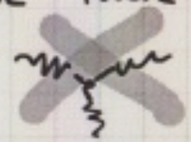
A'

(24)

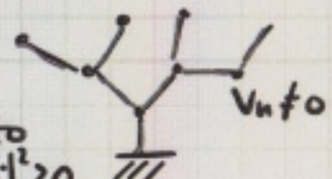
Claim: assume there are no nodes with only inductances 

Then C is invertible if (and only if) the circuit has a spanning tree which consists of capacitive branches only.

(other branches can also be capacitive)

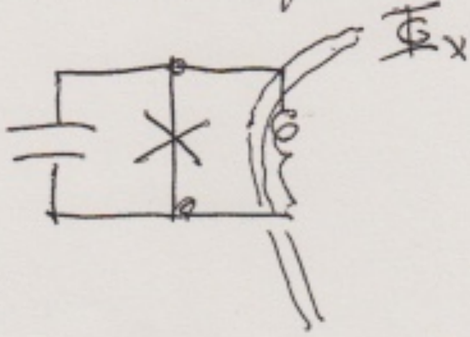
C has no zero eigenvalues : no $\{ \dot{\Phi}_n = V_n = \text{node voltage} \}$ with zero energy, except for $\forall n \; V_n = 0$ (all equal to ground)

$$T = \frac{1}{2} \sum_{b \in T} \dot{\Phi}_b^2 C_b + \underbrace{\sum_{b \notin T} \dots}_{\geq 0}$$

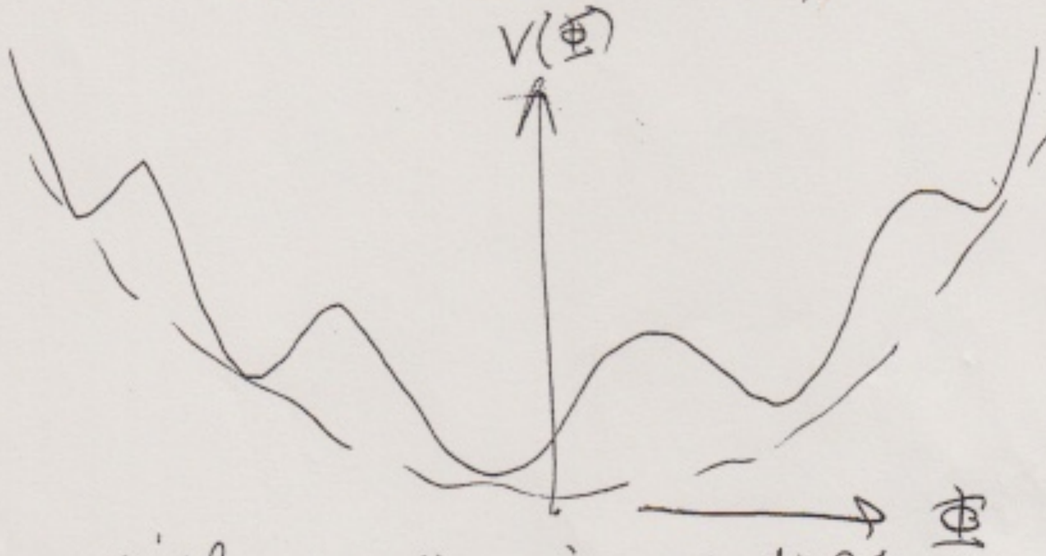


Let some $V_n \neq 0$, then on path (over spanning tree) to ground some $(V_k - V_j)^2 > 0$, so $T \geq C_{b=kj} \cdot (V_k - V_j)^2 > 0$

B. So, eg. Hamiltonian for



$$\hat{H} = -\frac{\hbar^2}{2C}\frac{\partial^2}{\partial \Phi^2} + \frac{\Phi_0^2}{L_J}\cos\left(\frac{\Phi - \Phi_x}{\Phi_0}\right) + \frac{1}{2L}\Phi^2 + \cancel{\frac{\Phi \Phi_x}{L}} \quad \circledast$$



Highly non-harmonic potential!

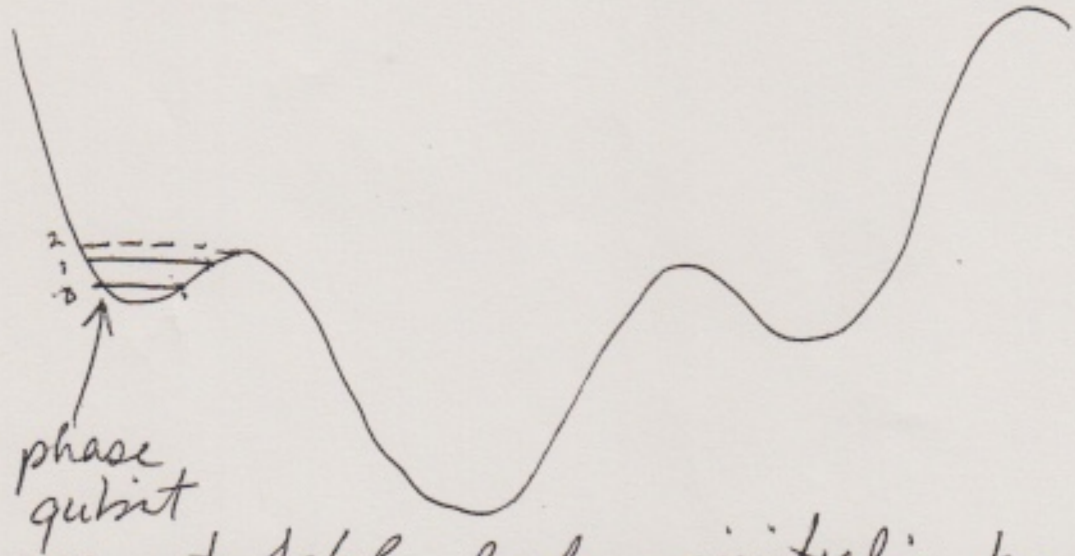
~~Linear term (*)~~ → characteristic "phase rigidity" — characteristic of superconductivity (not true of "perfect conductor")

MORE GENERAL THEORY FOR DERIVING CIRCUIT HAMILTONIAN:

Network Graph Theory.

Applications:

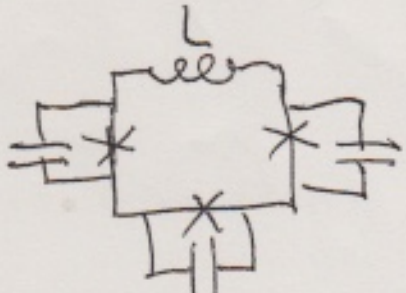
1. Phase qubit — Eq. (*) on p. 3.



- use metastable levels — initialize by ramping Φ_x
- read out by pumping to metastable state $|2\rangle$, observe "escape"

2. Flux qubit

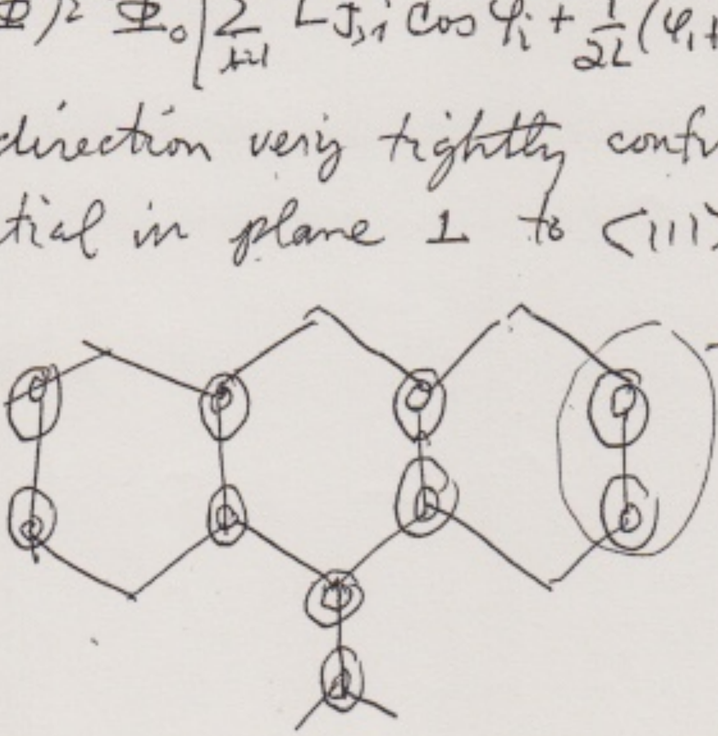
3D Hamiltonian



$$U(\Phi) = \tilde{\Phi}_0^2 \left[\sum_{i=1}^{3} L_{J,i} \cos\varphi_i + \frac{1}{2L}(\varphi_1+\varphi_2+\varphi_3)^2 + \frac{1}{L}(\varphi_1+\varphi_2+\varphi_3)\frac{\Phi_x}{\Phi_0} \right]$$

$\langle 111 \rangle$ direction very tightly confined.

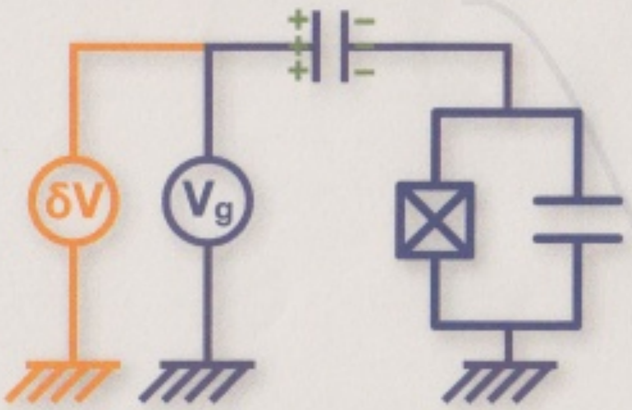
- Potential in plane $\perp$ to $\langle 111 \rangle$:

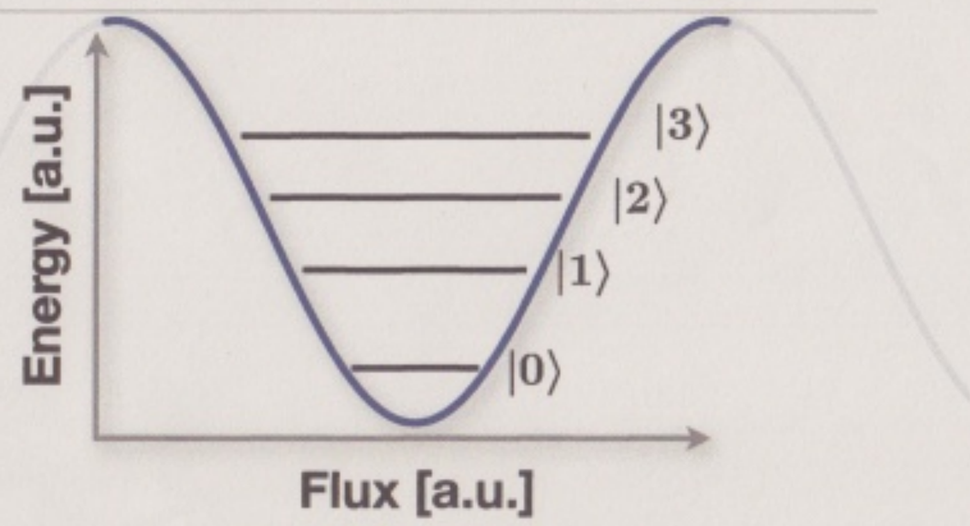


→ very controllable double-well potential

C.

Hamiltonian of a superconducting qubit





$$H = \frac{\hat{Q}^2}{2C} - E_J \cos\frac{2\pi\hat{\Phi}}{\Phi_0}$$

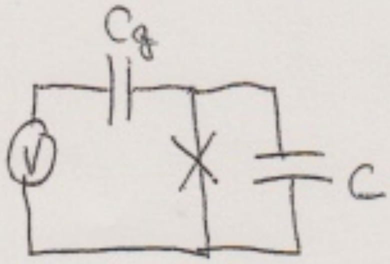
$$= \frac{(2e)^2}{2C}\hat{n}^2 - E_J\cos\hat{\phi}$$

$$= 4E_C(\hat{n}-n_g)^2 - E_J\cos\hat{\phi}$$

Optimal choice of E_J/E_C?

C

TRANSMON QUBIT
& COUPLED SUPERCONDUCTING QUBITS.



see:

www.qipc2011.ethz.ch/uploads/schoolpresentations/qipcAlexandre.pdf

$$H = \frac{1}{2(C+C_g)}\left(\hat{Q}-C_gV\right)^2 - \left(\frac{\Phi_0}{2\pi}\right)^2\frac{1}{L_J}\cos\hat{\varphi}$$

$$= \underbrace{\frac{(2e)^2}{2(C+C_g)}}_{4E_c}\Big(\hat{n}-\underbrace{\frac{C_gV}{2e}}_{n_{offset}}\Big)^2 - \underbrace{\left(\frac{\Phi_0}{2\pi}\right)^2\frac{1}{L_J}}_{E_J}\cos\hat{\varphi} \qquad \hat{n}=-i\frac{\partial}{\partial\varphi}$$

$$H = 4E_c(\hat{n}-n_{offset})^2 - E_J\cos\varphi$$

NOTE φ is only defined on interval $0 \le \varphi < 2\pi$.

Periodic boundary conditions if $n_{offset}=0$ (or integer

$n_{offset} \neq 0$ acts like Aharonov-Bohm flux

For $E_J=0$, eigenfunctions are plane waves:

$$H|\psi\rangle = E|\psi\rangle$$

$$|\psi\rangle = e^{im\varphi} \qquad m=0,\pm1,\pm2,\ldots$$

$$E = 4E_c(m-n_{offset})^2$$

D

$$H = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$ Harmonic Oscillator

Euler-Lagrange : $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\Phi}}\right) - \frac{\partial \mathcal{L}}{\partial \Phi} = 0 \Rightarrow C\ddot{\Phi} + \frac{\Phi}{L} = 0$

current through capacitor

current through inductor

$I = \frac{d}{dt}(CV)$..

Introduce $\hat{a} = \frac{i\hat{Q}}{\sqrt{2C\hbar\Omega}} + \frac{\hat{\Phi}}{\sqrt{2L\hbar\Omega}}$ $\quad [\hat{a}, \hat{a}^\dagger] = I$

to get $H = \hbar\Omega\,(a^\dagger a + 1/2)$ $\quad \Omega = \frac{1}{\sqrt{LC}}$