Boulder Theoretical Biophysics 2019

Neuroscience Mini-course: Exercise Set 2

Maximum entropy distributions:

1. Imagine a neural response that can take any real value, but is constrained to have a mean value, μ, and variance, σ^2. Derive the distribution over responses, $p(r)$, that maximizes the entropy of the distribution,

$$S(R) = -\int_{-\infty}^{+\infty} p(r)\log(p(r))dr,$$

subject to the constraints mentioned above, namely that $p(r)$ is normalized,

$$\int_{-\infty}^{+\infty} p(r)dr = 1$$

has mean value μ,

$$\int_{-\infty}^{+\infty} rp(r)dr = \mu$$

and variance σ^2,

$$\int_{-\infty}^{+\infty} (r-\mu)^2 p(r)dr = \sigma^2.$$

Use the method of Lagrange multipliers to solve this constrained maximization problem.

2. Compute the entropy of a Gaussian distribution with mean μ and variance σ^2: $p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-(x-\mu)^2}{2\sigma^2}$. Show your work.

Information in spike trains:

3. In class, we derived the following equation describing the information in a neuron's response $r(t)$, to a long repeated stimulus (long enough to sample the stimulus fully):

$$I(spikes; stimulus) = \frac{1}{T}\int_0^T dt \frac{r(t)}{\bar{r}} \log\left(\frac{r(t)}{\bar{r}}\right),$$

where information is expressed in units of bits per spike. Show that a neuron with a flat response, $r(t) = constant$, has no information about the stimulus. Sketch an $r(t)$ with a large amount of information about the stimulus and describe why it does so (in words).