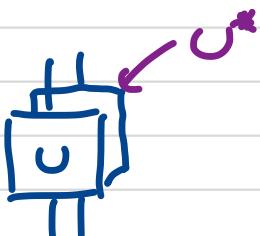


# Entanglement transitions in "Hybrid" Quantum circuits : HW

## (I) Haar calculus

i) We want:  $E_U$



Note that:

$$E_U \begin{array}{c} | \\ U \\ | \end{array} = E \begin{array}{c} | \\ V \\ | \end{array} \begin{array}{c} | \\ U \\ | \end{array} = E \begin{array}{c} | \\ U \\ | \end{array}$$

From Haar measure:

$$\int dU = \int d(UV) = \int d(VU)$$

$$E_U \begin{array}{c} | \\ U \\ | \end{array}_{\alpha \alpha'} = C \begin{array}{c} | \\ U \\ | \end{array}_{\alpha \alpha'} = C \delta_{\alpha \alpha'} \delta_{pp'}$$

$\uparrow$   
 $\alpha \alpha'$   
constant

and

$$\begin{array}{c} | \\ U \\ | \end{array}_{\alpha \alpha'} = \begin{array}{c} \curvearrowleft \\ \alpha \alpha' \end{array} = C \begin{array}{c} \curvearrowright \\ \alpha \alpha' \end{array} \Rightarrow C = \frac{1}{D}$$

loop =  $\sum_{p, p'=1}^D \delta_{pp'}$   
 $= D$

$$2) E_U = \sum_{i=1}^4 c_i \langle \text{Diagram}_i \rangle$$

Diagrams for  $E_U$ :

- $\beta\beta'ss'$ : Four vertical lines with horizontal bars at the top and bottom.
- $\alpha\alpha'gg'$ : Four vertical lines with diagonal bars at the top and bottom.
- $(U \otimes U^*)^{\otimes 2}$ : Two sets of two vertical lines each, with horizontal bars at the top and bottom.
- $\beta\beta'g\delta'$ : Two vertical lines with horizontal bars at the top and bottom, and a diagonal bar between them.
- $\alpha\alpha'gg'$ : Two vertical lines with diagonal bars at the top and bottom.
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Contact bottom and top by  $\langle UU \rangle = e \in S_2 = Z_2$

(4 equations, 4 unknowns)

$$\langle \psi \rangle = g_{S_{\text{swap}}} \in S_2$$

$$\frac{\langle e \rangle}{\langle e \rangle} = \langle \text{Diagram} \rangle = D^2 = c_1 \underbrace{\langle \text{Diagram} \rangle}_{D^4} + c_2 \langle \text{Diagram} \rangle + c_3 \langle \text{Diagram} \rangle + c_4 \langle \text{Diagram} \rangle$$

$$\text{using } \langle e | e \rangle = D^2, \langle g_{S_{\text{swap}}} | g_{S_{\text{swap}}} \rangle = \langle \text{Diagram} \rangle = D^2$$

$$\langle e | g_{S_{\text{swap}}} \rangle = \langle \text{Diagram} \rangle = D$$

$$\frac{\langle g_{S_{\text{swap}}} \rangle}{\langle g_{S_{\text{swap}}} \rangle} = \langle \text{Diagram} \rangle = D^2 = c_1 D^2 + D^3 (c_2 + c_3) + c_4 D^4$$

$$\frac{\Delta e}{\Delta \theta_{SWAP}} = \Delta = D = (c_1 + c_3)D^3 + c_2 D^2 + c_3 D^4$$

$$\frac{\Delta \theta_{SWAP}}{\Delta e} = D = (c_1 + c_4)D^3 + c_3 D^2 + c_2 D^4$$

We have :  $c_2 = c_3$  and  $c_1 = c_4$

and 2 equations:

$$D^2 = c_1(D^2 + D^4) + 2c_2 D^3$$

$$D = 2c_1 D^3 + c_2(D^2 + D^4)$$

$$\Rightarrow c_1 = c_4 = \frac{1}{D^2 - 1}, \quad c_2 = c_3 = \frac{-1}{D(D^2 - 1)}$$

## ② Purification transition

$$P_0 = \frac{1}{2^L}, \text{ Entropy of trajectory : } S_m^{(n)}(t) = \frac{1}{1-n} \log \frac{\int_t p_m^n}{(\int_t p_m^n)^n}$$

$$\text{Average : } S_n(t) = \mathbb{E}_U \sum_m P_m \sum_m^{(n)}(t) \quad n = \text{Renyi index}$$

↑  
average over circuit

$P_m$  = Born probability

$$S_n(H) = E_U \sum_m \frac{t_n p_m}{1-n} \lim_{K \rightarrow 0} \frac{\left( (t_n p_m^K)^K - (t_n p_m)^{nK} \right)}{K}$$

$$= \lim_{K \rightarrow 0} \frac{1}{K(1-n)} (Z - Z_0) = \lim_{K \rightarrow 0} \frac{1}{K(n-1)} (F - F_0)$$

$$F = -\log Z, \quad \text{for } K=0: Z = Z_0 = 1$$

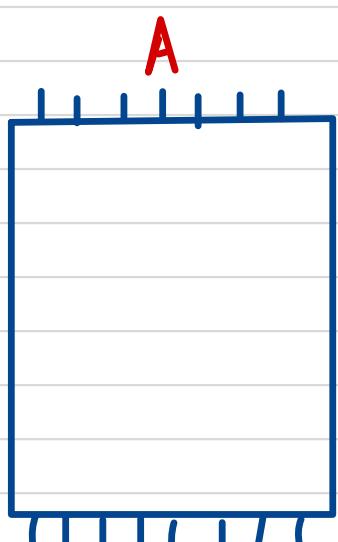
$$Z = E_U \sum_m (t_n p_m) (t_n p_m^n)^K$$

$$Z_0 = E_U \sum_m (t_n p_m)^{1+nK}$$

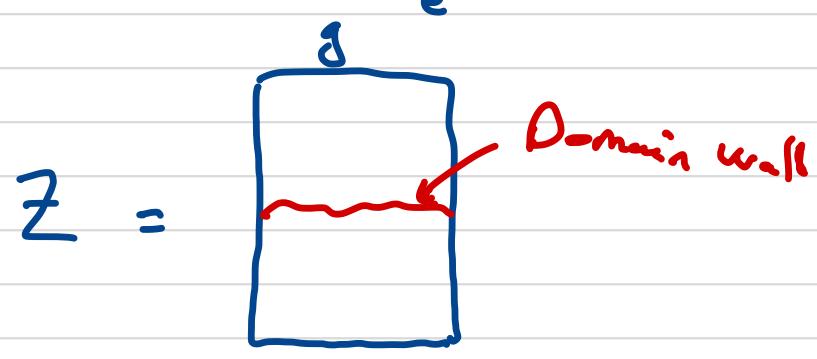
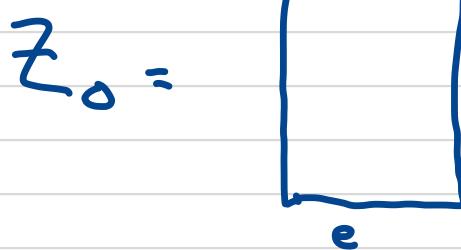
$$Q = 1 + nK$$

Replicas

. Purify initial state with  $L$  ancillas :  $S_n = E_n$  Entanglement between physical and ancilla spins.



$\Rightarrow \neq$  top/bottom Boundary conditions



$\otimes^K$

$g = g_{SWAP}$  on  $nK$  copies, identity on last replica

$g_{SWAP} = \text{XXX}$  on  $n$  copies

### III Percolation mapping

$$D = \zeta^2$$

1)  $d \rightarrow \infty$ :  $d^{C(g)}$   
 $\underset{d \rightarrow \infty}{\sim} d^Q \delta_{g,1}$

Since  $C(g)$  maximum for  $g=1$

Since  $W_{g_D}$  is the inverse of  $D^C$ , we also have  
 $W_{g_D}(g) \sim \delta_{g,e}$

$$\begin{cases} g_j \\ g_i \end{cases} \Rightarrow \begin{cases} g_i = g_j \end{cases}$$

$\Rightarrow$  the entire model reduces to a product of  $\delta$  function  
 $\Rightarrow$  forces all spins to be the same!

2) Finite  $d$ :  $C(g_i^{-1}, g_j)$   $C$  = Class Function

$$S_Q^L \times S_Q^R \text{ Symmetry}$$

$d \rightarrow \infty \cdot \delta_{g_i, g_j} : S_Q!$  Symmetry (permutations of  $Q!$  "spins")

3)  $Z_0$ :  $g_i = e$  everywhere!

$Z_A$ : Boundary forces DW between "e" and

$$g = \otimes_{i=1}^K g_{\text{swap}}$$

Each link on the DW costs:

$$d^{C(g)} \text{ vs } d^{C(z)} = d^Q$$

↑  
with  $C(g) = 1+K$

$$\Rightarrow \frac{Z_A}{Z_0} = \left( \frac{d}{\frac{C(\text{es})}{C(\text{gl})}} \right)^{\rho_{\text{DW}}} \quad \begin{matrix} \text{\# frustration links along DW} \\ \underbrace{\phantom{...}}_{\rho_{\text{DW}}} \end{matrix}$$

$$= d^{\frac{(K+1-Q)}{K(1-\alpha)}} \rho_{\text{DW}}(x)$$

where the DW picks the minimal cut through the diluted circuit to minimize energy.

$$4) S_A^{(n)} = \lim_{K \rightarrow 0} \frac{-1}{K(n-1)} \log \frac{Z_A}{Z_0}$$

$$= (\log d) \rho_{\text{DW}}(x)$$

$$= \log d \times \text{"length of minimal cut"}$$

in volume law phase,  $\rho_{\text{DW}} \sim L_A$

over law (non percolating) phase:  $\rho_{\text{DW}} \sim O(1)$