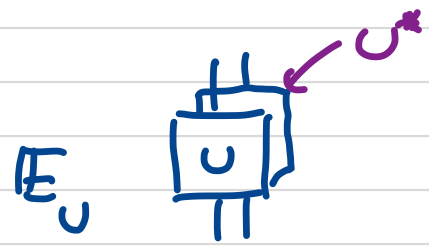
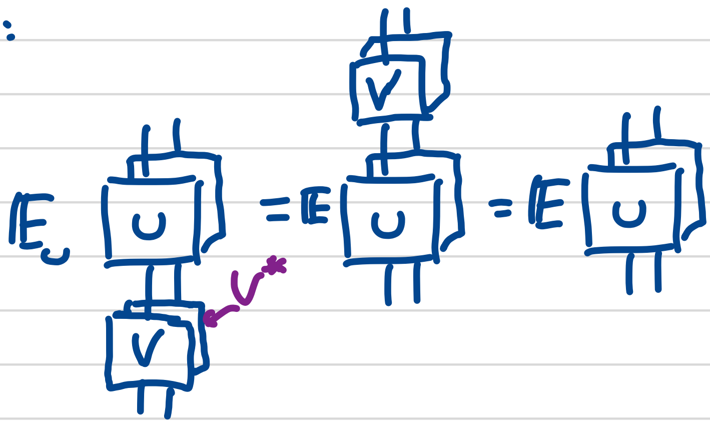
Entanglement transitions in "Hybrid" Quantum circuits : HW

(I) Haar calculus

1) We want: $\mathbb{E}_U$

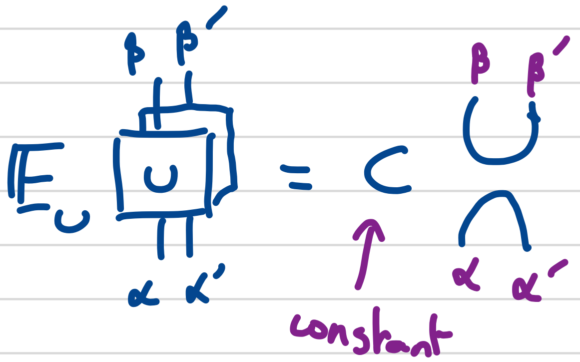


Note that:



from Haar measure:

$$\int dU = \int d(UV) = \int d(VU)$$

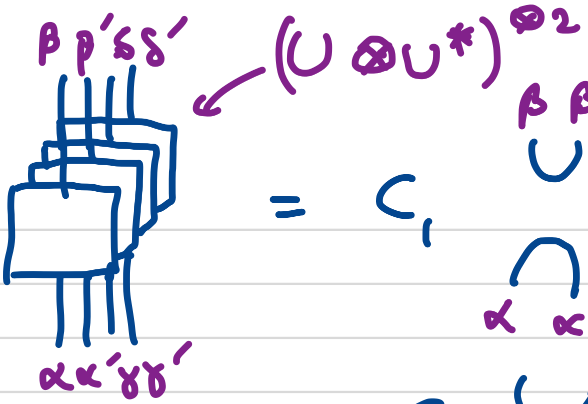


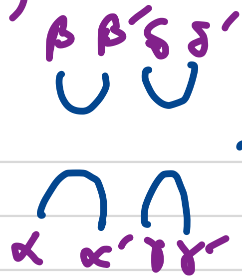
$$= C\, \delta_{\alpha\alpha'}\, \delta_{\beta\beta'}$$

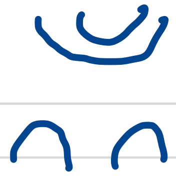
and

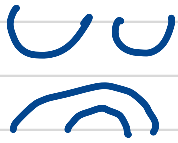
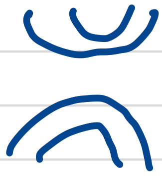
$$\text{loop} = \sum_{\beta,\beta'=1}^{D} \delta_{\beta\beta'} = D$$

$$\Rightarrow C = 1/D$$

2) $\mathbb{E}_U$ 

$= c_1$ 

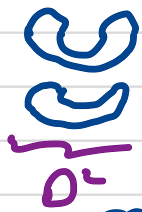
$+ c_2$ 

$+ c_3$  $+ c_4$ 

Contract bottom and top by $|\cup\cup\rangle = e \in S_2 = \mathbb{Z}_2$

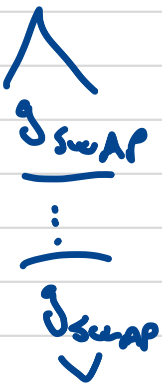
(4 equations, 4 unknown) $|$$\rangle = g_{SWAP} \in S_2$

$= \bigcirc\bigcirc = D^2 = c_1$  $+ c_2$ 

$+ c_3 \, D^3 + c_4$ 

using $\langle e|e\rangle = D^2$, $\langle g_{SWAP}|g_{SWAP}\rangle =$  $= D^2$

$\langle e|g_{SWAP}\rangle =$  $= D$



$=$  $= D^2 = c_1 D^2 + D^3(c_2 + c_3) + c_4 D^4$

$$\begin{array}{c}\wedge\\ e\\ \vdots\\ g_{SWAP}\\ \vee\end{array} = $$  $$= D = (c_1+c_4)D^3 + c_2 D^2 + c_3 D^4$$

$$\begin{array}{c}\wedge\\ g_{SWAP}\\ \vdots\\ e\\ \vee\end{array} = D = (c_1+c_4)D^3 + c_3 D^2 + c_2 D^4$$

We have : $c_2 = c_3$ and $c_1 = c_4$

and 2 equations:

$$D^2 = c_1(D^2+D^4) + 2c_2 D^3$$
$$D = 2c_1 D^3 + c_2(D^2+D^4)$$

$$\Rightarrow \quad c_1 = c_4 = \frac{1}{D^2-1} \quad , \quad c_2 = c_3 = \frac{-1}{D(D^2-1)}$$

(II) Purification transition

$\rho_0 = \mathbb{1}/2^L$, Entropy of trajectory : $S_m^{(n)}(t) = \frac{1}{1-n}\log\frac{\operatorname{tr}\rho_m^n}{(\operatorname{tr}\rho_m)^n}$

Average : $S_n(t) = \mathbb{E}_U \sum_m p_m S_m^{(n)}(t)$ $\quad n$ = Renyi index

average over circuits

P_m = Born probability

$$S_n(t) = \mathbb{E}_U \sum_m \frac{\mathrm{tr}\,\rho_m}{1-n} \lim_{K\to 0} \frac{\left((\mathrm{tr}\,\rho_m^n)^K - (\mathrm{tr}\,\rho_m)^{nK} \right)}{K}$$

$$= \lim_{K\to 0} \frac{1}{K(1-n)} (Z - Z_0) = \lim_{K\to 0} \frac{1}{K(n-1)} (F - F_0)$$

$$F = -\log Z \,, \qquad \text{for } K = 0 : Z = Z_0 = 1$$

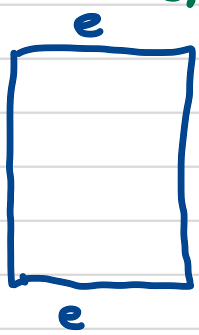
$$Z = \mathbb{E}_U \sum_m (\mathrm{tr}\,\rho_m)(\mathrm{tr}\,\rho_m^n)^K$$

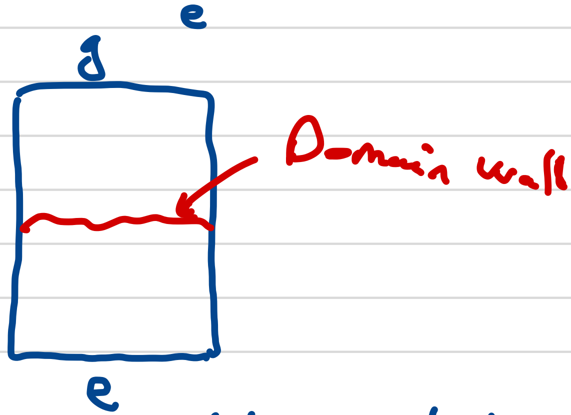
$$Z_0 = \mathbb{E}_U \sum_m (\mathrm{tr}\,\rho_m)^{1+nK}$$

$Q = 1 + nK$ replicas

. Purify initial state with L ancillas : S_n = Entanglement between physical and ancilla spins.

$\Rightarrow \neq$ top/bottom boundary conditions

$Z_0 =$ 

$Z =$ 

$g = g_{SWAP}^{\otimes K}$ on nK copies, identity on last replica

$g_{SWAP} =$ on n copies

III Percolation mapping

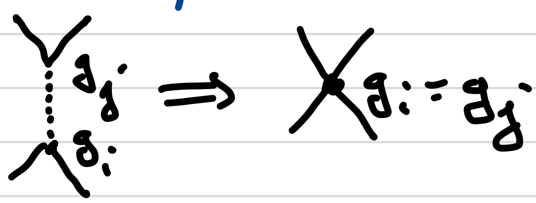
$$D = d^2$$

1) $d \to \infty$: $$d^{C(g)} \underset{d \to \infty}{\sim} d^Q \, \delta_{g,1}$$

since $C(g)$ maximum for $g = 1$

Since W_{g_D} is the inverse of D^C, we also have

$$W_{g_D}(g) \sim \delta_{g,e}$$



$\Rightarrow$ the entire model reduces to a product of δ functions & forces all spins to be the same!

2) Finite d: $C(g_i^{-1} g_j)$ $\quad C$ = class function

$S_Q^L \times S_Q^R$ symmetry

$d \to \infty$ $\cdot$ δ_{g_i, g_j} : $S_{Q!}$ symmetry (permutations of $Q!$ "spins")

3) Z_0 : $g_i = e$ everywhere!

Z_A: boundary forces DW between "e" and

$$g = g_{SWAP}^{\otimes K} \otimes 1$$

Each link on the DW costs: $d^{C(g)}$ vs $d^{C(e)} = d^Q$

with $C(g) = 1 + K$

$$\Rightarrow \quad Z_A / Z_0 = \left(\frac{d^{C(g)}}{d^{C(e)}} \right)^{\overbrace{\#\text{ frustrated links along DW}}^{\rho_{DW}}}$$

$$= d^{\underbrace{(K+1-Q)}_{K(1-n)} \rho_{DW}(x)}$$

where the DW picks the minimal cut through the diluted circuit to minimize energy.

4) $$S_A^{(n)} = \lim_{K\to 0} \frac{-1}{K(n-1)} \log Z_A/Z_0$$

$$= (\log d)\, \rho_{DW}(x)$$

$$= \log d \times \text{" length of minimal cut"}$$

in volume law phase, $\rho_{DW} \sim L_A$

area law (non percolating) phase: $\rho_{DW} \sim O(1)$