BOULDER LECTURE 3 SEP

$\boxed{\text{CB I } 1}$ + SEE $\LaTeX$ notes on efficient coding

information in a Gaussian channel

$$r = \underset{(m)}{f(\text{stim})} = \underset{(m)}{\phi(\text{stim})} + \underset{\text{noise}}{z}$$

$$I(R;M) = S(R) - \langle S(R|m) \rangle_m$$

$$= S(R) - \langle S(\phi(m) + z|m) \rangle_m$$

→ entropy is translation invariant

$$= S(R) - \langle S(z|m) \rangle_m$$

$$I(R;M) = S(R) - S(z)$$

if this is fixed, how can we max $I(R;M)$?

ans → max $S(R)$!

RECALL FROM LECTURE 1

$\boxed{\text{CBI } 6}$ Ilya said there's no HW, so I'll call this an exercise

12 coins, one counterfeit, simple balance

which coin? H or L?

minimal weighings?

WWSD?

recall: weighing problem

4 vs. 4 & 4 on the side has max entropy

$$-\sum_{\text{states}} p(\text{state}) \log(p(\text{state})) = -\sum p \log p$$

$1/3$ Δ

$1/3$ Δ

$1/3$ Δ

$$= -\sum \tfrac{1}{3} \log \tfrac{1}{3}$$

$$= 3 \cdot \tfrac{1}{3} \log(3)$$

* TALK ABOUT UNITS nat bits dats/dits
/nits xbats

~~NB INFO IN SPIKETRAINS (Brenner et al. PAPER)~~
~~see Lecture 1 notes & LaTeX notes~~

CBI 2

~~max ent in a gaussian channel~~

max ent dist'n w/o constraints

$$S(\text{response}) = -\sum_i p(r_i) \log p(r_i)$$

constraint $\sum_i p(r_i) = 1$

$$\mathcal{L} = -\sum p(r_i) \log p(r_i) - \lambda\left(1 - \sum_i p(r_i)\right)$$

$$\frac{\delta \mathcal{L}}{\delta p(r_i)} = -\frac{p(r_i)}{p(r_i)} - \log p(r_i) + \lambda = 0$$

$$p(r_i) = e^{-1+\lambda}$$

$$\sum_{r_i} e^{-1+\lambda} = 1 \;; \quad N e^{-1+\lambda} = 1 \Rightarrow e^{-1+\lambda} = 1/N$$

$$\Rightarrow p(r_i) = \frac{1}{N_{\text{bins}}}$$

if you constrain the mean firing rate, μ, &

let your response be positive (& continuous) $r \geq 0$

$$\rightarrow \text{max ent} \Rightarrow p(r) = \frac{1}{\mu} e^{-r/\mu}$$

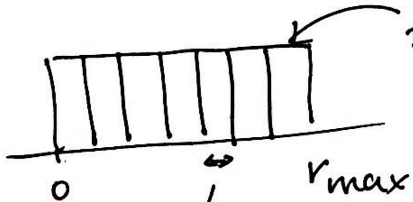
see the exercises for $r \in [-\infty, \infty]$ & constrained μ & σ^2

$\boxed{\text{CBI 3}}$

$$r = \Phi(m) + z$$

to max $I(M;R)$, we want to max $S(R)$; for r discrete between 0 & r_{max},

$$P(r) = \frac{\Delta r}{r_{max}}$$



$$P(r_i) = \frac{\Delta r}{r_{max}} = \frac{1}{N_{bins}}$$

$$N = \frac{r_{max}}{\Delta r}$$

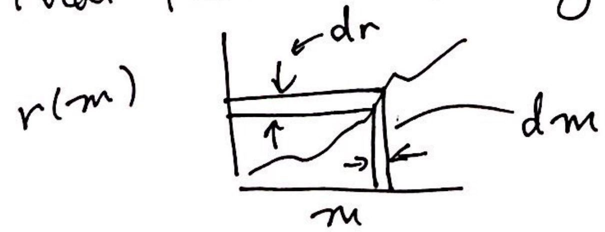
$p(r)$ prob density $= P(r)/\Delta r = \frac{1}{r_{max}}$

$$\text{s.t.} \int_0^{r_{max}} p(r)\, dr = 1$$

Now, assume a continuity condition on $r(m)$:

$$\underbrace{p(r)\, dr}_{\text{small change in } r} = \underbrace{p(m)\, dm}_{\text{small change in } m}$$

☆ need to assume noise is small ☆ so that this continuity holds!



$$p(r) = P(r)/\Delta r = \left(\frac{1}{r_{max}}\right)$$ for max entropy of R

$$p(r)\,\Delta r = \rho(m)\,\Delta m$$

$$p(r)\left(\Phi(m+\Delta m) - \Phi(m)\right) = p(m)\,\Delta m$$

now use max ent $p(r)$

$$\frac{1}{r_{max}}\left(\Phi(m+\Delta m) - \Phi(m)\right) = \rho(m)\,\Delta m$$

$$\frac{\Phi(m+\Delta m) - \Phi(m)}{\Delta m} = p(m)\, r_{max}$$

Φ monotonic & Δm small

$$\rightarrow \int \frac{d\Phi(m)}{dm} = \int r_{max}\, p(m)$$

$$\Phi(m) = r_{max} \underbrace{\int_{m_{min}}^{m} p(m')\,dm'}_{\text{CDF of the message dist'n}}$$

$$p(r) = \frac{1}{r_{max}} \quad \text{for } \max I(R;M) \ !$$

$$p(r)\,dr = p(m)\,dm$$

$$\frac{1}{r_{max}} \underbrace{\left(\Phi(m+dm) - \Phi(m) \right)}_{\text{assuming } \Phi(m+dm) > \Phi(m)} = p(m)\,dm$$

$\rightarrow \Phi$ is monotonic in m

$$\frac{\Phi(m+dm) - \Phi(m)}{dm} = p(m) \cdot r_{max}$$

$$\xrightarrow{dm \text{ small}} \approx \frac{d\Phi}{dm} = r_{max}\, p(m)$$

integrate both sides

$$\Phi(m) = r_{max} \underbrace{\int_{m_{min}}^{m} p(m')\,dm'}$$

= CDF of the stimulus dist'n !

→ see Laughlin 1981