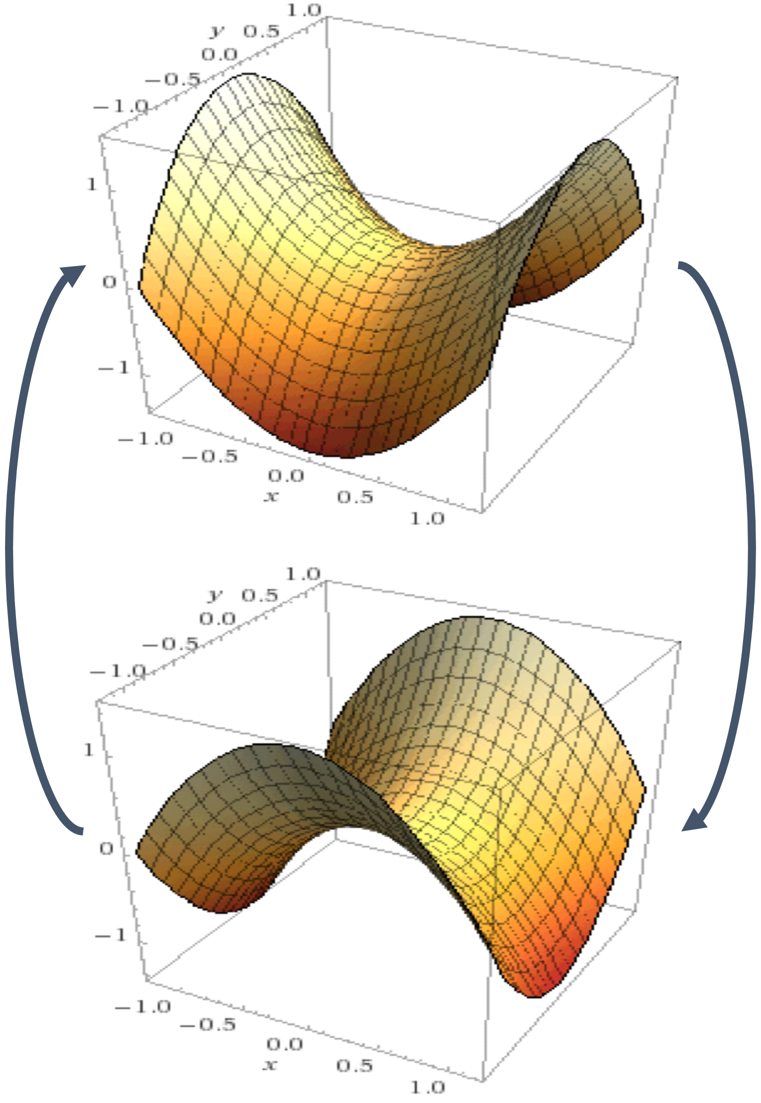




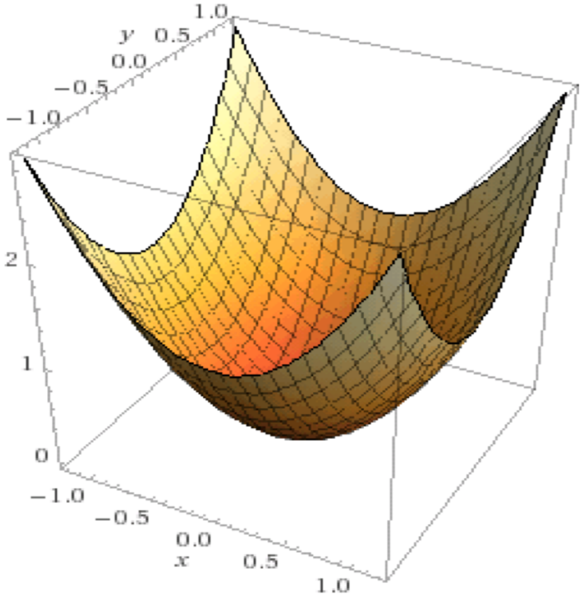
Paul Trap



GND
+V
+V
GND
GND
-V
-V
GND
y
x

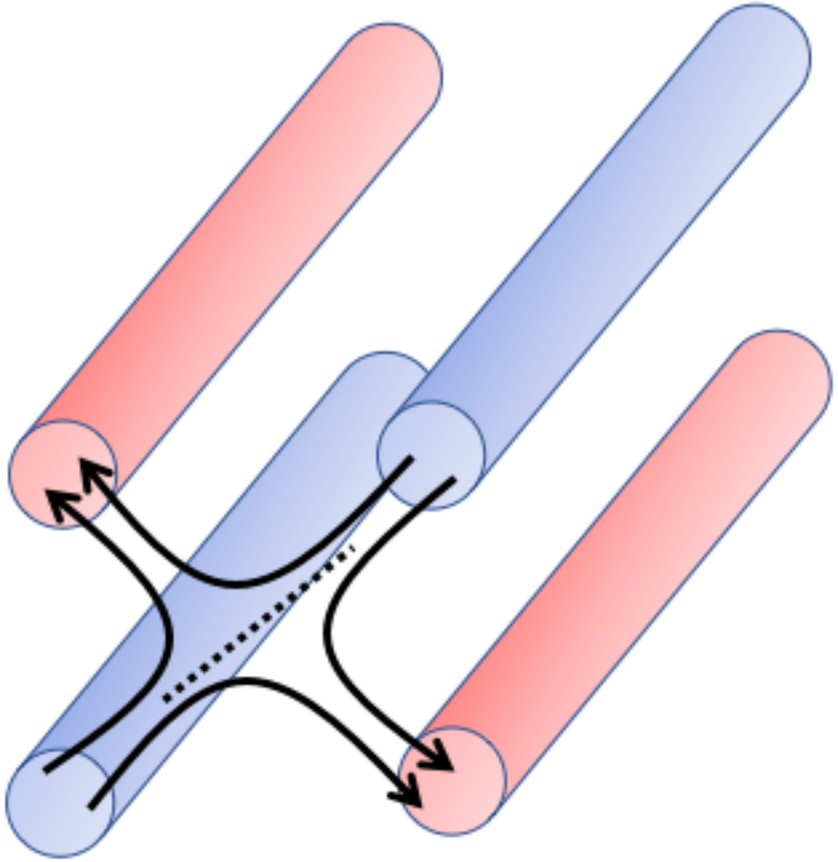


y
x
1.0
0.5
0.0
−0.5
−1.0
1
0
−1
−1.0
−0.5
0.0
0.5
1.0



y
x
1.0
0.5
0.0
−0.5
−1.0
2
1
0
−1.0
−0.5
0.0
0.5
1.0

Paul Trap – simple 4 rod



Mathieu Equation

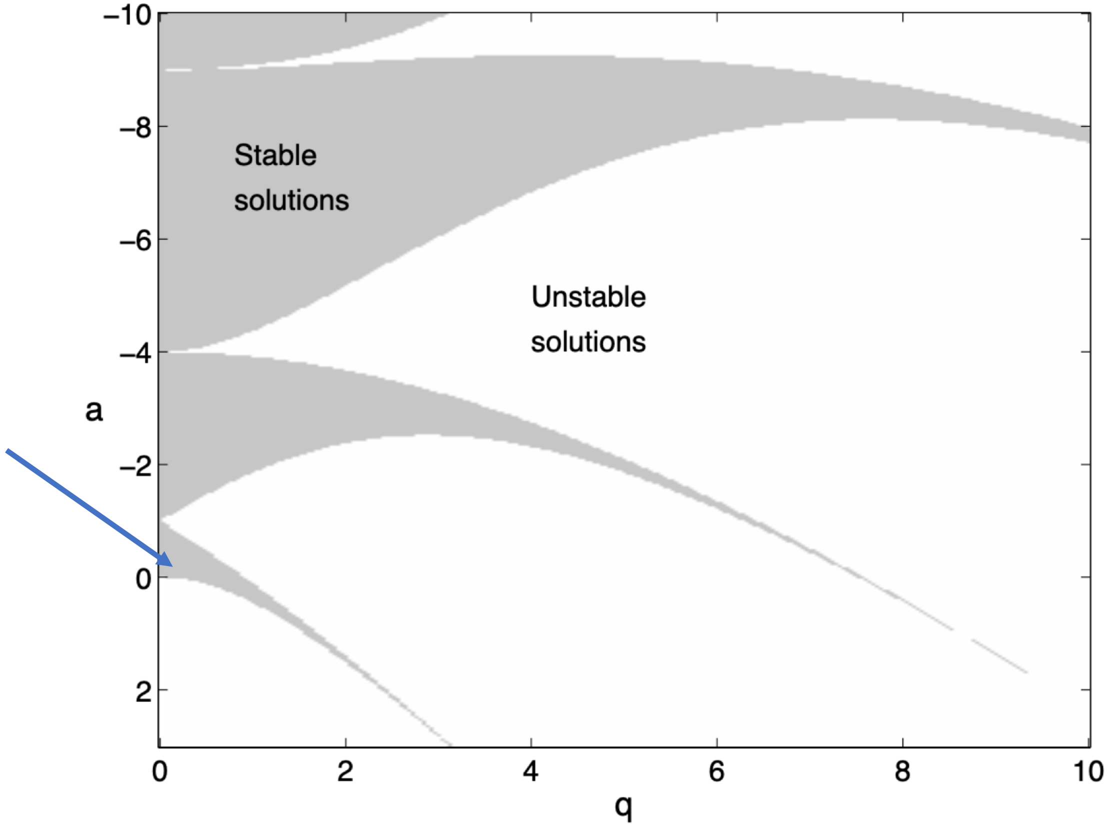
$$\Phi = \frac{\Phi_0}{r_0^2}\left(\alpha x^2 + \beta y^2 + \gamma z^2\right).$$

$$\Phi_0(t) = V_{\mathrm{DC}} + V_{\mathrm{RF}} \cos(\Omega_{\mathrm{RF}} t)$$

$$\frac{d^2 x}{d\tau^2} + (a - 2q\cos(2\tau))x = 0$$

$$a_x = -8\alpha Q V_{\mathrm{DC}} / m r_0^2 \Omega_{\mathrm{RF}}^2 \qquad q_x = 4\alpha Q V_{\mathrm{RF}} / m r_0^2 \Omega_{\mathrm{RF}}^2$$

Stability Parameters



$$a_x = -8\alpha Q V_{\mathrm{DC}}/m r_0^2 \Omega_{\mathrm{RF}}^2 \qquad q_x = 4\alpha Q V_{\mathrm{RF}}/m r_0^2 \Omega_{\mathrm{RF}}^2$$

Solution: Harmonic Oscillator

$$x(t) = x_0 \cos(\omega_x t + \phi_x) \left(1 + \tfrac{q_x}{2} \cos(\Omega_{\mathrm{RF}} t)\right)$$

$$\omega_x = \frac{\Omega_{\mathrm{RF}}}{2} \sqrt{a_x + q_x^2/2}$$

$$H_0 \quad = \quad \frac{p^2}{2m} + \tfrac{1}{2} m \omega^2 x^2$$

Mathieu parameters for ^{40}Ca$^+$

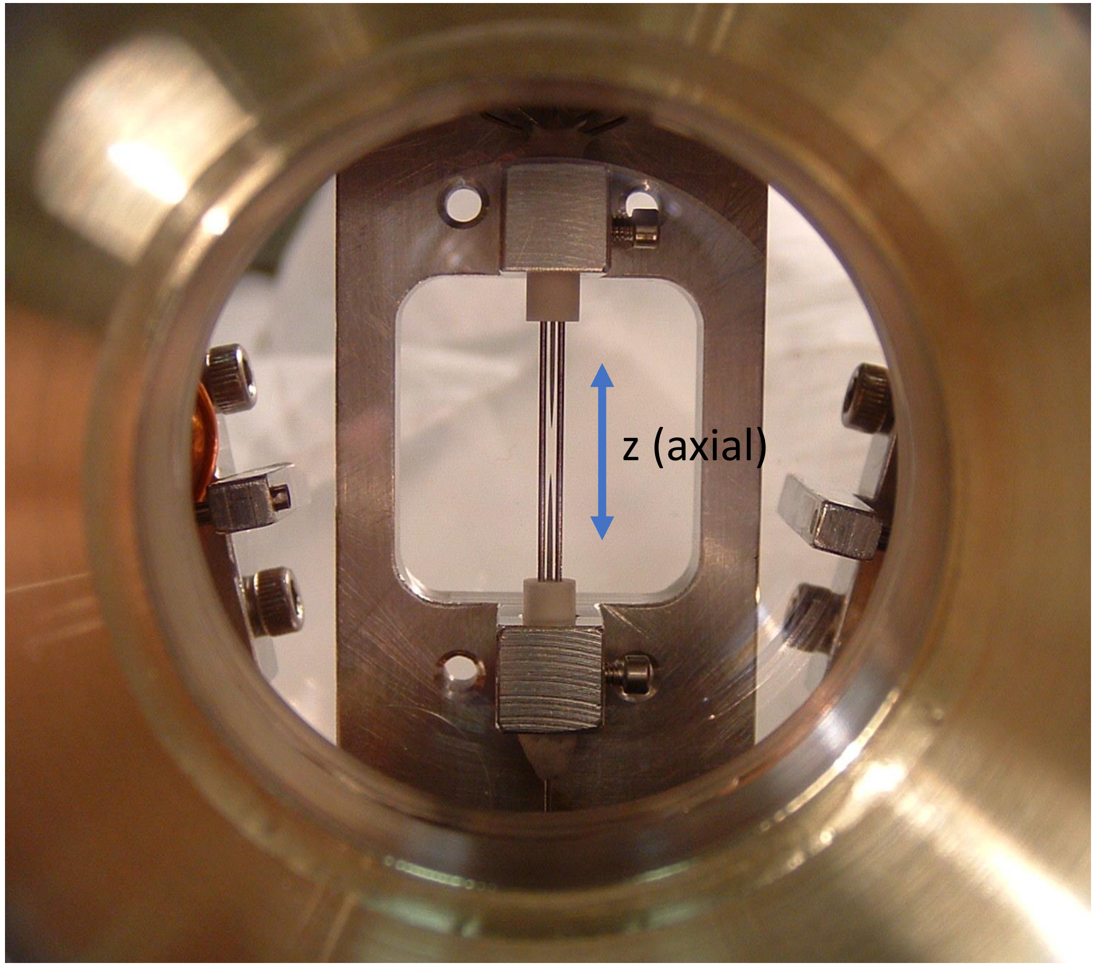
$$a_x = -8\alpha Q V_{\mathrm{DC}}/m r_0^2 \Omega_{\mathrm{RF}}^2$$

$$q_x = 4\alpha Q V_{\mathrm{RF}}/m r_0^2 \Omega_{\mathrm{RF}}^2$$

- m = 40 amu = 6.66 x 10^{-26} kg
- Q = e = 1.602 x 10^{-19} C
- r_0 ~ 100 um
- V ~ 100-300 V
- Ω_{RF} ~ 30-50 MHz

$$\omega_{radial} \sim 2 - 6 \text{ MHz}$$

Axial Confinement



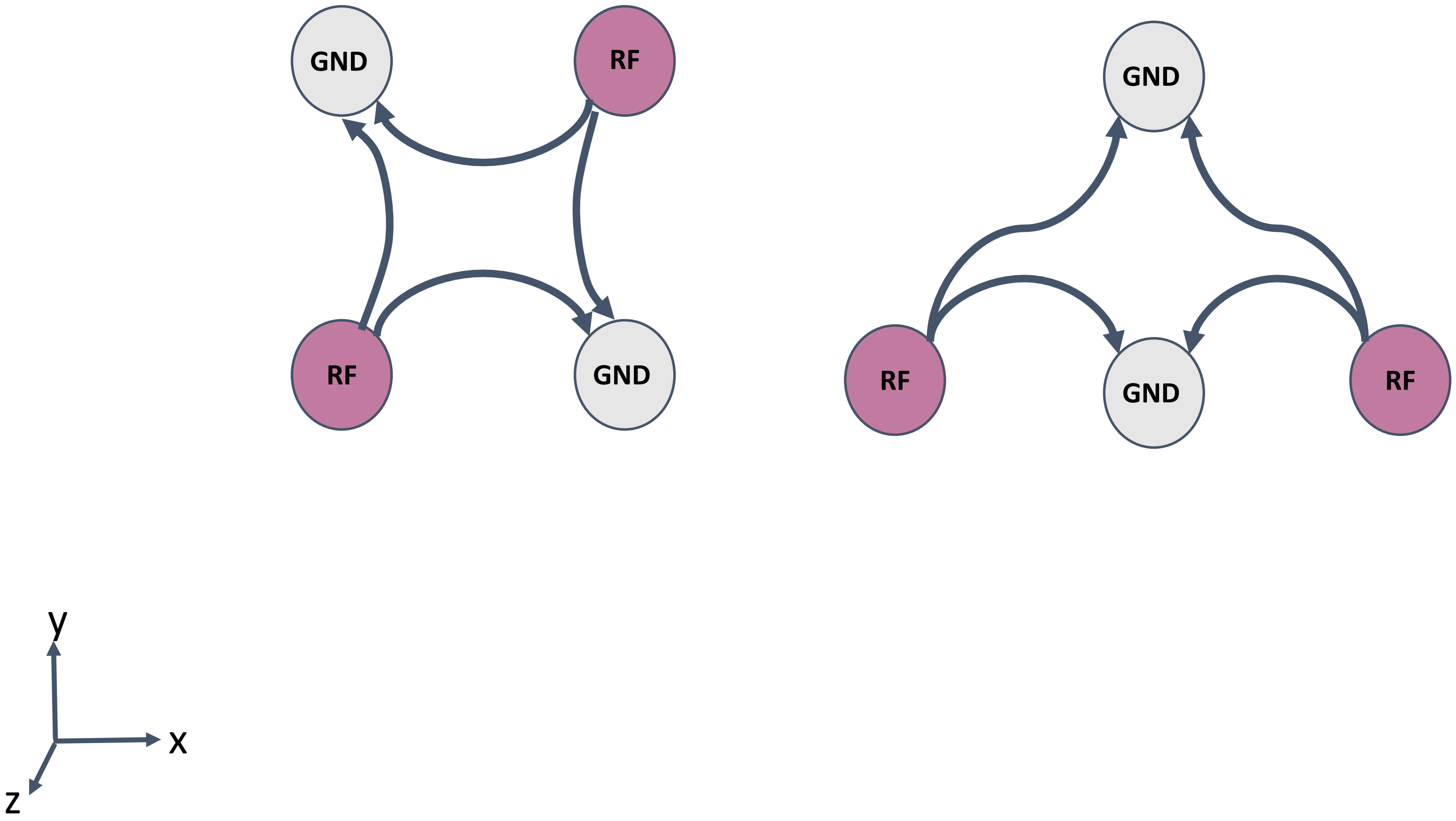
Also HO – static voltages

$$U_2 = \frac{2z^2 - x^2 - y^2}{2}$$

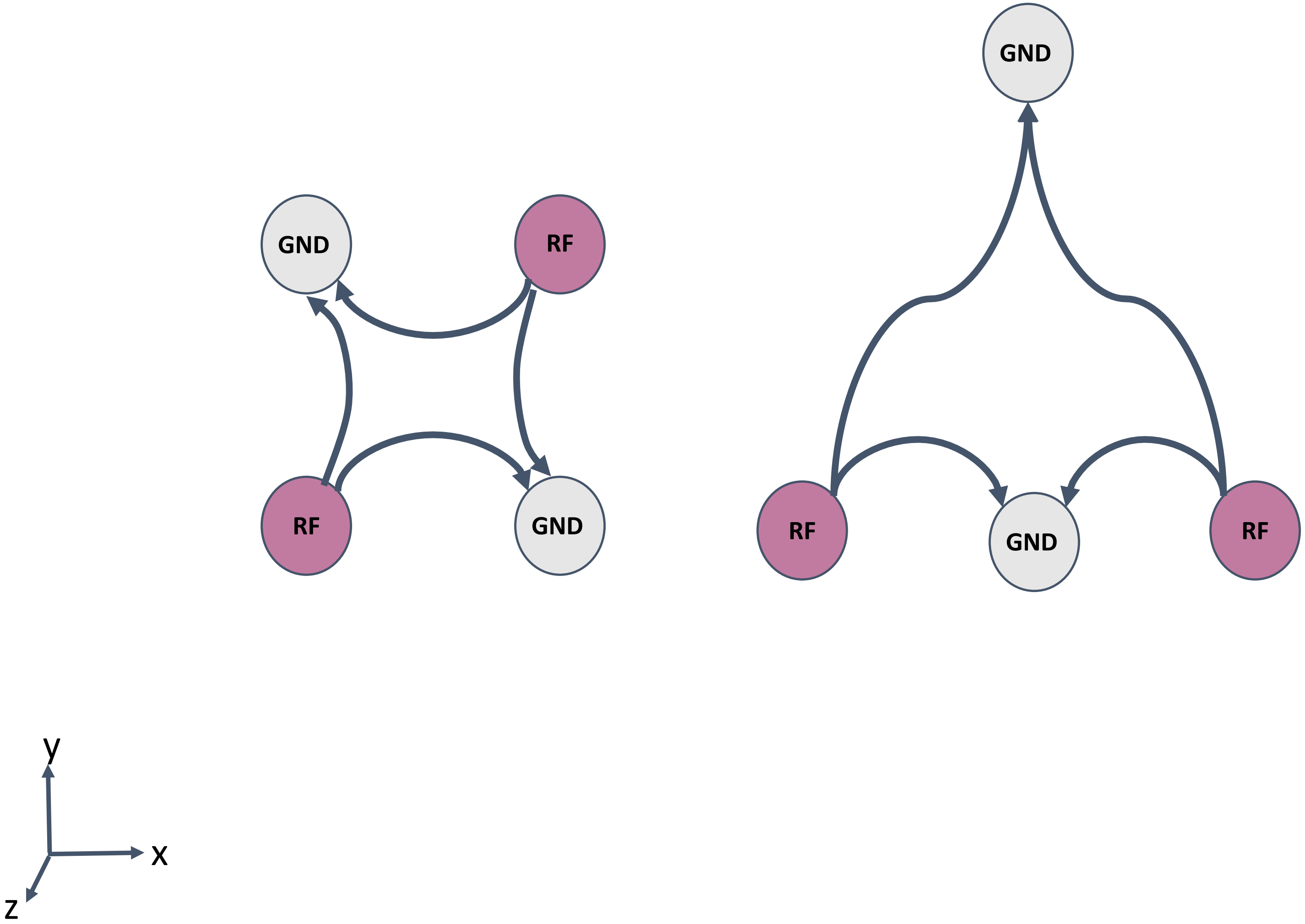
Surface traps

Scalability!

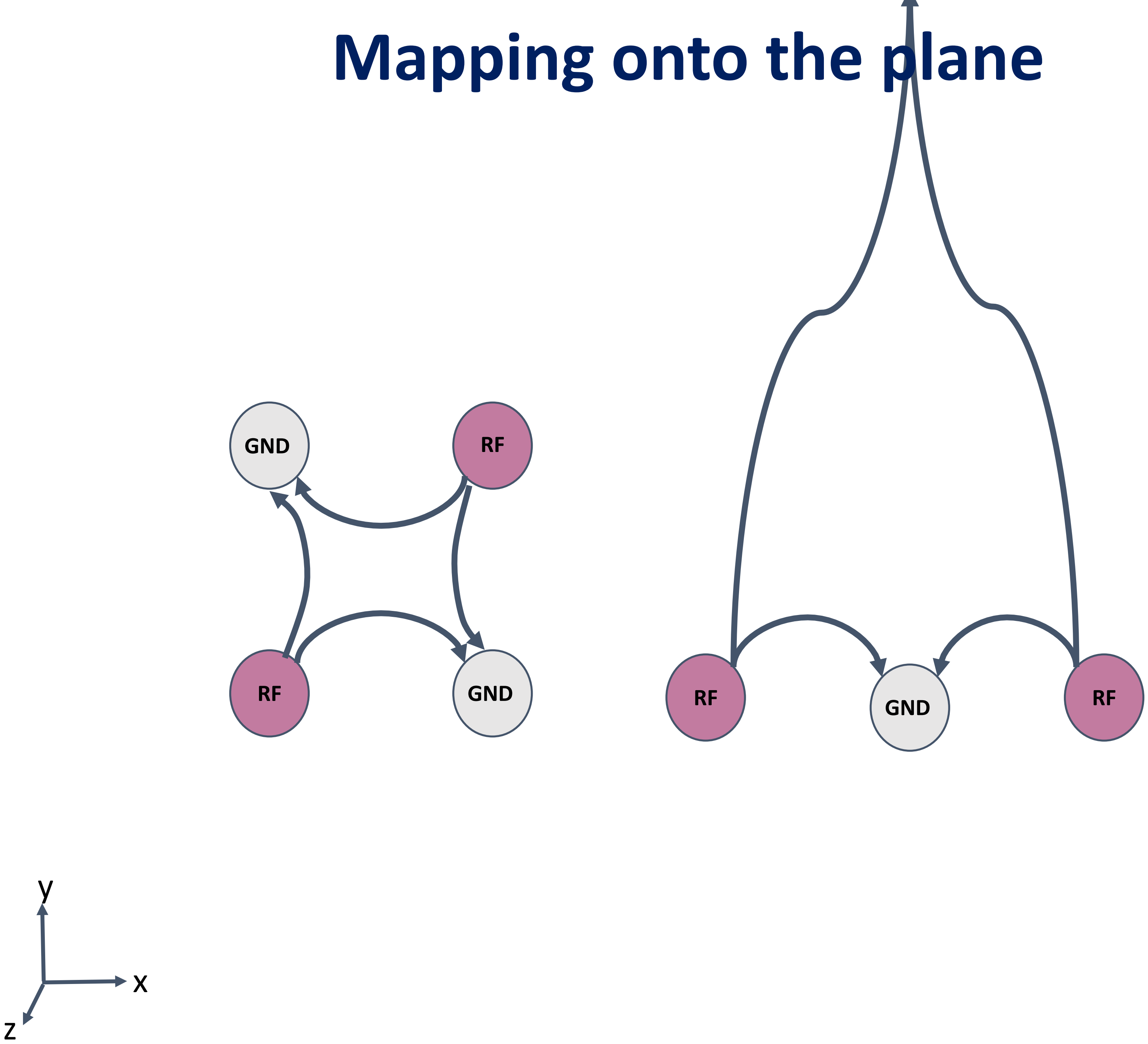
Mapping onto the plane



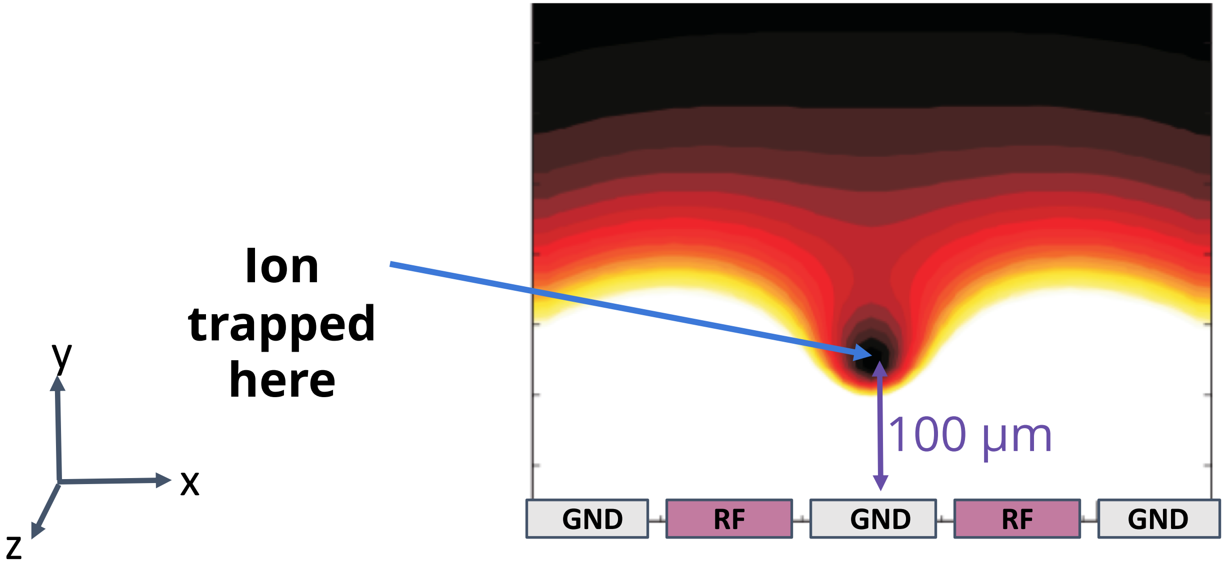
Mapping onto the plane



Mapping onto the plane



Surface Trap



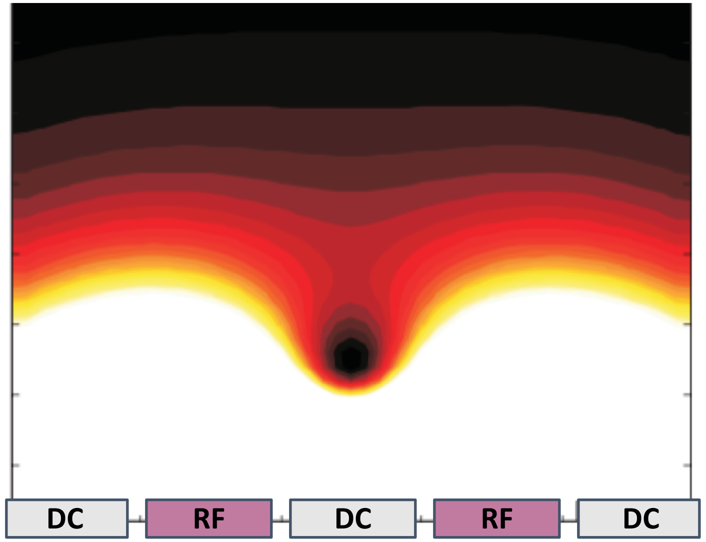
Ion trapped here

⬛ **Local minima**

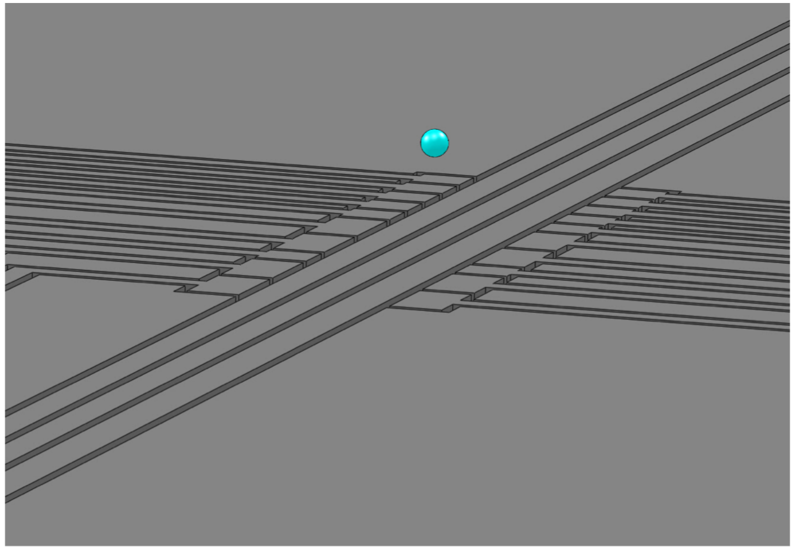
⬜ **Local maxima**

Chiaverini et al 2005

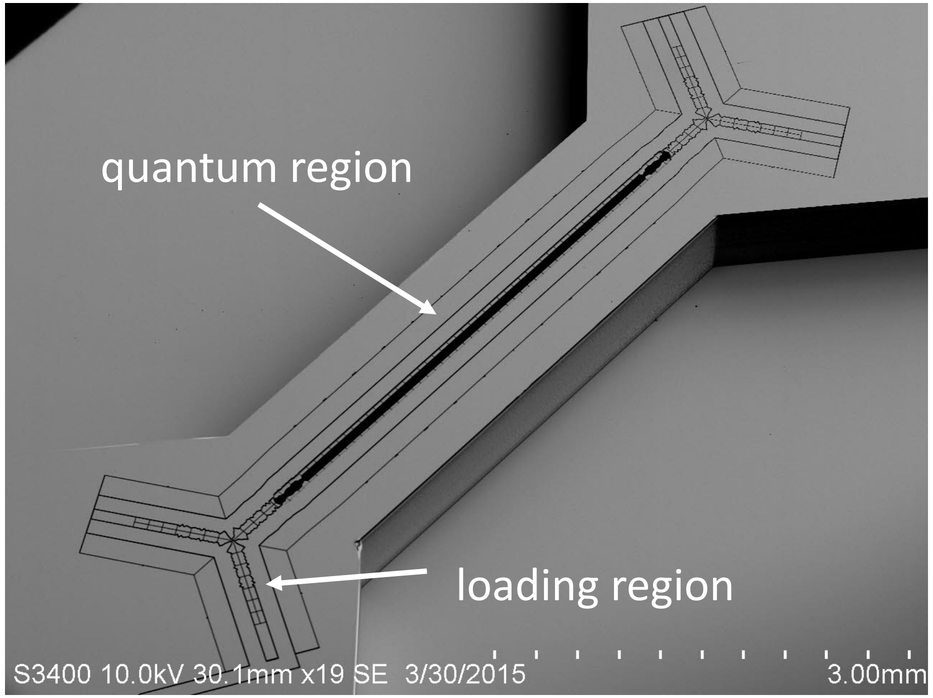
Surface Trap

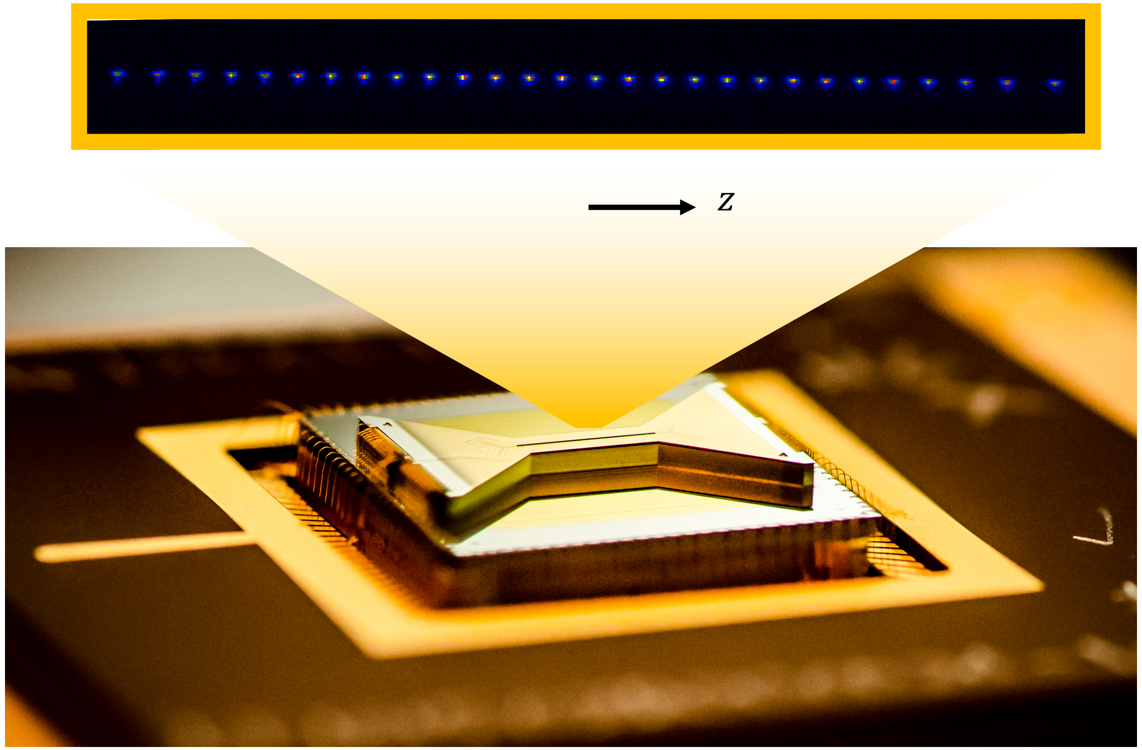


Above the trap



Trapping long chains

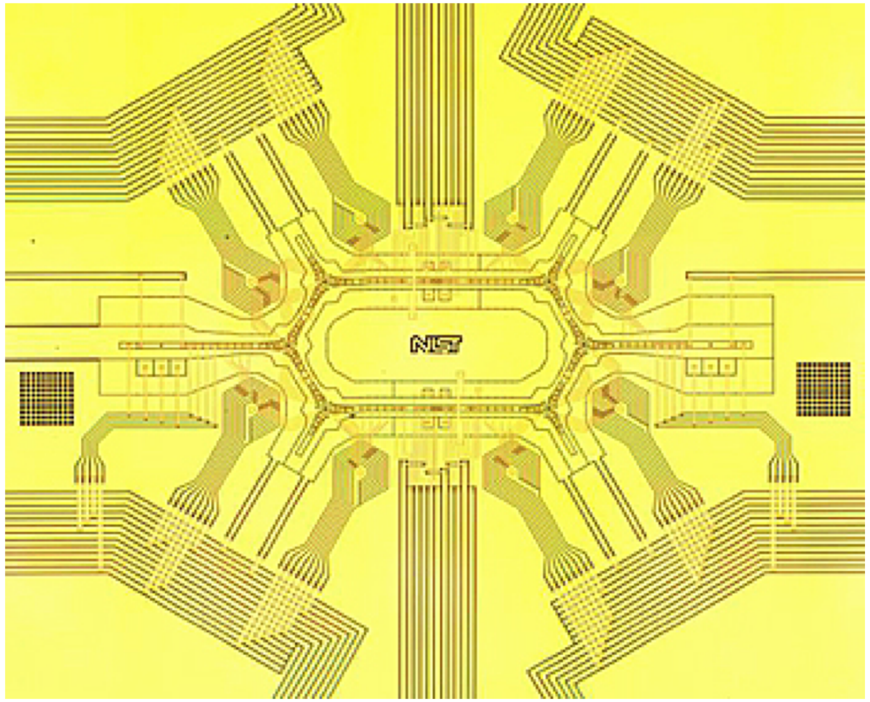




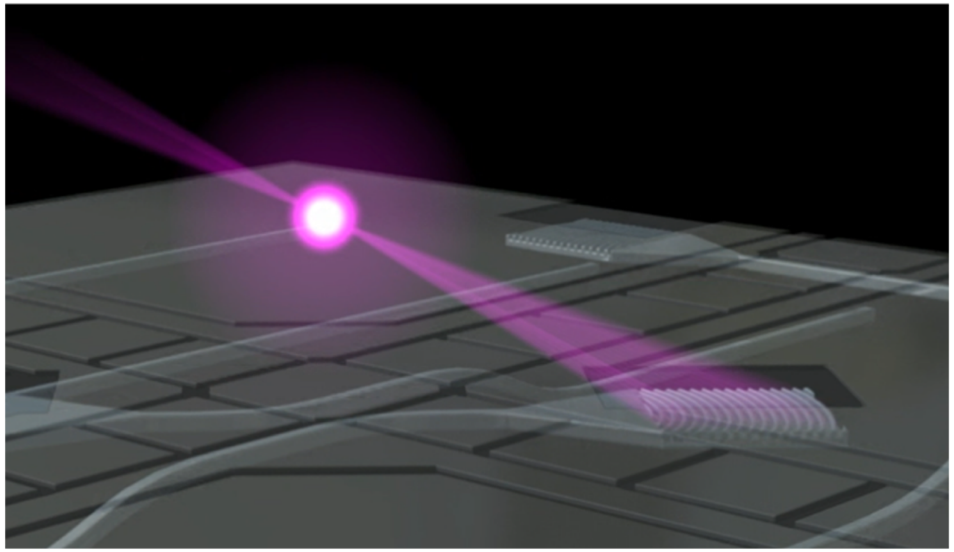
Sandia National Laboratories

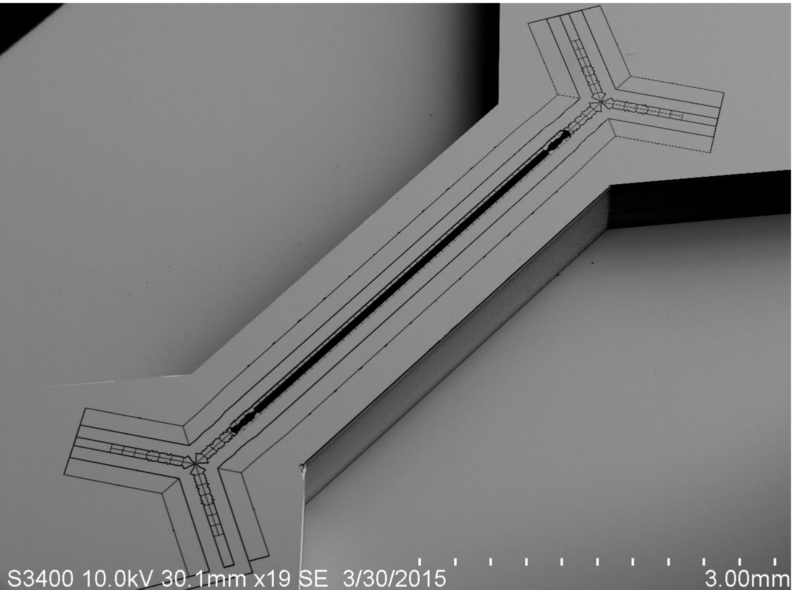
Surface Traps

NIST

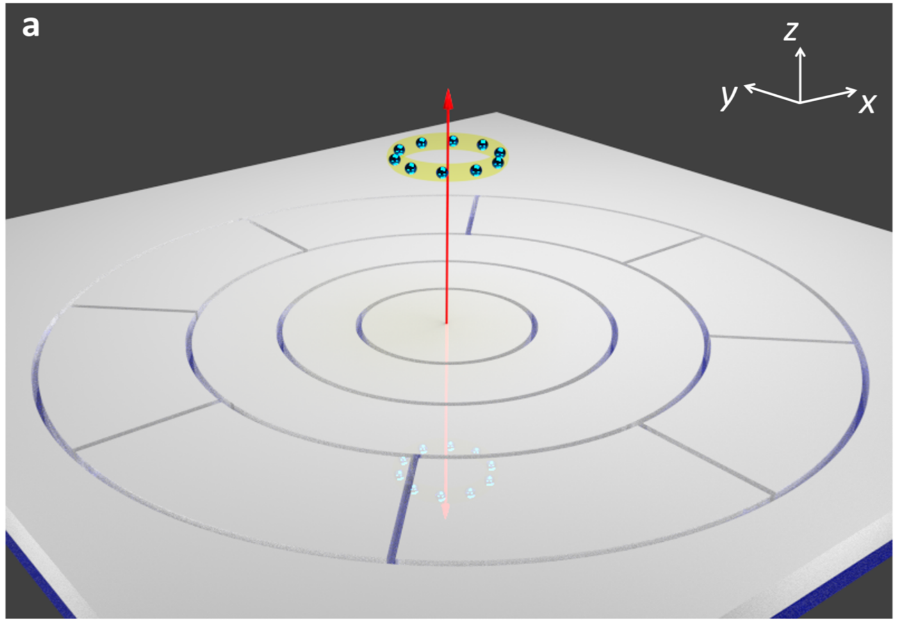


Lincoln Lab, MIT





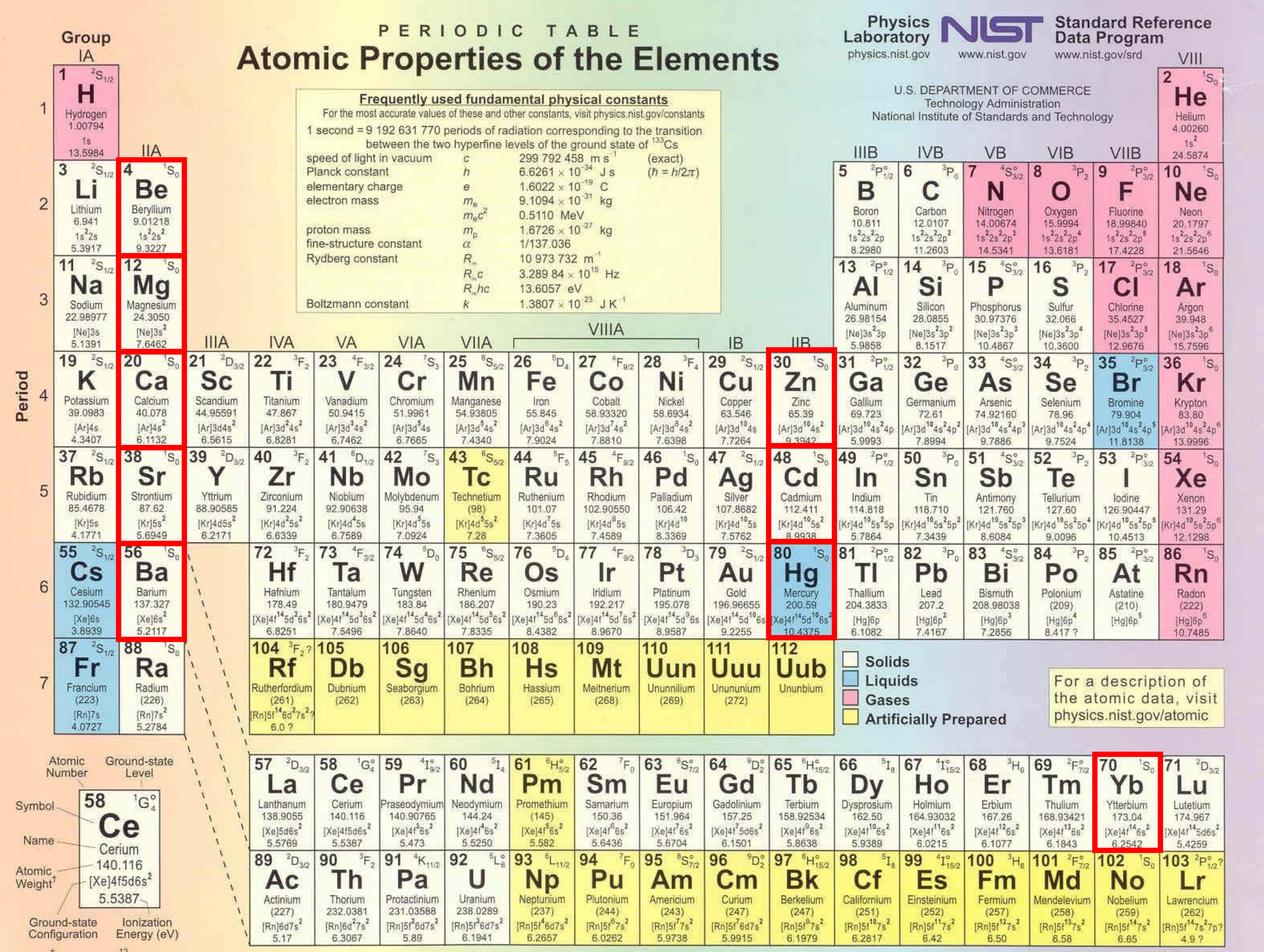
Sandia National Laboratories



UC Berkeley

Two Ion qubit examples

$^{40}Ca^+$ and $^{171}Yb^+$



Physics Laboratory
physics.nist.gov

NIST
www.nist.gov

Standard Reference Data Program
www.nist.gov/srd

U.S. DEPARTMENT OF COMMERCE
Technology Administration
National Institute of Standards and Technology

PERIODIC TABLE

Atomic Properties of the Elements

Frequently used fundamental physical constants

For the most accurate values of these and other constants, visit physics.nist.gov/constants

1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs

speed of light in vacuum	c	299 792 458 m s^{-1}	(exact)
Planck constant	h	6.6261 × 10^{-34} J s	($\hbar = h/2\pi$)
elementary charge	e	1.6022 × 10^{-19} C	
electron mass	m_e	9.1094 × 10^{-31} kg	
	m_ec^2	0.5110 MeV	
proton mass	m_p	1.6726 × 10^{-27} kg	
fine-structure constant	α	1/137.036	
Rydberg constant	R_∞	10 973 732 m^{-1}	
	$R_\infty c$	3.289 84 × 10^{15} Hz	
	$R_\infty hc$	13.6057 eV	
Boltzmann constant	k	1.3807 × 10^{-23} J K^{-1}	

Group

Period	IA	IIA	IIIA	IVA	VA	VIA	VIIA	VIIIA			IB	IIB	IIIB	IVB	VB	VIB	VIIB	VIII
1	1 $^2S_{1/2}$ **H** Hydrogen 1.00794 $1s$ 13.5984																	2 $^1S_0$ **He** Helium 4.00260 $1s^2$ 24.5874
2	3 $^2S_{1/2}$ **Li** Lithium 6.941 $1s^22s$ 5.3917	4 $^1S_0$ **Be** Beryllium 9.01218 $1s^22s^2$ 9.3227											5 $^2P^{\circ}_{1/2}$ **B** Boron 10.811 $1s^22s^22p$ 8.2980	6 $^3P_0$ **C** Carbon 12.0107 $1s^22s^22p^2$ 11.2603	7 $^4S^{\circ}_{3/2}$ **N** Nitrogen 14.00674 $1s^22s^22p^3$ 14.5341	8 $^3P_2$ **O** Oxygen 15.9994 $1s^22s^22p^4$ 13.6181	9 $^2P^{\circ}_{3/2}$ **F** Fluorine 18.99840 $1s^22s^22p^5$ 17.4228	10 $^1S_0$ **Ne** Neon 20.1797 $1s^22s^22p^6$ 21.5646
3	11 $^2S_{1/2}$ **Na** Sodium 22.98977 $[Ne]3s$ 5.1391	12 $^1S_0$ **Mg** Magnesium 24.3050 $[Ne]3s^2$ 7.6462											13 $^2P^{\circ}_{1/2}$ **Al** Aluminum 26.98154 $[Ne]3s^23p$ 5.9858	14 $^3P_0$ **Si** Silicon 28.0855 $[Ne]3s^23p^2$ 8.1517	15 $^4S^{\circ}_{3/2}$ **P** Phosphorus 30.97376 $[Ne]3s^23p^3$ 10.4867	16 $^3P_2$ **S** Sulfur 32.066 $[Ne]3s^23p^4$ 10.3600	17 $^2P^{\circ}_{3/2}$ **Cl** Chlorine 35.4527 $[Ne]3s^23p^5$ 12.9676	18 $^1S_0$ **Ar** Argon 39.948 $[Ne]3s^23p^6$ 15.7596
4	19 $^2S_{1/2}$ **K** Potassium 39.0983 $[Ar]4s$ 4.3407	20 $^1S_0$ **Ca** Calcium 40.078 $[Ar]4s^2$ 6.1132	21 $^2D_{3/2}$ **Sc** Scandium 44.95591 $[Ar]3d4s^2$ 6.5615	22 $^3F_2$ **Ti** Titanium 47.867 $[Ar]3d^24s^2$ 6.8281	23 $^4F_{3/2}$ **V** Vanadium 50.9415 $[Ar]3d^34s^2$ 6.7462	24 $^7S_3$ **Cr** Chromium 51.9961 $[Ar]3d^54s$ 6.7665	25 $^6S_{5/2}$ **Mn** Manganese 54.93805 $[Ar]3d^54s^2$ 7.4340	26 $^5D_4$ **Fe** Iron 55.845 $[Ar]3d^64s^2$ 7.9024	27 $^4F_{9/2}$ **Co** Cobalt 58.93320 $[Ar]3d^74s^2$ 7.8810	28 $^3F_4$ **Ni** Nickel 58.6934 $[Ar]3d^84s^2$ 7.6398	29 $^2S_{1/2}$ **Cu** Copper 63.546 $[Ar]3d^{10}4s$ 7.7264	30 $^1S_0$ **Zn** Zinc 65.39 $[Ar]3d^{10}4s^2$ 9.3942	31 $^2P^{\circ}_{1/2}$ **Ga** Gallium 69.723 $[Ar]3d^{10}4s^24p$ 5.9993	32 $^3P_0$ **Ge** Germanium 72.61 $[Ar]3d^{10}4s^24p^2$ 7.8994	33 $^4S^{\circ}_{3/2}$ **As** Arsenic 74.92160 $[Ar]3d^{10}4s^24p^3$ 9.7886	34 $^3P_2$ **Se** Selenium 78.96 $[Ar]3d^{10}4s^24p^4$ 9.7524	35 $^2P^{\circ}_{3/2}$ **Br** Bromine 79.904 $[Ar]3d^{10}4s^24p^5$ 11.8138	36 $^1S_0$ **Kr** Krypton 83.80 $[Ar]3d^{10}4s^24p^6$ 13.9996
5	37 $^2S_{1/2}$ **Rb** Rubidium 85.4678 $[Kr]5s$ 4.1771	38 $^1S_0$ **Sr** Strontium 87.62 $[Kr]5s^2$ 5.6949	39 $^2D_{3/2}$ **Y** Yttrium 88.90585 $[Kr]4d5s^2$ 6.2171	40 $^3F_2$ **Zr** Zirconium 91.224 $[Kr]4d^25s^2$ 6.6339	41 $^6D_{1/2}$ **Nb** Niobium 92.90638 $[Kr]4d^45s$ 6.7589	42 $^7S_3$ **Mo** Molybdenum 95.94 $[Kr]4d^55s$ 7.0924	43 $^6S_{5/2}$ **Tc** Technetium (98) $[Kr]4d^55s^2$ 7.28	44 $^5F_5$ **Ru** Ruthenium 101.07 $[Kr]4d^75s$ 7.3605	45 $^4F_{9/2}$ **Rh** Rhodium 102.90550 $[Kr]4d^85s$ 7.4589	46 $^1S_0$ **Pd** Palladium 106.42 $[Kr]4d^{10}$ 8.3369	47 $^2S_{1/2}$ **Ag** Silver 107.8682 $[Kr]4d^{10}5s$ 7.5762	48 $^1S_0$ **Cd** Cadmium 112.411 $[Kr]4d^{10}5s^2$ 8.9938	49 $^2P^{\circ}_{1/2}$ **In** Indium 114.818 $[Kr]4d^{10}5s^25p$ 5.7864	50 $^3P_0$ **Sn** Tin 118.710 $[Kr]4d^{10}5s^25p^2$ 7.3439	51 $^4S^{\circ}_{3/2}$ **Sb** Antimony 121.760 $[Kr]4d^{10}5s^25p^3$ 8.6084	52 $^3P_2$ **Te** Tellurium 127.60 $[Kr]4d^{10}5s^25p^4$ 9.0096	53 $^2P^{\circ}_{3/2}$ **I** Iodine 126.90447 $[Kr]4d^{10}5s^25p^5$ 10.4513	54 $^1S_0$ **Xe** Xenon 131.29 $[Kr]4d^{10}5s^25p^6$ 12.1298
6	55 $^2S_{1/2}$ **Cs** Cesium 132.90545 $[Xe]6s$ 3.8939	56 $^1S_0$ **Ba** Barium 137.327 $[Xe]6s^2$ 5.2117		72 $^3F_2$ **Hf** Hafnium 178.49 $[Xe]4f^{14}5d^26s^2$ 6.8251	73 $^4F_{3/2}$ **Ta** Tantalum 180.9479 $[Xe]4f^{14}5d^36s^2$ 7.5496	74 $^5D_0$ **W** Tungsten 183.84 $[Xe]4f^{14}5d^46s^2$ 7.8640	75 $^6S_{5/2}$ **Re** Rhenium 186.207 $[Xe]4f^{14}5d^56s^2$ 7.8335	76 $^5D_4$ **Os** Osmium 190.23 $[Xe]4f^{14}5d^66s^2$ 8.4382	77 $^4F_{9/2}$ **Ir** Iridium 192.217 $[Xe]4f^{14}5d^76s^2$ 8.9670	78 $^3D_3$ **Pt** Platinum 195.078 $[Xe]4f^{14}5d^96s$ 8.9587	79 $^2S_{1/2}$ **Au** Gold 196.96655 $[Xe]4f^{14}5d^{10}6s$ 9.2255	80 $^1S_0$ **Hg** Mercury 200.59 $[Xe]4f^{14}5d^{10}6s^2$ 10.4375	81 $^2P^{\circ}_{1/2}$ **Tl** Thallium 204.3833 $[Hg]6p$ 6.1082	82 $^3P_0$ **Pb** Lead 207.2 $[Hg]6p^2$ 7.4167	83 $^4S^{\circ}_{3/2}$ **Bi** Bismuth 208.98038 $[Hg]6p^3$ 7.2856	84 $^3P_2$ **Po** Polonium (209) $[Hg]6p^4$ 8.417 ?	85 $^2P^{\circ}_{3/2}$ **At** Astatine (210) $[Hg]6p^5$	86 $^1S_0$ **Rn** Radon (222) $[Hg]6p^6$ 10.7485
7	87 $^2S_{1/2}$ **Fr** Francium (223) $[Rn]7s$ 4.0727	88 $^1S_0$ **Ra** Radium (226) $[Rn]7s^2$ 5.2784		104 $^3F_2$? **Rf** Rutherfordium (261) $[Rn]5f^{14}6d^27s^2$? 6.0 ?	105 **Db** Dubnium (262)	106 **Sg** Seaborgium (263)	107 **Bh** Bohrium (264)	108 **Hs** Hassium (265)	109 **Mt** Meitnerium (268)	110 **Uun** Ununnilium (269)	111 **Uuu** Unununium (272)	112 **Uub** Ununbium						

57 $^2D_{3/2}$ **La** Lanthanum 138.9055 $[Xe]5d6s^2$ 5.5769	58 $^1G^{\circ}_4$ **Ce** Cerium 140.116 $[Xe]4f5d6s^2$ 5.5387	59 $^4I^{\circ}_{9/2}$ **Pr** Praseodymium 140.90765 $[Xe]4f^36s^2$ 5.473	60 $^5I_4$ **Nd** Neodymium 144.24 $[Xe]4f^46s^2$ 5.5250	61 $^6H^{\circ}_{5/2}$ **Pm** Promethium (145) $[Xe]4f^56s^2$ 5.582	62 $^7F_0$ **Sm** Samarium 150.36 $[Xe]4f^66s^2$ 5.6436	63 $^8S^{\circ}_{7/2}$ **Eu** Europium 151.964 $[Xe]4f^76s^2$ 5.6704	64 $^9D^{\circ}_2$ **Gd** Gadolinium 157.25 $[Xe]4f^75d6s^2$ 6.1501	65 $^6H^{\circ}_{15/2}$ **Tb** Terbium 158.92534 $[Xe]4f^96s^2$ 5.8638	66 $^5I_8$ **Dy** Dysprosium 162.50 $[Xe]4f^{10}6s^2$ 5.9389	67 $^4I^{\circ}_{15/2}$ **Ho** Holmium 164.93032 $[Xe]4f^{11}6s^2$ 6.0215	68 $^3H_6$ **Er** Erbium 167.26 $[Xe]4f^{12}6s^2$ 6.1077	69 $^2F^{\circ}_{7/2}$ **Tm** Thulium 168.93421 $[Xe]4f^{13}6s^2$ 6.1843	70 $^1S_0$ **Yb** Ytterbium 173.04 $[Xe]4f^{14}6s^2$ 6.2542	71 $^2D_{3/2}$ **Lu** Lutetium 174.967 $[Xe]4f^{14}5d6s^2$ 5.4259
89 $^2D_{3/2}$ **Ac** Actinium (227) $[Rn]6d7s^2$ 5.17	90 $^3F_2$ **Th** Thorium 232.0381 $[Rn]6d^27s^2$ 6.3067	91 $^4K_{11/2}$ **Pa** Protactinium 231.03588 $[Rn]5f^26d7s^2$ 5.89	92 $^5L^{\circ}_6$ **U** Uranium 238.0289 $[Rn]5f^36d7s^2$ 6.1941	93 $^6L_{11/2}$ **Np** Neptunium (237) $[Rn]5f^46d7s^2$ 6.2657	94 $^7F_0$ **Pu** Plutonium (244) $[Rn]5f^67s^2$ 6.0262	95 $^8S^{\circ}_{7/2}$ **Am** Americium (243) $[Rn]5f^77s^2$ 5.9738	96 $^9D^{\circ}_2$ **Cm** Curium (247) $[Rn]5f^76d7s^2$ 5.9915	97 $^6H^{\circ}_{15/2}$ **Bk** Berkelium (247) $[Rn]5f^97s^2$ 6.1979	98 $^5I_8$ **Cf** Californium (251) $[Rn]5f^{10}7s^2$ 6.2817	99 $^4I^{\circ}_{15/2}$ **Es** Einsteinium (252) $[Rn]5f^{11}7s^2$ 6.42	100 $^3H_6$ **Fm** Fermium (257) $[Rn]5f^{12}7s^2$ 6.50	101 $^2F^{\circ}_{7/2}$ **Md** Mendelevium (258) $[Rn]5f^{13}7s^2$ 6.58	102 $^1S_0$ **No** Nobelium (259) $[Rn]5f^{14}7s^2$ 6.65	103 $^2P^{\circ}_{1/2}$? **Lr** Lawrencium (262) $[Rn]5f^{14}7s^27p$? 4.9 ?

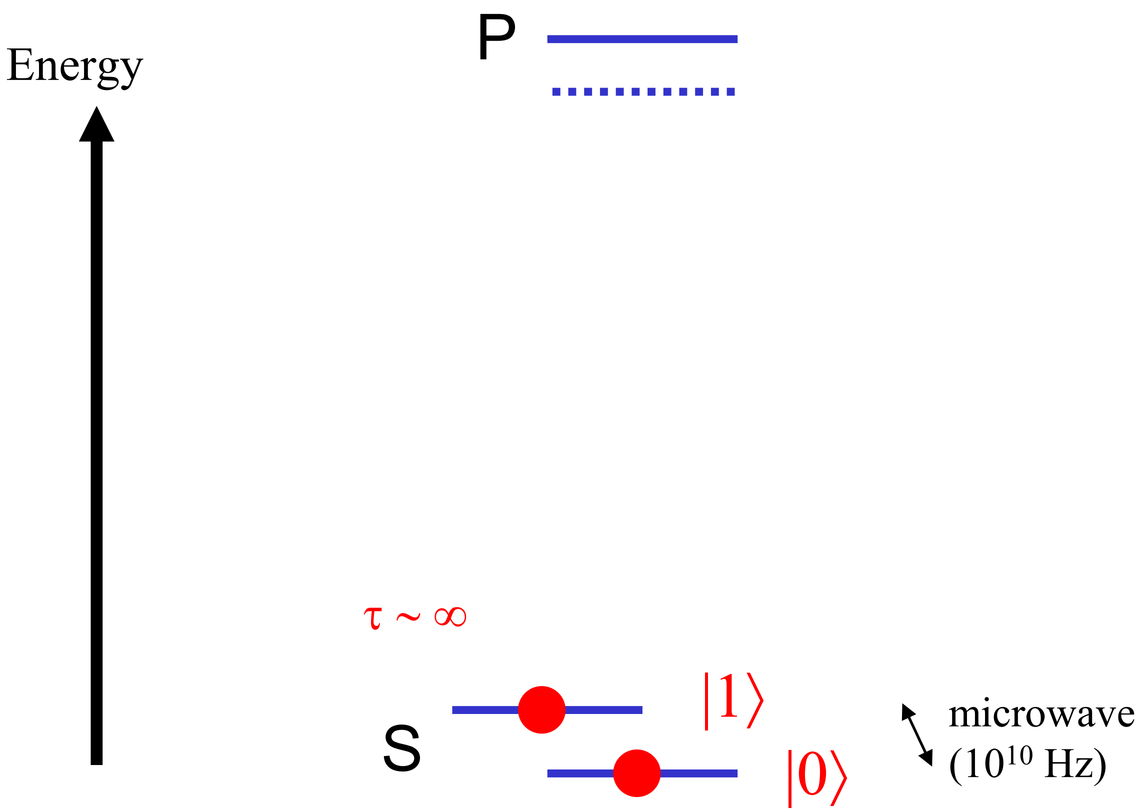
☐ Solids
🟦 Liquids
🟥 Gases
🟨 Artificially Prepared

For a description of the atomic data, visit physics.nist.gov/atomic

Atomic Number	58
Ground-state Level	$^1G^{\circ}_4$
Symbol	**Ce**
Name	Cerium
Atomic Weight†	140.116
Ground-state Configuration	$[Xe]4f5d6s^2$
Ionization Energy (eV)	5.5387

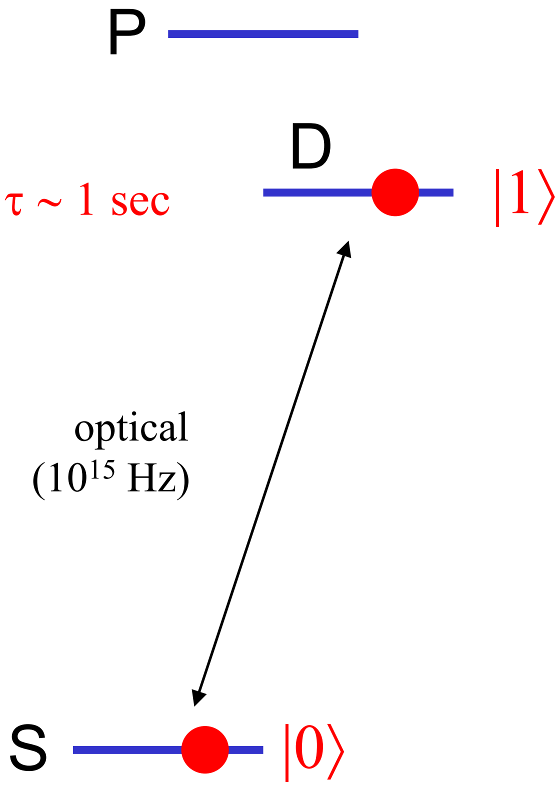
Trapped ion qubits

Ca$^+$, Sr$^+$, Ba$^+$, Yb$^+$
Be$^+$, Mg$^+$, Hg$^+$, Cd$^+$, Zn$^+$



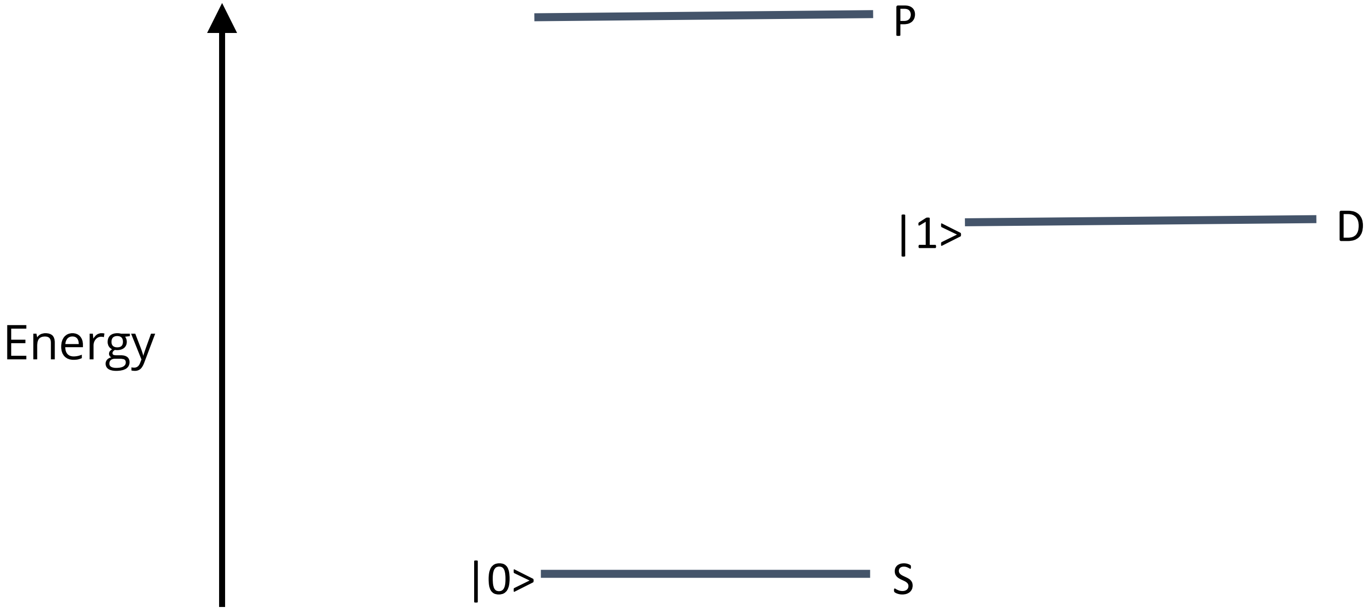
“**hyperfine**” qubit levels

Ca$^+$, Sr$^+$, Ba$^+$, Yb$^+$



“**optical**” qubit levels

Calcium ion energy diagram



P

|1>

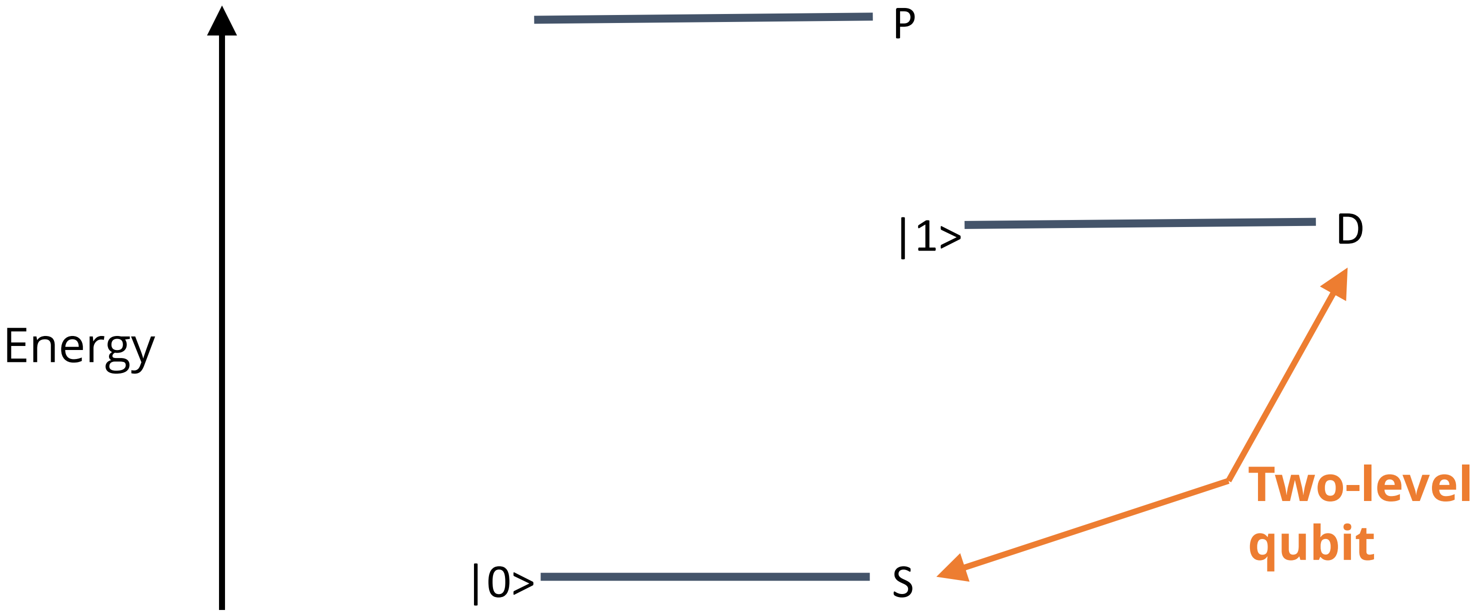
D

Energy

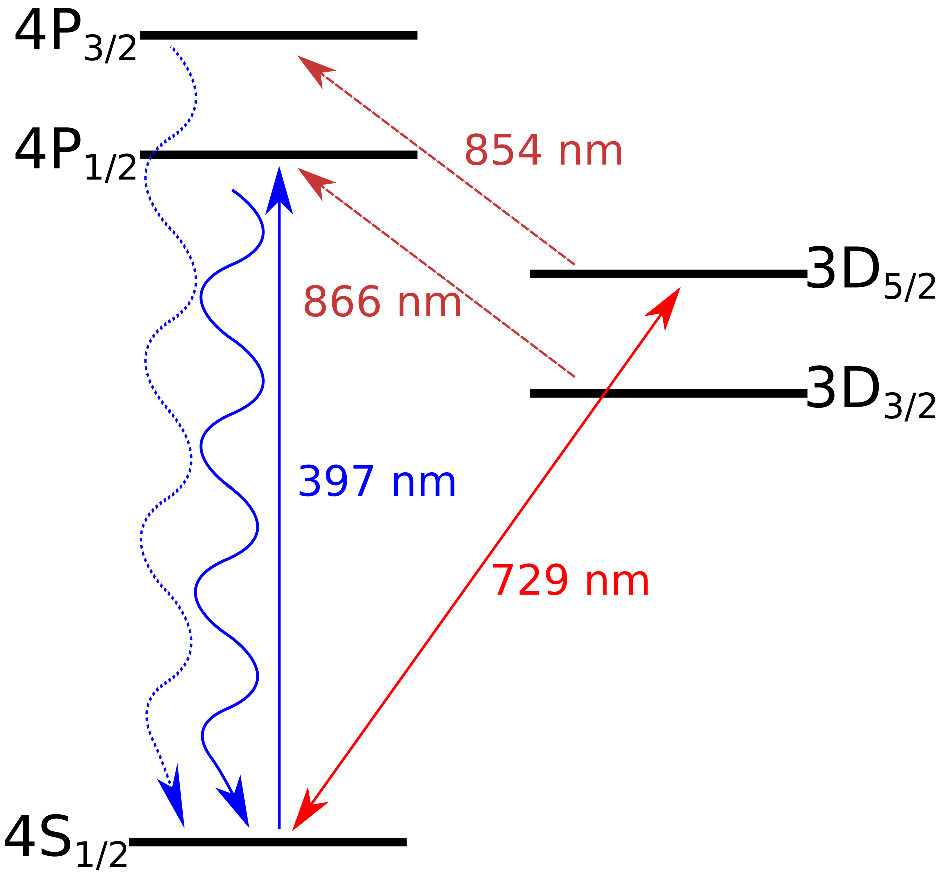
|0>

S

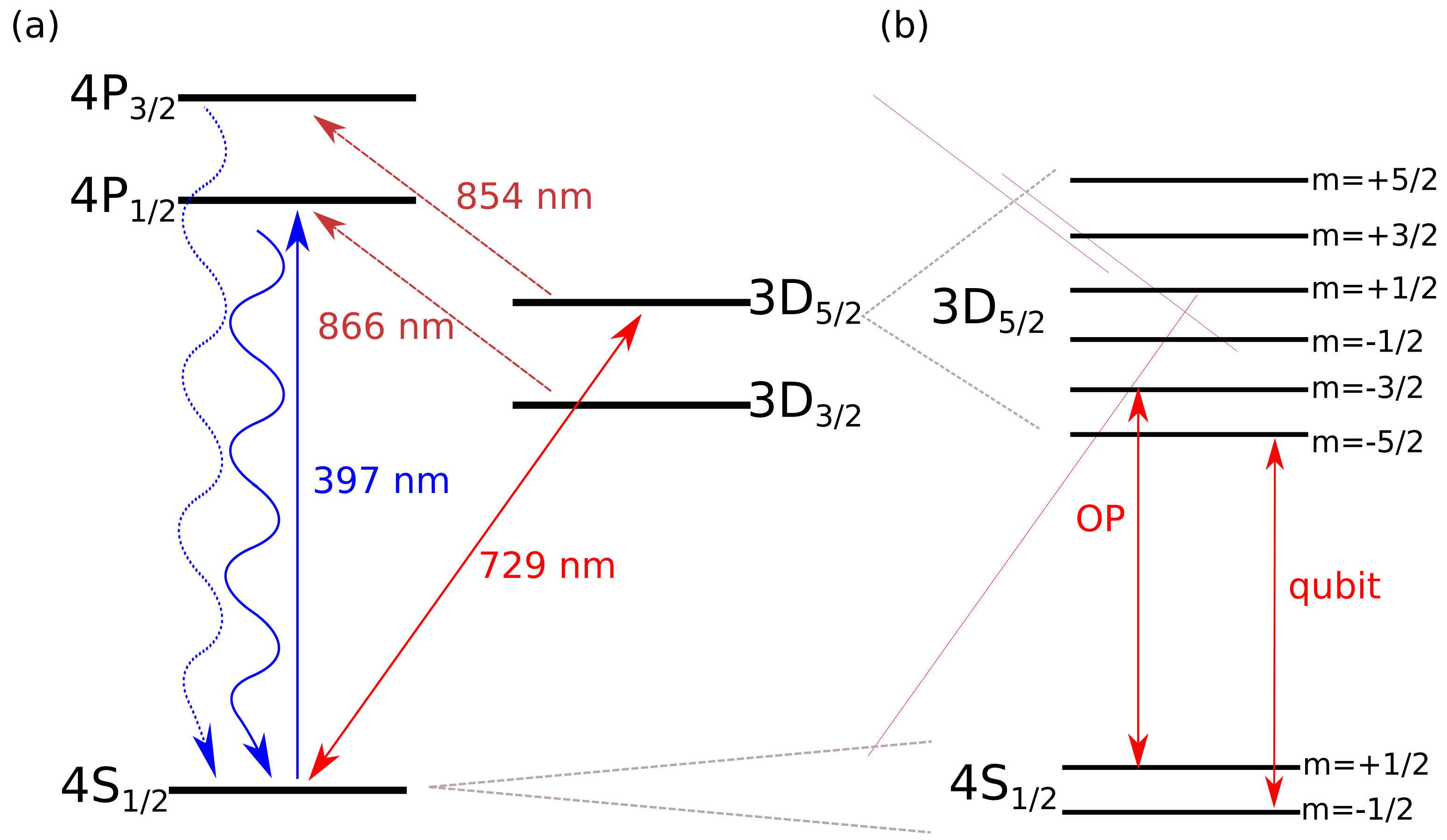
Calcium ion energy diagram



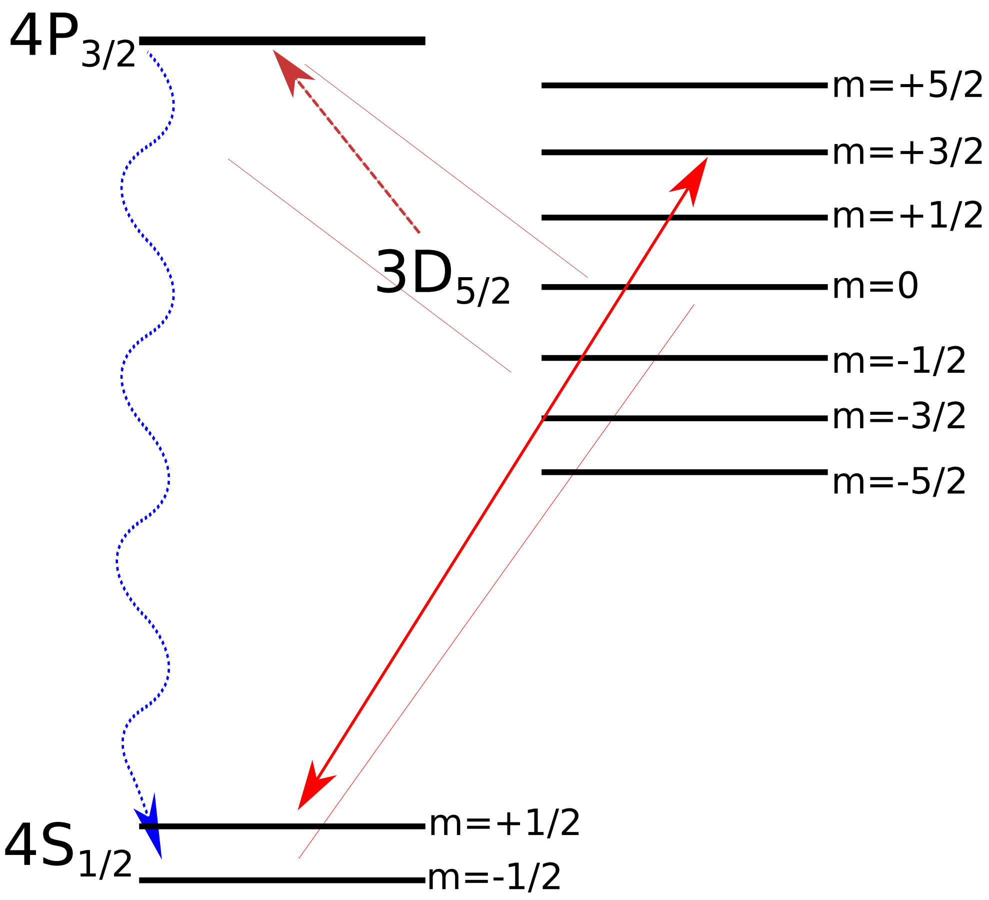
Detailed atomic structure



Zeeman levels

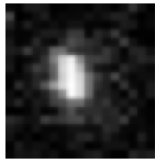


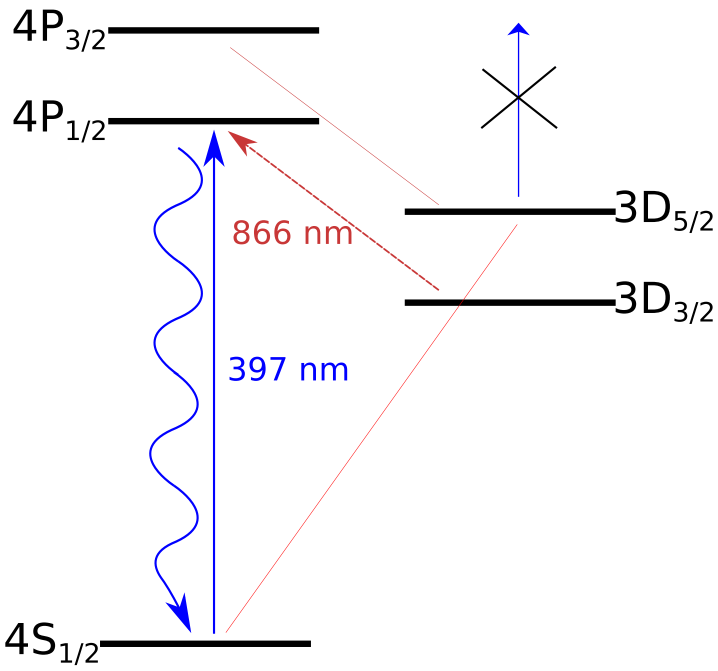
Initialization – optical pumping



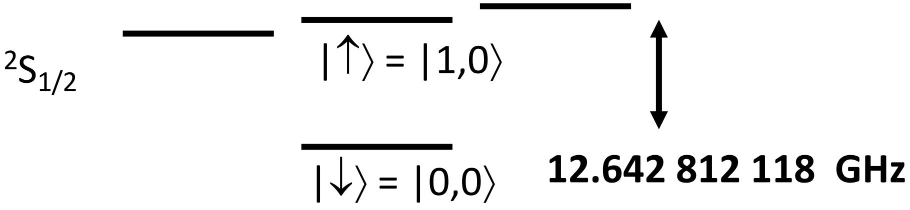
Reading out the qubit

- Electron Shelving
- P-state lifetime ~ 7 ns
- Fluorescence when ion is in S





$^{171}Yb^{+}$

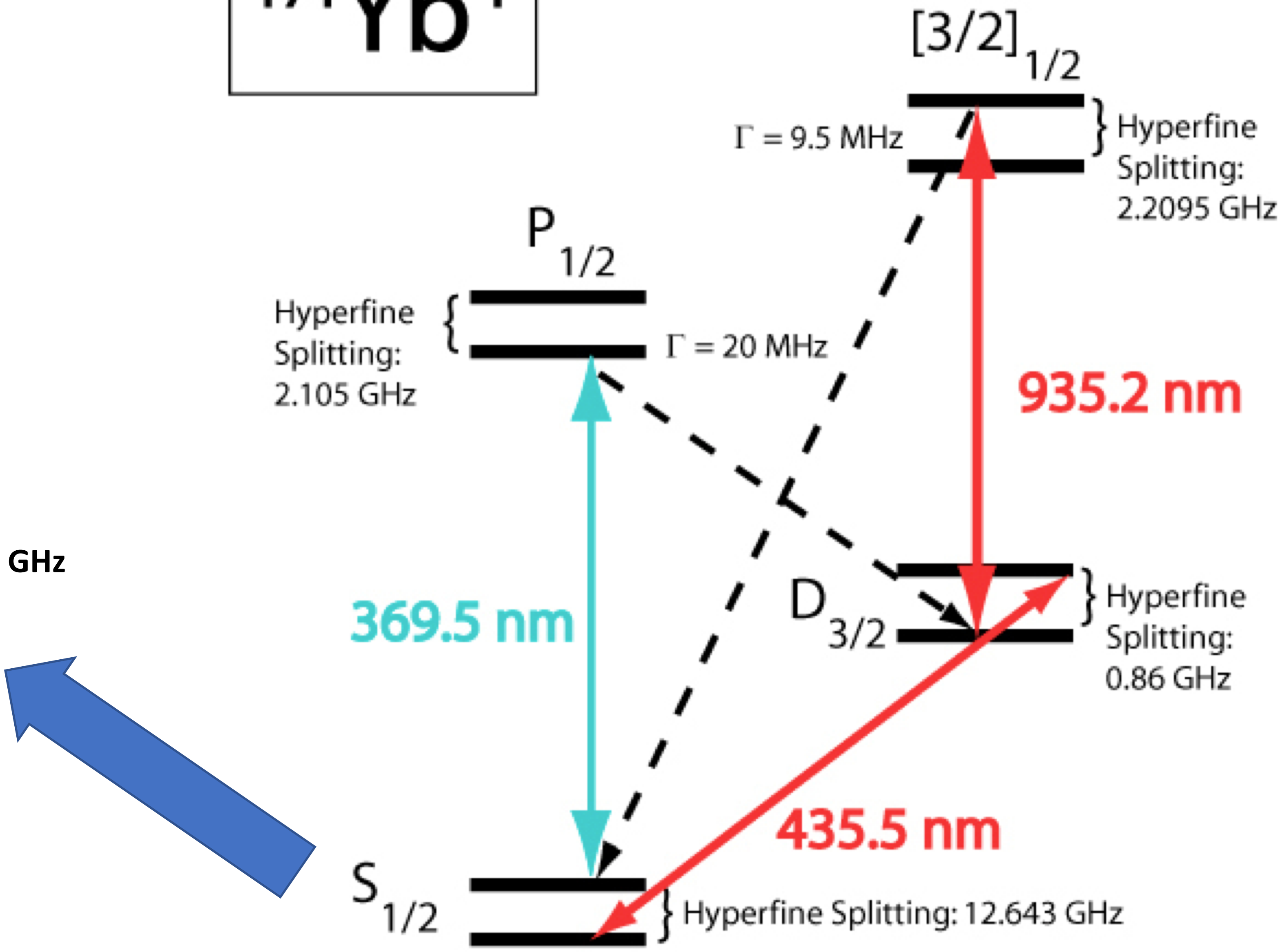


$^2S_{1/2}$

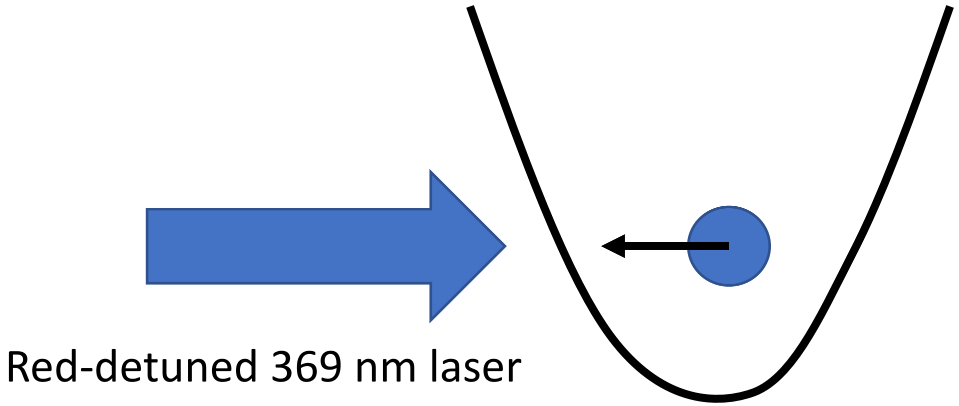
$|\uparrow\rangle = |1,0\rangle$

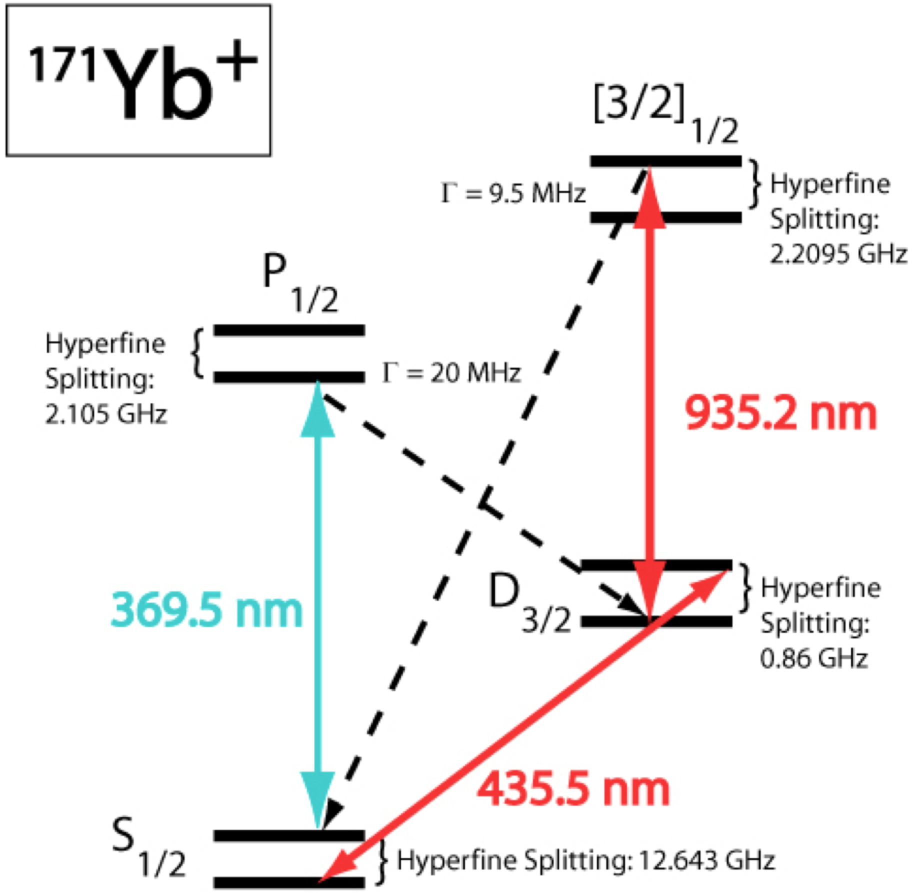
$|\downarrow\rangle = |0,0\rangle$

12.642 812 118 GHz



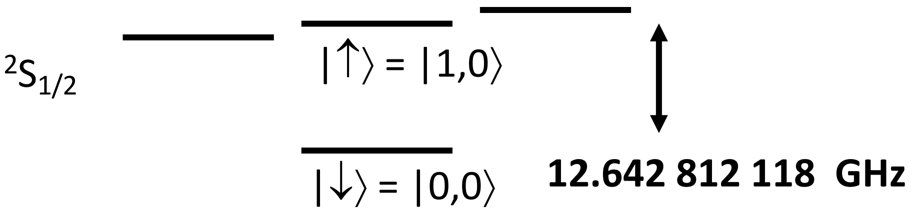
Doppler cooling and detection





- 0.6% of fluorescence detected
- 99.3% qubit detection fidelity

Atomic clock qubit (^{171}**Yb**$^{+}$)

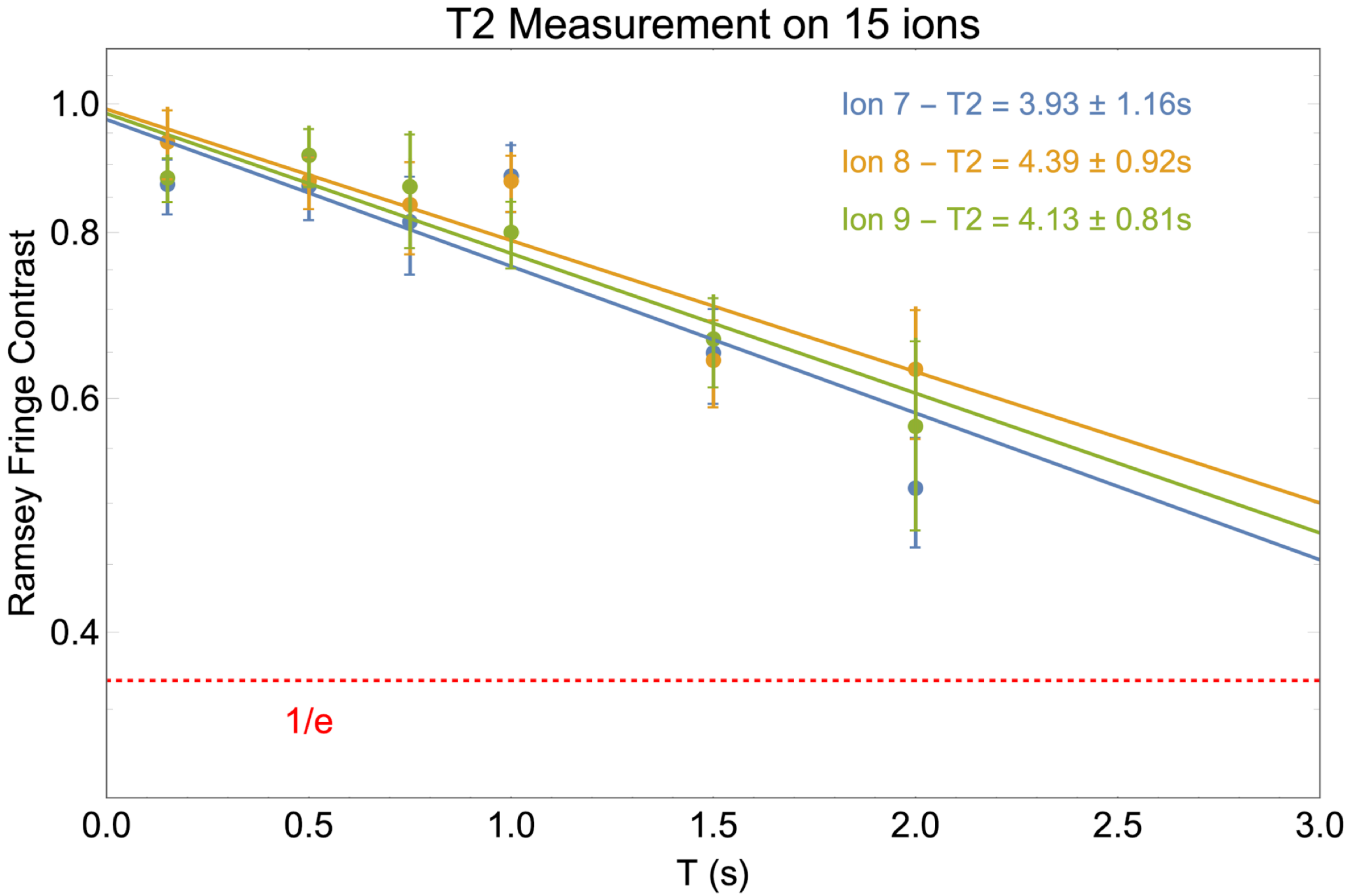


$^2S_{1/2}$

$|\uparrow\rangle = |1,0\rangle$

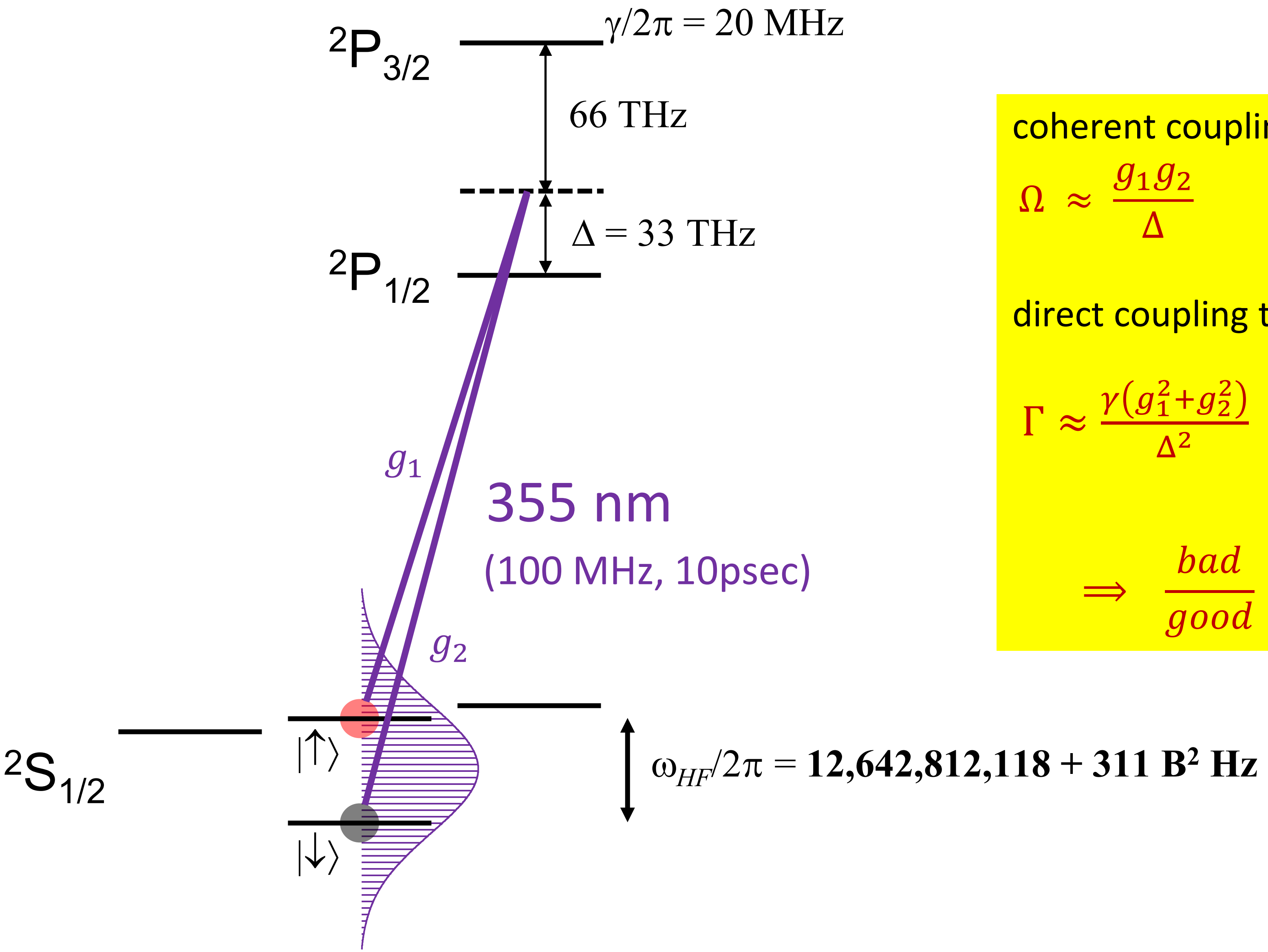
$|\downarrow\rangle = |0,0\rangle$

12.642 812 118 GHz



T_2 > 1 hour (P. Wang, K. Kim et al. arXiv:2008.00251)

^{171}Yb$^+$ Qubit Manipulation



coherent coupling rate (good):

$$\Omega \approx \frac{g_1 g_2}{\Delta}$$

direct coupling to P states (bad):

$$\Gamma \approx \frac{\gamma(g_1^2+g_2^2)}{\Delta^2} \sim \Omega\frac{\gamma}{\Delta} \quad (\text{for } g_1 \sim g_2)$$

$$\Rightarrow \quad \frac{bad}{good} \approx 10\frac{\gamma}{\Delta} \sim 10^{-5}$$

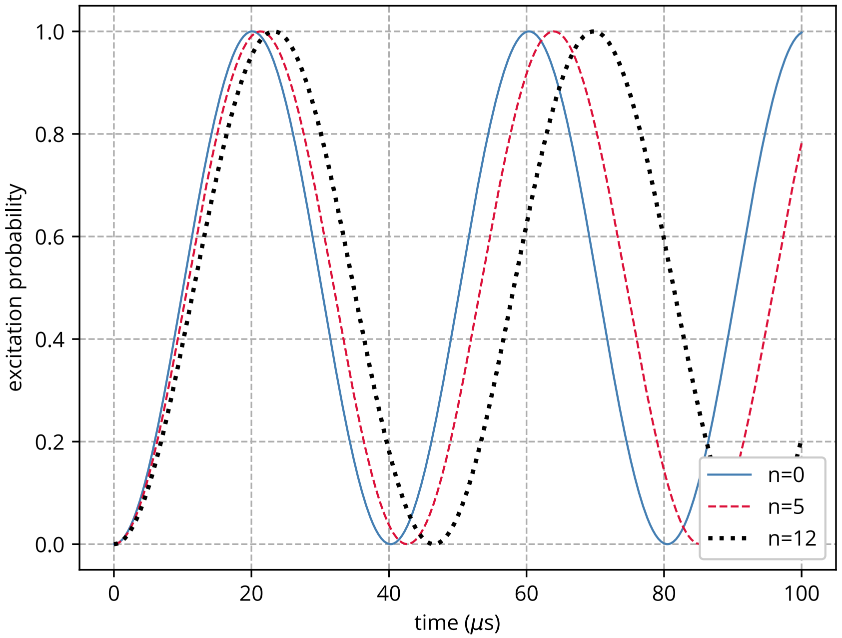
D. Hayes et al., PRL 104, 140501 (2010)

Single ion laser-ion interaction

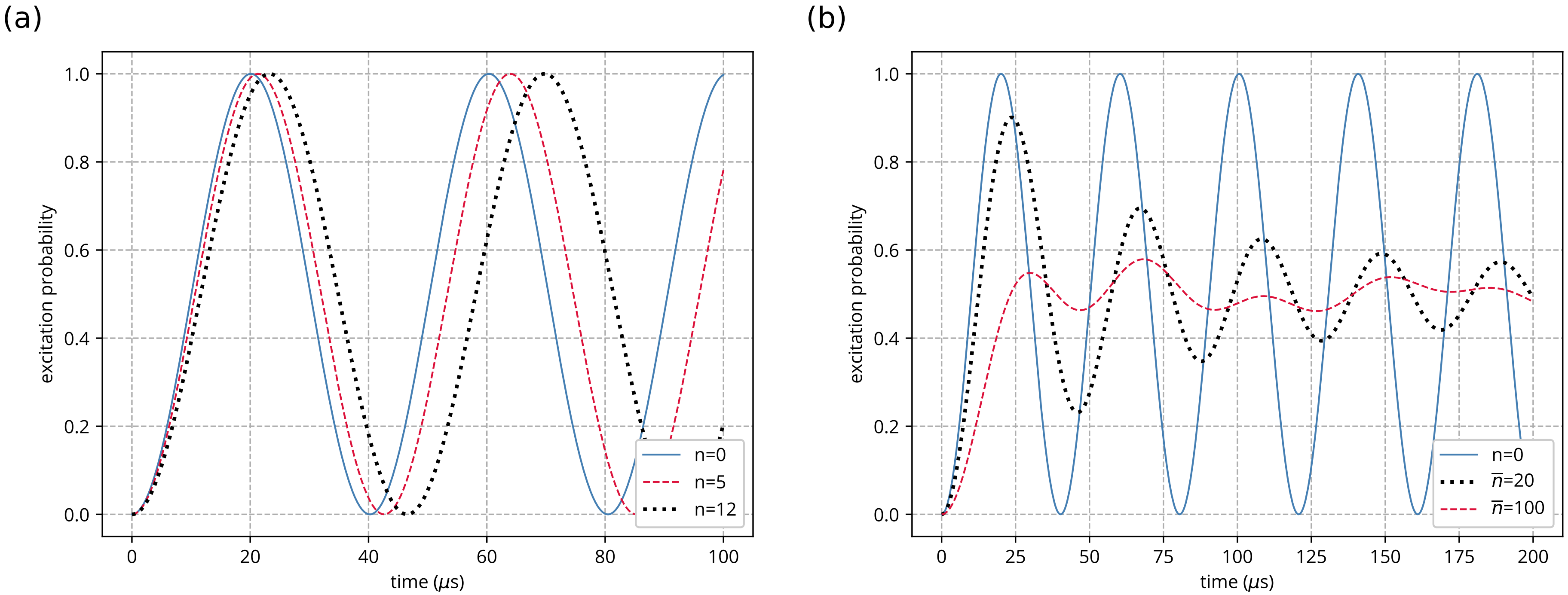
(break and go to notes)

Carrier Rabi Flopping

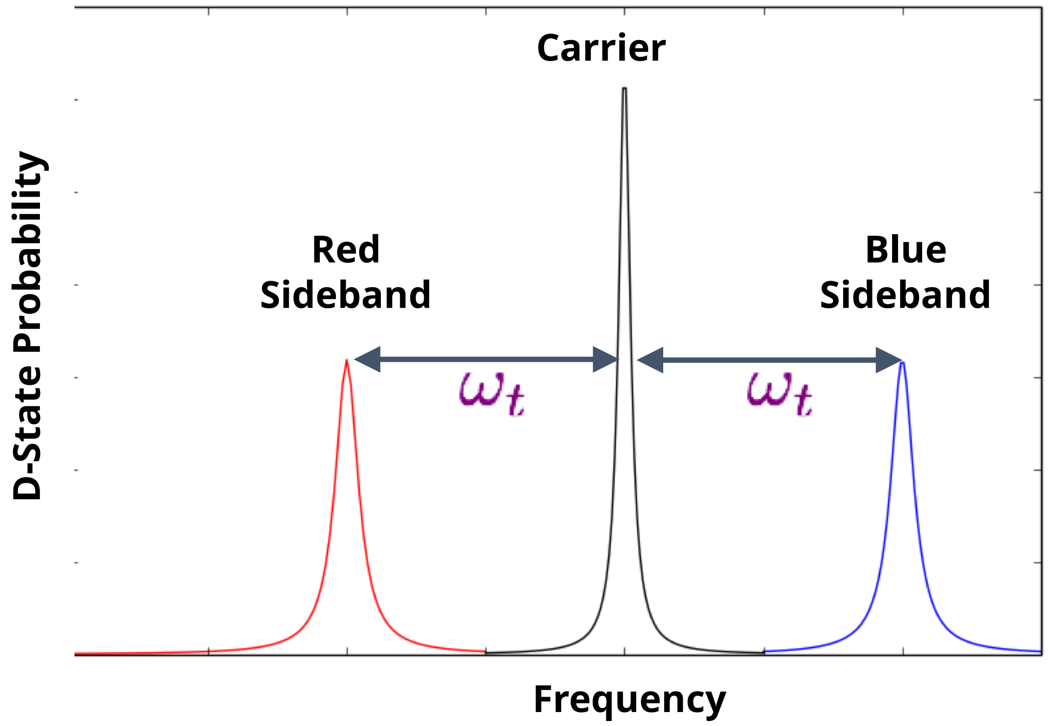
(a)

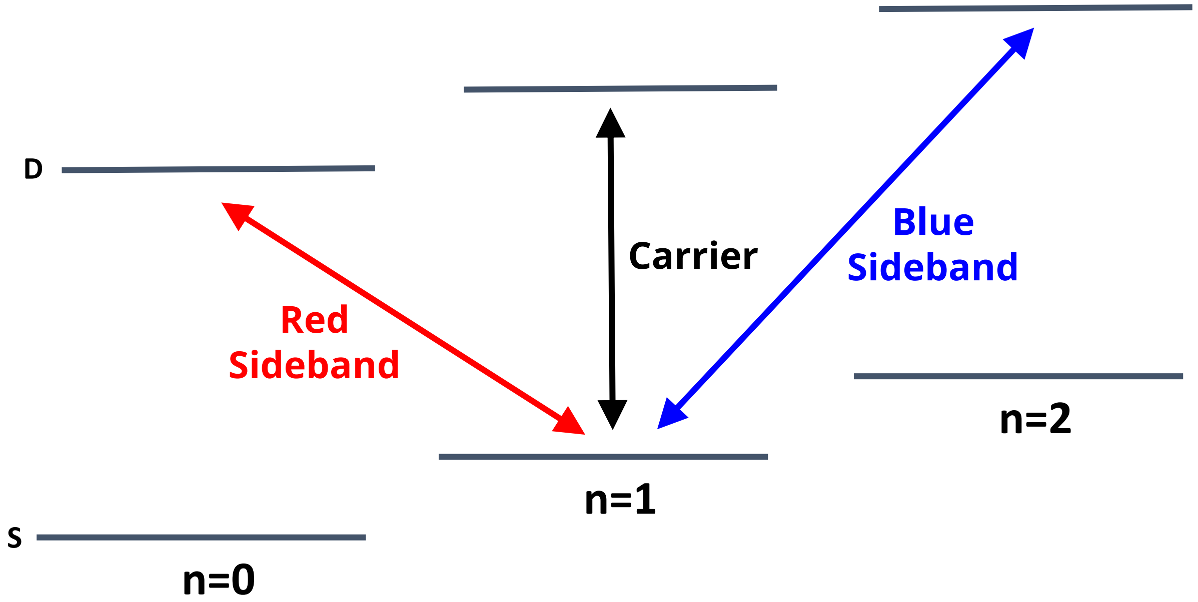


Carrier Rabi Flopping

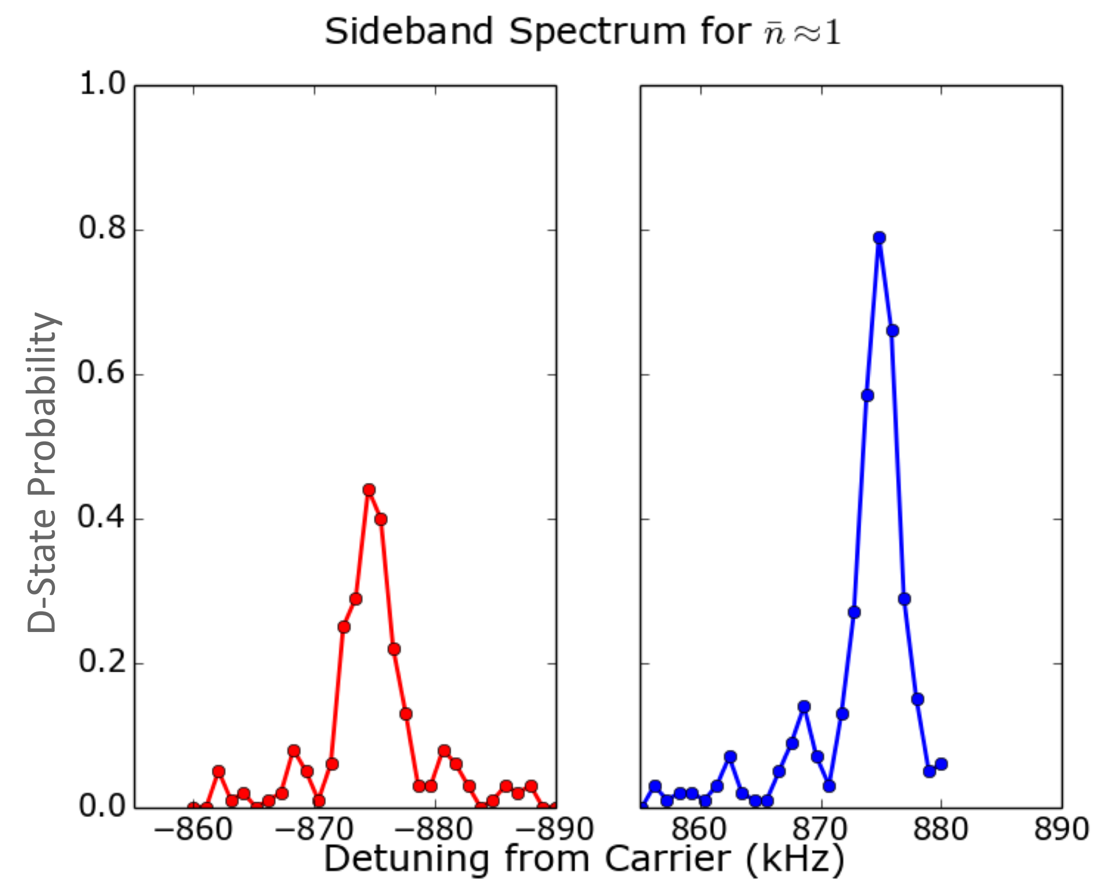


Single ion sideband spectrum





Measuring motional state



$$\frac{P_e^{rsb}}{P_e^{bsb}} = \frac{\bar{n}}{\bar{n}+1}$$

Apply red and blue sidebands simultaneously

$\Delta n = -1$

$$\Omega_{n-1,n} = \eta\sqrt{n}\Omega$$

$$H_I = \tfrac{1}{2} i\hbar\Omega_{n-1,n}(\hat{a}\sigma^+ - \hat{a}^\dagger\sigma^-)$$

$\Delta n = +1$

$$\Omega_{n+1,n} = \eta\sqrt{n+1}\Omega$$

$$H_I = \tfrac{1}{2} i\hbar\Omega_{n+1,n}(\hat{a}^\dagger\sigma^+ - \hat{a}\sigma^-)$$

$$\begin{aligned} H = {} & \eta\Omega\left(\sigma_+ a + \sigma_- a^\dagger\right) \\ & + \eta\Omega\left(\sigma_- a + \sigma_+ a^\dagger\right) \\ = {} & \eta\Omega\,\sigma_x\left(a + a^\dagger\right) \\ = {} & \Omega\,\sigma_x(k \cdot \hat{x}) \end{aligned}$$

Spin-dependent force!

Δk

For Raman qubits

End Lecture 1