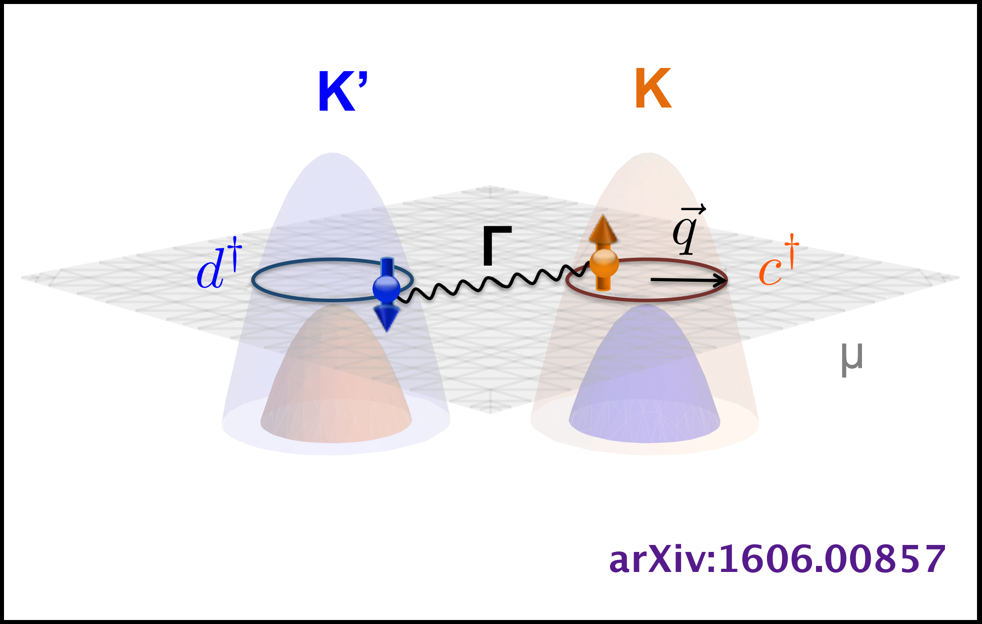
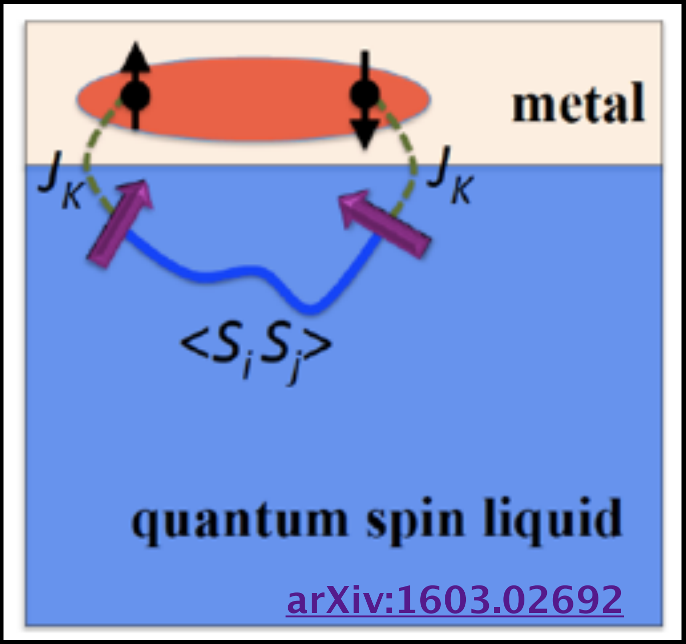
Let There Be Topological Superconductors



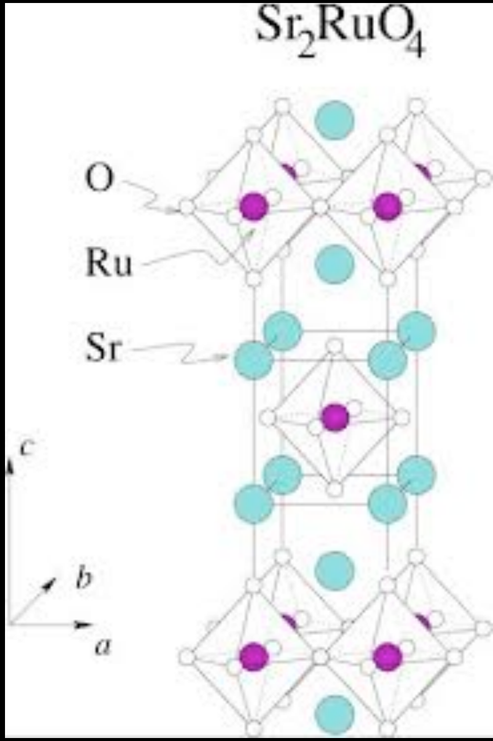


Eun-Ah Kim (Cornell)

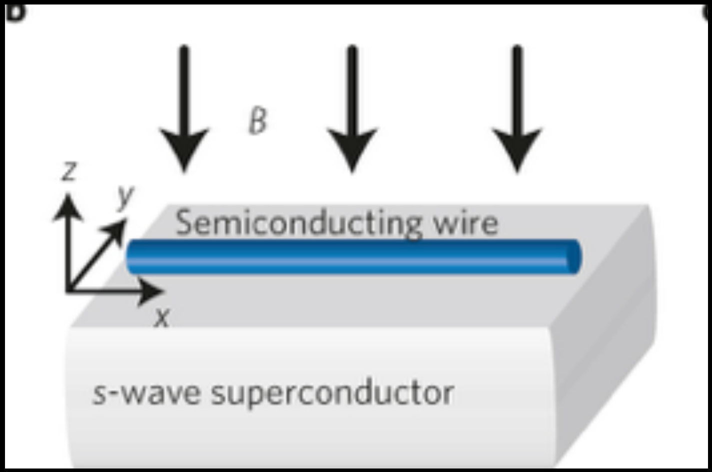
Boulder 7.21-22.2016

Q. Topological Superconductor material?

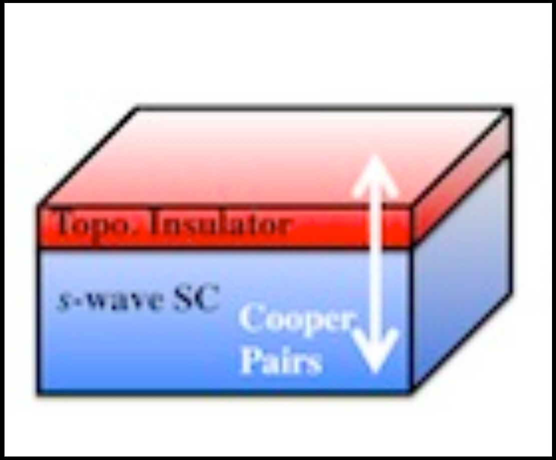
Bulk



1D proximity



2D proximity?



Designing 2D topological SC's

- 2D topological SC
  - odd-parity SC of spinless fermions
  - Majorana bound state

- Strategies:

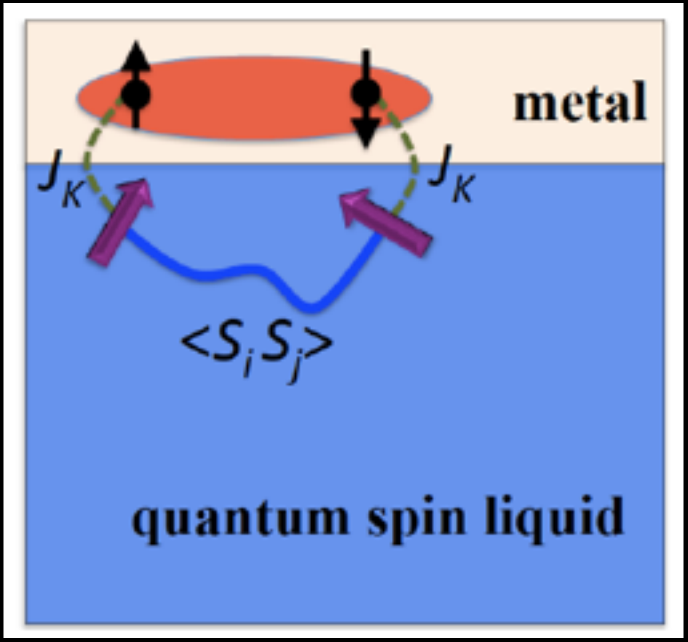
1) interaction,

2) spinlessness

Strategy I

- Manipulate the pairing interaction: target non-phononic mechanism

Topological Superconductivity in Metal/ Quantum-Spin-Ice Heterostructures

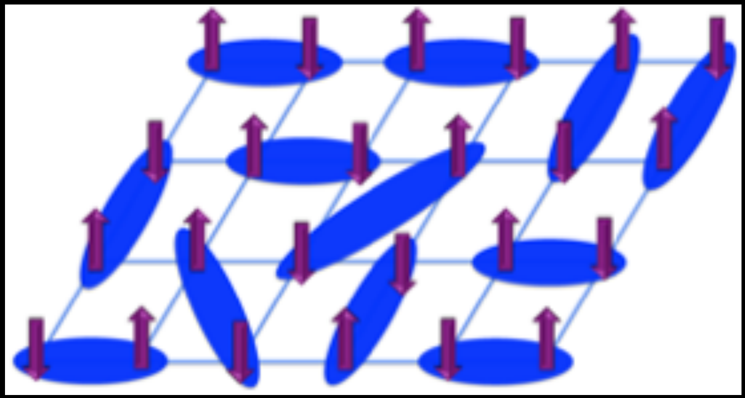


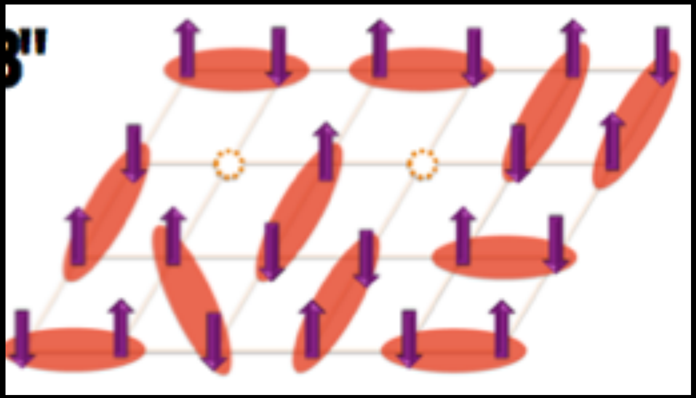
Jian-Huang She, Choonghyun Kim, Craig Fennie,
Michael Lawler, E-AK (arXiv:1603.02692)

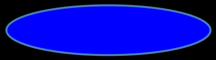
Wanted: non-phononic mechanism

Dope a Quantum spin liquid

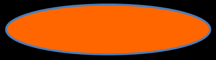
P.W.Anderson







RVB singlet

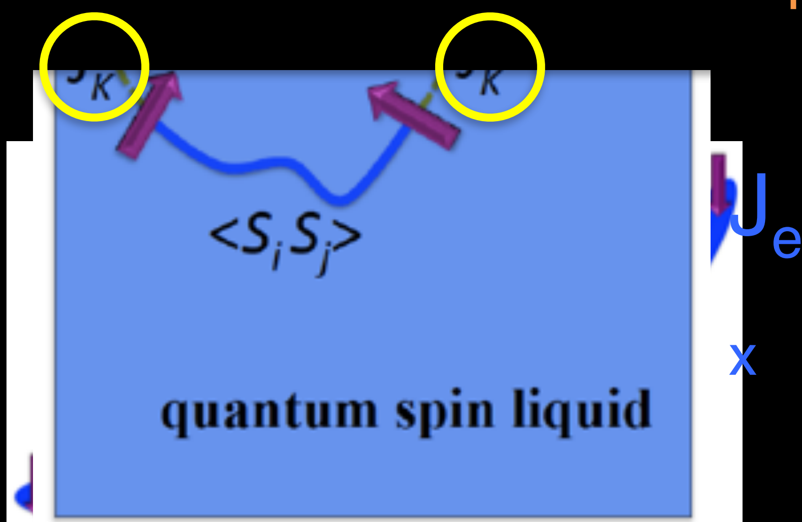


Cooper pair single

Wanted: non-phononic mechanism

Use Quantum spin liquid

E_F



- Characteristic energy scales:

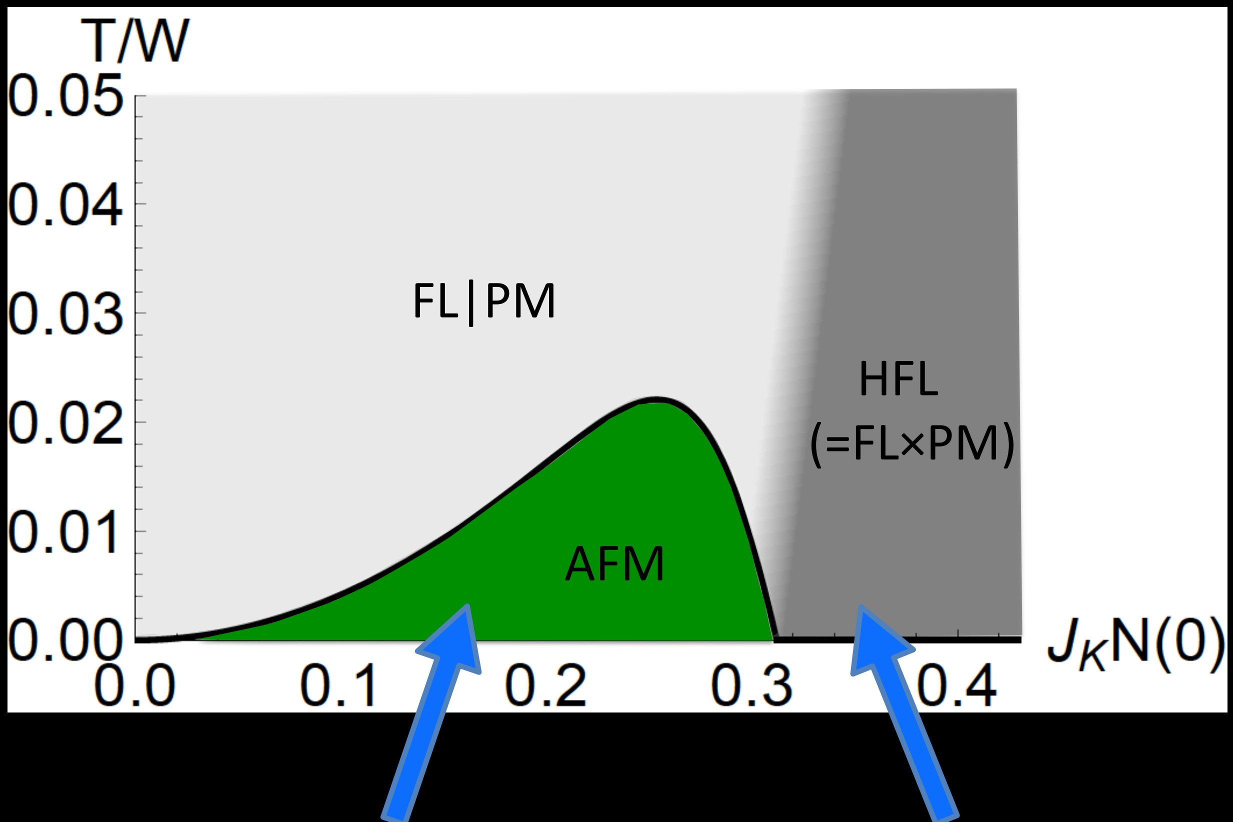
  $E_{F,}$ $J_{ex,}$ J_K

- Perturbative limit:

  $J_K / E_F << 1$

- Spin-fermion model

Spin-fermion model for J_{ex}=0



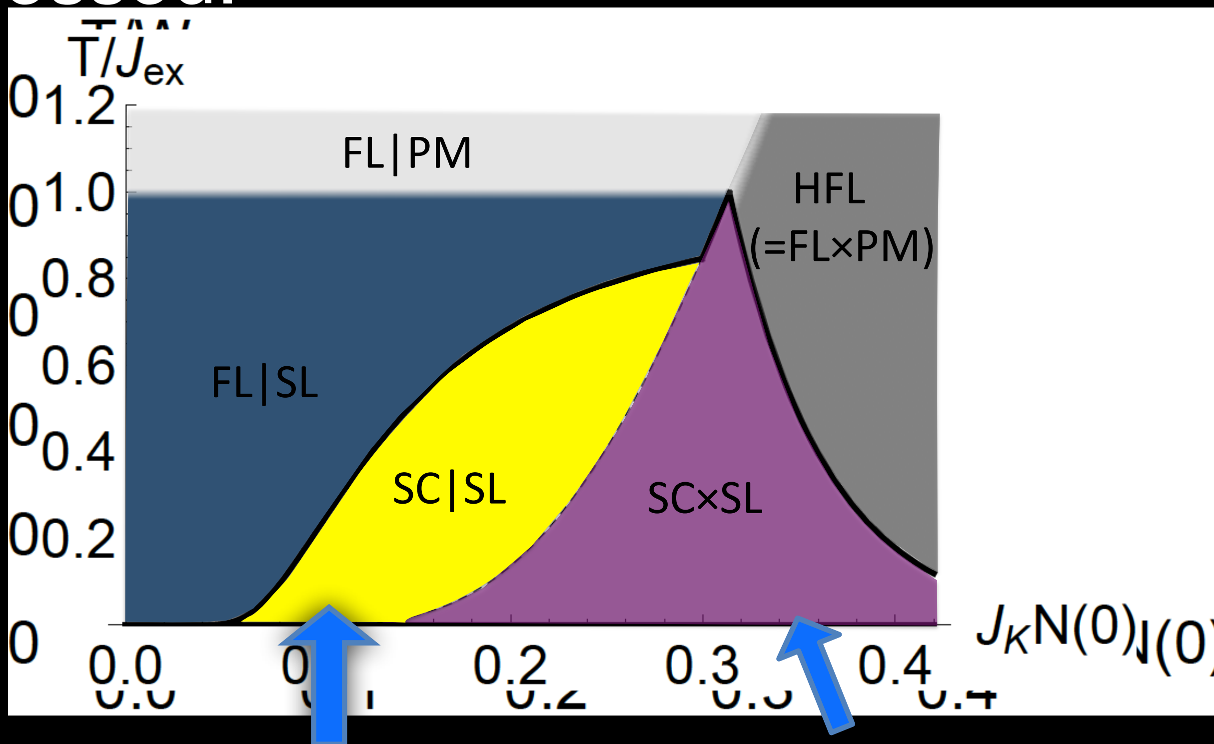
RKKY interaction

Kondo-Singlet

Doniach (1977)

Spin-fermion model for J_{ex} + Frustration

For $J_{RKKY} \sim J_K^2 N(0) < J_{ex}$ AFM order suppressed.



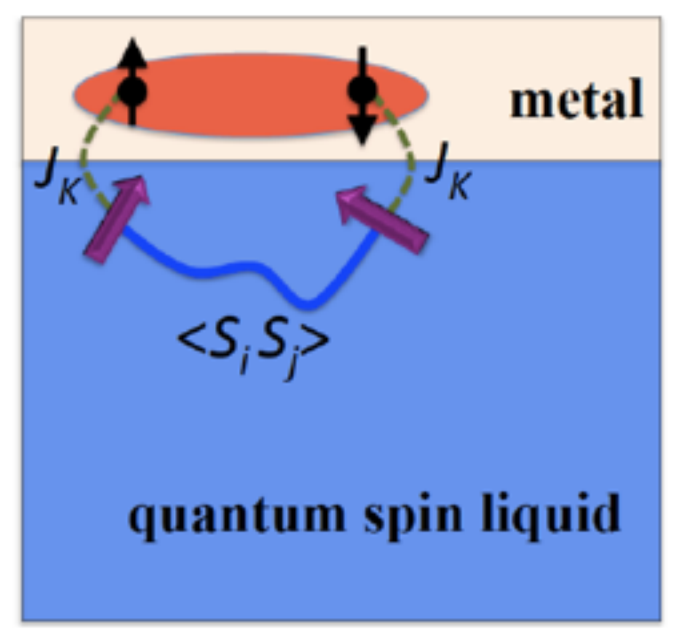
Superconduct or "riding" on QSL

Kondo-Singlet + RVB singlet+Cooper pair singlet

Coleman & Andrei (1989)

Senthil, Vojta, Sachdev (2003)

How to predictively materialize SCIQSL ?

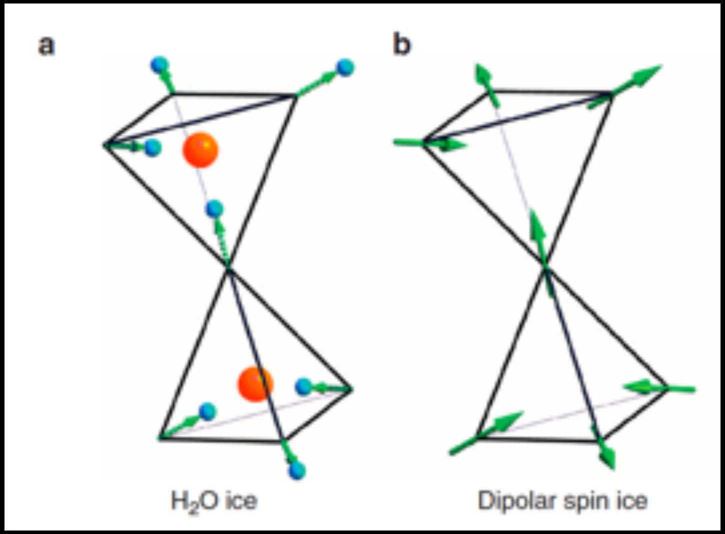


Simple isotropic metal

1. $\langle S \rangle = 0$
2. Dynamic spin fluctuation $\langle S_i S_j \rangle$
3. Gapped spectrum
4. Well understood

➡ Quantum Spin Ice

Emergent Gauge Field in Spin Ice



Kimura et al (2013)

- 2-in 2-out ice rule

$$\nabla \cdot \vec{S}(\boldsymbol{r}) = 0$$

$$\vec{S}(\boldsymbol{r}) = \nabla \times \vec{A}(\boldsymbol{r})$$

- Gauge Field Propagator

$$\langle A_a(\boldsymbol{q}) A_b(-\boldsymbol{q}) \rangle \sim \frac{1}{q^2} (\delta_{ab} - 2\hat{q}_a \hat{q}_b)$$

- Spin-spin correlation

$$\langle S_a(\boldsymbol{q}) S_b(-\boldsymbol{q}) \rangle \sim \delta_{ab} - \hat{q}_a \hat{q}_b$$

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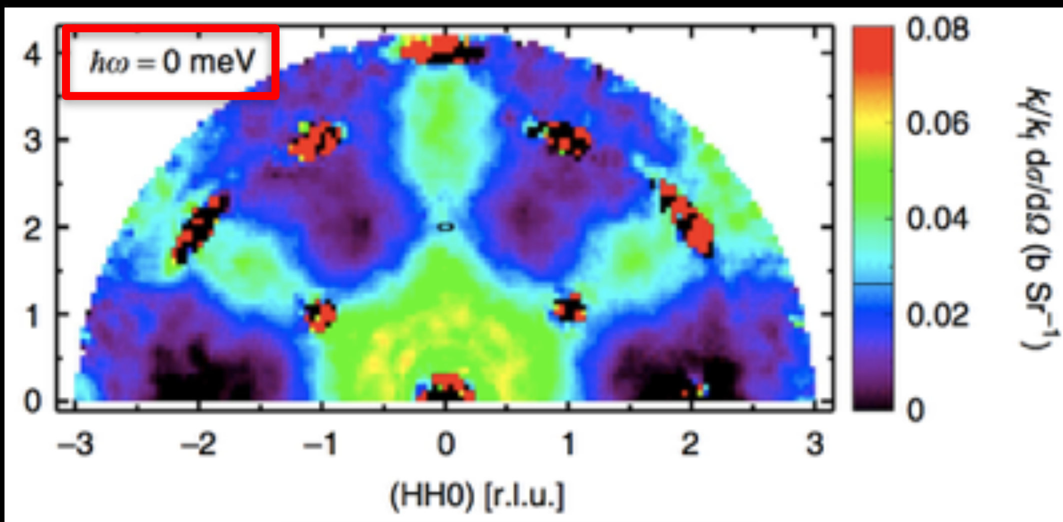
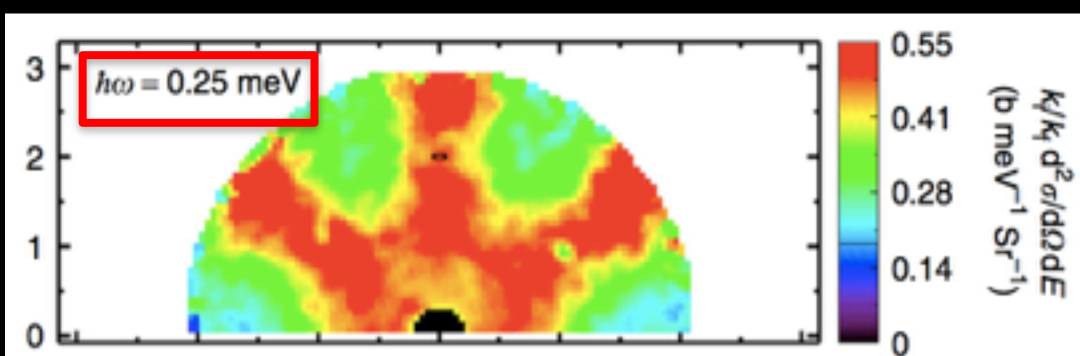
DOI: 10.1038/ncomms2914

Quantum fluctuations in spin-ice-like $Pr_2Zr_2O_7$

K. Kimura[1], S. Nakatsuji[1,2], J.-J. Wen[3], C. Broholm[3,4,5], M.B. Stone[5], E. Nishibori[6] & H. Sawa[6]



- Elastic neutron: pinch points (spin-ice like)



- Inelastic neutron: over 90% weight

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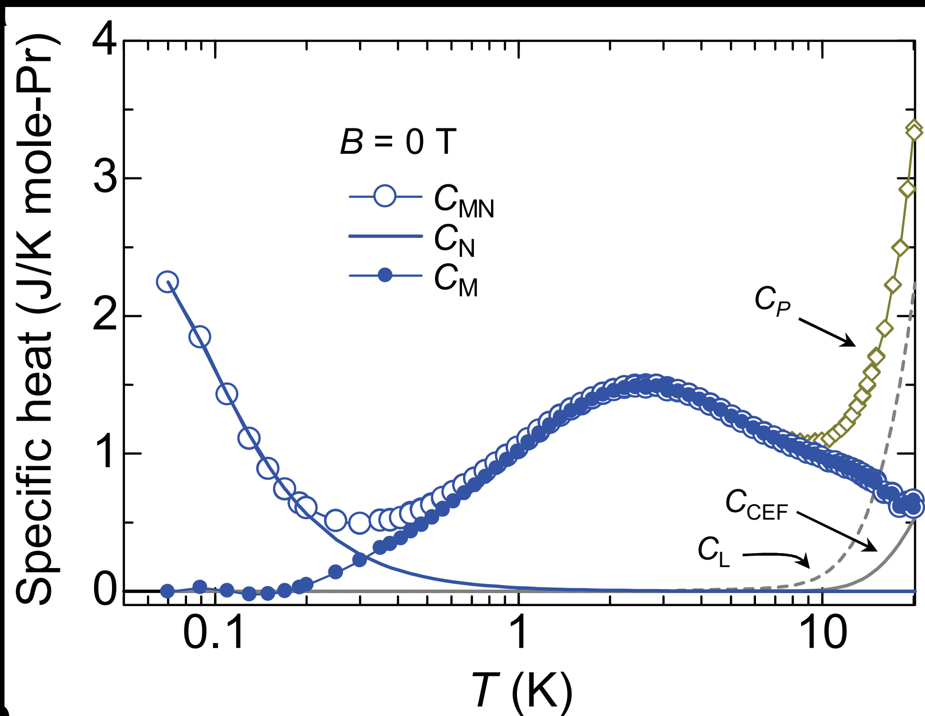
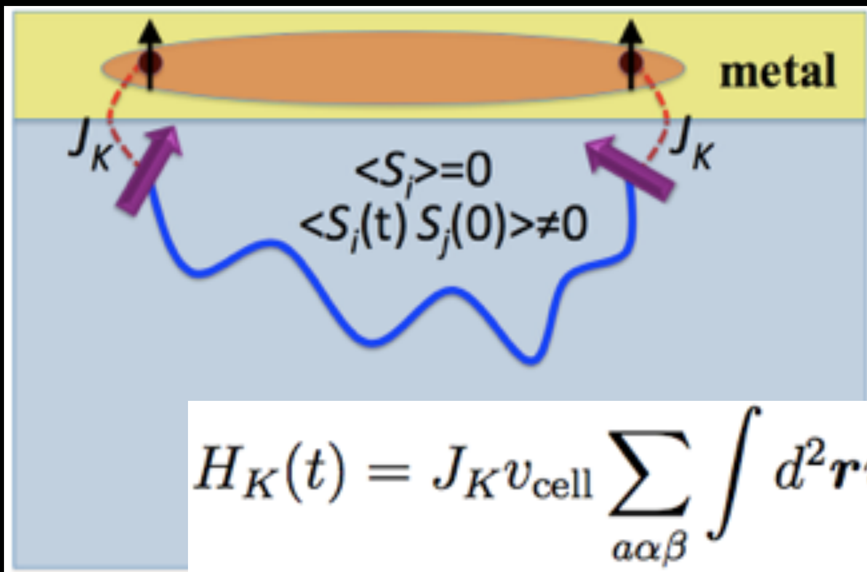


- No order down to 20mK
- Gapped quantum paramagnet ω_S=0.17meV
- Inelastic spectra peaked at Q=0

Effective Continuum Theory



$$H_c = \sum_{\boldsymbol{k}\alpha} \left(\frac{\hbar^2 k^2}{2m} - E_F \right) \psi^\dagger_\alpha(\boldsymbol{k}) \psi_\alpha(\boldsymbol{k})$$

$$H_K(t) = J_K v_{\text{cell}} \sum_{a\alpha\beta} \int d^2\boldsymbol{r} \psi^\dagger_\alpha(\boldsymbol{r}) \sigma^a_{\alpha\beta} \psi_\beta(\boldsymbol{r}) S_a(\boldsymbol{r}_\perp = \boldsymbol{r}, z = 0, t)$$

- Integrate out spins >> Effective e-e interaction

$$H_{\text{int}}(t) = -(J_K^2 v_{\text{cell}}^2 / 2\hbar) \sum_{ab} \int dt' \int d^2\boldsymbol{r} d^2\boldsymbol{r}' s_a(\boldsymbol{r}, t) \boxed{\langle S_a(\boldsymbol{r}, 0, t) S_b(\boldsymbol{r}', 0, t') \rangle} s_b(\boldsymbol{r}', t')$$

$$s_a(\boldsymbol{r}, t) = \sum_{\alpha\beta} \psi^\dagger_\alpha(\boldsymbol{r}, t) \sigma^a_{\alpha\beta} \psi_\beta(\boldsymbol{r}, t)$$

Unusual Gauge-Matter Coupling

- Minimal Coupling

$$\vec{j}(\boldsymbol{q}) \cdot \vec{A}(\boldsymbol{q}) = e \sum_{\boldsymbol{k}} \sum_{\alpha} \vec{A}(\boldsymbol{q}) \cdot \frac{\boldsymbol{k}}{m} \psi^{\dagger}_{\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\alpha} \psi_{\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\alpha}$$

- Repulsion against Cooper pairing

$$-\sum_{\boldsymbol{p}_1 \boldsymbol{p}_2 \boldsymbol{q} \alpha} D(q) \frac{(\boldsymbol{p}_1 \times \hat{\boldsymbol{q}}) \cdot (\boldsymbol{p}_2 \times \hat{\boldsymbol{q}}))}{m^2} \psi^{\dagger}_{\boldsymbol{p}_1+\boldsymbol{q},\alpha} \psi_{\boldsymbol{p}_1,\alpha} \psi^{\dagger}_{\boldsymbol{p}_2-\boldsymbol{q},\beta} \psi_{\boldsymbol{p}_2,\beta}$$

- Spin-ice/electron

$$J_K \sum_{\boldsymbol{r}\alpha\beta} \psi^{\dagger}_{\boldsymbol{r}\alpha} \vec{\sigma}_{\alpha\beta} \psi_{\boldsymbol{r}\beta} \cdot \left[\vec{\nabla} \times \vec{A}(\boldsymbol{r})\right]$$

Electrons are not magnetic monopoles

- Attractive equal-spin interaction!

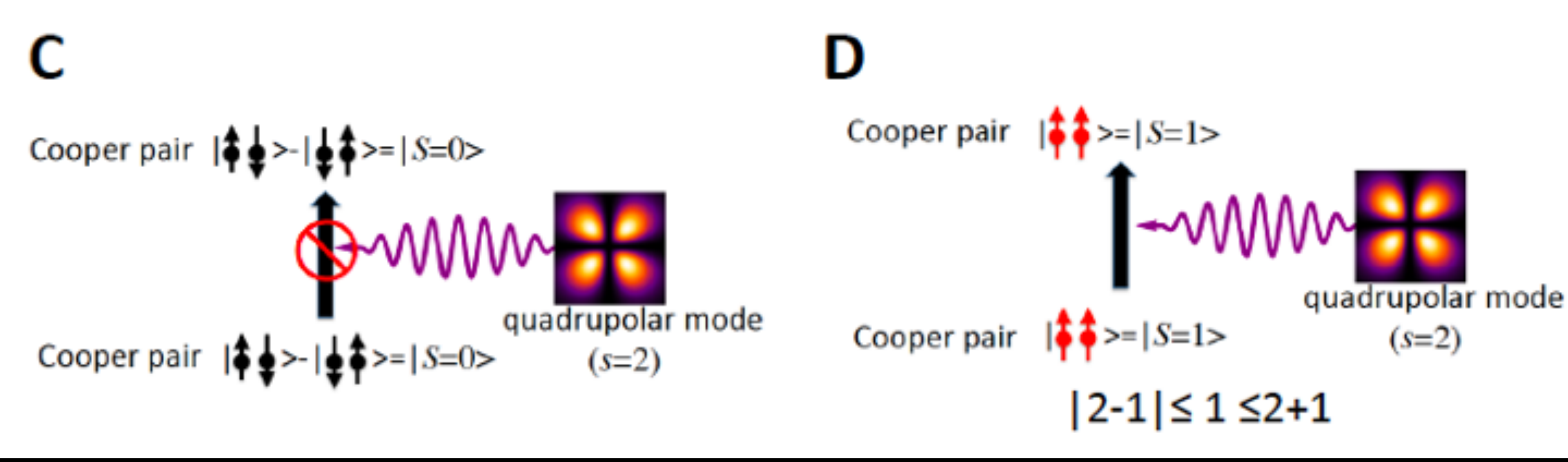
$$-J_K^2 D(q) \left(\vec{\sigma}_{\alpha\beta} \times \hat{\boldsymbol{q}}\right) \cdot \left(\vec{\sigma}_{\alpha'\beta'} \times \hat{\boldsymbol{q}}\right)$$

Selection Rule Dictated Odd-Parity

- Pair binding problem with dipole-dipole interactio

$$V_{\mathrm{dd}} = \frac{1}{r^3}[\vec{S}_1 \cdot \vec{S}_2 - 3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r})] \propto \mathcal{R}^{(2)}(r_1, r_2) \cdot \mathcal{S}^{(2)}(s_1, s_2)$$

- Wigner-Eckart thm: $\langle l' | \mathcal{T}^{(r)} | l \rangle = 0 \text{ unless } |r - l| \leq l' \leq (r + l)$

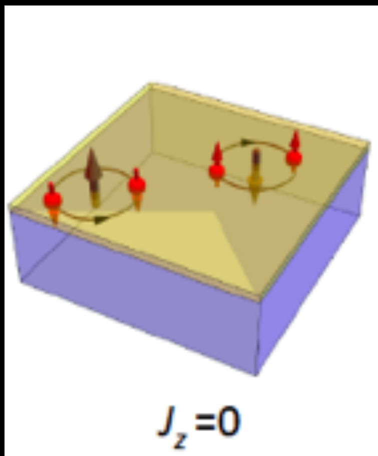


Dealing with interacting electrons?

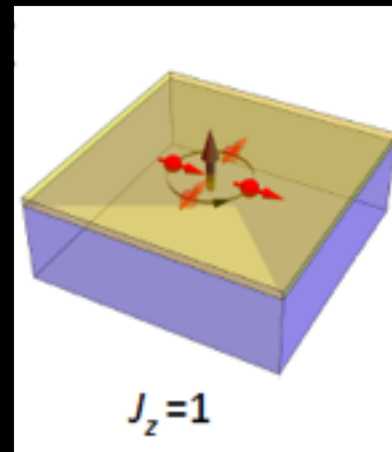
$$H_{\rm int}(t) = -(J_K^2 v_{\rm cell}^2/2\hbar)\sum_{ab}\int dt' \int d^2\boldsymbol{r} d^2\boldsymbol{r}' s_a(\boldsymbol{r},t)\langle S_a(\boldsymbol{r},0,t)S_b(\boldsymbol{r}',0,t')\rangle s_b(\boldsymbol{r}',t')$$

- Separation of scale: $\omega_S/E_F \ll 1$
  ➡“Migdal theorem”
- Dimensionless ratio: $\lambda \sim N(0)V \sim J_K^2 N(0)/J_{\rm ex} < 1$
- Full problem ≈
  solving the BCS mean-field theory $T_c \sim \omega_s e^{-1/\lambda}$

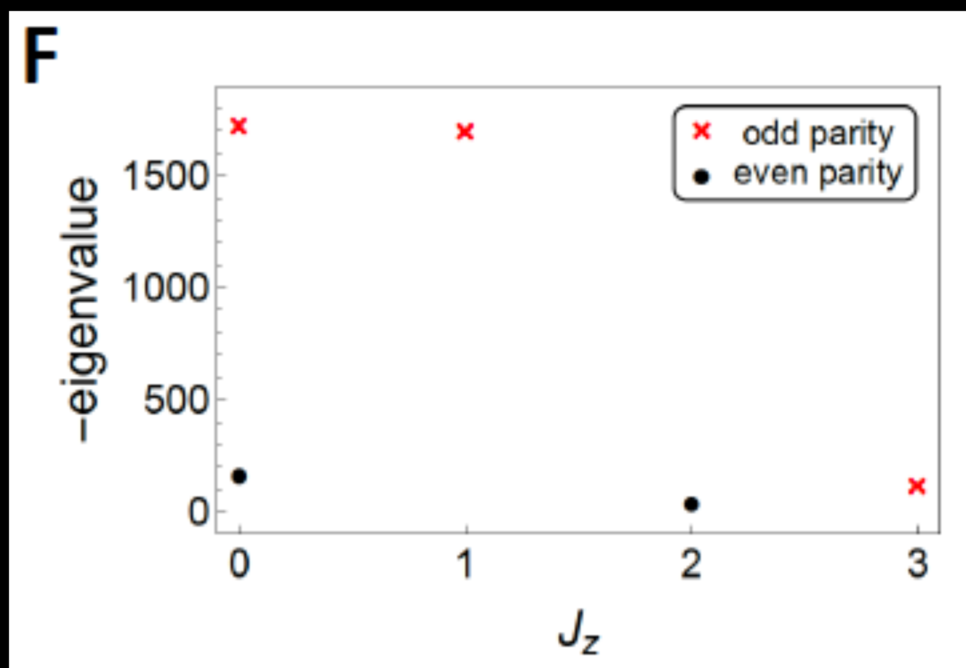
Leading channels



$$(k_x + ik_y)|\downarrow\downarrow\rangle + (k_x - ik_y)|\uparrow\uparrow\rangle$$



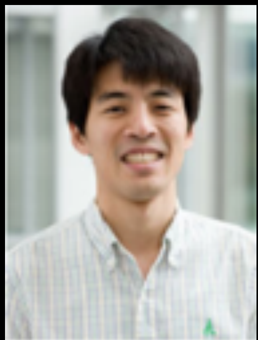
$$(k_x \pm ik_y)\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$$



Can we persuade a material synthesis person?

Criteria for Metal

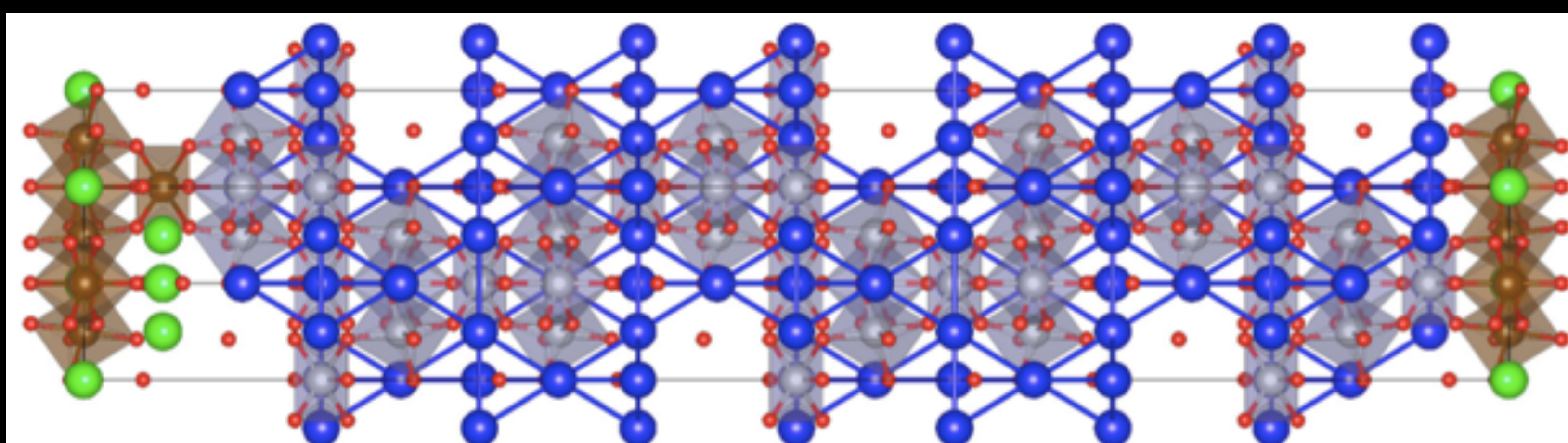
- Structural
  - Lattice match
    - ➡ $A_2B_2O_7$
  - No orphan bonds
- Electronic
  - Simple isotropic Fermi surface
  - Wave function penetration
  - Odd-# FS around high symmetry points



$$Pr_2Zr_2O_7/Y_2Sn_{2-x}Sb_xO_7 \ (111)$$

Non-magnetic

s-electrons:
large overlap,
isotropic FS.

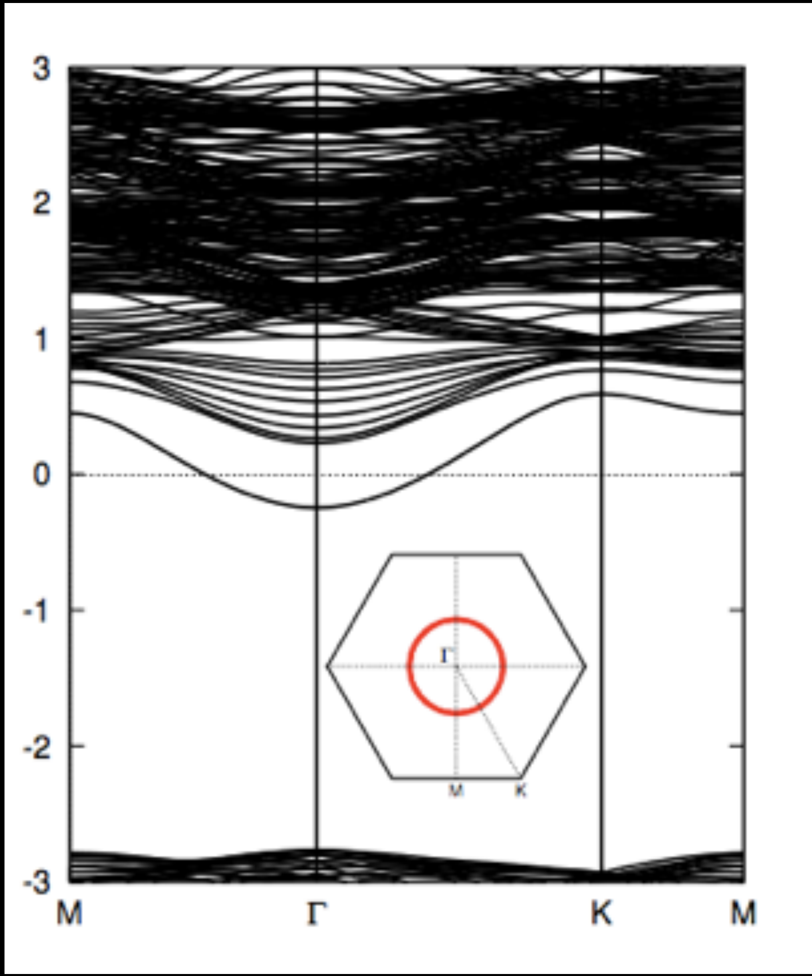


Band structure for the Proposal

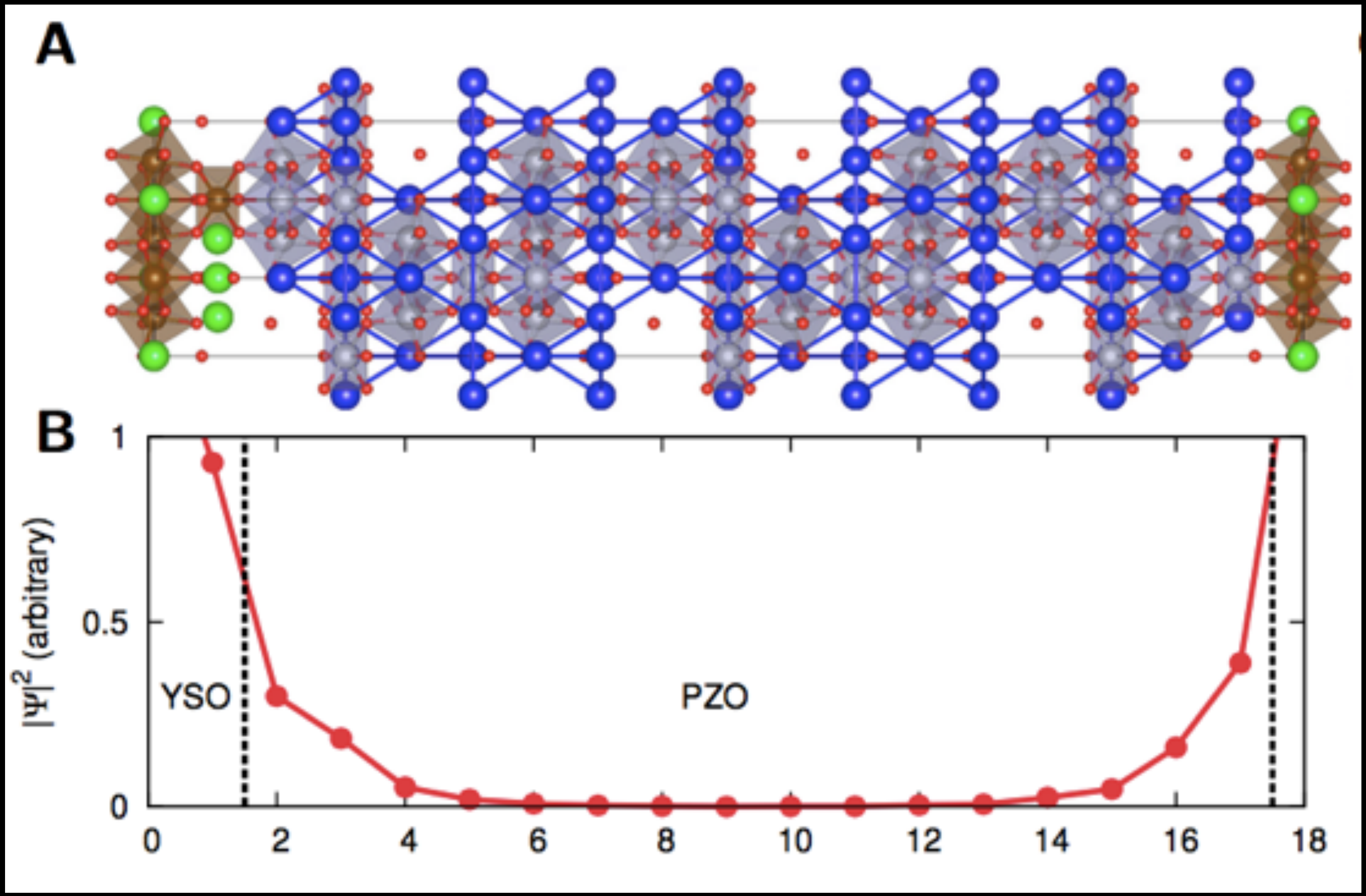
$Pr_2Zr_2O_7/Y_2Sn_{2-x}Sb_xO_7$ (111)

x=0.2

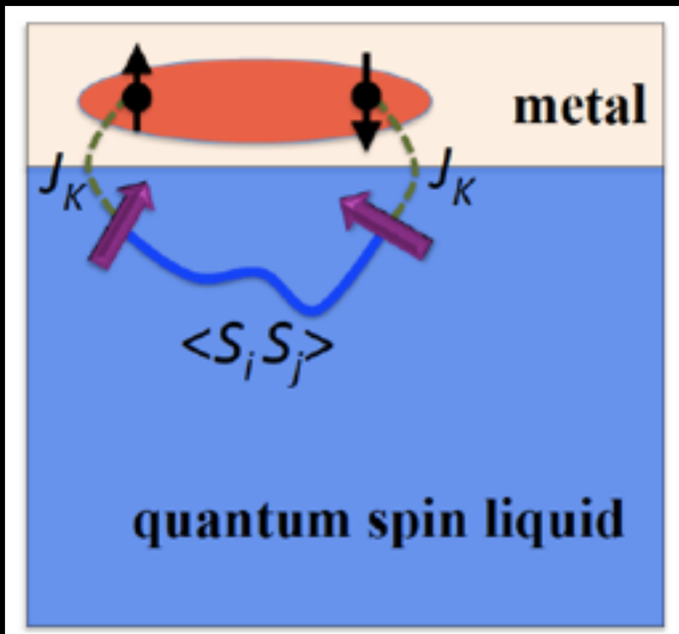
- Isotropic single pocket centered at Γ-point



Wave function penetration



Topological Superconductivity in Metal/Quantum-Spin-Ice Heterostructures



- Topological superconductor riding on QSL
- Selection Rule Dictated Intrinsic Topo SC.
- Substantial phase space.

Acknowledgements

Jian-huang She

Choonghyun Kim

Criag Fennie

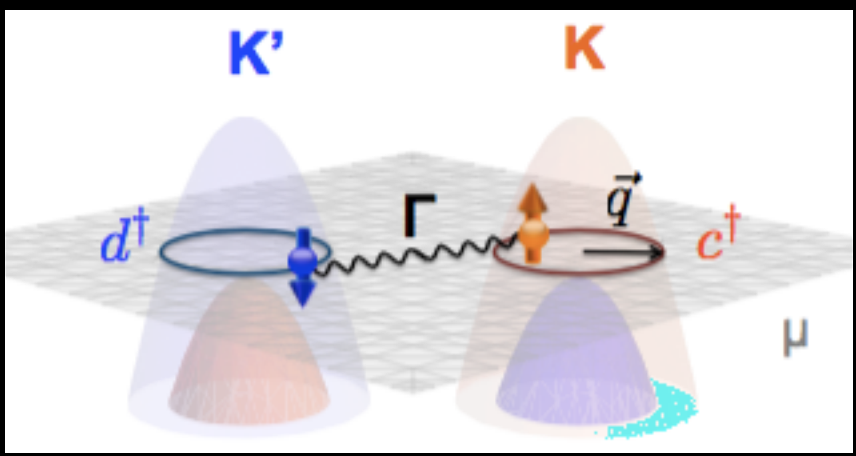
Michael Lawler

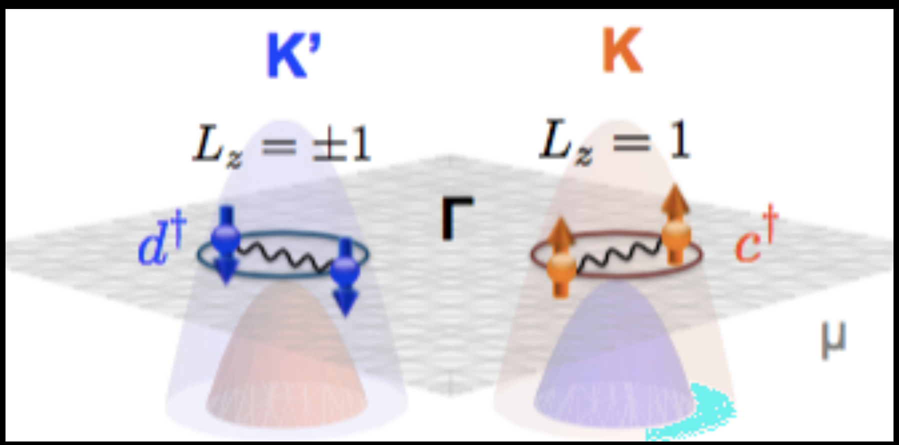
Funding: DOE, CCMR (NSF)

Strategy II

Manipulate the band structure

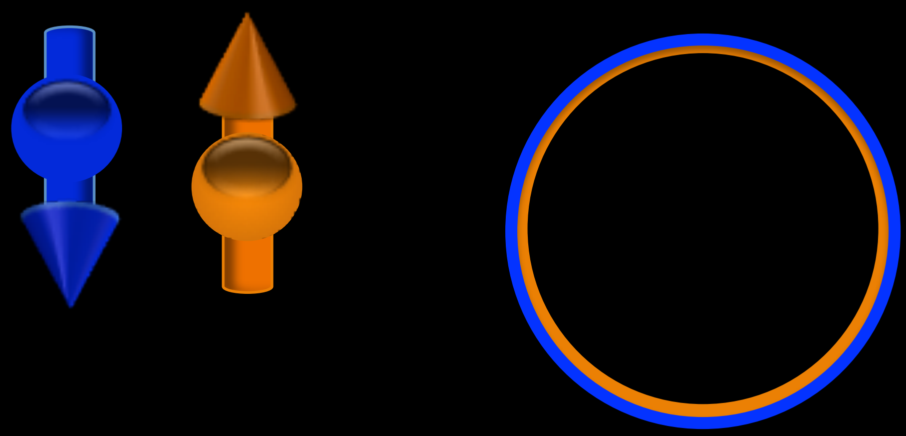
Topological superconductivity in group-VI TMDs





Yi-Ting Hsu, Abolhassan Vaezi, E-AK (arXiv:1606.00857)

Spin-degenerate Fermi surface



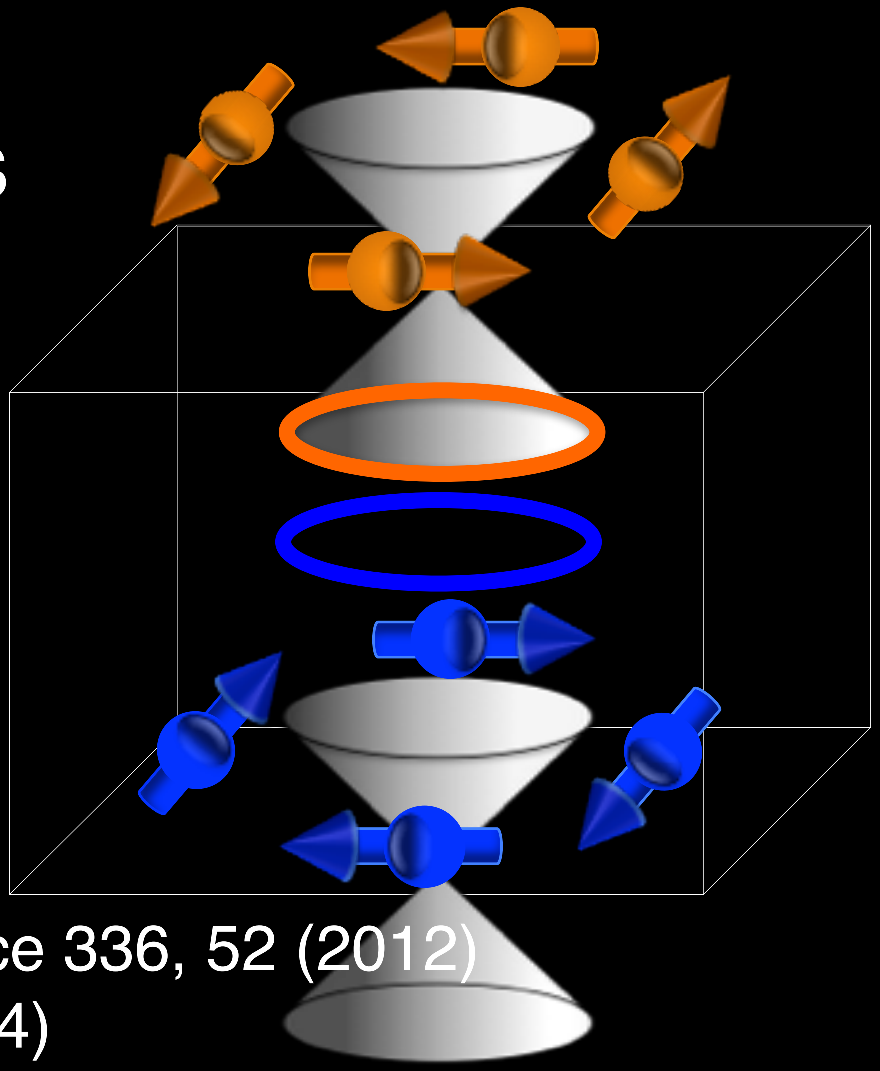
Singlet superconductor

Q. What if the band structure is spin-split?

Spinless fermion via real space splitting

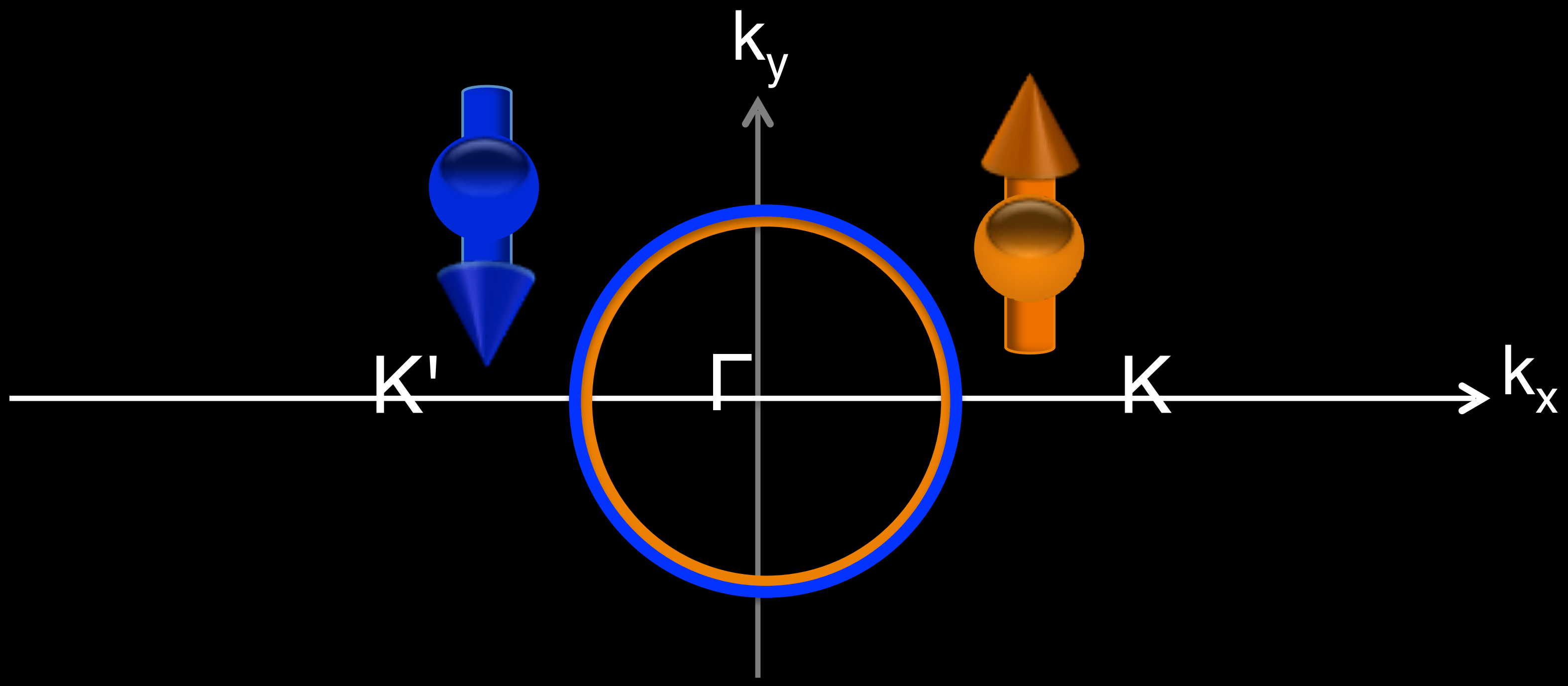
- TI surface states

- Proximity induce topo SC



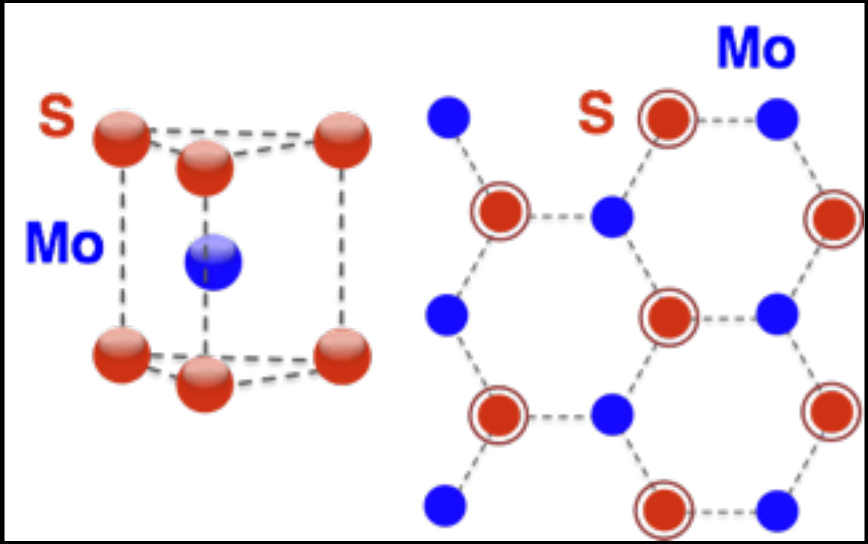
Fu & Kane, PRL (2008)
Experiments: Wang et al Science 336, 52 (2012)
Xu et al, Nat.Phys 10, 943 (2014)

Spinless fermion via k-space splitting?



Monolayer group VI TMD's

MoS_2, WS_2, $MoSe_2$, WSe_2



- **Non-centro symmetric**

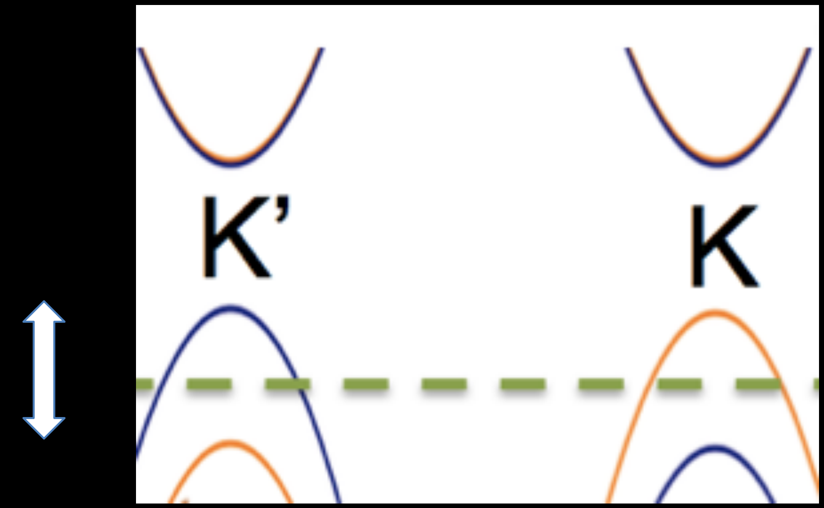
➡ Direct Gap ~2eV
➡ Dresselhaus spin-orbit

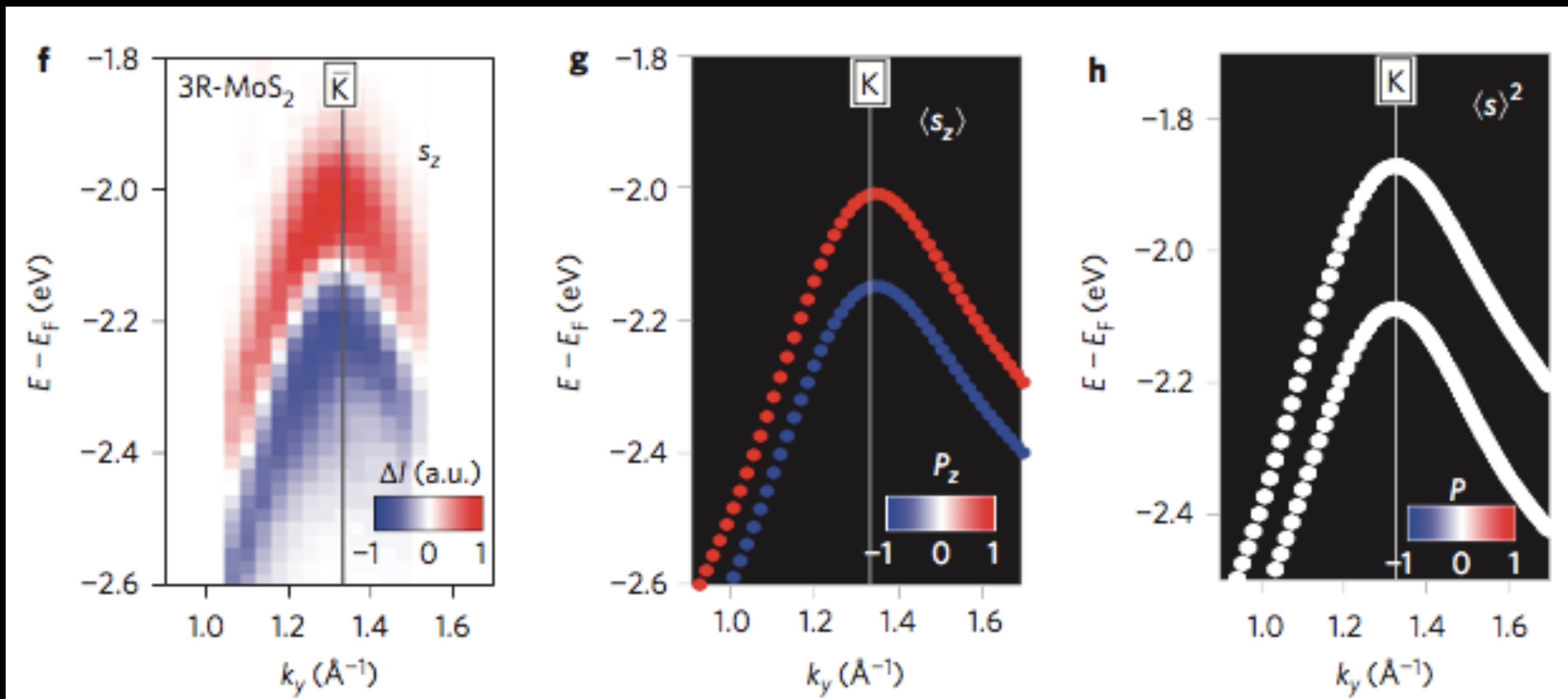
Band-selective spin-splitting

- Partially filled crystal-field-split d-bands
  - Conduction band $|d_{z^2}\rangle$: $l_z = 0$
  - Valence band $\frac{1}{\sqrt{2}}(|d_{x^2-y^2}\rangle \mp i|d_{xy}\rangle)$: $l_z = \mp 1$

- Spin-orbit coupling $\vec{L} \cdot \vec{S}$

150~460meV

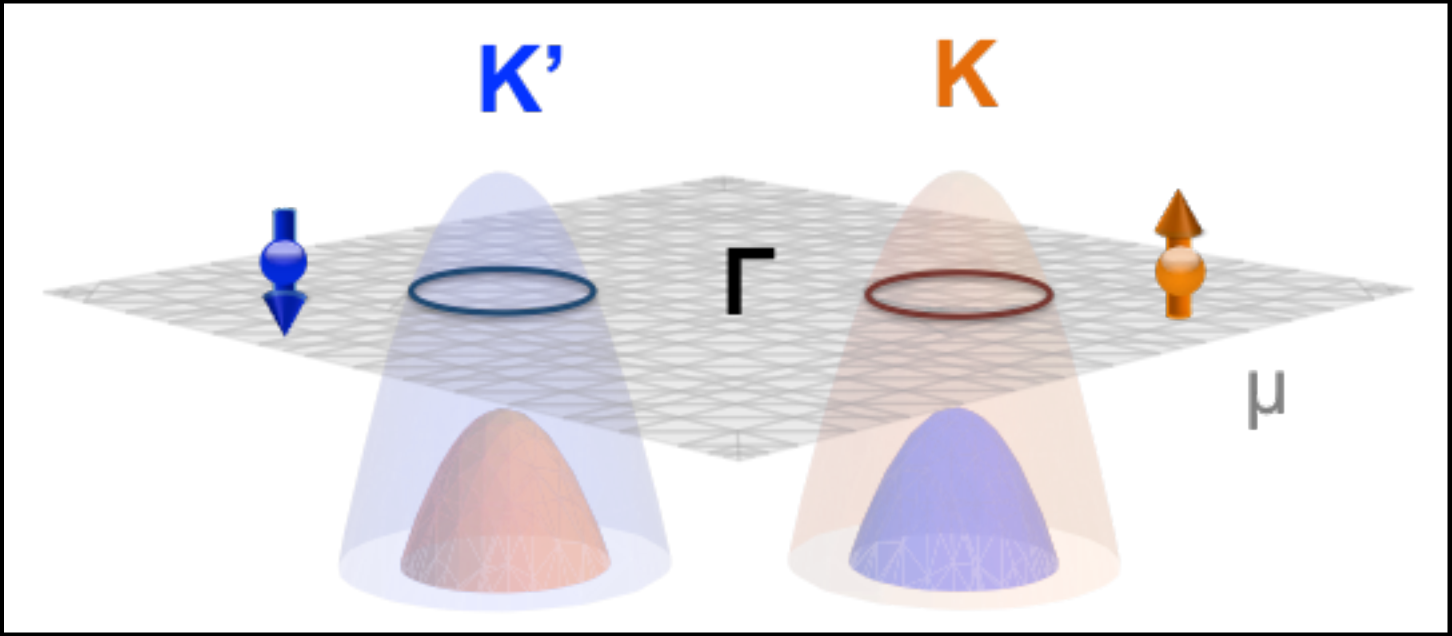




Iwasa group N. Nano (2014)

k-space spin-split FS?

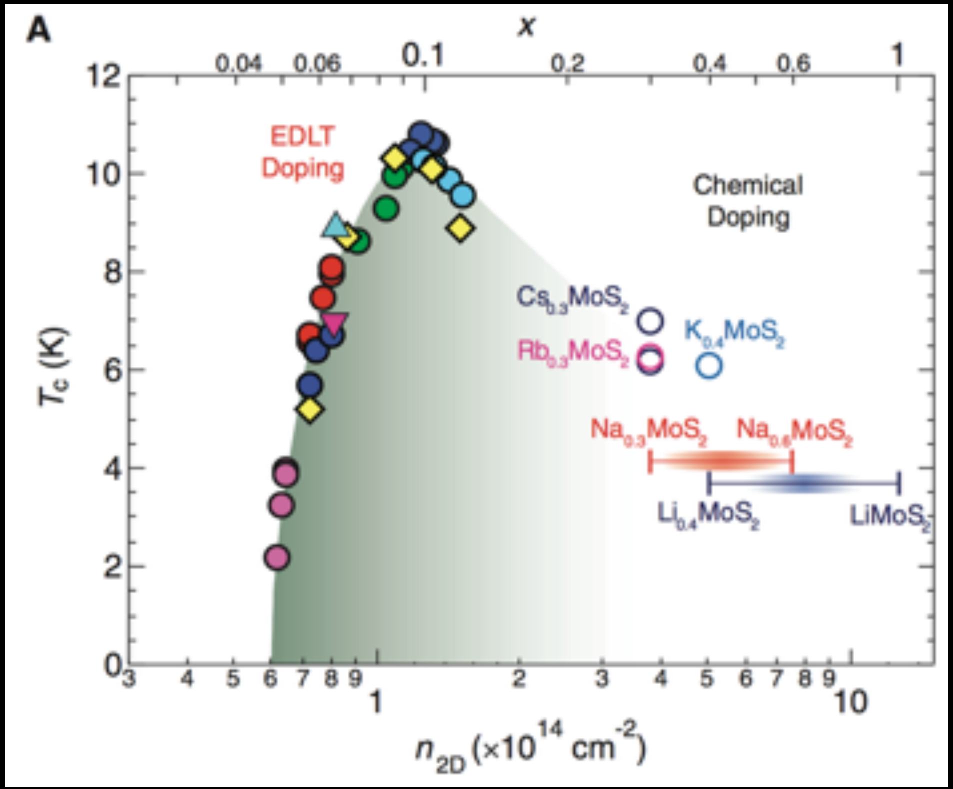
p-doped group VI- TMD!



Juice for superconductivity?

- d electrons => expect correlation effects
- n-doped

J.T.Ye *et al.* (Science 2012)



p-doped TMD

k-space spin-split Fermi surfaces

+

Moderate correlation (d-electron)

↓

Topological SC?

Yi-Ting Hsu

Mark Fischer

Abolhassan Vaezi

Model

- Kinetic term

$$H_0(\vec{q}) = at(\tau q_x \hat{\sigma_x} + q_y \hat{\sigma_y}) + \frac{\Delta}{2}\hat{\sigma_z} - \lambda\tau\hat{s_z} \otimes \frac{\hat{\sigma_z} - 1}{2}$$

Band-basis

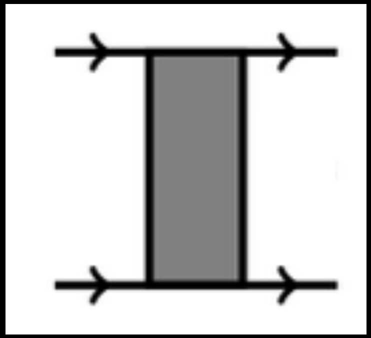
Spin-basis

- Repulsive interaction term

$$H'(W) = \sum_i U n_{i,\uparrow} n_{i,\downarrow}$$

Superconductivity out of repulsive interaction?

- Kohn-Luttiger: singularity in scattering amplitude $\Gamma(\vec{q})$



(Kohn &Luttinger 1965

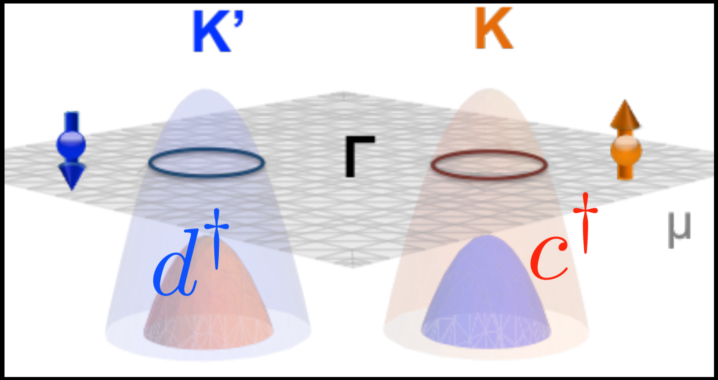
➡Non-s wave

- Two-step RG formulation
  : Fe-based SC, doped graphene, SrRu

Chubukov & Nandkishore, Raghu & Kivelson (2008 - 2012)

Two-step RG on p-doped TMD

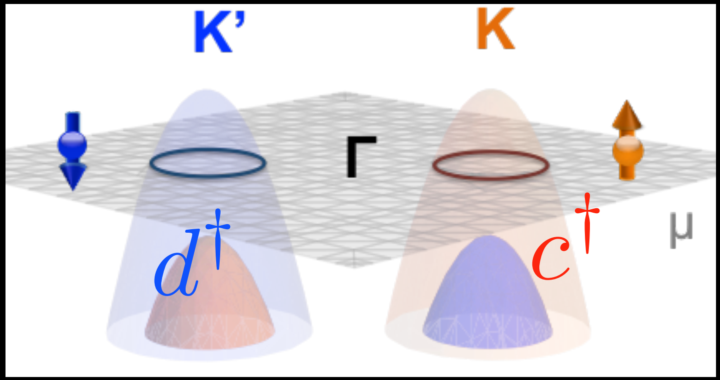
Step I: W -> Λ_0



- At scale W: Microscopic model
- At scale Λ_0: Effective model

$$H'_{eff}(\Lambda_0) = \sum_{\vec{q},\vec{q}'}{}' g^{(0)}_{\text{inter}}(\vec{q},\vec{q}')c^\dagger_{\vec{q}'}d^\dagger_{-\vec{q}'}d_{-\vec{q}}c_{\vec{q}} + g^{(0)}_{\text{intra}}(\vec{q},\vec{q}')d^\dagger_{\vec{q}'}d^\dagger_{-\vec{q}'}d_{-\vec{q}}d_{\vec{q}} + (c \leftrightarrow d)$$

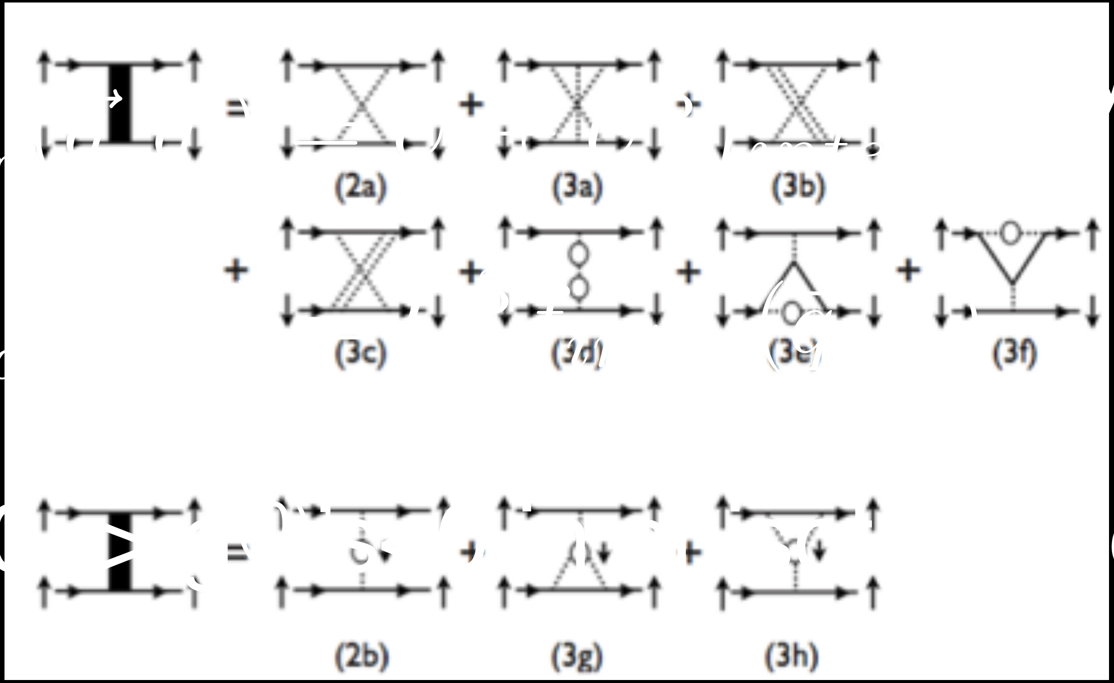
Step I: W -> Λ_0



- $g_{intra,0}$ and $g_{inter,0}$ at two-loop

$g^{(0)}_{inter}$ …)

$g^{(0)}_{intra}$ …



- f's <0 … channel

Step 2: Λ_0 ->0

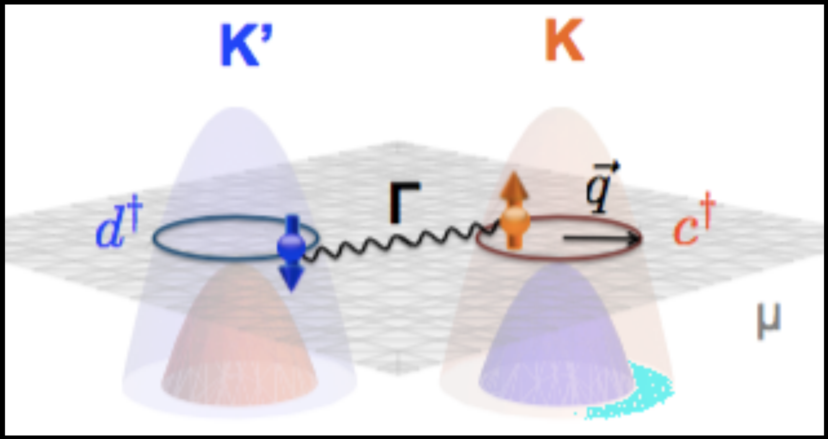
- RG flow $$\frac{d\lambda}{dy} = -\lambda^2$$ ⇒ $$\lambda(y) = \frac{\lambda^{(0)}}{1 + \lambda^{(0)} y}$$

$$y \equiv \nu_0 \mathrm{Log}(\Lambda_0/\mathrm{E})$$

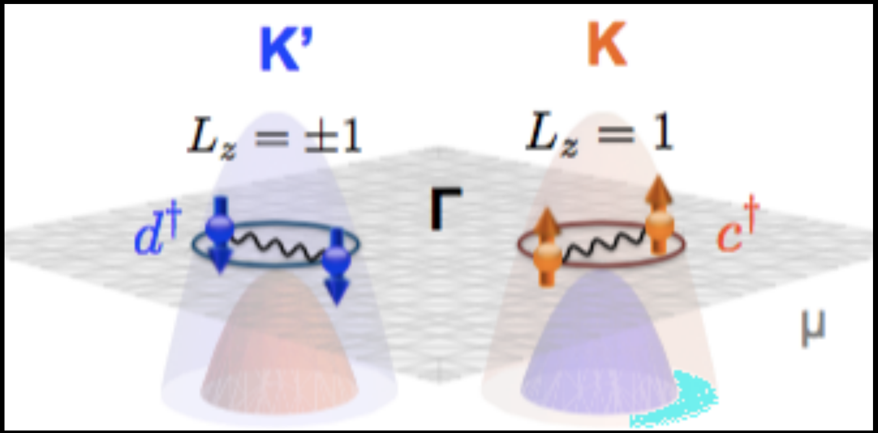
- Divergence if $\lambda^{(0)} < 0$

Two possibilities

- Intra-pocket p+ip



- Inter-pocket p'wave

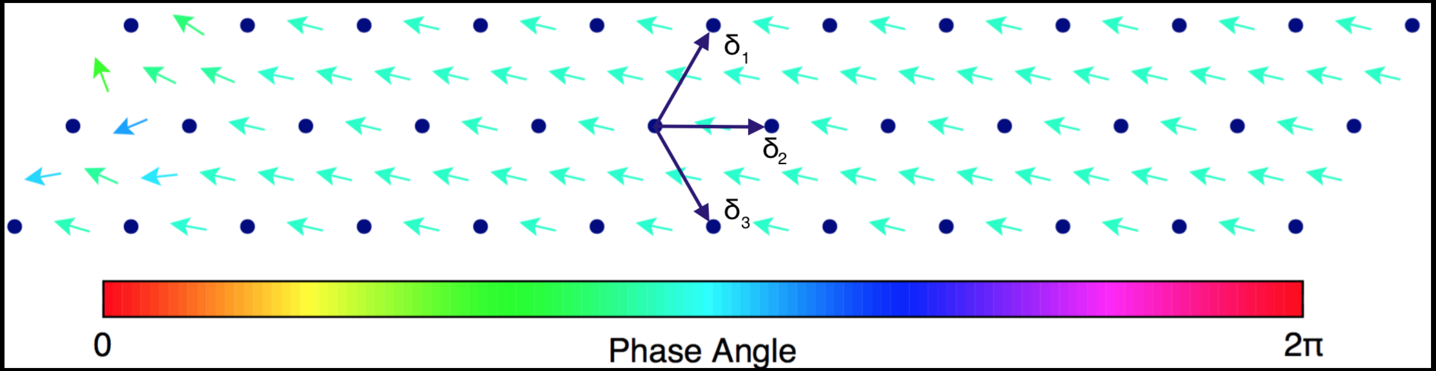


-T-breaking
- C=2

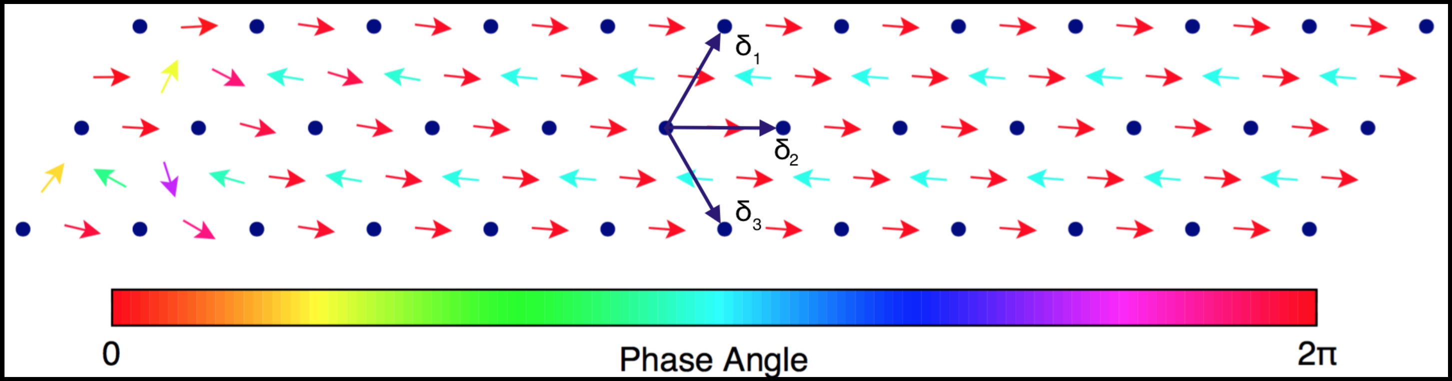
-Modulated
-C=\pm 1 per pocket

DMRG results

U=-2



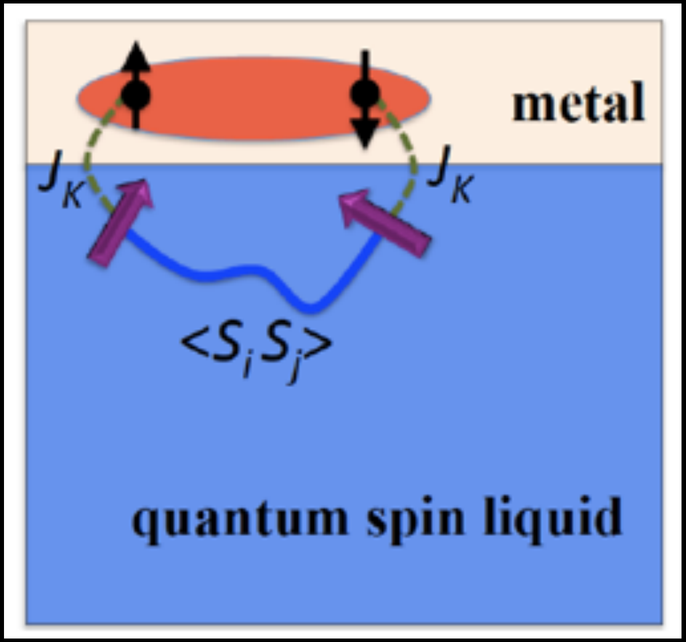
U=+4



Jordan Venderley, E-AK

Designing 2D topological SC's

- Control interaction



- k-space spin split TMD