Boulder summer school – Hydrodynamics Across Scales
July 2022

Large deviation theory applied to study rare and extreme events in turbulence, atmosphere, and climate dynamics

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Abstract: We will discuss a set of recent developments in non-equilibrium statistical physics, applied to turbulence, atmosphere and climate dynamics. To derive theory and algorithms, a key concept is large deviation theory. The first lecture will motivate the use of large deviations, explain the theory and how it can be used through the study of simple examples.

We will then discuss applications. The first one will be kinetic theory and the irreversibility paradox. The second one will be the study of rare trajectories that suddenly drive a turbulent flow from one attractor to a completely different one, related to abrupt climate changes on Jupiter or for the Earth troposphere dynamics. The third application will be extreme heat waves, as examples of rare events with huge impacts.

To understand these phenomena, we use on one hand theoretical approaches: based on path integrals, stochastic processes, and information theory. On the other hand, we use numerical tools: rare event simulations analogous to diffusion Monte-Carlo algorithms, and machine learning. All are related to large deviation theory.

Lecture notes and recommended reading

Following these links you will have access to files with lecture notes related to the theoretical part of the courses, key representative research papers with applications, and review articles.

1) Lecture notes for a basic introduction to large deviation theory

1a) For the sum on independent random variables:
https://www.dropbox.com/s/8j3zzhewo69hsik/Cours_Chapitre1_Etudiants_02_21.pdf?dl=0

1b) For dynamical systems with weak noise:
https://www.dropbox.com/s/kvhqrje5twupvak/Cours_Chapitre3_Etudiants_02_21.pdf?dl=0

2) Two research papers about large deviation and instanton theory for abrupt climate change:

https://mycore.cnrs.fr/index.php/s/udeG0XGMTUDen0c
https://mycore.cnrs.fr/index.php/s/1YIl9ssjehzt62X

3) A research paper about large deviation theory and rare event algorithms used to sample extreme heat waves:
https://mycore.cnrs.fr/index.php/s/wFd9G4JklcpZt0M

4) Large deviation for kinetic theories, related to the Boltzmann equation:
https://mycore.cnrs.fr/index.php/s/T9XEICCEJQHeCov

5) Review paper about equilibrium statistical mechanics for geophysical flows where large deviation theory plays a key role:
https://mycore.cnrs.fr/index.php/s/aPzDITOmjrFW6U8

6) Lecture notes about equilibrium statistical mechanics for geophysical flows and an introduction to dynamical large deviation and instanton theory applied to geophysical turbulent flows:
https://mycore.cnrs.fr/index.php/s/BzoUa8BP5851JIM

Detailed outline

Wednesday lecture: Introduction to large deviation theory and its applications to dynamical problems

1 Why study large deviations: rare events in complex dynamical systems
- 1.a Abrupt climate changes and transitions between turbulent attractors
- 1.b Rare events with a huge impact: extreme heat waves
- 1.c Rare and extreme events in astronomy

2 Introduction to large deviation theory
- 2.a What are large deviations?
- 2.b Large deviations for the sum of N i.i.d. random variables
- 2.c Large deviations for the empirical distribution

3 Dynamical large deviations (large deviations for path measures)
- 3.a Large deviations for stochastic differential equations with weak noises
- 3.b General dynamical large deviations
- 3.c Quasipotential, Hamilton–Jacobi equation, relaxation and fluctuation paths

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Thursday lecture: Large deviation theory for kinetic theories, geostrophic turbulence, and atmosphere dynamics

1 Large deviations for dilute gases (for the Boltzmann equation)
- 1.a Dilute gases and heuristic derivation of Boltzmann's equation
- 1.b Derivation of the large deviation action for dilute gas dynamics
- 1.c The irreversibility paradox revisited

2 Kinetic theory for two dimensional turbulent flows
- 2.a The barotropic quasi-geostrophic model and averaging
- 2.b Kinetic theory of the quasi-geostrophic model
- 2.c An explicit formula for the Reynolds stress for small scale forces

3 Rare transitions and Jupiter's abrupt climate changes
- 3.a Rare transitions for zonal jets
- 3.b Large deviations in the weak noise regime
- 3.c Rare event algorithms and rare transitions for turbulent flows

Friday lecture: Rare and extreme events in climate dynamics: sampling using rare event algorithms and machine learning

1) Introduction: rare events do matter - rare event algorithms
2) Rare events algorithms for predicting extreme heat waves
3) Coupling rare event algorithms with machine learning
4) Predicting extreme heat waves and committor functions using deep neural networks