Boulder Theoretical Biophysics 2019

Neuroscience Mini-course: Exercise Set 3

1. ***Data processing inequality:*** This inequality states that data processing cannot create information. Prove this theorem. Consider the ensemble STR, where s is the source of input from the external environment, t is the transmitted data gathered at the sensory periphery, and r is the processed data sent to the cortex. Assume these three variables form a Markov chain: $s \to t \to r$, such that

$$P(s,t,r) = P(s)P(t|s)P(r|t).$$

Show that the average information R conveys about S, $I(S;R)$, is less than or equal to the average information T conveys about S, $I(T;S)$.

2. Consider the so-called Z channel, with inputs x and outputs y, and with asymmetric bit-flip probability, $f = 0.2$. Compute the mutual information between inputs, X, and outputs, Y, in this channel. Plot $I(X;Y)$ as a function of the probability of the input, $P_X = \{p_0, p_1\}$. For what value of p_1 is this information maximized? What is the channel capacity, C?

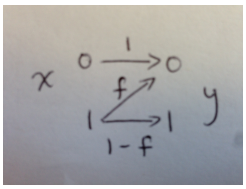

Figure 1. The Z-channel.

Capacity of a Gaussian channel

3. Consider a Gaussian channel with input x and signal-to-noise ratio σ_S^2/σ_N^2. What is the capacity of the channel, C?

4. If the input is constrained to be binary, $x \in \{\pm\sigma_S\}$, what is the capacity, C', of the channel?

5. If, additionally, the output of the channel is sent through a threshold function

$$y \to y' = \begin{cases} 1 & y > 0 \\ 0 & y \leq 0 \end{cases}$$

what is the capacity, C'', of the resulting channel?

6. Plot the three capacities as a function of the signal-to-noise ratio, with values ranging from 0.0 to 2.5, on the same plot.