Boulder Theoretical Biophysics summer school: Introduction to neuroscience and information theory

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RECAP: Lecture 1, intro to info theory

SHANNON-WEAVER’S MODEL OF COMMUNICATION
RECALL: Entropy as a measure of uncertainty:

\[ \text{uncertainty} = \log(n) \]
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\[
\text{uncertainty} = \log (n) \\
= \log \left( \frac{1}{p} \right) \\
= - \log (p)
\]
**RECALL:** Entropy as a measure of uncertainty:

\[ \text{uncertainty} = \log (n) = \log \left( \frac{1}{p} \right) = - \log (p) \]

\[ u_i = - \log (p_i) \]

\[ \langle u_i \rangle = - \sum_i p_i \log (p_i) \]
RECALL: Entropy as a measure of uncertainty:

uncertainty = log (n)

= log (1/p)

= − log (p)

\[ u_i = - \log (p_i) \]

\[ \langle u_i \rangle = - \sum_i p_i \log (p_i) \]

\[ S(X) = - \sum_x p(x) \log_2 (p(x)) \]
Recall: basics of probability theory

Product rule:

\[ P(a, b) = P(a|b)P(b) \]
Recall: basics of probability theory

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**Sum rule:**

\[ P(a) = \sum_b P(a, b) \]
\[ = \sum_b P(a|b)P(b) \]
Recall: basics of probability theory

**Product rule:**

\[ P(a, b) = P(a|b)P(b) \]

**Sum rule:**

\[ P(a) = \sum_b P(a, b) = \sum_b P(a|b)P(b) \]

**Bayes’ rule:**

\[ P(a|b) = \frac{P(b|a)P(a)}{P(b)} = \frac{P(b|a)P(a)}{\sum_{a'} P(b|a')P(a')} \]
Recall: basics of information theory
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Additivity:

\[ S(A, B) = S(A) + S(B) \]

\[ S(A, B) = S(A) + S(B) \iff P(a, b) = P(a)P(b) \]
**Recall:** basics of information theory

**Additivity:**

\[ S(A, B) = S(A) + S(B) \]

\[ S(A, B) = S(A) + S(B) \iff P(a, b) = P(a)P(b) \]

**Chain rule:**

\[ S(A, B) = S(A) + S(B|A) = S(B) + S(A|B) \]
Recall: basics of information theory

**Mutual information:**

\[
I(A; B) = S(A) - S(A|B) \\
= S(B) - S(B|A) \\
= \sum_{a,b} P(a, b) \log_2 \left( \frac{P(a, b)}{P(a)P(b)} \right) \\
= \sum_{a,b} P(a)P(b|a) \log_2 \left( \frac{P(b|a)}{P(b)} \right)
\]
**Recall: basics of information theory**

\[ I(X; Y) = S(X) - \langle S(X|Y) \rangle_y \]
**Recall:** basics of information theory

\[
I(X; Y) = S(X) - \langle S(X|Y) \rangle_y
\]

\[
I(X; Y) = S(Y) - \langle S(Y|X) \rangle_x
\]
Recall: basics of information theory

\[
I(X; Y) = S(X) - \langle S(X|Y) \rangle_y
\]

\[
I(X; Y) = S(Y) - \langle S(Y|X) \rangle_x
\]

\[
I(X; Y) = \sum_{x,y} P(X, Y) \log_2 \left( \frac{P(X,Y)}{P(X)P(Y)} \right)
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**Recall:** basics of information theory

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\]

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I(X; Y) = \sum_{x,y} P(X, Y) \log_2 \left( \frac{P(X, Y)}{P(X)P(Y)} \right)
\]

\[
P(X, Y) = P(X|Y)P(Y)
\]

\[
P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}
\]
Recall: basics of information theory

**Kullback-Liebler divergence ($D_{KL}$):**

\[ D_{KL}(P, Q) = \sum_a P(a) \log_2 \frac{P(a)}{Q(a)} \]
Recall: basics of information theory

**Mutual information ≥ 0:**

\[ I(A; B) = S(A) - S(A|B) \]
Information in single spikes:

\[
I(1 \text{ spike}; s) = \frac{1}{T} \int_0^T dt \left( \frac{r(t)}{\bar{r}} \right) \log_2 \left( \frac{r(t)}{\bar{r}} \right)
\]
Information in single spikes:

\[ I(1 \; \text{spike}; s) = \frac{T}{T} \int_0^T dt \left( \frac{r(t)}{\bar{r}} \right) \log_2 \left( \frac{r(t)}{\bar{r}} \right) \]

see Brenner et al., 2000
Searching for the symbols in the neural code:
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![Graphs showing neural activity and timing](image)

- **Pattern info**
  - Directed song
  - Undirected song

- **Information (bits/sec)**
  - Spike count
  - Rate (1/s)
  - Time in song (ms)
Searching for the symbols in the neural code:

![Graph showing neural activity over time](b)

- All spikes
- Time in song (ms)
- Rate (1/s)

Symbols highlighted:
- 10110
- 00111