Boulder Theoretical Biophysics summer school: Introduction to neuroscience and information theory

Lecturer:

Stephanie E. Palmer

Associate Professor

Dep't of OBA

Dep't of Physics

University of Chicago

sepalmer@uchicago.edu

stimuli → *neurons*

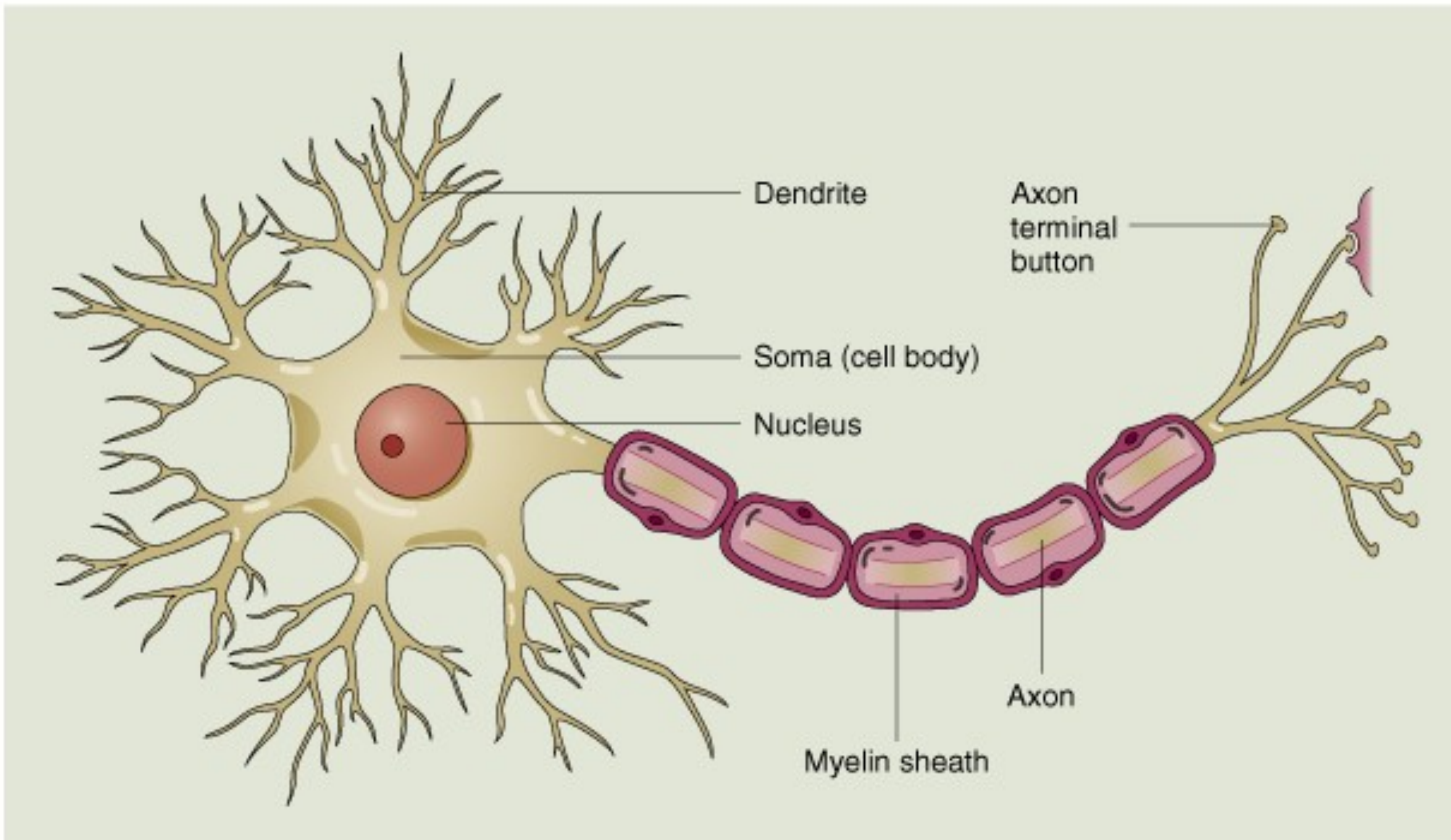
↘ *neurons*

↘ *neurons*

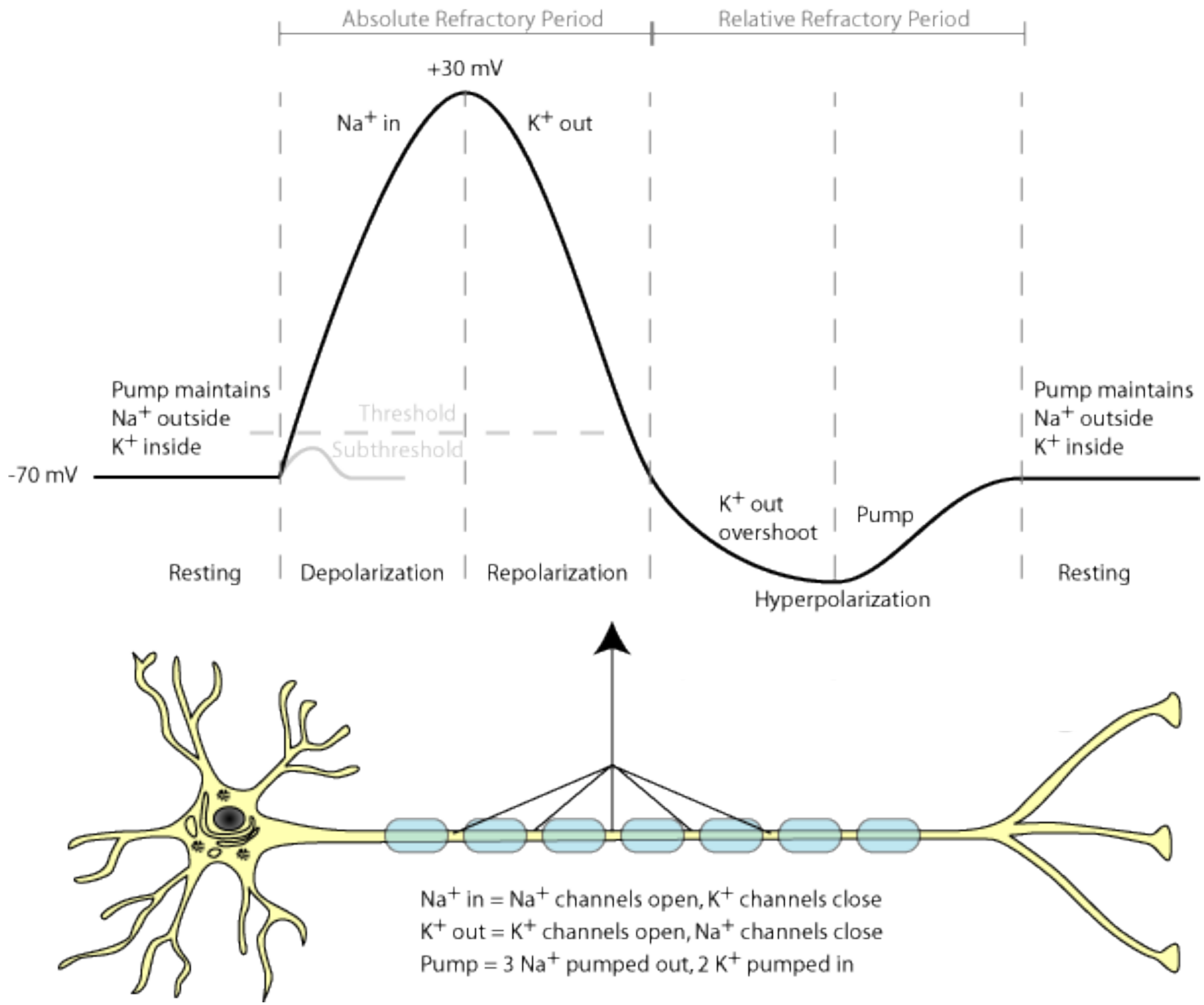
↘ ...

↘ *neurons* → *behavior*

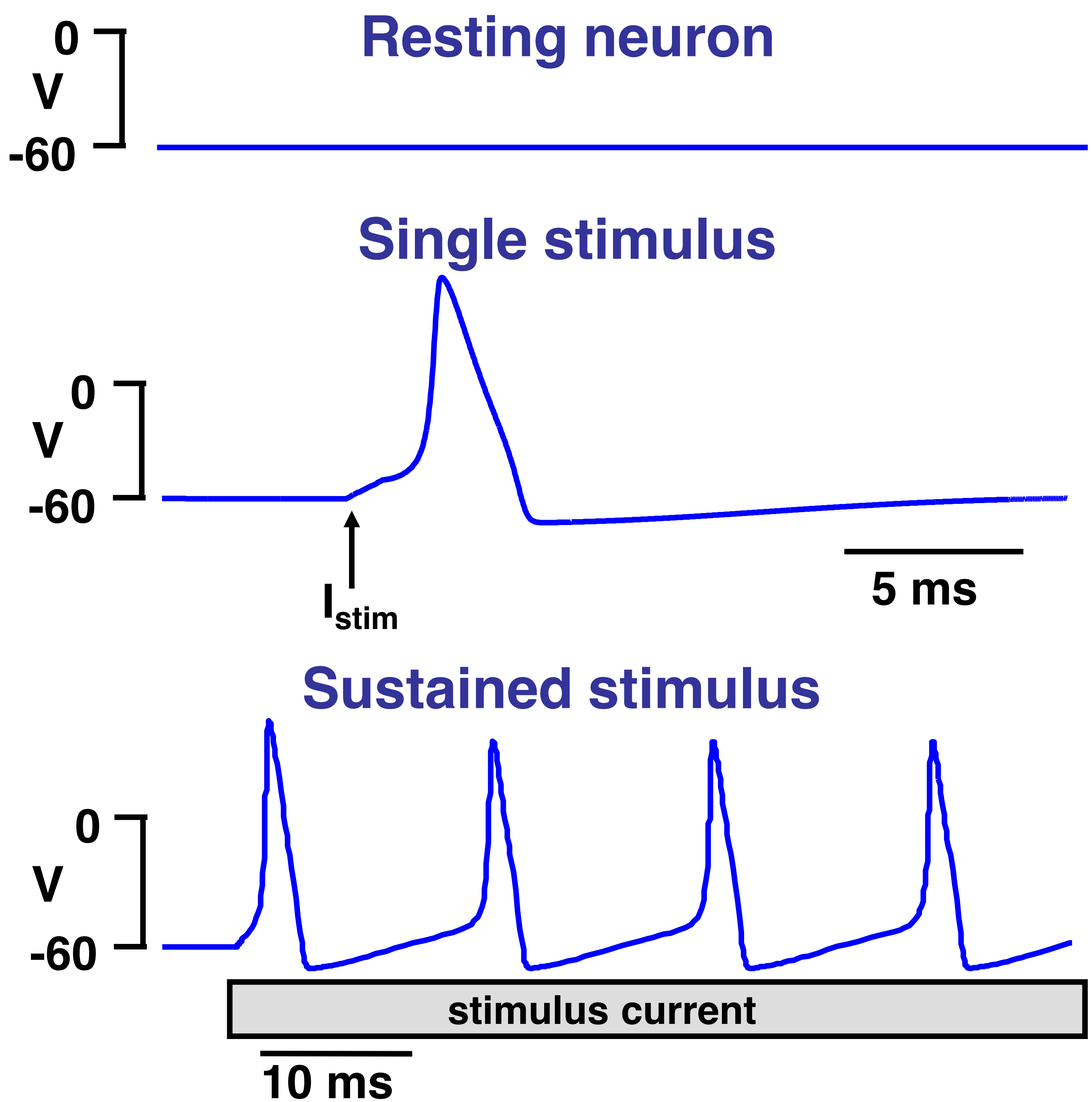
Anatomy of a neuron:



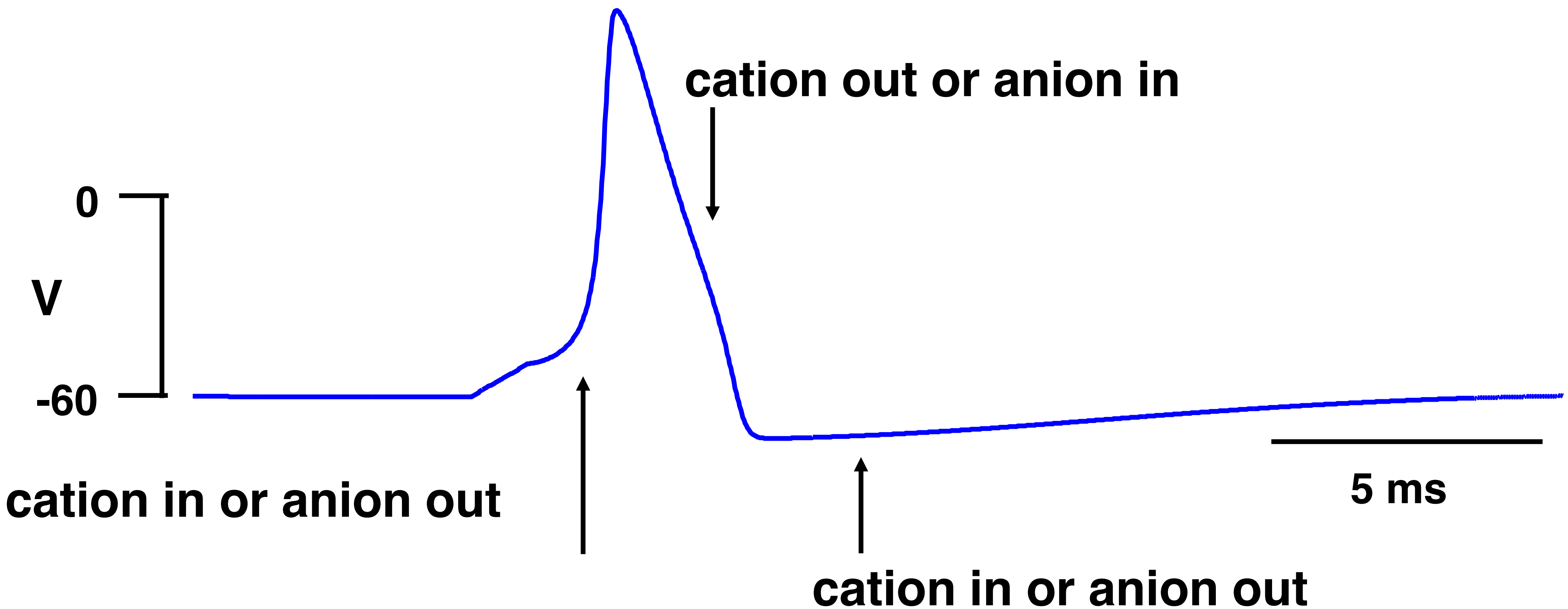
Anatomy of a spike:



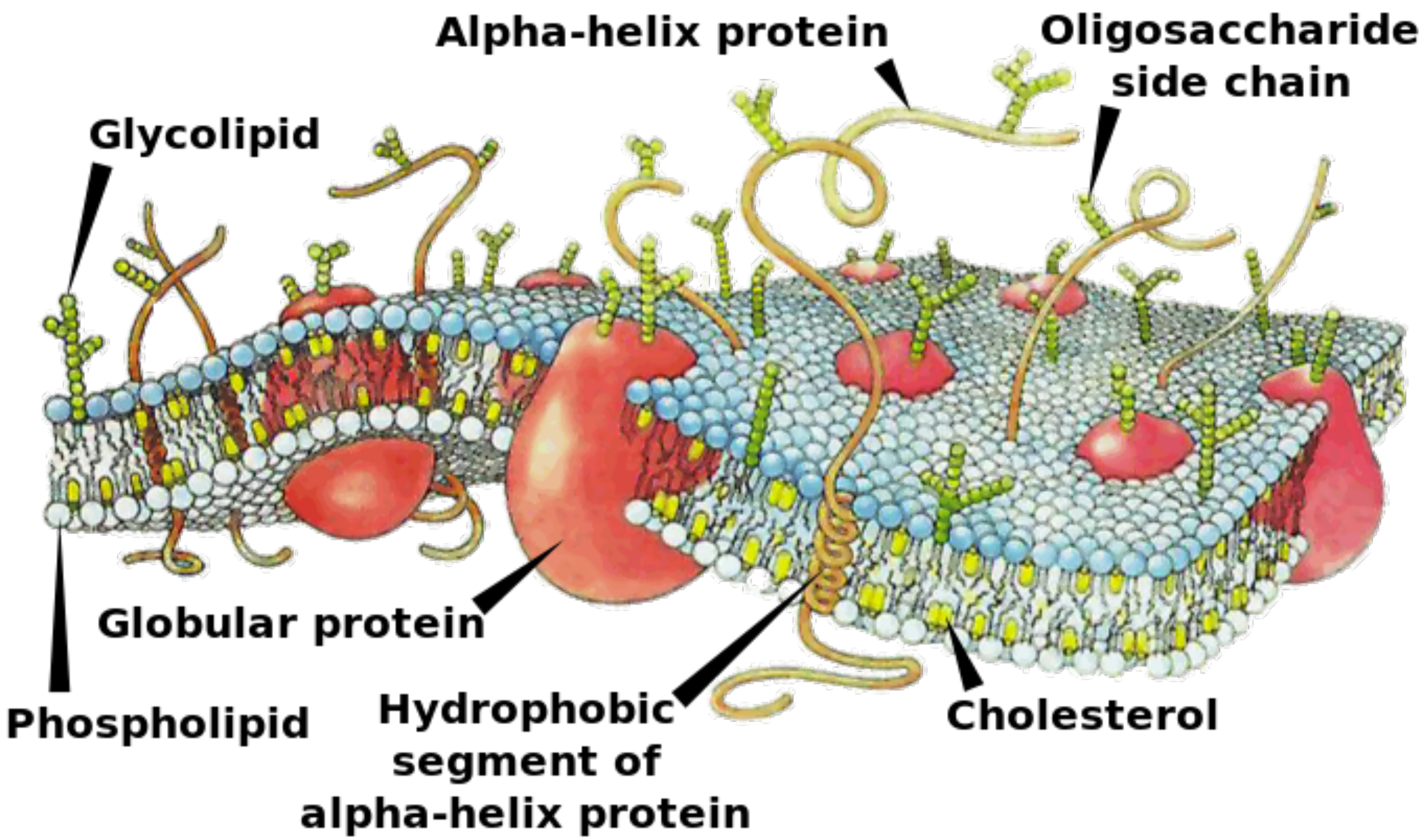
Properties of spiking:



We seek a quantitative description of this behavior:

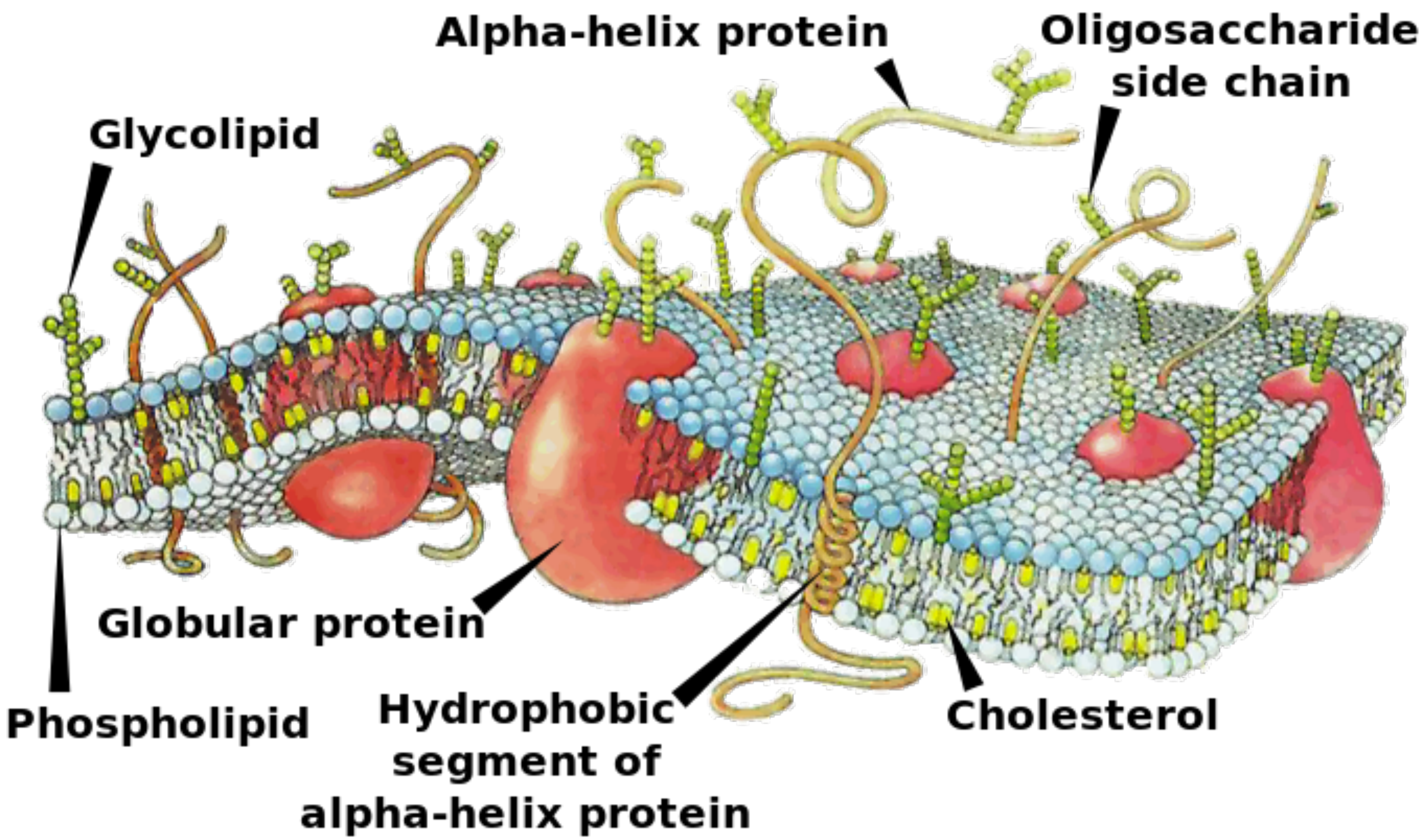


Some basics:



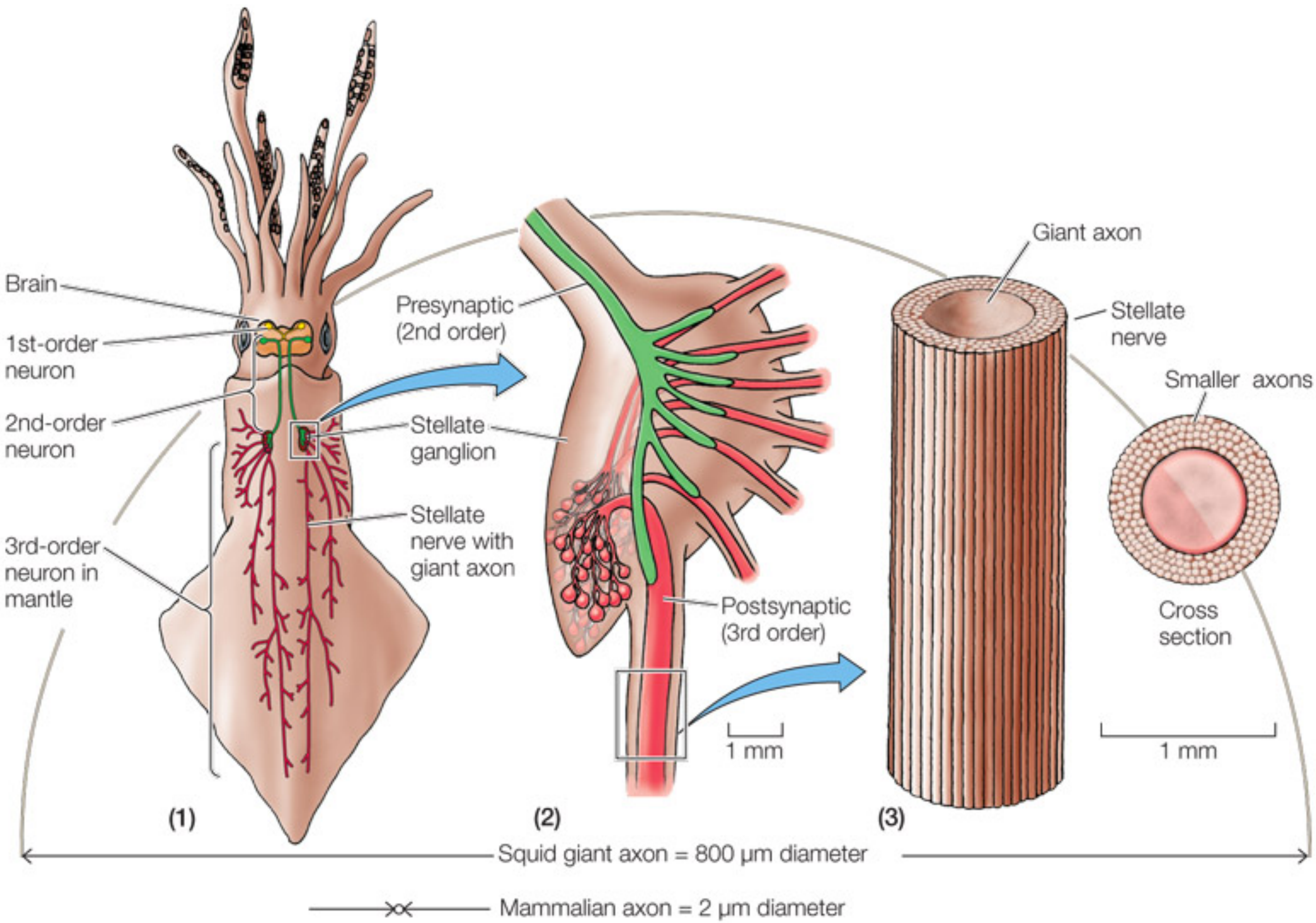
Alpha-helix protein
Oligosaccharide side chain
Glycolipid
Globular protein
Phospholipid
Hydrophobic segment of alpha-helix protein
Cholesterol

Some basics:

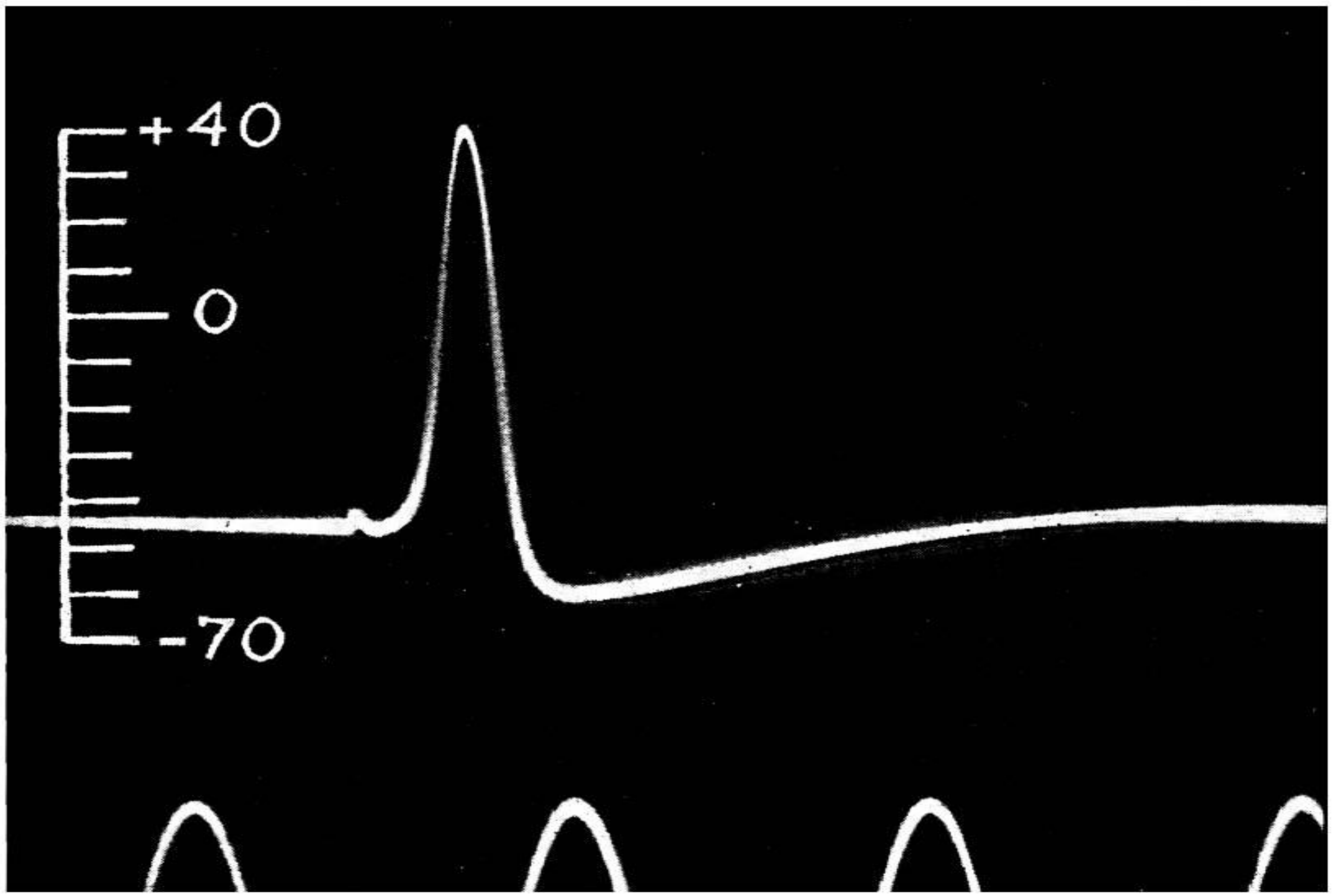


chalkboard interlude

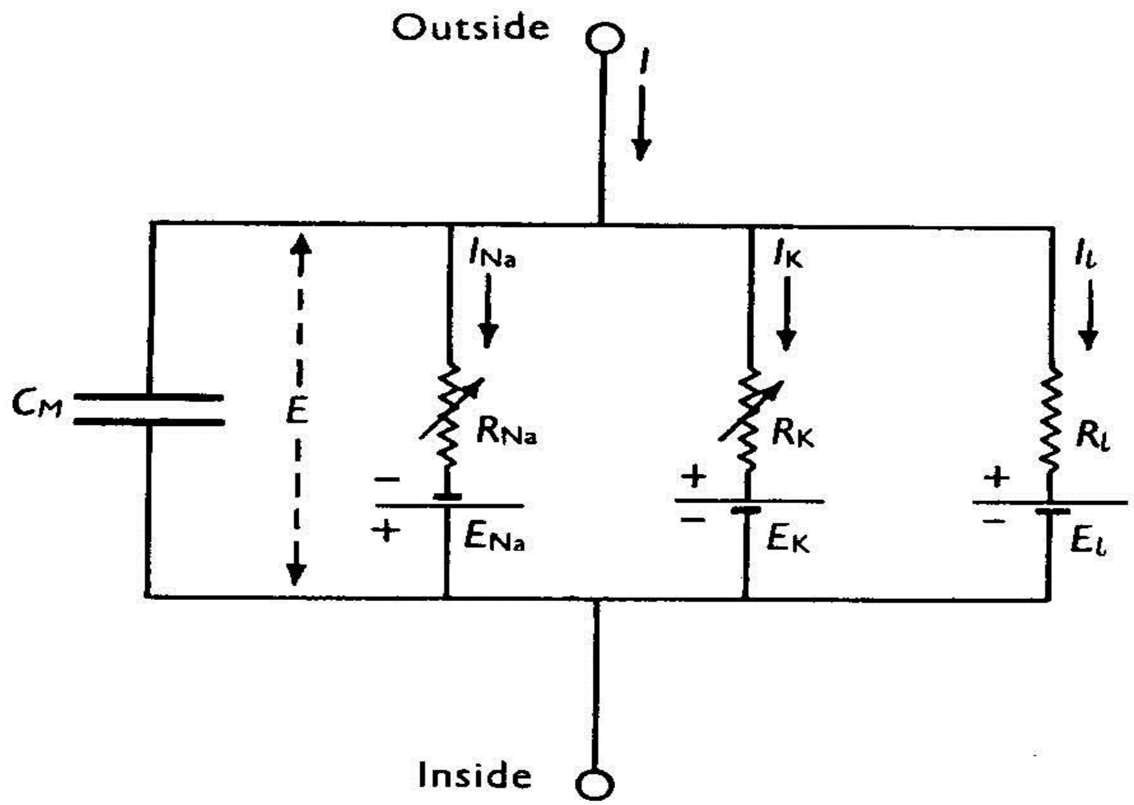
Squid giant axon:



The 1939 letter to Nature:

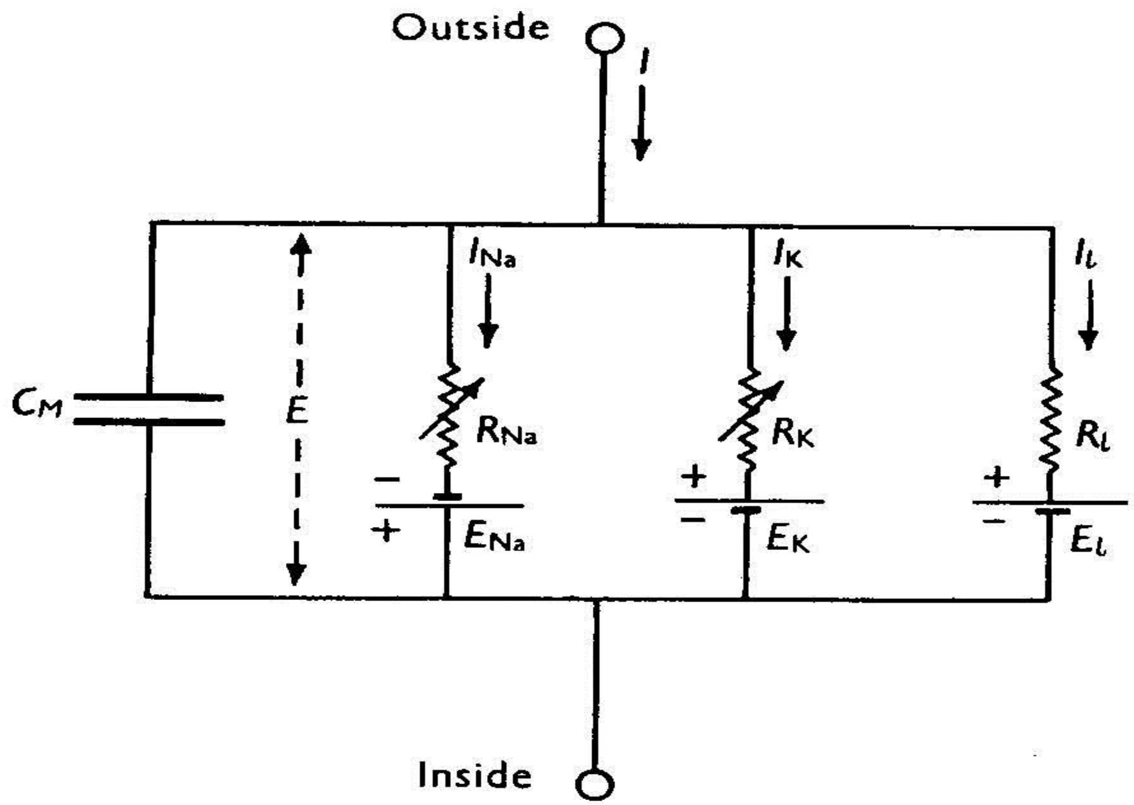


The final model:



Hodgkin & Huxley (1952), *J. Physiol.* 117:400.

The final model:



Hodgkin & Huxley (1952), *J. Physiol.* 117:400.

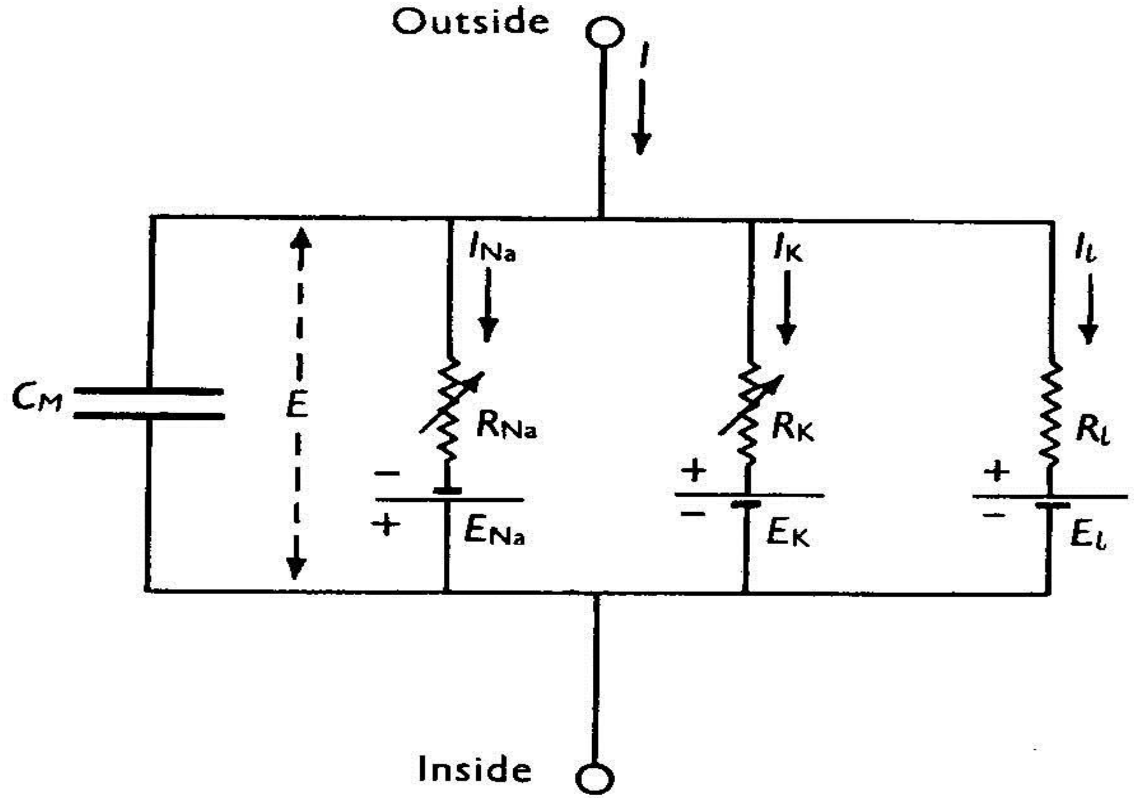
$$C_m \frac{dV}{dt} = -g_L\left(V - V_L\right) - \overline{g}_{Na} m^3 h \left(V - V_{Na}\right) - \overline{g}_K n^4 \left(V - V_K\right)$$

$$\frac{dm}{dt} = \alpha_m\left(V\right)\left(1-m\right) - \beta_m\left(V\right)m$$

$$\frac{dh}{dt} = \alpha_h\left(V\right)\left(1-h\right) - \beta_h\left(V\right)h$$

$$\frac{dn}{dt} = \alpha_n\left(V\right)\left(1-n\right) - \beta_n\left(V\right)n$$

The final model:



Hodgkin & Huxley (1952), *J. Physiol.* 117:400.

$$C_m \frac{dV}{dt} = -g_L (V - V_L) - \overline{g}_{Na} m^3 h (V - V_{Na}) - \overline{g}_K n^4 (V - V_K)$$

$$\frac{dm}{dt} = \alpha_m (V)(1-m) - \beta_m (V) m$$

$$\frac{dh}{dt} = \alpha_h (V)(1-h) - \beta_h (V) h$$

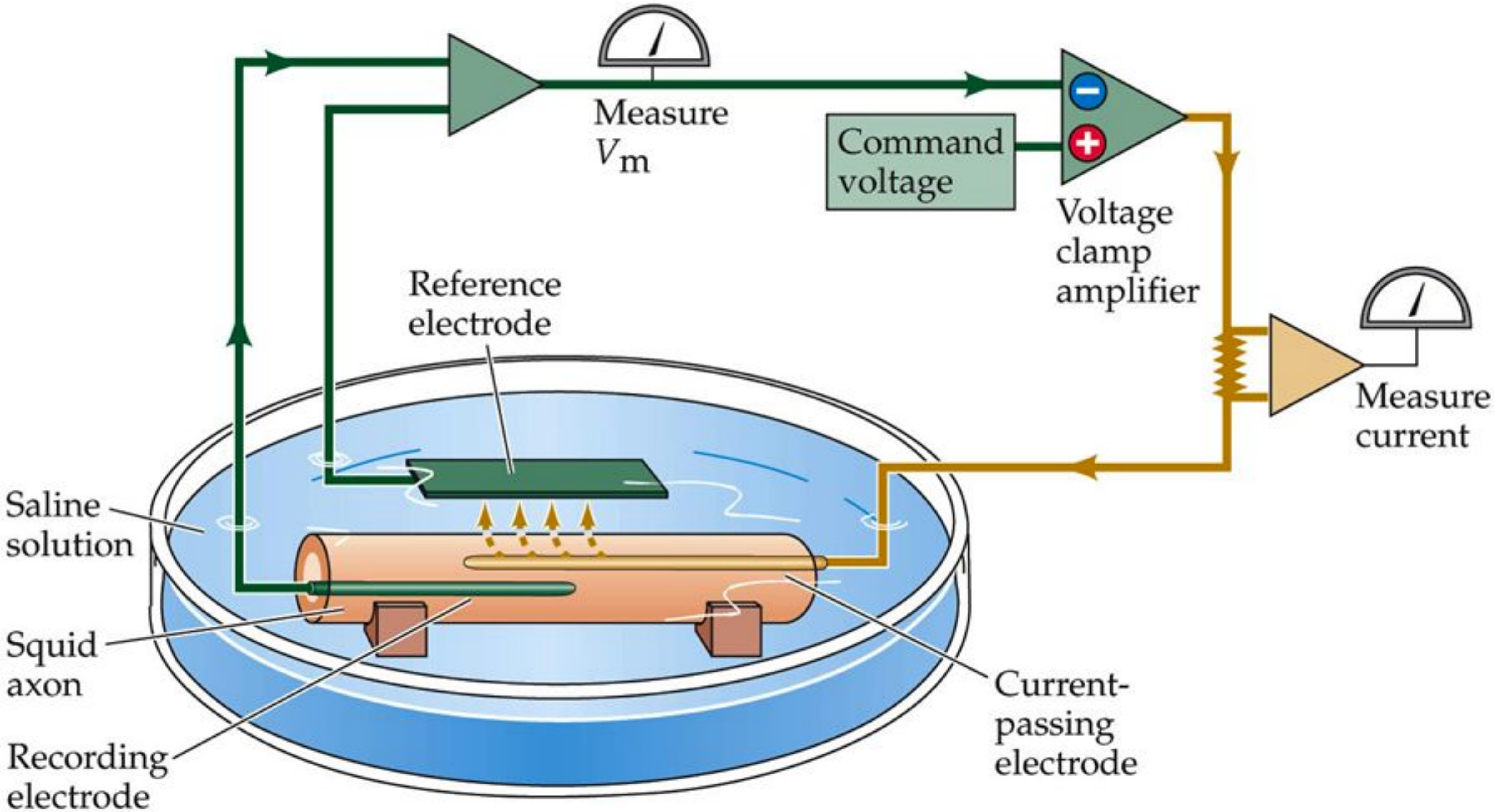
$$\frac{dn}{dt} = \alpha_n (V)(1-n) - \beta_n (V) n$$

$$\alpha_m = 0.1(V_m + 35.0)/(1. - e^{(-(V_m+35.0)/10.0)})$$
$$\beta_m = 4.0\, e^{(-(V_m+60.0)/18.0)}$$

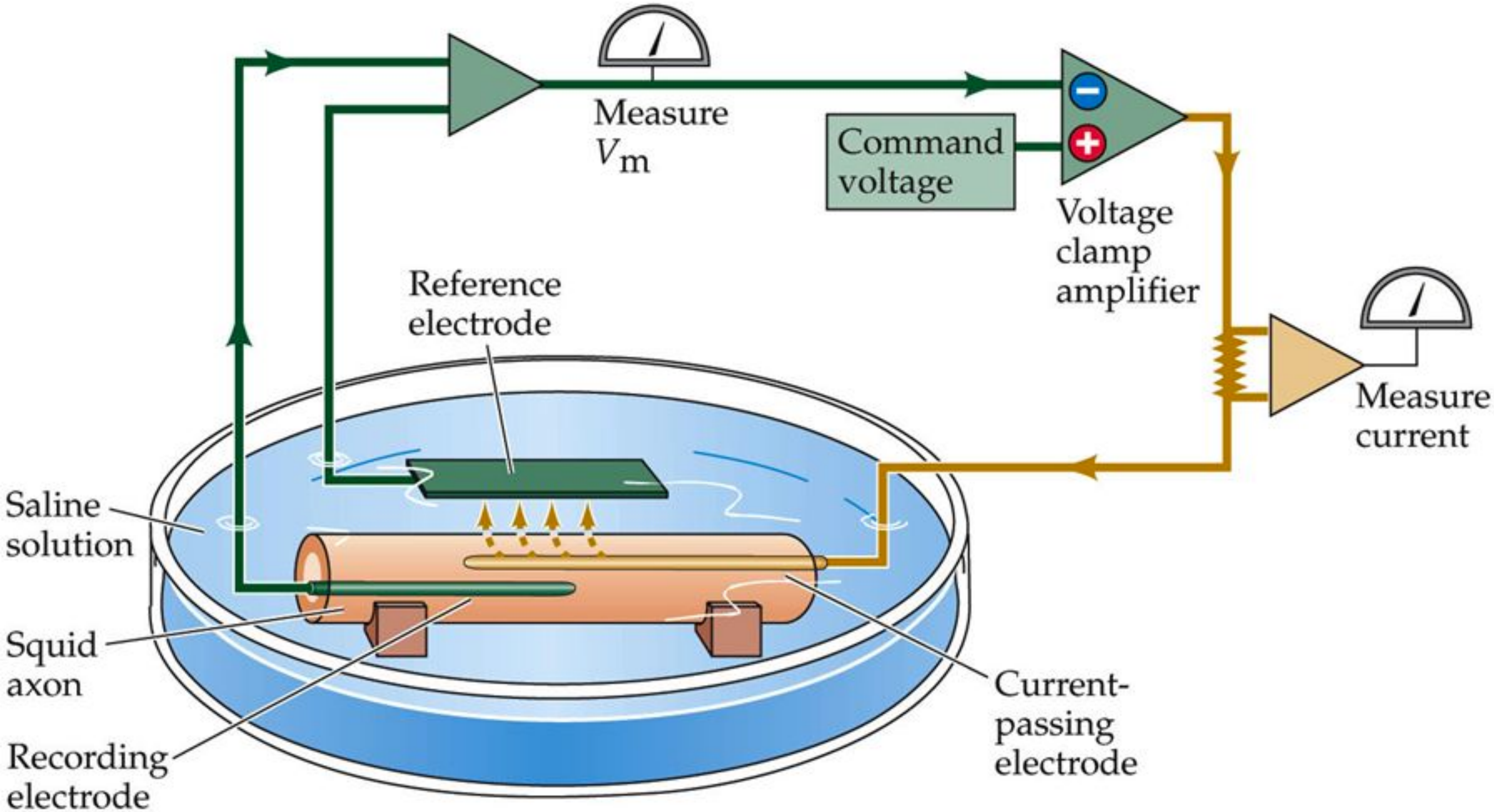
$$\alpha_h = 0.07\, e^{(-(V_m+60.0)/20.0)}$$
$$\beta_h = 1./(1 + e^{(-(V_m+30.0)/10.0)})$$

$$\alpha_n = 0.01(V_m + 50.0)/(1 - e^{(-(V_m+50.0)/10.0)})$$
$$\beta_n = 0.125\, e^{(-(V_m+60.0)/80.0)}$$

Box 3A The Voltage Clamp Technique

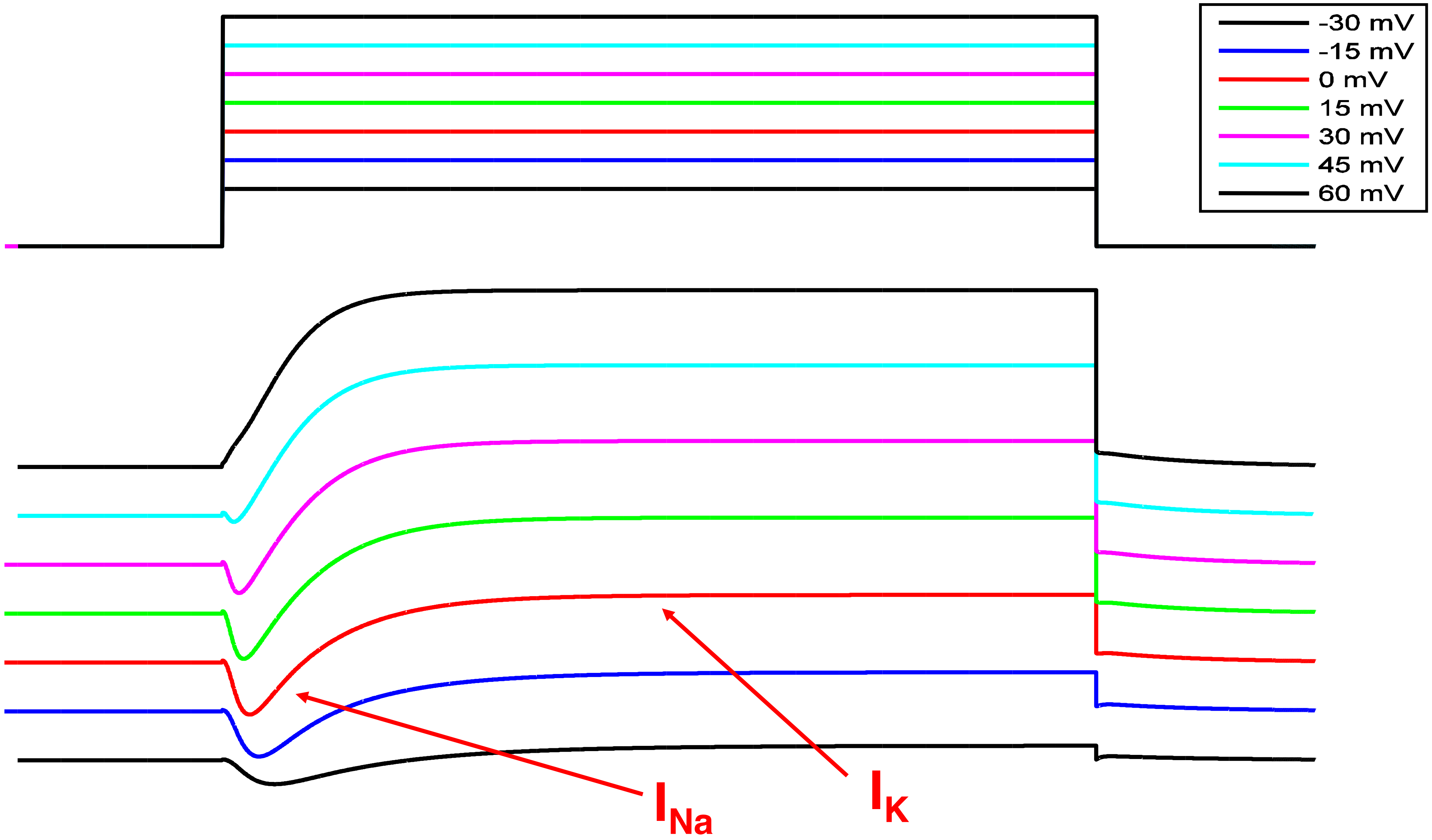


Box 3A The Voltage Clamp Technique

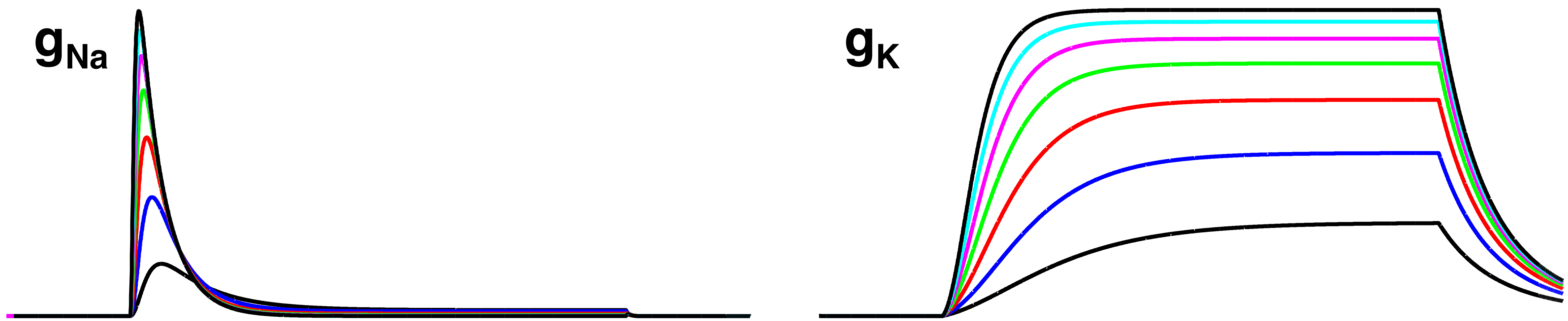


chalkboard interlude

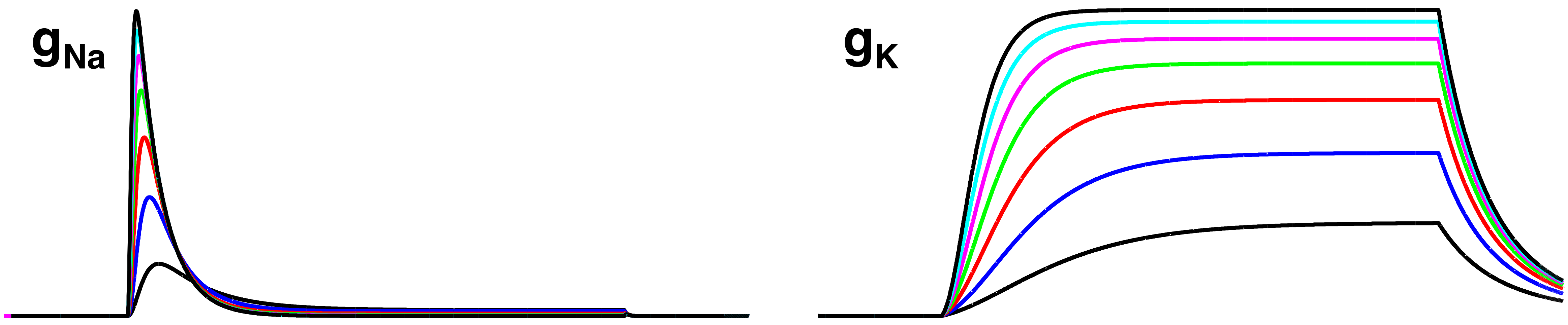
Make recordings, separate currents:



…and get conductances:

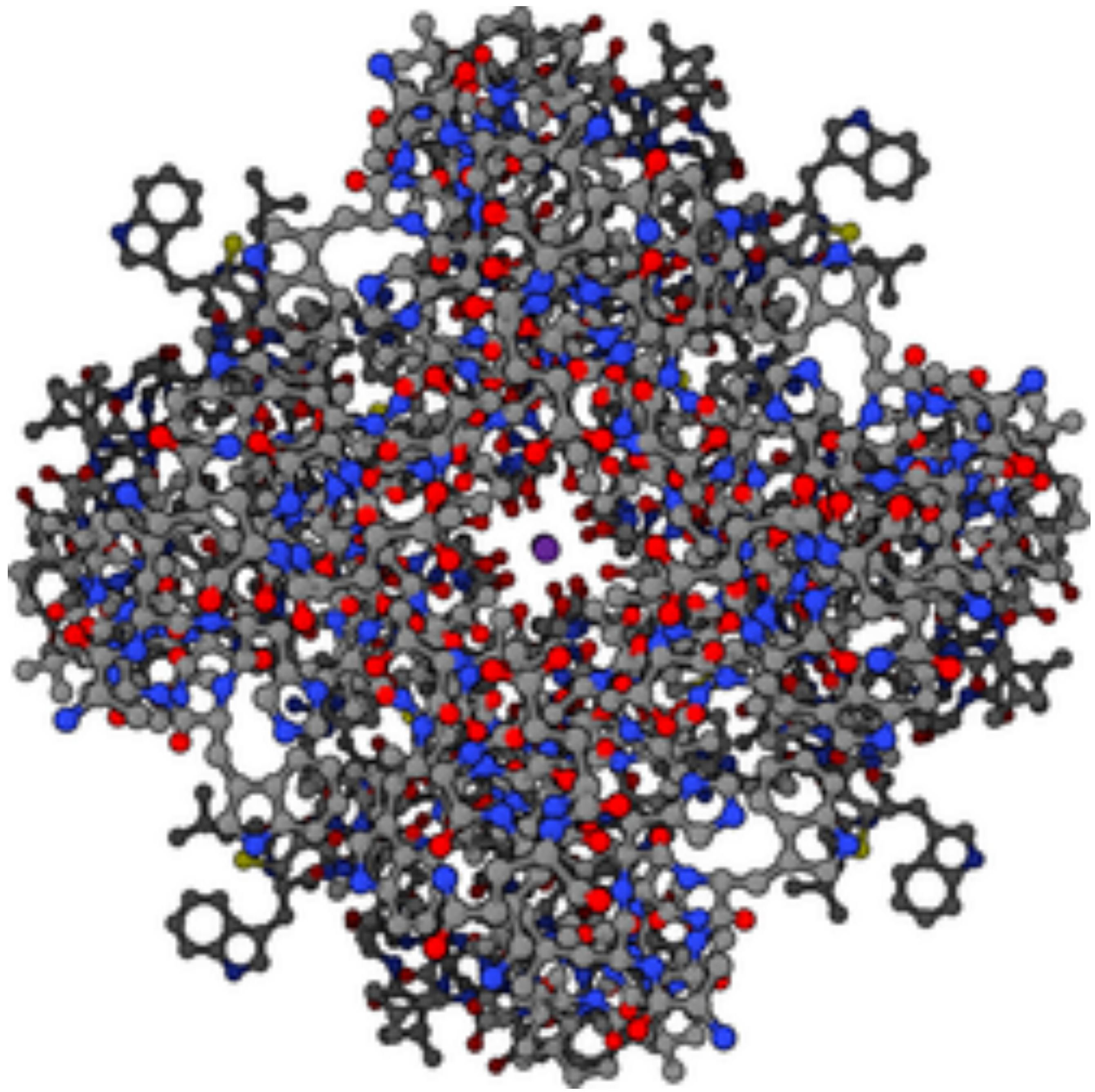


…and get conductances:



chalkboard interlude

Potassium channels are tetramers!



Structure of the potassium ion channel (tetramer)

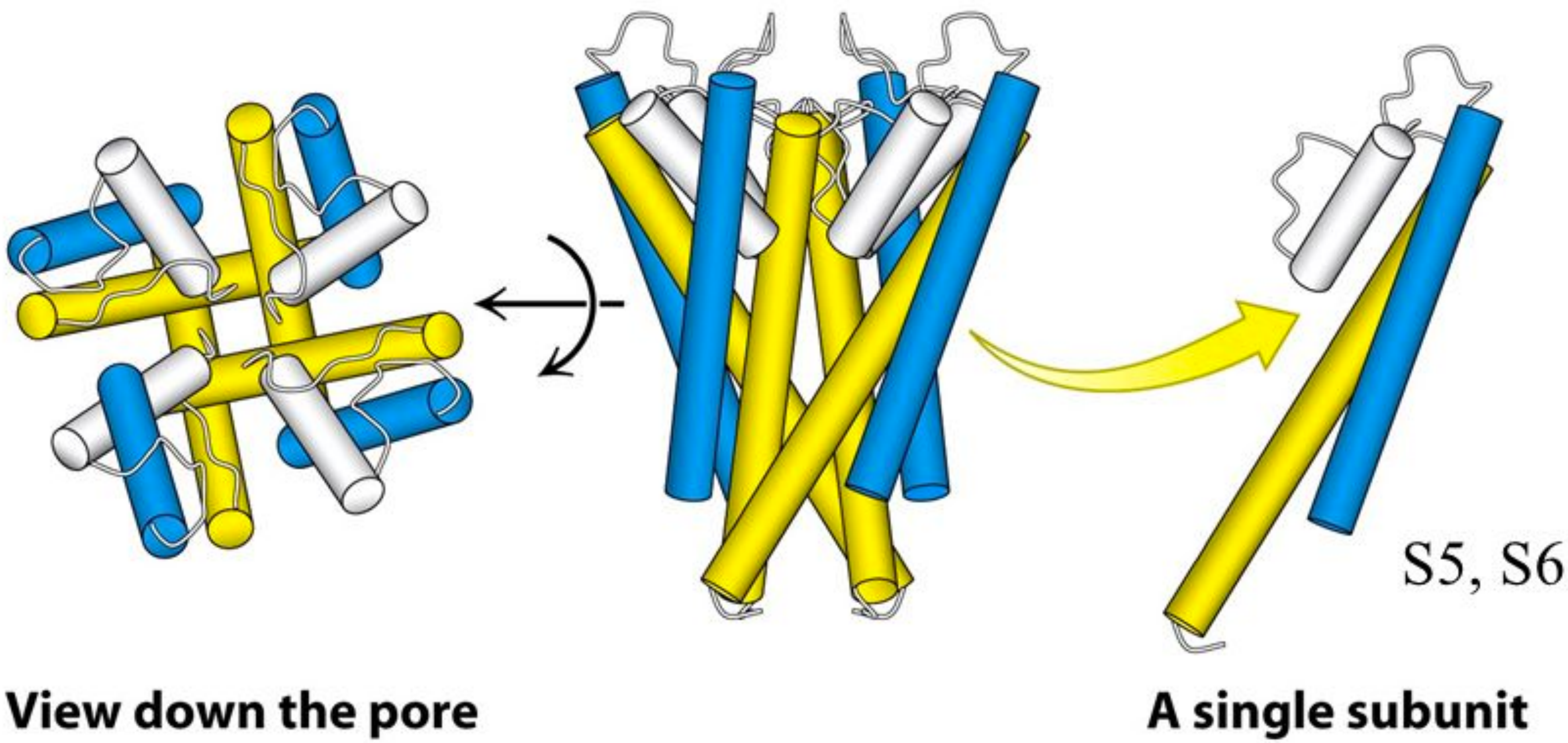
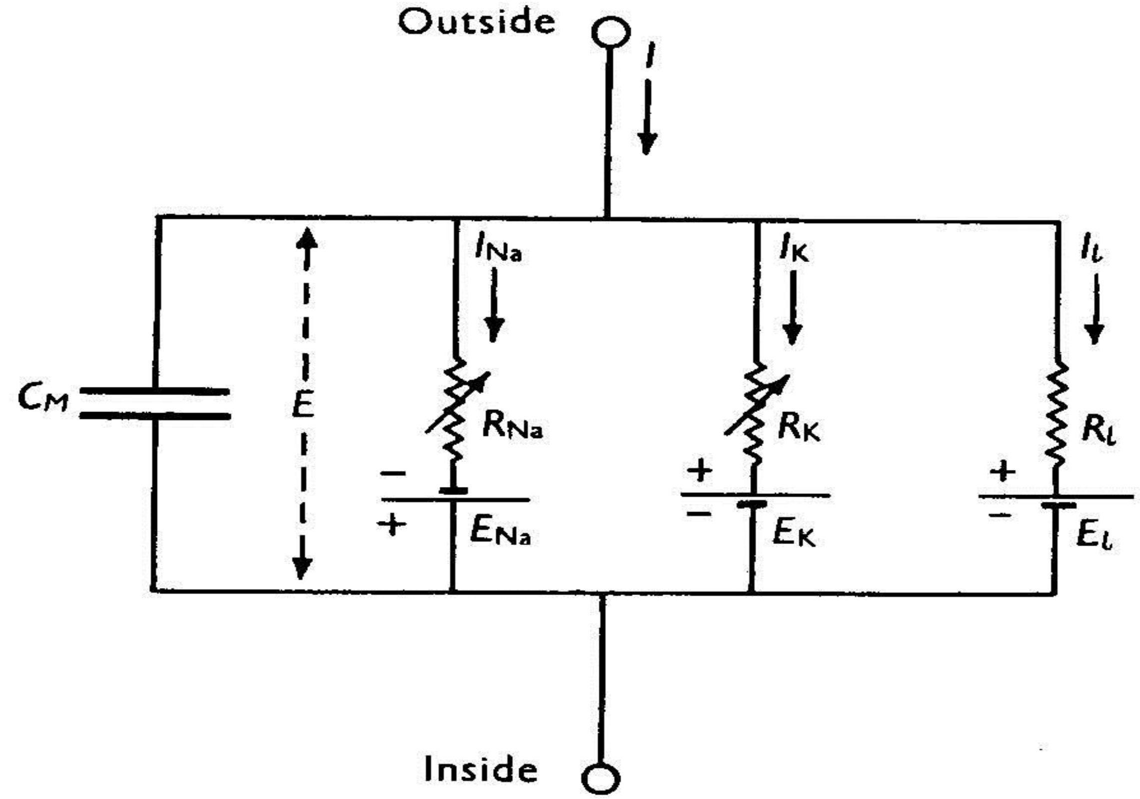


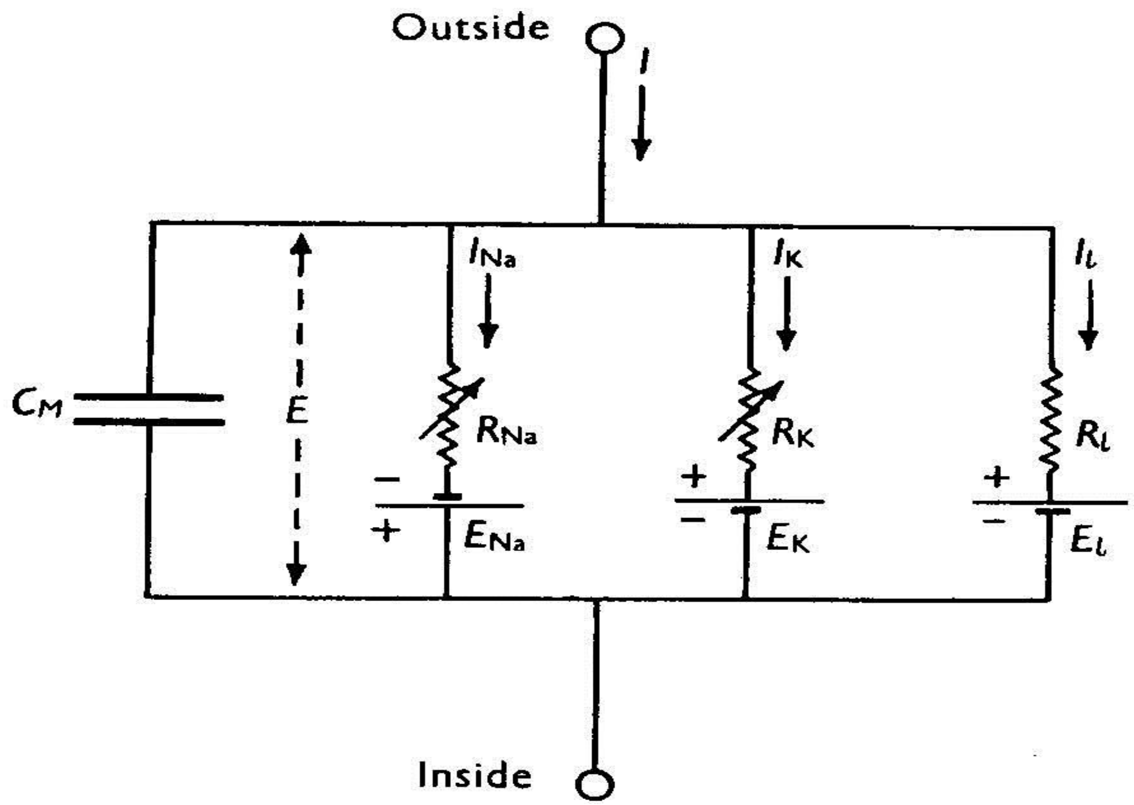
Figure 13-17
Biochemistry, Sixth Edition

The final model:



Hodgkin & Huxley (1952), *J. Physiol.* 117:400.

The final model:



Hodgkin & Huxley (1952), *J. Physiol.* 117:400.

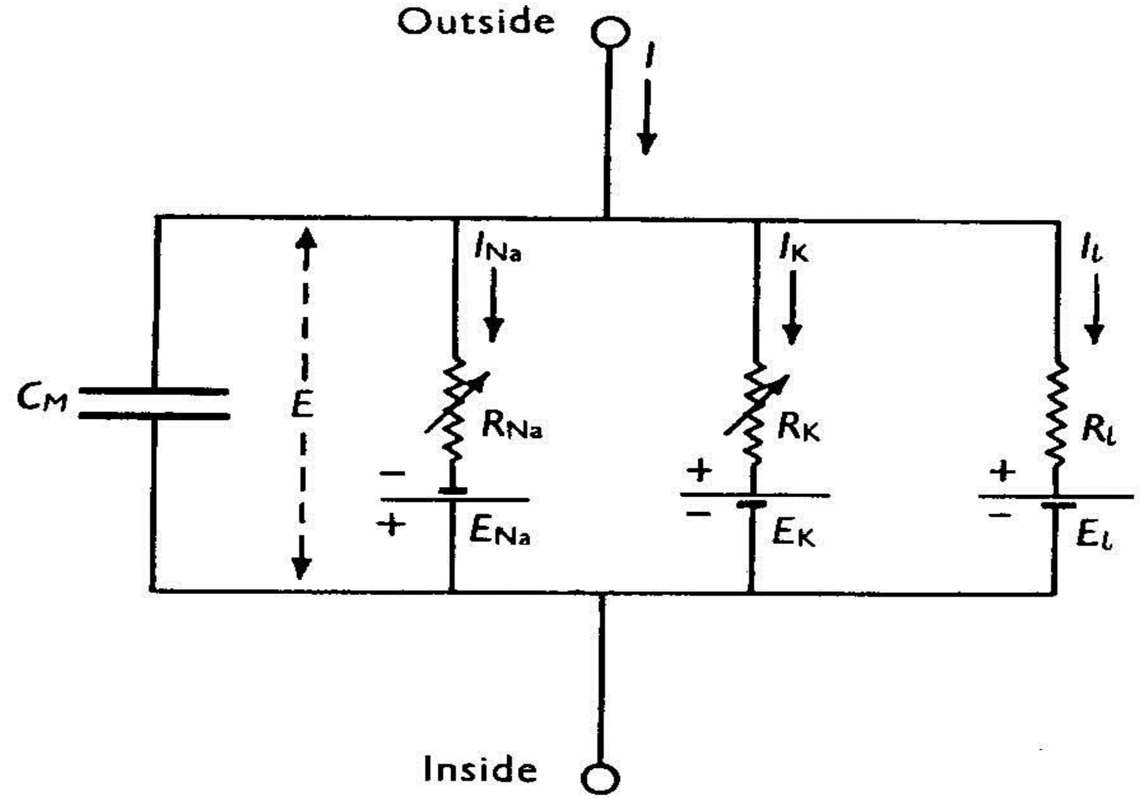
$$C_m \frac{dV}{dt} = -g_L\left(V - V_L\right) - \bar{g}_{Na} m^3 h\left(V - V_{Na}\right) - \bar{g}_K n^4\left(V - V_K\right)$$

$$\frac{dm}{dt} = \alpha_m(V)(1-m) - \beta_m(V)m$$

$$\frac{dh}{dt} = \alpha_h(V)(1-h) - \beta_h(V)h$$

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n$$

The final model:



Hodgkin & Huxley (1952), *J. Physiol.* 117:400.

$$C_m \frac{dV}{dt} = -g_L(V - V_L) - \overline{g}_{Na} m^3 h (V - V_{Na}) - \overline{g}_K n^4 (V - V_K)$$

$$\frac{dm}{dt} = \alpha_m(V)(1-m) - \beta_m(V)m$$

$$\frac{dh}{dt} = \alpha_h(V)(1-h) - \beta_h(V)h$$

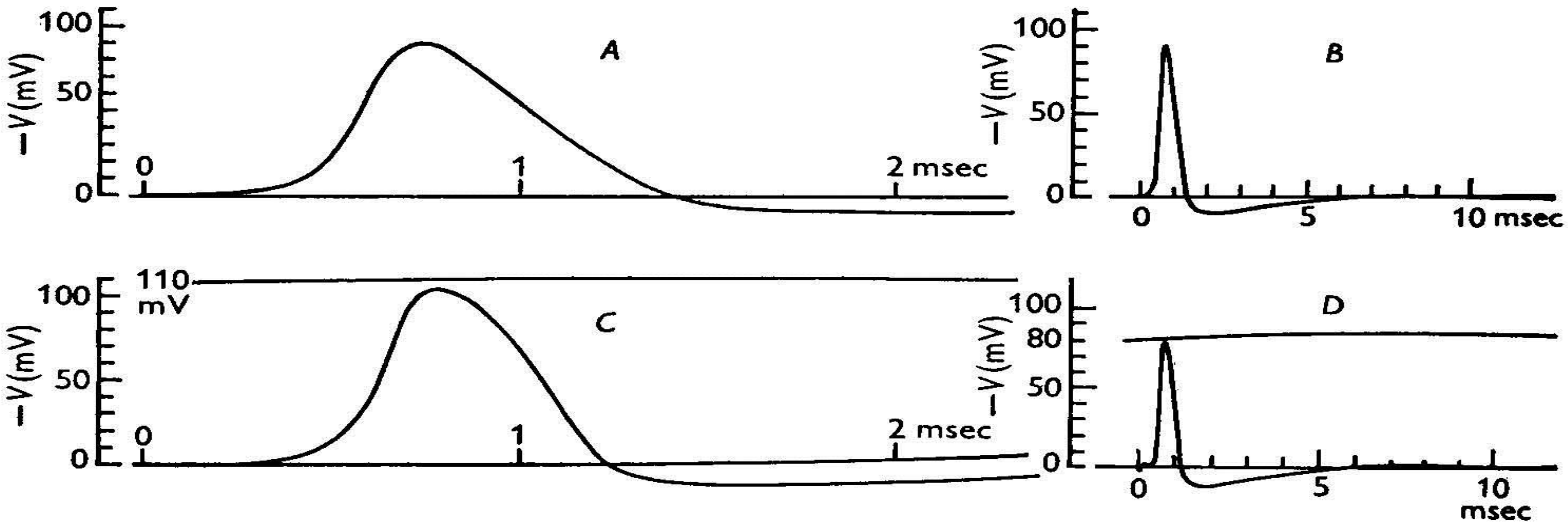
$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n$$

$$\alpha_m = 0.1(V_m + 35.0)/(1. - e^{(-(V_m+35.0)/10.0)})$$
$$\beta_m = 4.0\, e^{(-(V_m+60.0)/18.0)}$$

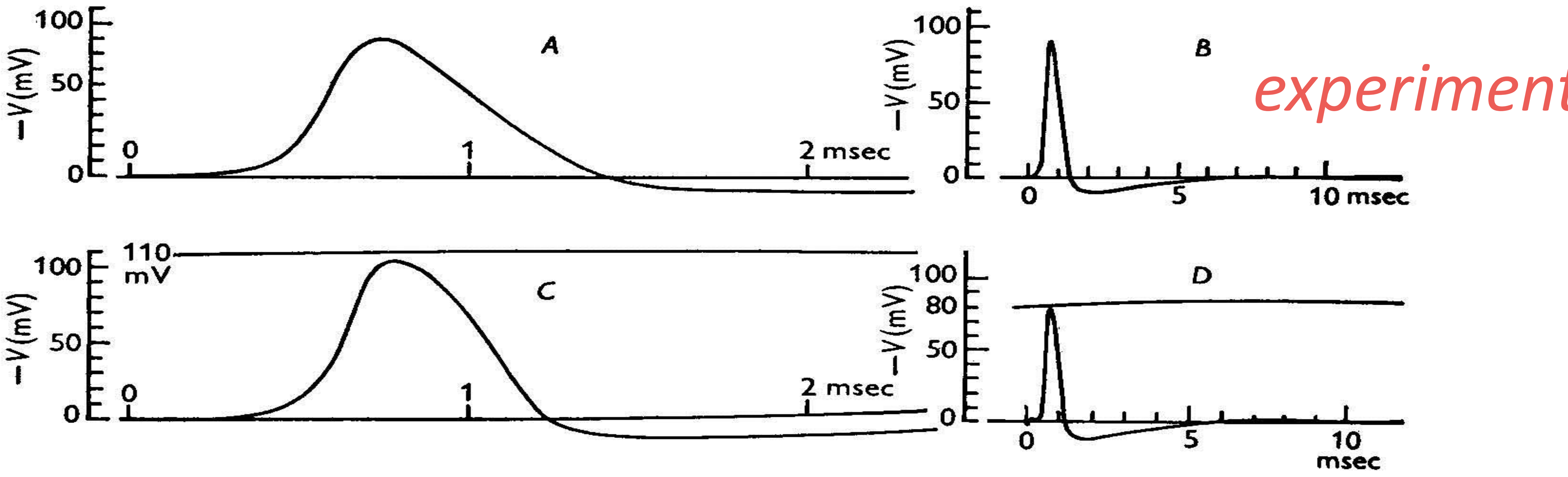
$$\alpha_h = 0.07\, e^{(-(V_m+60.0)/20.0)}$$
$$\beta_h = 1./(1 + e^{(-(V_m+30.0)/10.0)})$$

$$\alpha_n = 0.01(V_m + 50.0)/(1 - e^{(-(V_m+50.0)/10.0)})$$
$$\beta_n = 0.125\, e^{(-(V_m+60.0)/80.0)}$$

…works well:

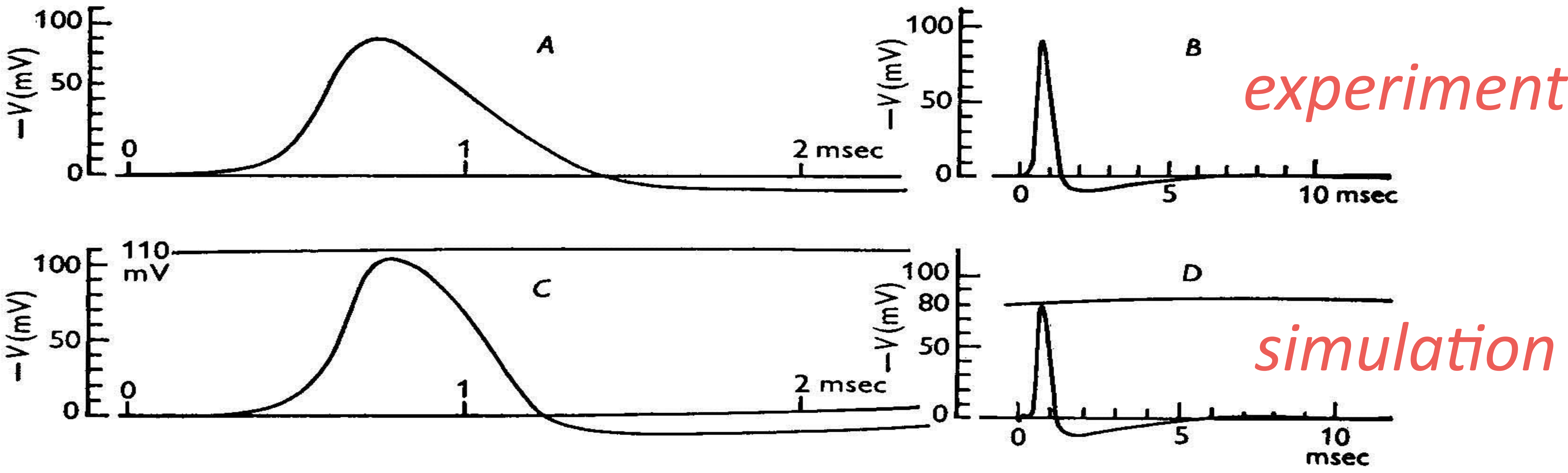

100
50
0
$-V$(mV)
A
0
1
2 msec
100
50
0
$-V$(mV)
B
0
5
10 msec
110 mV
100
50
0
$-V$(mV)
C
0
1
2 msec
100
80
50
0
$-V$(mV)
D
0
5
10 msec

…works well:



−V(mV)
100
50
0
0
1
2 msec
A
B
10 msec
5
experiment
C
110
mV
D
80
10
msec

…works well:

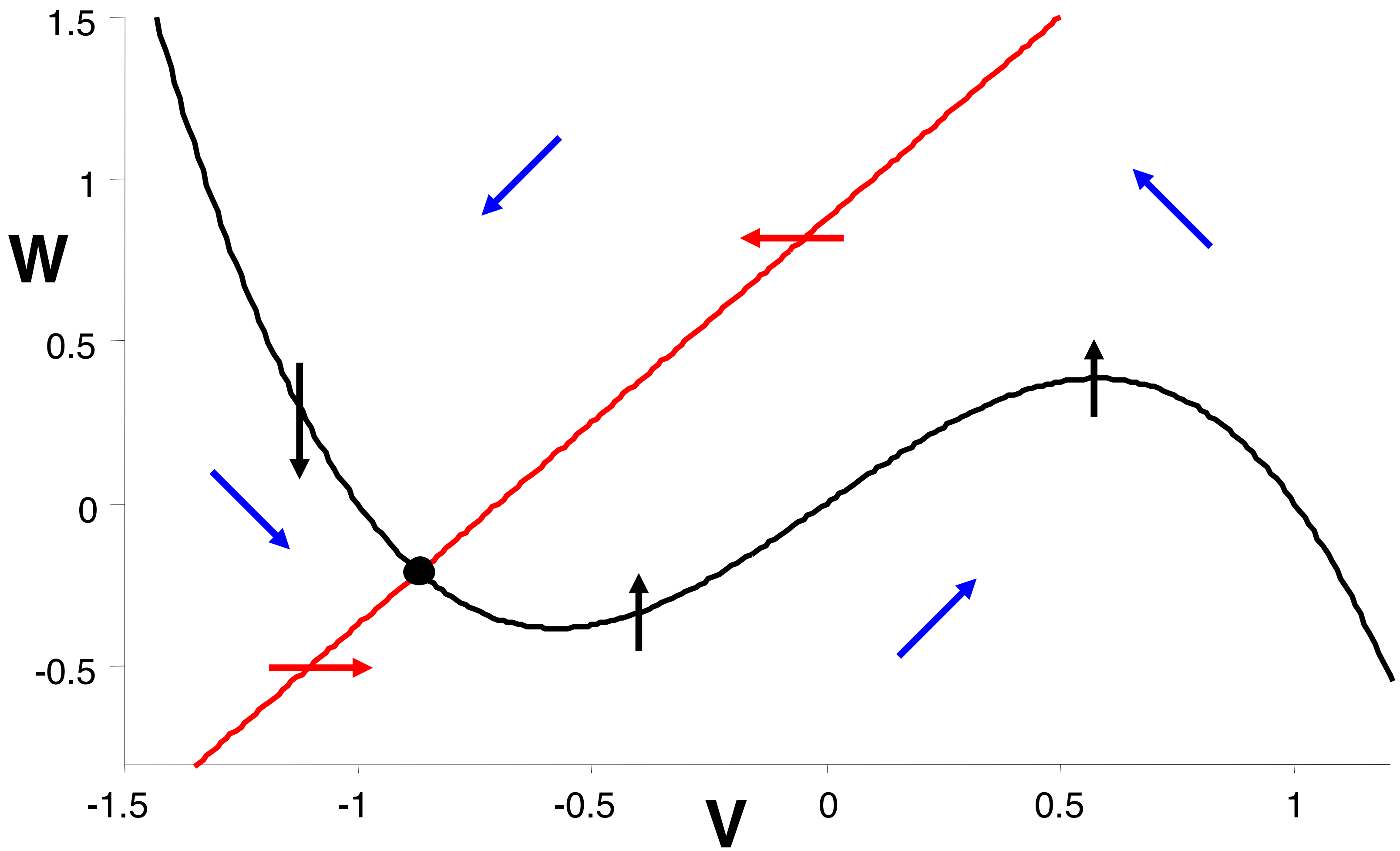

100
50
0
−V(mV)
0
1
2 msec
A
B
10 msec
5
experiment
110
mV
C
D
80
10
msec
simulation

The Fitzhugh-Nagumo model:

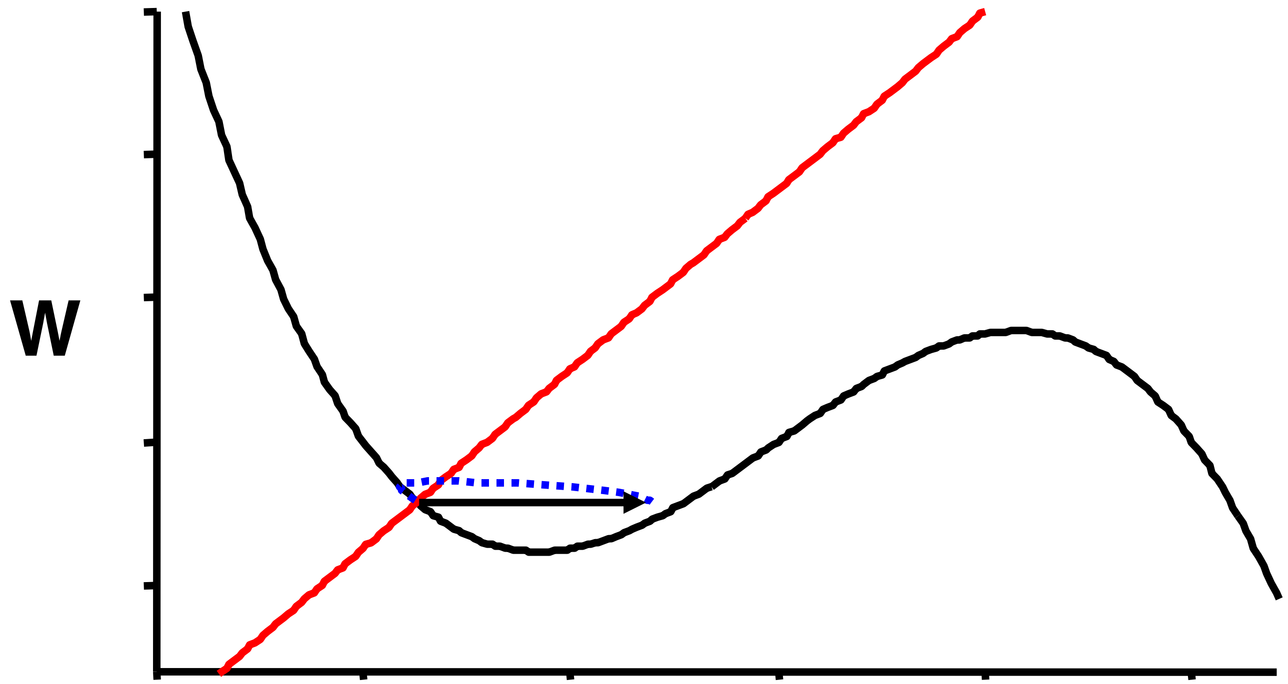
dV/dt = V - V^3 - W - I

dW/dt = 0.08*(V + 0.7-0.8W)

The Fitzhugh-Nagumo model is a useful simplification:

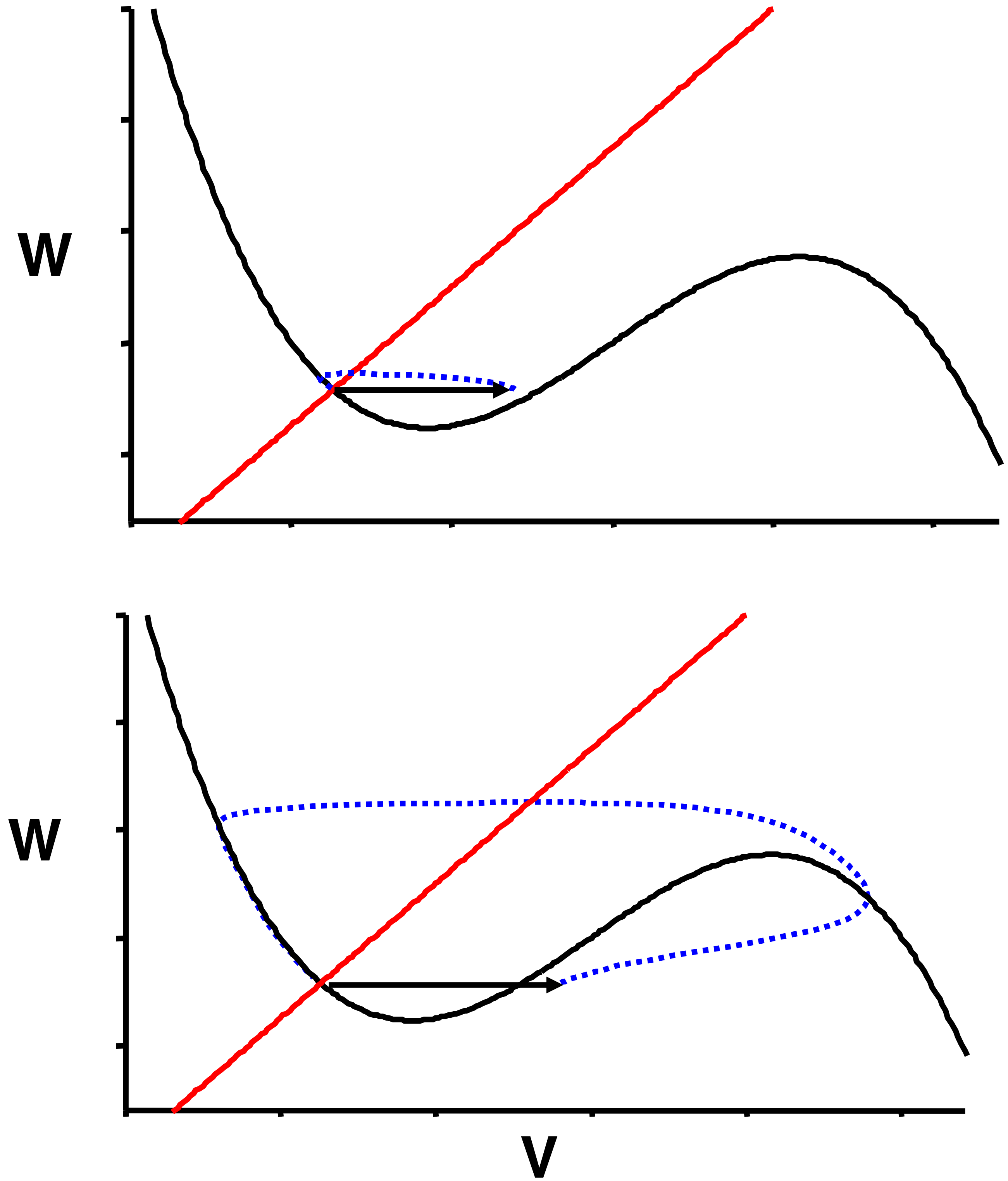


The Fitzhugh-Nagumo model is a useful simplification:



return to baseline

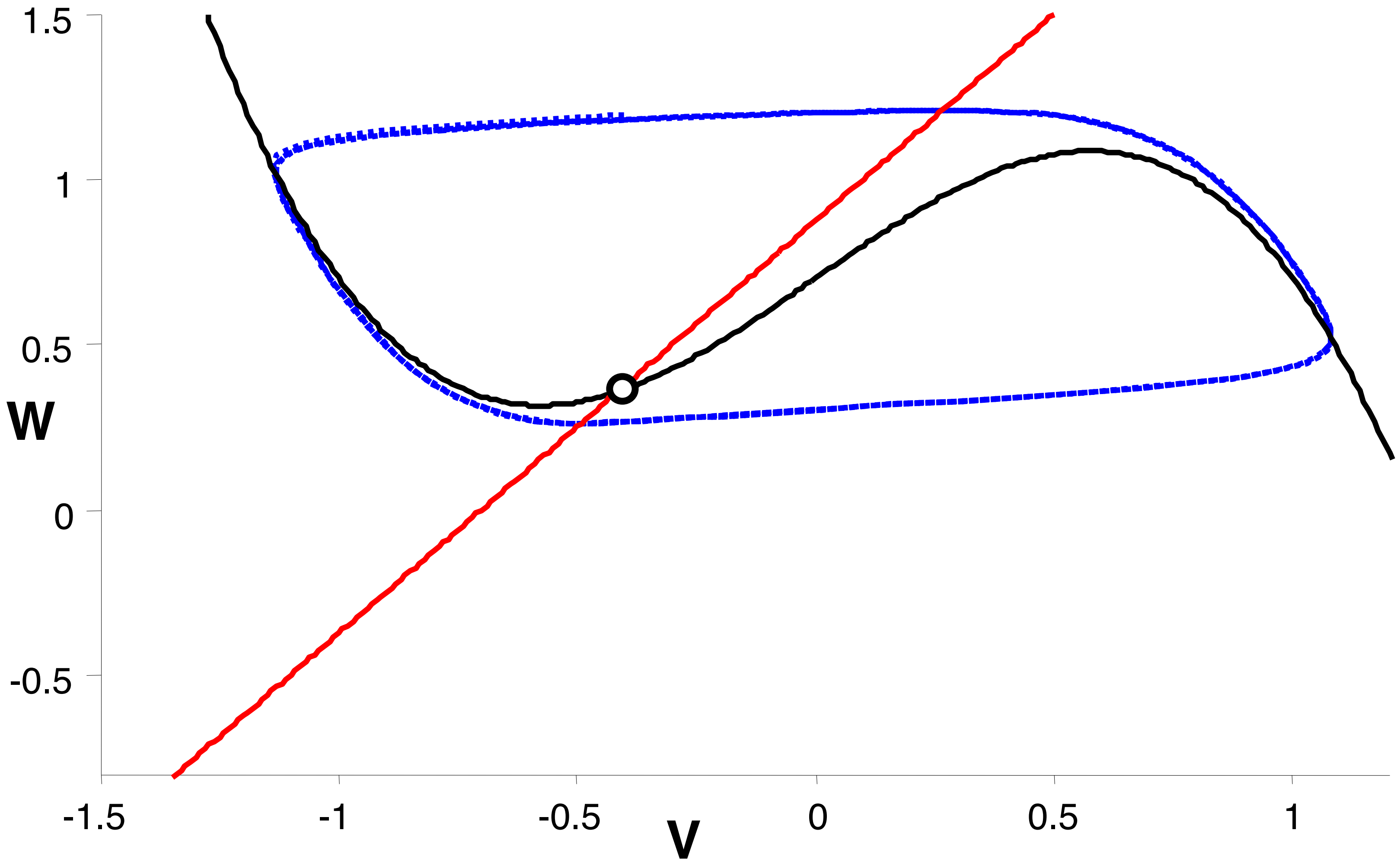
The Fitzhugh-Nagumo model is a useful simplification:



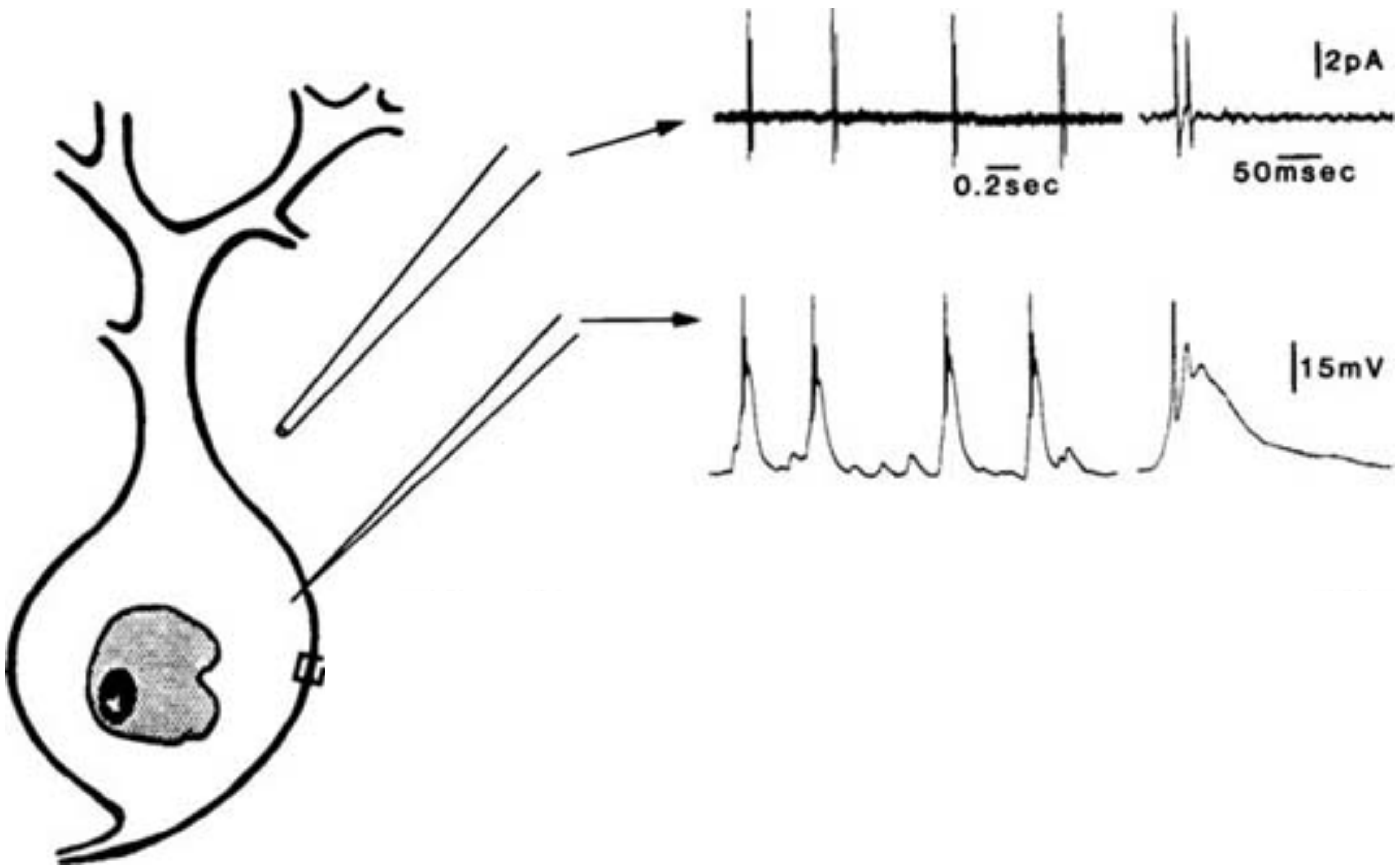
return to baseline

spike!

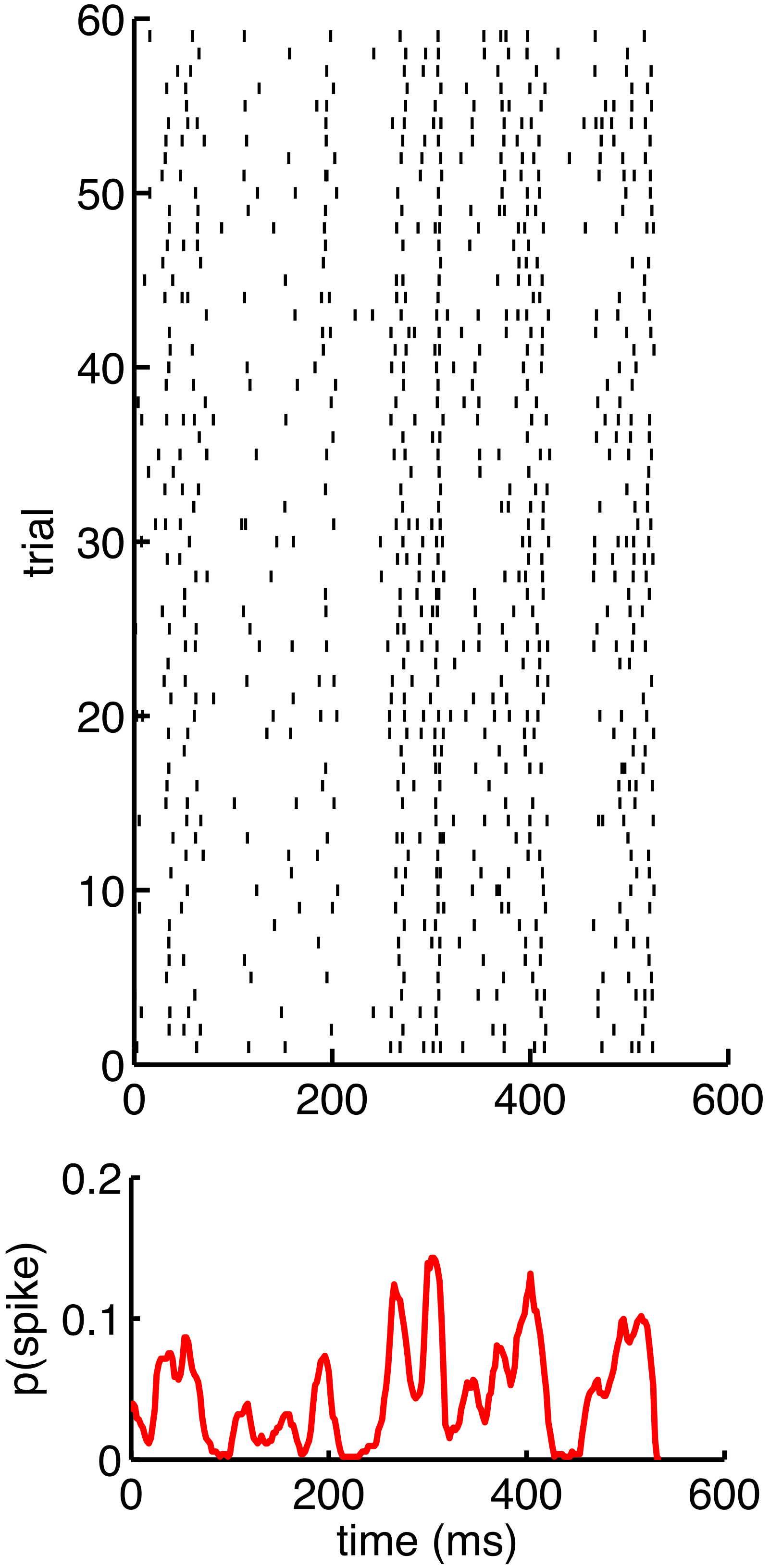
Even explains sustained firing!



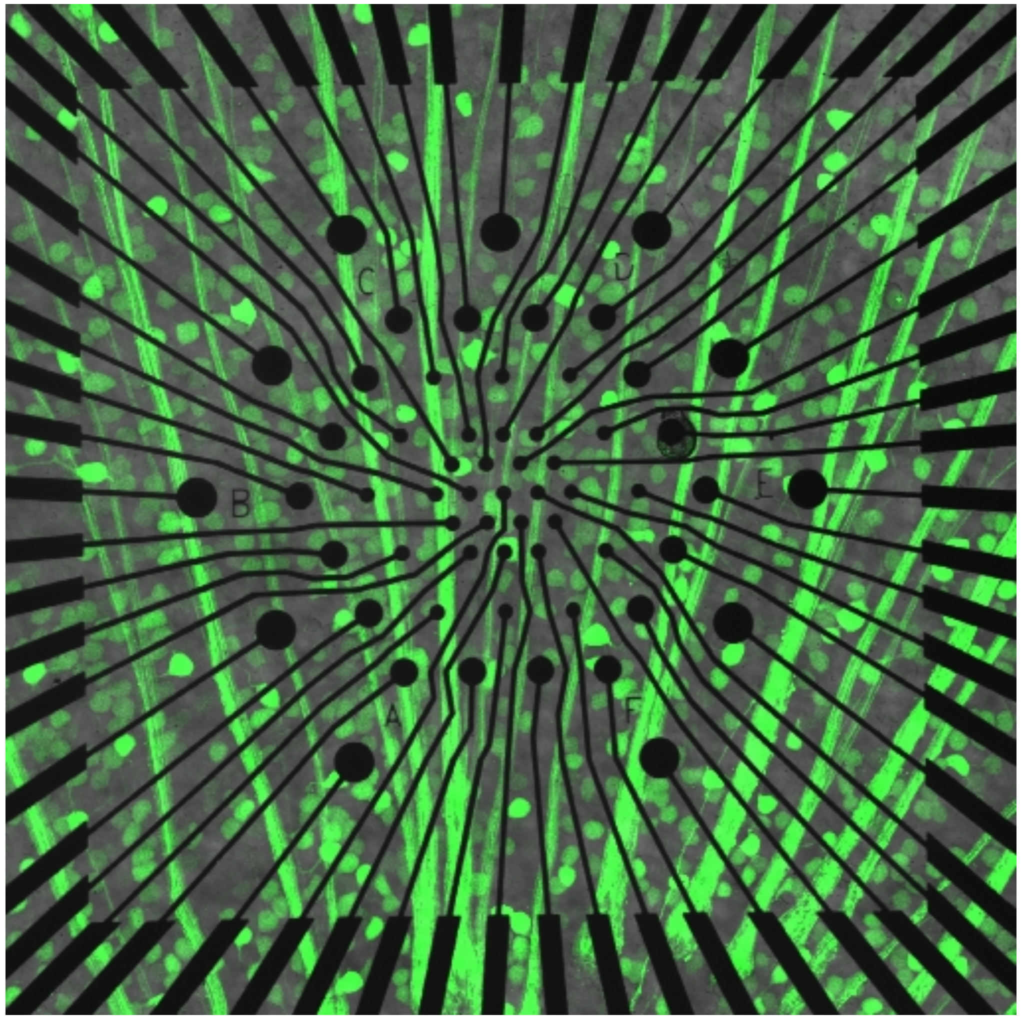
Extracellular recording:

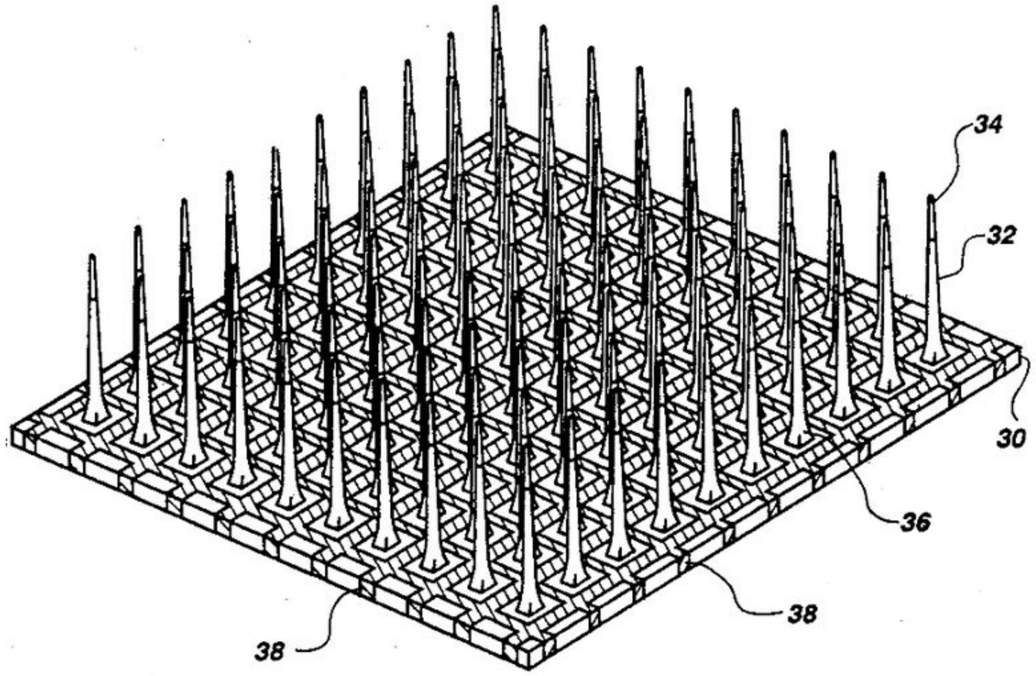


Neuronal response is ‘noisy’:



Population recordings:

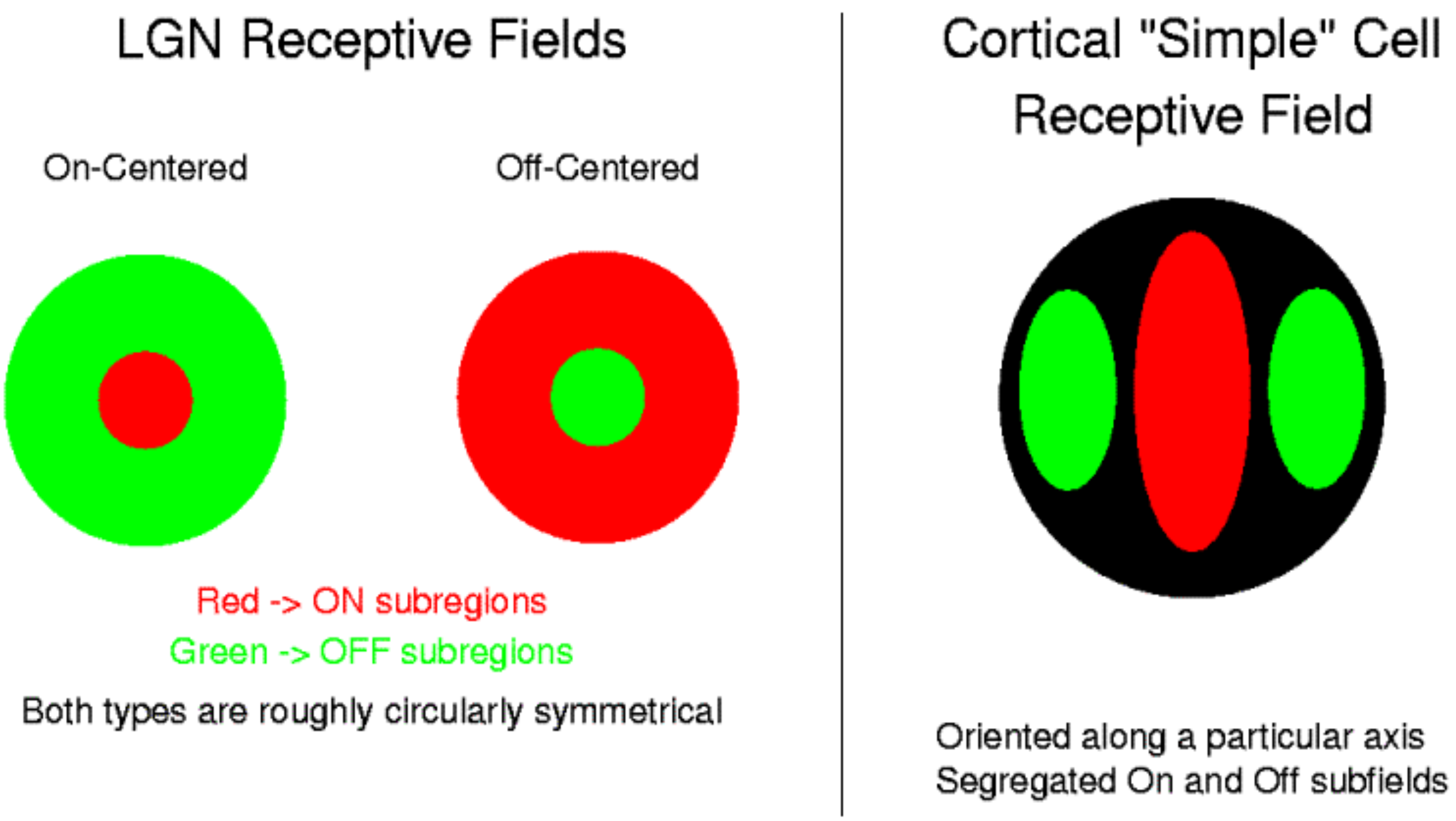




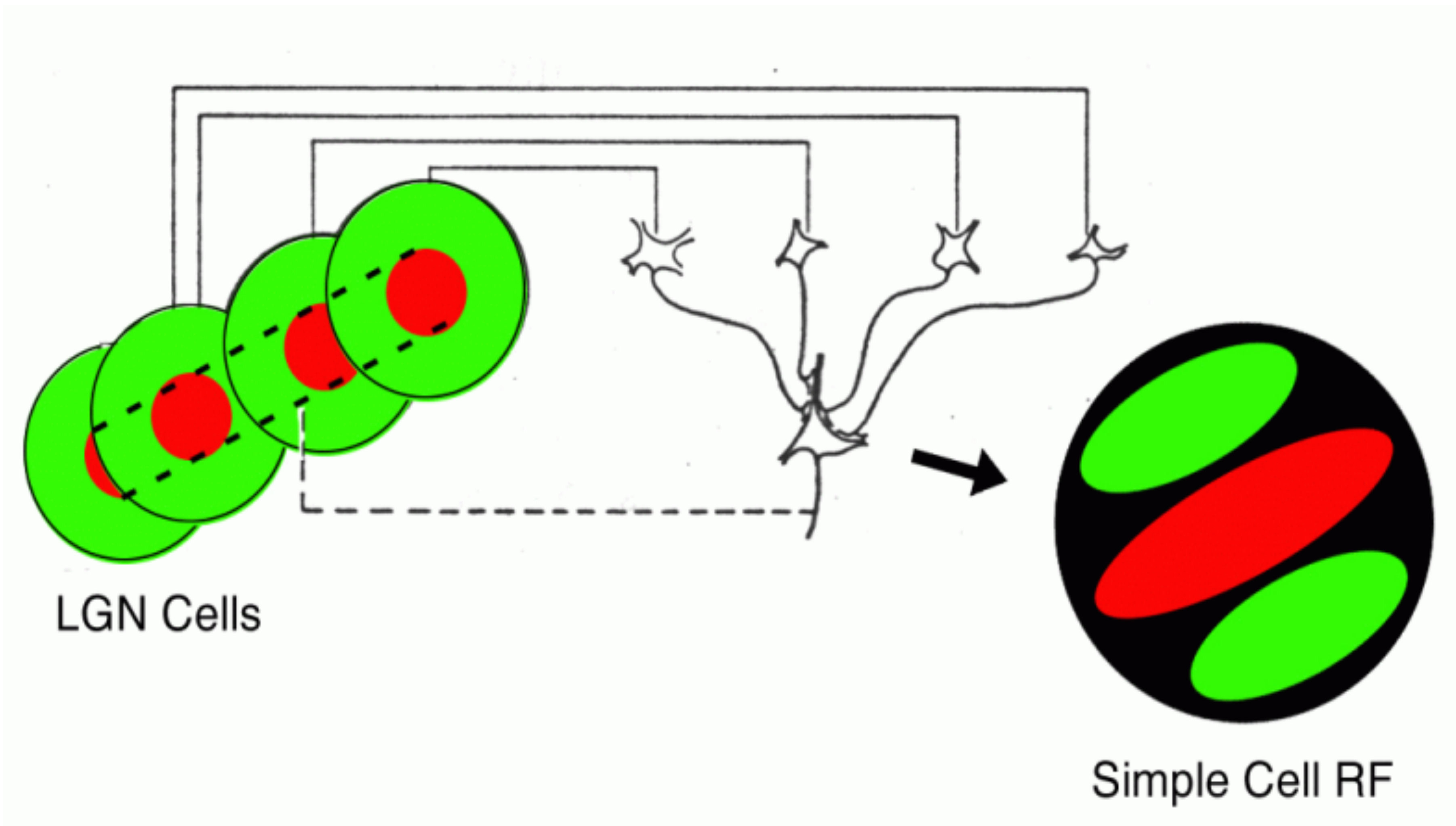
The basic learning problem: what about the stimulus do neurons respond to?

The basic learning problem: what about the stimulus do neurons respond to?

What about the stimulus do neurons respond to?



What about the stimulus do neurons respond to?



Tools we use:

- statistics
- learning / inference
- optimization
- dynamical systems
- information theory

A brief introduction to information theory:

When is information theory useful? When you...

- want to go beyond
linear correlation

- have enough data to
sample P(x,y)

- are not sure what your
'code' is

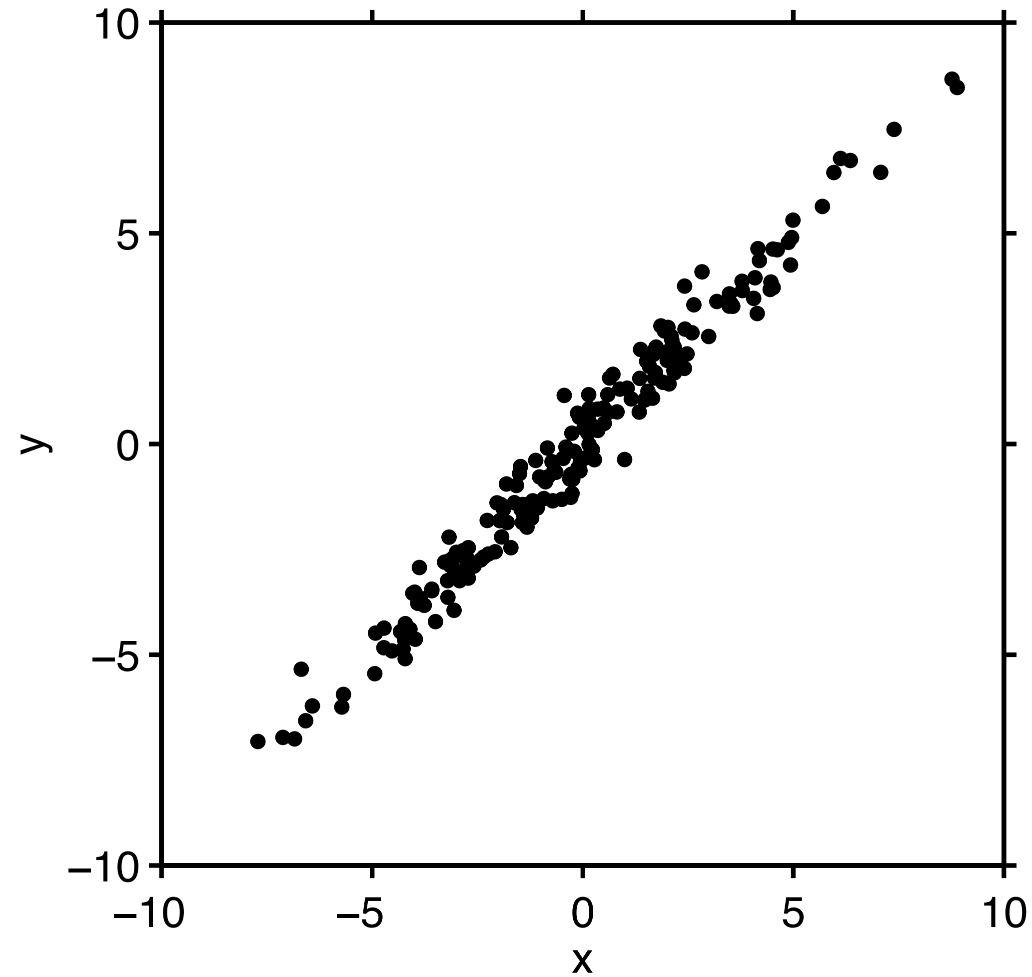
A brief introduction to information theory:

When is information theory useful? When you...

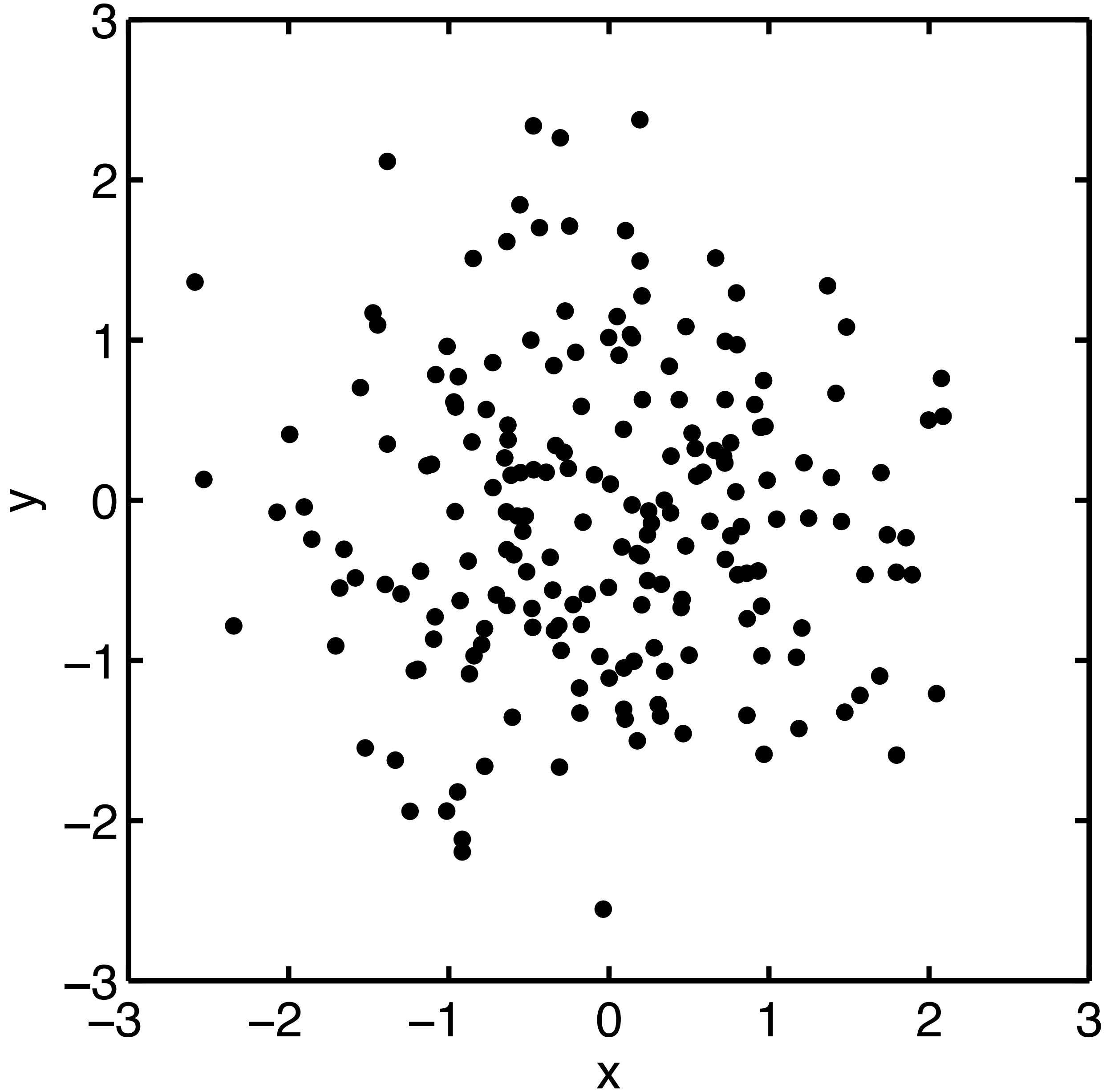
• want to go beyond linear correlation

• have enough data to sample P(x,y)

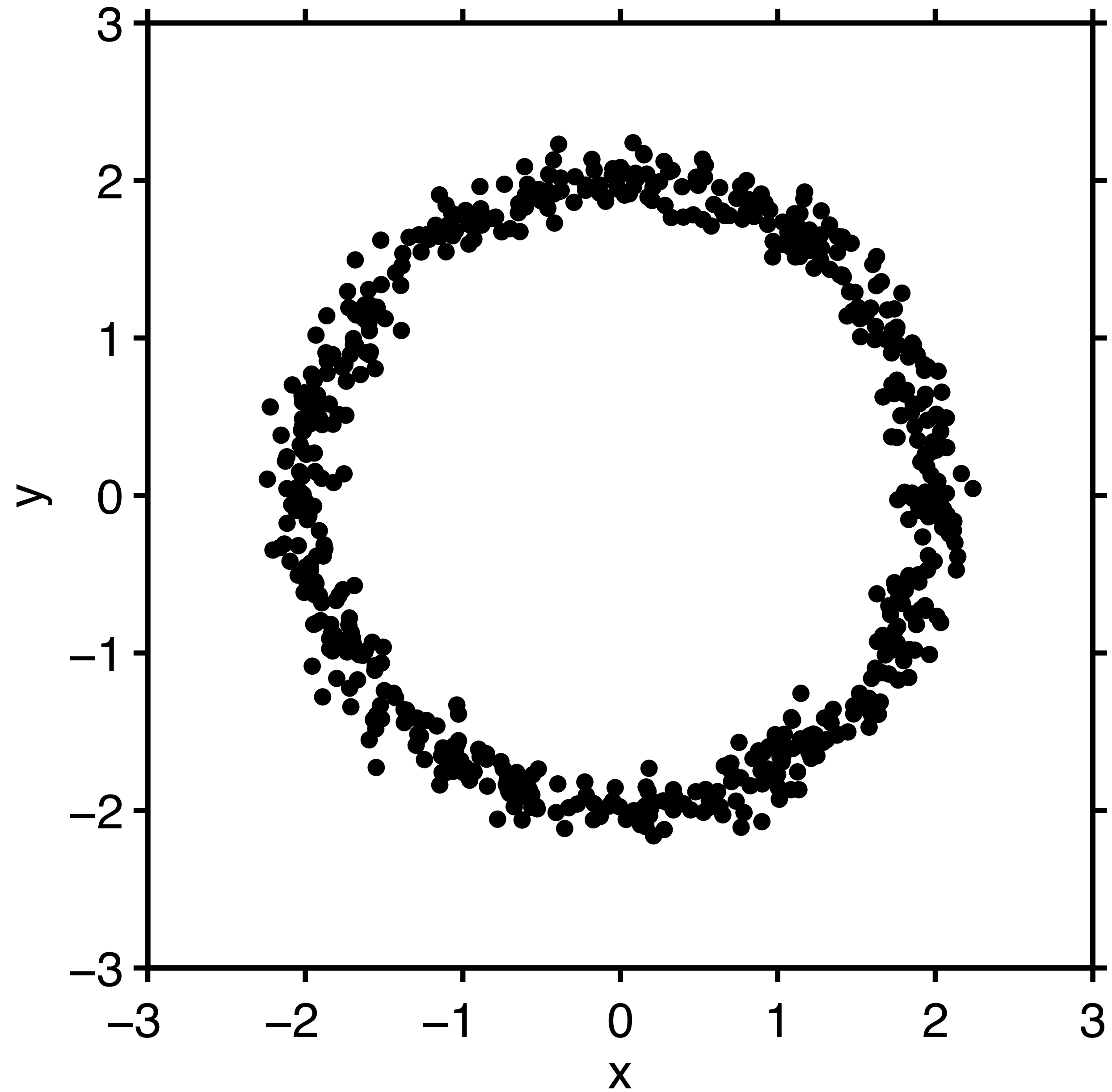
• are not sure what your 'code' is



Zero correlation, no information:



Zero correlation, but obvious information:



Entropy as a measure of uncertainty:

$$\text{uncertainty} = \log(n)$$

Entropy as a measure of uncertainty:

$$
\begin{aligned}
\text{uncertainty} &= \log\left(n\right) \\
&= \log\left(1/p\right) \\
&= -\log\left(p\right)
\end{aligned}
$$

Entropy as a measure of uncertainty:

$$\begin{aligned}\text{uncertainty} &= \log(n) \\ &= \log(1/p) \\ &= -\log(p)\end{aligned}$$

$$u_i = -\log(p_i)$$

$$\langle u_i \rangle = -\sum_i p_i \log(p_i)$$

Entropy as a measure of uncertainty:

$$\begin{aligned}\text{uncertainty} &= \log(n) \\ &= \log(1/p) \\ &= -\log(p)\end{aligned}$$

$$u_i = -\log(p_i)$$

$$\langle u_i \rangle = -\sum_i p_i \log(p_i)$$

$$S(X) = -\sum_x p(x) \log_2(p(x))$$

Information == reduction in uncertainty

Mutual information:

$$
\begin{aligned}
I(A;B) &= S(A) - S(A|B) \\
&= S(B) - S(B|A) \\
&= \sum_{a,b} P(a,b) \log_2 \left(\frac{P(a,b)}{P(a)P(b)} \right) \\
&= \sum_{a,b} P(a)P(b|a) \log_2 \left(\frac{P(b|a)}{P(b)} \right)
\end{aligned}
$$

Information == reduction in uncertainty

Mutual information:

$$
\begin{aligned}
I(A;B) &= S(A) - S(A|B) \\
&= S(B) - S(B|A) \\
&= \sum_{a,b} P(a,b)\log_2\left(\frac{P(a,b)}{P(a)P(b)}\right) \\
&= \sum_{a,b} P(a)P(b|a)\log_2\left(\frac{P(b|a)}{P(b)}\right)
\end{aligned}
$$

chalkboard interlude

useful formulae

Product rule:

$$P(a,b) = P(a|b)P(b)$$

useful formulae

Product rule:

$$P(a,b) = P(a|b)P(b)$$

Sum rule:

$$P(a) = \sum_b P(a,b)$$
$$= \sum_b P(a|b)P(b)$$

useful formulae

Product rule:

$$P(a,b) = P(a|b)P(b)$$

Sum rule:

$$P(a) = \sum_b P(a,b)$$
$$= \sum_b P(a|b)P(b)$$

Bayes' rule:

$$P(a|b) = \frac{P(b|a)P(a)}{P(b)}$$
$$= \frac{P(b|a)P(a)}{\sum_{a'} P(b|a')P(a')}$$

more useful formulae

more useful formulae

Additivity:

$$S(A,B) = S(A) + S(B) \iff P(a,b) = P(a)P(b)$$

more useful formulae

Additivity:

$$S(A,B) = S(A) + S(B) \iff P(a,b) = P(a)P(b)$$

Chain rule:

$$S(A,B) = S(A) + S(B|A) = S(B) + S(A|B)$$

more useful formulae

Additivity:

$$S(A,B) = S(A) + S(B) \iff P(a,b) = P(a)P(b)$$

Chain rule:

$$S(A,B) = S(A) + S(B|A) = S(B) + S(A|B)$$

Kullback-Liebler divergence (D_{KL}):

$$D_{\mathrm{KL}}(P,Q) = \sum_a P(a) \log_2 \frac{P(a)}{Q(a)}$$

more useful formulae

Additivity:

$$S(A,B) = S(A) + S(B) \iff P(a,b) = P(a)P(b)$$

Chain rule:

$$S(A,B) = S(A) + S(B|A) = S(B) + S(A|B)$$

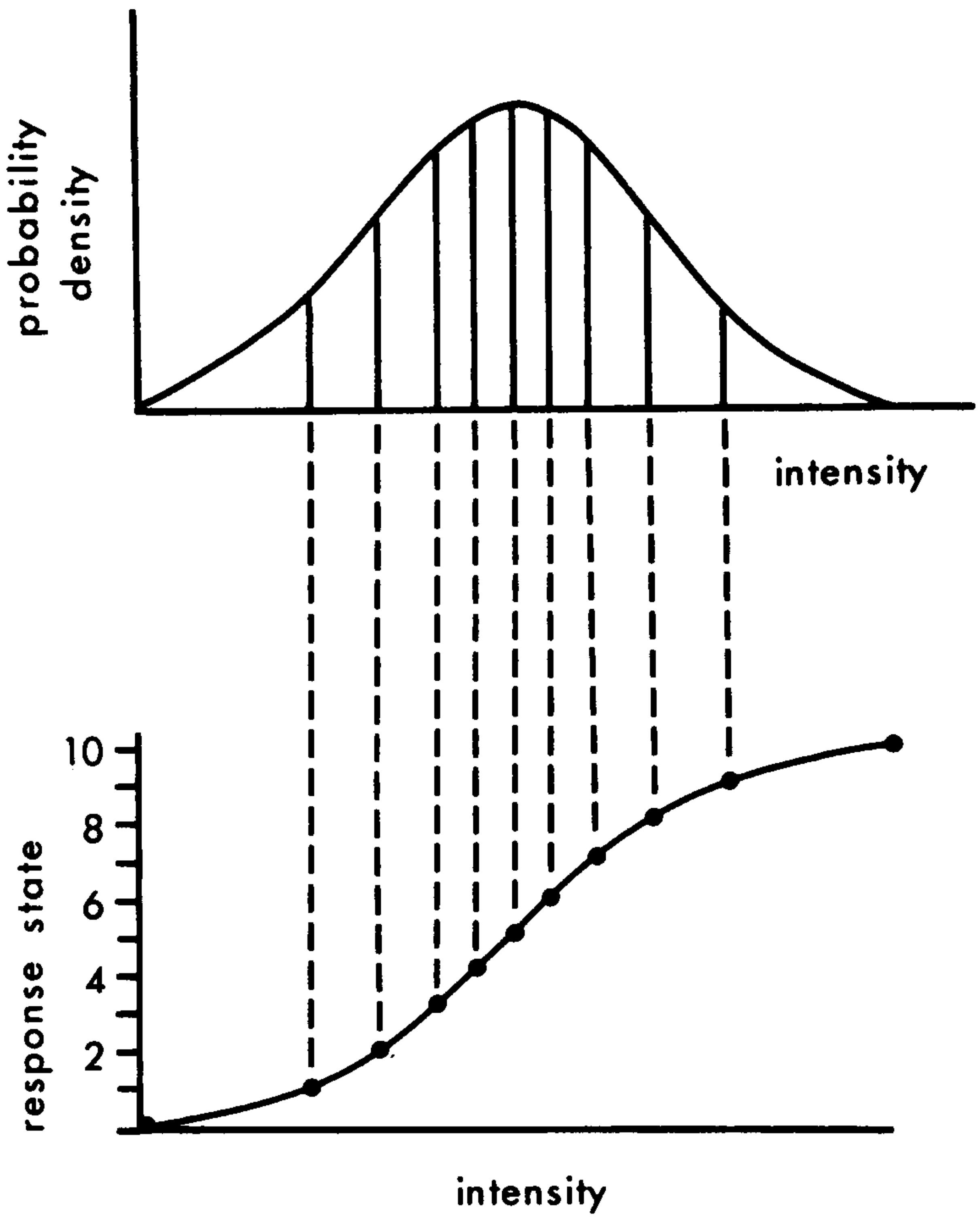
Kullback-Liebler divergence (D_{KL}):

$$D_{\mathrm{KL}}(P,Q) = \sum_a P(a)\log_2 \frac{P(a)}{Q(a)}$$

Mutual information ≥ 0:

$$I(A;B) = S(A) - S(A|B)$$

Efficient coding in single neurons



The efficient coding hypothesis, brief history:

- Claude Shannon (1948) *A Mathematical Theory of Communication*
- Fred Attneave (1954) *Some informational aspects of visual perception*
- Horace Barlow (1961) *Possible principles underlying the transformation of sensory messages*

Are sensory systems optimized for information transmission?

Information theory example, the weighing problem:

Information theory example, the weighing problem:

chalkboard interlude