Entanglement transitions in "Hybrid" Quantum circuits

↳ monitored, non unitary circuits, tensor networks

References: Pedagogical Book chapter / Reviews:

A.C. Potter and RV, 2111.08018

Fisher, Khemani, Nahum and Vijay, 2207.14280

Monitored circuits:

Li, Chen, Fisher 1808.06134 } 1st papers
Skinner, Ruhman, Nahum 1808.05953 }
Gullans, Huse 1905.05195, 1910.00020
Bao, Choi, Altman 1903.05124 + Qi
1908.04305 } Replica
Jian, You, Vasseur, Ludwig 1908.08051 } Stat. Mech. approach
Zabalo et al, 1911.00008
⋮

+ Many more!

Related papers on stat mech approach:

Random circuits (See Sagar's lectures):

Nahum, Ruhman, Vijay, Haah 1608.06950
Nahum, Vijay, Haah 1705.08975
Zhou, Nahum 1804.09737

Random tensor networks

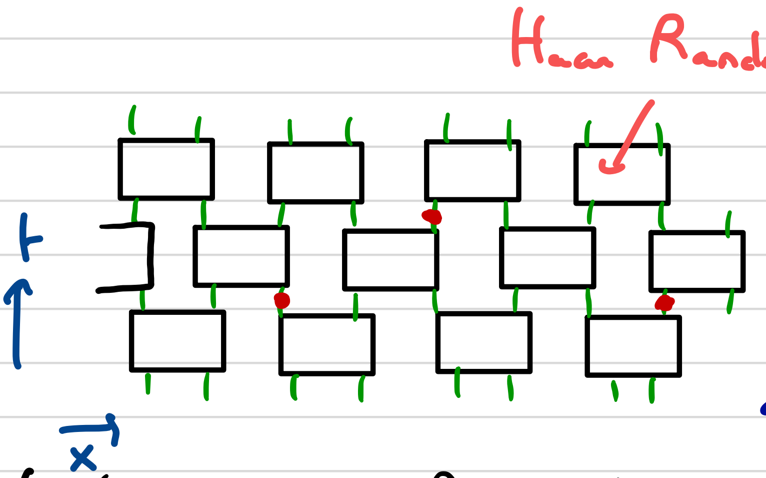
Hayden et al 1601.01694
Vasseur, Potter, You, Ludwig 1807.07082
Nahum, Roy, Skinner, Ruhman 2009.11311

Replica stat mech

I Entanglement transitions

a) Measurement induced phase transitions (MiPT's)

Chaotic dynamics (entanglement growth) vs local projective measurements.



- $\mathcal{H} = (\mathbb{C}^d)^N$

  qudit on each site (take $d=2$ for simplicity)

- Model quantum chaotic dynamics by random quantum circuit

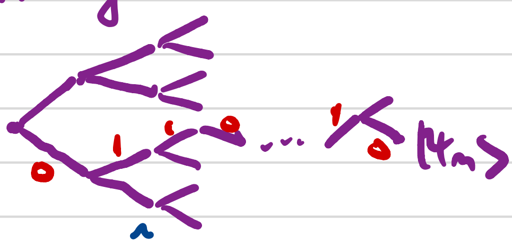
(not necessary, but makes problem tractable)

• = Random projective measurement with proba p.

Claim: "transition" = MiPT

"weak monitoring" $\;\; p_c \;\;$ "strong monitoring" $\;\; p$

. transition in quantum trajectories



- Single measurement: $\hat{\Pi}_\uparrow = |\uparrow\rangle\langle\uparrow|$, $\hat{\Pi}_\downarrow = |\downarrow\rangle\langle\downarrow|$

normalized

$$|\psi\rangle \to |\tilde{\psi}_\uparrow\rangle = \frac{\hat{\Pi}_\uparrow|\psi\rangle}{\|\hat{\Pi}_\uparrow|\psi\rangle\|}, \quad |\psi_\uparrow\rangle = \hat{\Pi}_\uparrow|\psi\rangle \qquad p_\uparrow = \langle\psi|\hat{\Pi}_\uparrow|\psi\rangle$$

$$|\psi\rangle \to |\tilde{\psi}_\downarrow\rangle = \frac{\hat{\Pi}_\downarrow|\psi\rangle}{\|\hat{\Pi}_\downarrow|\psi\rangle\|}, \quad |\psi_\downarrow\rangle = \hat{\Pi}_\downarrow|\psi\rangle \qquad p_\downarrow = \langle\psi|\hat{\Pi}_\downarrow|\psi\rangle$$

Born Probability

What if we "throw away" the measurement outcomes?

$\rightarrow$ decoherence / Open system

$$\rho \mapsto \mathcal{N}(\rho) = p_\uparrow |\tilde{\psi}_\uparrow\rangle\langle\tilde{\psi}_\uparrow| + p_\downarrow |\tilde{\psi}_\downarrow\rangle\langle\tilde{\psi}_\downarrow|$$

$$= \hat{\Pi}_\uparrow \rho \hat{\Pi}_\uparrow + \hat{\Pi}_\downarrow \rho \hat{\Pi}_\downarrow \quad \text{with } \rho = |\psi\rangle\langle\psi|$$

$$= \sum_{\sigma=\uparrow,\downarrow} \underbrace{\langle\sigma|\rho|\sigma\rangle}_{p_\sigma} |\sigma\rangle\langle\sigma| \quad \text{dephasing channel}$$

Full evolution: $m = \{1,0,0,1,\dots\}$ measurement outcomes

Trace out measurement outcomes: $\bar{\rho}_t = \sum_{\{m\}} |\psi_m\rangle\langle\psi_m|$

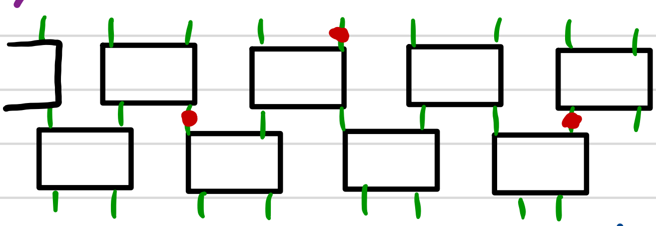
Quantum channel: $\bar{\rho}_t = \mathcal{N}_t(\rho) = \sum_{\{m\}} K_m \rho K_m^\dagger, \quad \sum_m K_m^\dagger K_m = \mathbb{1}$

unitaries

K_m: Kraus operator, $K_m = \prod_{\tau=1}^{t} \hat{\Pi}^{m'_\tau} U_\tau^{\text{odd}} \hat{\Pi}^{m_\tau} U_\tau^{\text{even}}$

projections onto measurement outcomes

non unitary circuit (tensor network)

$K_m =$ 

- Each summand: $K_m \rho K_m^\dagger$ = quantum trajectory

$$\mathrm{Tr}(K_m \rho K_m^\dagger) = p_m \quad \text{Born probability}$$

linear quantity: $\langle O \rangle$, average over measurement outcomes:

$$\overline{\langle O \rangle} = \sum_{\{m\}} \cancel{p_m} \frac{\langle \psi_m | O | \psi_m \rangle}{\cancel{\langle \psi_m | \psi_m \rangle}} = \mathrm{Tr}(\bar{\rho}_t O)$$

sum over quantum trajectories

. At long times: combined action of random Haar unitaries + measurements drive $\bar{\rho}_t$ toward a featureless / infinite T state

$$\bar{\rho}_t \longrightarrow \rho_\infty = \frac{1}{d^L} \mathbb{I}$$ (unital channel $\sum_{\{m\}} K_m K_m^\dagger = \mathbb{I}$)

$\Rightarrow$ linear observables trivial!

. What about non-linear observables?

Nonlinear quantity:

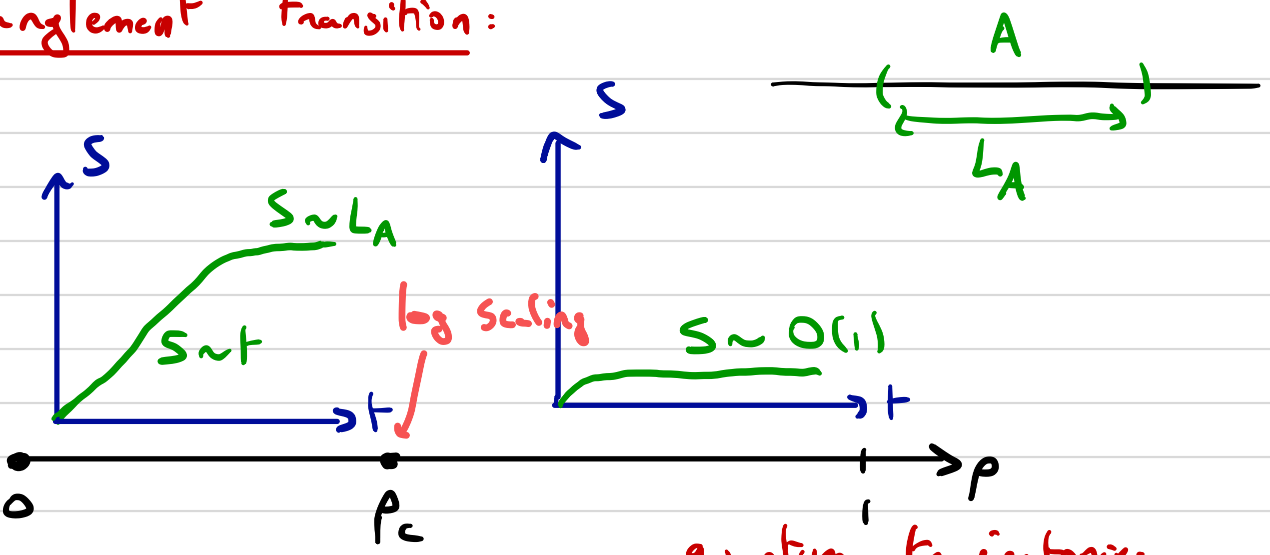
$$\overline{\langle O \rangle^2} = \sum_{\{m\}} \cancel{p_m} \frac{\langle \psi_m | O | \psi_m \rangle^2}{\langle \psi_m | \psi_m \rangle^{\cancel{2}}},$$ can't be expressed in terms of $\bar{\rho}_t$!

Claim: Interesting phase structure, transitions, in quantum trajectories $\{p_m, |\psi_m\rangle\}$ invisible in $\bar{\rho}_t$!

Post-selection issue: Measuring $\overline{\langle O \rangle^2}$ requires preparing the state $|\psi_m\rangle$ many times: requires post selecting over $\exp(2pLt)$ measurements!!!

↳ more on this later: see Sarang + Ehud's lectures

Entanglement transition:



quantum trajectories

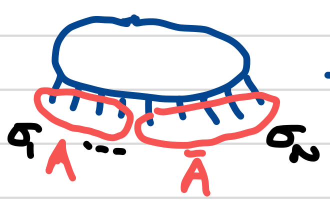
$$\boxed{S_n = \mathbb{E}_{\text{circuits}} \sum_{\{m\}} p_m \frac{1}{1-n} \log\left[\frac{\operatorname{tr} \rho_{A,m}^n}{(\operatorname{tr} \rho_m)^n}\right]}$$

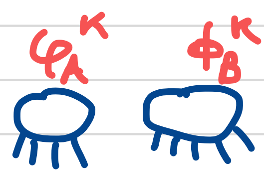
Average over Haar unitaries, measurement locations

Renyi Entropy of $|\psi_m\rangle$ (nonlinear in ρ_m)

$$\rho_m = |\psi_m\rangle\langle\psi_m| , \quad \rho_{A,m} = \operatorname{tr}_{\bar{A}} \rho_m$$

Reminder:

$|\psi\rangle = \sum_{\{\sigma\}}$ 

$= \sum_K \lambda_K$ 

$\lambda_K = \sqrt{p_K}$ = schmidt values, $\sum_K p_K = 1$

$$S_n = \frac{1}{1-n} \log \sum_K p_K^n = \frac{1}{1-n} \log \operatorname{Tr} \rho_A^n , \quad \rho_A = \operatorname{tr} |\psi\rangle\langle\psi|$$

Renyi entanglement Entropy

Entanglement entropy: $S = \lim_{n\to 1} S_n = -\sum_k p_k \log p_k$

$= -\,\mathrm{tr}\, \rho_A \log \rho_A$

Alternatively: S_n = expectation value of a permutation operation on n-folded replicated space:

$$\boxed{S_n = \frac{1}{1-n} \log \mathrm{tr}\left(\rho^{\otimes n}\, \hat{S} \right)}$$

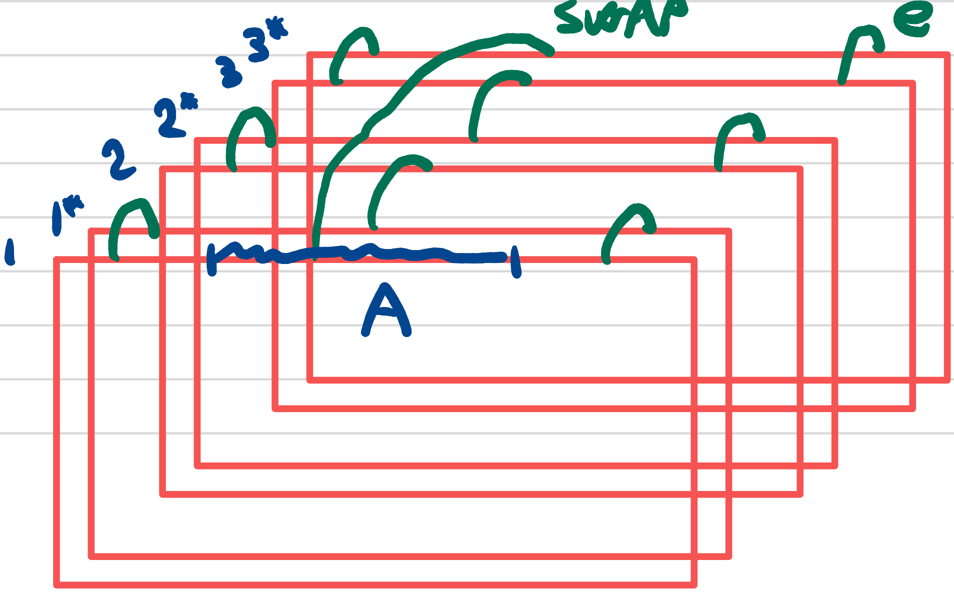
with: $\hat{S} = \prod_x \hat{S}_{g_x}$, $\quad g_x = \underset{\text{SWAP}}{(12\ldots n)} = \cancel{XX},\ x \in A$

$= e,\ x \in \bar{A}$

$$\hat{S}_{g_x} = \sum_{\{i\}} |i_{g_x(1)}\, i_{g_x(2)} \ldots i_{g_x(n)}\rangle\langle i_1\, i_2 \ldots i_n|$$

= representation of g_x on $(\mathbb{C}^d)^{\otimes n}$

Pictorially:
$(n=3)$

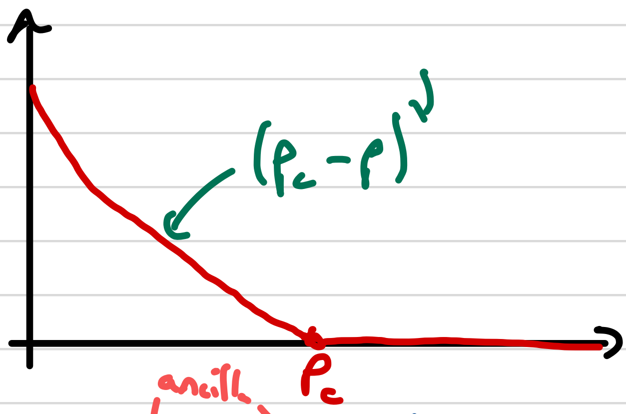


② Purification and learnability transitions

- Start with initial state $\rho_0 = \rho_\infty = \frac{1}{2^L}\mathbb{1}$. $d=2$ ρ_{cc}

Mesurements will purify state : $S = -\operatorname{tr} \rho_m \log \rho_m$ (Entropy of mixed state)

$t \sim \mathrm{poly}(L)$: S/L

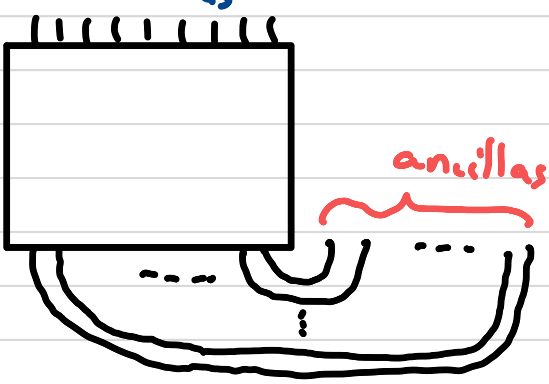


Purify ρ_0 using ancillas: $|\psi_0\rangle = \left(\frac{|\uparrow\Uparrow\rangle + |\downarrow\Downarrow\rangle}{\sqrt{2}}\right)^{\otimes L}$ ancilla

$$= \frac{1}{2^{L/2}} \sum_{\{\sigma\}} |\sigma\rangle \otimes \underbrace{|\sigma\rangle}_{\text{ancillas}}$$

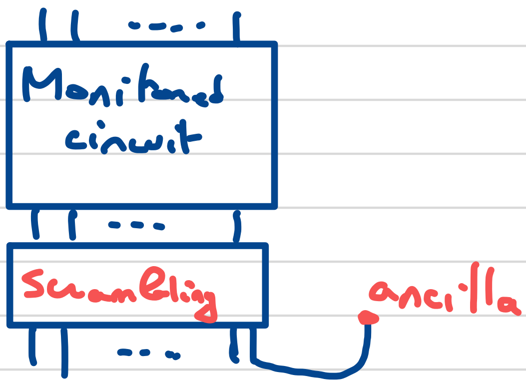
$$\rho_0 = \operatorname{Tr}_{\text{ancillas}} |\psi_0\rangle\langle\psi_0| ,$$

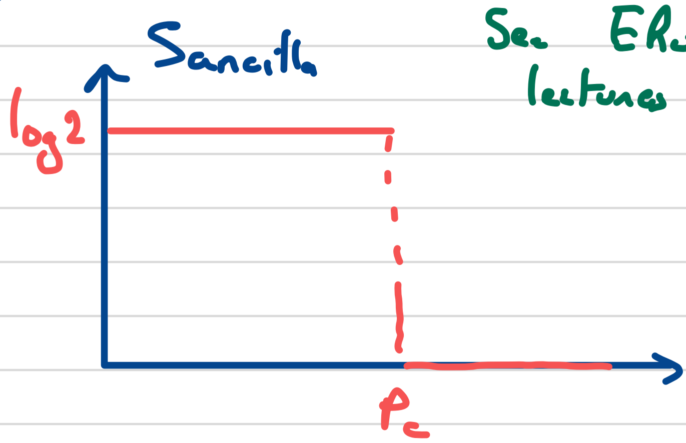
$S(\rho) =$ Entanglement between system and ancillas



$S/L = \#$ "encoded" qubits

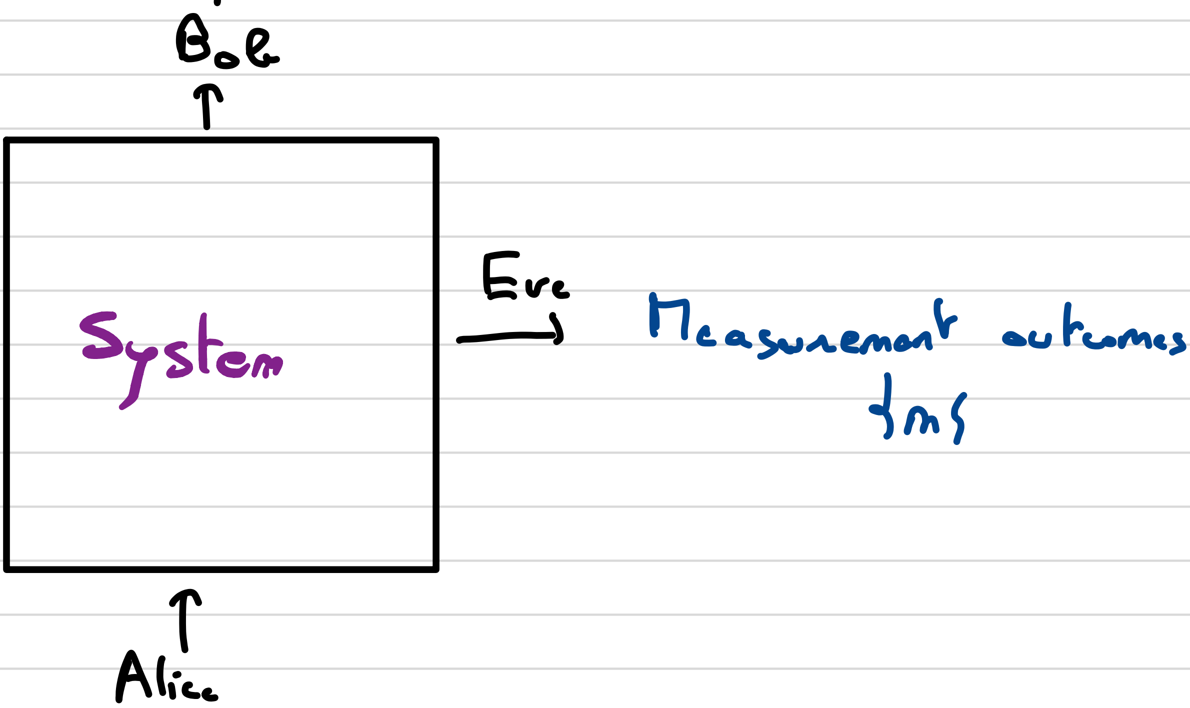
- Order parameter (Gullans - Huse)





See Ehud's lectures

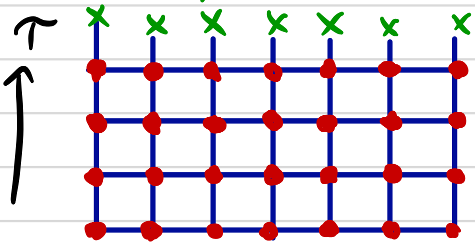
- Learnability perspective: See Sarang's lectures



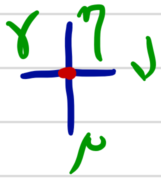
- Mixed / Entangled phase: $\{m\}$ random, contain no info! Dynamics scrambles information, not accessible to local measurements
- Pure phase: Measurements effectively extract info from system

ⓒ Random tensor networks (RTN)

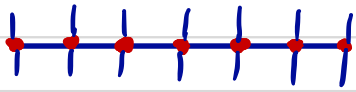
physical degrees of freedom



Similar to random circuits, but no unitarity.



$= T_{\mu\nu\eta\gamma} =$ Random



$=$ Random MPO

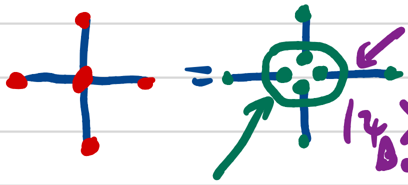
↳ matrix product operator

$\mu = 1, \dots, D$ bond dimension

Look at $S_A(\tau \to \infty)$ vs D

Complexity transition

Area law $S_A \sim cst$ "Easy" — D_c — Volume law $S_A \sim L_A$ "Hard"

> D can be made continuous:
>
> 
>
> $\langle \psi_{Bell}|$
>
> $|\psi_{Bell}\rangle = \sum_{i=1}^{D} \frac{|ii\rangle}{\sqrt{D}}$
>
> $|T\rangle = \sum T_{\mu\nu\eta\gamma} |\mu\nu\eta\gamma\rangle$

- Very similar to MIPT. Key difference: no Born probability

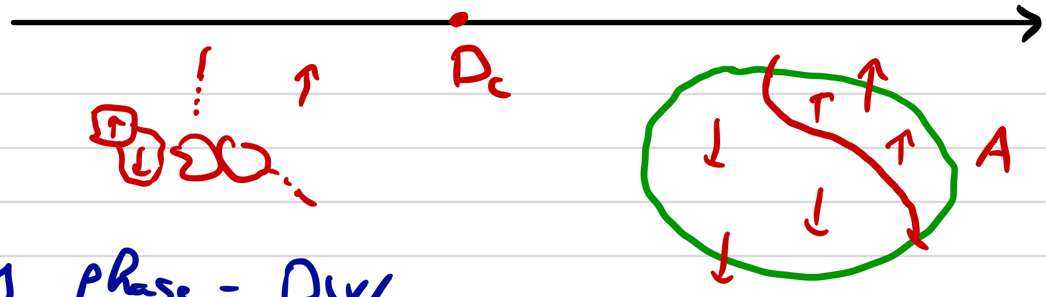
Ⓘ Statistical mechanics mapping for RTN

→ What do we know?:

- Exact Mapping onto (replica) 2D Stat. Mech. Model
- Qualitative picture:

$$S_A = \Delta F\,(\text{insert DW})$$

All Renyi Entropies have the same D_c



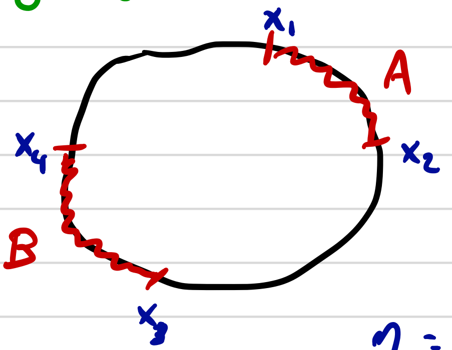
$\Delta F \propto L_A$
FM phase

PM phase = DW condensate
$\Delta F \propto$ cst

- Critical point = CFT in 2d with $c=0$ } non unitary field theory
  can write down a Lagrangian etc.
- Conformal invariance at transition : $\boxed{z=1}$
- $S_A \sim \log L_A$ at criticality

$$S_A \sim \log \langle \phi_{Bcc} \phi_{Bcc} \rangle \quad \text{in CFT}$$

Conformal invariance: (seen numerically for Clifford + Measurements)

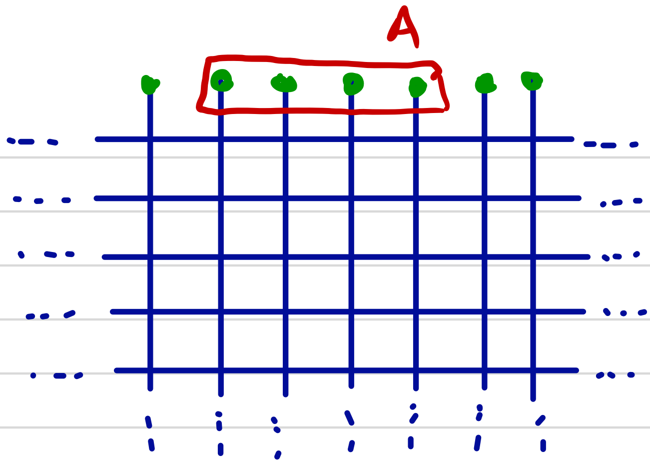


$$I_{AB} = S_A + S_B - S_{A\cup B}$$
$$= f(\eta)$$

$$\eta = \frac{x_{12}\, x_{34}}{x_{13}\, x_{24}} \qquad x_{ij} = \frac{L}{\pi} \sin\left(\frac{\pi}{L}|x_i - x_j|\right)$$

Mapping onto Stat. Mech model

- Focus on RTN for simplicity. (Measurement transition similar but model more involved)
  ↳ next section



G : Square lattice on the lower half plane.

Let's consider a Random tensor Network (RTN) defined on G.

$\bullet$ = physical degrees of freedom $\longrightarrow$ 1d boundary system (defined on ∂G)

average over RTN

Compute: $$\overline{S_A^n} = \frac{1}{1-n}\,\overline{\log\frac{\mathrm{tr}\,\rho_A^n}{(\mathrm{tr}\,\rho)^n}}$$ ← Normalization!

$$= \lim_{m\to 0}\frac{1}{1-n}\frac{\partial}{\partial m}\left[\overline{(\mathrm{tr}\,\rho_A^n)^m} - \overline{(\mathrm{tr}\,\rho^n)^m}\right]$$

Replica approach! Let $Z_A = \overline{(\mathrm{tr}\,\rho_A^n)^m}$

$Z_0 = \overline{(\mathrm{tr}\,\rho^n)^m} = \overline{\mathrm{tr}\,\rho^{\otimes nm}}$

We have:

$$\boxed{\overline{S_A^n} = \lim_{m\to 0}\frac{1}{n-1}\frac{\partial}{\partial m}\left(F_A - F_0\right)}$$, w/ $F = -\log Z$

$Z_A = Z_0 = 1$ as $m \to 0$

⚠ Replica limit can be problematic

$$f(n) = \frac{\sin(\pi n)}{\pi n}$$

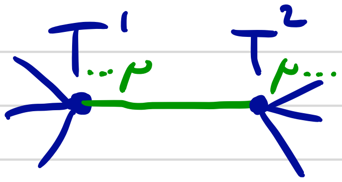
$f(n) = 0$ for $n \in \mathbb{N}^*$

but $f(n\to 0) = 1$!

Let's now compute $\mathrm{tr}\,\rho^{\otimes Q}$ $\qquad Q = nm$

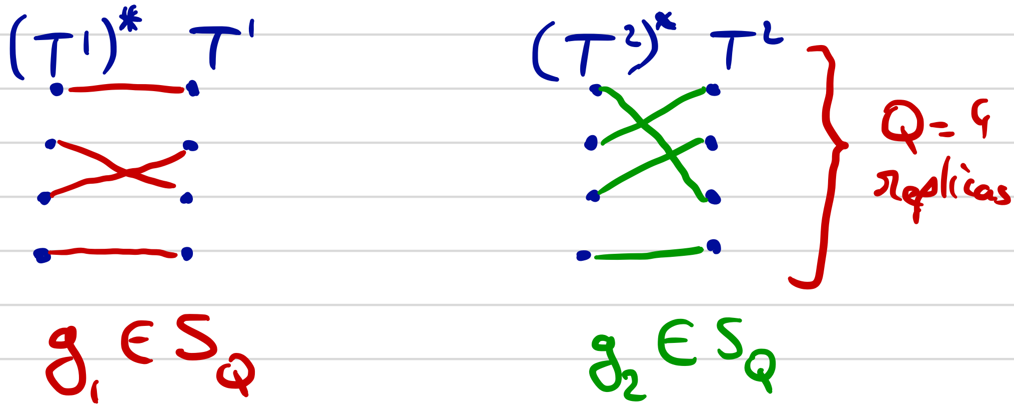
(ignore boundary for now)

and Gaussian moments



$$\overline{(T^{i}_{\mu\nu..})^{*}\, T^{i'}_{\mu'\nu'}} = \delta_{ii'}\,\delta_{\mu\mu'}\,\delta_{\nu\nu'}\dots$$

Now $\rho^{\otimes Q}$:



Match T^i with $(T^i)^*$ in different replicas. (Wick's theorem!)

$$\overline{\rho^{\otimes Q}} = \sum_{\{g_i\}} w(\{g_i\})$$

+ weight invariant under L/R multiplication by $h \in S_Q$ (reordering identical factors in $\rho^{\otimes Q}$)

$\Rightarrow$ $S_Q \times S_Q$ symmetry. ($\rtimes \mathbb{Z}_2$: $g \to g^{-1}$)

$\Rightarrow$ Factor into product of pairwise weights

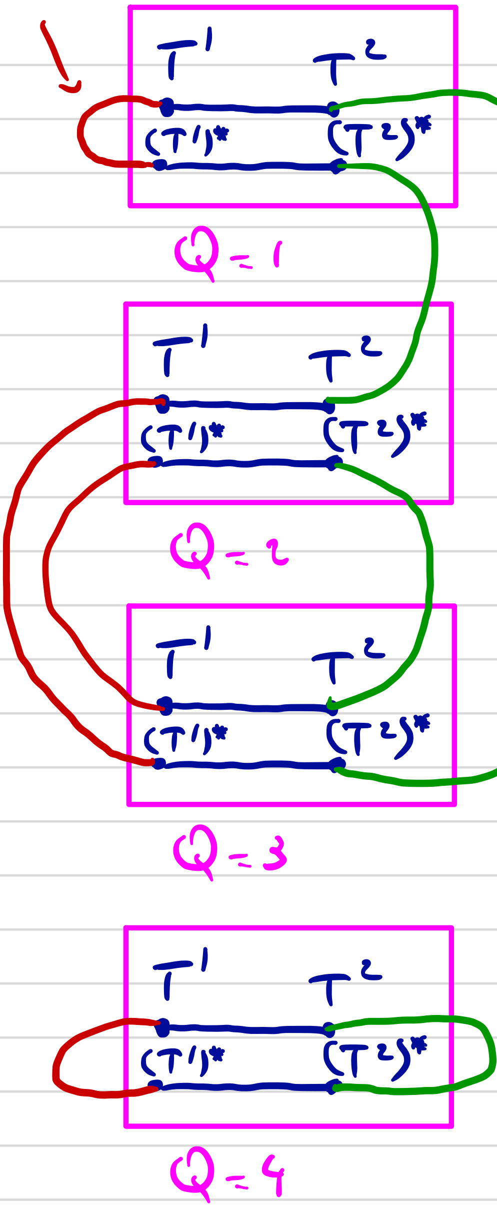
$$w = \prod_{\langle ij \rangle} C(g_i^{-1} g_j)$$

C class function $C(h^{-1} g h) = C(g)$

Weight: Given in terms of Cycle counting function $\chi(g) = \#$ cycles in g.

Choice of pairing on site 1

choice of pairing on site 2



physical link = contraction

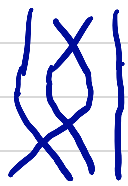
$$\text{Weight} = D^{\#\,\text{loops}}$$

$$= D^3 \quad \text{here}$$

$$\#\,\text{loops} = \chi(g_1^{-1} g_2)$$

$$g_1 = |X|$$

$$g_2 = X|$$

$g_1^{-1} g_2 =$  $= X||$

↳ 3 cycles

$$\boxed{Z = \sum_{\{g_i \in S_Q\}} \prod_{\langle i,j \rangle} D^{\chi(g_i^{-1} g_j)}}$$

D = Bond dimension

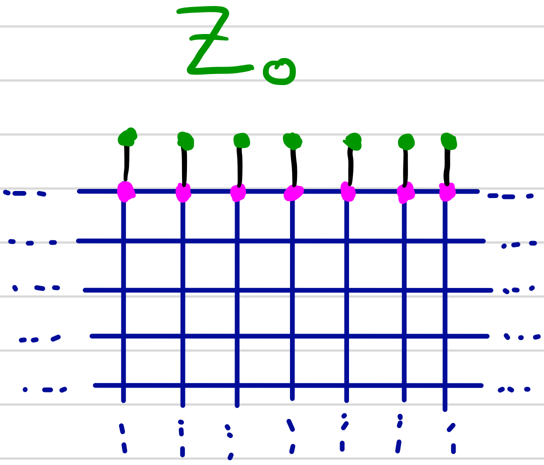
Q = nm

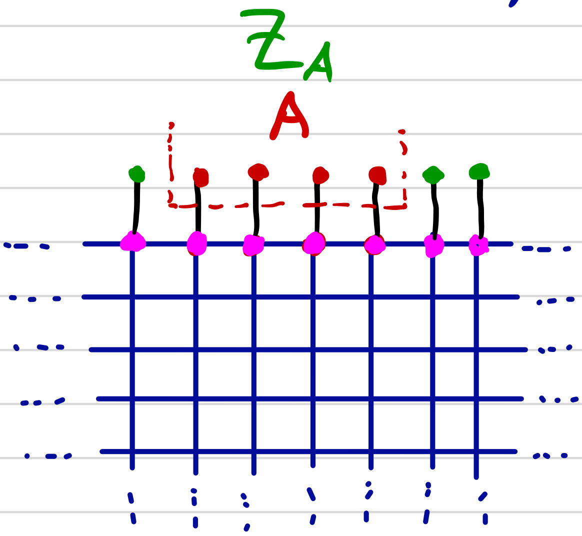
χ = # cycles

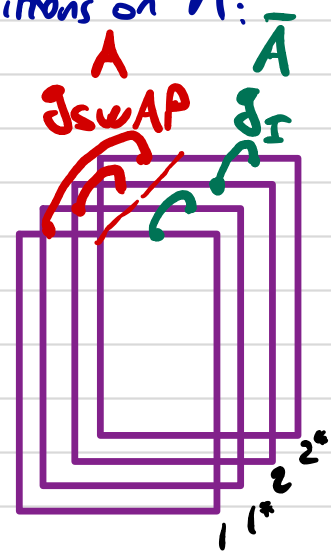
↳ classical stat mech model

"spins" $g_i \in S_{Q=nm}$

Z_A and Z_0 differ by their boundary conditions on A:







● = "spin" fixed to $g_I = ||\cdots|$ (identity: enforces $\mathrm{tr}\, \rho^{\otimes Q}$)

● = "spin" fixed to $g_{SWAP} = \underbrace{\underbrace{X}_{n=3} \; X \cdots X}_{m \text{ replicas}}$: implements $\underline{(\mathrm{tr}\, \rho_A^n)^m}$

n = Renyi index
m = Replica index

g_i ●—● : boundary link $= d^{\chi(g_i)}$

●—● : $= d^{\chi(g_{SWAP}^{-1} g_i)}$

} boundary "fields"

d = dimension of the physical Hilbert space

__Physical picture:__

Large D: Stat mech model is ordered : FM

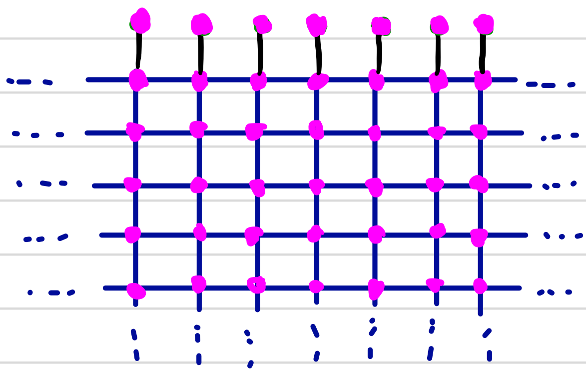
$\Delta F \propto L_A$: volume law phase

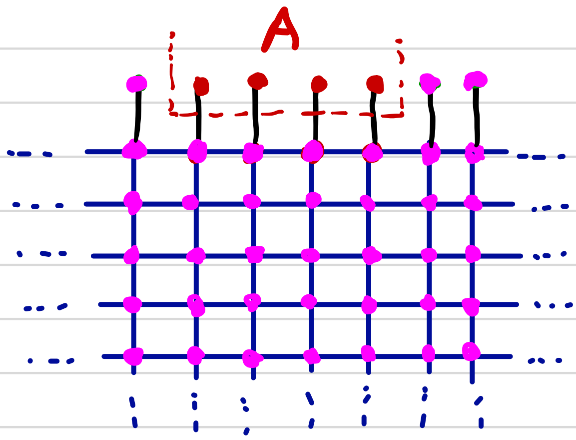
$D \to \infty$ (low T for the stat. mech. model).

We want to maximize $\chi(g_i^{-1} g_j) \Rightarrow g_i = g_j$ (FM interaction)

Z_A and Z_0 are dominated by a single configuration

Z_0 Z_A





g_I = identity everywhere

$$Z_0 = D^{\chi(g_I)(\#\text{bulk links})} \, d^{\chi(g_I)\,\overbrace{\#\text{boundary links}}^{L}}$$

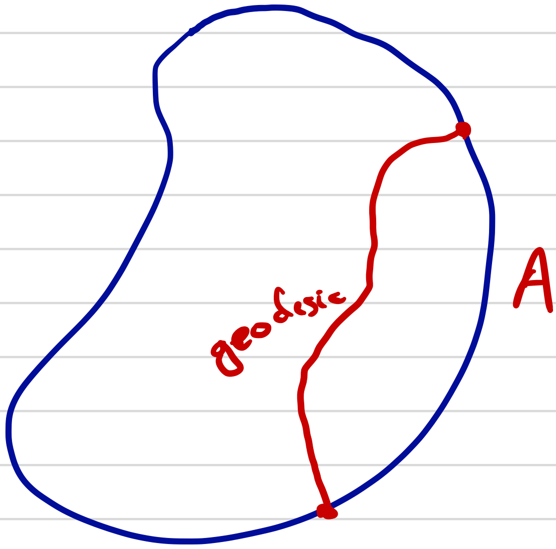
$$Z_A = Z_0 \, d^{L_A(\chi(g_{SWAP}) - \chi(g_I))}$$
$$= Z_0 \, d^{L_A(m - nm)}$$

$$\Rightarrow F_A - F_0 = \Delta F = -\log\frac{Z_A}{Z_0} = m(n-1) L_A \log d$$

$$\Rightarrow \boxed{\overline{S_A^n} \underset{D\to\infty}{=} (\log d) L_A}$$

Volume law, maximally entangled!

In general, L_A = minimal number of bonds that have to be cut (to minimize the energy cost of the domain wall in Z_A)



S = minimal cut

= Ryu-Takayanagi Formula

- At small Δ: PM phase, DW condensate.

  $S_A^n \sim$ constant : area law

- Entanglement transition = ordering transition in stat mech model

  ↳ same for measurement transition!

- At criticality: $c=0$ CFT in 2d $(z=1)$

  $\uparrow$ $z=1$

$$Z_A/Z_0 = \langle \phi_{Bcc}(L_A)\phi_{Bcc}(0)\rangle \rightsquigarrow \text{general scaling theory.}$$

$$\sim \log L_A \quad \text{at criticality}$$

- <u>Remaining Question</u>: Universality class / Critical exponent

$\Rightarrow$ Hard! $c=0$ (logarithmic) CFT ($Z=1$ trivial Z / non trivial correlations)

(III) Statistical mechanics model for MiPT's

- Replica trick: $\log x = \lim_{K\to 0} \frac{x^K - 1}{K}$

$$S_A^{(n)} = \lim_{K\to 0} \mathbb{E}_{\text{circuits}} \sum_{\{m\}} \frac{p_m}{(1-n)K} \Big[\underbrace{\left(\mathrm{tr}\, \rho_{A,m}^n\right)^K - \left(\mathrm{tr}\, \rho_m^n\right)^K}_{\text{"easy" to average if } n,K \text{ integers}} \Big]$$

$$= \lim_{K\to 0} \frac{1}{(1-n)K} (Z_A - Z_0) = \lim_{K\to 0} \frac{1}{(n-1)K} (F_A - F_0)$$

$F = -\log Z$

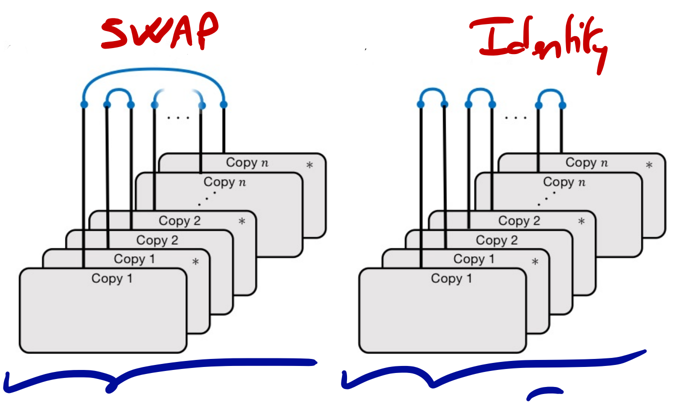
$\Rightarrow$ Need to average: $\rho^{\otimes Q}$, $Q = nK + 1$ ← Born proba

$(Q \to 1$ vs $Q \to 0$ for RTN$)$

(Other nonlinear observables: $\overline{\langle O^2\rangle} = \sum_m \frac{\langle O\rangle_m^2}{p_m} = \lim_{K\to 0} \sum_m \langle O\rangle_m^2 (\mathrm{tr}\, \rho_m)^{K-1}$, $Q = K+1$ copies)

with top boundary contraction:

(Fig from Bao et al)

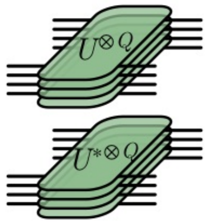


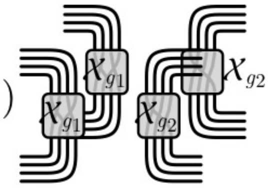
$\otimes K$ times in region A — in $\bar{A}$

Different "boundary conditions" (top layer) in Z_A, Z_0

(Just like in the RTN problem)

- Haar average (see problem set)

$$\mathbb{E}_U$$ 

$$= \sum_{g_1, g_2 \in S_Q} \mathrm{Wg}_{d^2}(g_1^{-1} g_2)$$ 

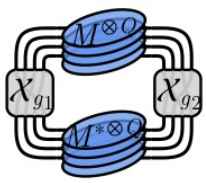
permutations ("Schur-Weyl" duality)

$\rightarrow$ degrees of freedom: $g \in S_Q$, permutations "spins" (Compare RTN: non trivial Weingarten functions)

Contract $U^{\otimes Q}$ with $U^{*\otimes Q}$ with permutation g_3:

$$\sum_{g_1 \in S_Q} W_g(g_1^{-1} g_2)\, D^{\chi(g_3^{-1} g_1)} = \delta_{g_2, g_3} \quad \text{: "} W_g = (D^{\chi})^{-1} \text{"}$$

$$D = d^2$$

- Contracting unitaries: 

$$= \mathrm{Tr}\, \hat{X}_{g_1} M^{\otimes Q} \hat{X}_{g_2} M^{\dagger \otimes Q},$$

measurement

if no measurement: $\mathrm{Tr}\left[\hat{X}_{g_1^{-1} g_2}\right] = d^{\chi(g_1^{-1} g_2)}$ # of cycles

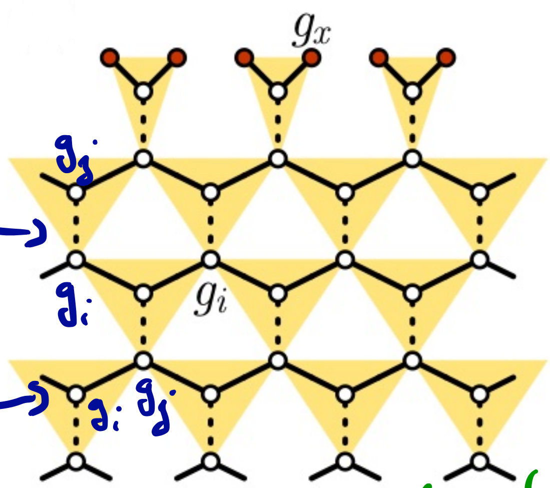
$Q = 2$: $\bigcirc = d^2$ $\quad \bigcirc = d^1$

if measurement: all replicas are forced to agree : weight $d' = d$

⇒ Stat mech model:

$W_g(g_i^{-1} g_j)$

$pd + (1-p)\, d^{\chi(g_i^{-1} g_j)}$



$d \quad d \to \infty$

$$\boxed{S_n = \lim_{k \to 0} \frac{1}{k(n-1)} \left(F_A - F_0 \right)}$$

$g_i \in S_Q$

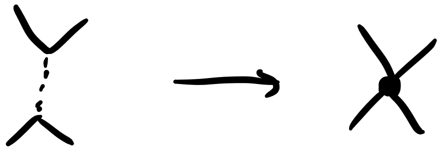
$Q = nk+1$
$\to 1$

$\neq$ boundary conditions in A

→ Explains most qualitative features of transition, entanglement scaling etc. Volume law phase: "Spontaneous Symmetry breaking"

$S_Q \times S_Q \qquad g_i \to g_L^{-1} g_i g_R$

→ Replica limit tricky in general $(k \to 0,\ Q \to 1)$ except $d \to \infty$



Large onsite Hilbert space dimension limit : $d \to \infty$

$$d^{C(g)} \sim d^{Q} \delta_{g,1} \quad \text{as } d \to \infty$$

$$W_g(g_i^{-1} g_j) \underset{d\to\infty}{\sim} \delta_{g_i, g_j} \quad (\text{up to } d^Q \text{ factors})$$

square lattice

$$\boxed{Z_{d\to\infty} = \sum_{\{g_i \in S_Q\}} \prod_{\langle i,j \rangle} \left((1-p)\, \delta_{g_i, g_j} + p \right)}$$

(For the measurement transition $Q = nm+1 \to 1$ instead of 0)

Enlarged symmetry: $S_{Q!}$ (permutation of all g_i's)

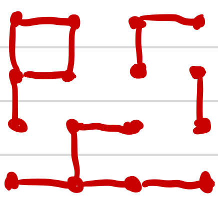
- This is a $Q!$ - Potts model. $Q! \to 1$ corresponds to percolation ($c=0$ CFT)

Expand product: Fortuin - Kasteleyn clusters (FK)

$$\bullet\!\!-\!\!\!-\!\!\bullet = (1-p)\, \delta_{g_i, g_j}$$

$$\bullet \quad \bullet = p$$

$\sum_{g_i}$: $Q!$ per cluster
all spins the same



$$Z = \sum_{\text{clusters}} (1-p)^{\#\text{links}}\, p^{\#\text{empty links}}\, \underbrace{(Q!)}_{\substack{\downarrow \\ 1}}{}^{\#\text{clusters}}$$

in replica limit

$$\boxed{p_c = 1/2}$$

$$S_Q \subset S_{Q!}$$

Exact results: $\nu = 4/3$, etc...

$1/d$ corrections? Hard! $S_{Q!} \longrightarrow S_Q \times S_Q$

$$\mathcal{L} = \mathcal{L}_{\text{Potts}}[\phi_a] + \sum_{a,b \in S_Q} C(a^{-1}b)\, \phi_a \phi_b$$

class function

: Relevant! IR fixed point?

Crossover: $\xi(d) \sim d^{4/3}$

For $L_A \ll \xi$, min-cut applies: $S_n \overset{p=p_c}{\sim} \frac{\sqrt{3}}{\pi} \log d \log L_A$

(See problem set)

<u>Free Energy</u>: $Z_Q^0 = \sum_m p_m^Q$ (average over circuits implicit)

Conformal invariance:

$$F_Q = -\log Z_Q^0 = F_\infty(Q) L - \frac{\pi c(Q)}{6L} + \dots$$

up to anisotropy factor ignored here

at $p = p_c$ (critical point)

$$\boxed{\lim_{Q \to 1} \frac{d}{dQ} F_Q = -\sum_m p_m \log p_m = \bar{F}}$$

"Free Energy" = Entropy of measurement record

$$\bar{F} = \bar{F}_\infty L - \frac{\pi c_{\text{eff}}}{6L} + \dots, \qquad c_{\text{eff}} = c'(Q=1)$$

Effective central charge

(terminology from disordered systems)