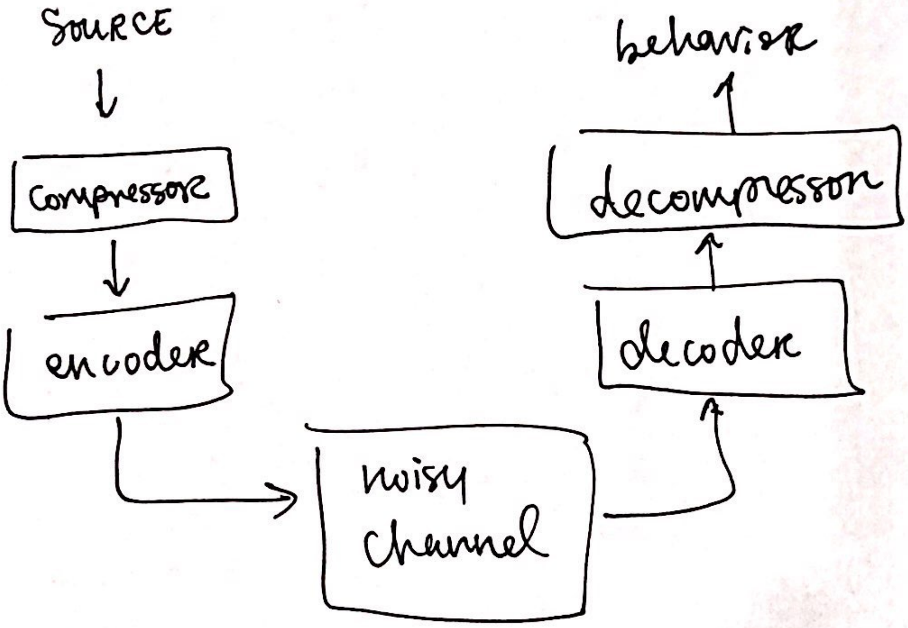
BOULDER LECTURE 4 SEP



SOURCE CODING

CHANNEL CODING

The Channel Coding Th^{m}:

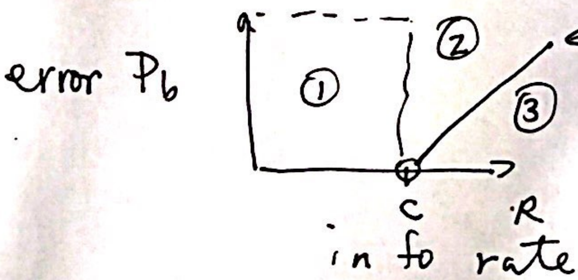
FOR ANY DISCRETE, MEMORYLESS CHANNEL, $\exists$ A $C \geq 0$ called the channel capacity s.t.:

info rate

$\forall$ $\varepsilon > 0$ & $R < C$, for large enough N,

$\exists$ a code of length N and trans rate $\geq R$ and a decoding algorithm s.t. the max. prob. of bit error is $< \varepsilon$.

$X \Rightarrow Y$



$$R(p_b) = \frac{C}{1 - H_2(P_b)}$$

$$C = \max_{p(x)} I(X;Y)$$

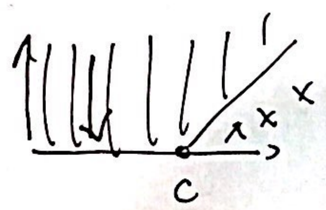
CONSIDER SENDING INFO DOWN A NOISY CHANNEL in the brain

$$r = \Phi(m) + \underbrace{z}_{\text{noise}}$$

objective is to optimize $\Phi(m)$

S.T. $I(R;M)$ is maximal

If instead you get to mess with the messages, or the inputs you're doing channel coding.



← rate-distortion theory

$$C = \max_{p(x)} I(X;Y)$$

$$I(R;M) = S(R) - S(R|S)$$

$$= \underset{\text{max entropy}}{S(R)} - \underset{\text{fixed}}{S(z)}$$

if all is gaussian

$$I(R;M) = \frac{1}{2}\log\left[2\pi e\left[\sigma_r^2 + \sigma_z^2\right]\right] - \frac{1}{2}\log\left[2\pi e\,\sigma_z^2\right]$$

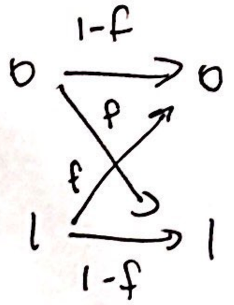
$$= \frac{1}{2}\log\left[1 + \frac{\sigma_r^2}{\sigma_z^2}\right] = \frac{1}{2}\log\left[1 + SNR\right]$$

Scanned with CamScanner

BSC: binary symmetric channel

$$X \quad \begin{matrix} 0 \longrightarrow 0 \\ \\ 1 \longrightarrow 1 \end{matrix} \quad Y$$

bit flip + noise :



$\boxed{\text{take } f = 0.15}$

optimal input distribution is

$$P(x=0) = P(x=1) = 1/2$$

$$I(x;y) = S(Y) - S(Y|X)$$

$$p(y=0) = (p(y=0|X=1)\cdot p(x=1)) + (p(y=0|x=0)\cdot p(x=0))$$

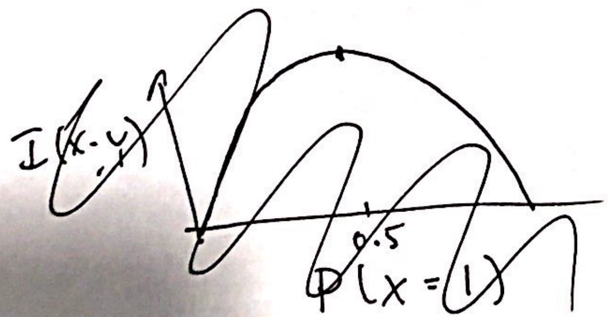
$$= \frac{1-f}{2} + \frac{f}{2} = \frac{1}{2}$$

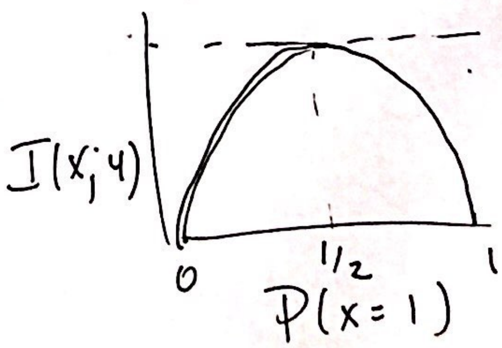
& similarly for $y=1$

$$I(x;y) = S(\tfrac{1}{2}, \tfrac{1}{2}) - S(f, 1-f)$$

$$= 1 - 0.61 = 0.39 \text{ bits}$$

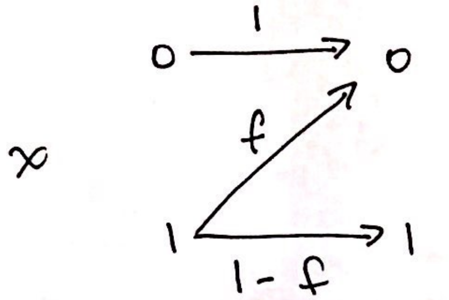
binary entropy
$S(p_0, p_1) = -p_0 \log_2 p_0 - p_1 \log_2 p_1$





0.39 bits for $f = 0.15$

Z channel



$$I(x;Y) = S(Y) - S(Y|x)$$

$$P(x) = (P_0, P_1)$$ not clear what the optimum is.

$$S(y) = \cancel{S(P_1(1-f), 1-P_1(1-f))}$$

$$= S(\underbrace{1-P_1(1-f)}_{y=0}, \underbrace{P_1(1-f)}_{y=1})$$

$$S(Y|x) = S(Y|x=0)P_0 + S(Y|x=1)P_1$$

$$= P_0 \cdot \underbrace{S(1,0)}_{0} + P_1 S(f, 1-f)$$

$$= P_1 S(f, 1-f)$$