7-9-18 Majorana-based Quantum Computing

Outline

I. Big Picture (non-Abelian anyons / TQC)

II. Toy Models
  A. Kitaev chain
  B. 2D topo SC

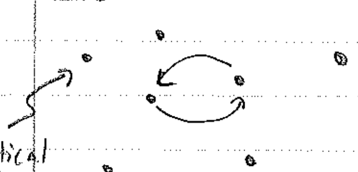
III. Experimental Blueprints

IV. Majorana Detection

V. Quantum state readout

VI. Qubit characterization

Big Picture



identical particles

Possible exchange statistics

$\psi \to \pm\psi$ (bosons/fermions; all elementary particles)

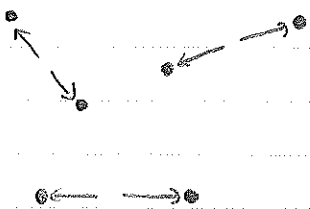
$\psi \to e^{i\alpha}\psi$ (Abelian anyons)

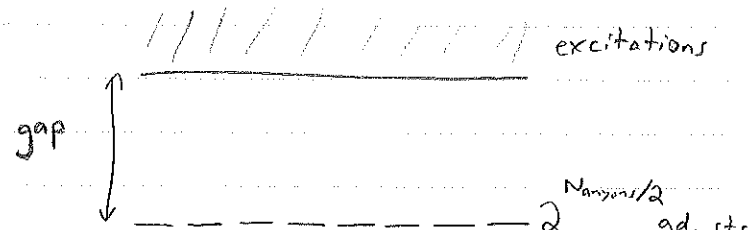
Focus here $\longrightarrow$ $\boxed{\psi_i \to U_{ij}\psi_j \text{ (non-Abelian anyons)}}$

(Ising) non-Abelian anyons

Three hallmarks:

1. Ground state degeneracy





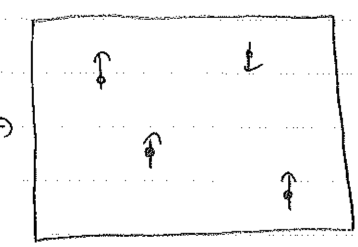
locally indistinguishable, robust!

$\Rightarrow$ anyons carry unusual zero-energy D.O.F. — "Majorana zero modes"

$\boxed{\text{Fault-tolerant qubit states}}$

[motivate through absence of dephasing]

cf. free spin-$\frac{1}{2}$'s

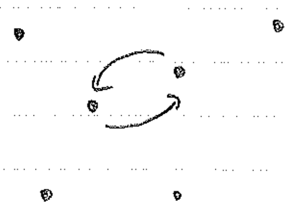


$\Rightarrow$ $2^{N_{spin}}$ gd. sts., but qualitatively different!

- locally distinguishable, not robust

[e.g., measure magnetizations, apply Zeeman field]

2. Non-Abelian statistics



adiabatic braid $\Rightarrow$ $\boxed{\psi_i \to U_{ij}\,\psi_j}$

braid matrix — rigid!

gd. sts.

[i.e., depends on topology of braid, not detailed path chosen]

$\boxed{\text{Fault-tolerant quantum gates}}$

3. Nontrivial fusion

$\bullet \longrightarrow \longleftarrow \bullet$

can "fuse" in multiple ways

$$\boxed{\overbrace{\sigma \times \sigma}^{\text{2 Ising anyons...}} = I + \psi}$$

... or form a fermion

... can annihilate

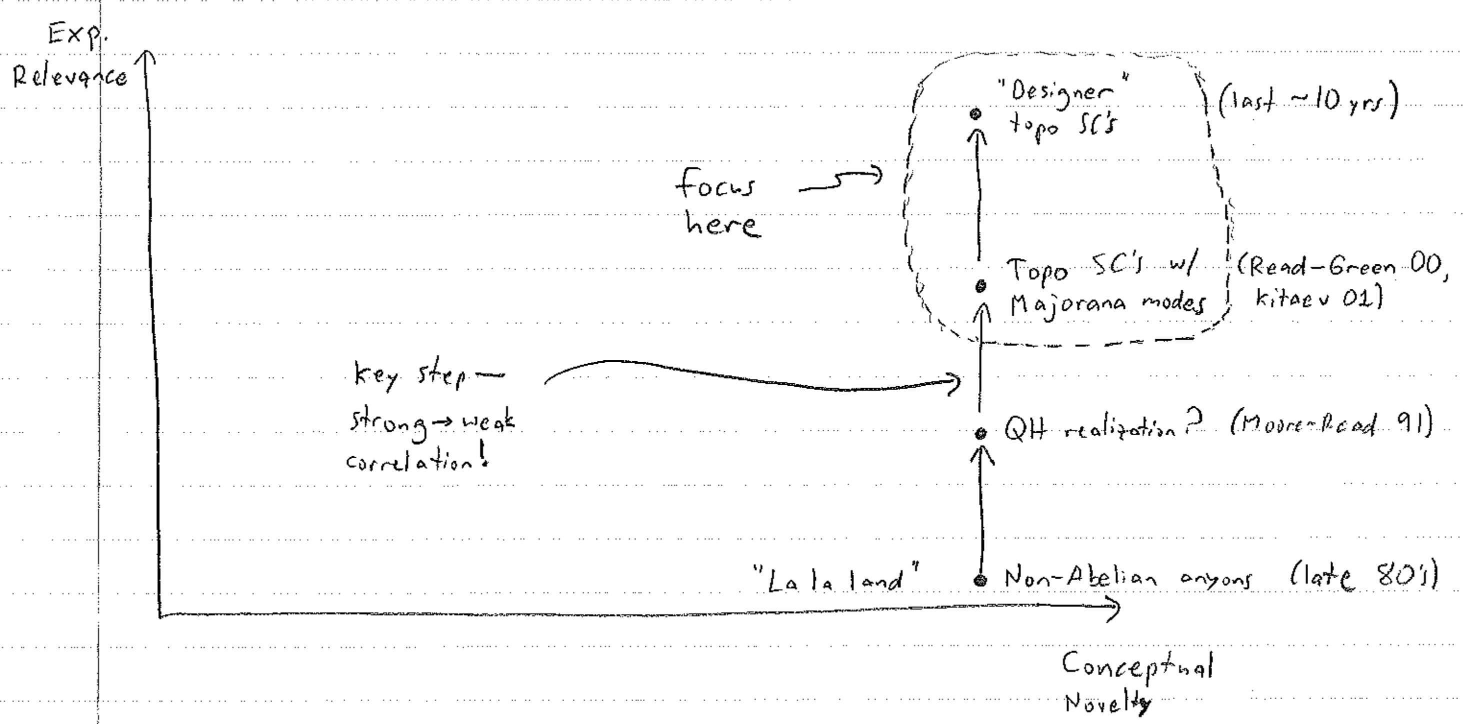
[Note that non-Abelian statistics $\Leftrightarrow$ nontrivial fusion; these properties also rely critically on peculiar gd. st. deg.]

$\boxed{\boxed{\text{Readout}}}$

Kitaev — exploit for "topological quantum computing"

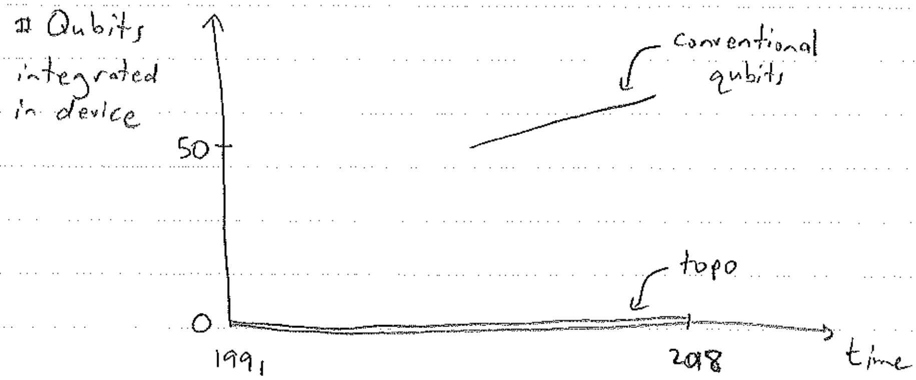
But how to build the hardware???

"Fisher plot"



[QH, topo SC's realize slightly different non-Abelian-anyon physics; SC's harbor "extrinsic non-Abelian defects".]

Where are we now?



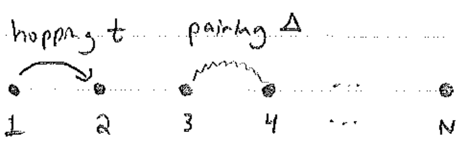
Motivation for TQC:

(1) Awesome physics, discoverable even @ 1-qubit level [efforts underway, e.g., @ MS]

(2) Mitigating error correction [which is daunting]

Toy Models I: Kitaev chain

Spinless fermions on N-site chain w/ P.B.C.'s (for now).



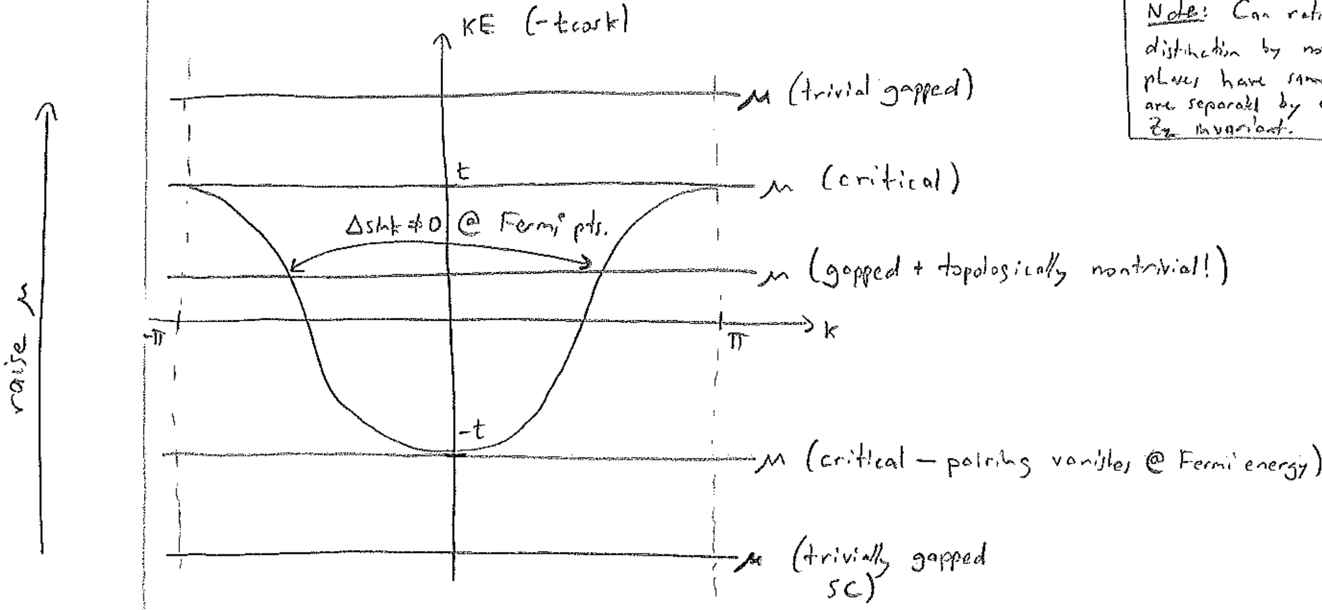
$$\boxed{H = \sum_x \left[-\mu c_x^\dagger c_x - \frac{1}{2}\left(t c_x^\dagger c_{x+1} + \Delta c_x c_{x+1} + h.c. \right) \right]}$$

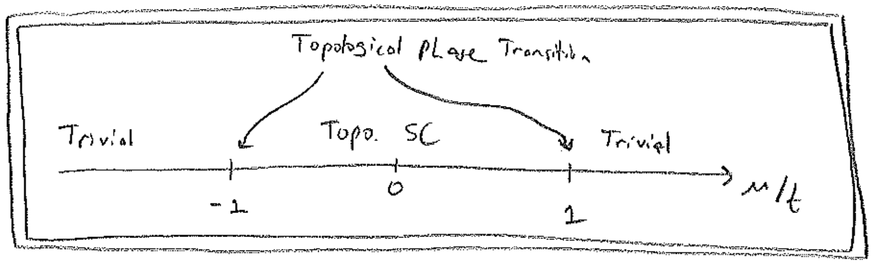
P.B.C. ⇒ go to k-space,

$$\boxed{H = \sum_{k \in BZ} \left[(-\mu - t\cos k)\, c_k^\dagger c_k + \left(\frac{i\Delta}{2} \underbrace{\sin k}_{\substack{\text{odd parity} \\ \text{(required by} \\ \text{spinlessness)}}} c_k c_{-k} + h.c. \right) \right]}$$

Phase Diagram as f^n of μ?

Note: Can rationalize topo distinction by noting that gapped phases have same symmetry, yet are separated by critical pt. Mention $\mathbb{Z}_2$ invariant.





Want to explore <u>universal</u> properties of phases/transitions. Convenient to take

$\boxed{\Delta = t}$ hereafter.

hopping

$$\Rightarrow \boxed{H = \sum_x \left[-\mu c_x^\dagger c_x - \frac{1}{2} t \left(c_x^\dagger + c_x\right)\left(c_{x+1} - c_{x+1}^\dagger\right)\right]}$$

pairing

Gapped phases

Take open B.C.'s now + use Majorana rep.:

$$\boxed{c_x = \frac{1}{2}\left(\gamma_{B,x} + i\gamma_{A,x}\right)}$$

Majorana ops.
(only pairs have well-defined occupation #'s)

$$\boxed{\begin{gathered}\gamma_\alpha = \gamma_\alpha^\dagger, \quad \gamma_\alpha^2 = I \\ \{\gamma_\alpha, \gamma_{\alpha' \neq \alpha}\} = 0\end{gathered}}$$

(Majorana op. algebra — reproduces $c_x^2 = (c_x^\dagger)^2 = 0$, $\{c_x, c_{x'}^\dagger\} = \delta_{x,x'}$)

[Note: this is always a legitimate rep. of any ordinary fermion op, like c_x. Does not however guarantee that a system supports Majorana-like excitations as we'll see!]

Rewrite H:

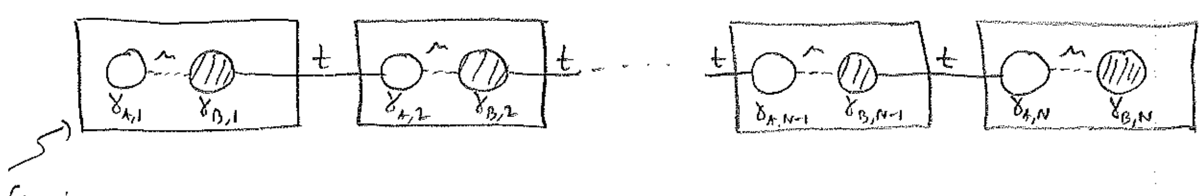
$$c_x^\dagger c_x = \frac{1}{4}\left(\gamma_{B,x} - i\gamma_{A,x}\right)\left(\gamma_{B,x} + i\gamma_{A,x}\right)$$

$$= \frac{1}{4}\left(1 + 1 + i\gamma_{B,x}\gamma_{A,x} - i\gamma_{A,x}\gamma_{B,x}\right)$$

$$= \frac{1}{2}\left(1 + i\gamma_{B,x}\gamma_{A,x}\right)$$

$$\boxed{(c_x^\dagger + c_x)(c_{x+1} - c_{x+1}^\dagger) = i\gamma_{B,x}\gamma_{A,x+1}}$$

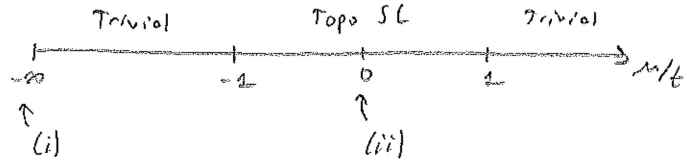
$$\Rightarrow \boxed{H = -\frac{i}{2}\sum_x\left(\mu\gamma_{B,x}\gamma_{A,x} + t\gamma_{B,x}\gamma_{A,x+1}\right)}$$



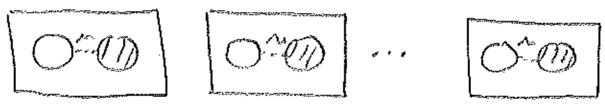
c_x fermions

Majorana chain w/ competing dimerizations μ, t!

For revealing snapshots of gapped phases, examine 2 limits:

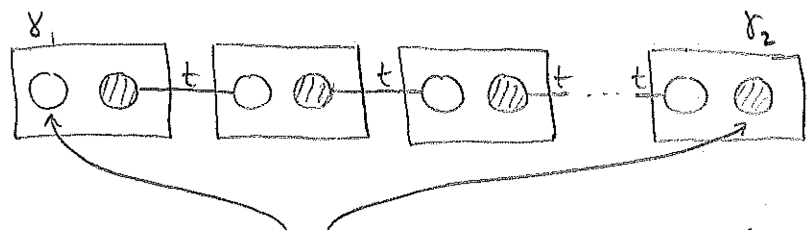


(i) $\mu < 0,\ t = 0$



Trivial product state w/ no entanglement between sites. Unique gd. state. (Vacuum of c_x fermions.)

(ii) $\mu = 0,\ t > 0$



unpaired Majorana ops — zero modes!
[fermion split into two well-separated halves]

Several comments in order:

- $\boxed{\gamma_1 = \frac{c_1 - c_1^\dagger}{i}, \quad \gamma_2 = c_N + c_N^\dagger}$. $\boxed{[H, \gamma_{1,2}] = 0}$

- Non-local fermion $\boxed{d = \frac{\gamma_1 + i\gamma_2}{2}}$ can be filled, empty w/ no energy cost.

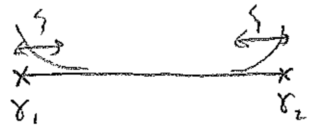
  $\Rightarrow$ 2-fold topological gd. state deg.!

  $\boxed{|0\rangle, \ d^\dagger|0\rangle \equiv |1\rangle}$

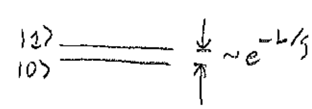
  carry opposite fermion parity

  [Very unusual — most SC's prefer even parity so that all e^-'s can pair. Excitation energy for unpaired e^- vanishes for topo. reason, here.]

- For $\mu \neq 0, t \neq \Delta$ zero modes decay exponentially into bulk



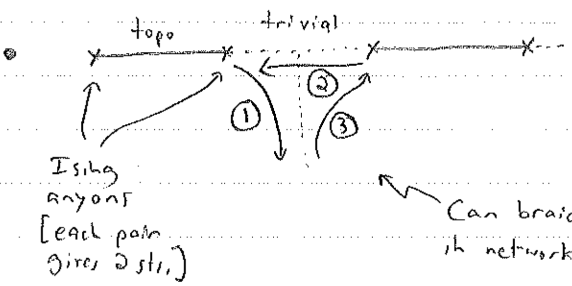
$\Rightarrow$



[most statements here should be understood as "up to exponentially small corrections"]

- Deg. robust to local perturbations; local measurements can't distinguish gd. sts.
  [Fermion parity not locally detectable! contrast to deg. ↑,↓ spin states]

- Ends of topo SC ≃ Ising non-Abelian anyons; γ's are "internal" D.O.F. encoding deg.
  [shouldn't view them as particles!]

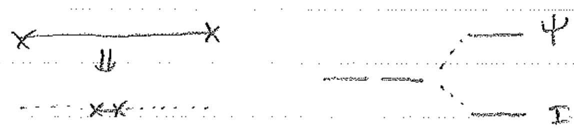
- 

$$\boxed{\psi_i \to U_{ij}\psi_j}$$

<u>HW</u>: Use fermion parity conservation, locality to argue that $\boxed{U = e^{\pm\frac{\pi}{4}\gamma_a\gamma_b}}$ for γ_a, γ_b swap.

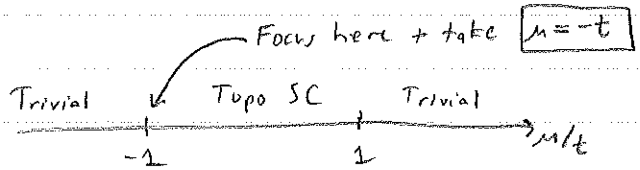
[Braids exchange "half" of one fermion w/ "half" of another — hence nontriviality.]

- Braiding alone <u>not</u> universal [Deficit of Ising; missing gates have high error tolerance]

- 

fusion channels!

<u>Topo Phase Transition</u>



Low-energy physics @ criticality?

$$H \to \boxed{H_{crit}} = \frac{-it}{2}\sum_x\left(-\gamma_{B,x}\gamma_{A,x} + \gamma_{B,x}\gamma_{A,x+1}\right) = \frac{-it}{2}\sum_x \gamma_{B,x}\left(\gamma_{A,x+1} - \gamma_{A,x}\right)$$

$$\sim \frac{-it}{2}\int_x \gamma_B \partial_x \gamma_A$$

continuum limit

Write $\boxed{\gamma_{A/B} = \gamma_R \pm \gamma_L} \Rightarrow H_{crit} = \frac{-it}{2}\int_x (\gamma_R - \gamma_L)\partial_x(\gamma_R + \gamma_L)$

$$= \frac{-it}{2}\int_x \Big(\gamma_R\partial_x\gamma_R - \gamma_L\partial_x\gamma_L + \underbrace{\gamma_R\partial_x\gamma_L - \gamma_L\partial_x\gamma_R}_{\text{cancel}}\Big)$$

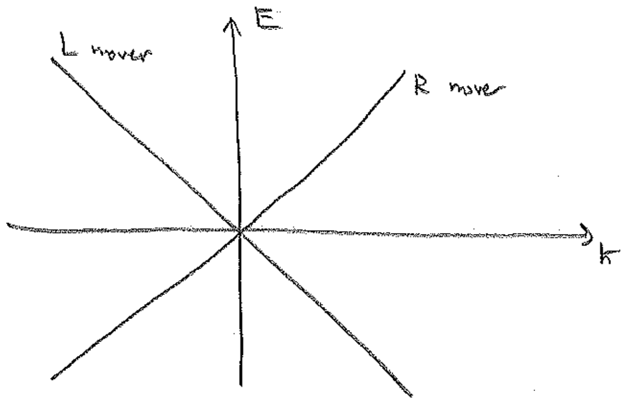
So we get $\boxed{H_{crit} = \int_x \left(-iv\gamma_R\partial_x\gamma_R + iv\gamma_L\partial_x\gamma_L\right)}$ $(v \propto t)$

chiral, gapless Majorana fermions!

Go to k-space: $\boxed{\gamma_{R/L}(x) = \int_k e^{ikx}\gamma_{R/L}(k)}$ [Implicitly thinking about P.B.C.'s again.]

Hermicity $\Rightarrow$ $\boxed{\gamma_{R/L}(k) = \gamma^{\dagger}_{R/L}(-k)}$ (*)

$$\Rightarrow \boxed{H_{crit} = \int_k \left[vk\,\gamma_R^{\dagger}(k)\gamma_R(k) - vk\,\gamma_L^{\dagger}(k)\gamma_L(k)\right]}$$



not distinct from E>0 levels.

By (*), E>0, E<0 states not distinct!

$\Rightarrow$ "half" of usual single-channel wire.

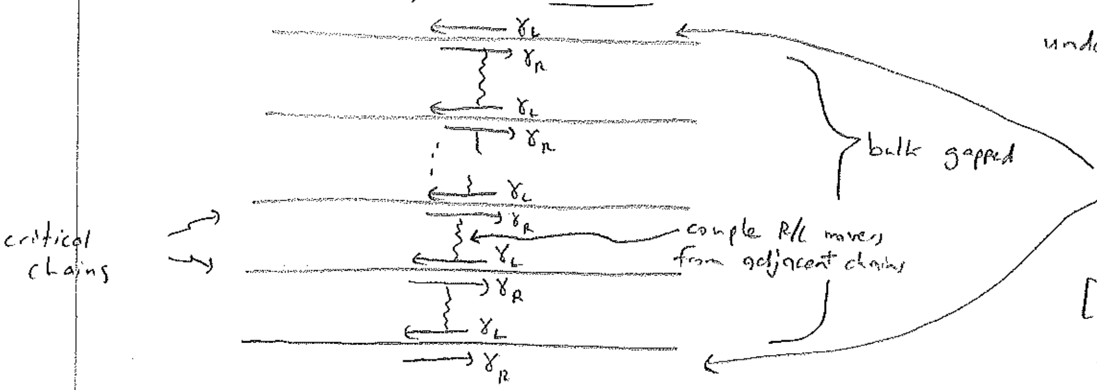
[can be made very precise; e.g., thermal transport exactly half that in a usual wire ($c = \frac{1}{2}$ vs. 1).]

So Majorana physics in Kitaev chain appears in 2 ways:

(i) Localized zero modes in topo. phase
(ii) Gapless propagating deg. of freedom at criticality.

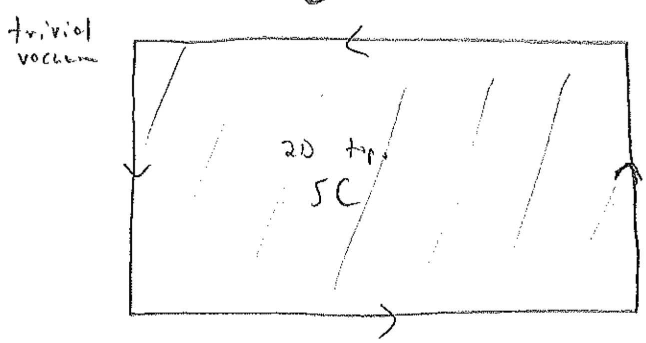
Toy Models II: 2D Topo. SC's

Build from array of critical Kitaev chains [efficiently leverages our understanding of 1D case.]



[hallmark of 2D topo SC. Note similarity to nontrivial dimerization in Kitaev chain.]

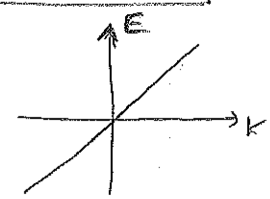
$\Downarrow$



edge coord. / chiral Majorana field

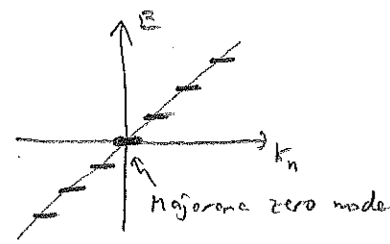
$$\boxed{H_{edge} = \int dx \left(-iv\,\gamma\,\partial_x \gamma\right)}$$

$$\boxed{E = vk}$$



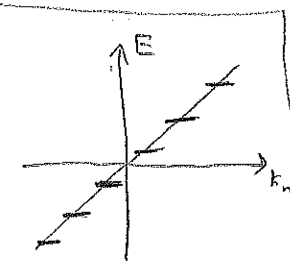
Important Q: Spectrum for finite perimeter L?

i.e. is k quantized to (i) $k_n = \frac{2\pi}{L} n$ (PBC)



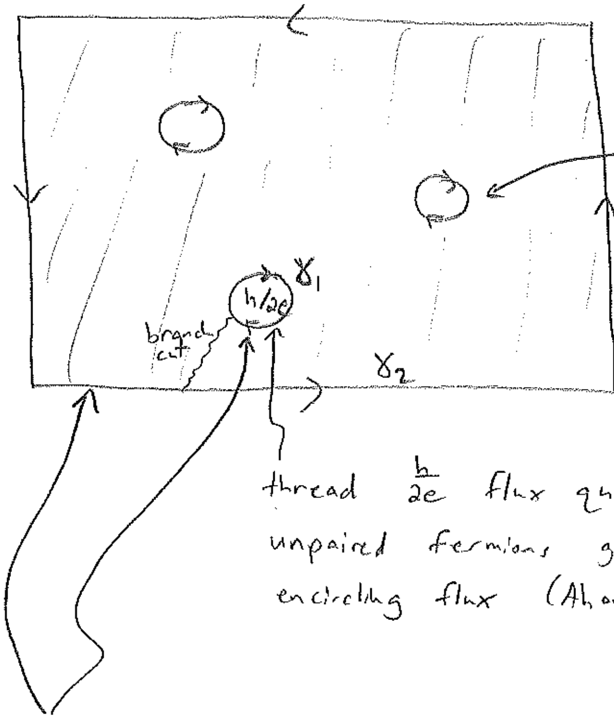
or

(ii) $k_n = \frac{2\pi}{L}(n+\frac{1}{2})$ (anti-PBC)



correct answer; PBC ruled out because you can't have just one Majorana zero mode! [Hilbert space wouldn't make sense.]

Drill holes:



thread $\frac{h}{2e}$ flux quantum;
unpaired fermions get − sign when encircling flux (Aharanov−Bohm)

Majorana B.C.'s shift from anti-periodic → periodic!
⇒ zero modes γ_1, γ_2!

Lessons: (i) boundaries of 2D topo SC host chiral Majorana fermions [cousins of Majorana end-states in 1D]

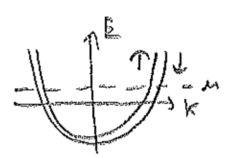
(ii) $\frac{h}{2e}$ flux localizes a Majorana zero mode [deeply related to (i)!]
+ ∴ forms Ising non-Abelian anyon

Experimental Blueprints

Wanted: "Spinless" 1D, 2D SC's ← [both harbor Ising anyons, albeit in different ways]

Challenges: (1) We live in 3D

(2) e^-'s carry spin



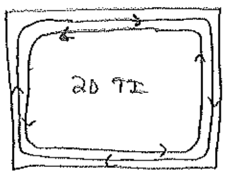
← Need to remove half of the carriers.

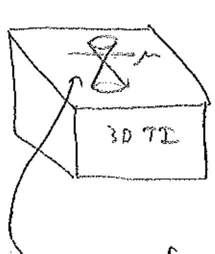
(3) Nearly all SC's arise from spin singlet Cooper pairs.

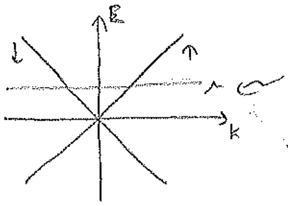
Likely no "intrinsic" realizations in solid state − despite 1000's of known SC's!

Can instead "engineer" topo SC's! O(100) papers revealing various strategies. Most follow a common recipe:

Step I <u>Use TI boundary</u> <u>or</u> <u>Break $\mathcal{I}$ in low-D systems w/ SOC</u>

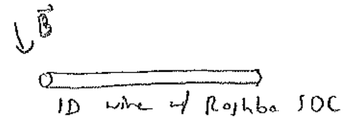


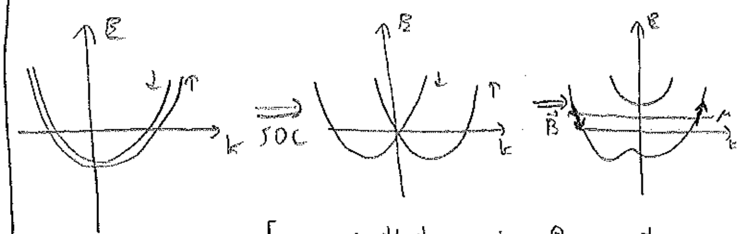




one pair of Fermi pts./one Fermi surface as desired

e.g.,



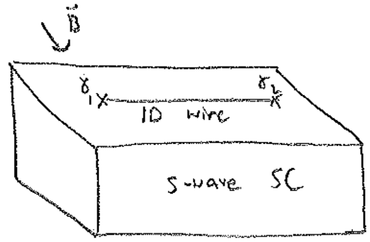


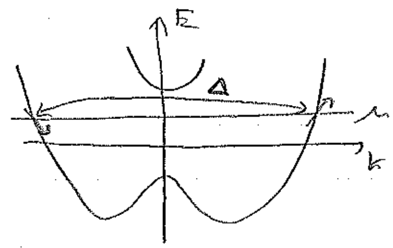
[crucial that carriers @ μ not spin polarized; c.f. 2D TI]

Solves challenges 1, 2! [in a way that makes challenge 3 "easy"]

Step II Couple systems above to <u>s-wave</u> SC.

e.g.,





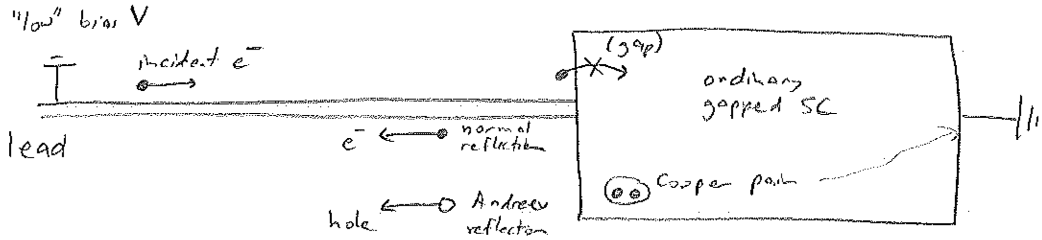
"proximity effect" drives 1D, 2D topo SC!!

Lots of expts followed, but first...

Majorana Detection*

Focus on tunneling methods – most common so far.

Primer — SC tunneling as scattering problem



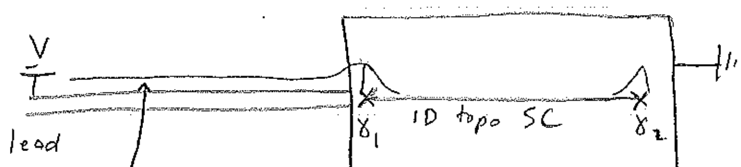
Conductance $\boxed{G = \frac{dI}{dV} = 2 \times \frac{e^2}{h} \times (\text{Andreev ref. prob.})}$

reflects pair injection — conductance quantum

Solve just like potential barrier scattering in intro QM
– get wavefn, extract normal/Andreev ref. coefficients.

skip

Result for topo SC



γ_1 delocalizes into lead

[generic property of localized modes coupled to gapless deg. of freedom]

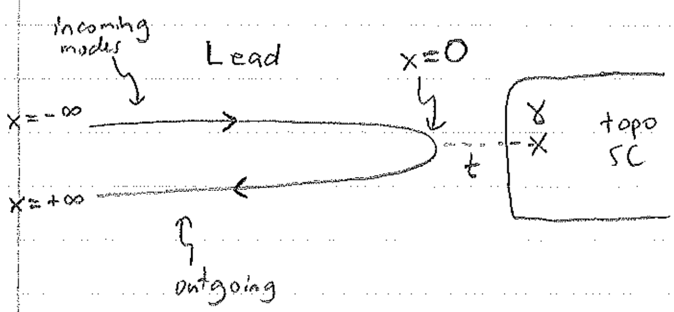
$\Rightarrow$ low-energy wavefn acquire equal e^-/hole character

$\Rightarrow$ Majorana-mediated "perfect Andreev reflection"!

$$\boxed{G = \frac{2e^2}{h} \quad (V \to 0)}$$

*We'll probe the existence of Majorana modes (which is possible w/ local measurements) and not the quantum info they encode!

Topo SC case



[γ could arise from 1D or 2D topo SC.]

$$\boxed{\begin{aligned} H &= H_{lead} + H_t \\ H_{lead} &= \int_x \left[-iv\psi^\dagger \partial_x \psi\right] \\ H_t &= t\gamma\left[\psi(x=0) - \psi^\dagger(x=0)\right] \end{aligned}}$$

HW: Diagonalize, find Andreev ref. prob. for any energy.

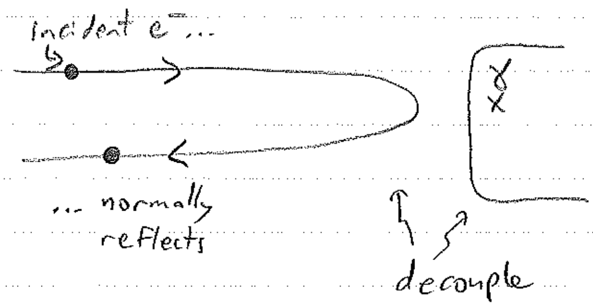
We'll look @ $E \gg \frac{t^2}{v}$, $E \ll \frac{t^2}{v}$

[t^2/v looks strange, but is dimensionally correct]

- $E \gg \frac{t^2}{v}$

Here $H \simeq H_{lead}$ $\Rightarrow$ plane-wave eigensts. created by

$$\boxed{\Gamma_E^\dagger = \int_x e^{i\frac{E}{v}x}\,\psi^\dagger(x)}$$



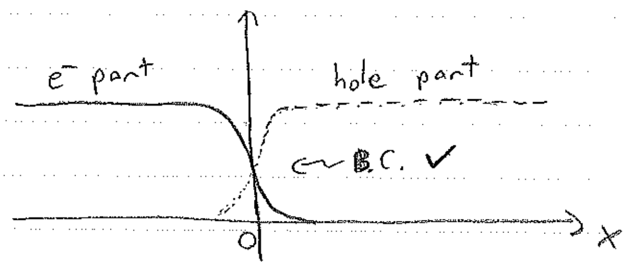
$\Rightarrow$ $\boxed{G(E \gg \frac{t^2}{v}) \simeq 0}$

[could get $G(E \gg \frac{t^2}{v})$ non-zero if we included more terms beyond t though; this result is not universal contrary to $E \to 0$ limit]

- $\underset{\sim\sim}{E \ll \frac{t^2}{v}}$ "Large" H_t imposes $\boxed{\psi(x=0) = \psi^\dagger(x=0)}$ B.C.'s!

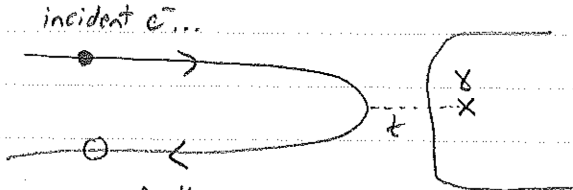
$\Rightarrow$ plane waves w/ $e^- \leftrightarrow$ hole conversion @ $x=0$,

$$\boxed{\Gamma_E^\dagger = \int_x e^{i\frac{E}{v}x}\left[\Theta(-x)\psi^\dagger(x) + \Theta(x)\psi(x)\right]}$$



[or, invert to get $\psi(x)$ in terms of Γ_E's; will see $\psi(x=0) = \psi(x=0)^\dagger$.]

incident e^-...



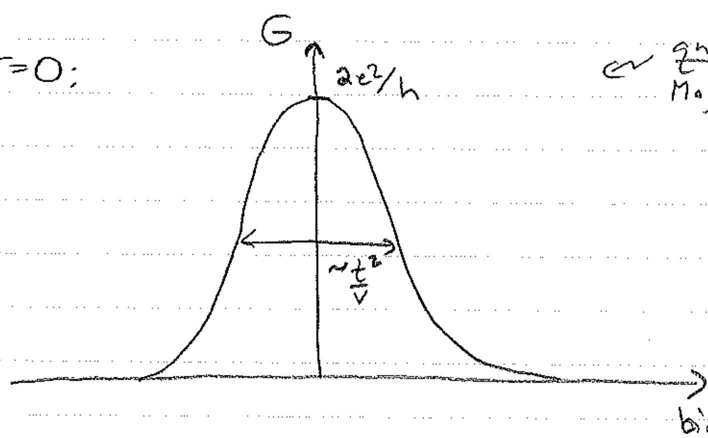
... perfectly Andreev reflects due to γ!

$$\Rightarrow \boxed{G(E \to 0) = \frac{2e^2}{h}}$$

universal!

Summary

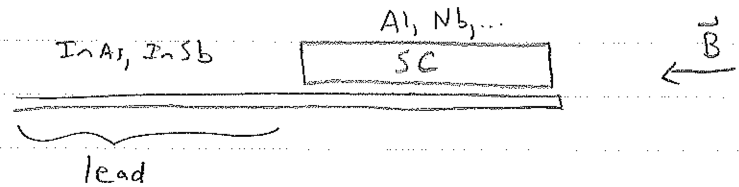
$T=0$:

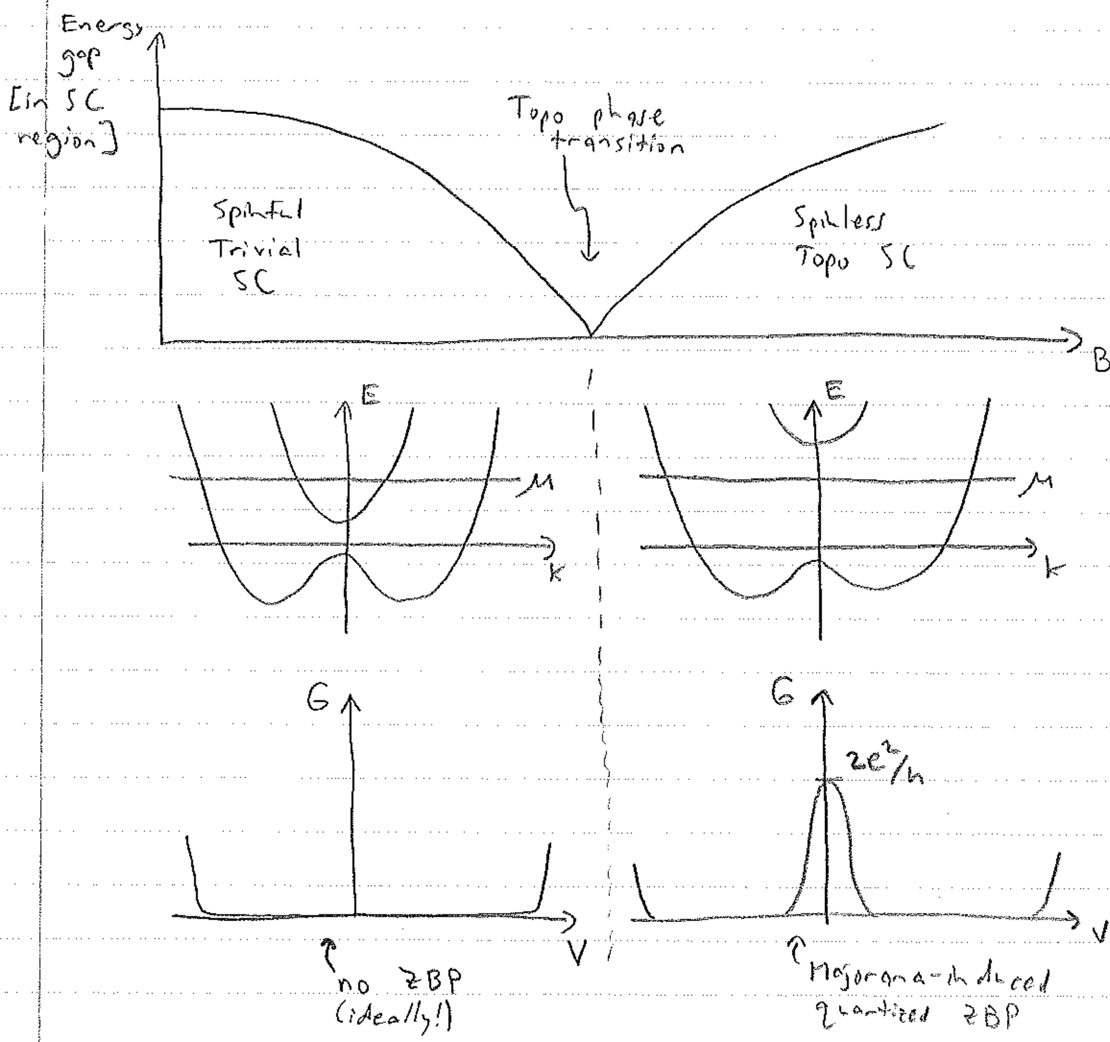


quantized ZBP — appealing Majorana signature

Experiments

- 1D wires (Kouwenhoven, Heiblum, Marcus, van Harlingen, ...)

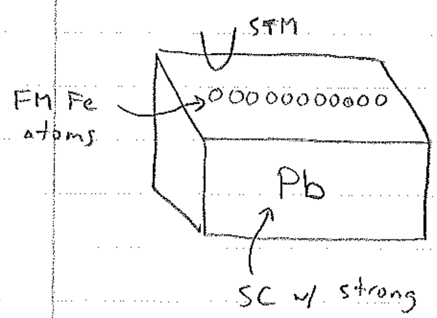




Pros — high-quality devices, many gps. see expected ZBP — even quantized (as of 2018)

Cons — phase transition usually not seen, only probes wire ends, hard to distinguish Majorana ZBP from trivial "Andreev bound st." ZBP, tuning required to access topo phase

• Fe chains (Yazdani, Berlin gp., ...)



No B-field required, topo phase "automatic", spatial resolution of G, ZBP's seen @ edges [non-quantized].

Path forward? ⤳ Interesting problem!

• Vortices in Fe-based SC's (see, eg, arXiv: 1807.01278)

Interesting systems / ZBP data... [similar to Fu-Kane 3DTI story, but w/ intrinsic superconductivity]

After Majorana detection, natural to ask a subtler question:

How to reveal information encoded in Majoranas?

[Now looking ahead to future expts.]

Quantum State Readout

2 strategies:

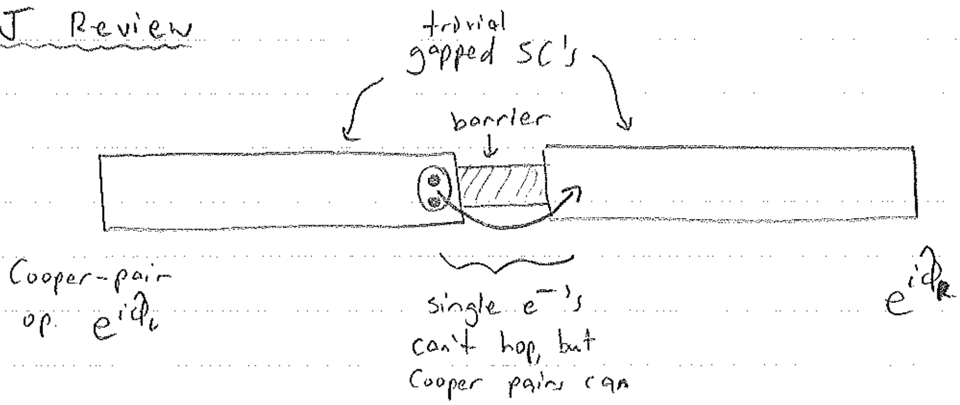
(i) Spoil degeneracy or (ii) interference expt.

[converts locally indistinguishable states → locally distinguishable] [essentially non-local measurement]

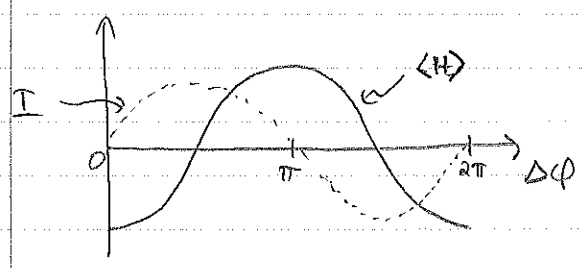
Look @ examples of both, focusing on 1D wires.

Fractional Josephson Effect

JJ Review

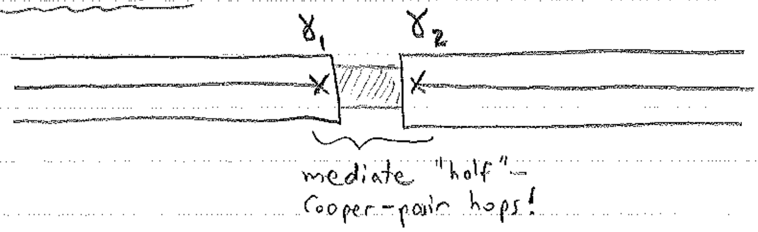


$$\boxed{H = -J\cos(\hat\phi_L - \hat\phi_R)} \Rightarrow \text{current} \quad \boxed{I = \frac{2e}{\hbar}\frac{d\langle H\rangle}{d\Delta\phi} \propto \sin(\Delta\phi)}$$



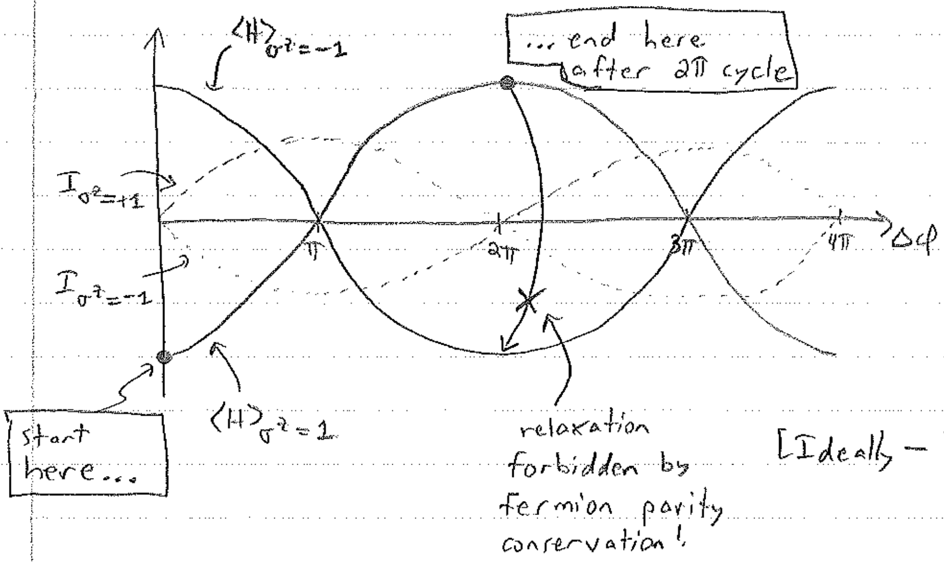
← both 2π periodic

Topo SC JJ



$$\boxed{\begin{aligned} &H_{12} = -t\,\sigma^z \cos\left(\frac{\hat\phi_L - \hat\phi_R}{2}\right) \\ &\sigma^z = i\gamma_1\gamma_2 \end{aligned}} \Rightarrow \boxed{I \propto \langle\sigma^z\rangle \sin\left(\frac{\Delta\phi}{2}\right)}$$

HW – Derive H_{12} from JJ made of Kitaev chains.



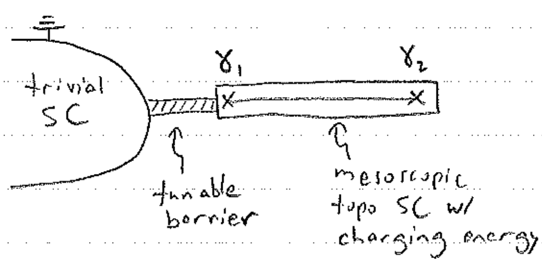
w/ fixed σ^z, I is 4π periodic!
(fractional JE)

[Ideally — "poisoning" is a challenge...]

$$\boxed{\text{Measure } I(\Delta\phi) \Rightarrow \text{infer } \sigma^z!}$$

[Can connect w/ fusion channels I, ψ, corresponding to gd. states / excited states of $H_{1,2}$. Note that 2π $\Delta\phi$ winding cycles fusion channels.]

Parity → Charge Conversion

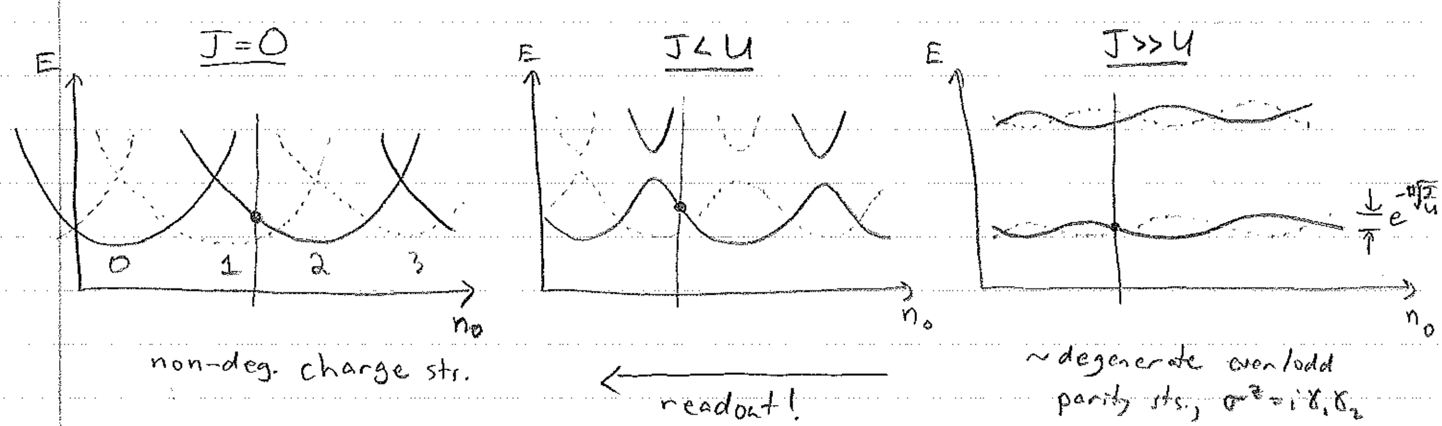


$$\boxed{H = -J\cos(\Delta\hat{\phi}) + U(\hat{n} - n_0)^2}$$

tunable offset charge

hops Cooper pairs, $\hat{n} \to \hat{n} \pm 2$

$\hat{n} \in \mathbb{Z}$, charge on topo SC

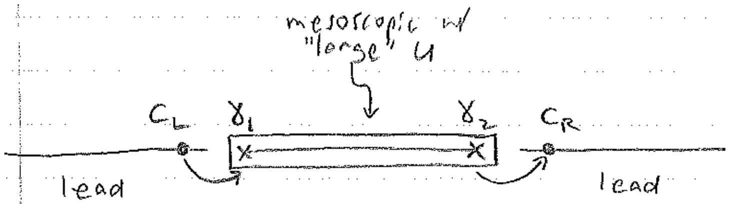


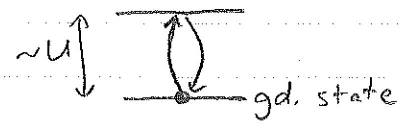
$$\boxed{\text{measure } \hat{n} \Rightarrow \text{infer } \sigma^z}$$

[tuning across these regimes is tricky; also possible to explore transmon-based readout.]

Interference

[It's crucial that γ_1, γ_2 encode a single fermion state shared between 2 ends.]



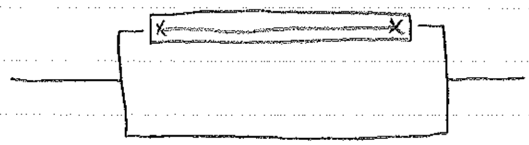


$$\boxed{H_{eff} \sim \frac{t^2}{U} \sigma^z c_L^\dagger c_R + h.c.}$$

[Note that tunneling term w/out σ^z is disallowed by symmetry; e.g., $c_L \to -c_L$, $\gamma_1 \to -\gamma_1$]

e^- acquires $\sigma^z = i\gamma_1\gamma_2$ dependent phase.
How to measure? Interference!

Add reference arm:



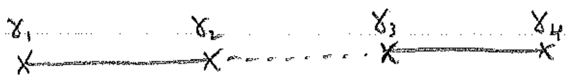
Conductance $\boxed{G = G_0 + \sigma^z \delta G}$

$$\boxed{\text{Measure } G \Rightarrow \text{infer } \sigma^z}$$

[Very powerful if realized. Also allows multi-Majorana measurements.]

Qubit Characterization

Elementary topo qubit:



$$\boxed{\begin{aligned} |0\rangle &\equiv |i\gamma_1\gamma_2 = 1,\ i\gamma_3\gamma_4 = 1\rangle \\ |1\rangle &\equiv |\quad -1 \qquad\quad -1\rangle \end{aligned}}$$

← logical qubit sts.
[which have same total fermion parity; w/ only 2 Majoranas we're stuck in one sector]

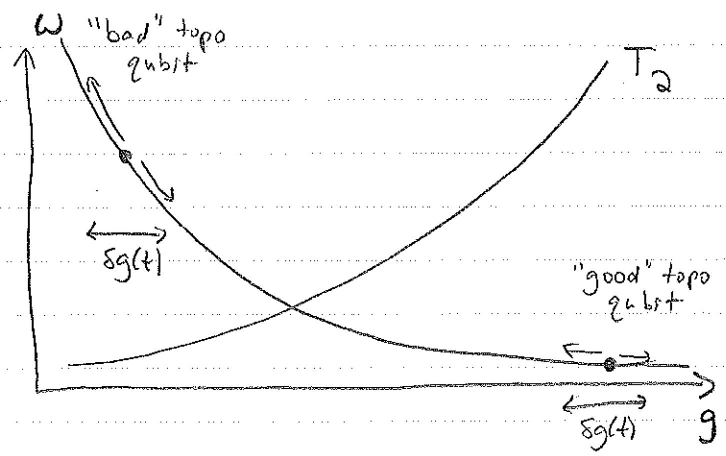
Explore (i) topological protection of quantum info

(ii) non-Abelian stat. / Fault-tolerant gates

Topological protection

What to look for experimentally?

$$\begin{matrix} |1\rangle \\ \updownarrow \; \omega \sim e^{-L/\xi(\underbrace{g_1, g_2, \ldots}_{\text{e.g., } \mu, B, \ldots})} \\ |0\rangle \end{matrix}$$



$$\boxed{\omega \sim \frac{1}{T_2} \xrightarrow[\substack{\text{perfect} \\ \text{topo qubit}}]{} 0}$$

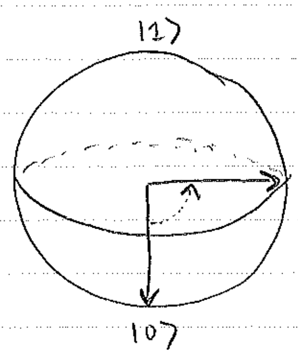
[measurable in Ramsey-like protocols; contrast to, say, spin qubit where $\omega = 0$ does not constrain T_2]

[Note also that g stands for all possible couplings. Some qubits (e.g., transmon) enjoy insensitivity to certain noise sources. Topo qubits are insensitive to arbitrary noise, provided freq / amplitude aren't too high.]

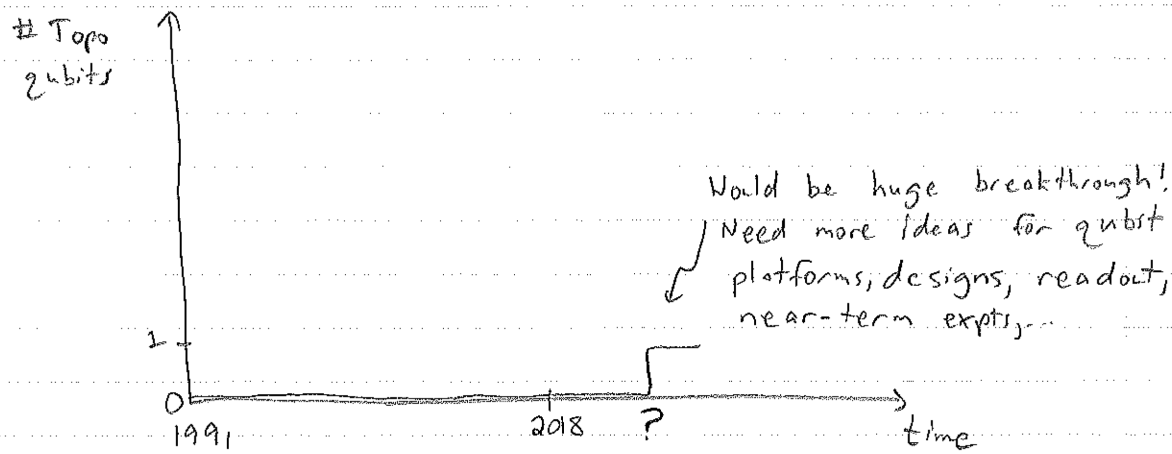
Gates

Swap $\gamma_2 \longleftrightarrow \gamma_3 \Rightarrow$ unitary rot. $\boxed{U = e^{\pm\frac{\pi}{4}\gamma_2\gamma_3} = e^{\pm i\frac{\pi}{4}\sigma^x}}$

recall HW



$\boxed{\text{"perfect" } \frac{\pi}{2} \text{ rotation}}$



Topo qubits
1
0
1991
2018
?
time
Would be huge breakthrough! Need more ideas for qubit platforms, designs, readout, near-term expts, ...