“Introduction to cold atoms and Bose-Einstein condensation (II)”

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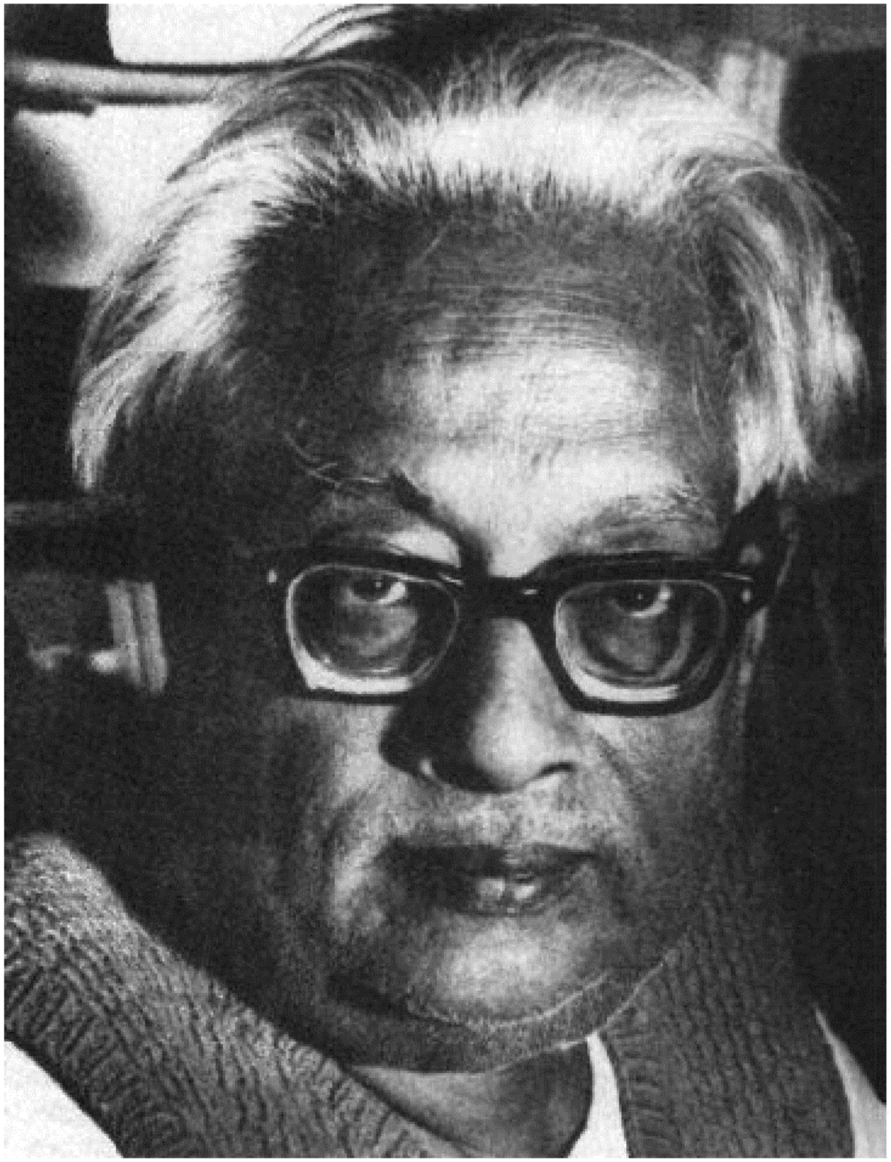
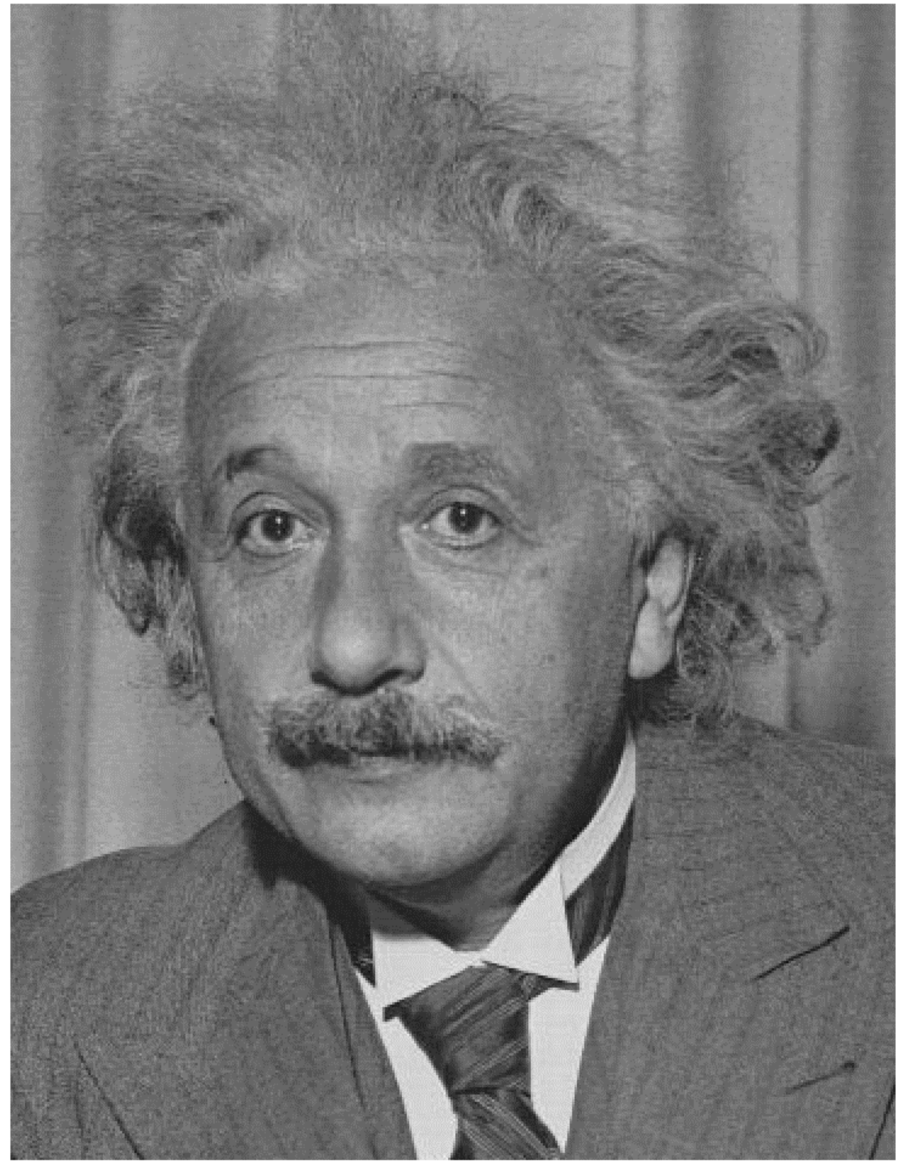
7/7/04
Boulder Summer School

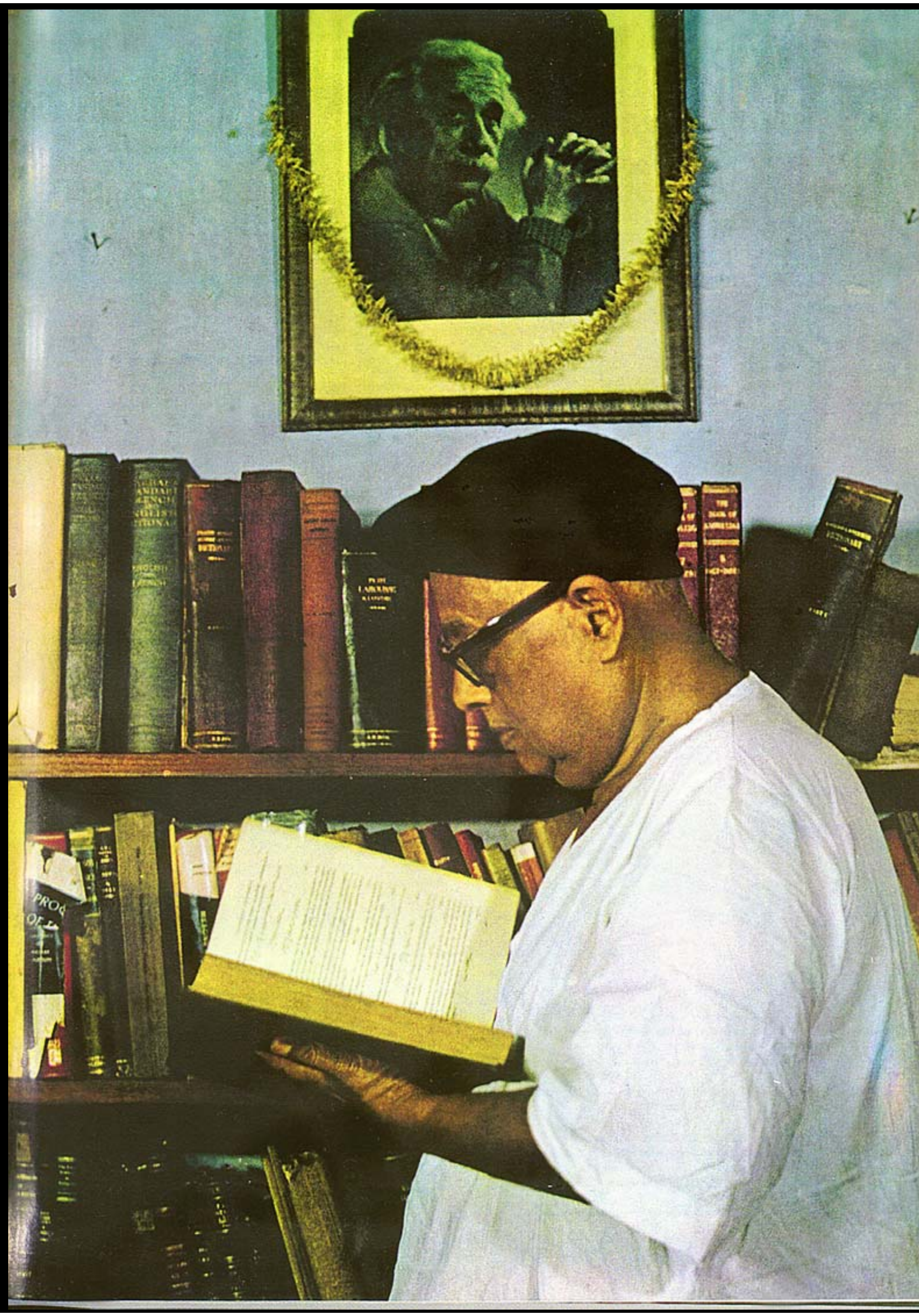
Bose-Einstein condensation *** 1925**



Satyendra Nath Bose



Albert Einstein



History of Bose-Einstein condensation (mainly exp.)

- Theoretical prediction

  1924/25 Bose and Einstein
- Superfluidity in liquid helium

  1938 Fritz London

  1983 Reppy et al. (Cornell): BEC of helium in vycor
- Excitons (complicated interactions - no BEC observed)
- Dilute atomic gases

Spin-polarized hydrogen:

agenda & experimental techniques (since late ‘70s)

MIT *(Greytak, Kleppner)* BEC ‘98

Amsterdam *(Silvera, Walraven)*

also: Harvard, BC, Turku, Cornell, Moscow

2D quantum degeneracy ‘98

Alkali atoms:

Laser cooling (‘80s)

Focused programs in Boulder and at MIT (since early ‘90s)

June ‘95: Boulder *(Cornell/Wieman)*

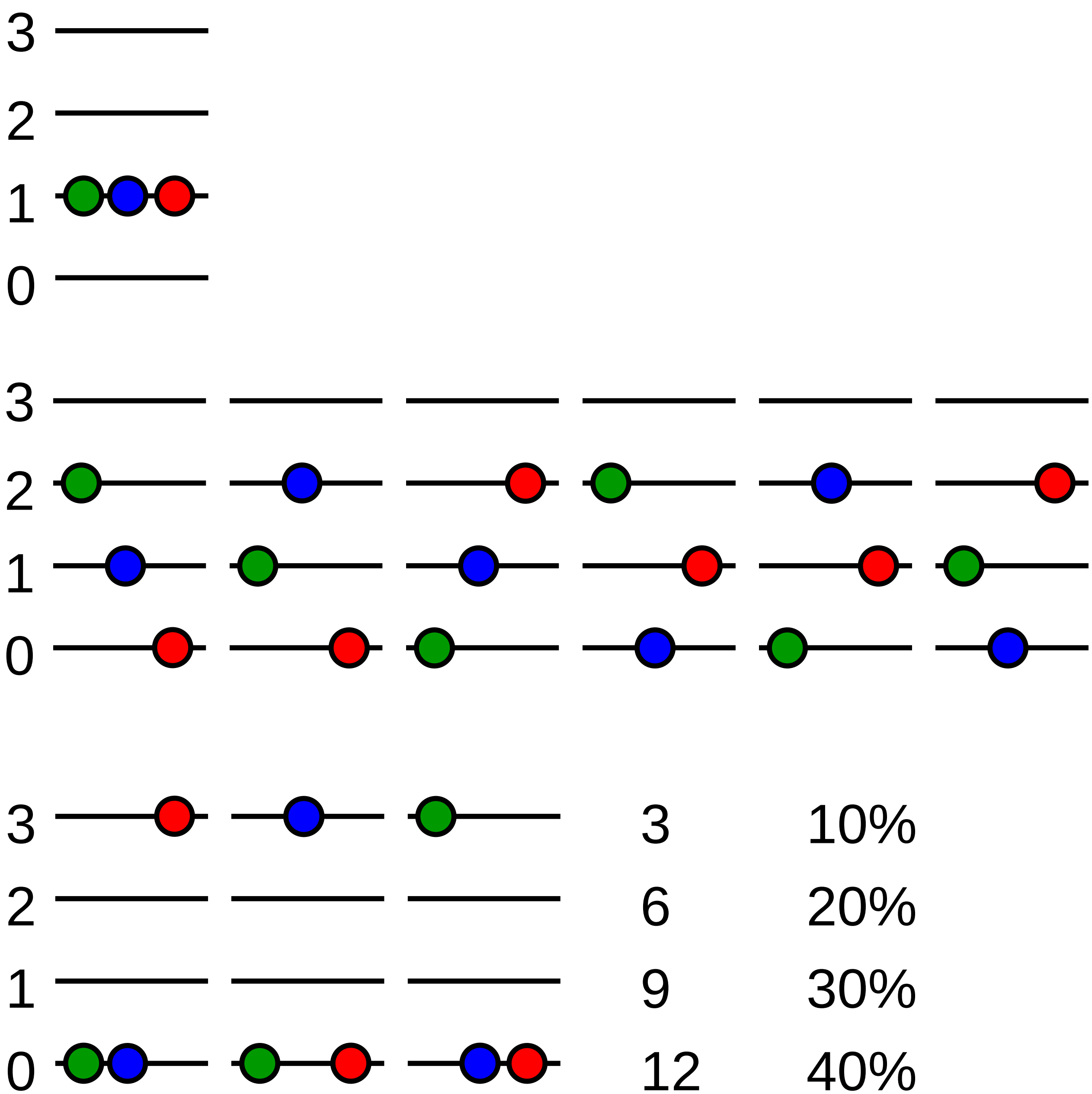
Sept. ‘95: MIT *(W.K.)*

July ‘95 [indirect evidence]: Rice *(Hulet)*

now:
many experiments

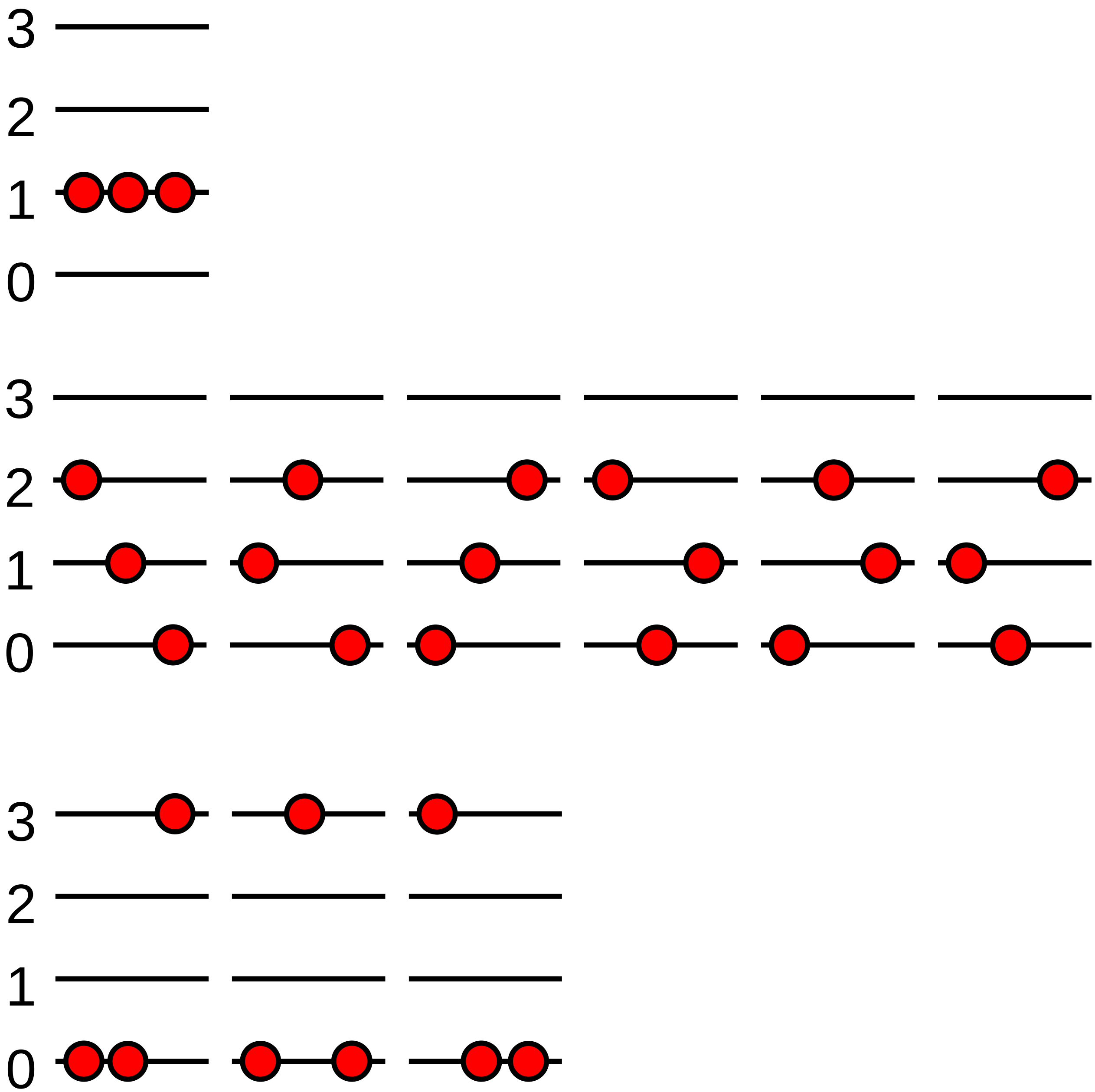
3 particles, total energy = 3

Classical



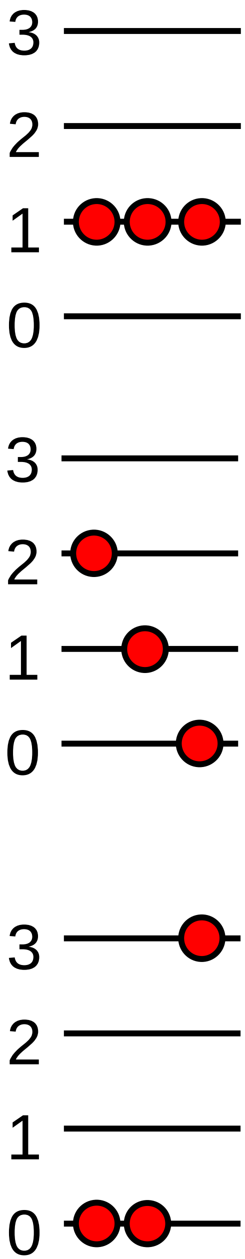
3 particles, total energy = 3

Identical



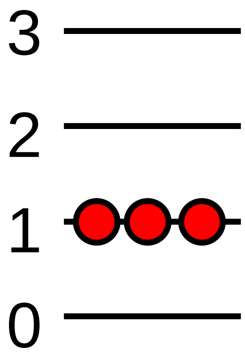
3 particles, total energy = 3

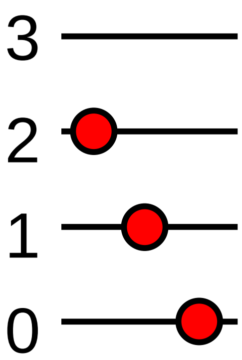
Bosons

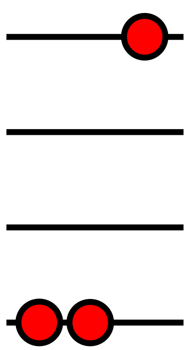


3 particles, total energy = 3

Bosons



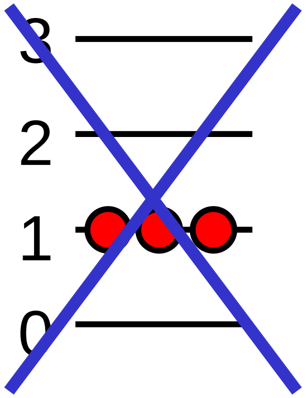


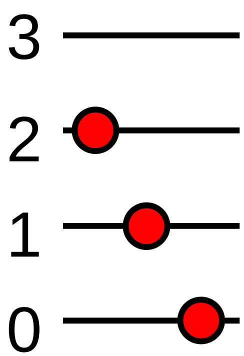


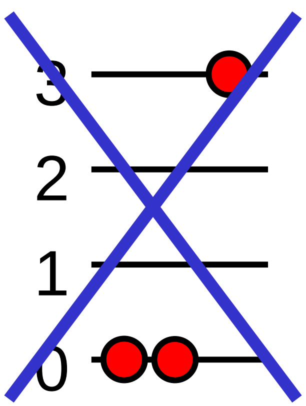
3	1	11%
2	1	11%
1	4	44%
0	3	33%

3 particles, total energy = 3

Fermions





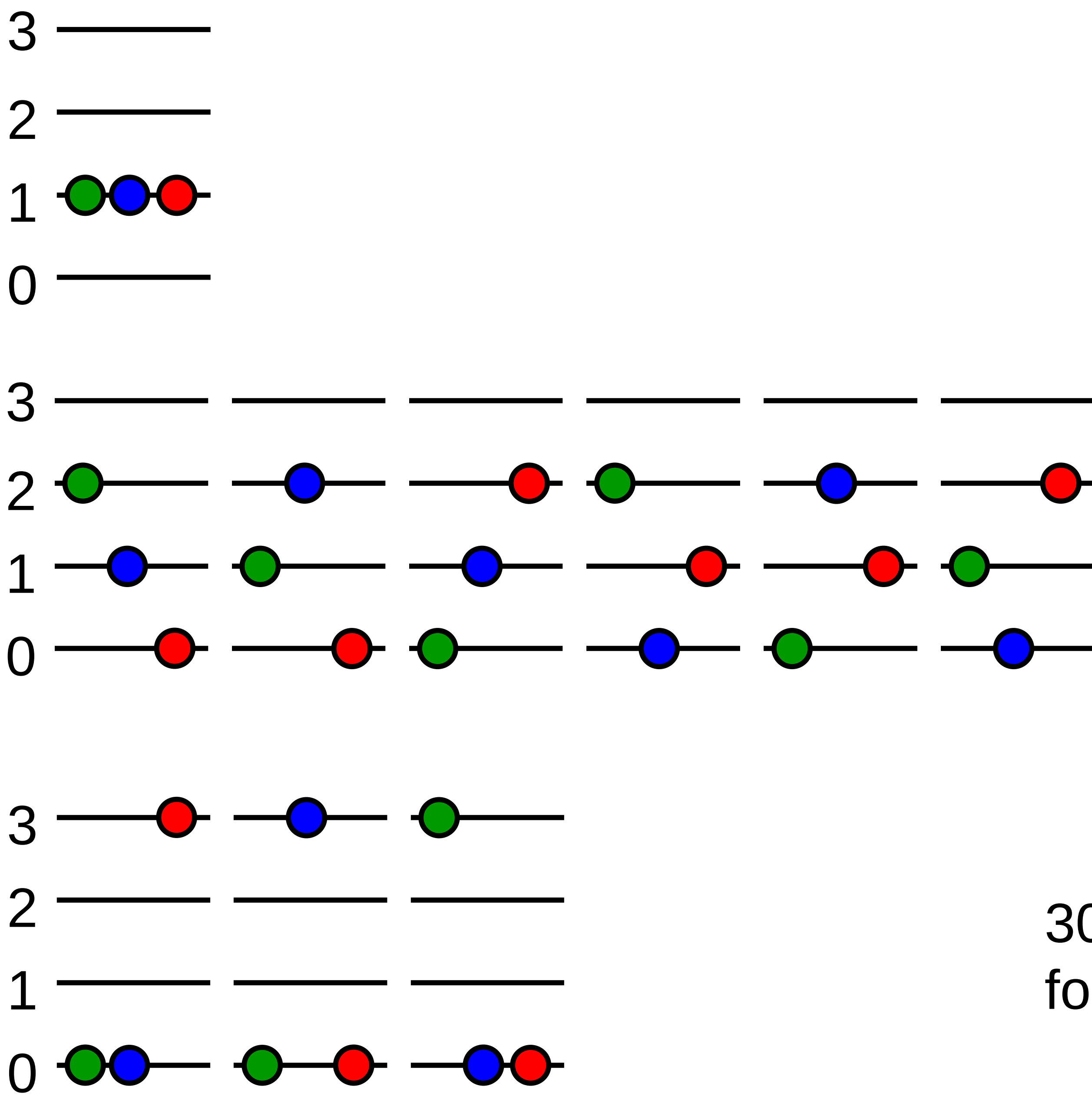


	fermions	bosons	classical
0	0%	11%	10%
1	33%	11%	20%
1	33%	44%	30%
1	33%	33%	40%

3 particles, total energy = 3

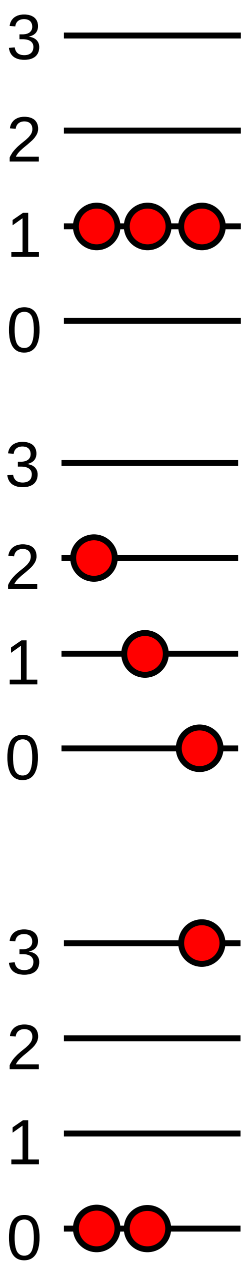
Classical

10 % probability
for triple occupancy



30 % probability
for double occupancy

3 particles, total energy = 3



Bosons

33 % probability
for triple occupancy

Bosons are gregarious!
Fermions are loners!

33 % probability
for double occupancy

$$n(\varepsilon) = \begin{cases} \dfrac{1}{e^{(\varepsilon-\mu)/k_B T}} & \text{classical particles Maxwell-Boltzmann statistics} \\[2ex] \dfrac{1}{e^{(\varepsilon-\mu)/k_B T}-1} & \text{bosons Bose-Einstein statistics} \\[2ex] \dfrac{1}{e^{(\varepsilon-\mu)/k_B T}+1} & \text{fermions Fermi-Dirac statistics} \end{cases}$$

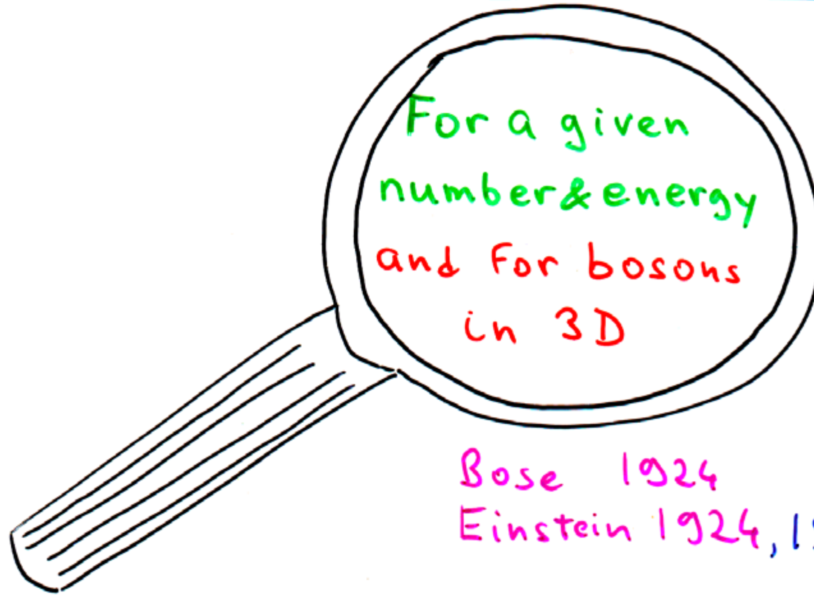
$$n(\varepsilon) = \begin{cases} \dfrac{1}{e^{(\varepsilon-\mu)/k_BT}-1} \\[2em] \dfrac{1}{e^{\varepsilon/k_BT}-1} \end{cases}$$

bosons
Bose-Einstein
statistics

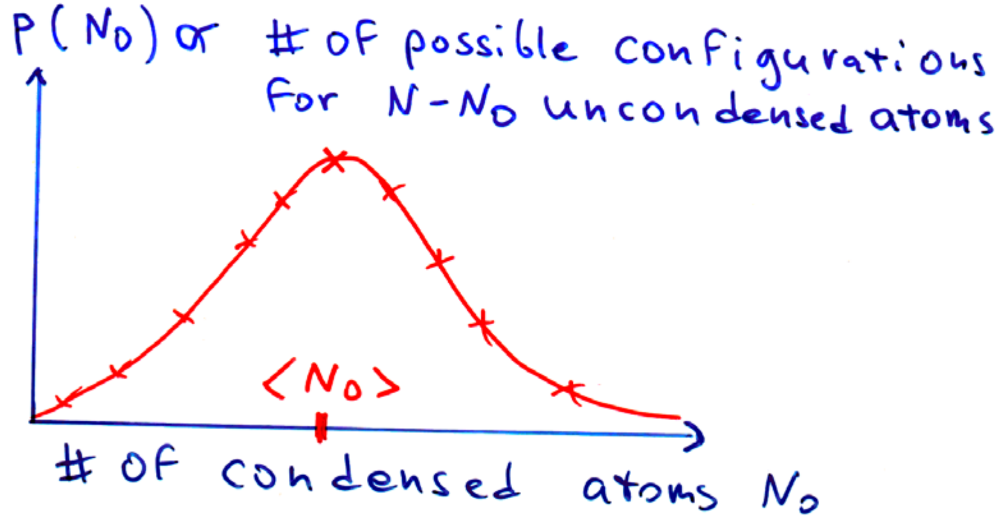
μ=0
(photon number
not conserved)

photons
Planck's blackbody
spectrum

BEC is the most random state in nature

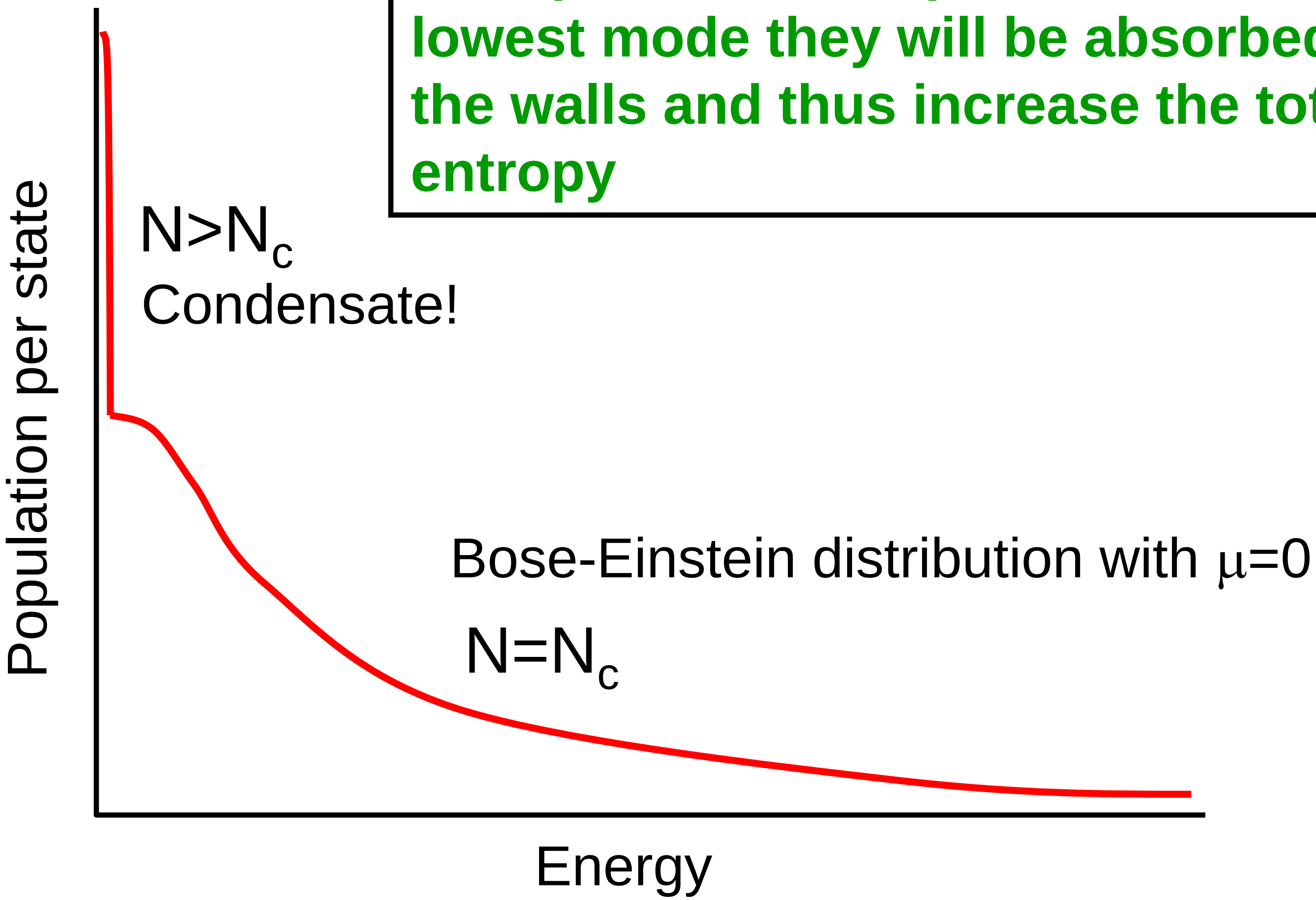


Bose 1924
Einstein 1924, 1925



Why do photons not Bose condense?

If we put in "extra" photons into the lowest mode they will be absorbed by the walls and thus increase the total entropy



Are different particles absolutely identical?

Necessary assumption for indistinguishability

In quantum field theory they are excitations of the same field

Tests of the (anti-) symmetry of the state for bosons/fermions at the level of 10^{-9} and 10^{-26}

Development of quantum statistics in three years

1924 Bose's paper
1924/25 Three papers by Einstein

Particles are no longer statistically independent!

Einstein mentioned hydrogen, helium and the electron gas as possible candidates for BEC

1925 Pauli exclusion principle

1926 Fermi-Dirac statistics

Confusion about which statistics to apply

1927 Things were cleared up

On the Theory of Quantum Mechanics.

By P. A. M. DIRAC, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received August 26, 1926.)

If now we adopt the solution of the problem that involves symmetrical eigenfunctions, we should find that all values for the number of molecules associated with any wave have the same *a priori* probability, which gives just the Einstein-Bose statistical mechanics.* On the other hand, we should obtain a different statistical mechanics if we adopted the solution with antisymmetrical eigenfunctions, as we should then have either 0 or 1 molecule associated with each wave. The solution with symmetrical eigenfunctions must be the correct one when applied to light quanta, since it is known that the Einstein-Bose statistical mechanics leads to Planck's law of black-body radiation. The solution with antisymmetrical eigenfunctions, though, is probably the correct one for gas molecules, since it is known to be the correct one for electrons in an atom, and one would expect molecules to resemble electrons more closely than light-quanta.

* Bose, 'Zeits. f. Phys.,' vol. 26, p. 178 (1924); Einstein, 'Sitzungsb. d. Preuss. Ac.,' p. 261 (1924) and p. 3 (1925).

History of BEC

W. Pauli, Z. Phys. **41**, 81 (1927):

"We shall take the point of view also advocated by Dirac, that the Fermi, and not the Einstein-Bose, statistics applies to the material gas."

A. Einstein (December 1924) about BEC:

“The theory is pretty, but is there also some truth to it?”

Fritz London

He realized in 1938 that BEC is an observable phenomenon

Criterion for BEC

Thermal de Broglie wavelength ($\propto T^{-1/2}$)

equals

distance between atoms ($= n^{-1/3}$)

$n_{crit} \propto T^{3/2}$

"High" density: n_{water}: $T = 1$ K

BUT: molecule/cluster formation, solidification

$\Rightarrow$ no BEC ☹

STATISTICAL THERMODYNAMICS

Erwin Schrödinger

(1952)

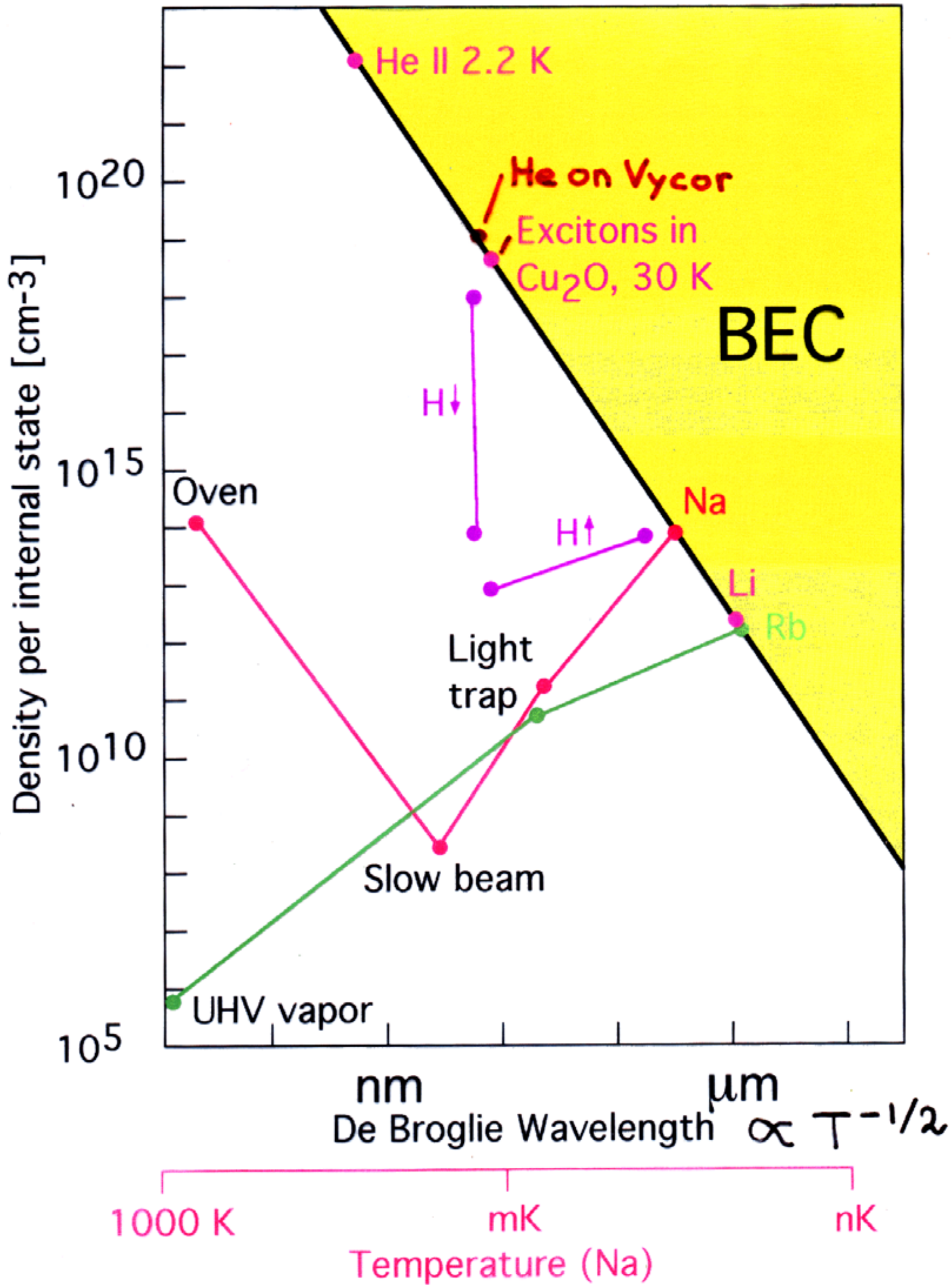
(*b*) The densities are so high and the temperatures so low—those required to exhibit a noticeable departure—that the van der Waals corrections are bound to coalesce with the possible effects of degeneration, and there is little prospect of ever being able to separate the two kinds of effect.

Interactions

BEC

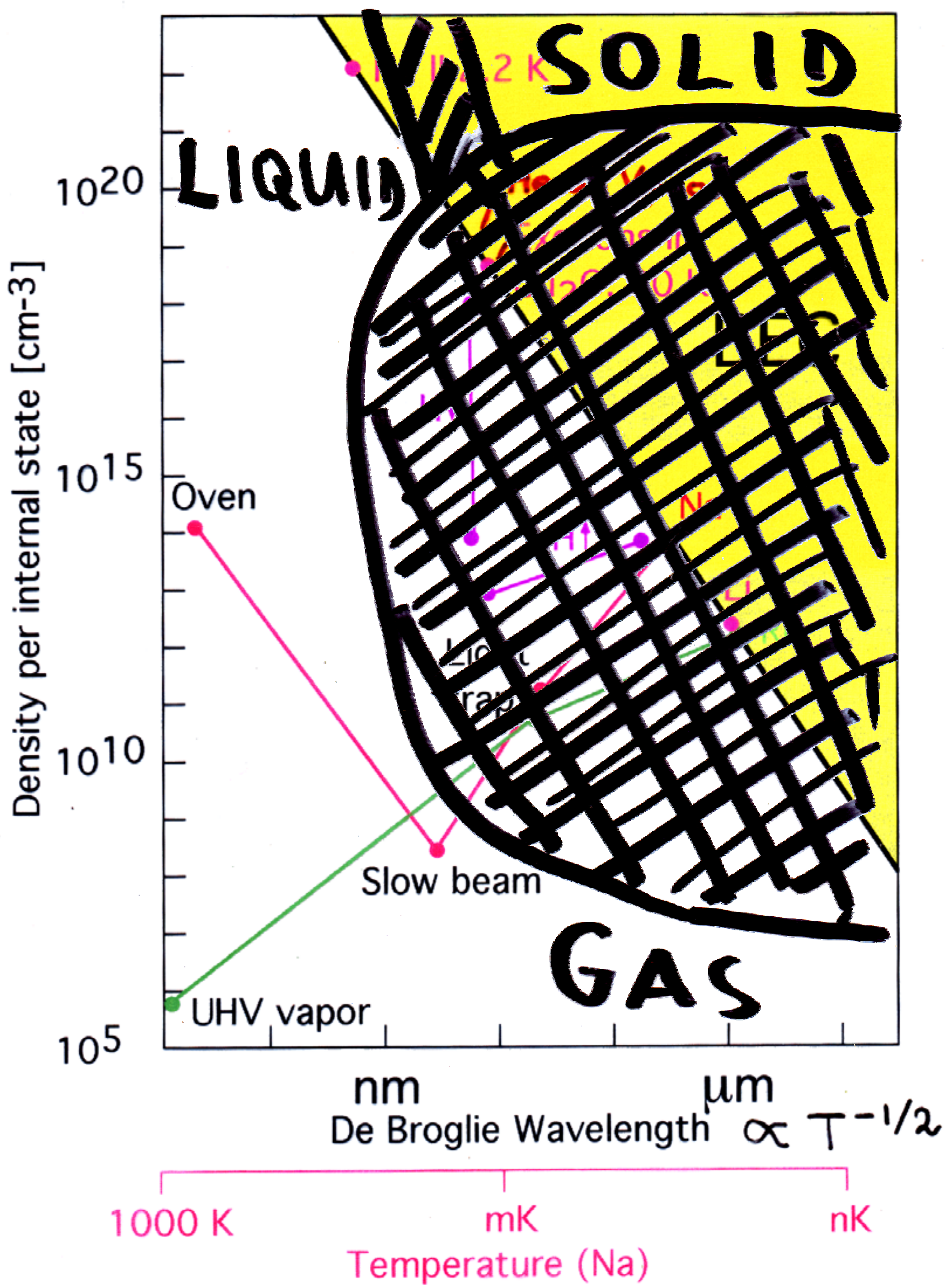
BEC:

$\lambda_{dB} > n^{-1/3}$



BEC:

$\lambda_{dB} > n^{-1/3}$



Criterion for BEC

Thermal de Broglie wavelength ($\propto T^{-1/2}$)
equals
distance between atoms ($= n^{-1/3}$)
$n_{crit} \propto T^{3/2}$

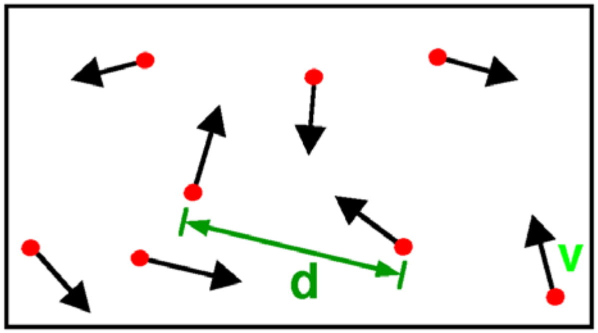
"High" density: n_{water}: $T = 1$ K

BUT: molecule/cluster formation, solidification
$\Rightarrow$ no BEC ☹

"Low" density: $n_{water}/10^9$: $T = 100$ nK - 1 μK

seconds to minutes lifetime of the atomic gas
$\Rightarrow$ BEC ☺

What is Bose-Einstein condensation (BEC)?

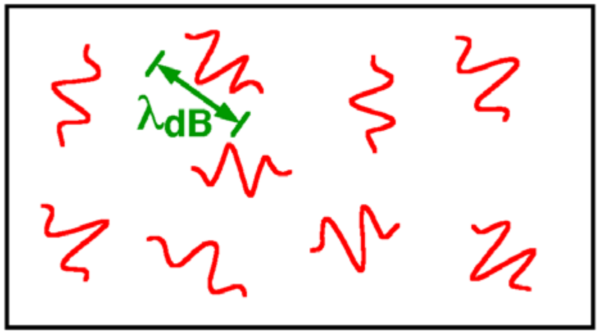


High Temperature T:

thermal velocity v

density d^{-3}

"Billiard balls"

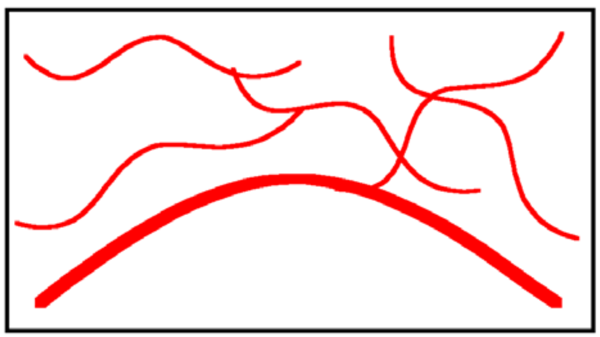


Low Temperature T:

De Broglie wavelength

$\lambda_{dB}=h/mv \propto T^{-1/2}$

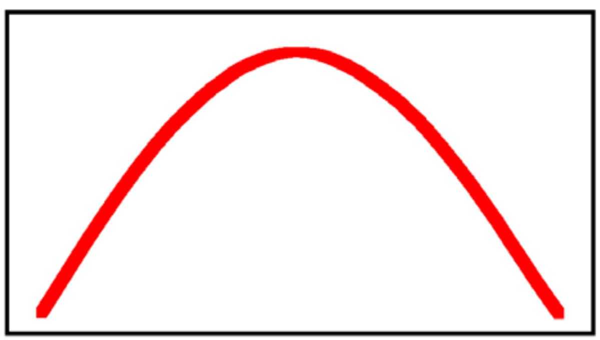
"Wave packets"



$T=T_{crit}$: Bose-Einstein Condensation

$\lambda_{dB} \approx d$

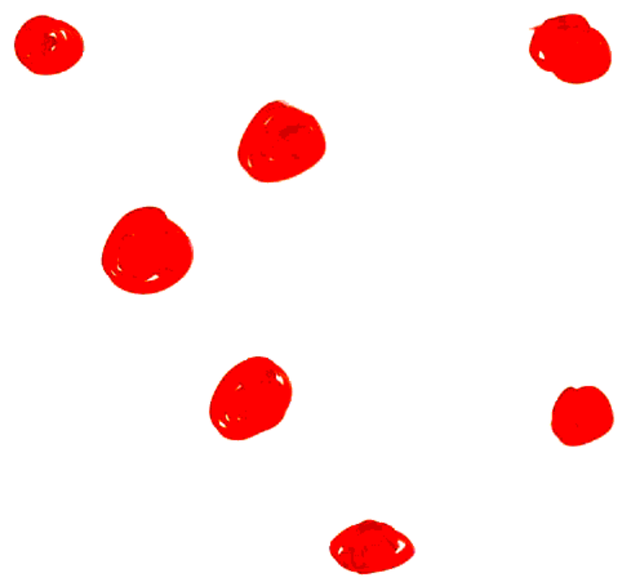
"Matter wave overlap"



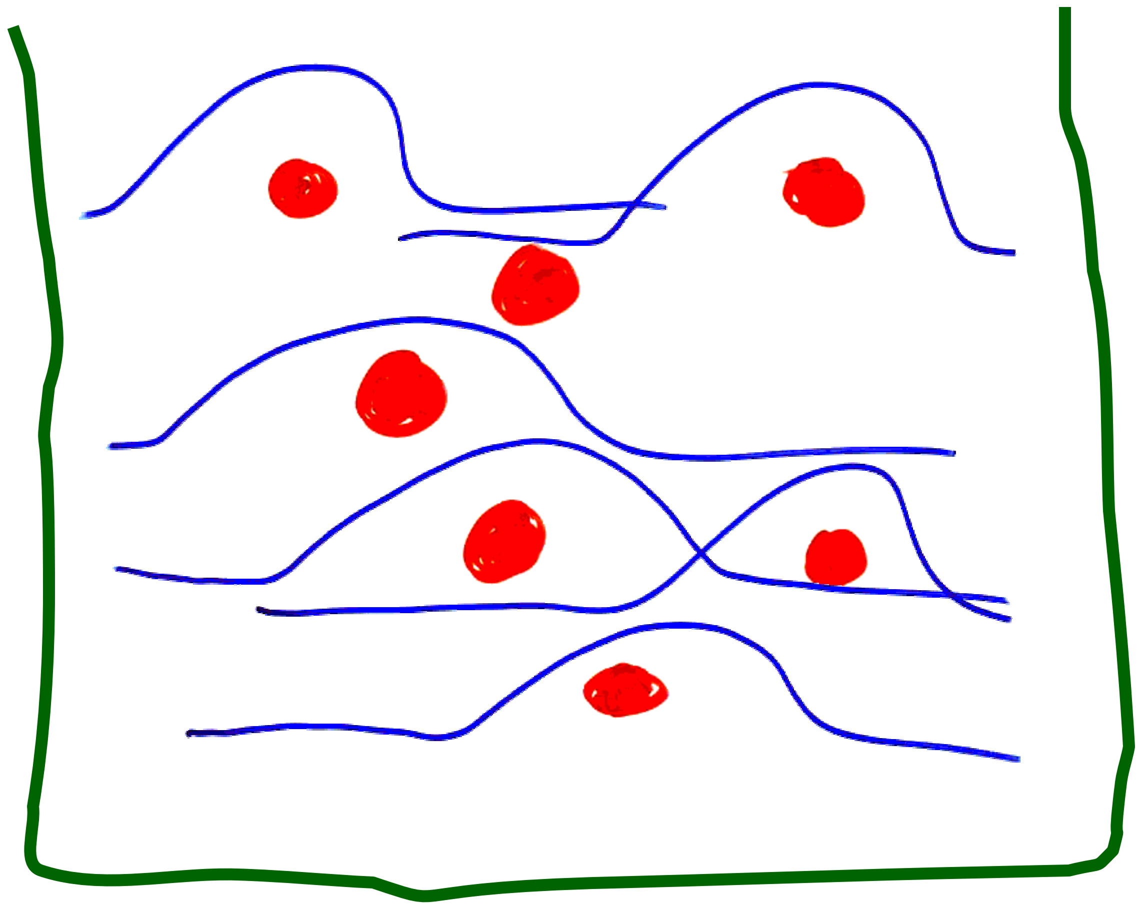
T=0: Pure Bose condensate

"Giant matter wave"

Length Scales @BEC



Length Scales @BEC



Length and energy scales in BEC

Size of the atom	a	3 nm
Separation between atoms	$n^{-1/3}$	200 nm
Matter wavelength	λ_{dB}	1 μm
Healing length	$2\pi\xi$	2μm
Size of confinement	a_{osc}	30μm

$$a \ll n^{-1/3} \leq \lambda_{dB} < 2\pi\xi < a_{osc}$$

$$k_B T_{s\text{-wave}} \gg k_B T_c \geq k_B T > U_{int} > \hbar\omega$$

Gas!

BEC

$=(\hbar^2/m)na$

Cast of characters: nK tools

Cooling

Laser cooling
Evaporative cooling

Atoms for BEC

Traps

Magnetic traps
Optical traps

How to observe BEC

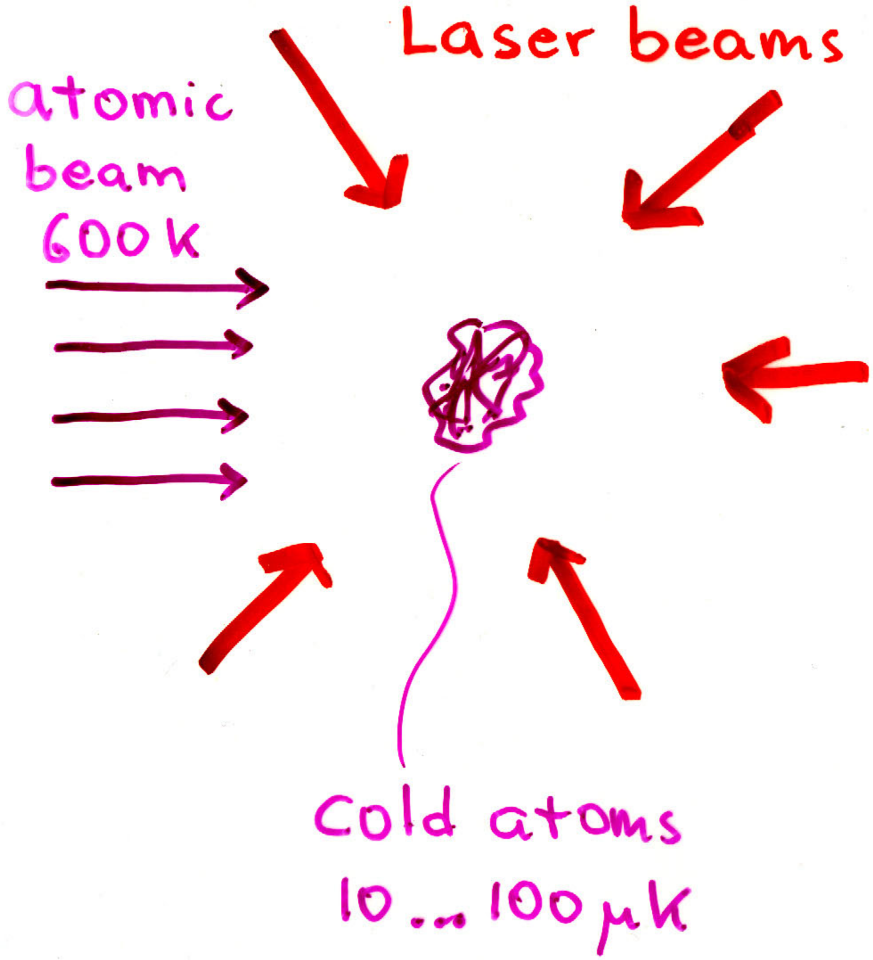
Absorption imaging
Dispersive imaging

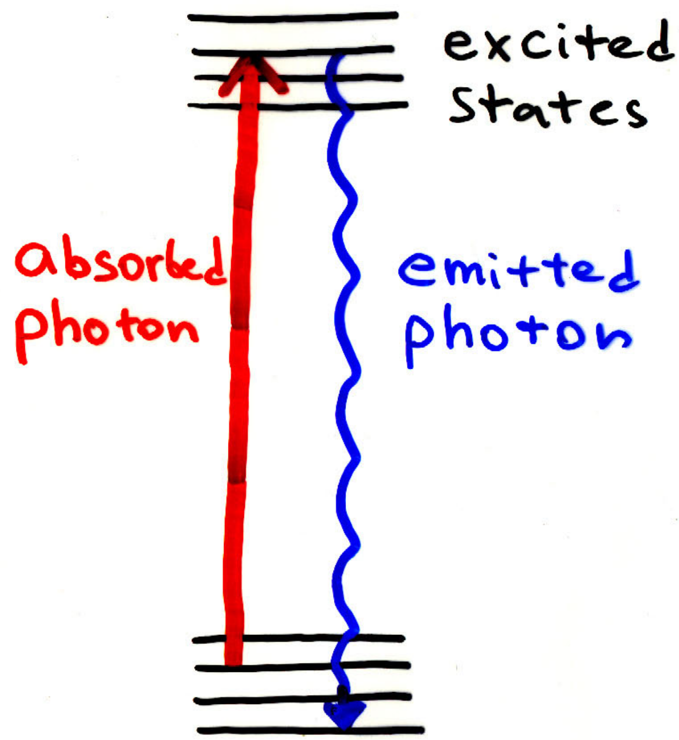
Manipulation of BEC

Magnetic fields
Rf
Optical dipole force

The cooling methods

Laser cooling = Precooling

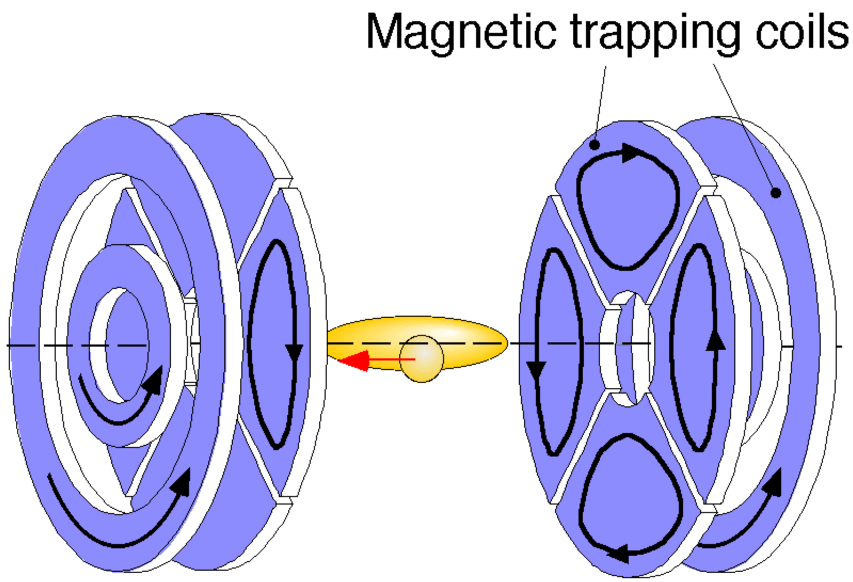


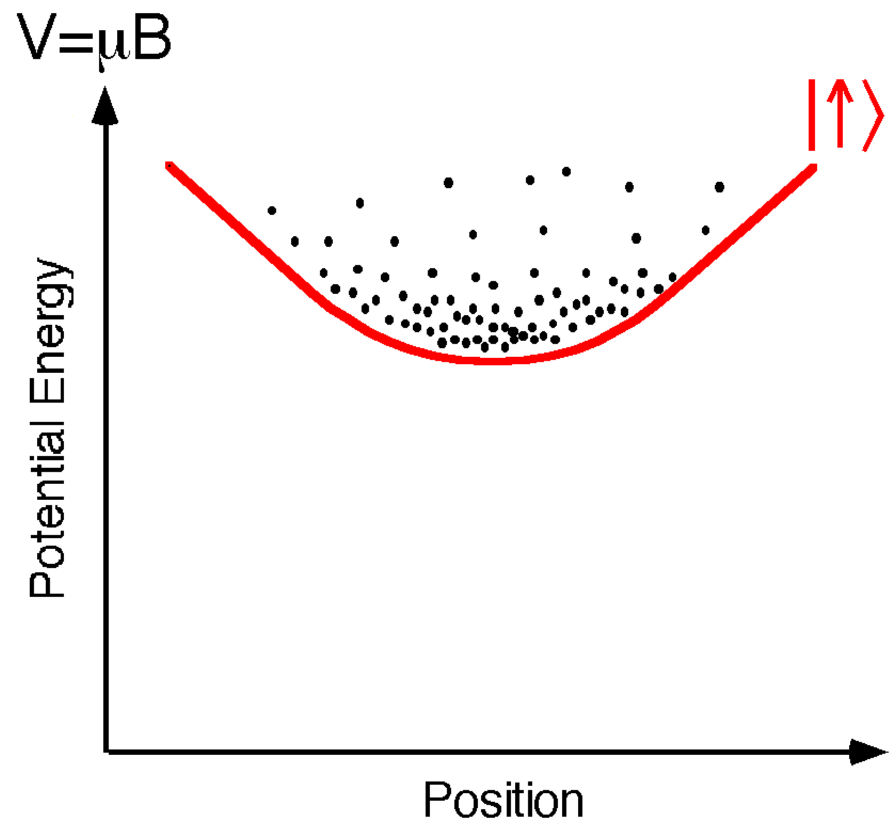


$$\hbar\omega_{abs} < \hbar\omega_{em}$$

$\Rightarrow$ cooling!

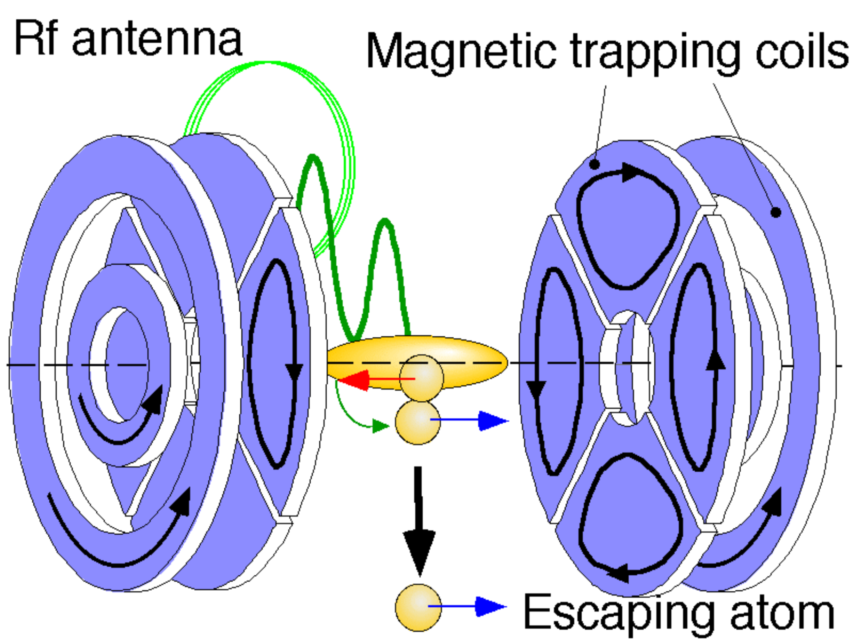
Magnetic trapping -
"thermos" for nanokelvin atoms

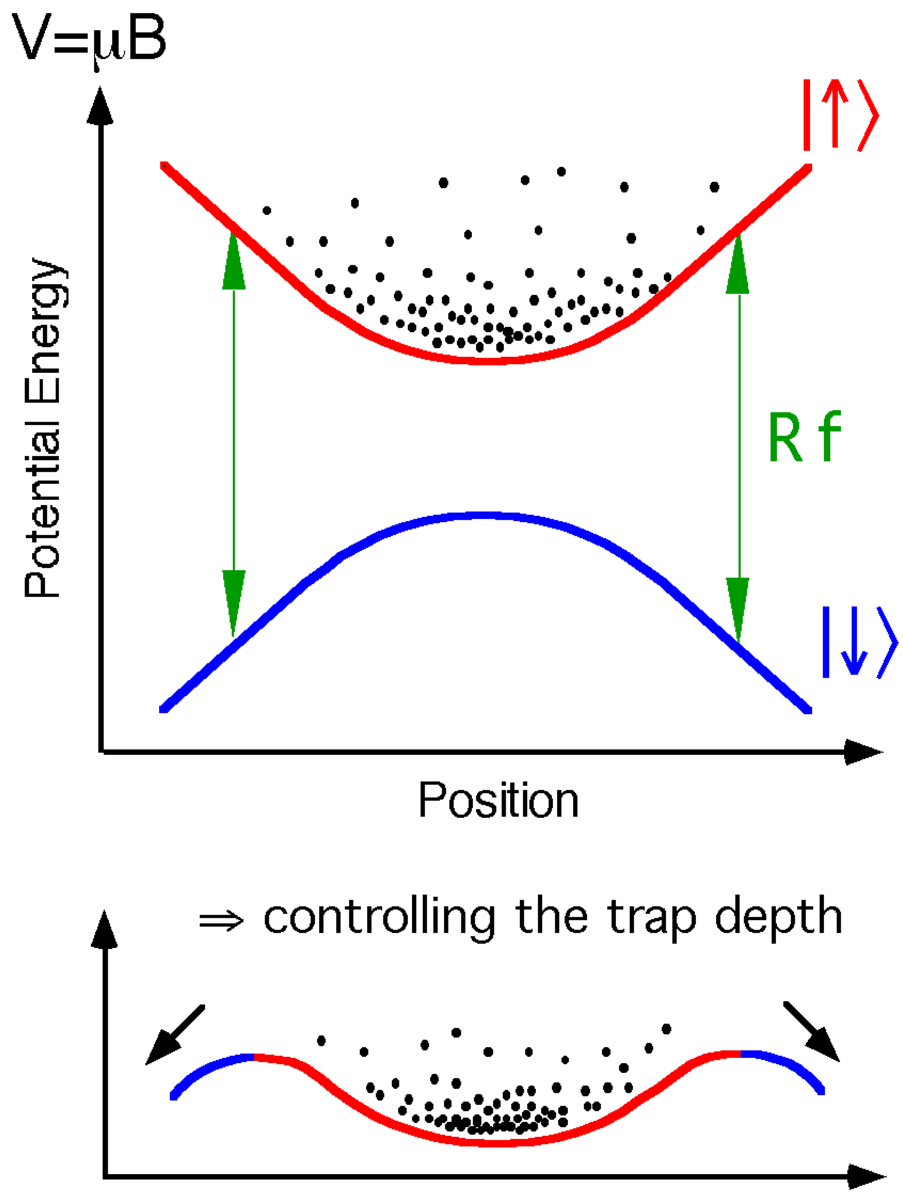




Phillips et al. (1985)
Pritchard et al. (1987)

Evaporative cooling using rf induced spinflips



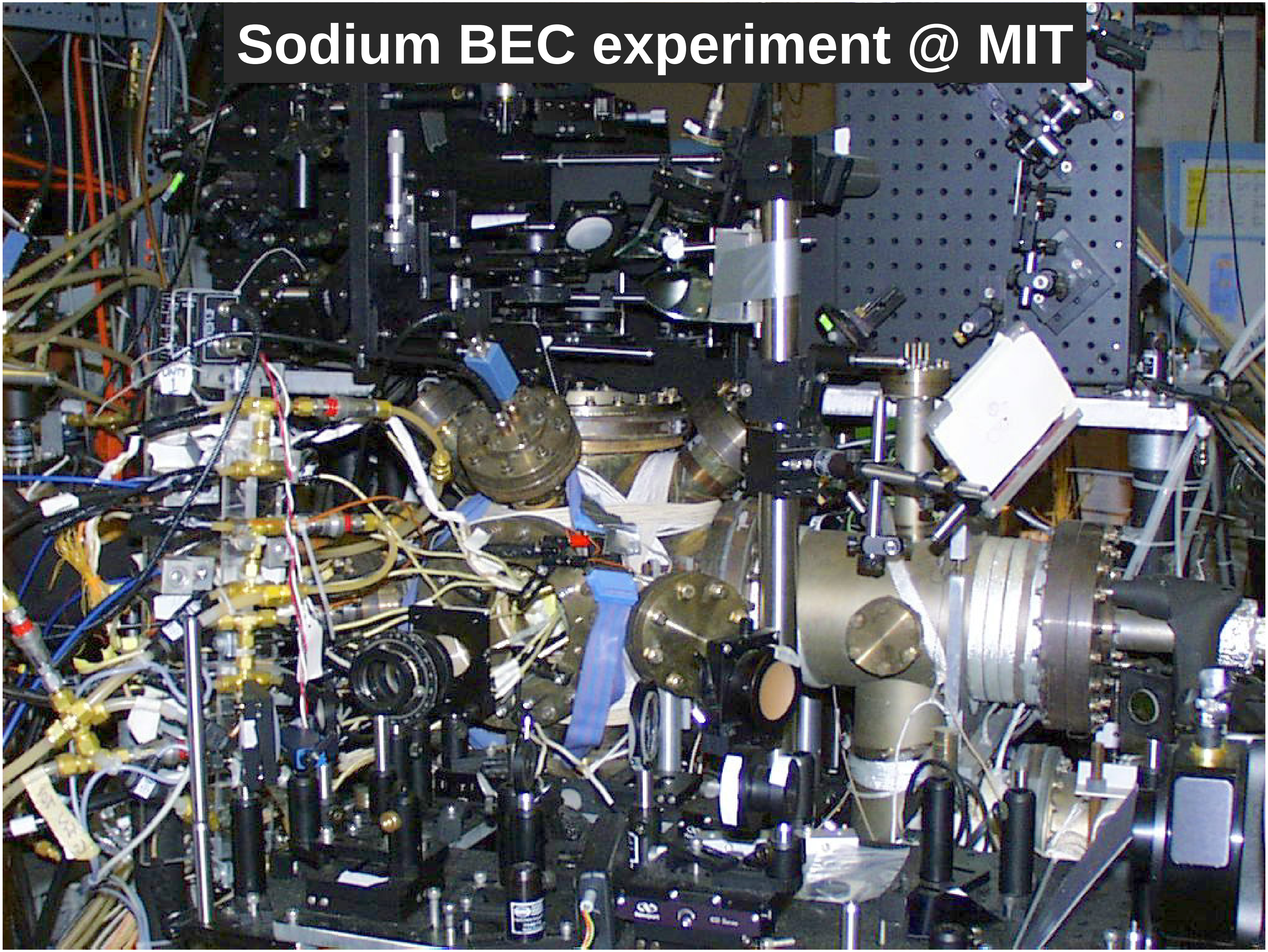


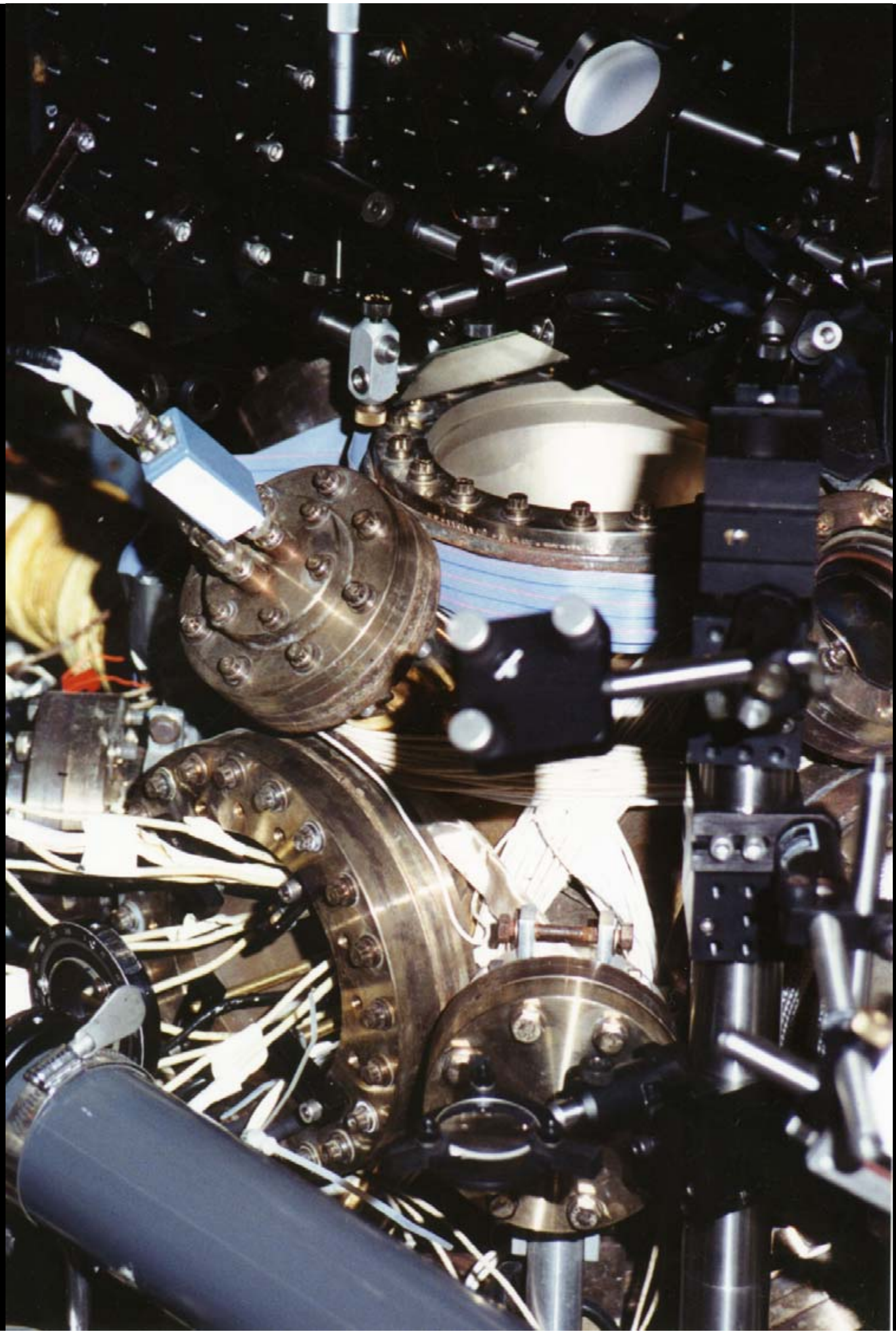
Hess, Kleppner, Greytak et al. (1986/7)
Pritchard et al. (1989)
Ketterle et al. (1993/4)
Cornell et al. (1994)

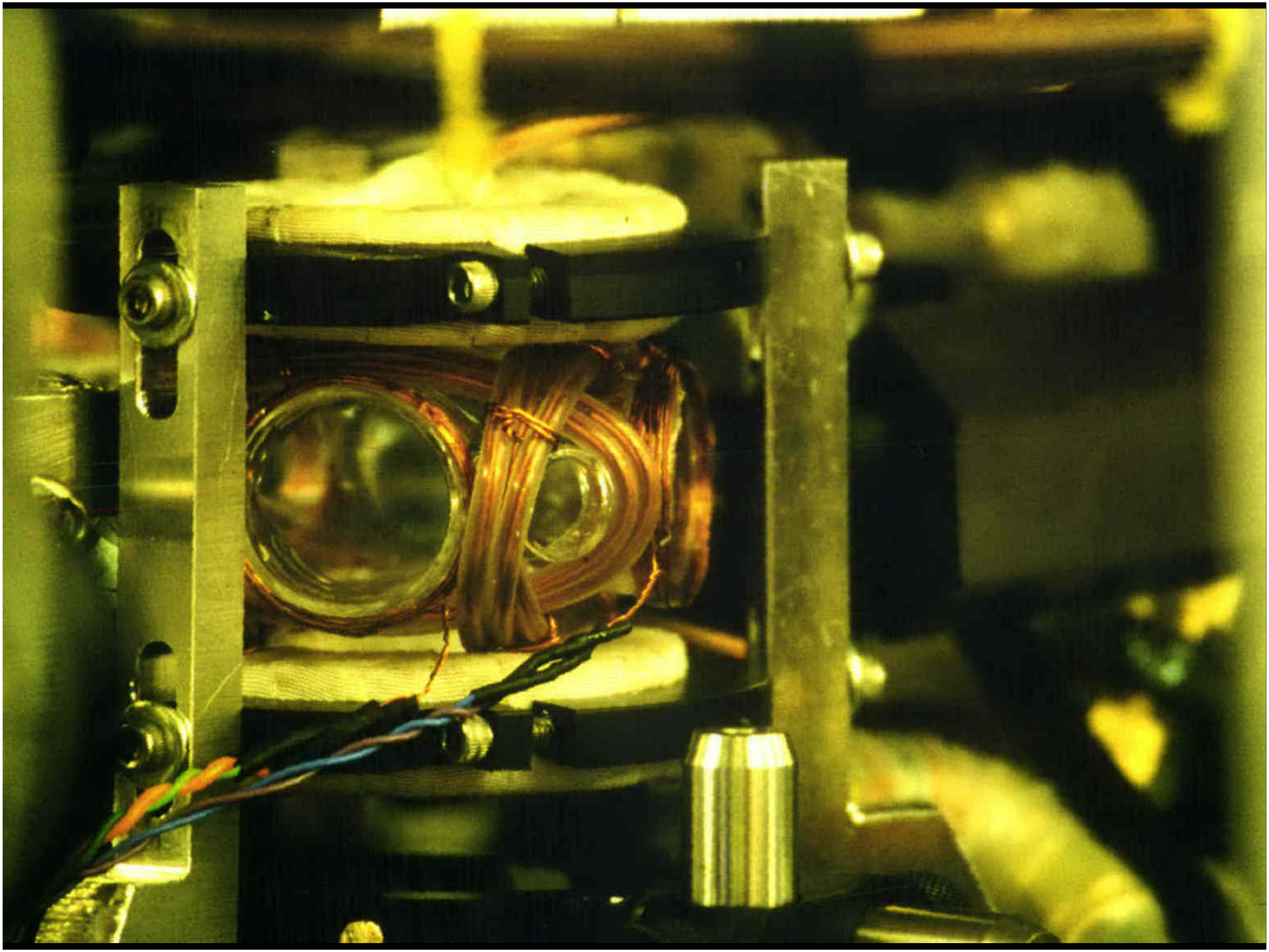
Multi-stage cooling to BEC

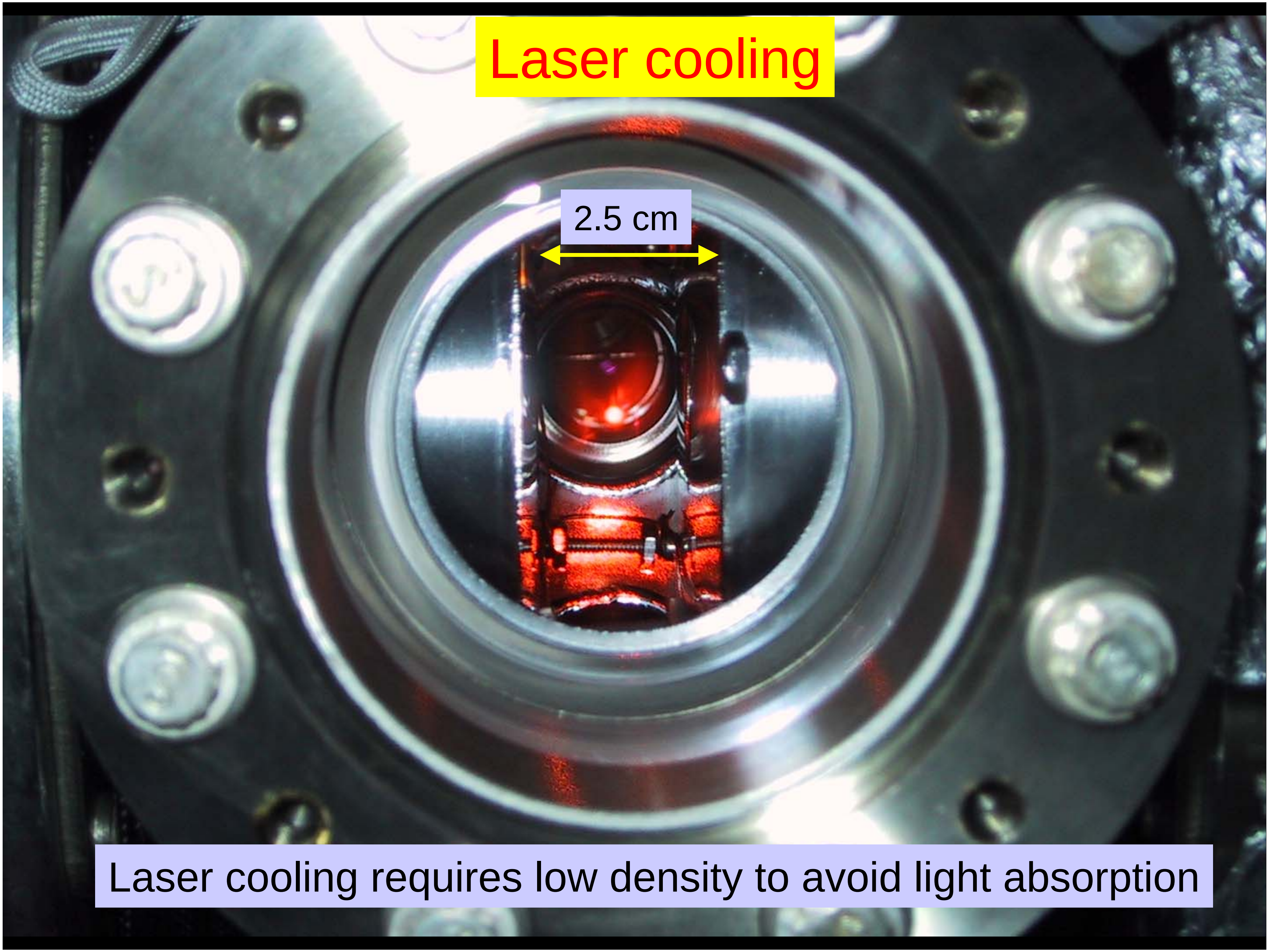
	Temp. T	Density n [cm^{-3}]	Phase space density $nT^{-3/2}$
Oven	500 K	10^{14}	10^{-13}
Laser cooling	50 μK	10^{11}	10^{-6}
Evap. cooling	500 nK	10^{14}	2.6
BEC	(10 - 100 nK)	$3 \cdot 10^{14}$	10^{7}

Sodium BEC experiment @ MIT

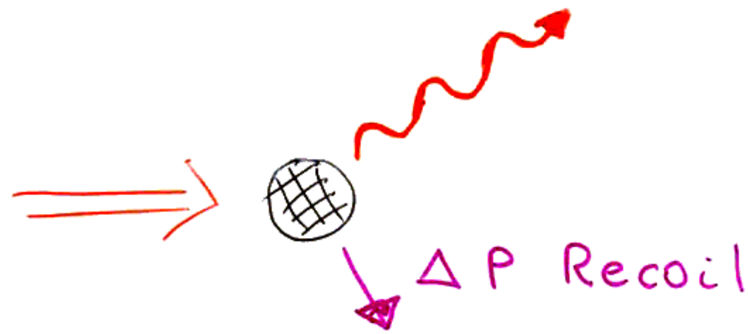




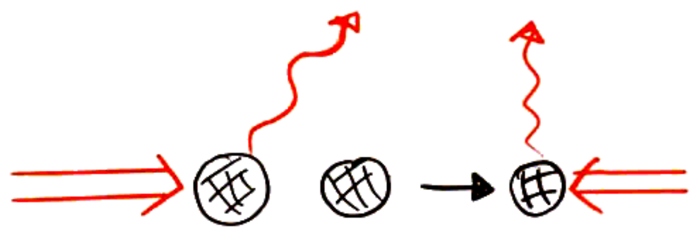



Laser cooling
2.5 cm
Laser cooling requires low density to avoid light absorption

Limitations of Laser Cooling

- 

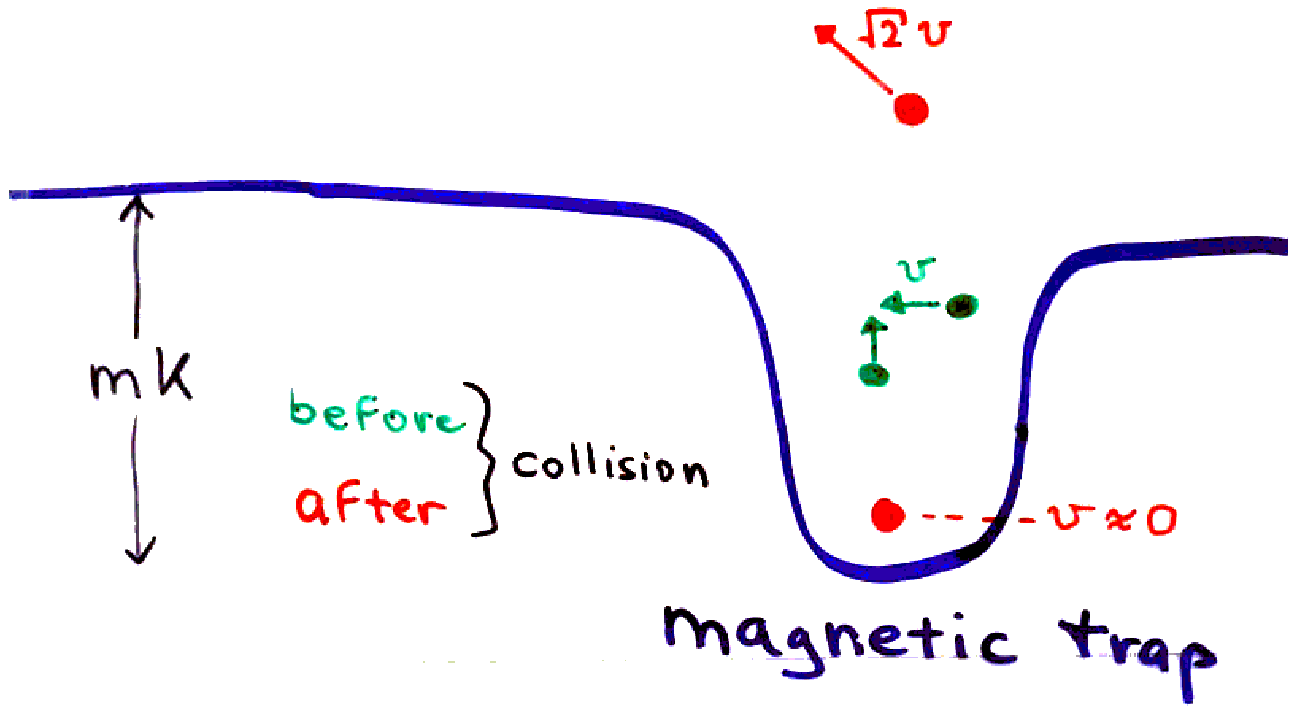
  $T \sim \mu K$

- 

  Absorption
  excited state collisions } $n \le 10^{12}\,\mathrm{cm}^{-3}$

$\Rightarrow$ Phase space density 10^{-5}

Evaporative cooling
is driven by
elastic collisions



$$\Gamma_{ec} = n\,\sigma_{ec}\,v \gg \Gamma_{Loss}$$

H. Hess (1986)

The problem ...

Absorption cross section for light:

$$\sigma_{opt} = \frac{3}{2\pi}\lambda^2 \approx 2\cdot 10^{-9}\,\mathrm{cm}^2$$

Elastic collision cross section:

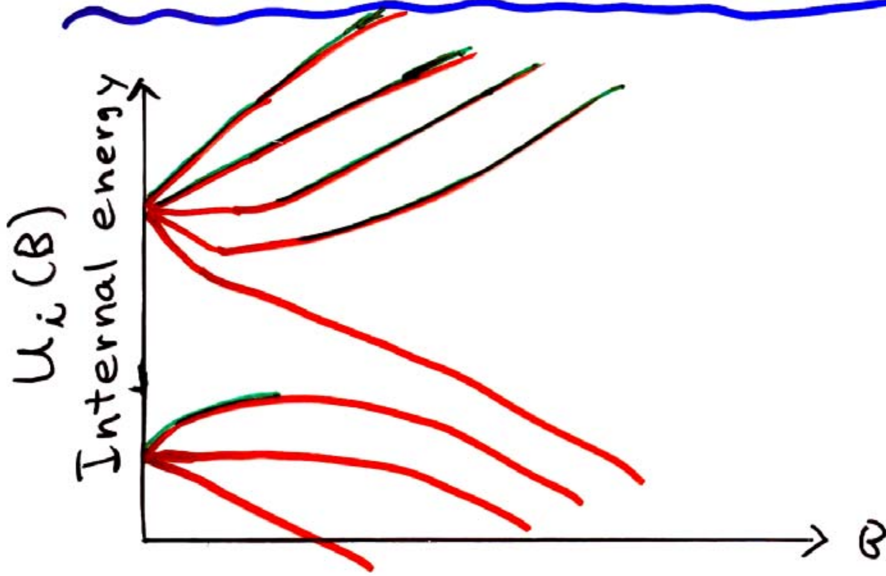
$$\sigma_{coll} = 8\pi a^2 \approx 2\cdot 10^{-12}\,\mathrm{cm}^2$$

The solution …

- Dark light trap (“Dark SPOT MOT”)
- Tight magnetic confinement
- ULTRA-high vacuum
- … a few years of engineering

Magnetic trapping

Magnetic trapping



Stability: $\vec{\nabla}\, U_i(B(\vec{r})) = 0$

$$\Delta U_i(B(\vec{r})) > 0$$

Unless $\dfrac{d\,U_i(B)}{dB} = 0$

$\Rightarrow$ Local ~~extremum~~ minimum of $|\vec{B}|$ ← Wing's theorem

→ Only weak-field seeking states

Most common situation

$$U_i(r) = \mu_i \cdot |\vec{B}(\vec{r})|$$

$$\frac{dU_i(B_0)}{dB}$$

adiabatic condition:

Atom stays in HFS i.

classical analogon (cf. Levitron)

$$U(r) = -\vec{\mu}\vec{B} = -\mu B \cos\Theta$$

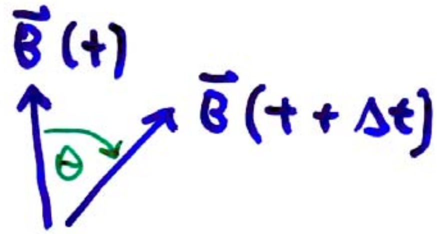
$= \text{const}$ (precession)

choose $\cos\Theta < 0$

q.m., weak field:

$$U(r) = \mu_B B g \overset{\text{const}}{\overbrace{m_F}}$$

Adiabatic condition

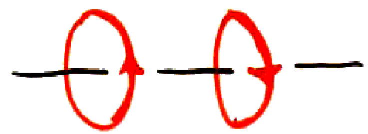


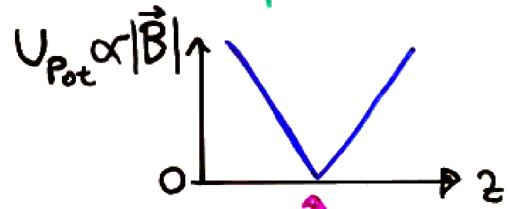
$$\omega_{ROT} = \frac{d\theta}{dt} \ll \frac{U_i - U_f}{\hbar} = g\mu_B B/\hbar$$

$$= \omega_{Larmor}$$

Magnetic Traps

- Spherical Quadrupole





Linear potential

$B = 0$

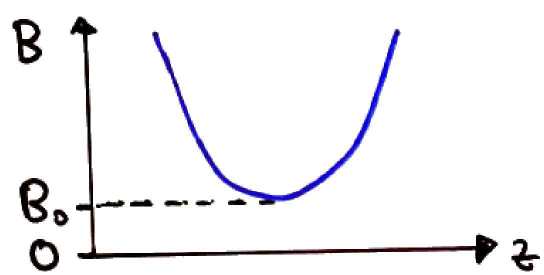
$\Rightarrow$ leaky!

Solutions:

rotating B Field
"TOP" trap
JILA

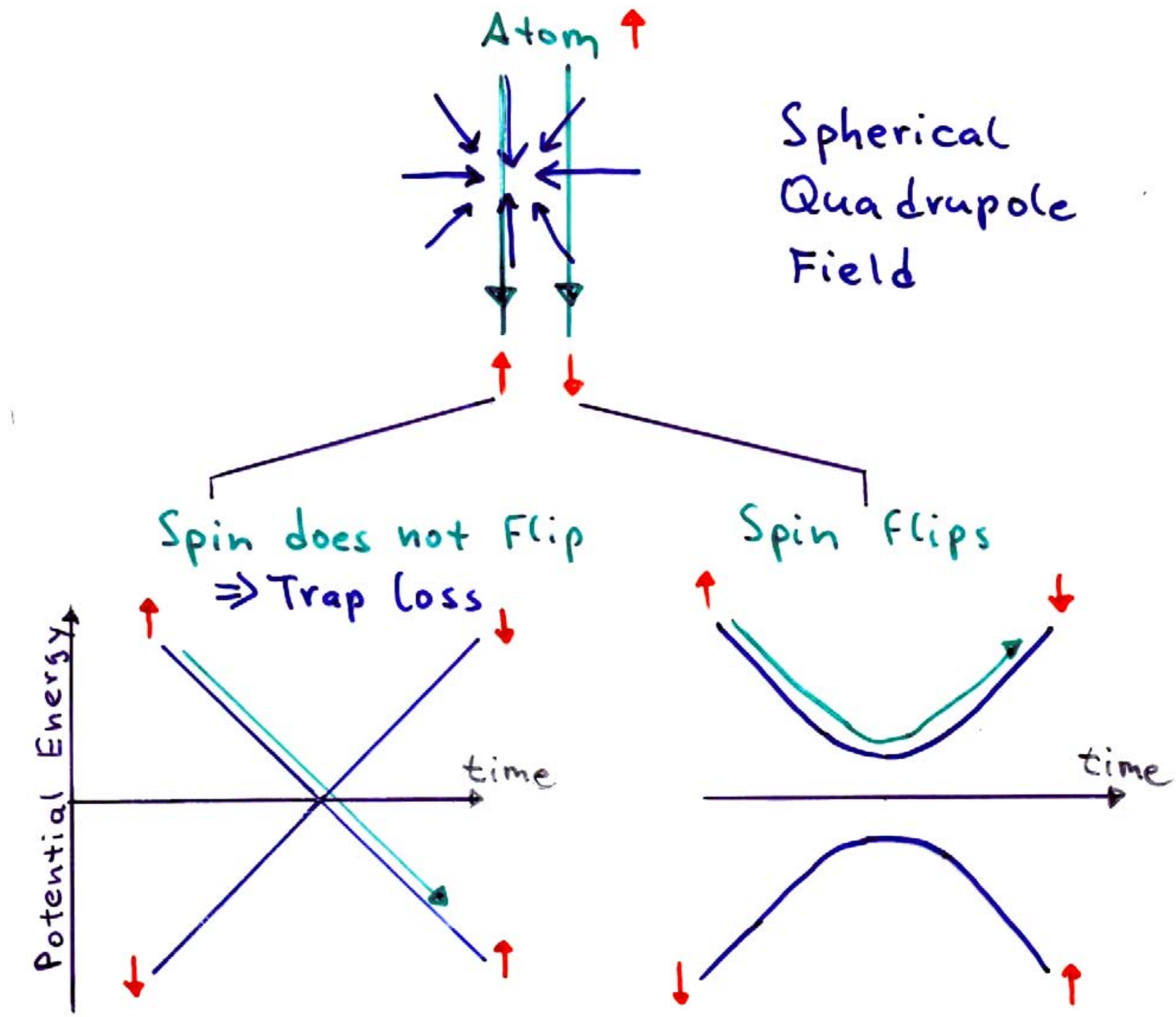
optical plug
MIT

- Ioffe-Pritchard Trap



harmonic potential

Majorana Flops



Landau-Zener crossing

Landau-Zener trap loss probability

$$e^{\left[\frac{-2\pi |V_{12}|^2}{v \hbar\, d(V_2-V_1)/dz}\right]} \simeq e^{-\frac{2\pi \mu B' x^2}{\hbar v}}$$

$$= e^{-2\pi (x/x_0)^2}$$

$x_0 \approx 1\,\mu m$
"hole size"

@ $x = x_0$:

Larmor Freq. $\frac{\mu B' x}{\hbar}$

$=$ orbital Freq. $\frac{d\theta}{dt} = \frac{v}{x}$

$\Rightarrow$ Trapping Time $\propto$ (cloud diameter)^2

TOP trap (JILA '94)

$$\vec{B}_{Stat} = B' \begin{pmatrix} x \\ y \\ -2z \end{pmatrix}$$ Quadrupole Field

$$\vec{B}_{rot} = B_0 \begin{pmatrix} \cos\omega t \\ \sin\omega t \\ 0 \end{pmatrix}$$ rotating bias Field

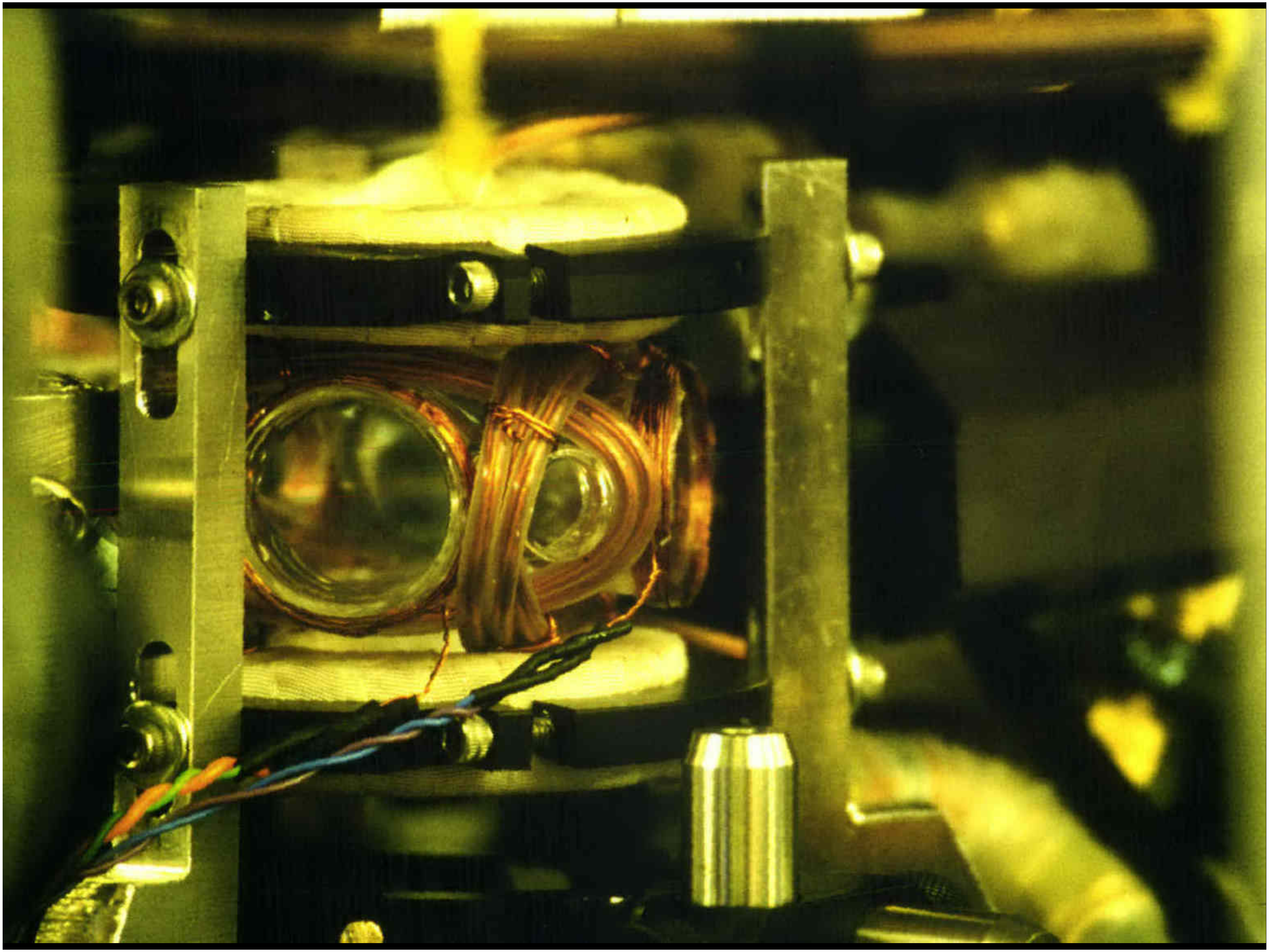
Time-averaged potential

$$U_{TOP} = \frac{\mu}{2}\left(B_r'' r^2 + B_z'' z^2 \right)$$

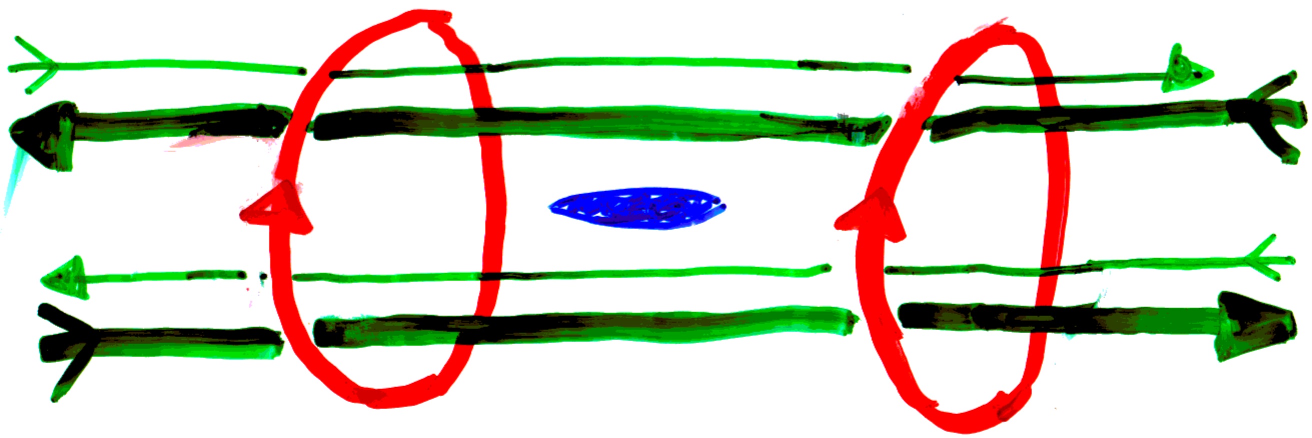
$$B_r'' = \frac{B'^2}{2B_0} \qquad B_z'' = \frac{4B'^2}{B_0}$$

Circle of death $r_D = B_0/B'$

$$U_{TOP}(r_D) = \mu B_0/4$$



Ioffe-Pritchard Trap



4 "Ioffe" bars

$\Rightarrow B'_{radial}$

2 "Pinch" coils

$\Rightarrow B_0, B''_{axial}$

$B_0 \neq 0$ traps

Ioffe - Pritchard configuration

$$\vec{B} = B_0 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + B' \begin{pmatrix} x \\ -y \\ 0 \end{pmatrix} + \frac{B''}{2} \begin{pmatrix} -zy \\ -zy \\ z^2 - \frac{1}{2}(x^2+y^2) \end{pmatrix}$$

$$B(x,y,z) = \left\{ \left[B_0 + \frac{1}{2} B'' z^2 - \frac{B''}{4}(x^2+y^2) \right]^2 + \left(B' - B'' z/2\right)^2 x^2 + \left(B' + B'' z/2\right)^2 y^2 \right\}^{\frac{1}{2}}$$

$$\approx \frac{1}{2} \left[\underbrace{\frac{B'^2}{B_0}}_{= B''_{\mathrm{radial}}} r^2 + B'' z^2 \right]$$

$\uparrow$ large B_0, B'

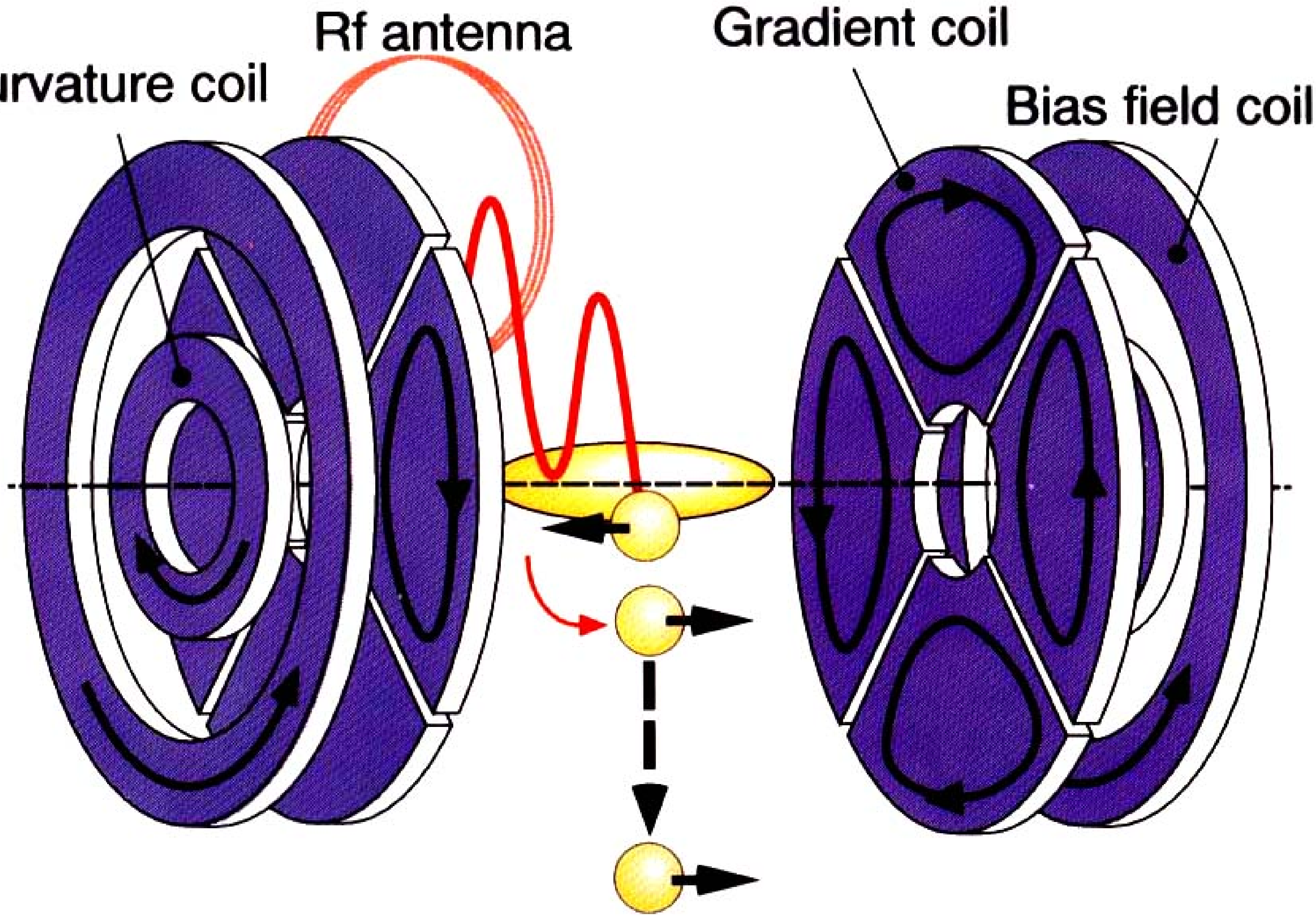
WARNING: Green terms limit trapping volume! (important for hot clouds)

IP - Trappology

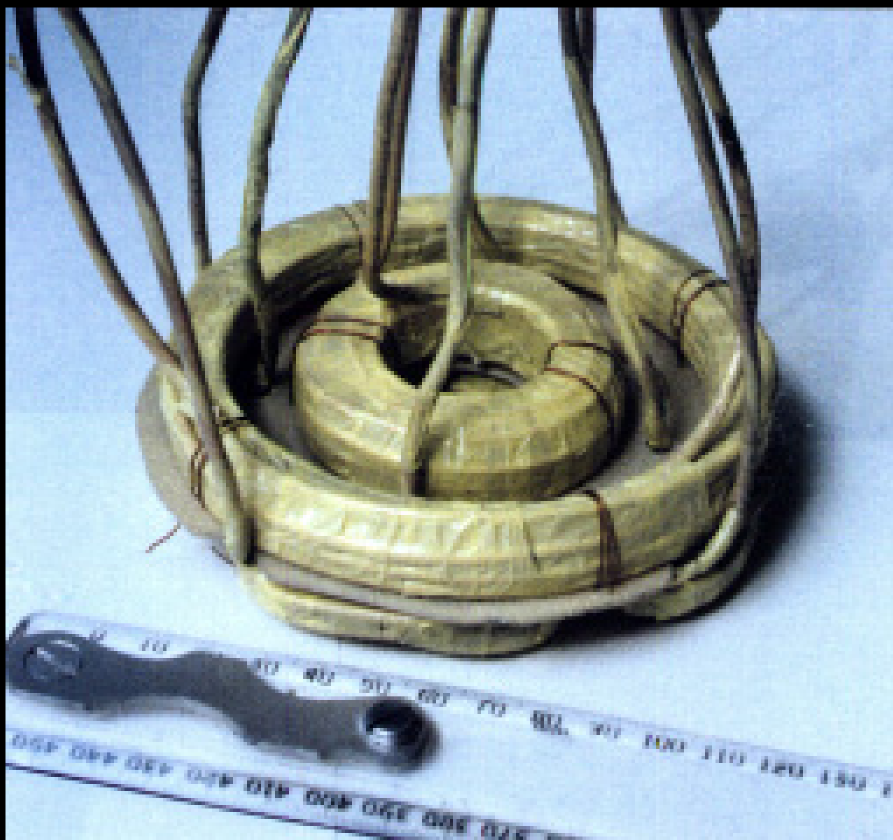
in order of appearance

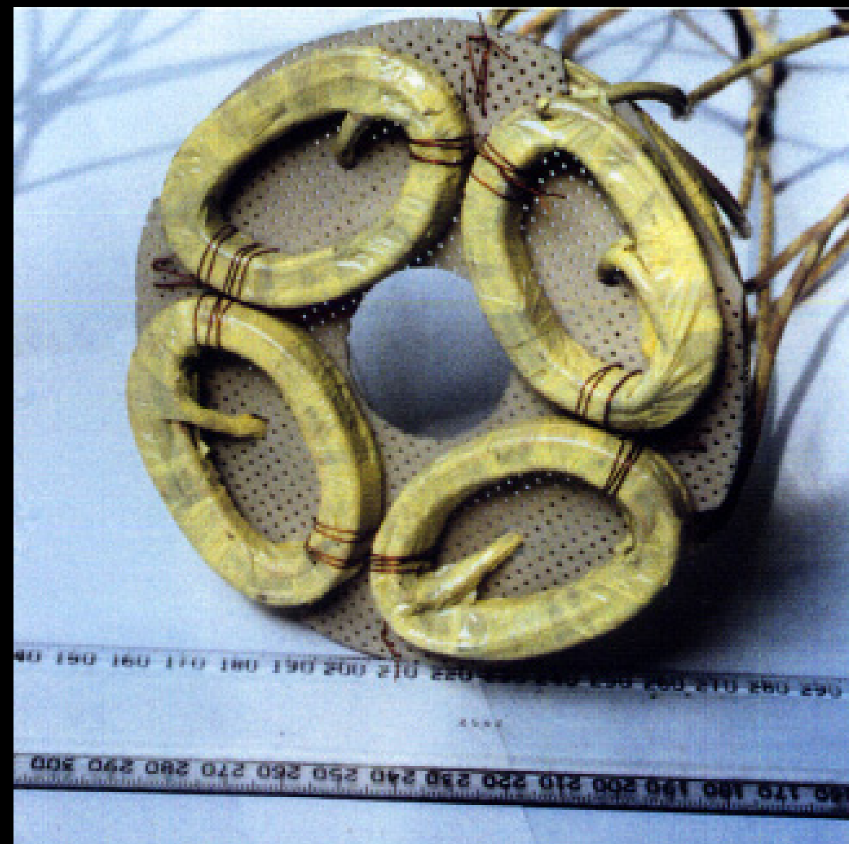
Permanent magnets	Rice
Cloverleaf trap	MIT
Baseball trap	Boulder
Four-dee	Rowland
Ioffe bars	Konstanz
3 coils, no extra bias	Munich
3 coils + extra bias	Paris
Ioffe bars, superconducting	MIT
Pole piece	Orsay

BEC in a "cloverleaf" magnetic trap

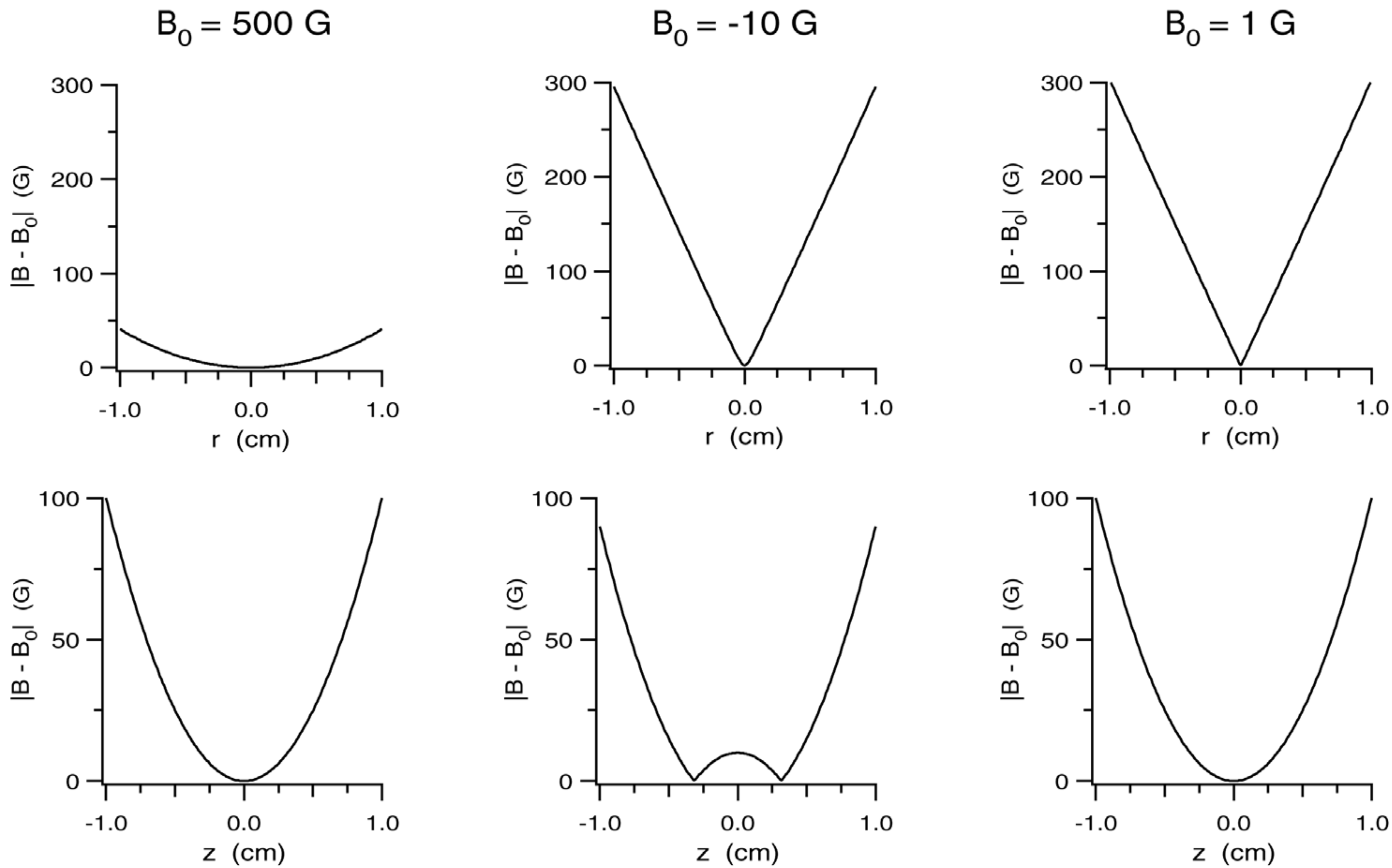


MIT, March '96 [M.-O. Mewes et al., PRL 77, 416 (1996)]

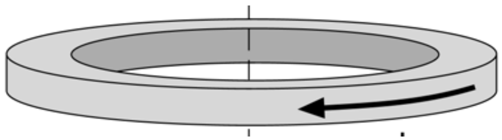


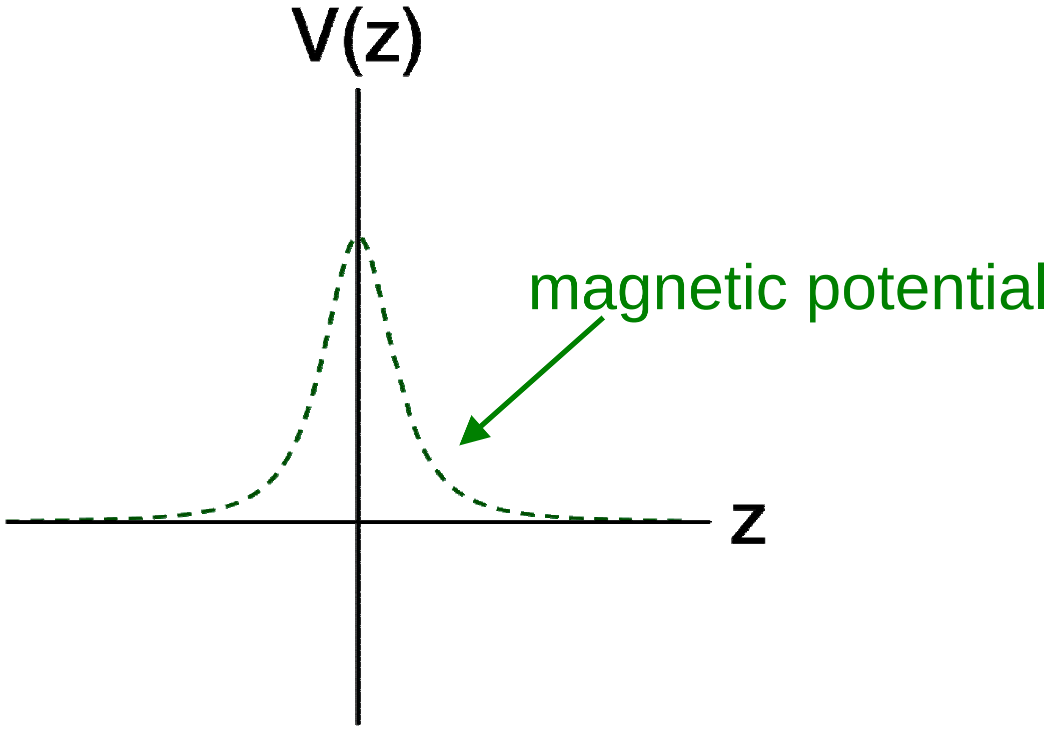


Bias field adjustment is critical



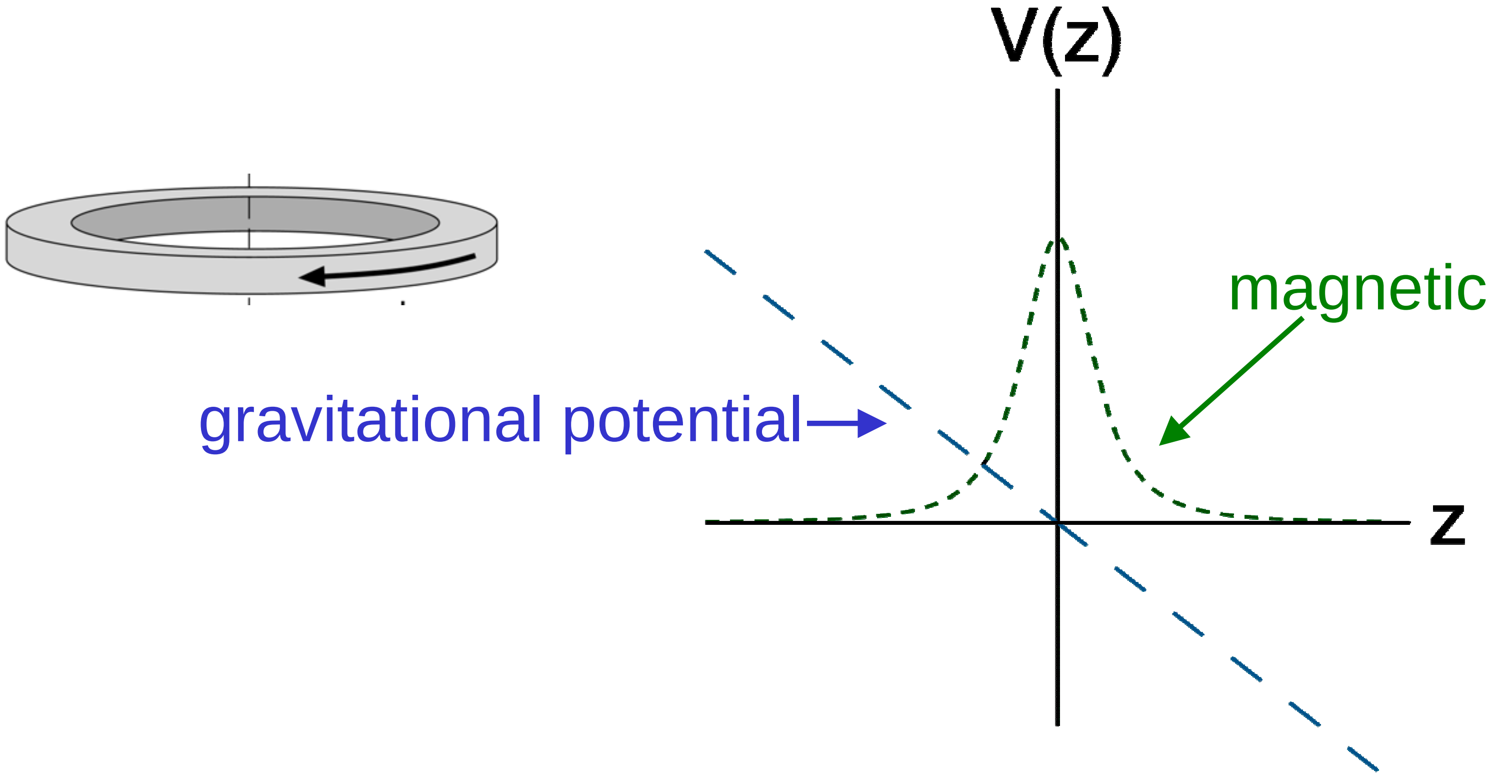
Gravito-Magnetic Trap





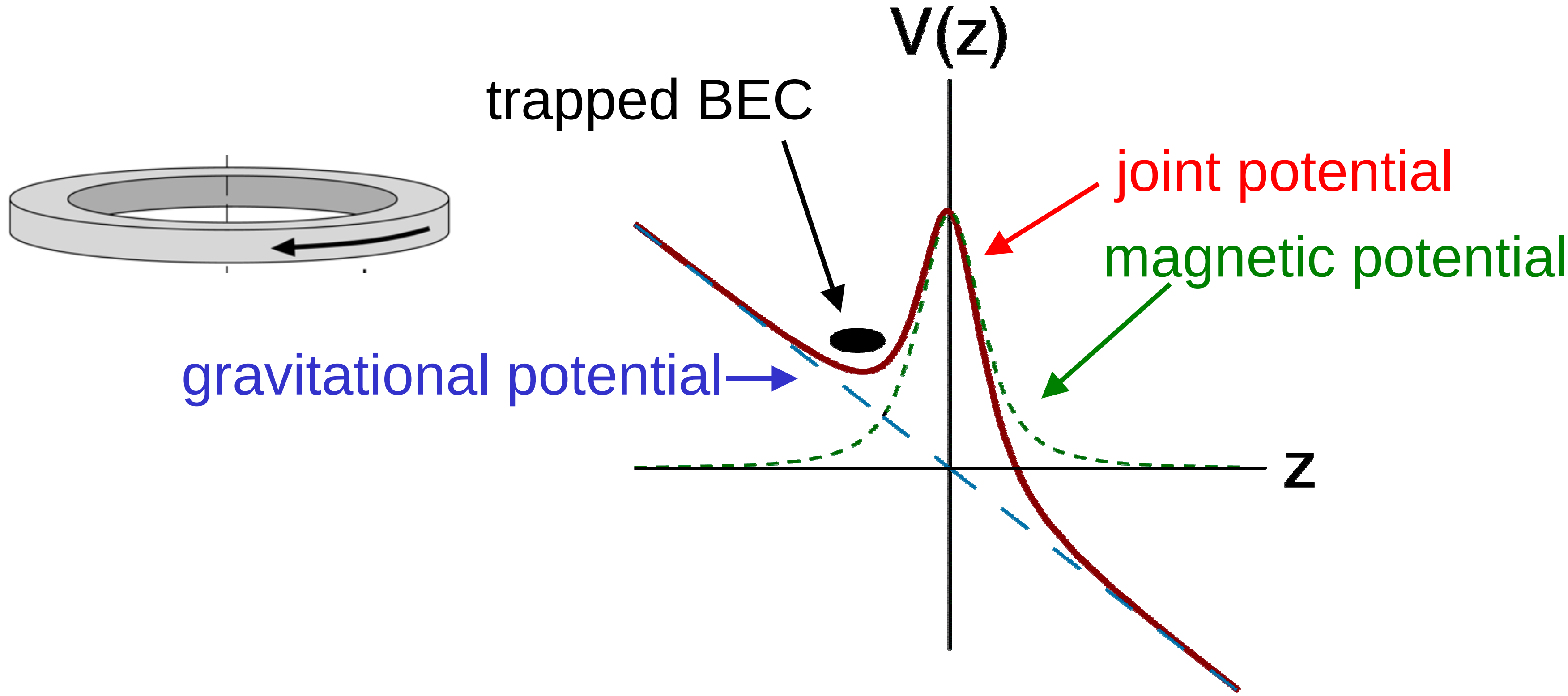
• Single coil carrying current I_S levitates atoms against gravity with magnetic field gradient ~8 G/cm.

Gravito-Magnetic Trap

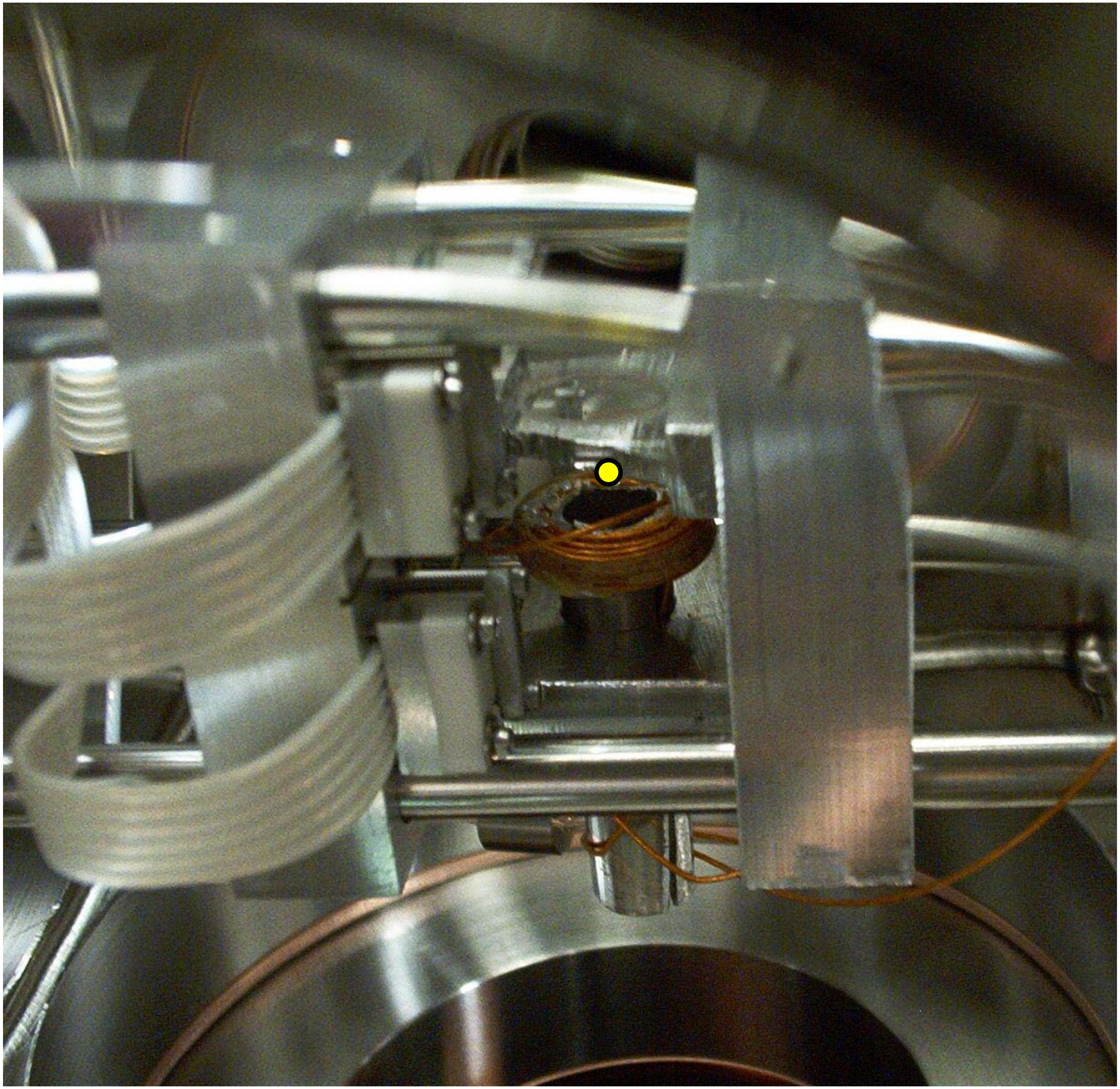


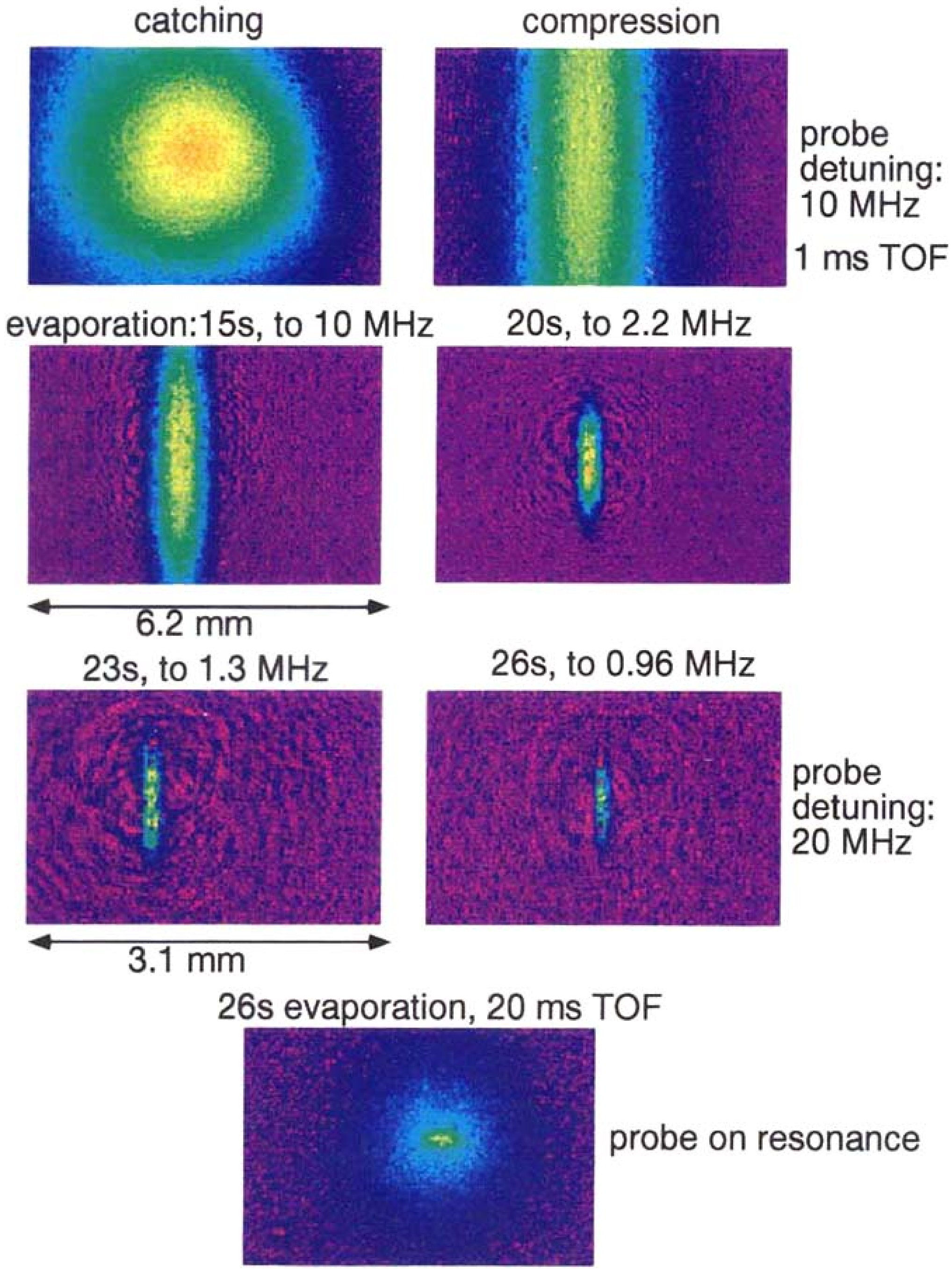
• Single coil carrying current I_S levitates atoms against gravity with magnetic field gradient ~8 G/cm.

Gravito-Magnetic Trap



- Single coil carrying current I_S levitates atoms against gravity with magnetic field gradient ~8 G/cm.
- Stable vertical confinement for |z| > R/2 above coil.




catching
compression
probe detuning: 10 MHz
1 ms TOF
evaporation:15s, to 10 MHz
20s, to 2.2 MHz
6.2 mm
23s, to 1.3 MHz
26s, to 0.96 MHz
probe detuning: 20 MHz
3.1 mm
26s evaporation, 20 ms TOF
probe on resonance