Boulder Theoretical Biophysics summer school: Introduction to neuroscience and information theory

Lecturer:

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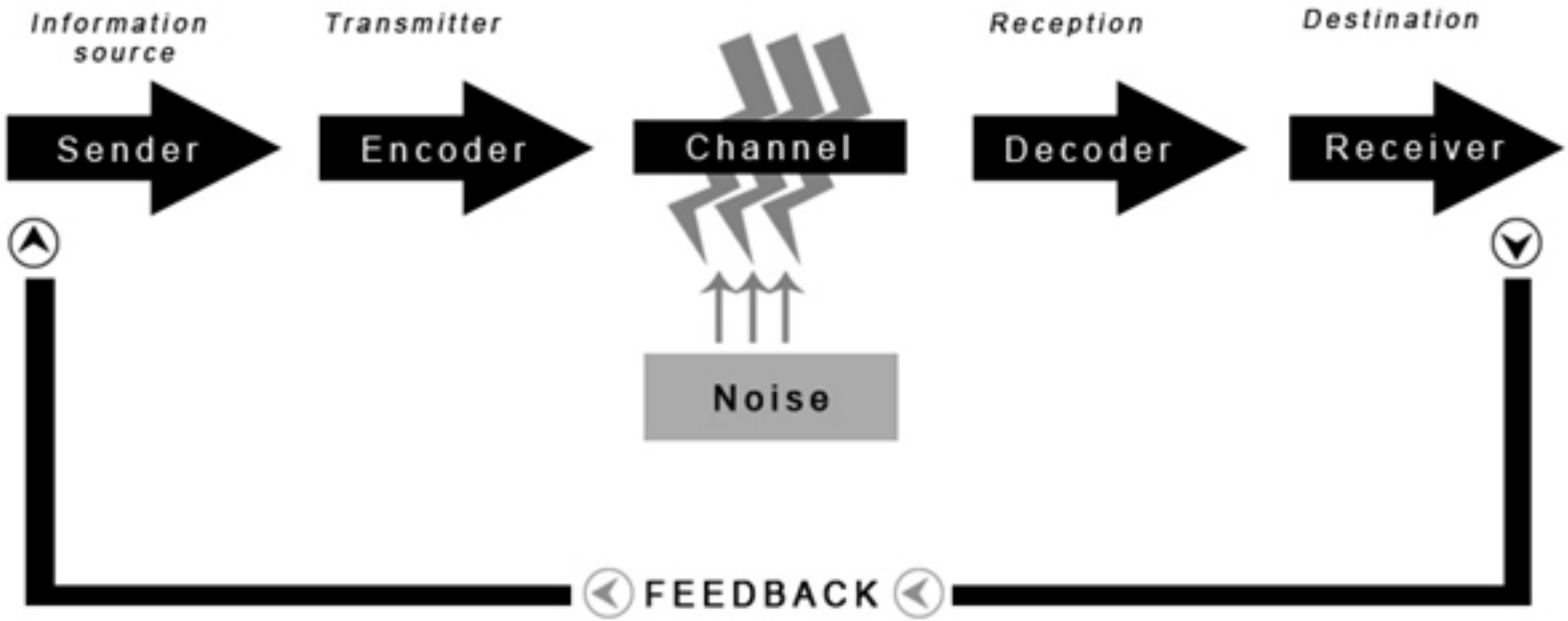
RECAP: *Lecture 1, intro to info theory*



Shannon-Weaver's Model of Communication

RECALL: *Entropy as a measure of uncertainty:*

$$\text{uncertainty} = \log(n)$$

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$$S(X) = -\sum_x p(x) \log_2(p(x))$$

Recall: *basics of probability theory*

Product rule:

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Sum rule:

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Bayes' rule:

$$\begin{aligned} P(a|b) &= \frac{P(b|a)P(a)}{P(b)} \\ &= \frac{P(b|a)P(a)}{\sum_{a'} P(b|a')P(a')} \end{aligned}$$

Recall: *basics of information theory*

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Additivity:

$$S(A, B) = S(A) + S(B)$$

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Chain rule:

$$S(A,B) = S(A) + S(B|A) = S(B) + S(A|B)$$

Recall: *basics of information theory*

Mutual information:

$$
\begin{aligned}
I(A;B) &= S(A) - S(A|B) \\
&= S(B) - S(B|A) \\
&= \sum_{a,b} P(a,b) \log_2 \left(\frac{P(a,b)}{P(a)P(b)} \right) \\
&= \sum_{a,b} P(a) P(b|a) \log_2 \left(\frac{P(b|a)}{P(b)} \right)
\end{aligned}
$$

Recall: *basics of information theory*

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$$I(X;Y) = \sum_{x,y} P(X,Y) \log_2 \left(\frac{P(X,Y)}{P(X)P(Y)} \right)$$

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$$P(X,Y) = P(X|Y)P(Y)$$

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

Recall: *basics of information theory*

Kullback-Liebler divergence (D_{KL}):

$$D_{\mathrm{KL}}(P, Q) = \sum_a P(a) \log_2 \frac{P(a)}{Q(a)}$$

Recall: *basics of information theory*

Mutual information ≥ 0:

$$I(A;B) = S(A) - S(A|B)$$

Information in single spikes:

$$I(1\ \text{spike; s}) = \frac{1}{T}\int_0^T dt\ \left(\frac{r(t)}{\bar{r}}\right)\log_2\left(\frac{r(t)}{\bar{r}}\right)$$

see Brenner et al., 2000

Information in single spikes:

chalkboard interlude

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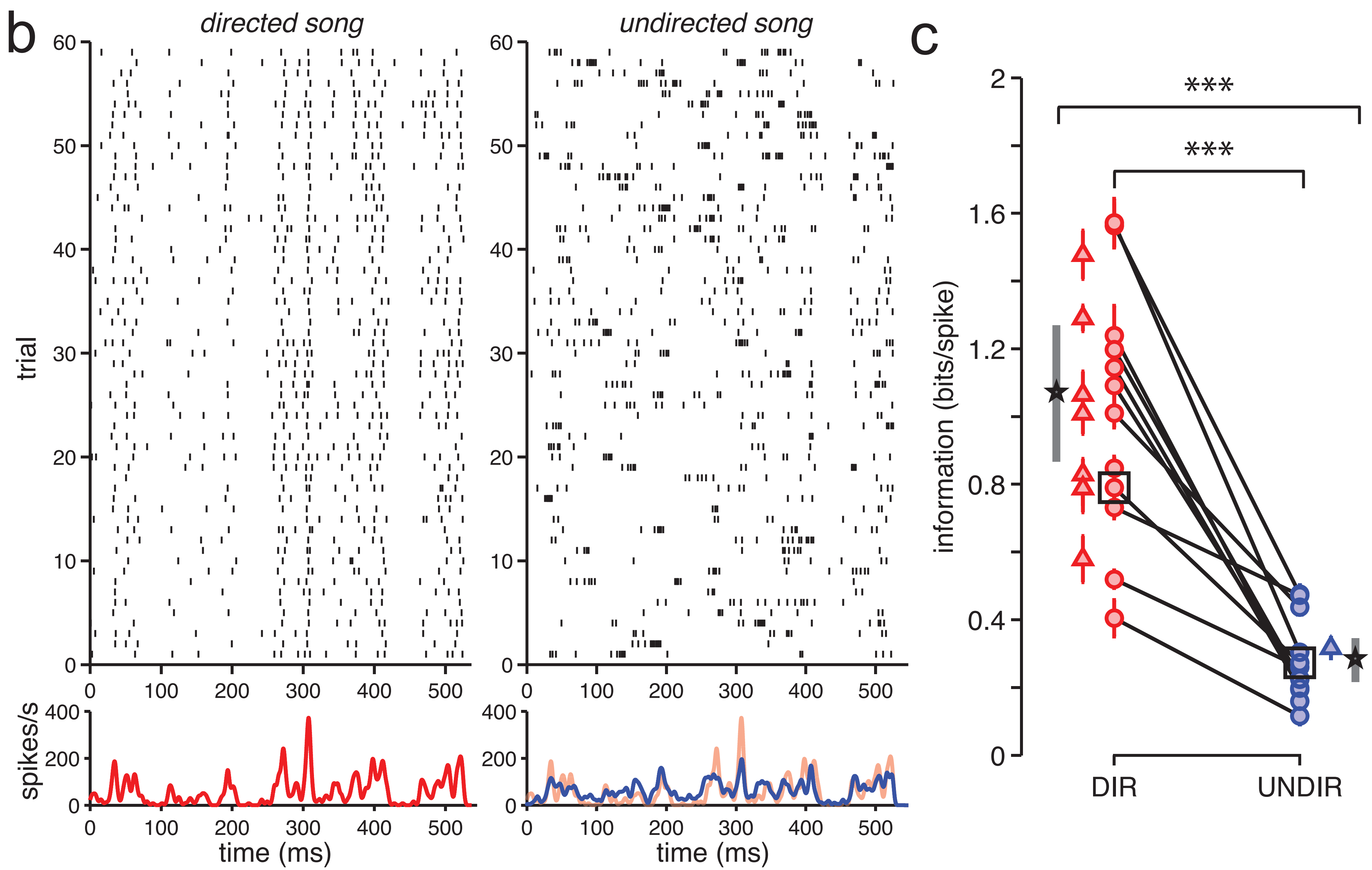
Searching for the symbols in the neural code:



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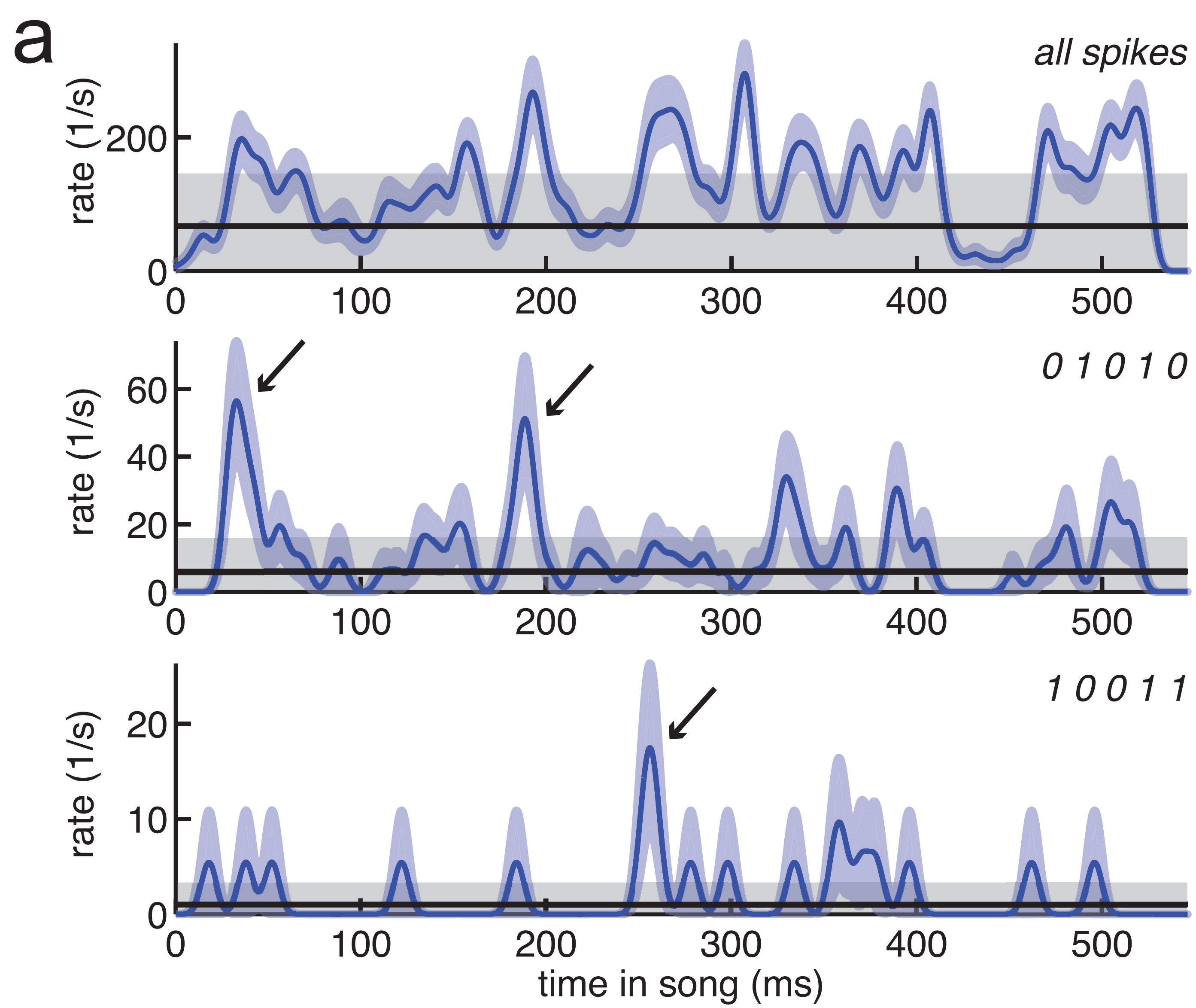

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