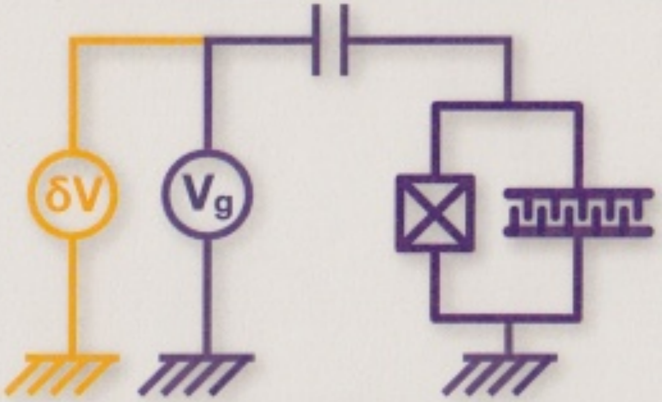
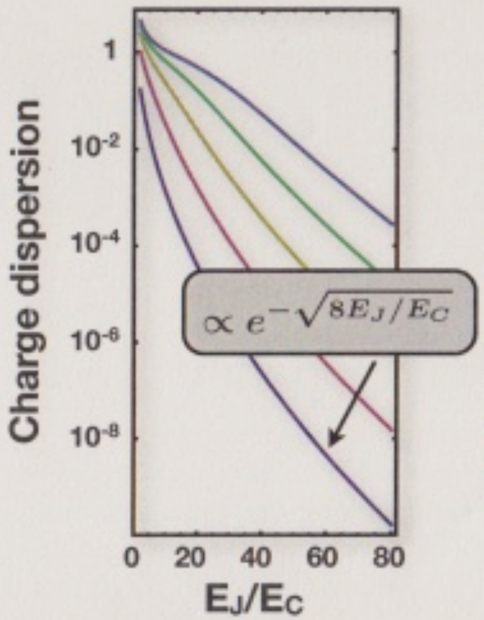
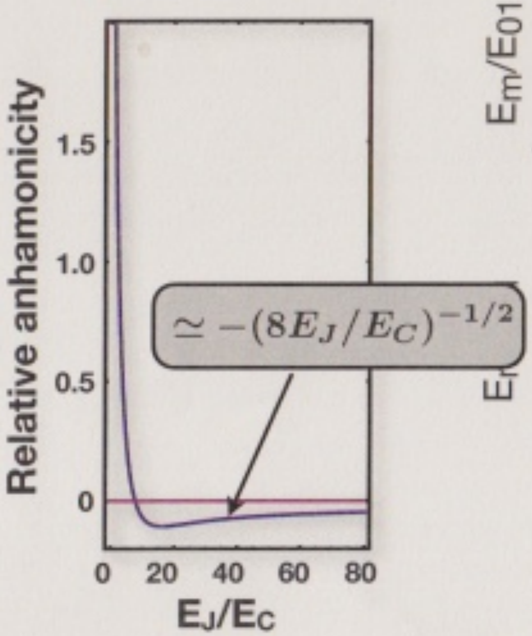
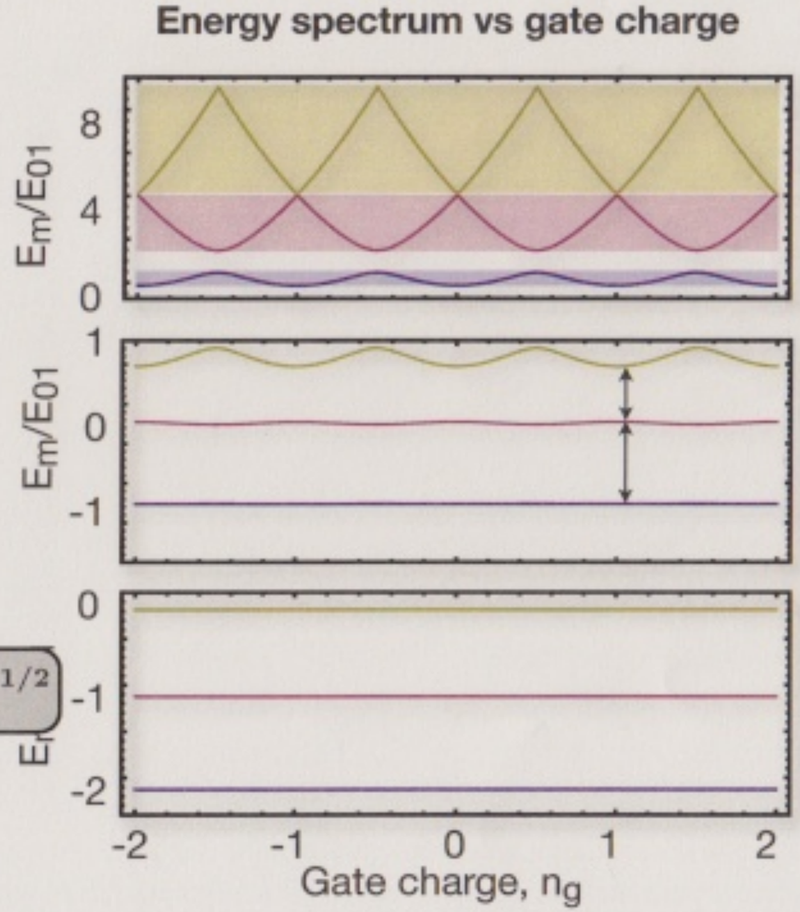
E.

Superconducting qubit: transmon regime









Transmon: J. Koch et al. Phys. Rev. A **76**, 042319 (2007)
Sweet spot: D. Vion et al. Science **296**, 886 (2002)

F.

Transmon regime: anharmonic oscillator

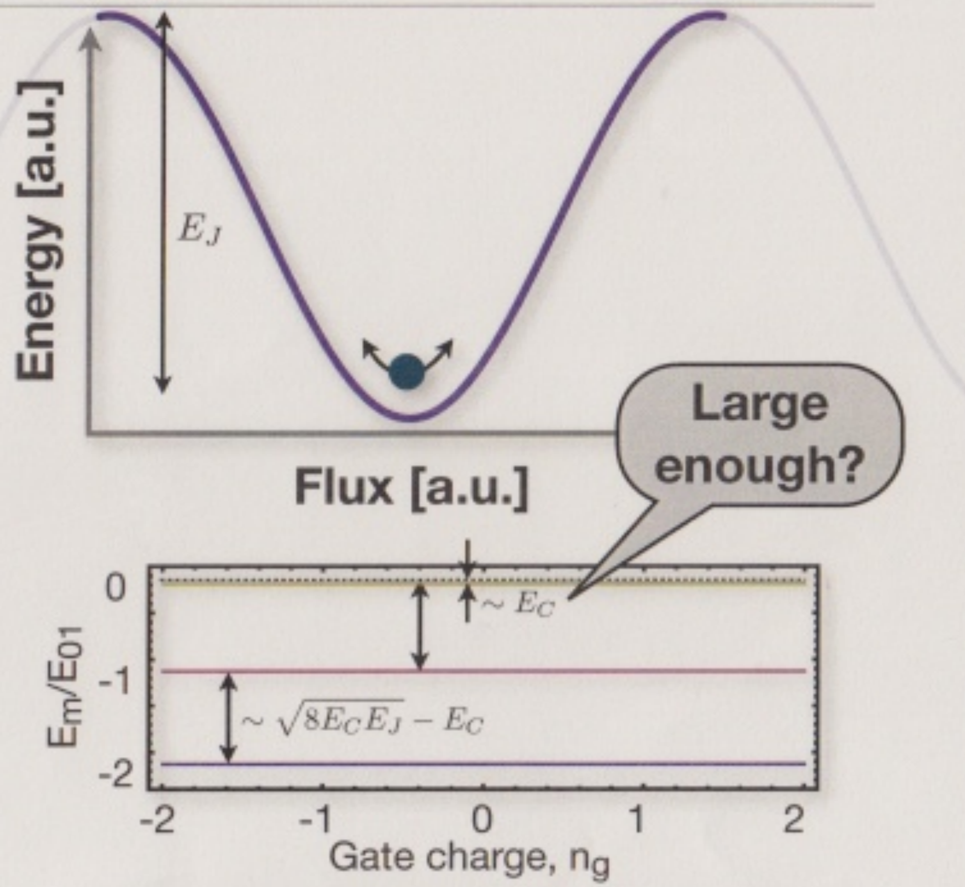
$$H = 4E_C\hat{n}^2 - E_J\cos\hat{\phi}$$

$$\approx 4E_C\hat{n}^2 + \frac{1}{2}E_J\hat{\phi}^2 - \frac{1}{24}E_J\hat{\phi}^4$$

$$\equiv \frac{\hat{n}^2}{2C'} + \frac{\hat{\phi}^2}{2L'} - \frac{1}{24}E_J\hat{\phi}^4$$

$$\hat{n} = i\sqrt{\hbar}\left(\frac{E_J}{32E_C}\right)^{1/4}(\hat{b}^\dagger - \hat{b})$$

$$\hat{\phi} = \sqrt{\hbar}\left(\frac{2E_C}{E_J}\right)^{1/4}(\hat{b}^\dagger + \hat{b})$$



$$H \approx \sqrt{8E_CE_J}\hat{b}^\dagger\hat{b} - \frac{E_C}{2}(\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b} + 2\hat{b}^\dagger\hat{b})$$

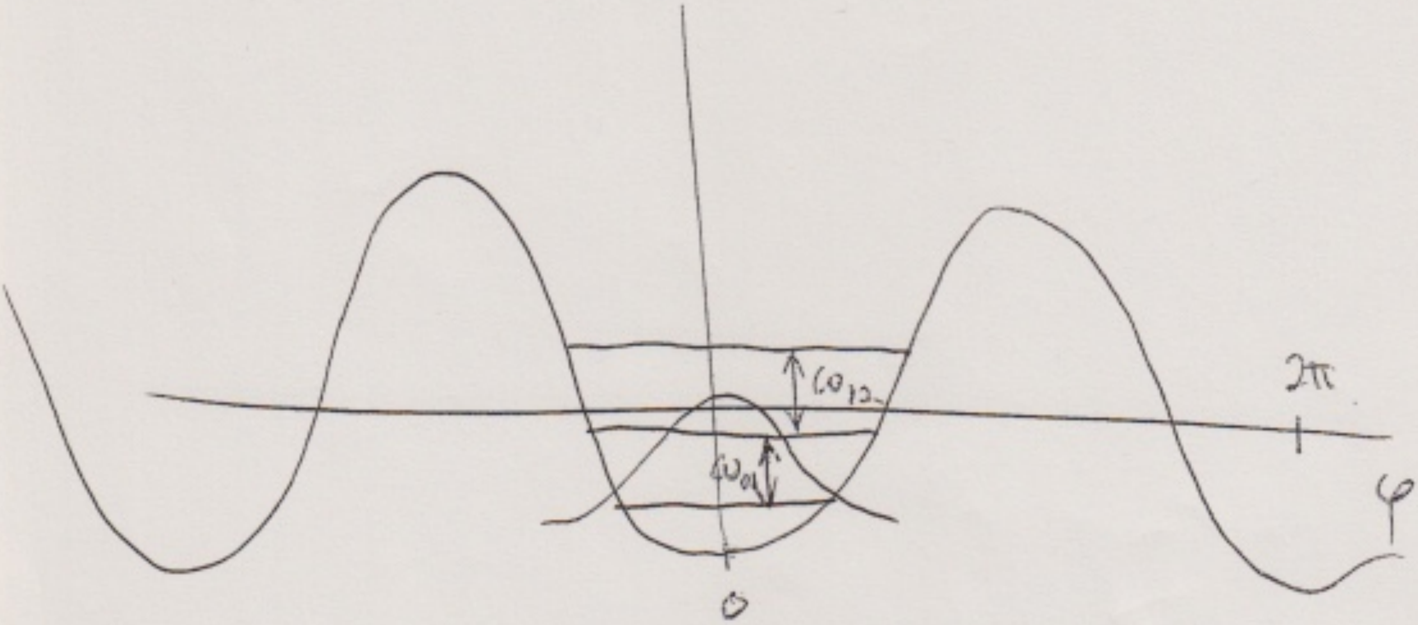
$$= (\sqrt{8E_CE_J} - E_C)\hat{b}^\dagger\hat{b} - \frac{E_C}{2}\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b}$$

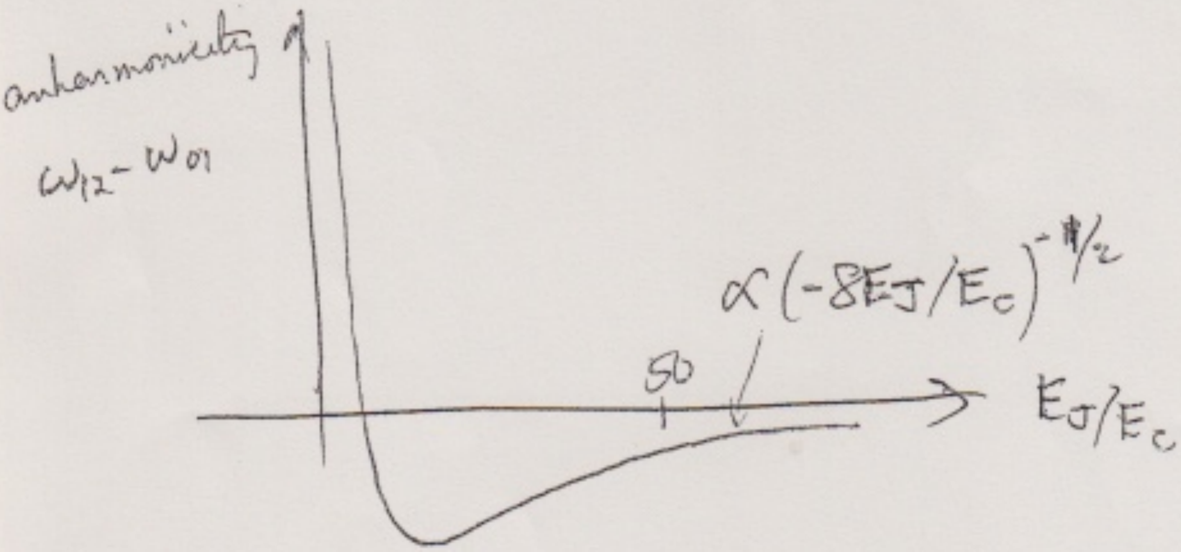
Typical values:

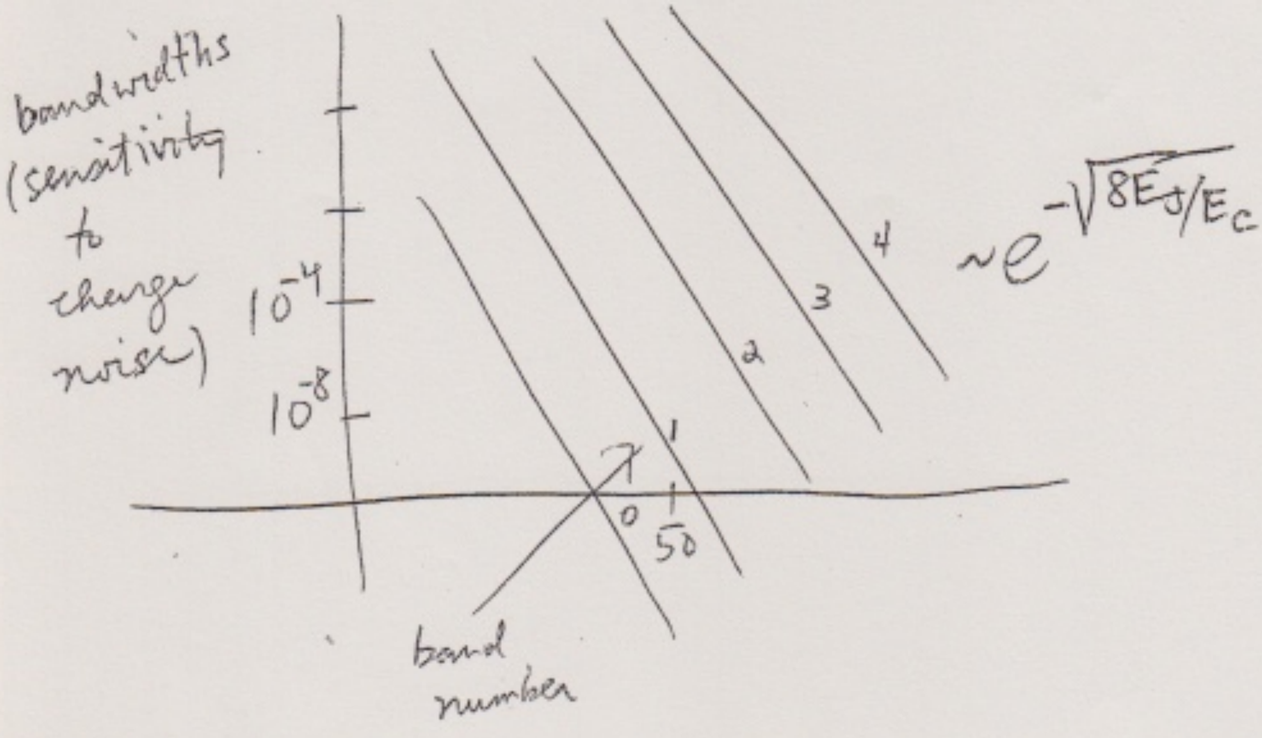
$E_J \sim 20$ GHz

$E_C \sim 400$ MHz

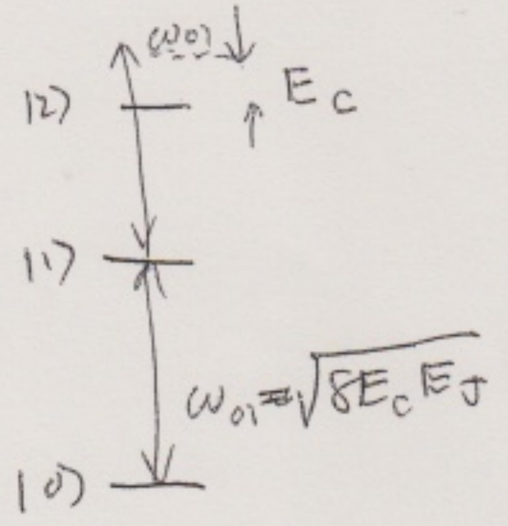
G


ω_{12}
ω_{01}
2π
φ
0


anharmonicity
$\omega_{12}-\omega_{01}$
$\propto(-8E_J/E_c)^{-1/2}$
50
E_J/E_c


bandwidths (sensitivity to charge noise)
10^{-4}
10^{-8}
0
1
2
3
4
50
band number
$\sim e^{-\sqrt{8E_J/E_c}}$

H/o i.e., charge sensitivity exponentially suppressed, anharmonicity "reasonable".

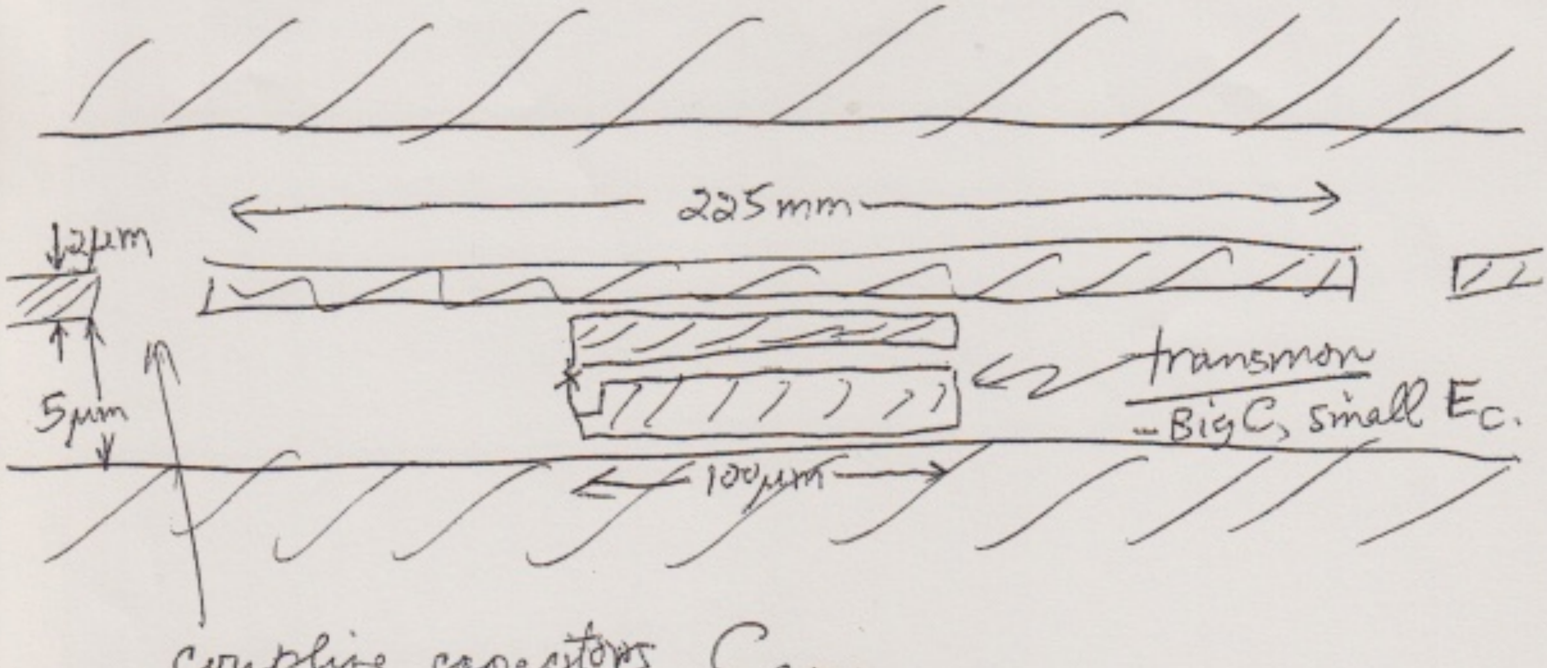


- Very accurate 1-qubit gates,

$$F = 99.993\%$$

- COUPLING TRANSMONS.

coplanar waveguide:

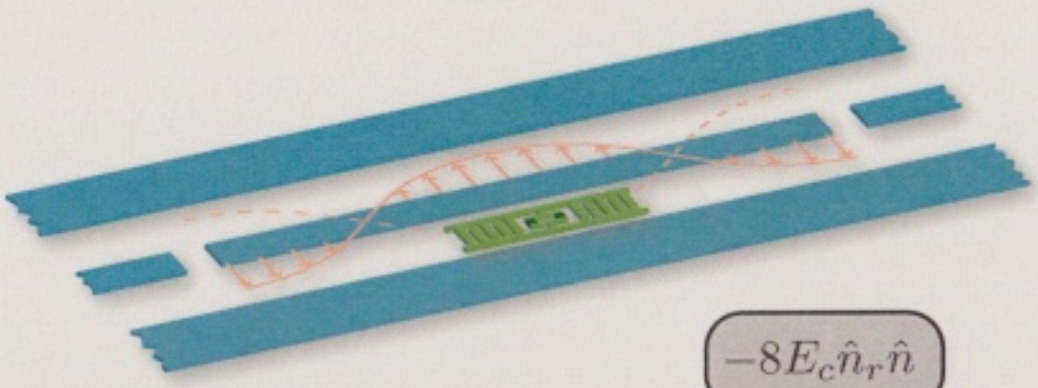


coupling capacitors C_{coup}.

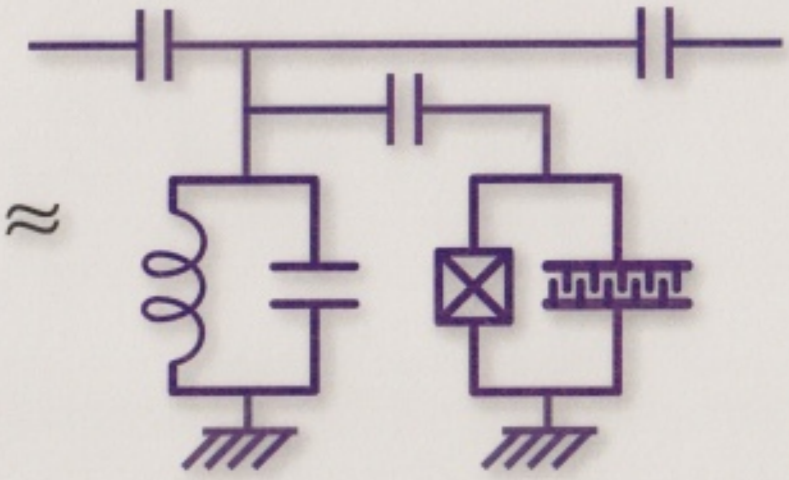
→ slightly transparent mirror for microwaves

CPW is cavity for microwave photons, $\frac{\omega_r}{2\pi} = 2$-$20$ GHz.

Bringing it all together: Jaynes-Cummings model



$$\approx$$



$$-8E_c\hat{n}_r\hat{n}$$

$$\hat{H} = \left(\frac{\hat{Q}_r^2}{2C_r} + \frac{\hat{\Phi}_r^2}{2L_r}\right) + \left(4E_C[\hat{n} - \hat{n}_r]^2 - E_J \cos\hat{\phi}\right)$$

$$= \omega_r \hat{a}^\dagger \hat{a} + \left(\omega_{01}\hat{b}^\dagger\hat{b} - \frac{E_c}{2}\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b}\right) - g(\hat{a}^\dagger - \hat{a})(\hat{b}^\dagger - \hat{b})$$

TLS approx.

$$= \omega_r \hat{a}^\dagger \hat{a} + \frac{\omega_{01}}{2}\hat{\sigma}_z - g(\hat{a}^\dagger - \hat{a})(\hat{\sigma}_+ - \hat{\sigma}_-)$$

RWA

$$\approx \omega_r \hat{a}^\dagger \hat{a} + \frac{\omega_{01}}{2}\hat{\sigma}_z + g(\hat{a}^\dagger\hat{\sigma}_- + \hat{a}\hat{\sigma}_+)$$

Energy exchange between 'light' and 'matter' at rate $2g$

$$g_{\text{circuit}}/2\pi \sim [0-1]\ \text{GHz}$$
$$g_{\text{cavity}}/2\pi \sim 50\ \text{kHz}$$

Blais, Huang, Wallraff, Girvin & Schoelkopf, Phys. Rev. A **69**, 062320 (2004) calculate Jaynes Cummings coupling g:

$$8E_c \hat{n}\hat{n}_r = \frac{8E_c}{(2e)^2}\hat{Q}\hat{Q}_r$$

$$\frac{\hat{Q}}{2e} = i\left(\frac{E_J}{32E_c}\right)^{1/4}(b^\dagger - b)$$

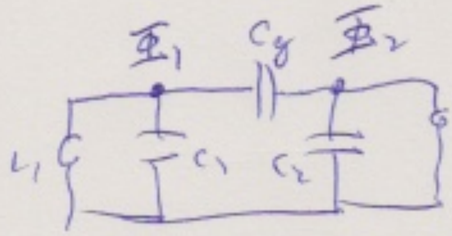
$$\frac{\hat{Q}_r}{2e} = i\left(\frac{E_{Lr}}{32E_{Cr}}\right)^{1/4}(a^\dagger - a)$$

$$\boxed{g = E_{coup}\left(\frac{E_J}{32E_c}\right)^{1/4}\left(\frac{E_{Lr}}{32E_{Cr}}\right)^{1/4}}$$

coupling $= -$ $E_{C_{tot}}$

$$\frac{e^2}{C} \qquad \propto C_g$$

start over:



$$C_{node} = \begin{pmatrix} C_1 + C_g & -C_g \\ -C_g & C_2 + C_g \end{pmatrix}$$

in H, coupling term is $(C_g \ll C_1, C_2)$:

$$E_{coup}\hat{n}\hat{n}_r = \frac{\hat{Q}_1\hat{Q}_2 C_g}{C_1 C_2} \qquad E_{coup} = \frac{(2e)^2 C_g}{C_r C}$$

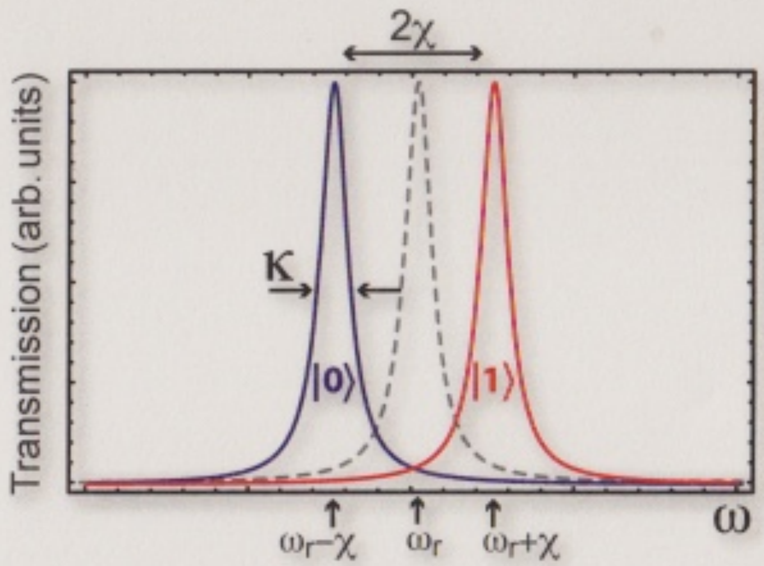
Dispersive regime: Qubit readout $|\Delta| = |\omega_{01} - \omega_r| \gg g$

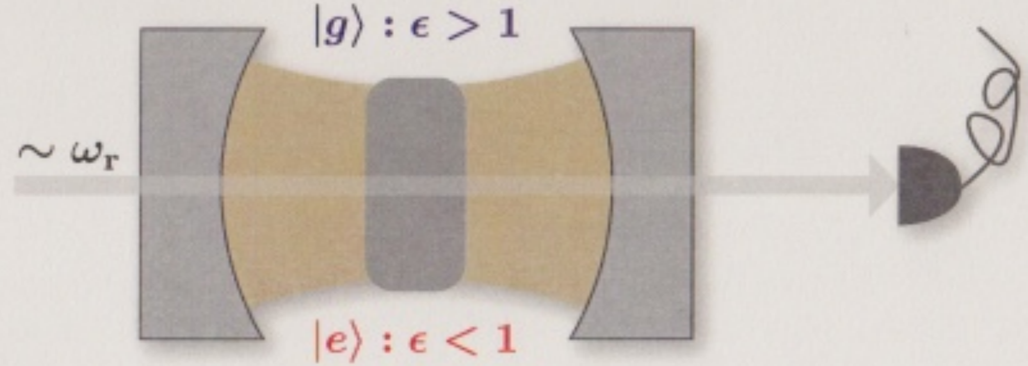
Dispersive interaction: $H \approx (\omega_r + \chi\hat{\sigma}_z)\hat{a}^\dagger\hat{a} + \frac{\tilde{\omega}_{01}}{2}\hat{\sigma}_z$

Resonator frequency

Dispersive interaction strength
$\chi = g^2/\Delta$

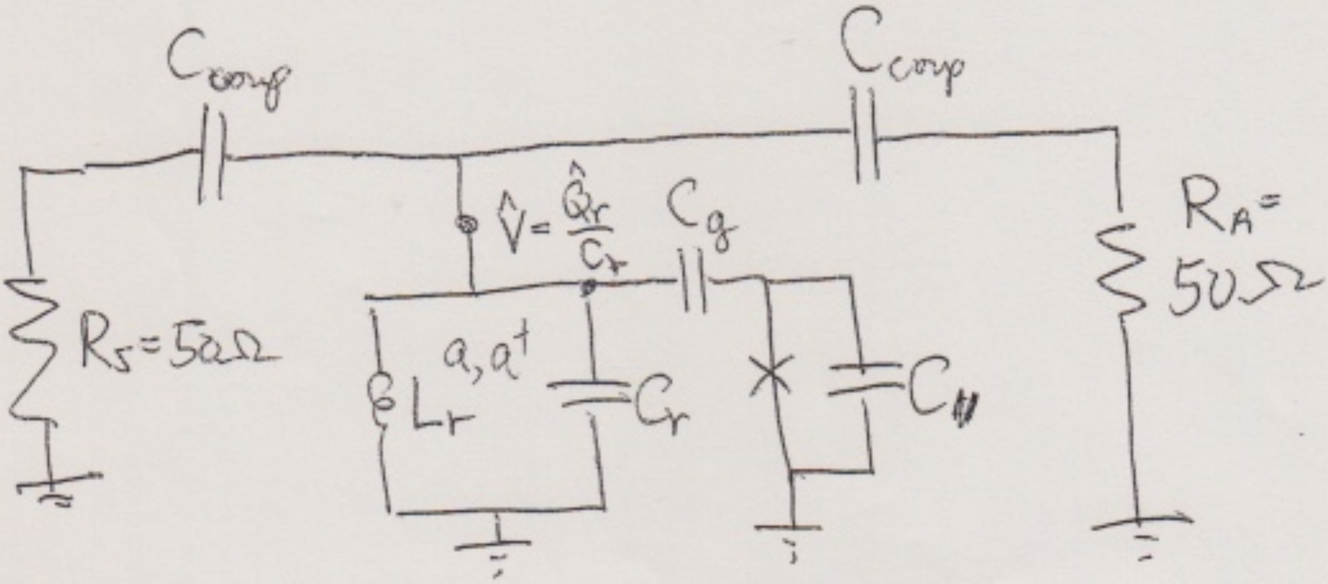
Qubit transition frequency





Blais, Huang, Wallraff, Girvin & Schoelkopf, Phys. Rev. A **69**, 062320 (2004)

I. Circuit model:



R_S, R_A: Source & amplifier impedances

L_r, C_r: model for first resonant mode of transmission line resonator

circuit Hamiltonian

$$H = \frac{\hat{Q}_r^2}{2C_r} + \frac{\Phi_r^2}{2L_r} + \left[4E_c(\hat{n} - \hat{n}_{offset})^2 - E_J \cos\hat{\varphi}\right]$$

$$\hat{n}_{offset} = \frac{C_g}{2e}\hat{V} = \frac{C_g}{2eC_r}\hat{Q}_r \Leftarrow$$ offset-charge operator

coupling term $-8E_c \hat{n}\hat{n}_{offset}$

2nd quantize: rotating-wave approx.

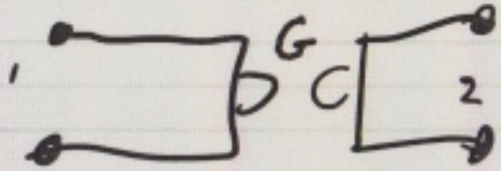
$$H = \omega_r a^\dagger a + \frac{\omega_{01}}{2}Z + g\left(a^\dagger\sigma_- + a\sigma_+\right)$$

Jaynes-Cummings model.

So, we come to what is needed:

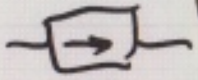
THD connection

Gyrator: (Tellegen 1948))



(circulator is Hogan (1952))

or



port rotation

$I_1 = -GV_2$

$I_2 = GV_1$

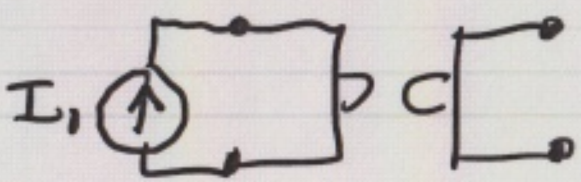
$Y_{port} = \begin{pmatrix} 0 & -G \\ +G & 0 \end{pmatrix}$

if $G = \frac{1}{Z_0}$: $S_{port} = \begin{pmatrix} 0 & -1 \\ +1 & 0 \end{pmatrix}$

- Tellegen said that gyrator is lossless.
- Lagrangian. We will explore

$$L_{\text{gyrator}} = \frac{1}{2} G \left[(1+\kappa)\Phi_1 \dot{\Phi}_2 - (1-\kappa)\Phi_2 \dot{\Phi}_1 \right]$$

consider circuit



full Lagrangian:

current source

$$L = \frac{1}{2}G\left[(1+k)\Phi_1\dot{\Phi}_2 - (1-k)\Phi_2\dot{\Phi}_1\right] + I_1\Phi_1$$

$\frac{\partial L}{\partial \Phi_2} = -\frac{1}{2}(1-k)G\dot{\Phi}_1$ | $\frac{\partial L}{\partial \dot{\Phi}_2} = \frac{1}{2}(1+k)G\Phi_1$ | $\frac{\partial}{\partial t} \to \frac{1}{2}(1+k)G\dot{\Phi}_1$

$\frac{\partial L}{\partial \Phi_1} = \frac{1}{2}(1+k)G\dot{\Phi}_2 + I$ | $\frac{\partial L}{\partial \dot{\Phi}_1} = -\frac{1}{2}(1-k)G\Phi_2$ | $\frac{\partial}{\partial t} \to \frac{1}{2}(1-k)G\dot{\Phi}_2$

Φ_2: $\dot{\Phi}_1 = 0 \Longrightarrow V_1 = 0.$

Φ_1:

$$-\frac{G}{2}(1-k)\dot{\Phi}_2 - \frac{G}{2}(1+k)\dot{\Phi}_2 - I = 0 \Longrightarrow V_2 = \frac{-I}{G}$$

we would find:

$$V_2 = -\frac{1}{G} I_1$$

$$V_1 = +\frac{1}{G} I_2 \qquad \text{(current source @ 2)}$$

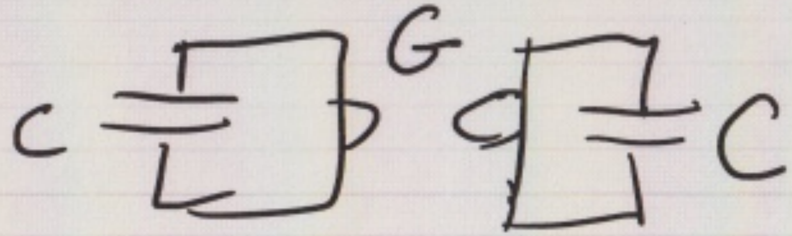
$$Z = \begin{pmatrix} 0 & \frac{+1}{G} \\ -\frac{1}{G} & 0 \end{pmatrix} = Y^{-1}_{\text{gyrator}} \checkmark$$

- NB: independent of K!

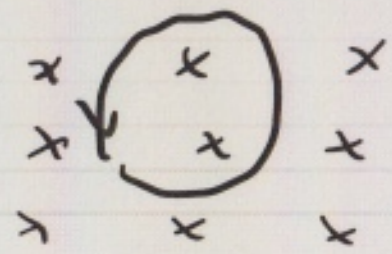
K is a <u>gauge</u> degree of freedom.

Tellegen said gyrator does not store energy, but it does have a term in the Lagrangian.

Interpretation of this is given by final example:



doing the Euler-Lagrange equations (homework), we get the equations of motion of a charged particle in 2D, with mass $\propto C$, in a uniform magnetic field $\propto G$

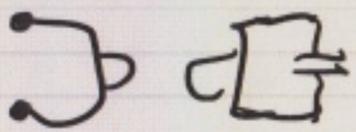


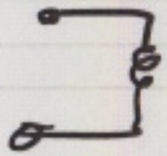
solution – cyclotron motion,
frequency is

$$\omega = \frac{G}{C}$$

explained by electrical engineers
with identity

$$\omega = \frac{1}{\sqrt{LC}}$$



$=$

$L = \frac{1}{G^2 C}$ $= \frac{G}{C}$