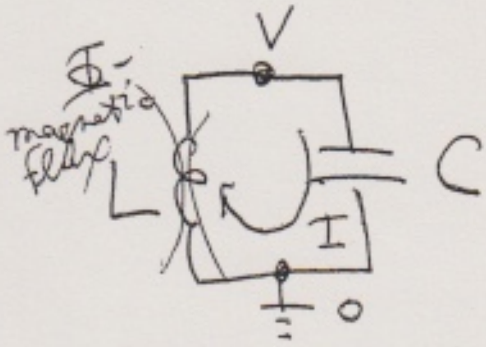
SUPERCONDUCTING QUBITS

- BASIS IN ELECTRIC CIRCUIT THEORY

$$\Phi = -LI$$
$$\dot{\Phi} = V \quad \text{(Faraday)}$$



$$(*) \quad Q = CV = C\dot{\Phi}$$
$$\dot{Q} = I = C\ddot{\Phi}$$

$$\Rightarrow \boxed{C\ddot{\Phi} = -\frac{1}{L}\Phi} \quad \text{equation of motion}$$

$\rightarrow$ resonates @ $\omega_0 = \frac{1}{\sqrt{LC}}$

Lagrange function $\mathcal{L}(q, \dot{q})$
(conservative mechanics)

equation of motion from Lagrangian:

Euler-Lagrange equation:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial q} \qquad q \Leftrightarrow \Phi$$

q ← canonical coordinate

<u>good guess</u>

$$\mathcal{L} = \underbrace{\tfrac{1}{2}C(\dot{\Phi})^2}_{\text{electric energy}} - \underbrace{\tfrac{1}{2L}\Phi^2}_{\text{magnetic energy}}$$

prescription for obtaining Hamiltonian function:

$$p \equiv \frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}} = C\dot{\Phi} \qquad C \text{ like a mass}$$

Legendre transformation

$H = \dot{q}p - \mathcal{L}$, plus replace $\dot{q} = f(p)$

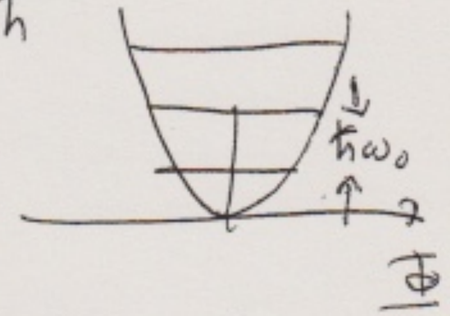
$\dot{\Phi} = \frac{1}{C}p$ NB $p = Q$ from $(*)$

$$H = \frac{1}{2} C \dot{\Phi}^2 + \frac{1}{2L}\Phi^2$$

$$\boxed{H = \frac{Q^2}{2C} + \frac{1}{2L}\Phi^2}$$

quantum postulate: $[\hat{Q}, \hat{\Phi}] = -i\hbar$

$$\hat{H} = \frac{-\hbar^2}{2C}\frac{\partial^2}{\partial \Phi^2} + \frac{1}{2L}\Phi^2 \longrightarrow$$



To make a qubit, make
non-harmonic

$\Longrightarrow$ nonlinear inductance

$I = \frac{1}{L}\Phi$ linear

$= f(\Phi)$ nonlinear $\rightarrow$ still non-dissipative

particular nonlinearity in
superconducting tunneling device:

Josephson Junction:

$$I = \frac{\tilde{\Phi}_0}{L_J}\sin\left(\Phi/\tilde{\Phi}_0\right) \qquad \tilde{\Phi}_0 = \frac{\hbar}{2e}$$

usually just h.

"Josephson inductance"

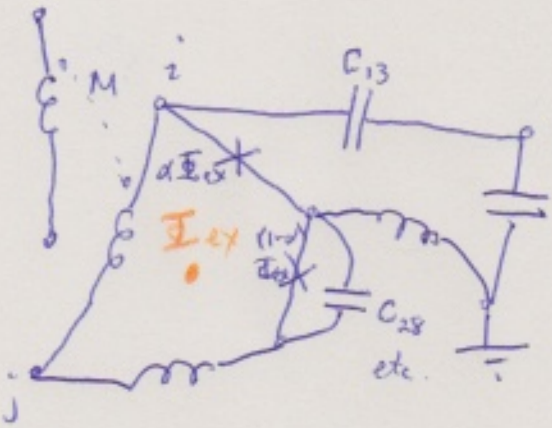
$\frac{\tilde{\Phi}_0}{L_J} = I_c$ critical current

1. <u>general theory for quantizing electric circuits:</u>

Vool & Devoret

arXiv:1610.03438

$N+1$ nodes



Pick one node, call it "ground": $(\Phi_{gr.} = 0)$

all other nodes (N): node fluxes Φ_i

NB $\dot{\Phi}_i = V_i$ more familiar node voltages.

Procedure:

1) Kinetic energies = capacitors.

$$T = \sum_{ij}^{\text{edges}} \frac{1}{2} C_{ij} \left(\dot{\Phi}_i - \dot{\Phi}_j\right)^2 = \frac{1}{2} \dot{\underline{\Phi}}^T \underline{C}\, \dot{\underline{\Phi}} = \frac{1}{2} \sum_{ij}^{N} C_{ij} \dot{\Phi}_i \dot{\Phi}_j$$

can be $=0$ (ground)

nodes

non-diagonal node matrix

$$\text{e.g.} \quad C = \begin{pmatrix} C_{13} & 0 & -C_{13} & \\ 0 & 0 & 0 & \\ -C_{13} & 0 & C_{13} & \ddots \\ & & & \ddots \end{pmatrix} + \dots$$

2. potential energies:

a) individual $^{\text{linear}}_{\wedge}$ inductances

$$U = \sum \frac{1}{2L_{ij}} (\Phi_i - \Phi_j)^2$$

susceptance = inverse inductance.

b) coupled inductors:

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} L_1 & M \\ M & L_2 \end{pmatrix}^{-1} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

M = mutual inductance

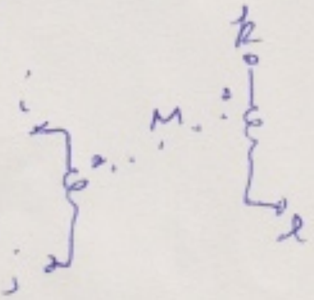
can be + or −

$|M| \leq \sqrt{L_1 L_2}$ stability

$\hookrightarrow$ tight coupling limit — special

$$\begin{pmatrix} L_1 & M \\ M & L_2 \end{pmatrix}^{-1} \equiv \begin{pmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{pmatrix}$$

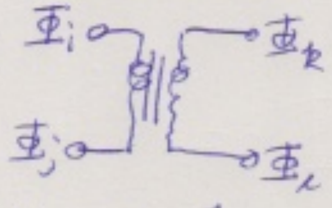
<u>not</u> tight coupled.



$$U = \frac{1}{2} S_{11} (\Phi_i - \Phi_j)^2$$
$$+ S_{12} (\Phi_i - \Phi_j)(\Phi_k - \Phi_l)$$
$$+ \frac{1}{2} S_{22} (\Phi_k - \Phi_l)^2$$

c) tight coupling — special case:

transformer



<u>no</u> potential energy contribution

Φ_k is <u>not</u> independent variable:

$$(\Phi_k - \Phi_l) = N(\Phi_i - \Phi_j)$$

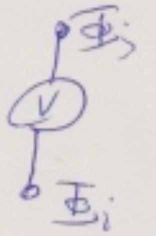
N = turns ratio

$$\pm\sqrt{\frac{L_2}{L_1}} = N$$

comes from above with $M = \pm\sqrt{L_1 L_2}$, $L_1, L_2 \to \infty$,

d) ~~Josephson junction~~

~~(4)~~ voltage source



Φ_j not independent.

$$\Phi_j = \int V\,dt + \Phi_i$$

$$= Vt + \Phi_i \quad \text{for DC.}$$

can make $\mathcal{L}$ or H explicitly time dependent.

e) Josephson junctions:

: nonlinear inductance:

$$I_b(t') = f(\overset{\Phi_i - \Phi_j}{\Phi_b}(t'))$$

$$U = \int_{-\infty}^{t} I_b(t') V_b(t')\,dt'$$

general def. of inductance

NB t' same

J.J. $I_b = \left(\frac{\Phi_0}{2\pi L_J}\right) \sin\left(\frac{2\pi \Phi_b}{\Phi_0}\right)$ $\quad \Phi_0 = \frac{h}{2e}$

I_c

do integral:

$$U = \left(\frac{\Phi_0^2}{(2\pi)^2 L_J}\right) \cos\left(\frac{2\pi\Phi_b}{\Phi_0}\right)$$

E_J Josephson energy.

4. f) J.J.s with external fluxes:

gauge choice (see diagram) for inductor only loops.

$$U = E_J \cos\left(\frac{2\pi}{\Phi_0}(\Phi_b - \alpha \Phi_{ext})\right)$$

$\therefore$

$$\mathcal{L} = T - U$$

$$= \tfrac{1}{2}\sum C_{ij}\dot\Phi_i\dot\Phi_j - U(\Phi)$$ [no V branches]

Legendre transform:

$$\frac{\partial \mathcal{L}}{\partial \dot\Phi_i} = \sum_j C_{ij}\dot\Phi_j \equiv Q_i$$ ($Q = CV$) dimensionally correct.

$$H = \sum_i \dot\Phi_i Q_i - \mathcal{L}$$

$$= (1)\sum C_{ij}\dot\Phi_i\dot\Phi_j - \tfrac{1}{2}\sum C_{ij}\dot\Phi_i\dot\Phi_j + U(\Phi)$$

$$= +\tfrac{1}{2}\sum C_{ij}\dot\Phi_i\dot\Phi_j + U(\Phi)$$

$$= \tfrac{1}{2}\sum_j Q_j\dot\Phi_j + U(\Phi)$$

solve: $$\dot\Phi_j = (C)^{-1}_{ji}Q_i$$

$$H = \tfrac{1}{2}\sum_{ij}(C^{-1})_{ij}Q_iQ_j + U(\Phi)$$

if C is not invertible — start over! (add C_{J}'s)