Boulder Theoretical Biophysics summer school: Introduction to neuroscience and information theory

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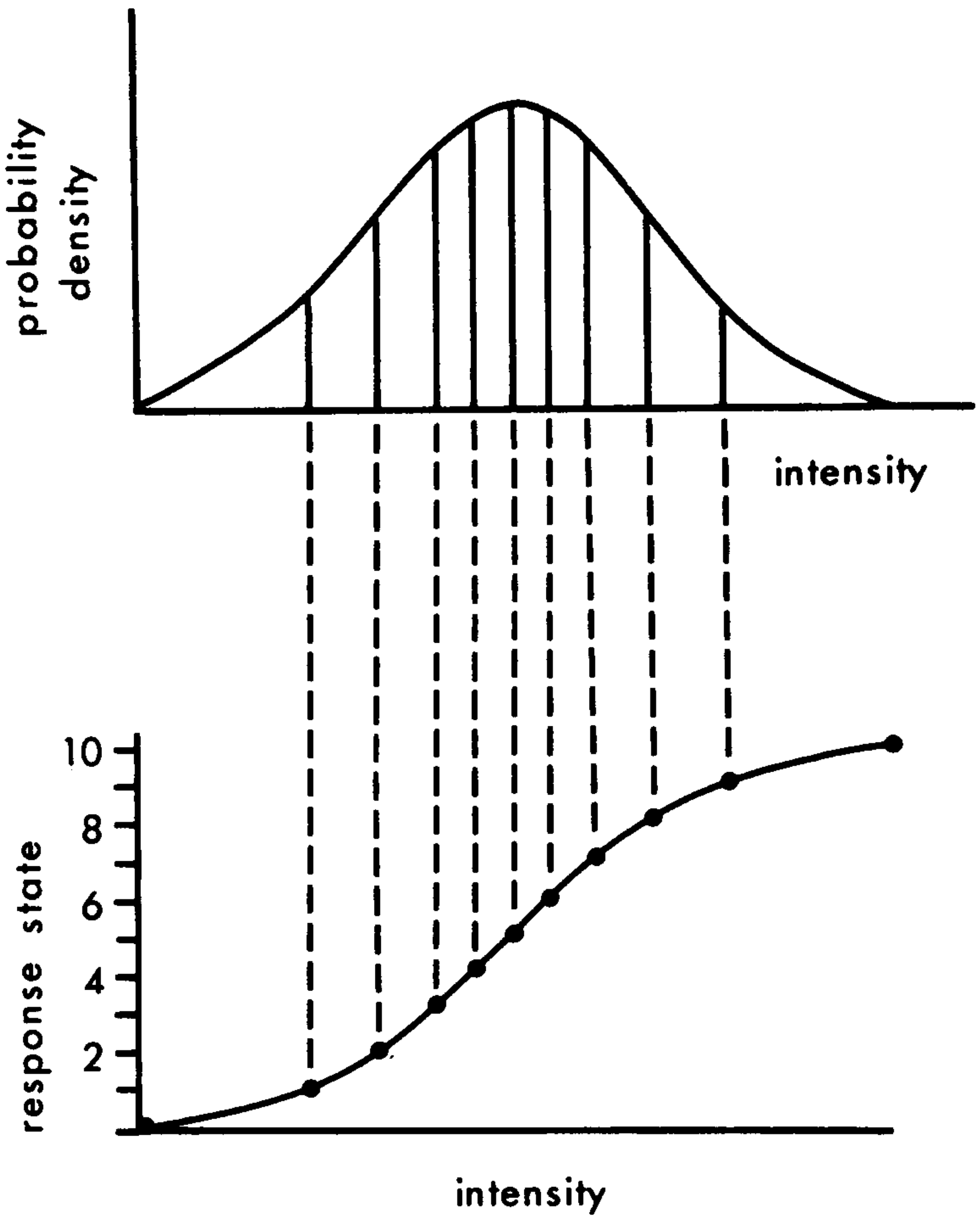
Lecture 3: *Maximum entropy and efficient coding in single neurons*



The efficient coding hypothesis, brief history:

- Claude Shannon (1948) *A Mathematical Theory of Communication*
- Fred Attneave (1954) *Some informational aspects of visual perception*
- Horace Barlow (1961) *Possible principles underlying the transformation of sensory messages*

Are sensory systems optimized for information transmission?

Recall: info theory basics

- Stimulus s drawn from $P(s)$;
- $\Rightarrow$ neural response y, $P(y|s)$
- Mutual information:

$$
\begin{aligned}
I &= \int ds dy P(s,y) \log_2 \left(\frac{P(s,y)}{P(s)P(y)} \right) \\
&= \int ds P(s) \int dy P(y|s) \log_2 \left(\frac{P(y|s)}{P(y)} \right) \\
&= -\int dy P(y) \log_2 P(y) + \int ds P(s) \int dy P(y|s) \log_2 P(y|s)
\end{aligned}
$$

- 1 bit of information reduces the uncertainty about the stimulus by a factor 2.
- n bits of information reduce the uncertainty about the stimulus by a factor 2^n
- Linear Gaussian channel $y = s + z$ where s is Gaussian with variance S^2, z is a Gaussian noise with variance N^2:

$$I = \frac{1}{2} \log_2 \left(1 + \frac{S^2}{N^2} \right)$$

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chalkboard interlude

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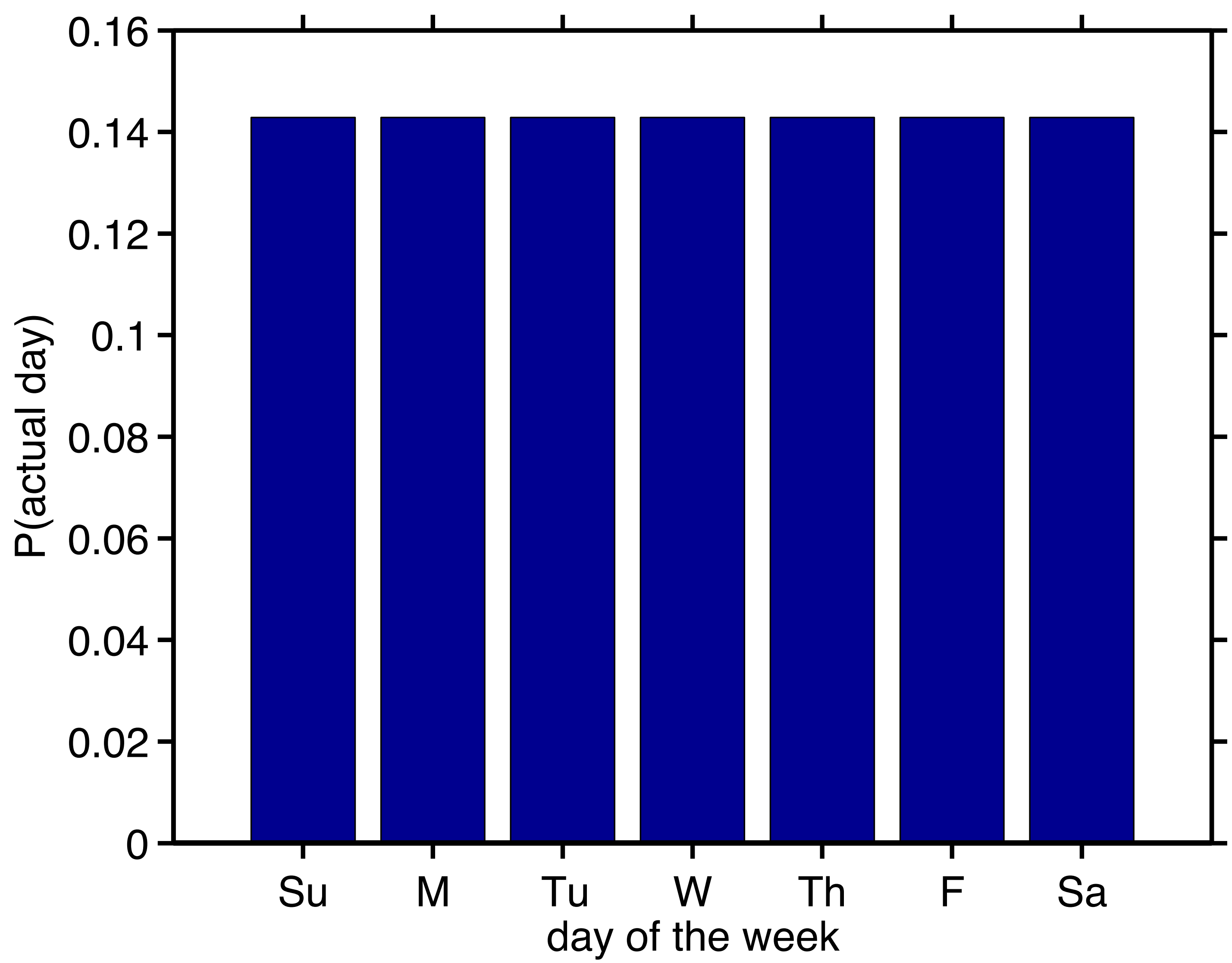
Histogram equalization:

For discrete variables, the ***uniform*** *distribution has the maximum entropy.*



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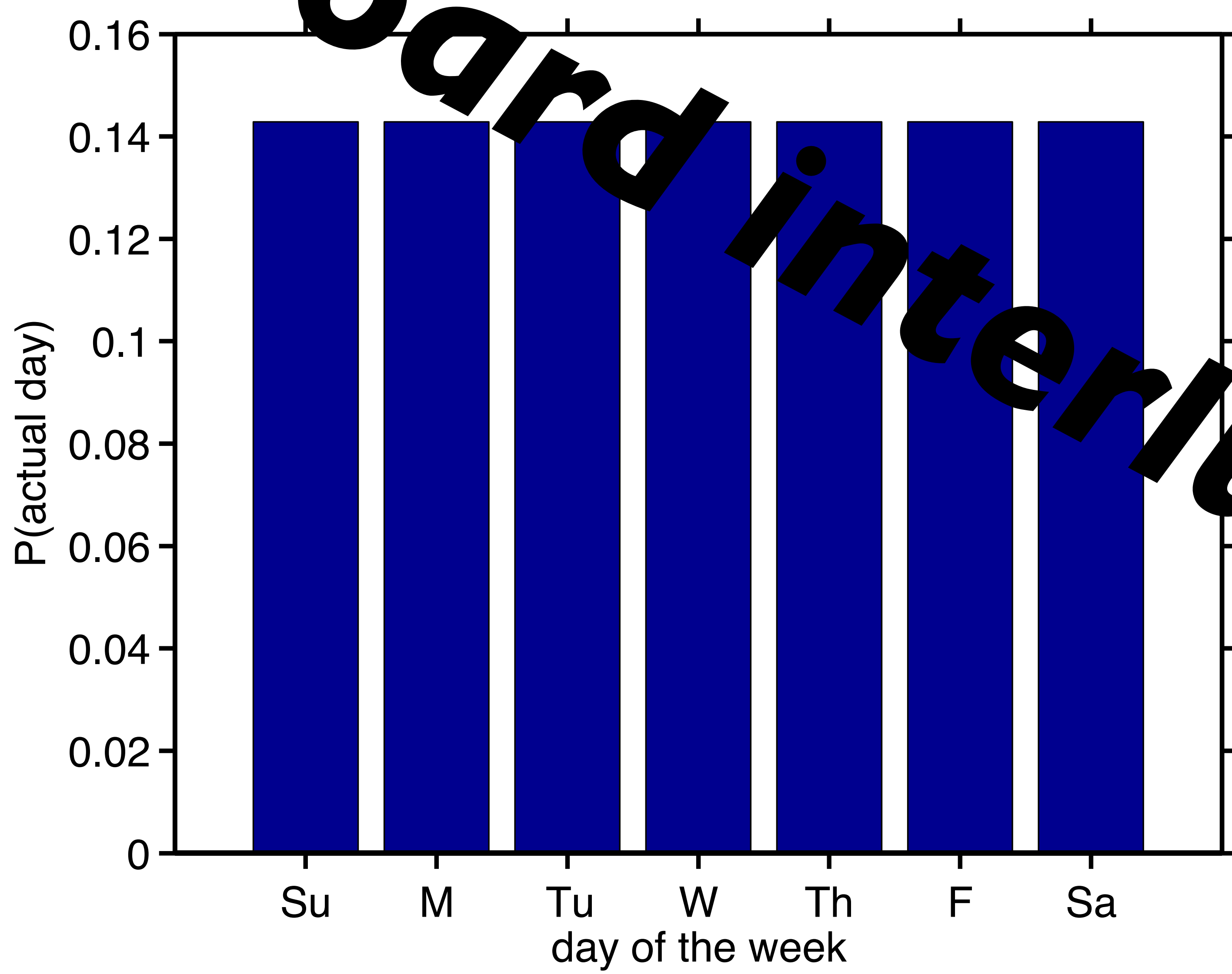

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Histogram equalization:

$$p[r] = \frac{1}{r_{\max}}$$

$$|f(s+\Delta s) - f(s)|/r_{\max} = p[s]\Delta s.$$

$$\frac{df}{ds} = r_{\max} p[s]$$

$$f(s) = r_{\max} \int_{s_{\min}}^{s} ds' \, p[s']$$

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chalkboard interlude

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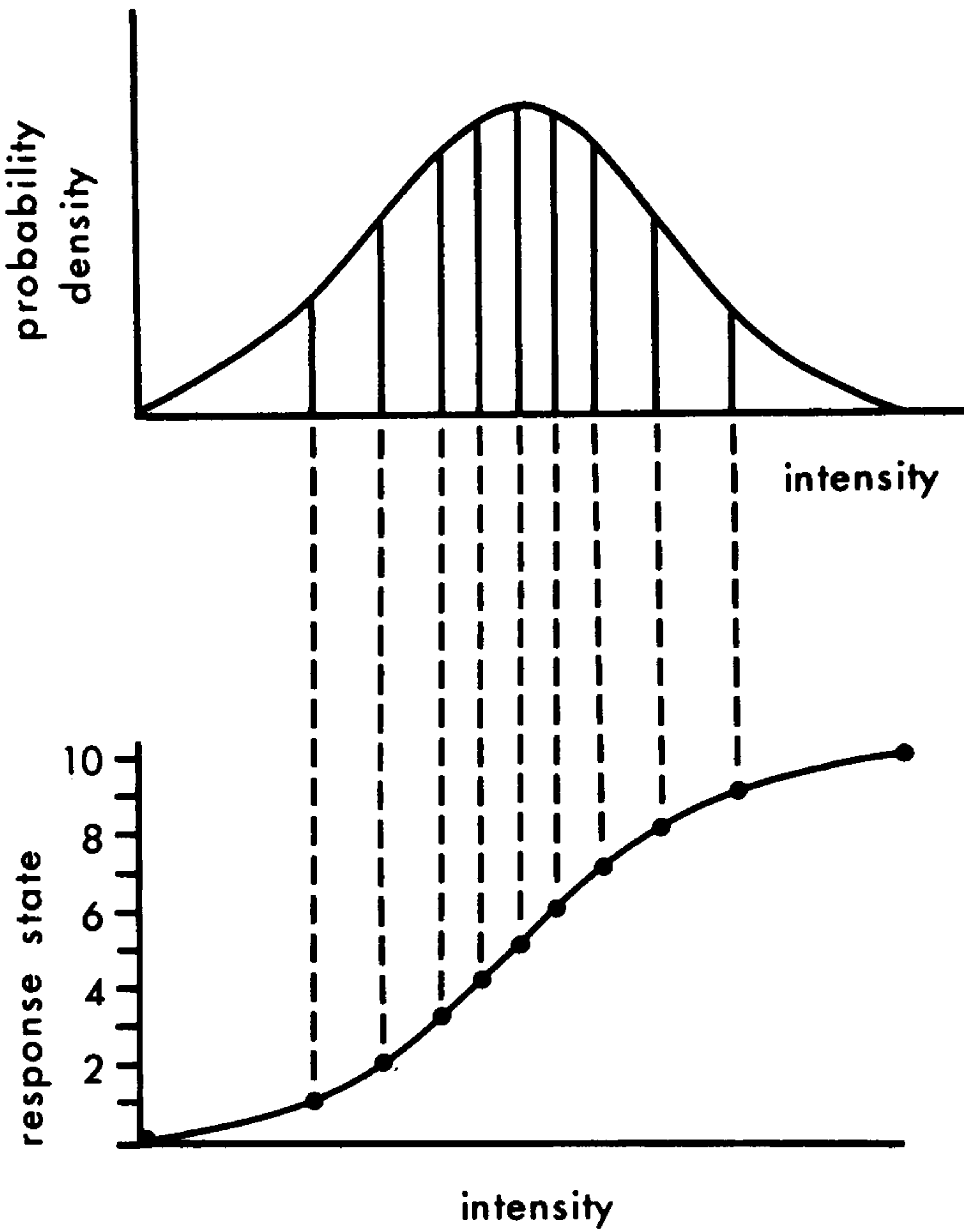
Evidence for entropy maximization in the fly:



Laughlin, 1981

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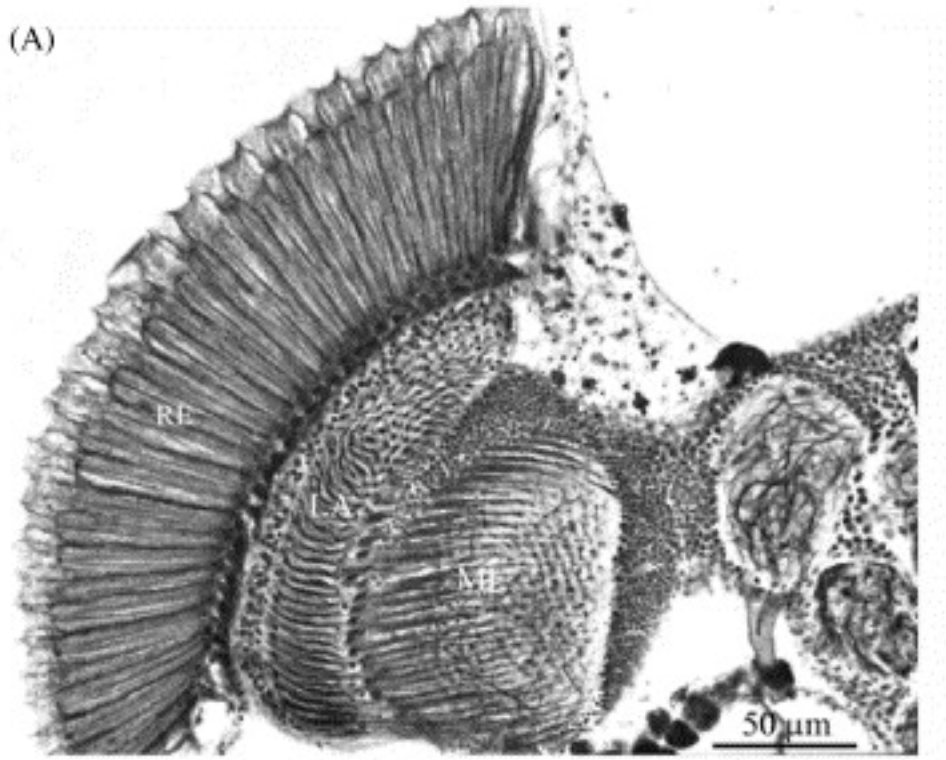
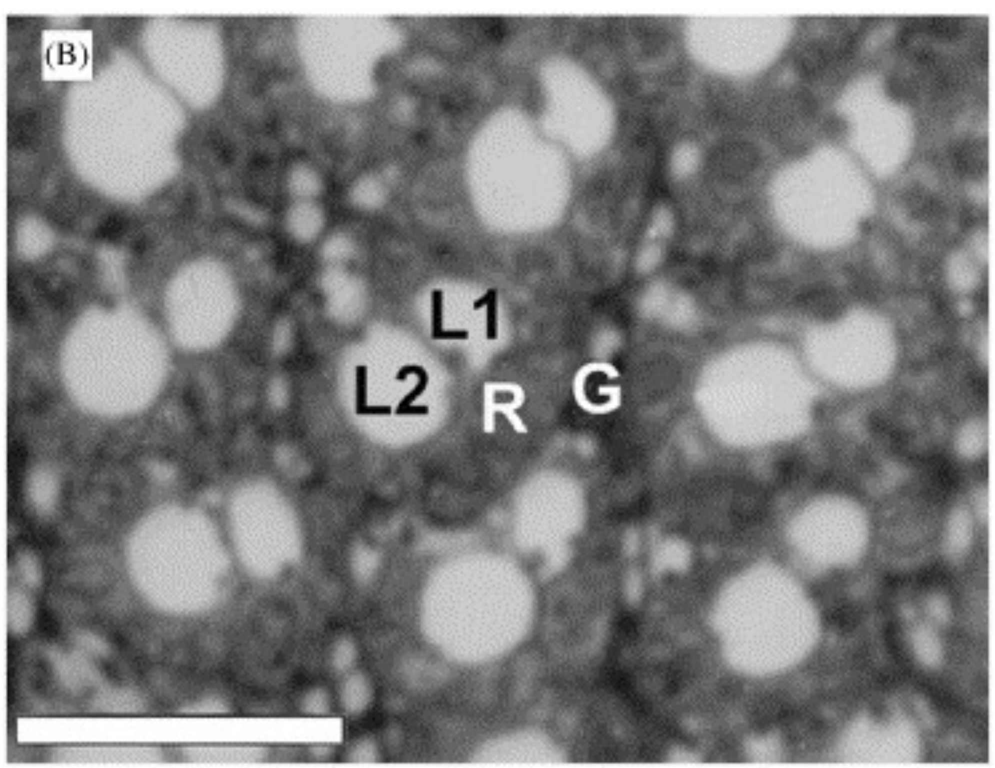
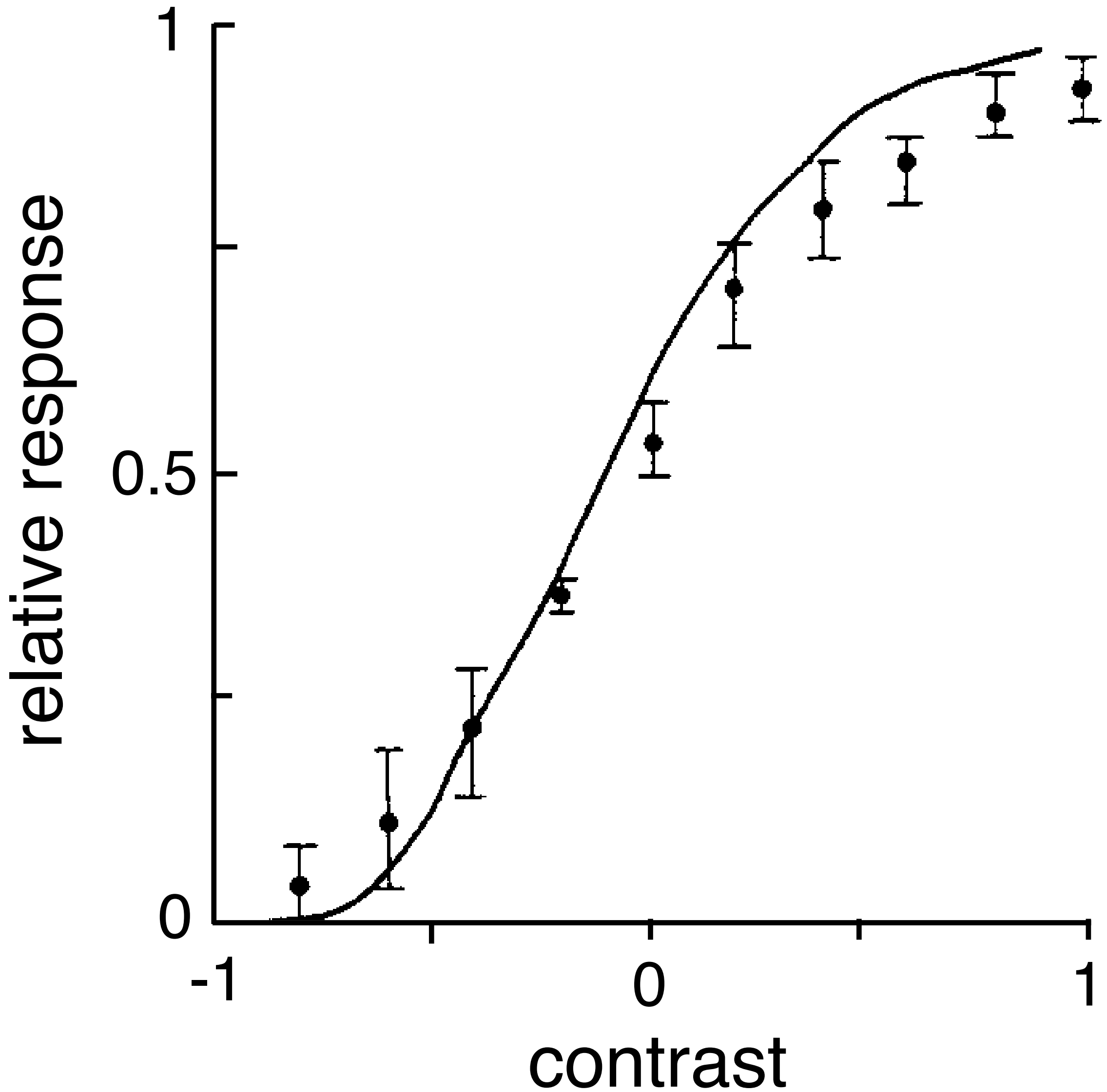


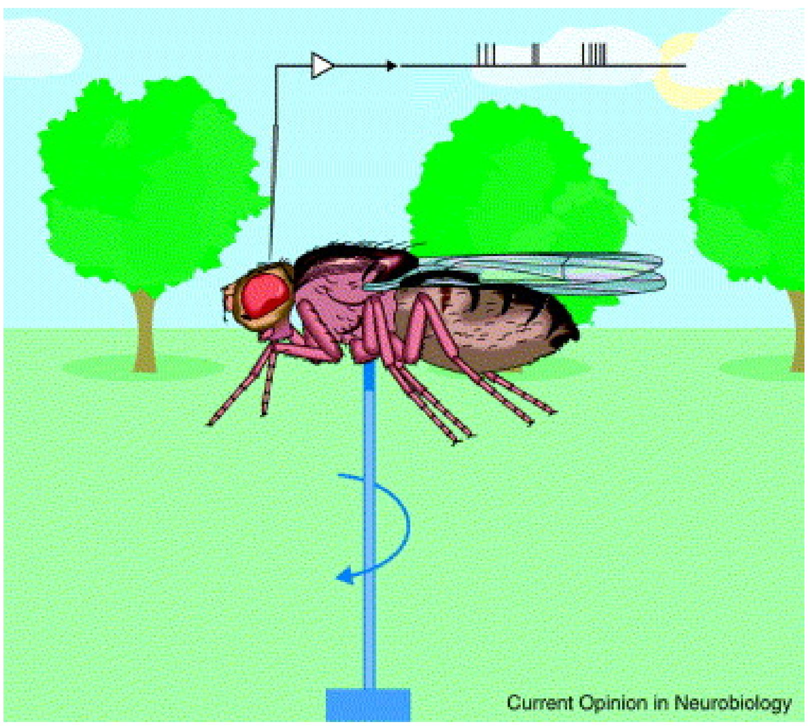


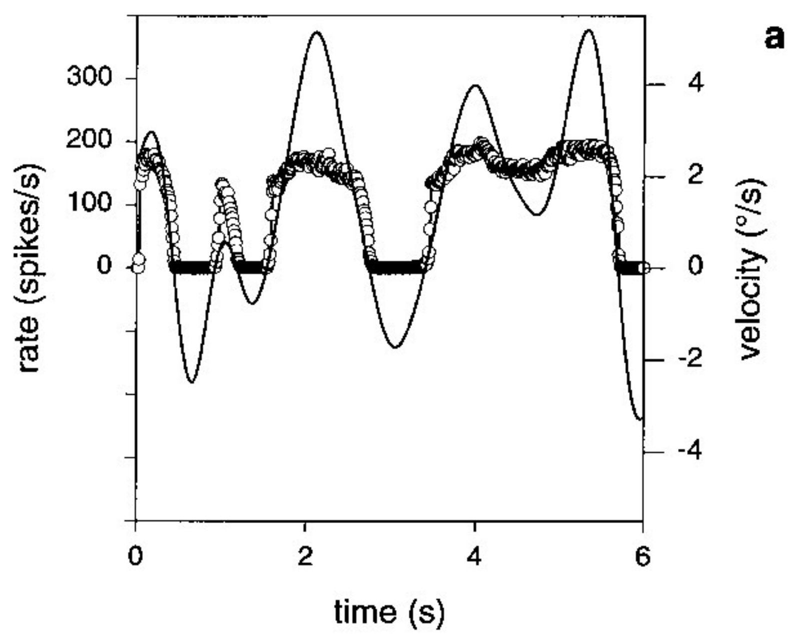


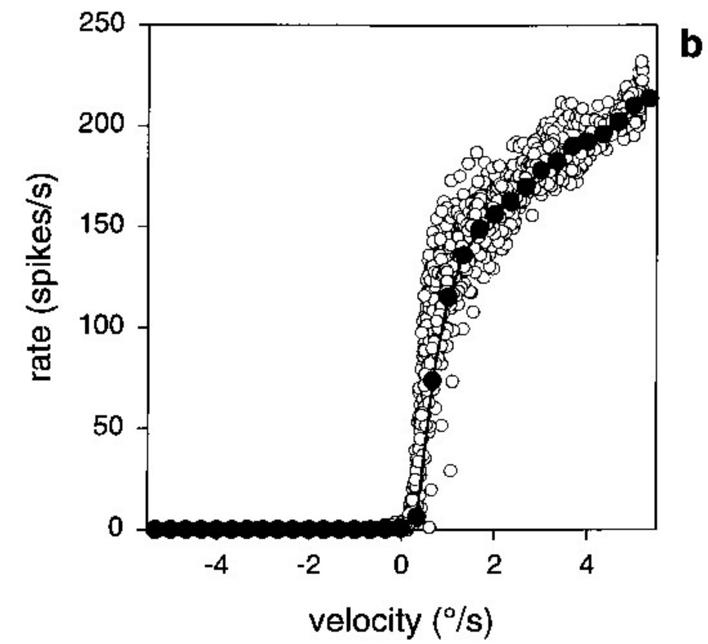
Laughlin, 1981

Adaptive rescaling in the fly H1 neuron:

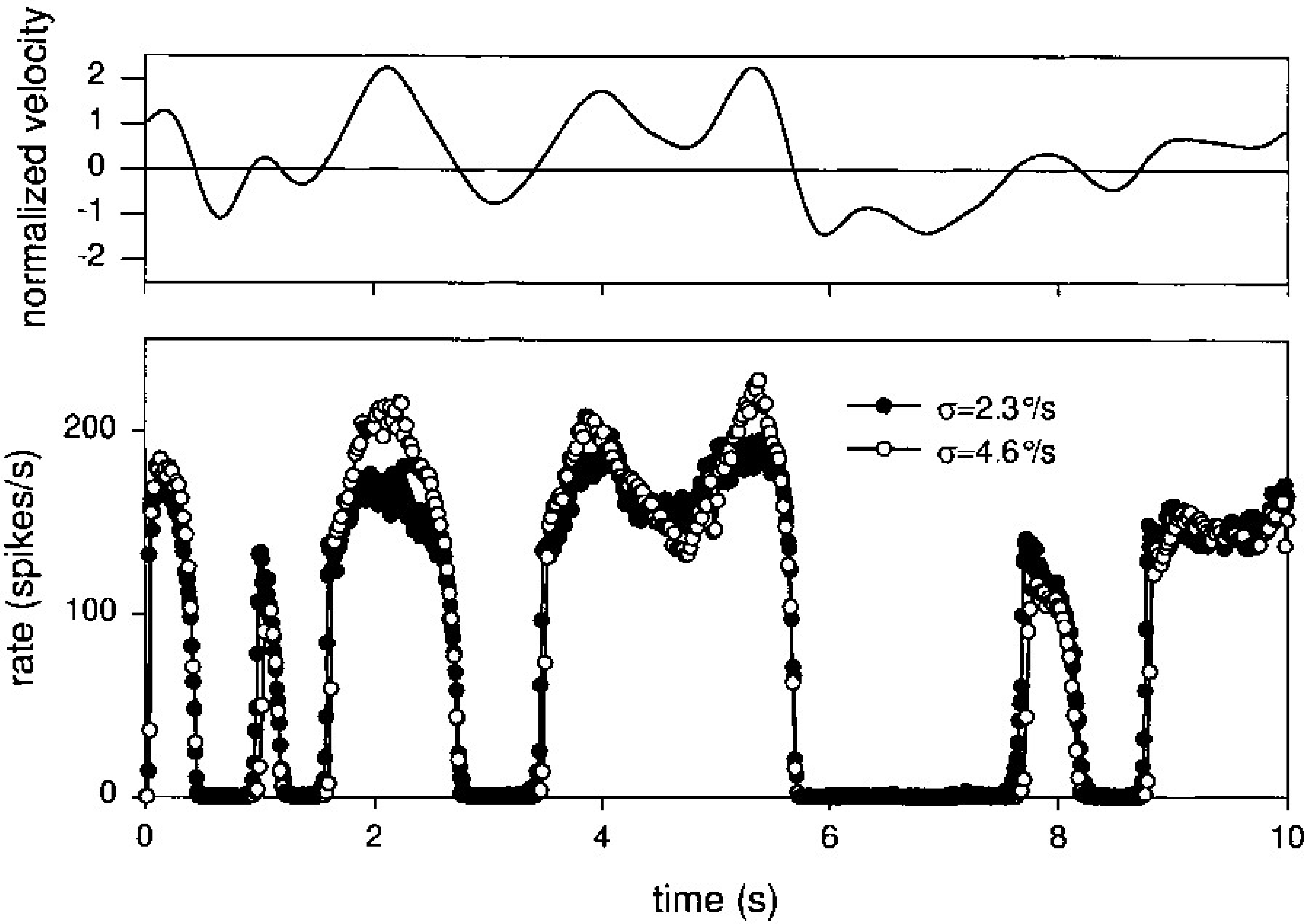
Brenner et al 2000 (H1 neuron, visual system of the blowfly)



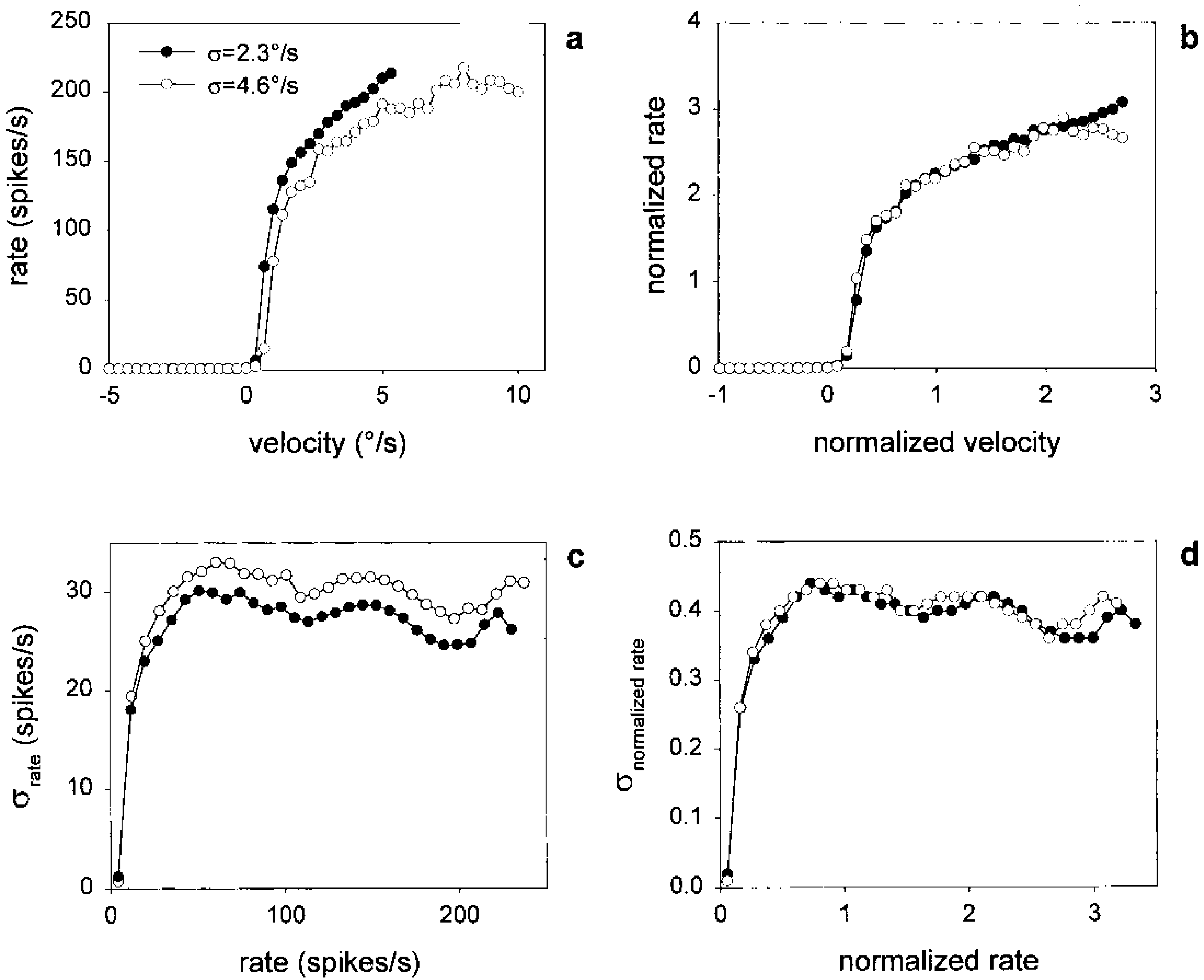




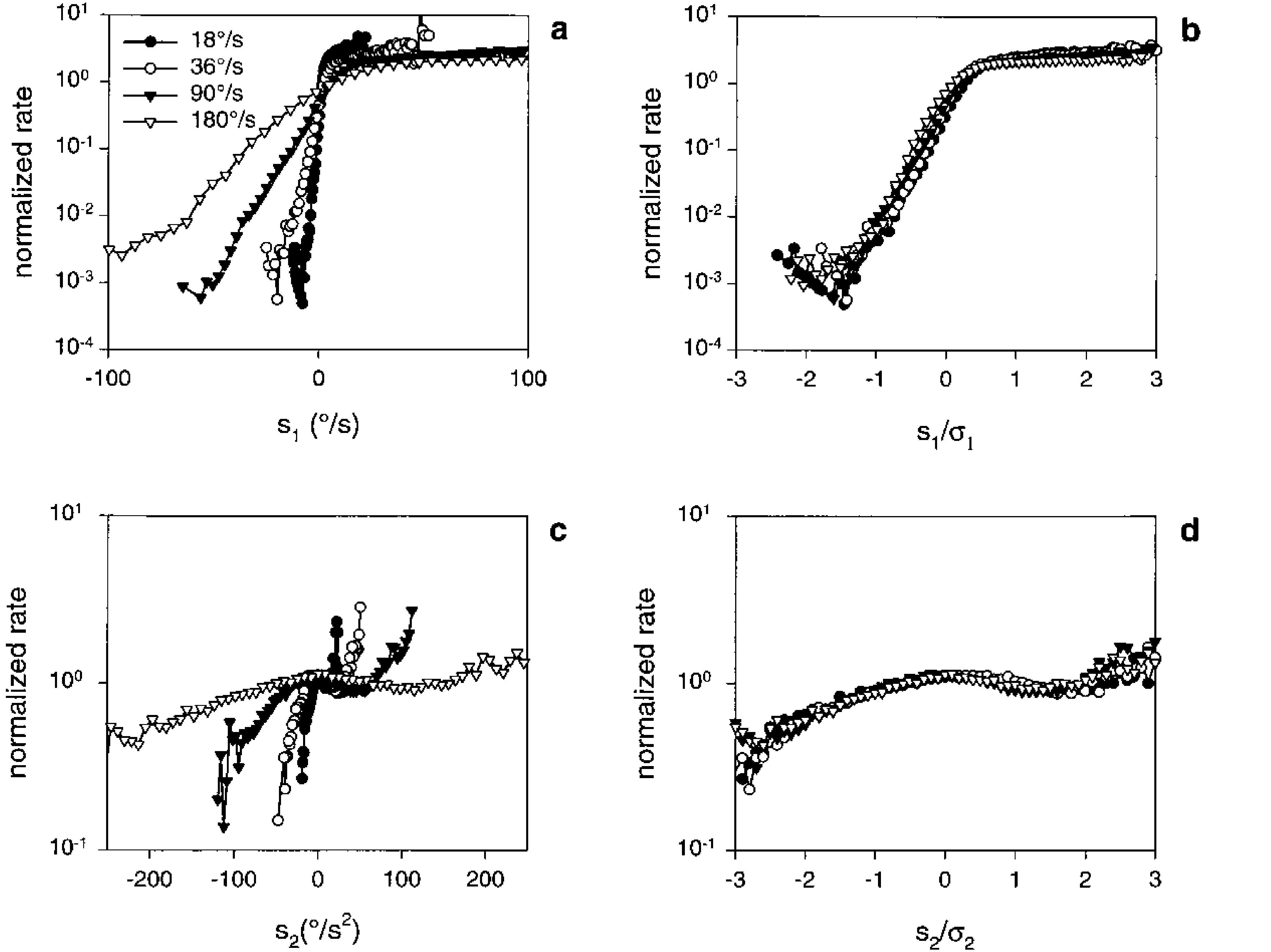
Responses to stimuli with 2x standard deviation change are nearly identical:



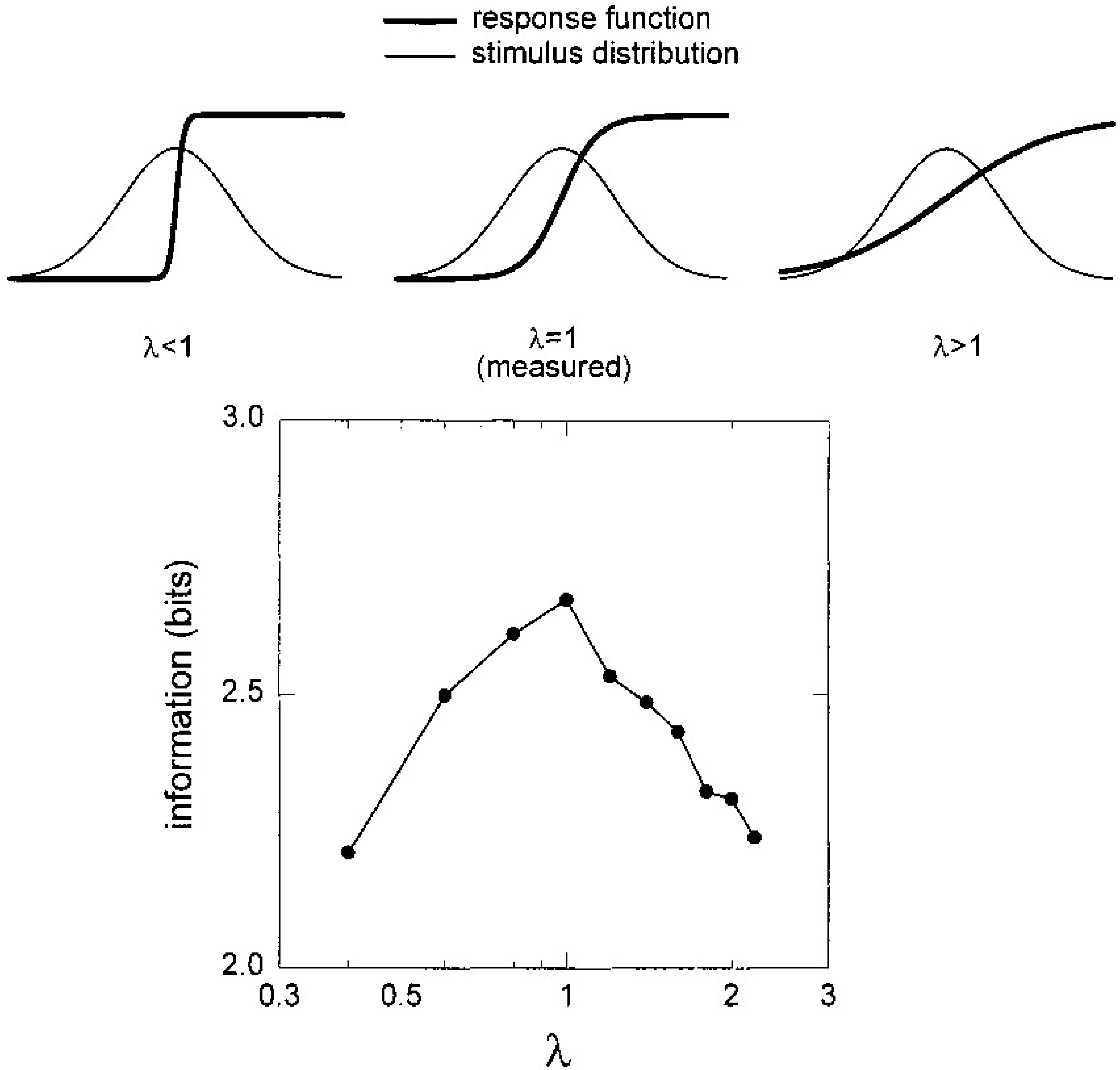
Signal and noise in the two stim conditions:



Adaptive rescaling to fast varying inputs:



Rescaling maximizes information transmission:



Optimal filter: whitening

- Optimize mutual information:

$$I = \frac{1}{2} \int \frac{d\omega}{2\pi} \log_2 \left(1 + \frac{|\tilde{K}(\omega)|^2 S(\omega)}{N(\omega)} \right)$$

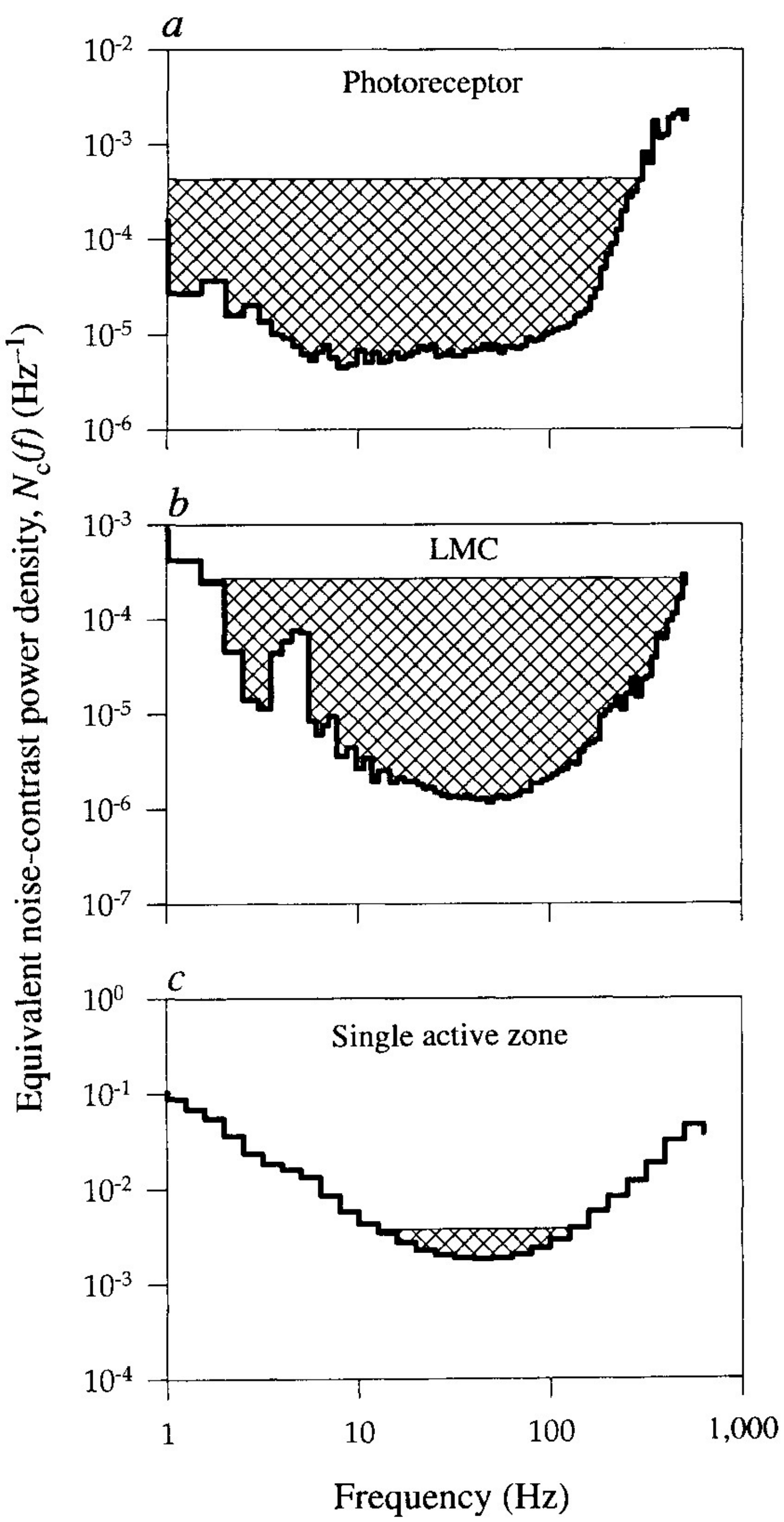
subject to constraint

$$\int d\omega |\tilde{K}(\omega)|^2 S(\omega) = \text{constant}$$

- Solution

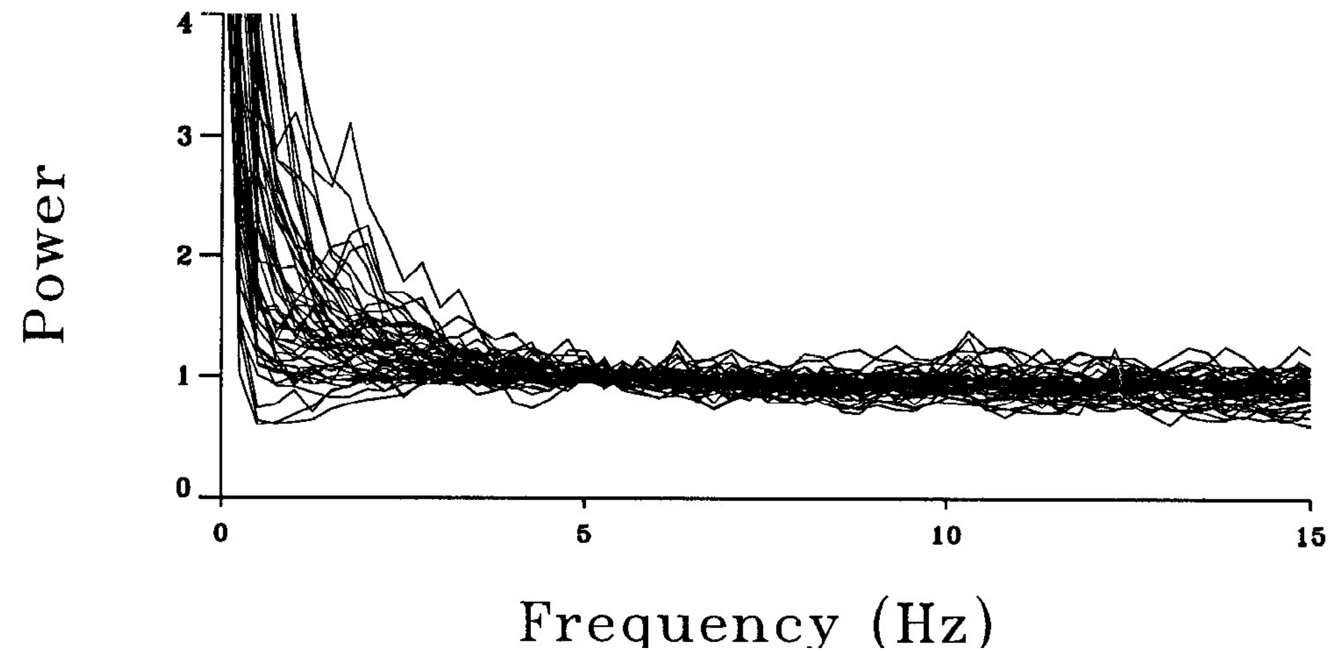
$$|\tilde{K}(\omega)|^2 S(\omega) = [A - N(\omega)]_+$$

- Whitening (water-filling analogy)

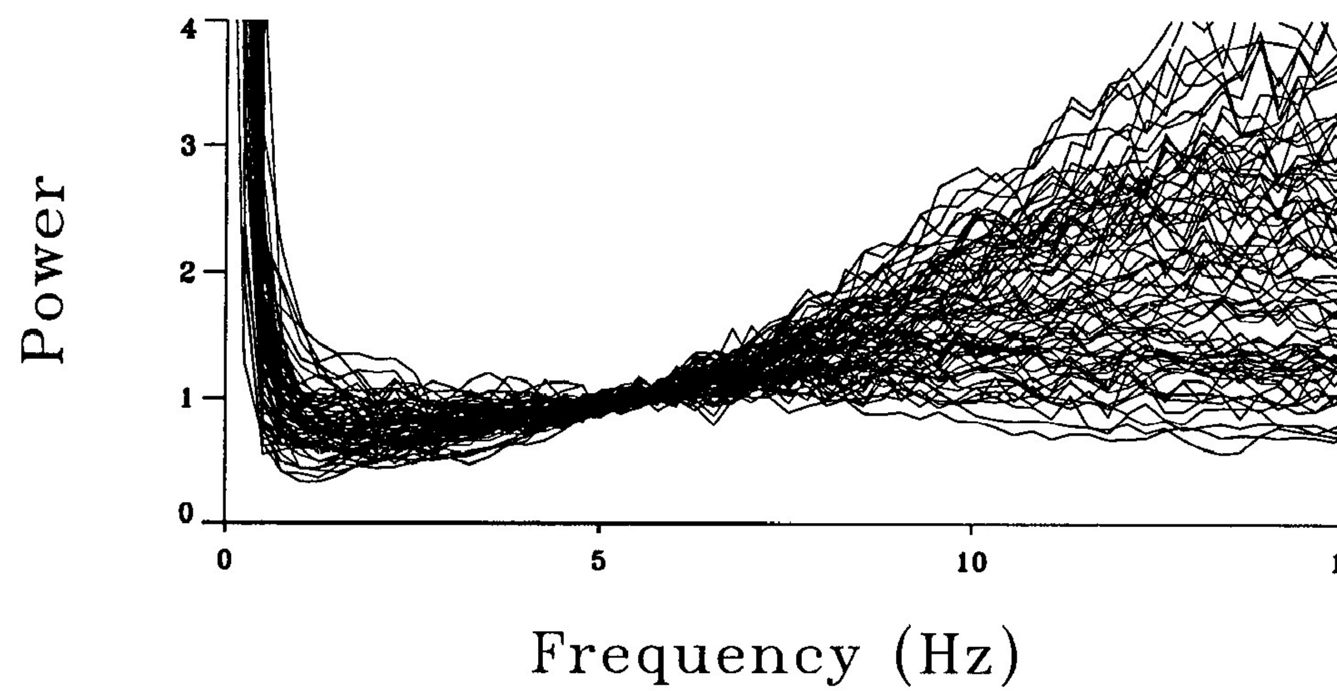


Whitening in the LGN (Dan et al 1996)

- Natural stimuli



- White-noise stimuli



A lightning fast introduction to channel coding:

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