Boulder Summer School 2005 – Lectures 2 & 3

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Electron Dephasing and Energy Exchange in Diffusive Metal Wires:

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With special thanks to Hugues Pothier!

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Prologue

Resistivity of metals

Pt

High T, phonons

Low T, impurities & disorder

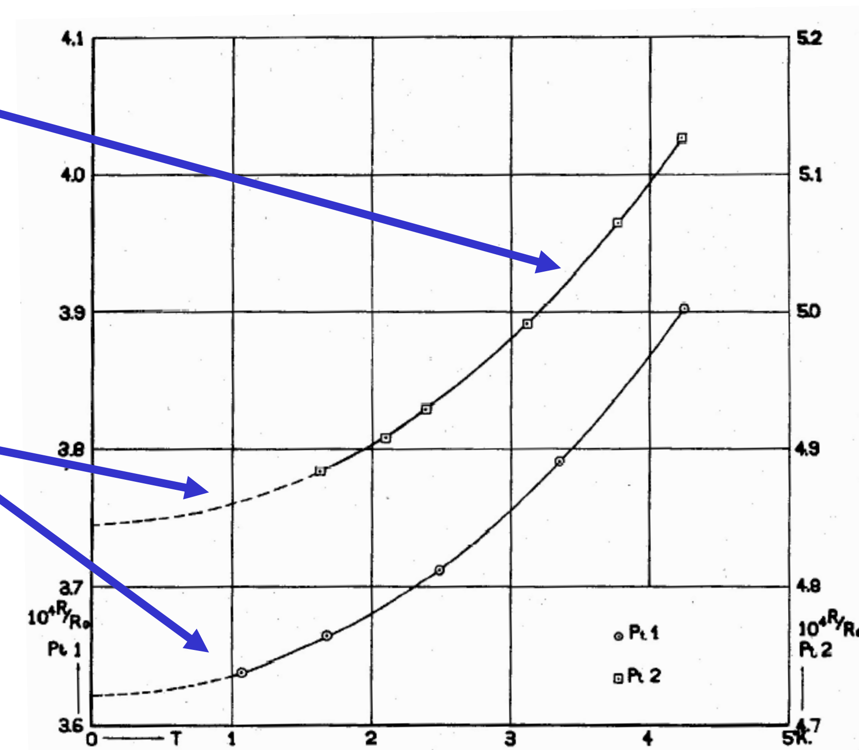


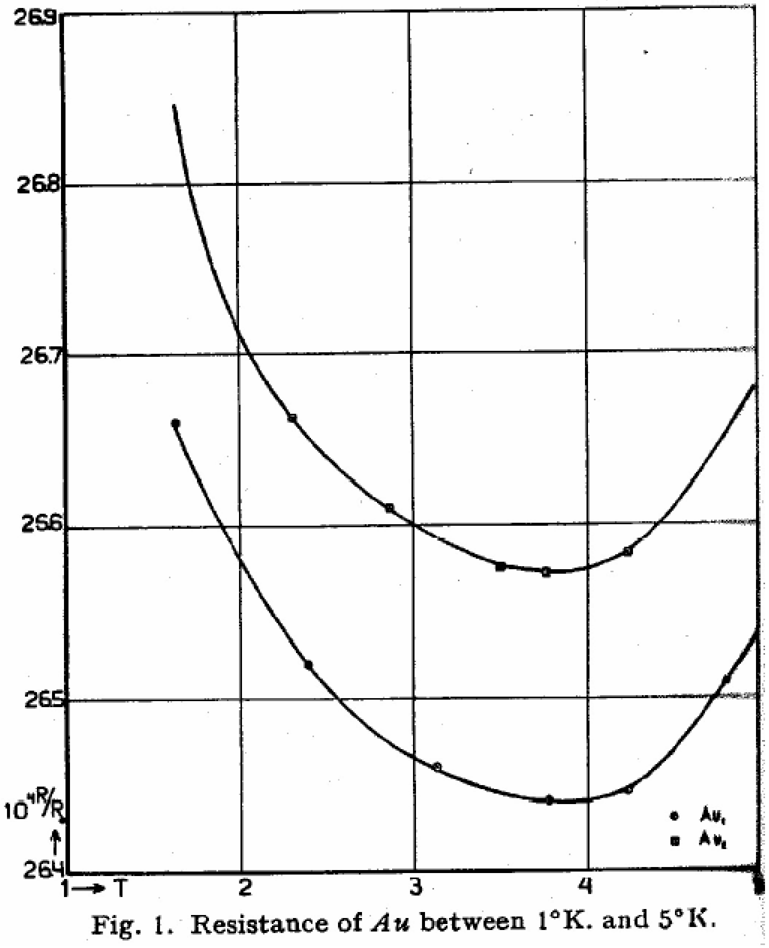
Fig. 1. Electrical resistance of Pt between 0°K. and 4.2°K.

De Haas & de Boer, 1934

dR/dT>0 always

But dR/dT<0 in some samples!

Au



De Haas, de Boer, & van den Berg, 1934

Suspect magnetic impurities

Fe in Cu:

J.P. Franck, Manchester, Martin (1961)

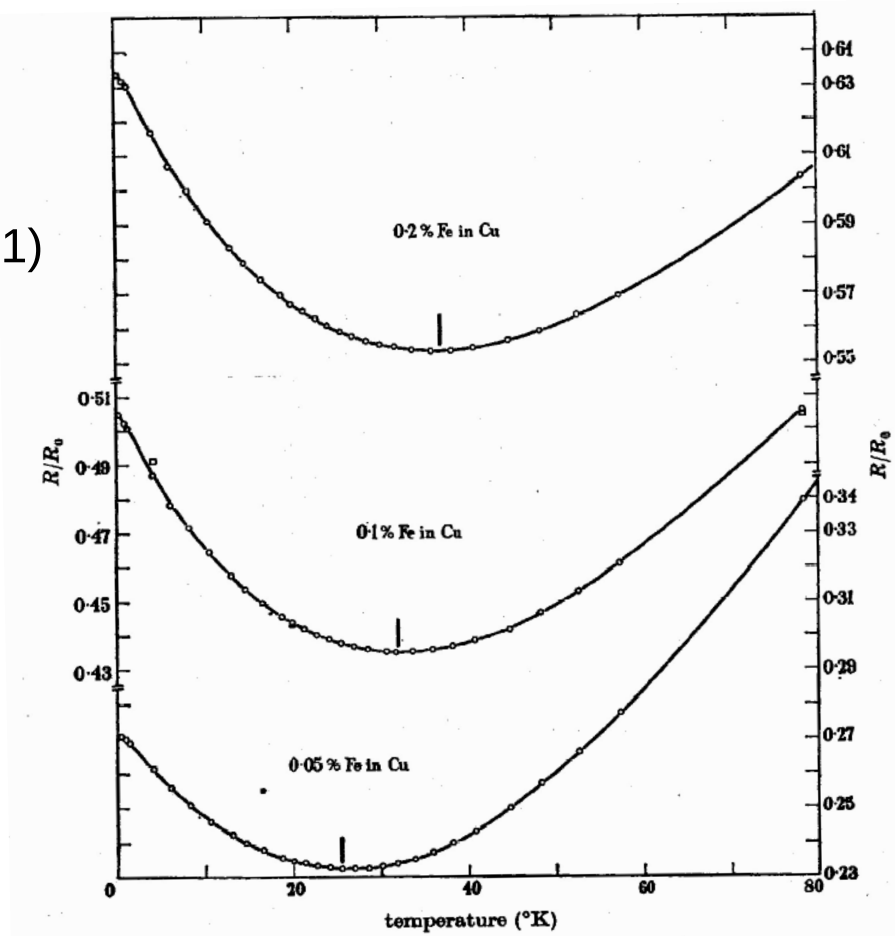


FIGURE 3. The electrical resistance of dilute copper + iron alloys. The bars indicate the point of minimum resistance. The points shown □ were taken after re-annealing the 0·1% alloy.

But how do they work?

The solution

Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

Resistance Minimum in Dilute Magnetic Alloys

Jun KONDO

The s-d exchange model: $H = \sum_i J\, \vec{s}_i \cdot \vec{S}$

Kondo's result: $\delta\rho \propto -B \log\left(\frac{T}{T_K}\right) \Rightarrow \frac{d\rho}{dT} < 0 \quad !!$

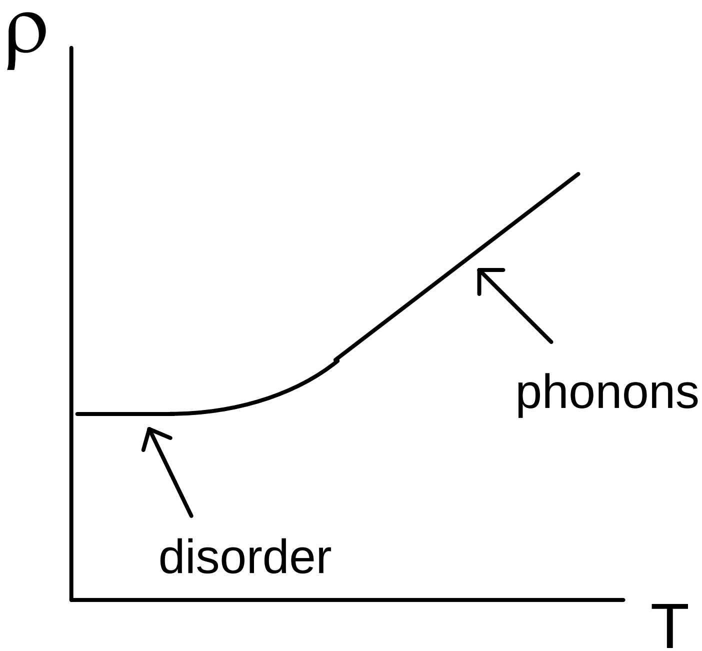
Kondo temperature: $k_B T_K \approx E_F\, e^{-\frac{1}{\nu J}}$

ν = density of states at E_F

1960’s

Moral of the story: magnetic impurities dominate the low-temperature resistivity of metals, even at concentrations as low as 0.01%

Jump ahead 20 years … 1980's



WRONG!

$$\rho = \frac{m}{ne^2}\frac{1}{\tau}$$

Matthiessen's rule:

$$\frac{1}{\tau} = \frac{1}{\tau_{disorder}} + \frac{1}{\tau_{el-ph}} + \ldots$$

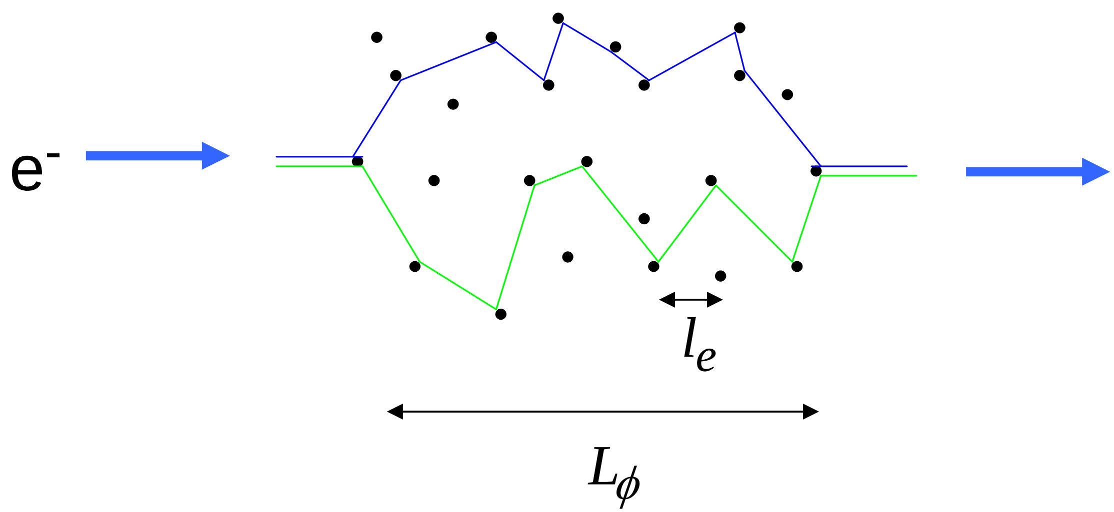
elastic — preserves quantum phase coherence

inelastic — destroys quantum phase coherence

Low T: $$\frac{1}{\tau_{inelastic}} << \frac{1}{\tau_{elastic}}$$

Electrons maintain quantum phase coherence over distance $L_\phi >> l_e$

Electron transport in diffusive regime



1. Elastic scattering (film boundaries, impurities) $l_e = v_F \tau_e$

   ⇨ diffusive states $D = \frac{1}{3} v_F l_e$

2. Inelastic scattering (phonons, other electrons, spins)

   ⇨ loss of phase coherence $L_\phi = \sqrt{D\tau_\phi}$

   ⇨ energy exchange between electrons

Why Is the Phase Coherence Time Important?

- τ_ϕ limits quantum transport phenomena:
  - normal metals: weak localization, UCF, Aharonov-Bohm
  - superconductors: proximity & Josephson effects
- Localization theory assumes $L_\phi > \xi$
  - no M-I transition if τ_ϕ saturates at low temperature
- example of quantum system coupled to environment

Predictions for τ_ϕ at low T

(Altshuler & Aronov, 1979)

At low T, τ_ϕ limited by e-e interactions

Screening depends on dimensionality

(At energy E, compare $\sqrt{\hbar D / E}$ with transverse dimensions)

$$\left(\sum_{q_x, q_y, q_z} \frac{\cdots}{\left(Dq^2 + i\omega\right)^{\cdots}} \right)$$

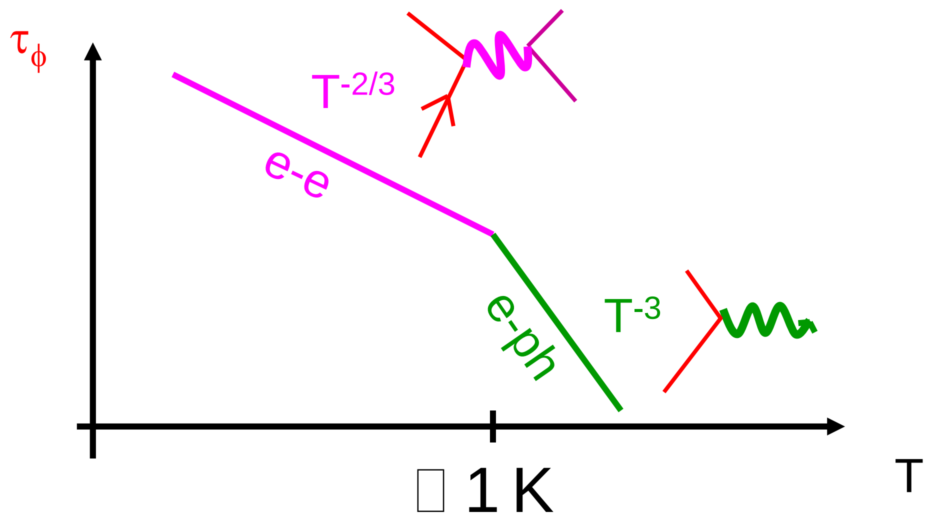
τ_ϕ depends on dimensionality

« wires » (1d regime) : $L_\phi = \sqrt{D\tau_\phi} >$ transverse dimensions

($E \sim \hbar / \tau_\phi$ rule the game)

$\tau_\phi(T)$ in wires

(Altshuler, Aronov, Khmelnitskii, 1982)



$$\tau_\phi = (A\,T^{2/3} + B\,T^{3})^{-1}$$

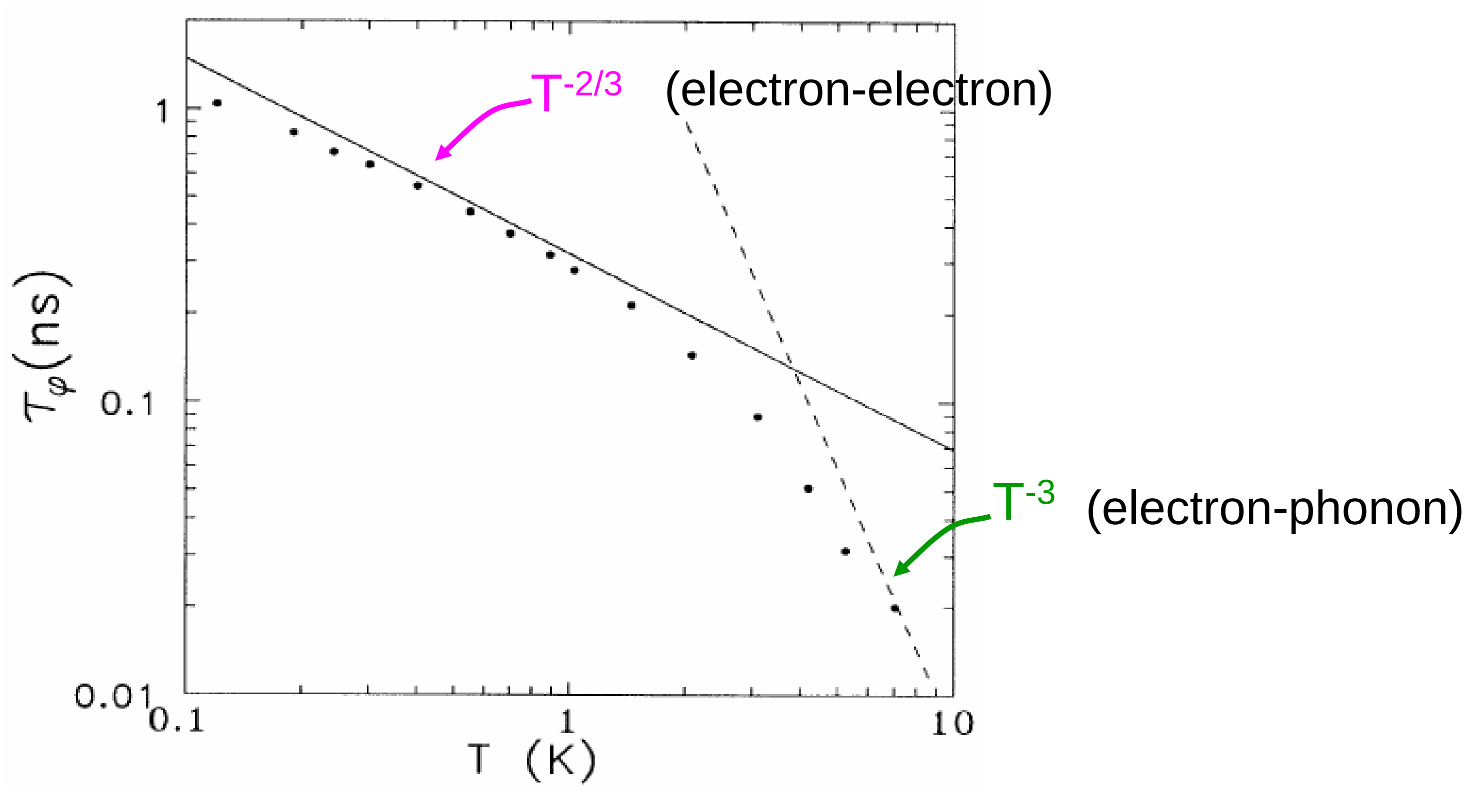
$$A = \frac{1}{\hbar}\left(\frac{\pi k_B^2}{4\nu_F L w t} \frac{R}{R_K} \right)^{1/3}$$

Screened Coulomb interaction at d=1

Temperature dependence of τ_ϕ

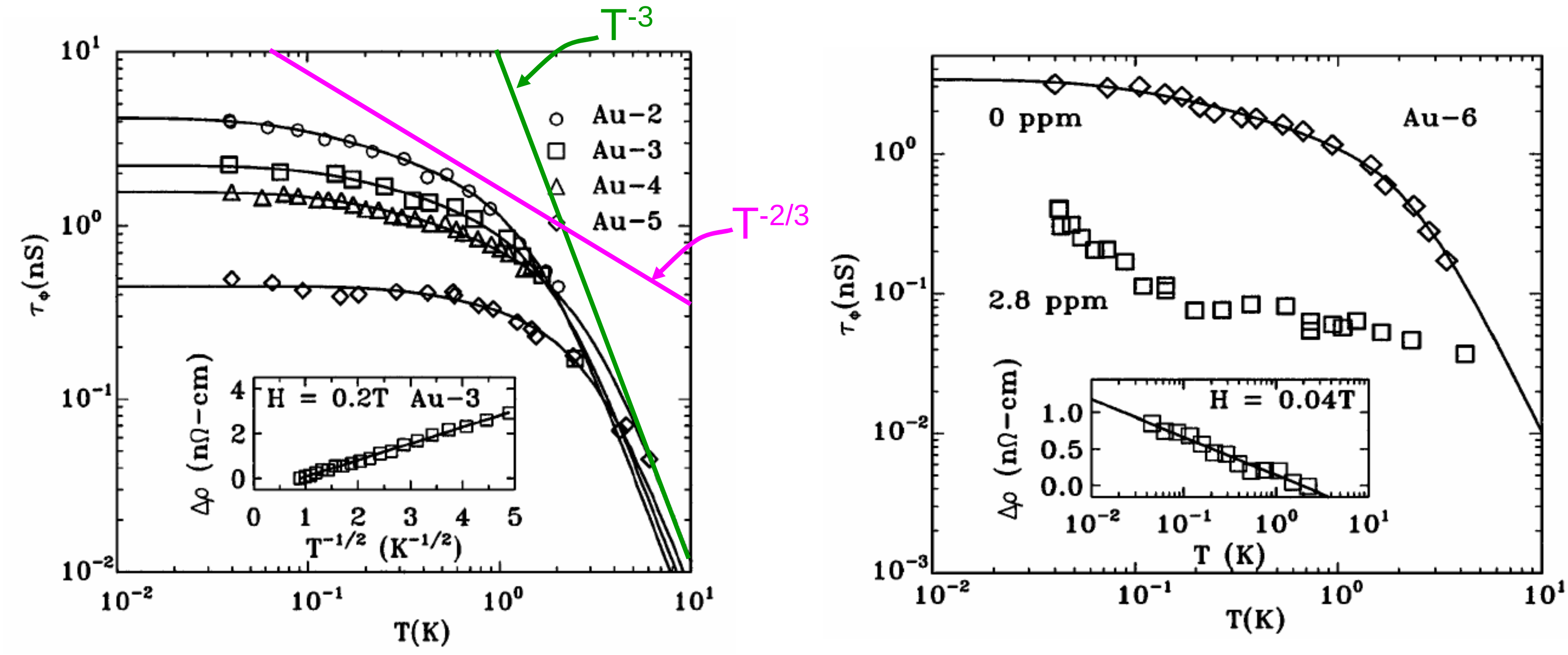
confirmation of AAK theory in quasi-1D

Echternach, Gershenson, Bozler, Bogdanov & Nilsson, PRB **48**, 11516 (1993)



The Experimental Controversy

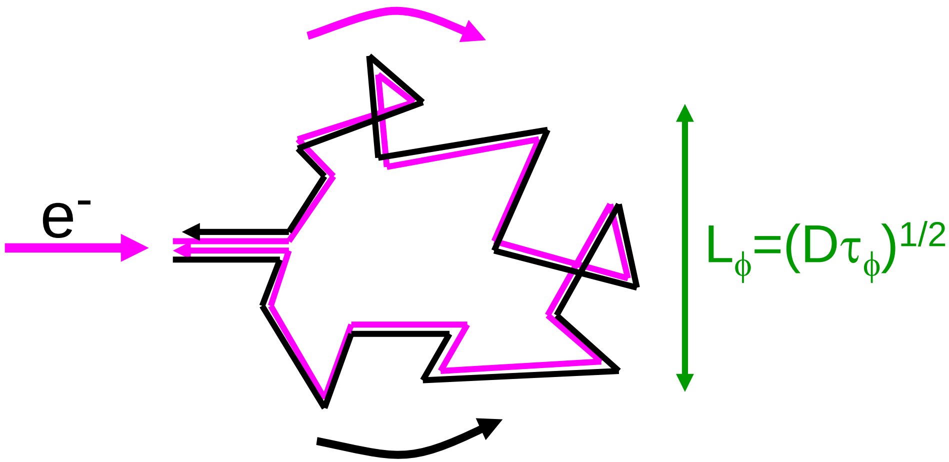
Mohanty, Jariwala and Webb, PRL **78**, 3366 (1997)



Saturation of τ_ϕ:

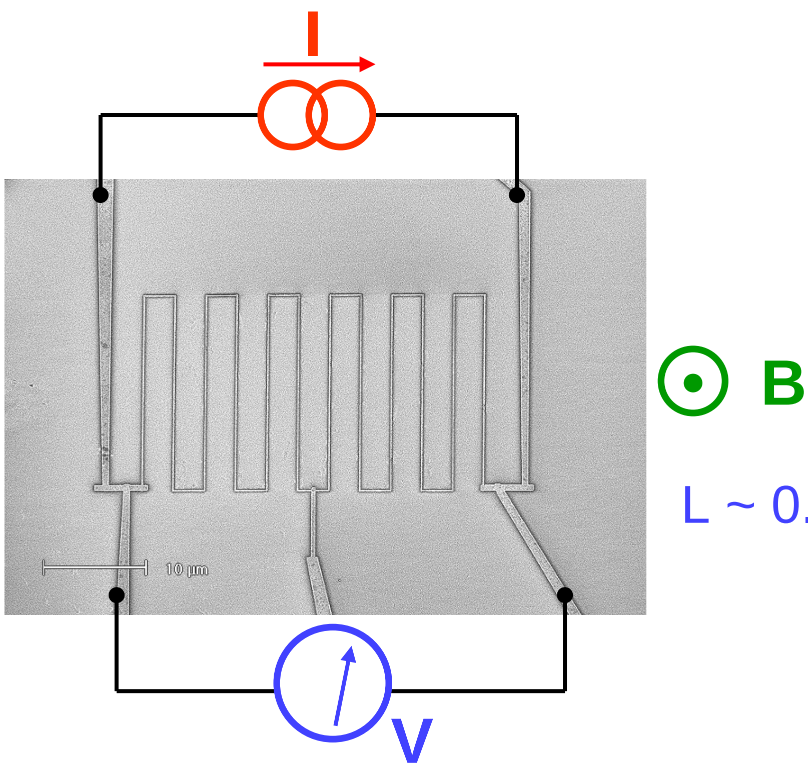
Artifact of measurement ?
If not, is it intrinsic ?

Measuring $\tau_\phi(T)$

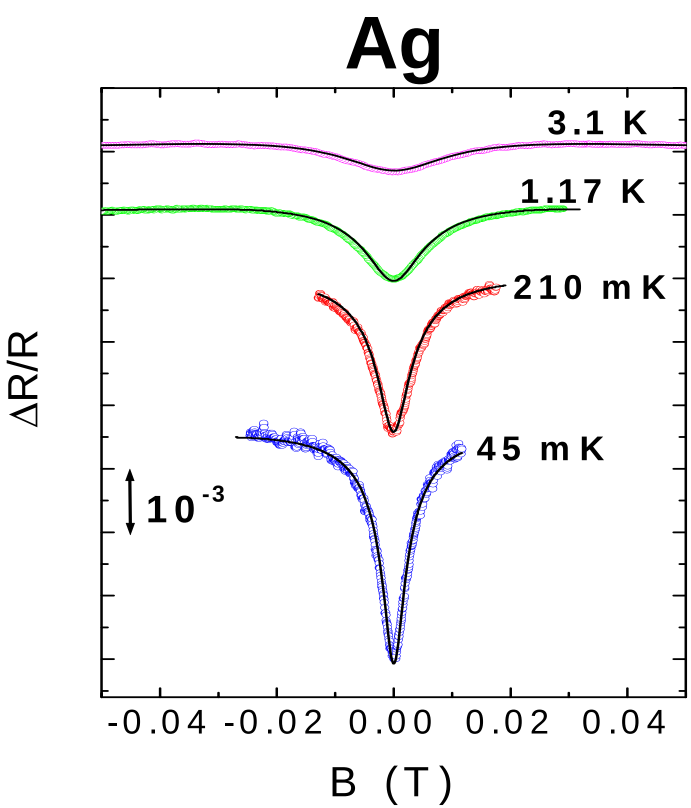


Interference of time-reversed paths
⇒ "weak-localization" correction to R

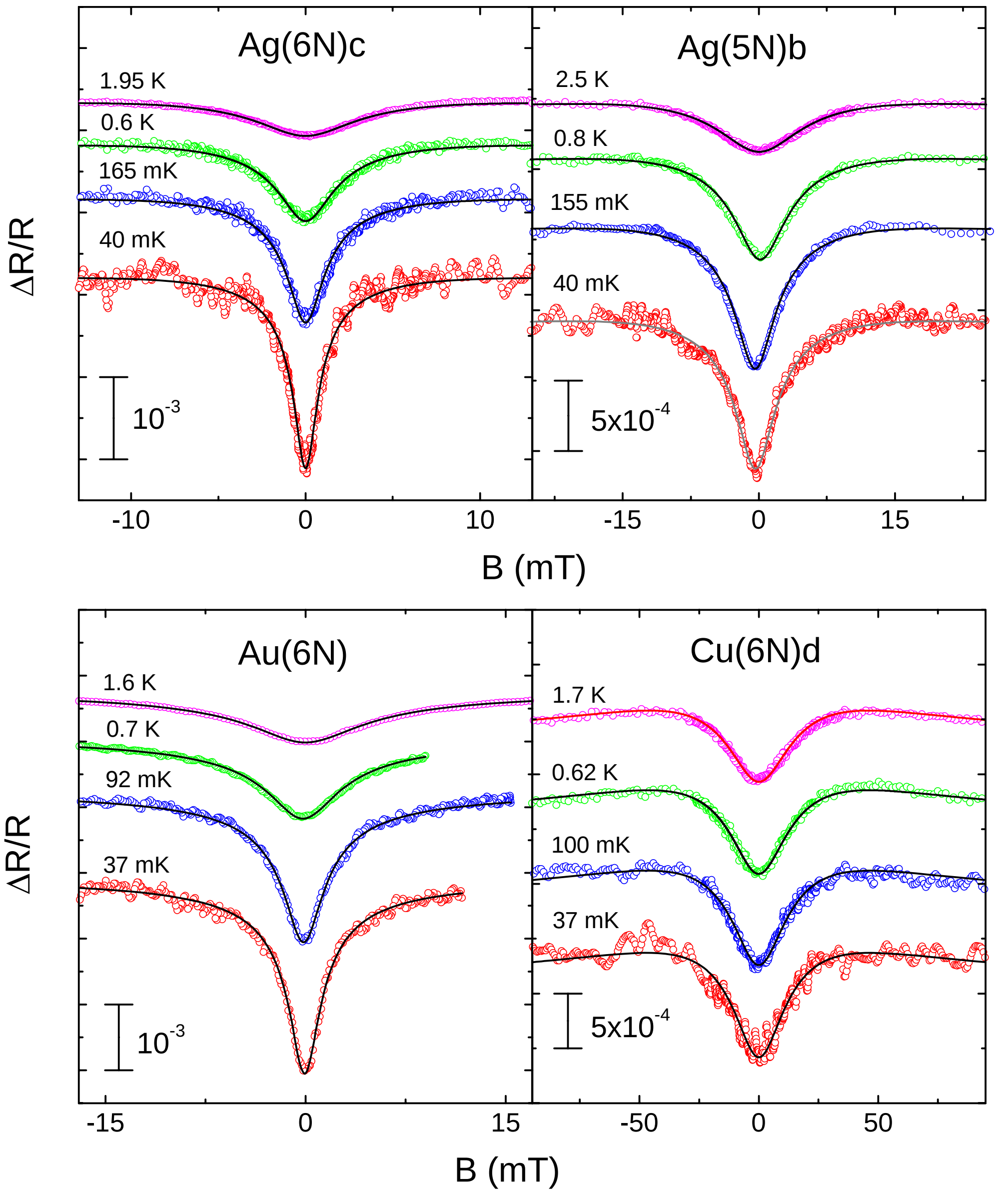
B reduces weak-loc. correction



L ~ 0.25 mm



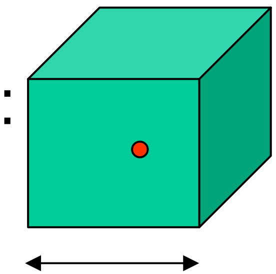
Measuring $\tau_\phi(T)$: raw data



5N = 99.999 % source purity

6N = 99.9999 % " "

1 ppm of *impurities* :



100 atoms ~ 25 nm

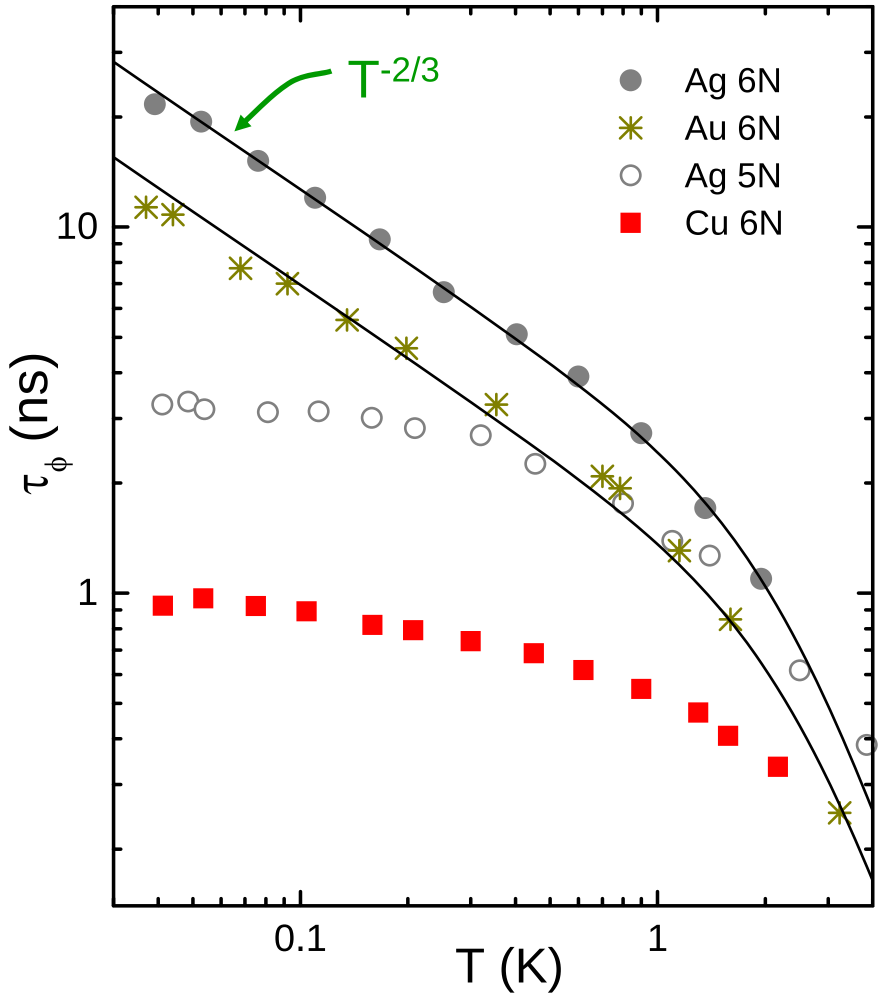
Ag(6N) & Au(6N):

ΔR grows as T decreases

Ag(5N) & Cu(6N):

ΔR saturates below ~ 100mK

τ_ϕ(T) in Ag, Au & Cu wires



5N = 99.999 % source material purity
6N = 99.9999 % " " "

Low T behavior vs. Purity:

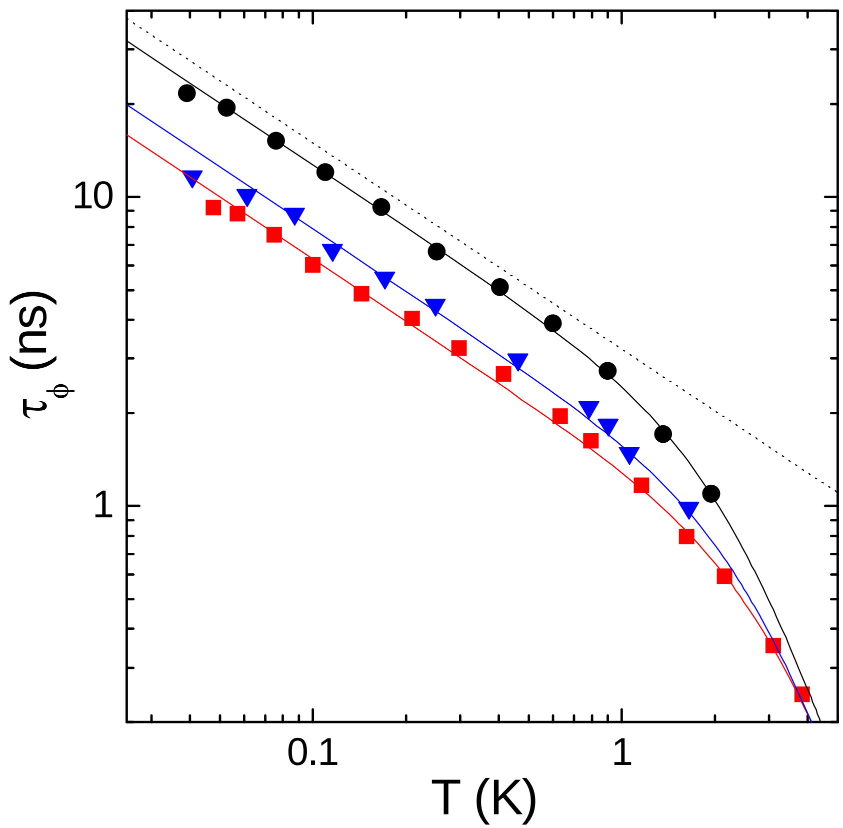
- **Ag 6N, Au 6N**
  → agreement with AAK theory

- **Ag 5N, Cu 6N**
  → saturation of τ_ϕ(T)

Saturation of τ_ϕ is sample dependent

Quantitative comparison with AAK theory for clean samples

$$\tau_\phi = (A\,T^{2/3} + B\,T^3)^{-1}$$



Sample	A_{thy} ($\text{ns}^{-1}\,\text{K}^{-2/3}$)	A ($\text{ns}^{-1}\,\text{K}^{-2/3}$)
■ Ag(6N)a	0.55	0.73
▼ Ag(6N)b	0.51	0.59
● Ag(6N)c	0.31	0.37
Ag(6N)d	0.47	0.56
Au(6N)	0.40	0.67

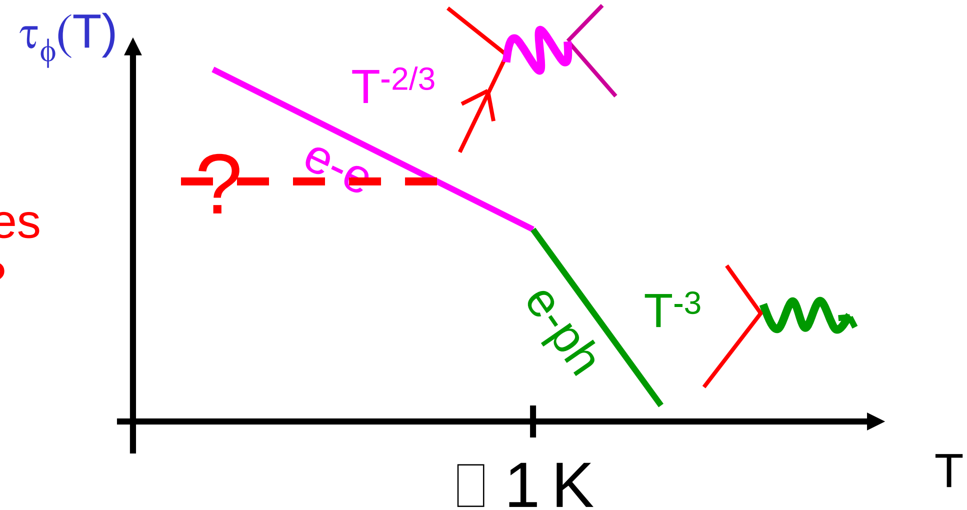
F. Pierre *et al.*,
PRB **68**, 0854213 (2003)

$$A_{thy} = \frac{1}{\hbar}\left(\frac{\pi k_B^2}{4\nu_F L w t}\frac{R}{R_K}\right)^{1/3}$$

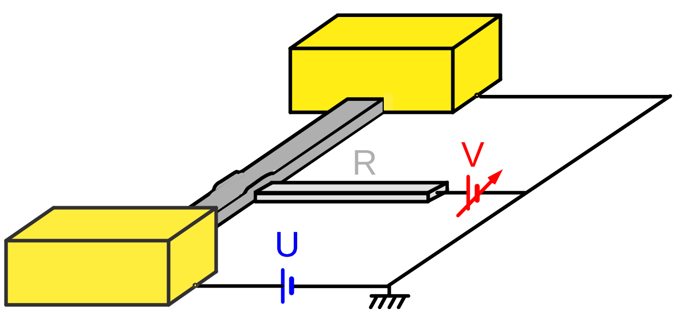
Investigation of inelastic processes

1st method : τ_ϕ

Another process dominates
in not-so-pure samples?



2nd method : measure energy exchange rates



Distribution f(E)
reflects the
exchange rates

Background: Shot noise in diffusive metal wires

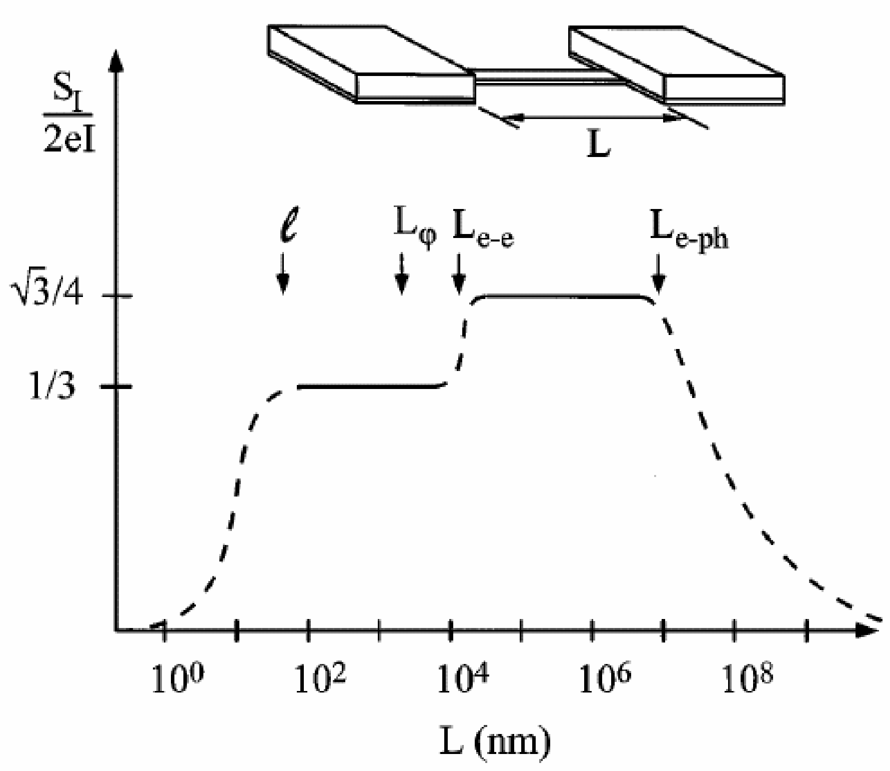
Steinbach, Martinis and Devoret, PRL **76**, 3806 (1996)

Theory:

Nagaev 1992,1995

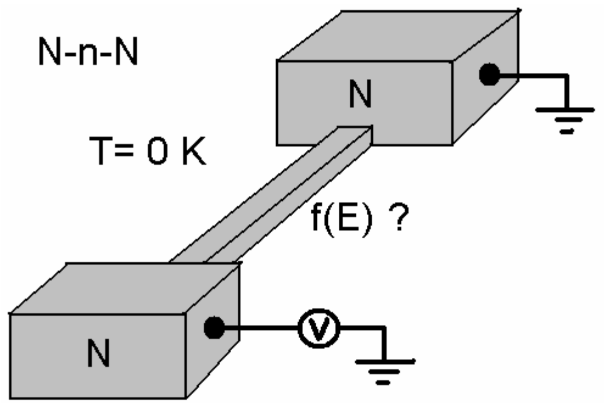
Kozub & Rudin, 1995

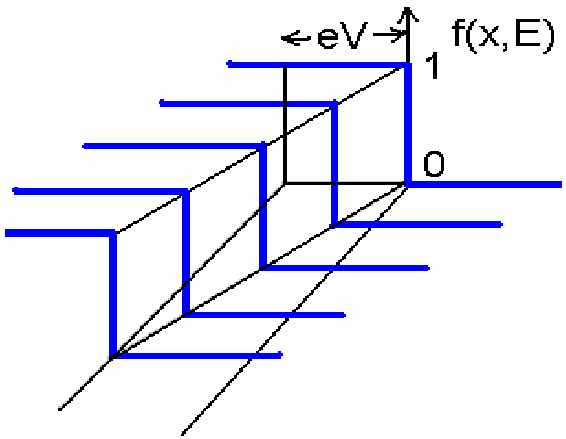
$$S_I = \frac{4}{RL} \iint dx dE \; f(x,E)(1 - f(x,E))$$



What does f(x,E) look like?

Distribution function -- textbook case (no shot noise)



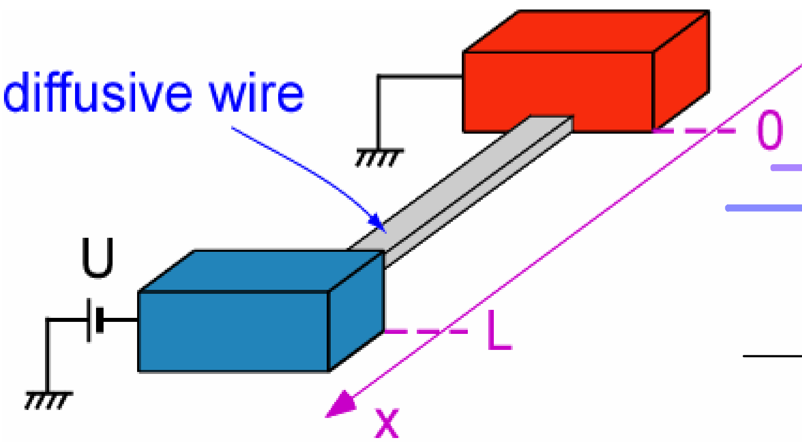


$\tau_D = L^2/D$

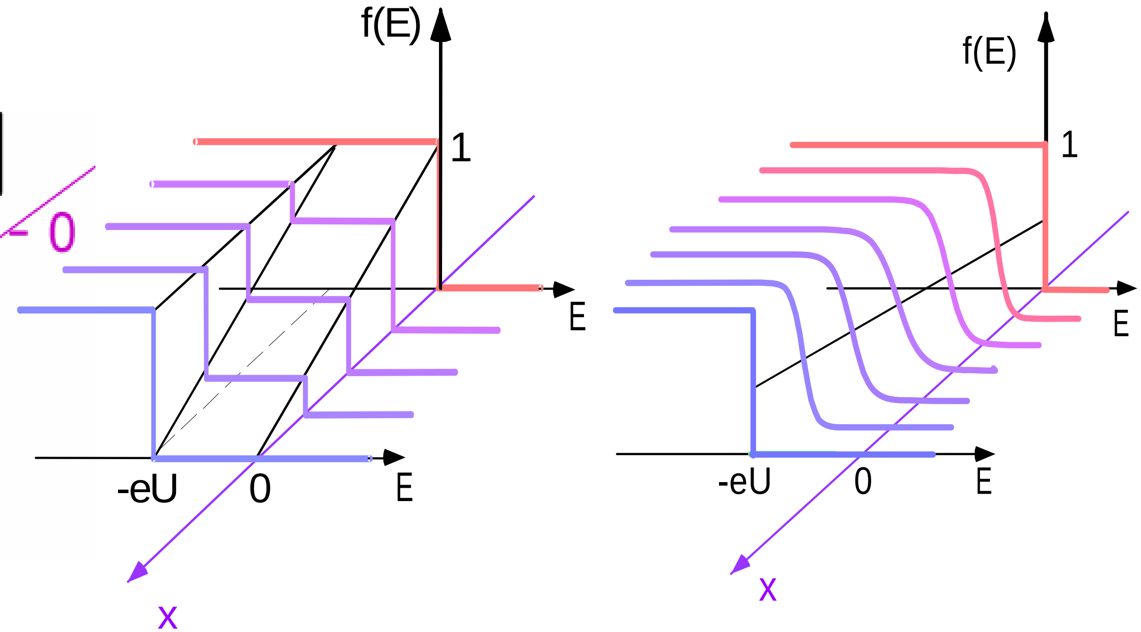
Assumes complete thermalization -- $t_D >> t_{electron-phonon}$

Never true in mesoscopic metal samples at low T!

Distribution function for $\tau_D << \tau_{electron\text{-}phonon}$



free electrons | "hot" electrons

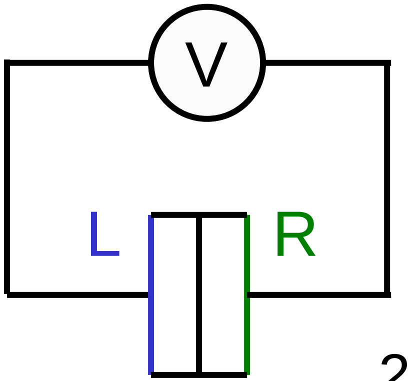


$\tau_D = L^2/D$

$\tau_D << \tau_{interaction}$ | $\tau_D >> \tau_{interaction}$

f(x,E) shaped by energy exchange

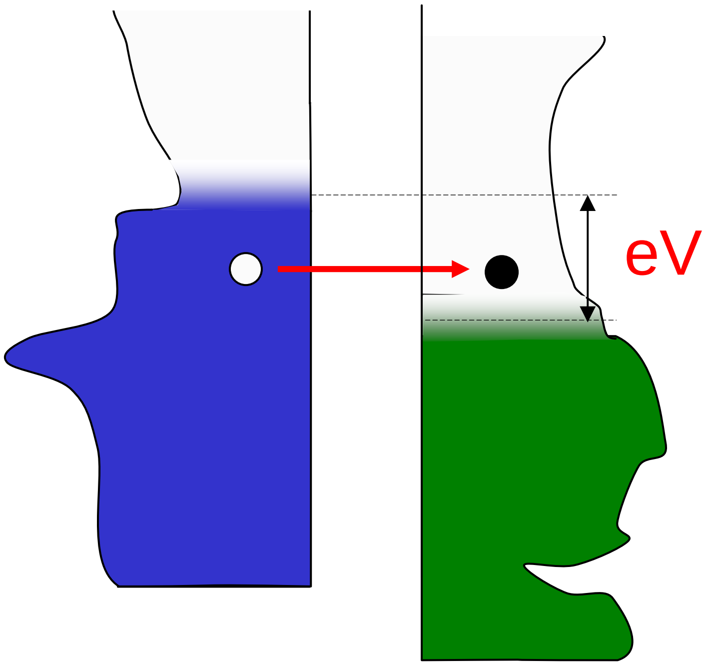
Aside 1: Current through a tunnel junction



$$I = e\left(\Gamma_{\rightarrow} - \Gamma_{\leftarrow}\right)$$

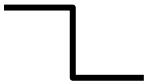
$$\Gamma_{\rightarrow} = \frac{2\pi\nu_F^2}{\hbar}\int dE \left|\langle M\rangle\right|^2 n_L(E)\, n_R(E+eV)\, f_L(E)\left(1-f_R(E+eV)\right)$$

$$\Gamma_{\leftarrow} = \qquad \left(1-f_L(E)\right) f_R(E+eV)$$



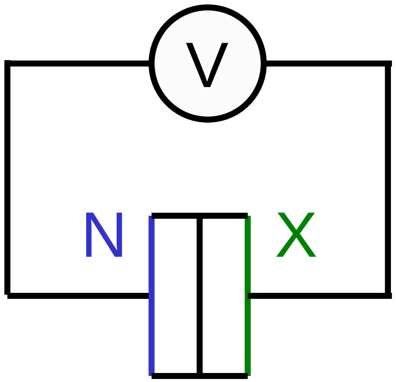
$$I = \frac{1}{eR_T}\int dE\, n_L(E)\, n_R(E+eV)$$
$$\text{x}\ \left(f_L(E) - f_R(E+eV)\right)$$

NN junction: $n(E) = 1 \quad f(E) =$



$$\Rightarrow I = \frac{V}{R_T}$$

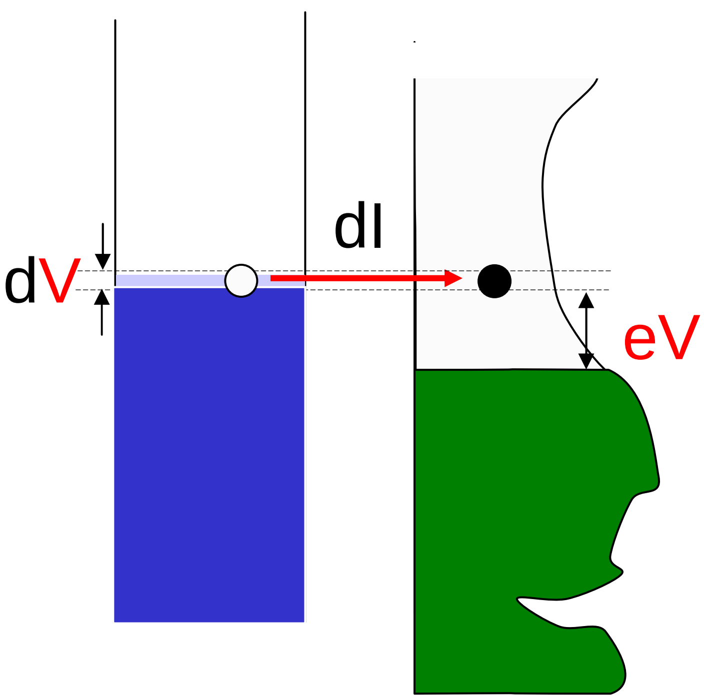
Conductance of an N-X junction at T=0



$$I = \frac{1}{eR_T}\int dE\, n_L(E)\, n_R(E+eV) \times \left(f_L(E) - f_R(E+eV)\right)$$

$$= \frac{1}{eR_T}\int_{-eV}^{0} dE\, n_X(E+eV)$$

$$= \frac{1}{eR_T}\int_{0}^{eV} dE\, n_X(E)$$

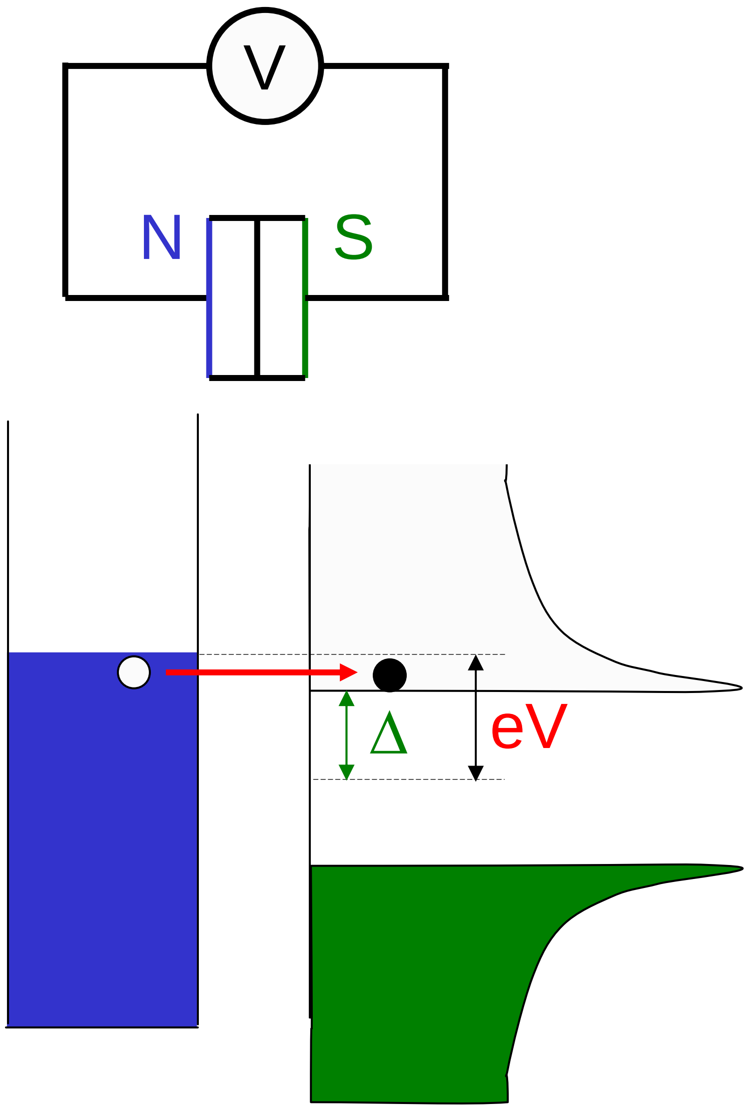


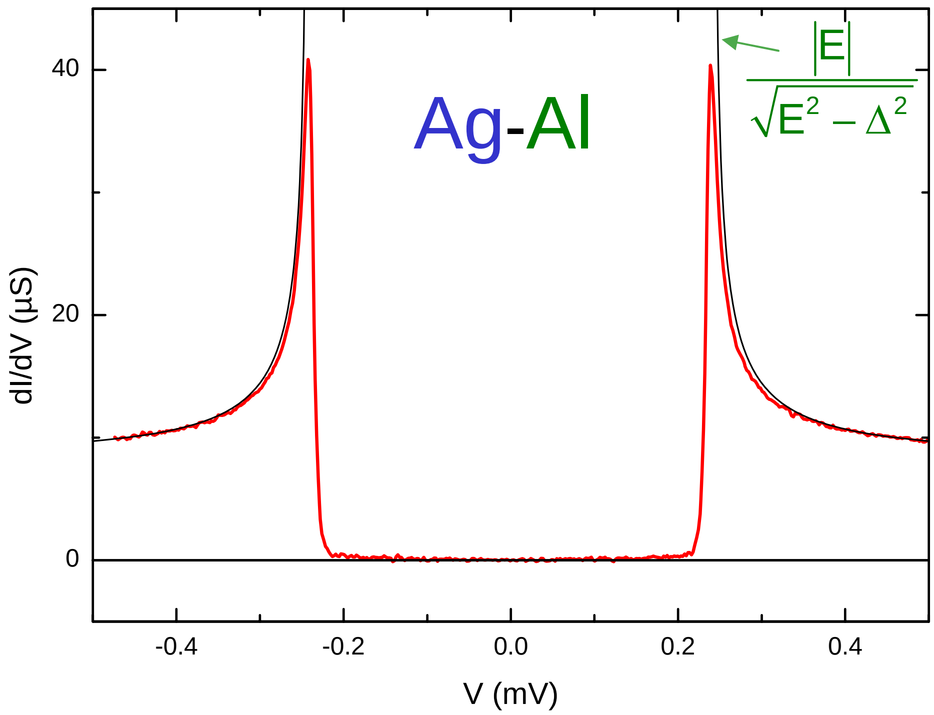
$$\frac{dI}{dV} = \frac{1}{R_T}\, n_X(eV)$$

Spectroscopy of n_X

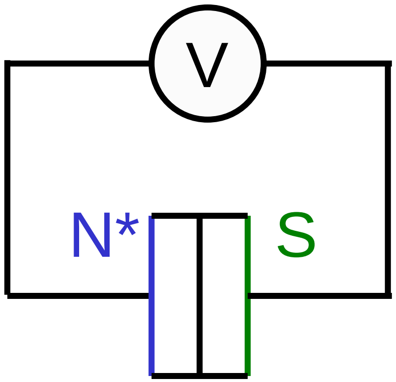
How to measure f(E):

tunnel spectroscopy using an N-S junction



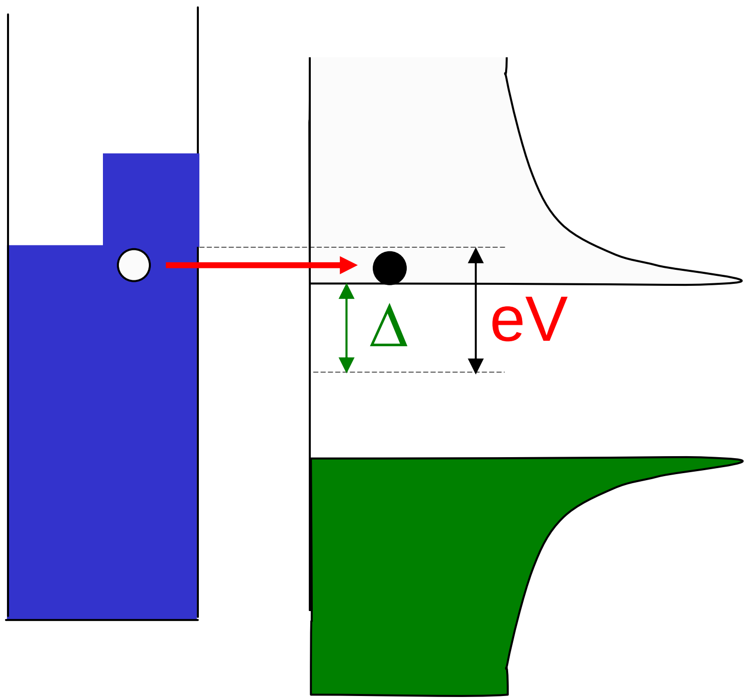


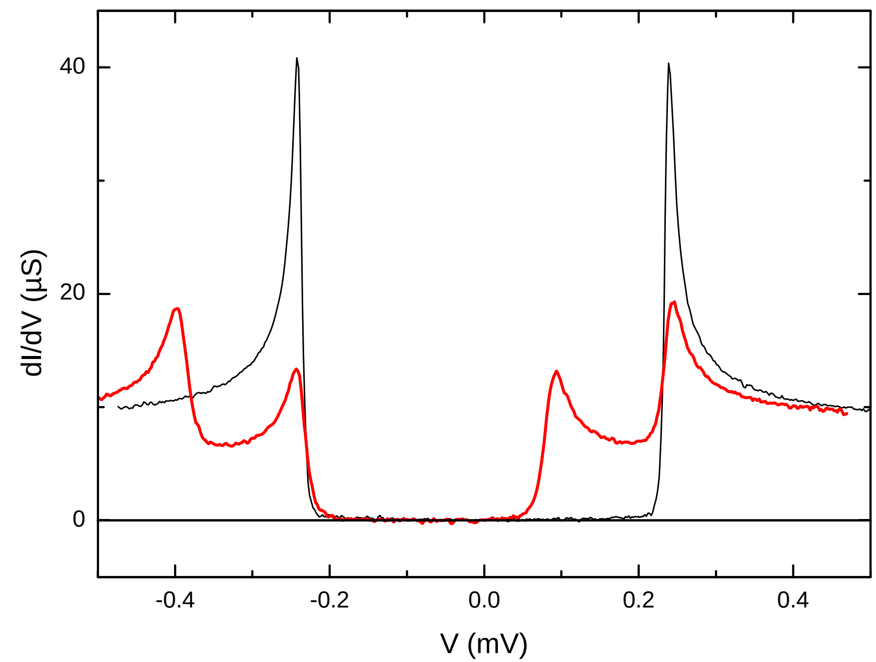
N out of equilibrium:
spectroscopy of f(E)



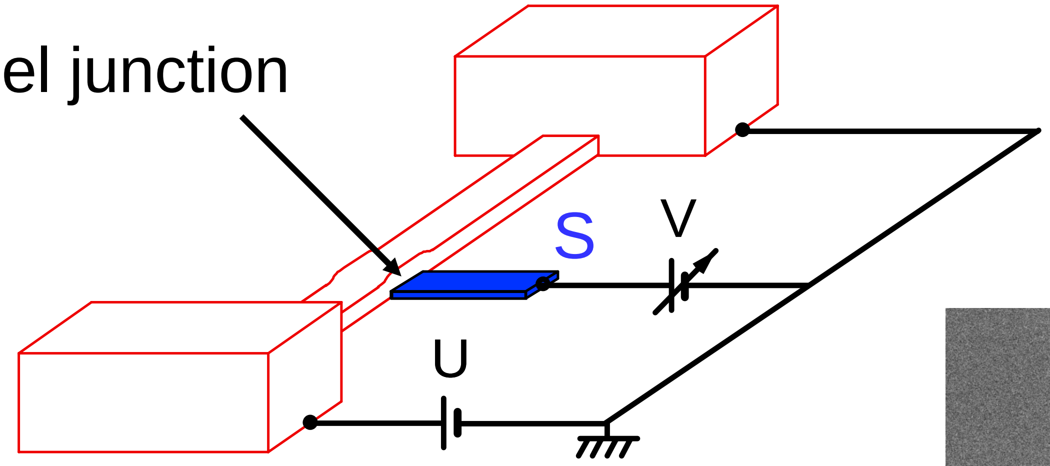
$$I = \frac{1}{eR_T}\int dE\, n_S(E)\left(f_N(E - eV) - f_S(E)\right)$$

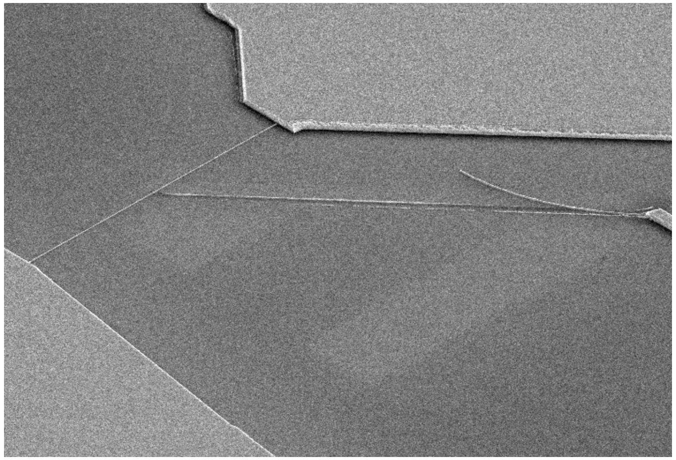
$$\frac{dI}{dV} = \frac{-1}{R_T}\int dE\, n_S(E)\, f_N'(E - eV)$$





Experimental setup



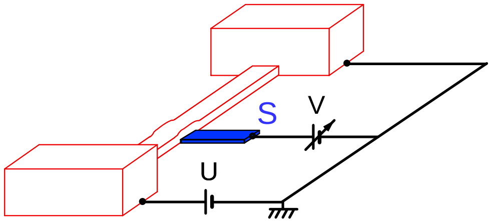


L=5 to 40 µm

Diffusion time: $\tau_D = \frac{L^2}{D} = 1$ to 60 ns

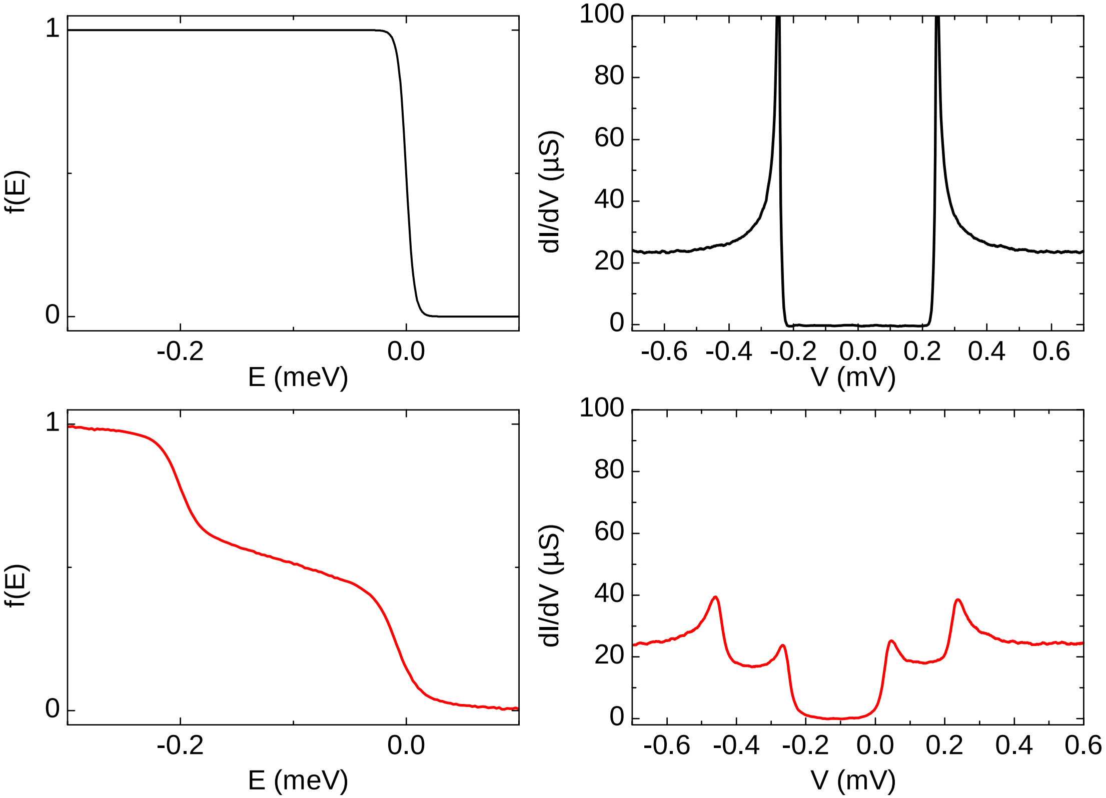
$$\frac{dI}{dV}(V) \xrightarrow[\text{deconvolution}]{\text{numerical}} f(E)$$

Summarize how to measure f(E):



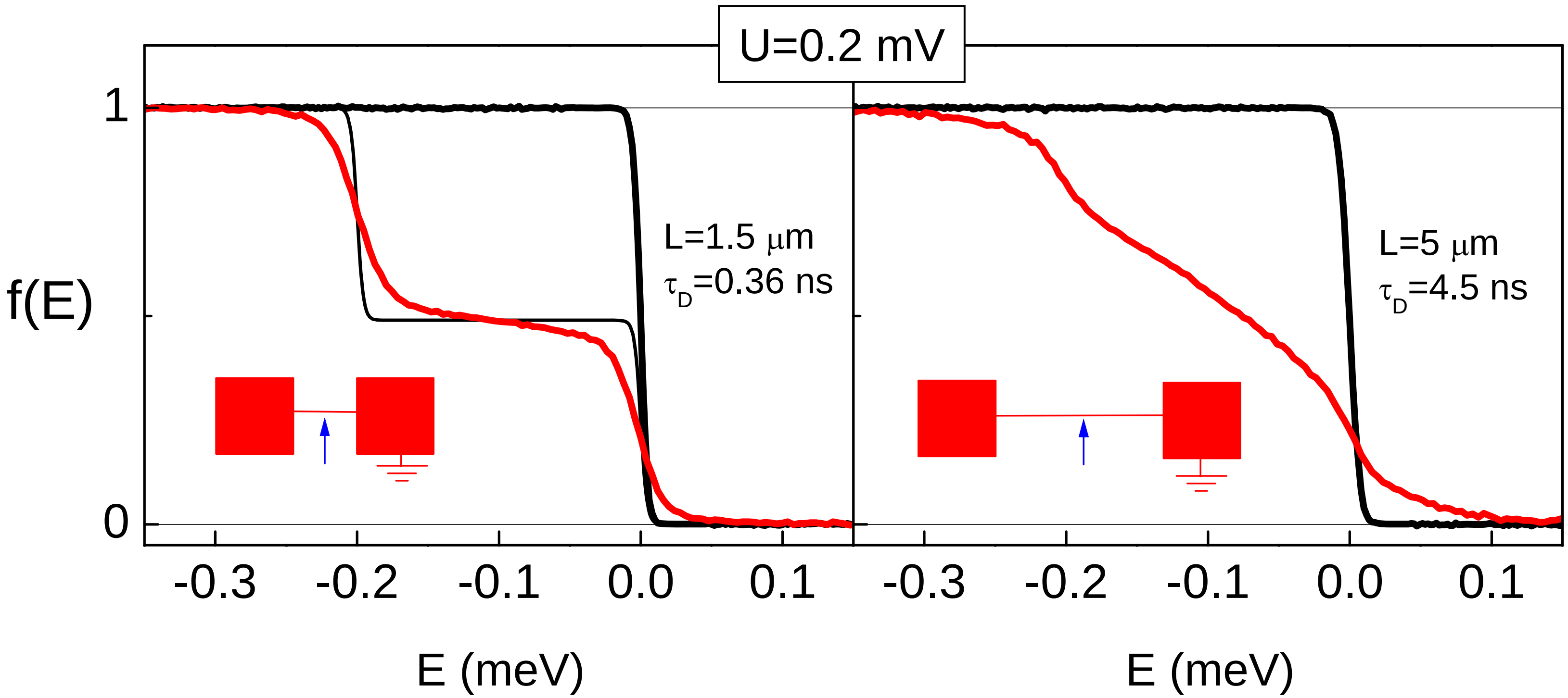
U=0 mV

U=0.2 mV



f(E) dI/dV

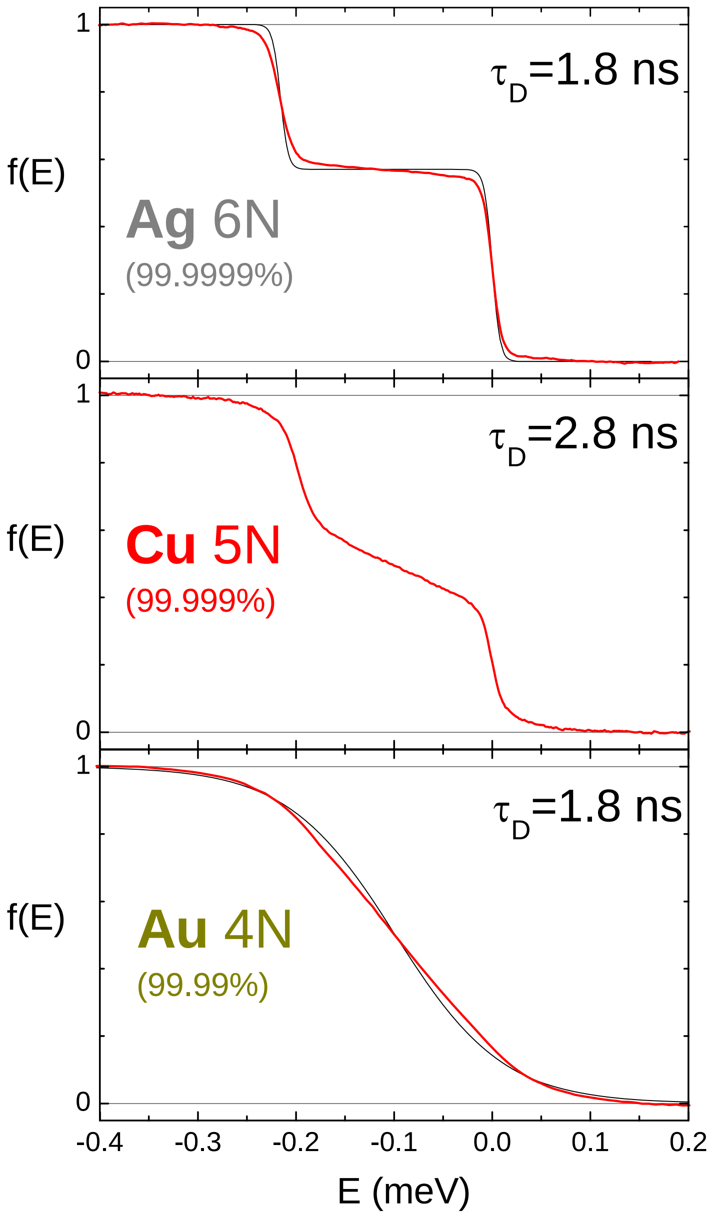
Effect of the diffusion time τ_D on f(E)



longer interaction time $\Rightarrow$ more rounding

H. Pothier et al., PRL 79, 3490 (1997)

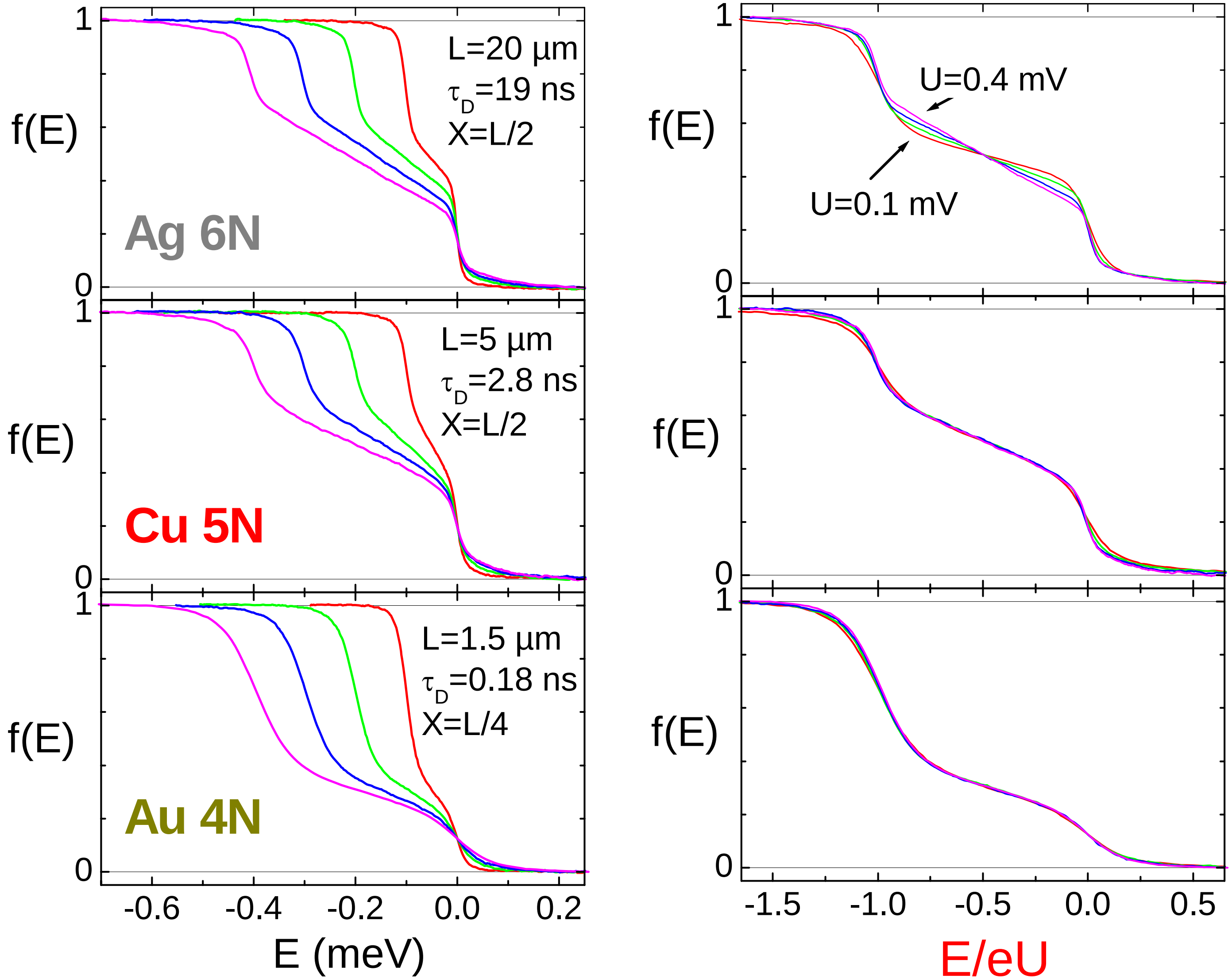
Compare strength of interactions



effect of material ?
effect of purity ?

Compare Dependence on U

U=0.1, 0.2, 0.3 & 0.4 mV

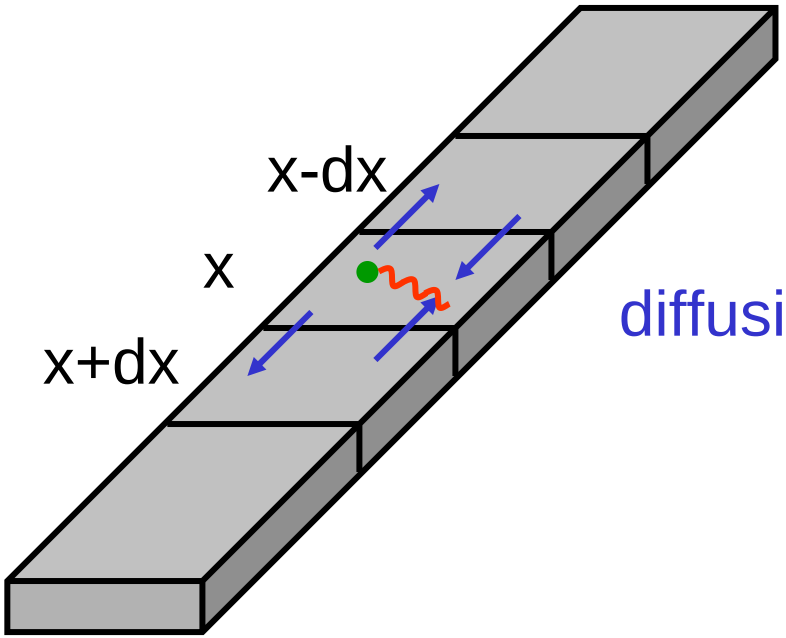


Observe scaling law in **Au 4N** & **Cu 5N** but not in **Ag 6N**

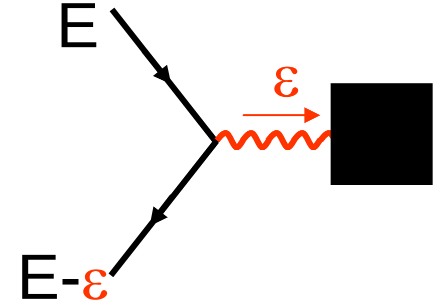
Calculation of f(x,E)

Boltzmann equation in the diffusive regime (Nagaev, Phys. Lett. A, 1992):

$$\frac{\partial f(E)}{\partial t} + D\frac{\partial^2 f(E)}{\partial x^2} = I_{in}\left(x,E,\{f\}\right) - I_{out}\left(x,E,\{f\}\right)$$



diffusion in real space



transfers of population
in the energy space

Boundary conditions :

$$f_{x=0}(E) = f_{x=L}(E) = \text{Fermi function}$$

Calculation of f(x,E)

Boltzmann equation in the diffusive regime (Nagaev, Phys. Lett. A, 1992):

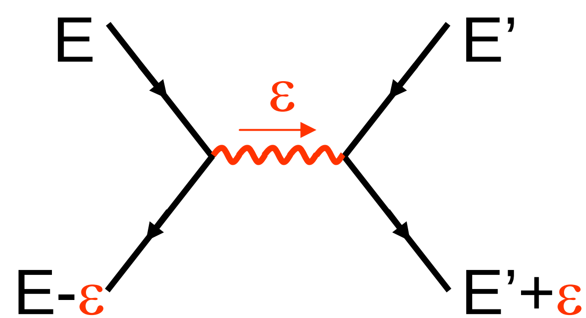
$$D \frac{\partial^2 f(E)}{\partial x^2} = I_{in}\left(x, E, \{f\}\right) - I_{out}\left(x, E, \{f\}\right)$$

e-e interactions :

$$\frac{\kappa}{\varepsilon^{3/2}}$$

(Altshuler, Aronov, Khmelnitskii, 1982)

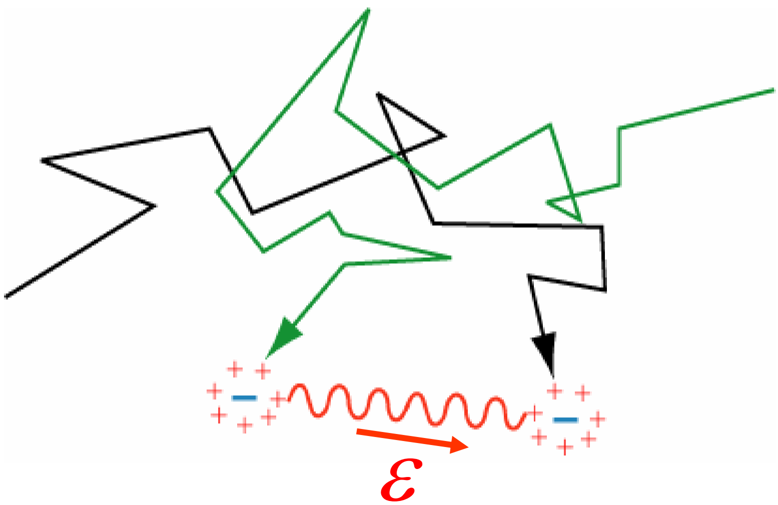
$$I_{out}\left(x, E, \{f\}\right) = \int dE' d\varepsilon \, K(\varepsilon) \, f(E) \left[1 - f(E - \varepsilon)\right] f(E') \left[1 - f(E' + \varepsilon)\right]$$



fn3D

Theory of screened Coulomb interaction in the diffusive regime

(Altshuler & Aronov, 1979)



ingredients:

polarisability ↘

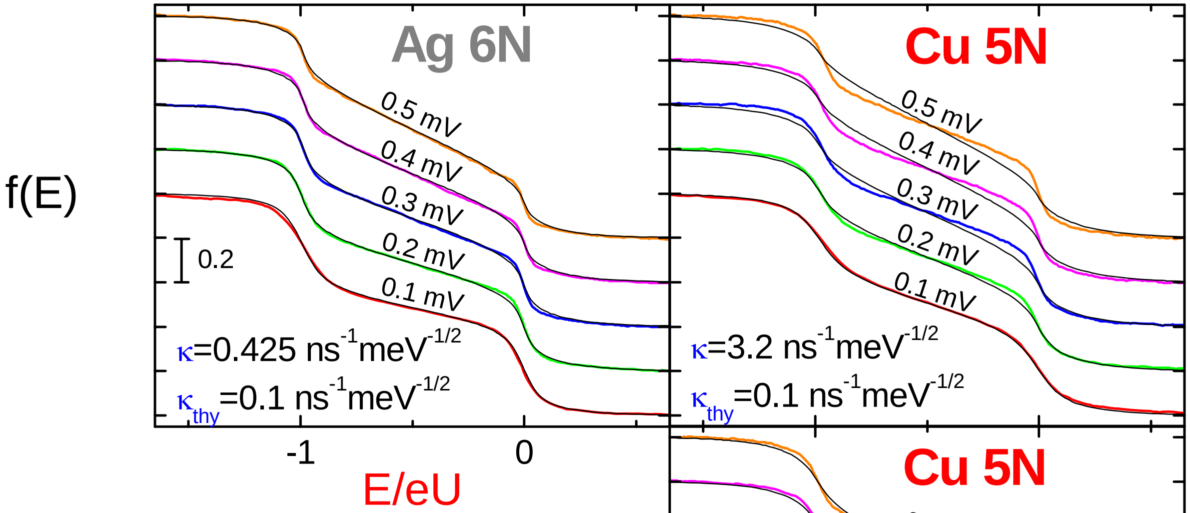
overlap ↗

Prediction for 1D wire :

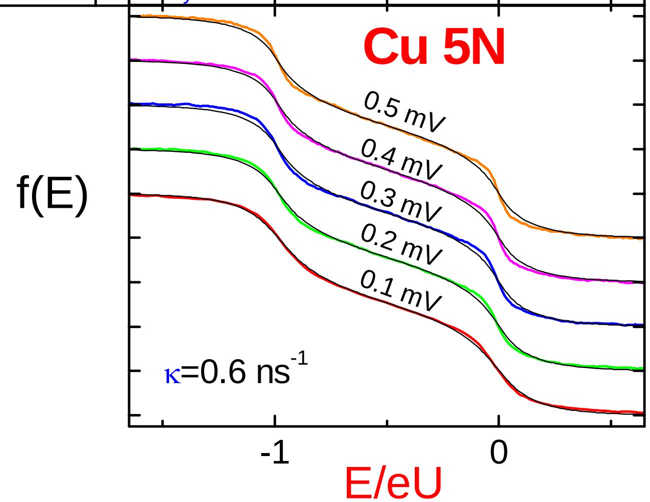
$$K(\varepsilon) = \frac{\kappa}{\varepsilon^{3/2}} \qquad \left(\propto \int \frac{dq}{D^2 q^4 + \omega^2} \right)$$

$$\kappa = \left(\sqrt{2D}\ \pi\ \hbar^{3/2}\ \nu_F\ S_e \right)^{-1}$$

Experiment vs. Theory



$K(\varepsilon)=\kappa\,\varepsilon^{-3/2}$



$K(\varepsilon)=\kappa\,\varepsilon^{-2}$

Ag 6N:

experiment agrees with theory

Cu 5N, **Au 4N**, **Ag 5N**:

- energy exchange stronger than predicted
- $K(\varepsilon)=\kappa\,\varepsilon^{-2}$ fits data

Comparison of the results of the two methods

	$\tau_\phi(T)$	$f(E)$
Ag_{6N}	$\propto T^{-2/3}$	$K(\varepsilon) = \frac{\kappa}{\varepsilon^{3/2}}$
Ag_{5N} $Cu_{6N,5N,4N}$ Au_{4N}	saturation	fast relaxation rates $K(\varepsilon) \not\propto \frac{1}{\varepsilon^{3/2}}$

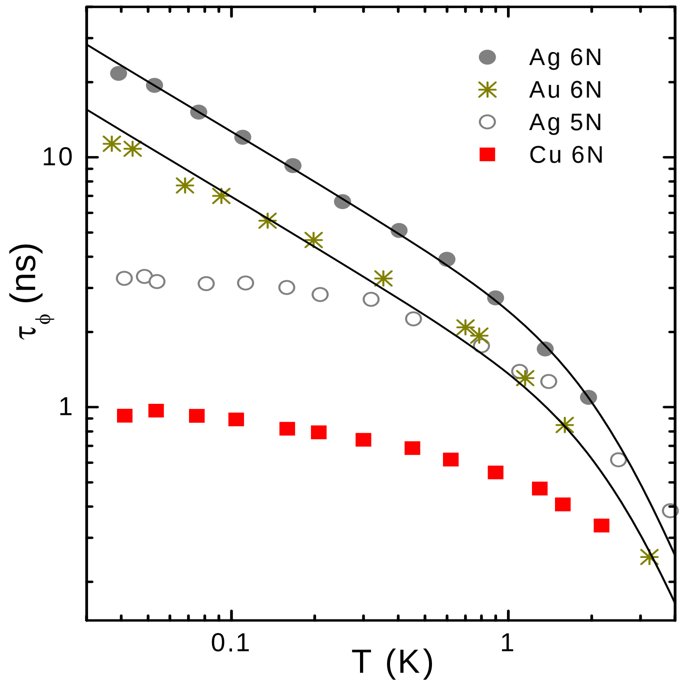
Comparison of the results of the two methods

	$\tau_\phi(T)$		$f(E)$
Ag_{6N}	$\propto T^{-2/3}$	*Coulomb interactions*	$K(\varepsilon) = \dfrac{\kappa}{\varepsilon^{3/2}}$
Ag_{5N} $Cu_{6N,5N,4N}$ Au_{4N}	saturation	*Other mechanism*	fast relaxation rates $K(\varepsilon) \not\propto \dfrac{1}{\varepsilon^{3/2}}$

ROLE OF RESIDUAL IMPURITIES ?

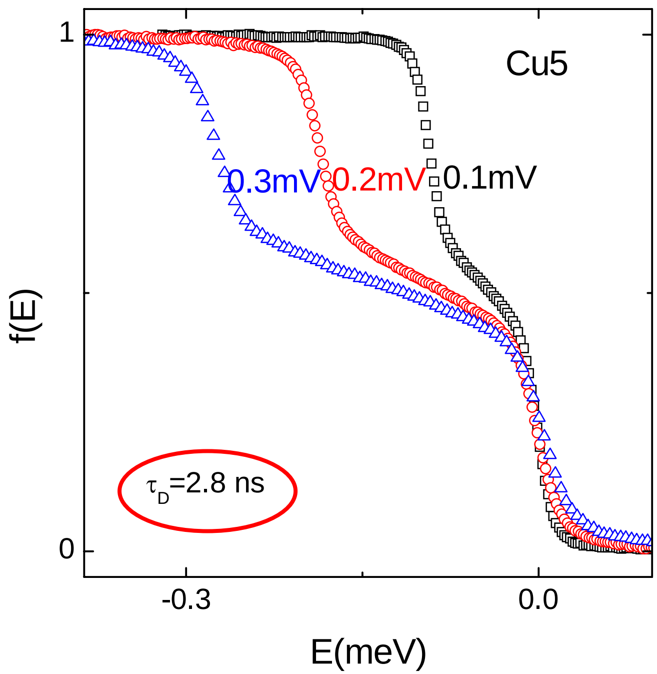
The two puzzles

τ_ϕ(T) measurements



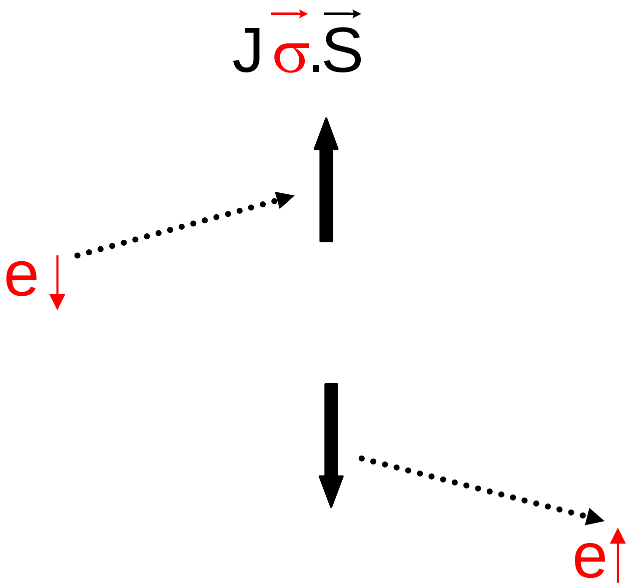
f(E) measurements

Ag_{5N} - $Cu_{5N,4N}$ - Au_{4N}



Anomalous interactions in the less pure samples

The Kondo effect again

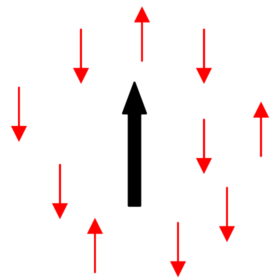


Spin-flip scattering

⇨ increased resistivity

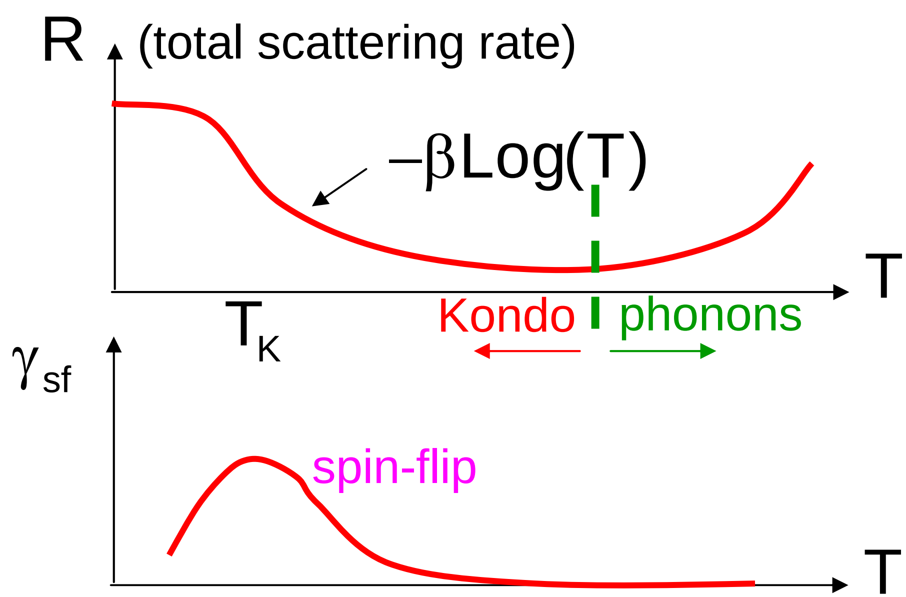
⇨ reduction of τ_ϕ

Collective effect:

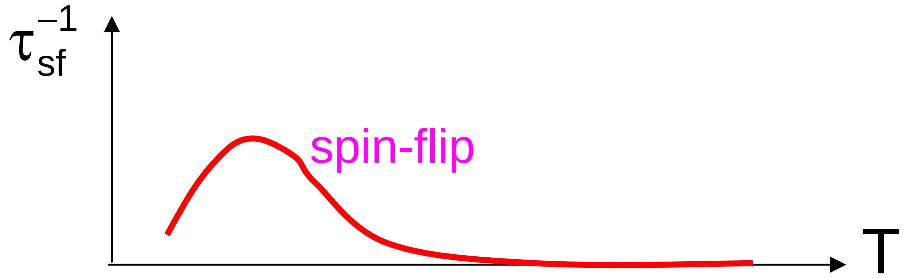


Formation of a singlet spin state

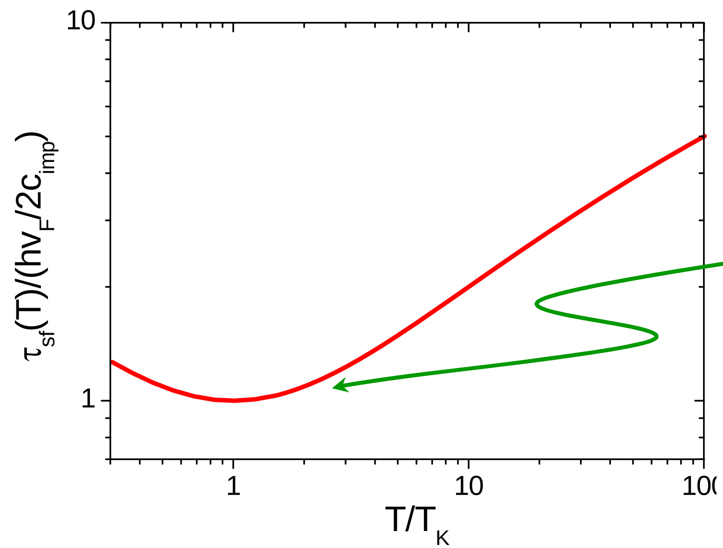
$$k_B T_K \approx E_F e^{-1/\nu J}$$



Nagaoka-Suhl expression of the spin-flip scattering rate near T_K



$$\frac{1}{\tau_{\mathrm{sf}}} = \frac{c_{\mathrm{mag}}}{\pi \hbar \nu_F} \frac{\pi^2 S(S+1)}{\pi^2 S(S+1) + \ln^2(T/T_K)}$$



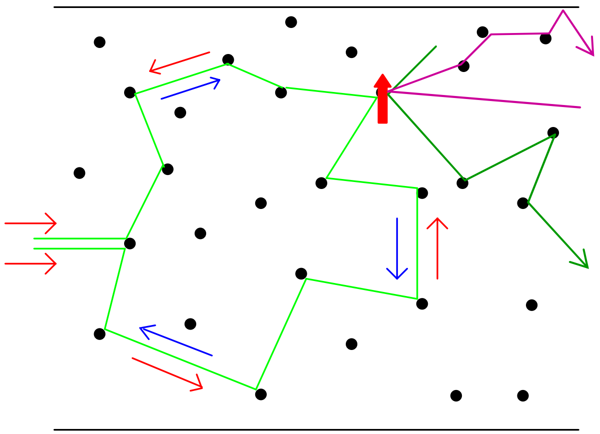
Weak temperature dependance near T_K !!

Link to τ_ϕ(T) saturation?

From τ_{sf} to τ_ϕ

Another important timescale: τ_K

Lifetime of the spin state
of a magn. imp.



(V.I. Fal'ko, JETP Lett. **53**, 340 (1991))

If $\tau_K < \tau_{sf}$ *Other electrons matter*

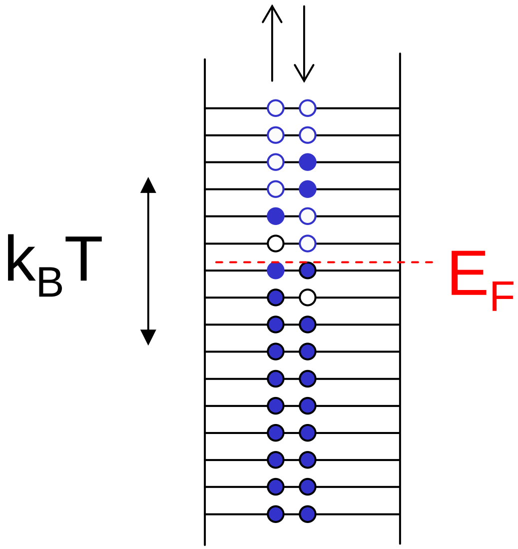
⇨ Randomising effect

$$\frac{1}{\tau_\phi} = \frac{1}{\tau_{ee}} + \frac{1}{\tau_{e-ph}} + \frac{1}{\tau_{sf}}$$

If $\tau_K > \tau_{sf}$ *The spin states of the mag. imp. seen by time-reversed electrons are correlated*

$$\frac{1}{\tau_\phi} = \frac{1}{\tau_{ee}} + \frac{1}{\tau_{e-ph}} + \frac{2}{\tau_{sf}}$$

Comparison of τ_{sf} and τ_K



$\nu_F k_B T$ concentration of electrons that can spin-flip

c_{imp} concentration of magnetic impurities

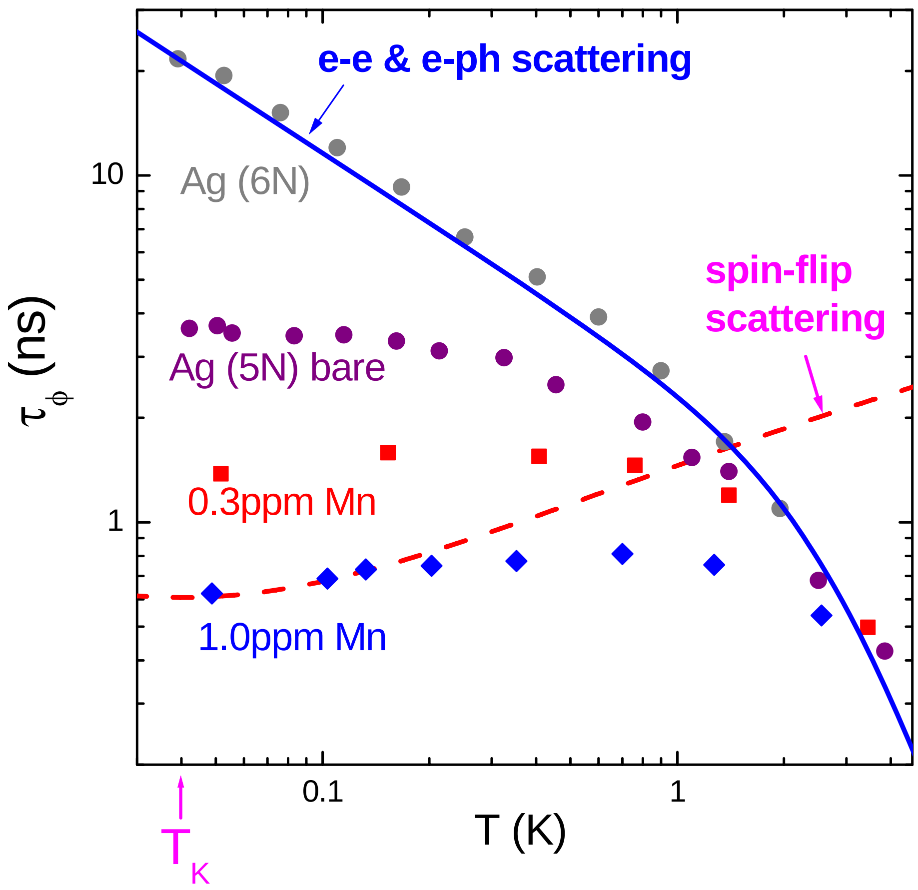
If $\nu_F k_B T > c_{imp}$: $\tau_K < \tau_{sf}$

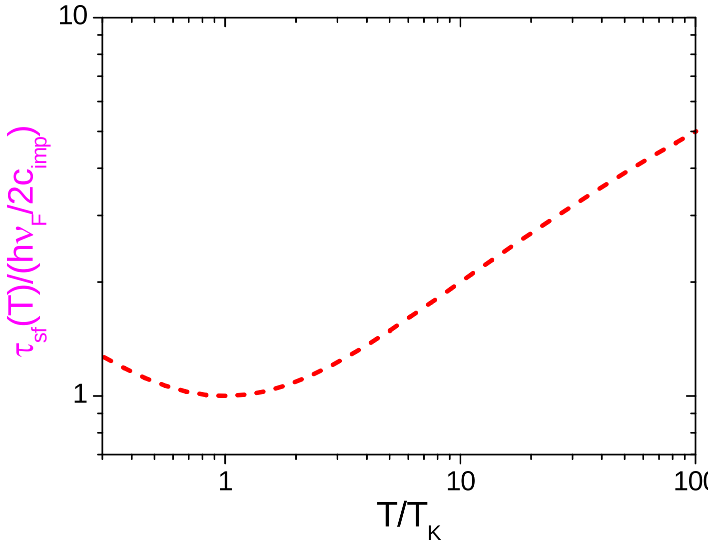
$$\frac{1}{\tau_\phi} = \frac{1}{\tau_{ee}} + \frac{1}{\tau_{e-ph}} + \frac{1}{\tau_{sf}}$$

Numerically,
for Au, Ag, Cu, …

$$T > 40 \text{ mK } \times c_{imp}(\text{ppm})$$

Effect of magnetic impurities on τ_ϕ



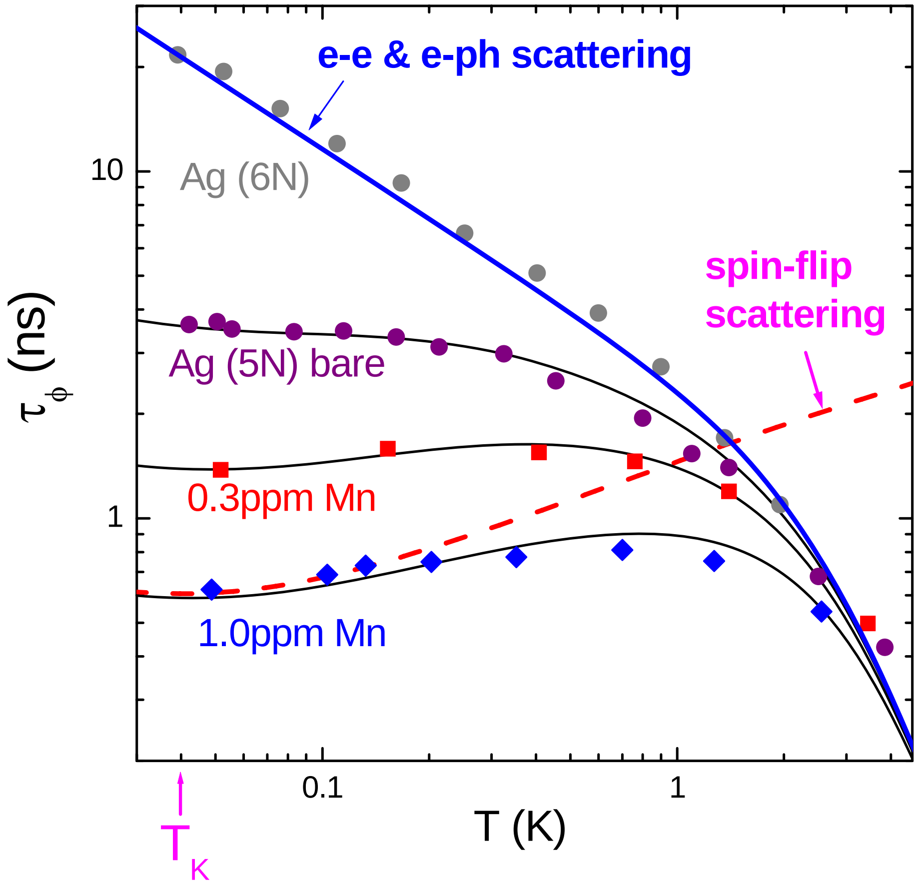


Spin-flip rate peaks at T_K:

$$\tau_\phi(T_K) = \frac{0.6 \text{ ns}}{c_{imp}\text{ (ppm)}}$$

$$\frac{1}{\tau_\phi} = \frac{1}{\tau_{ee}} + \frac{1}{\tau_{e-ph}} + \frac{1}{\tau_{sf}}$$

Effect of magnetic impurities on τ_ϕ



F. Pierre *et al.*,
PRB **68**, 0854213 (2003)

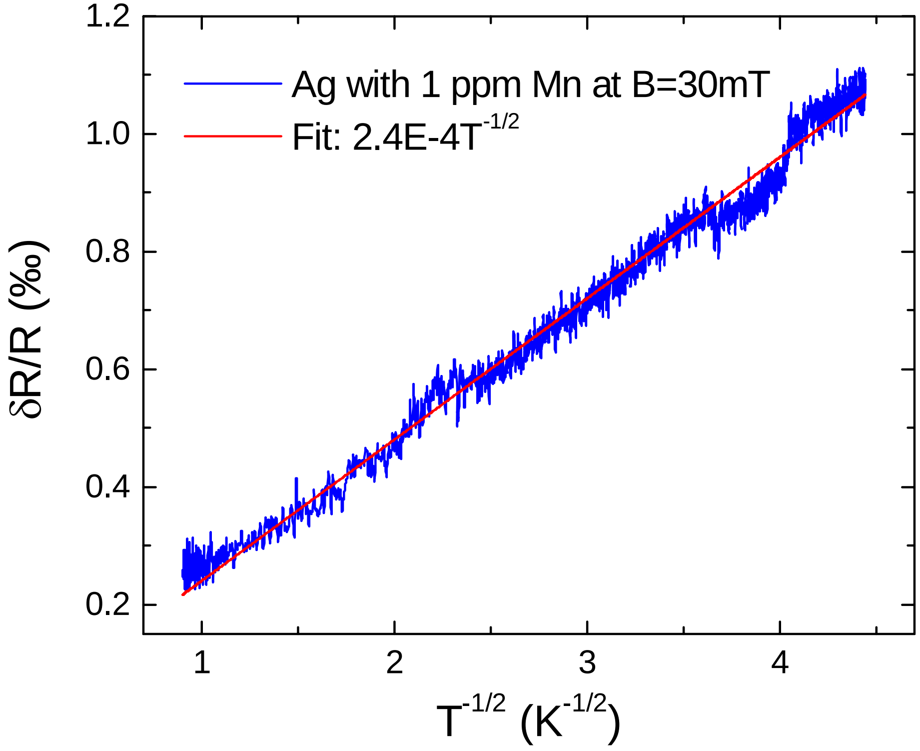
Fit parameters:

Ag(5N) bare: *0.13 ppm*
+ 0.3 ppm : *0.40 ppm*
+ 1 ppm : *0.96 ppm*

Above T_K : partial compensation of e-e and s-f

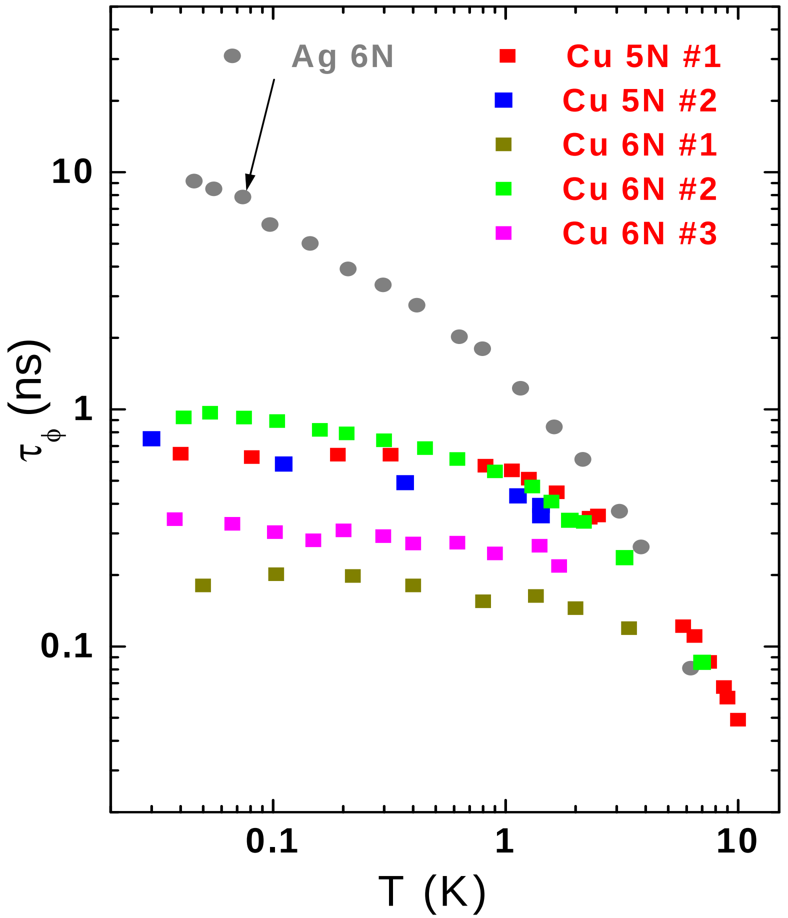
→ apparent saturation

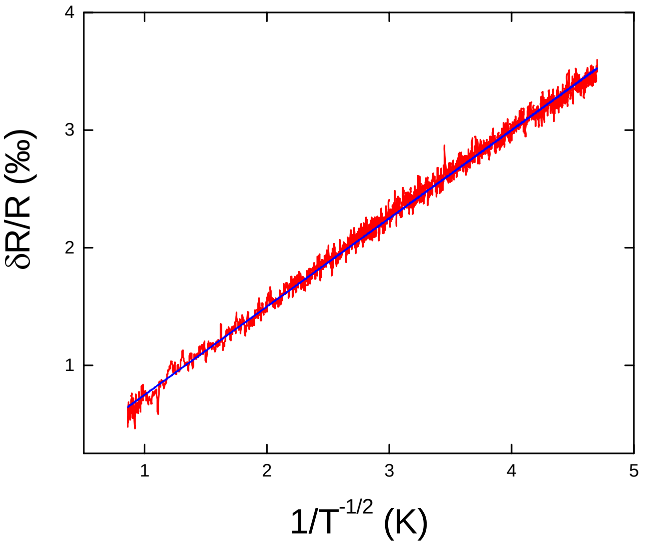
Why can't we just detect magnetic impurities with R(T) (the original Kondo effect)?



1 ppm of Mn is <u>invisible</u> in R(T)
(hidden by e-e interactions)

Source material purity vs. sample purity: Cu samples

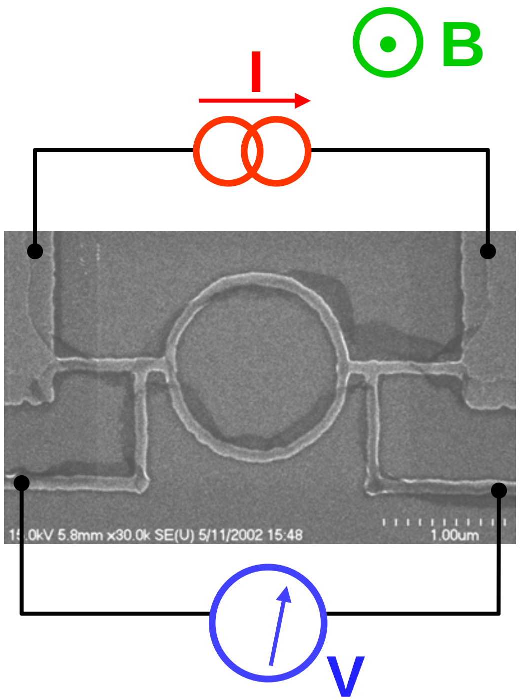




Magnetic impurities are invisible in R(T)

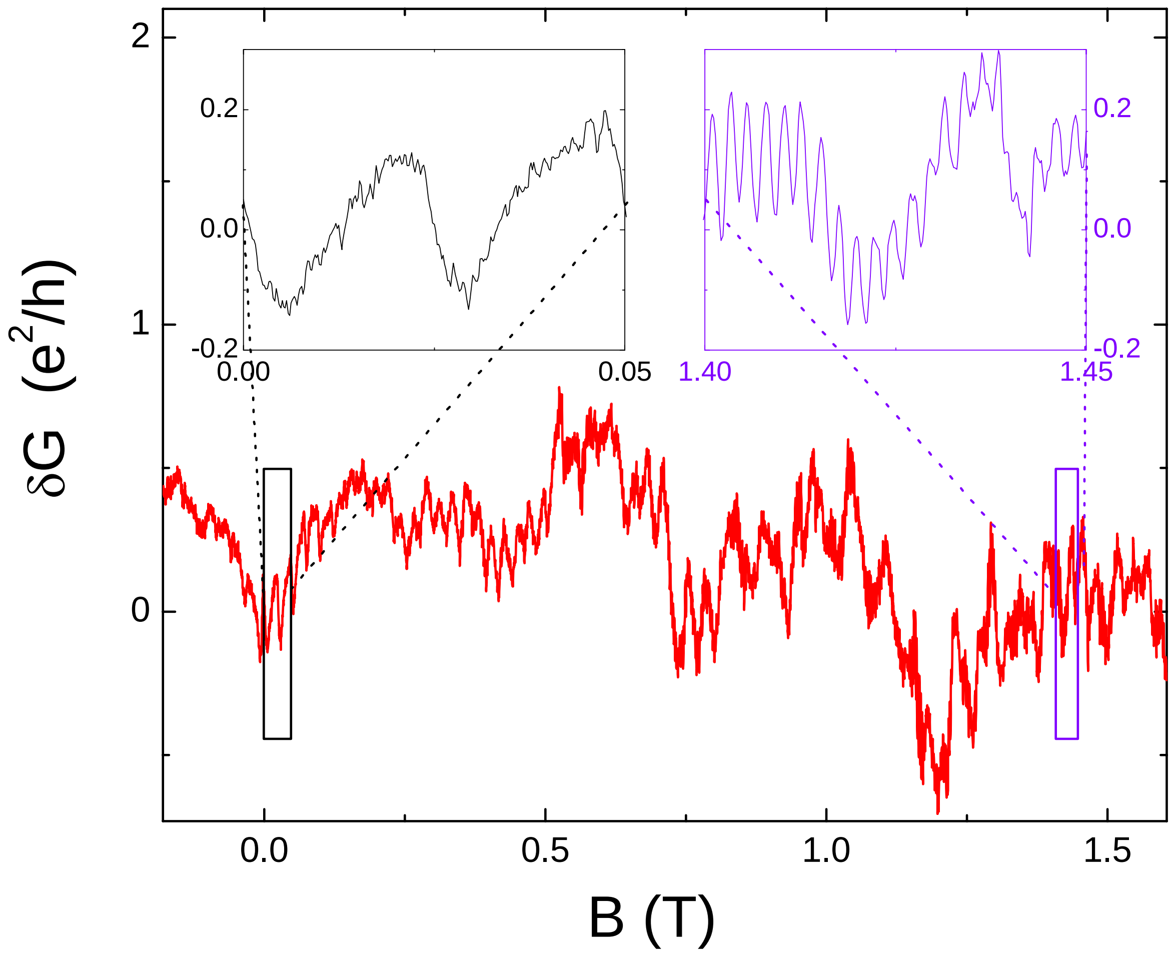
- In all **Cu** samples $\tau_\phi(T)$ saturates at low T
- $\tau_\phi(T)$ is strongly reduced but shows no dip

Measure $\tau_\phi(B)$ from Aharonov-Bohm oscillations

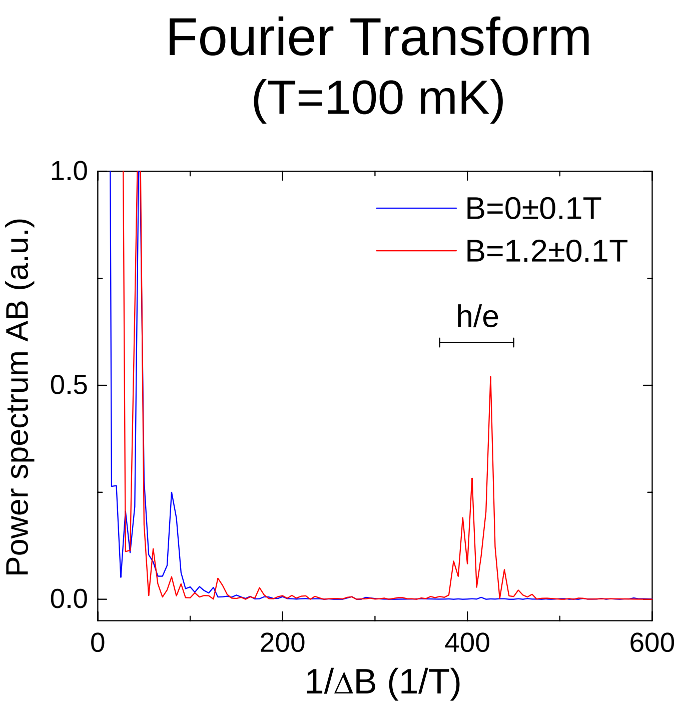


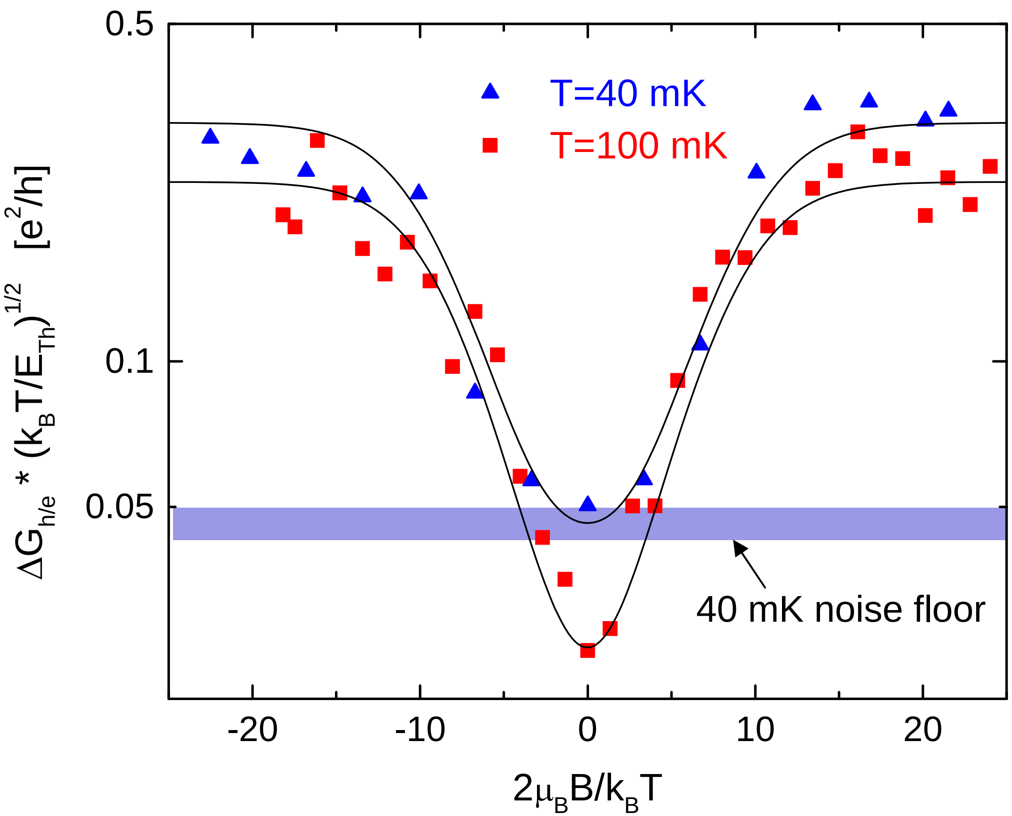
Cu ring

T=100 mK



A.B. Oscillations vs. Magnetic Field



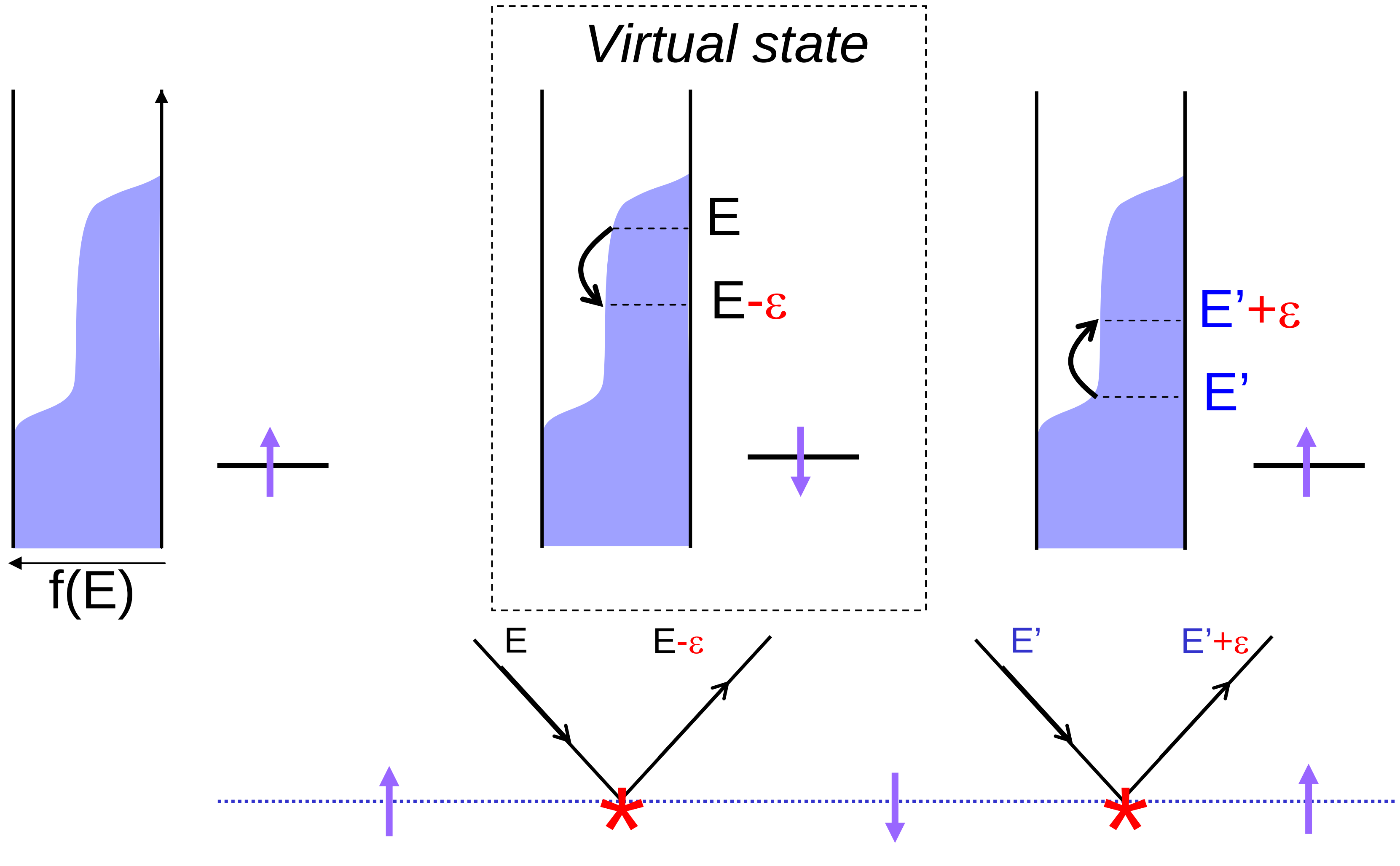


AB oscillations increase with B
⇒ presence of magnetic "impurities" !

What about energy exchange ?

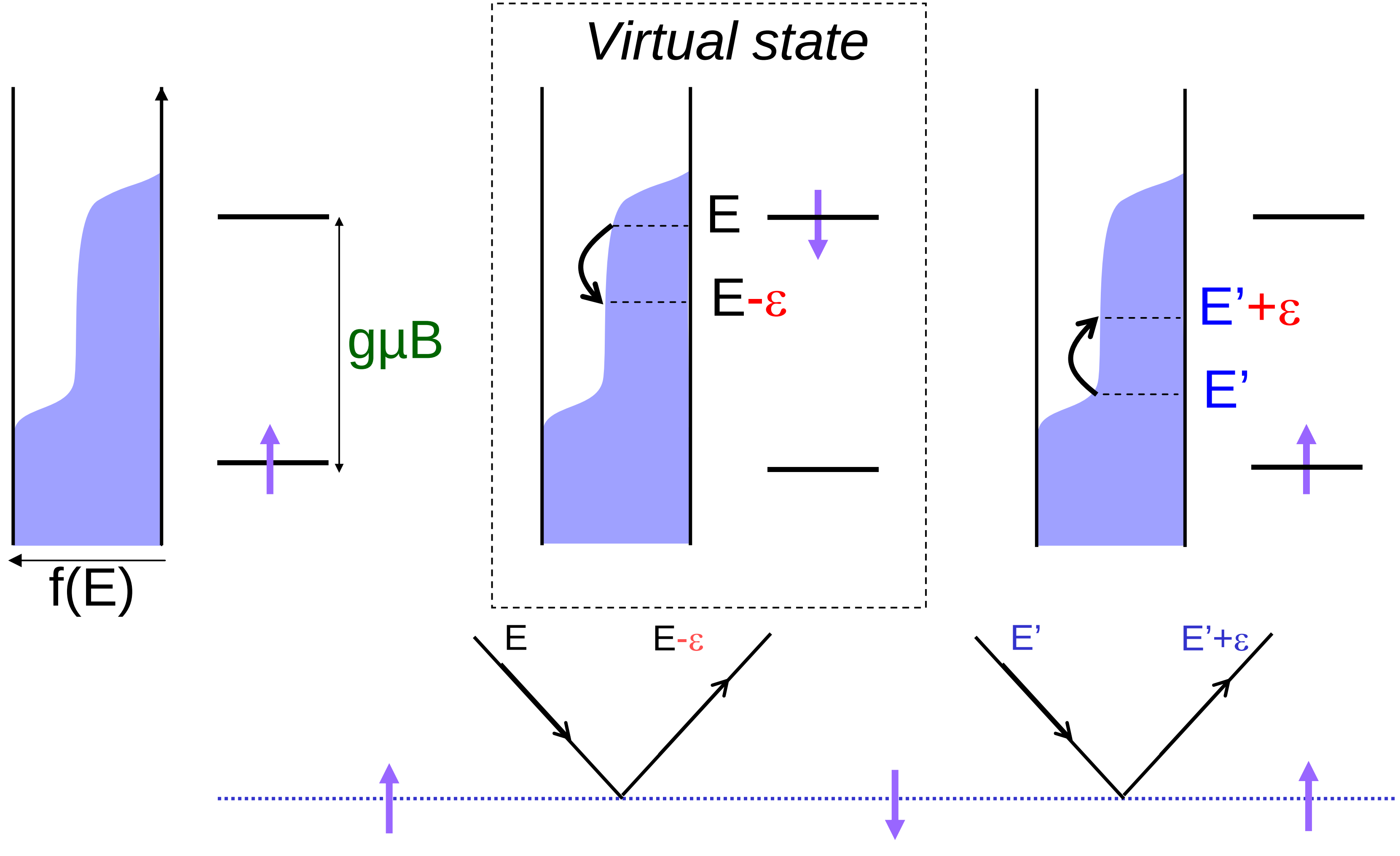
Energy exchange mediated by magnetic impurities

Kaminski and Glazman, PRL **86**, 2400 (2001)



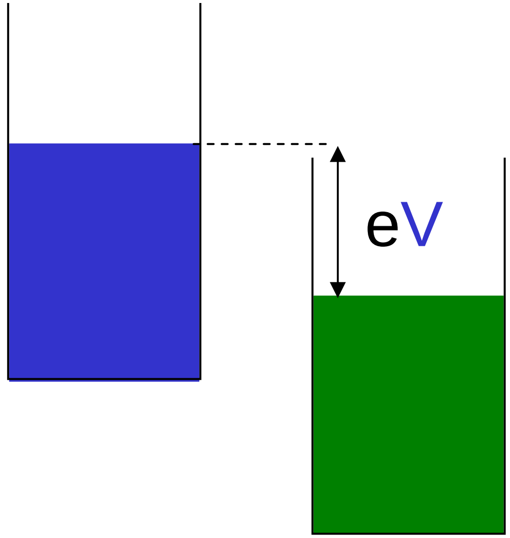
Reinforced by Kondo effect

Energy exchange mediated by magnetic impurities vanishes when $g\mu_B B \gg eU$



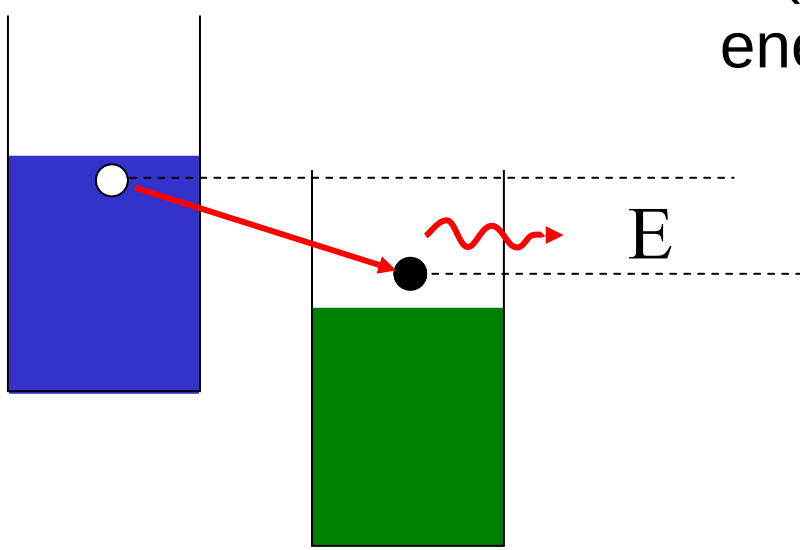
Aside 2: Inelastic tunneling

initial state:

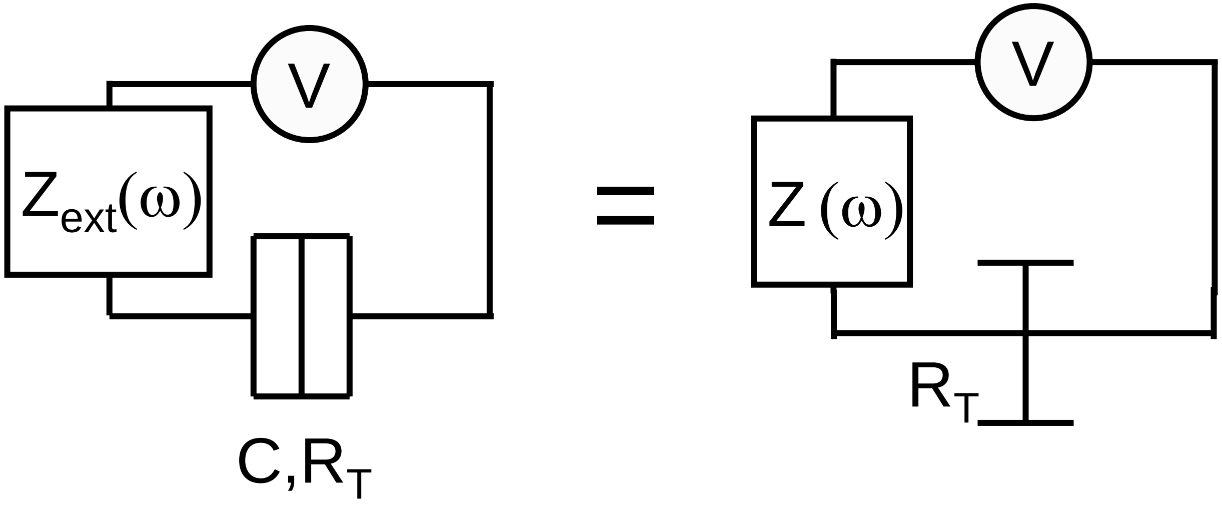


P(E)=probability to give energy E to environment

final state:



P(E) depends on environmental impedance



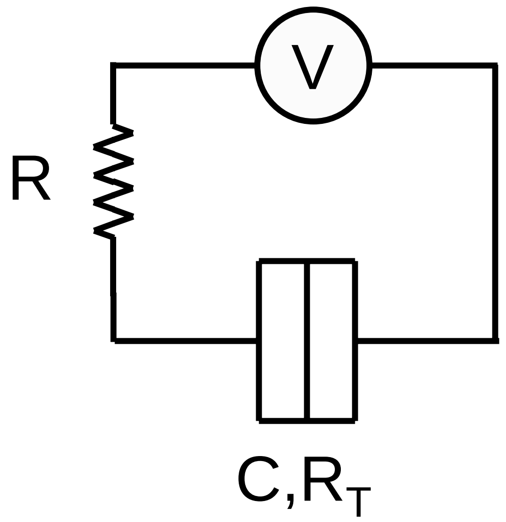
At T=0, one obtains :

$$\frac{dI}{dV} = \frac{1}{R_t}\int_0^{eV} P(E)\,dE$$

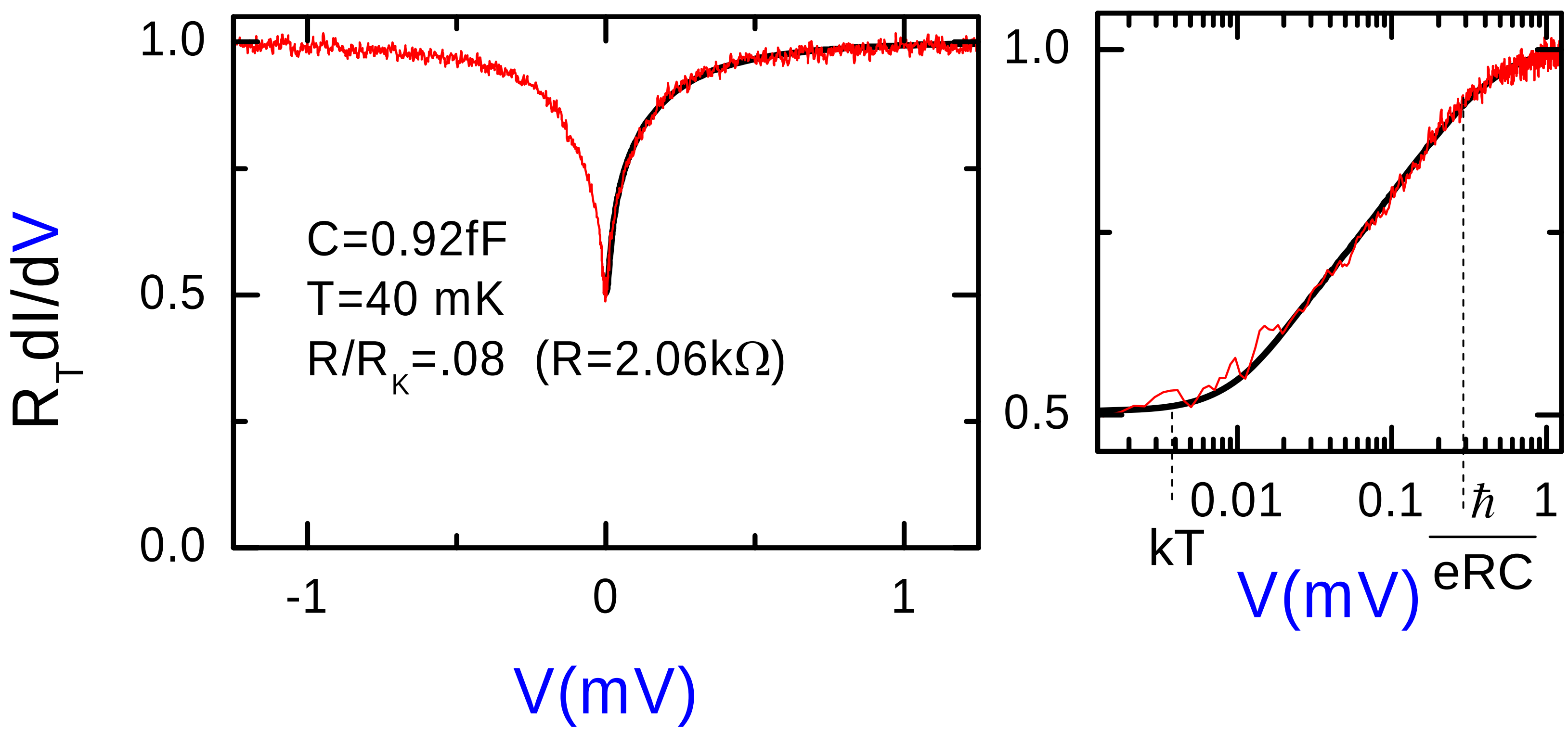
$$P(E) = \frac{1}{2\pi\hbar}\int e^{iEt/\hbar + J(t)}\,dt$$

$$J(t) = 2\int_0^{+\infty}\frac{d\omega}{\omega}\frac{\mathrm{Re}\left[Z(\omega)\right]}{R_K}\left(e^{-i\omega t} - 1\right)$$

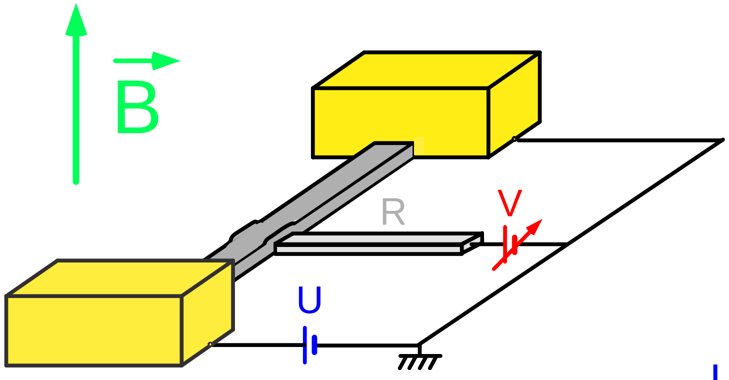
Resistive environment



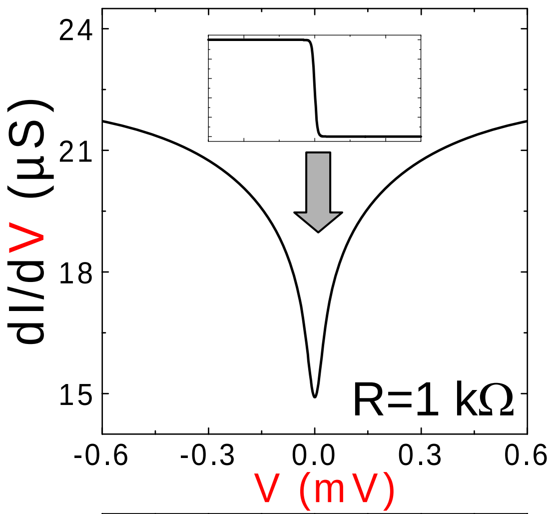
For $eV < \frac{\hbar}{RC}$ $\quad \frac{dI}{dV} \propto \left(V^{\frac{2R}{R_K}} + cst. \right)$



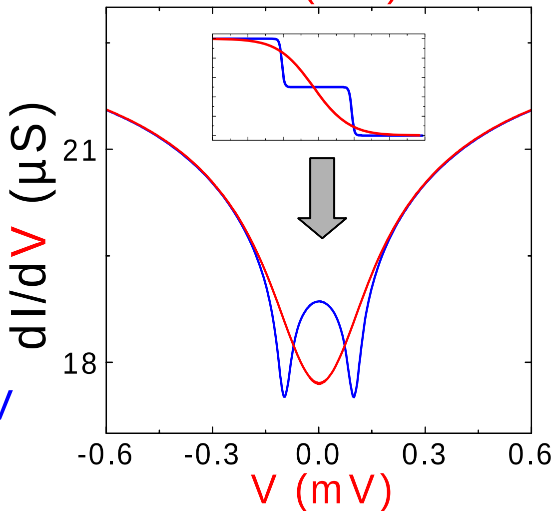
Measure f(E) at B ≠ 0 using Dynamical Coulomb Blockade



U=0 mV

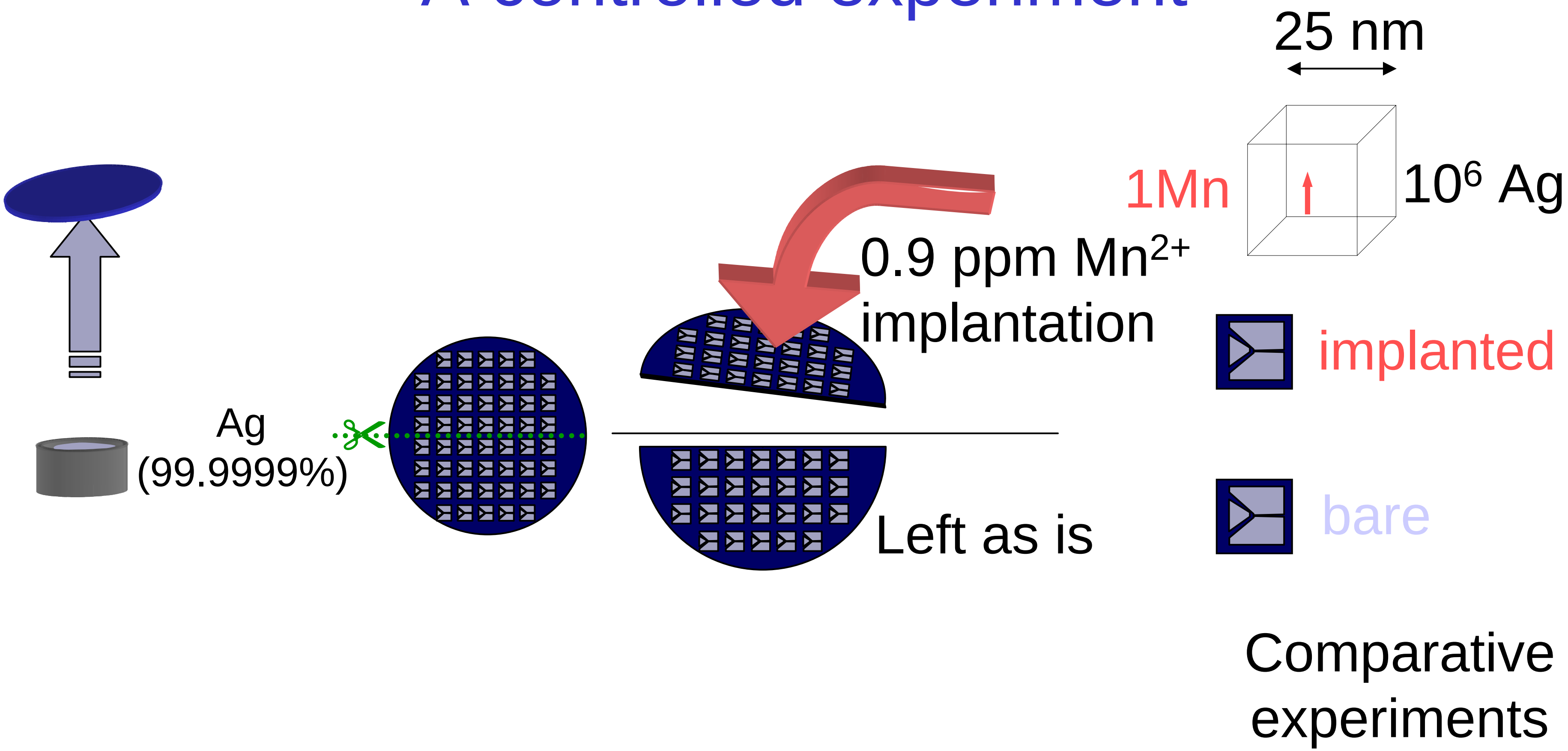


U=0.2 mV



dI/dV → f(E) → electron-electron interactions

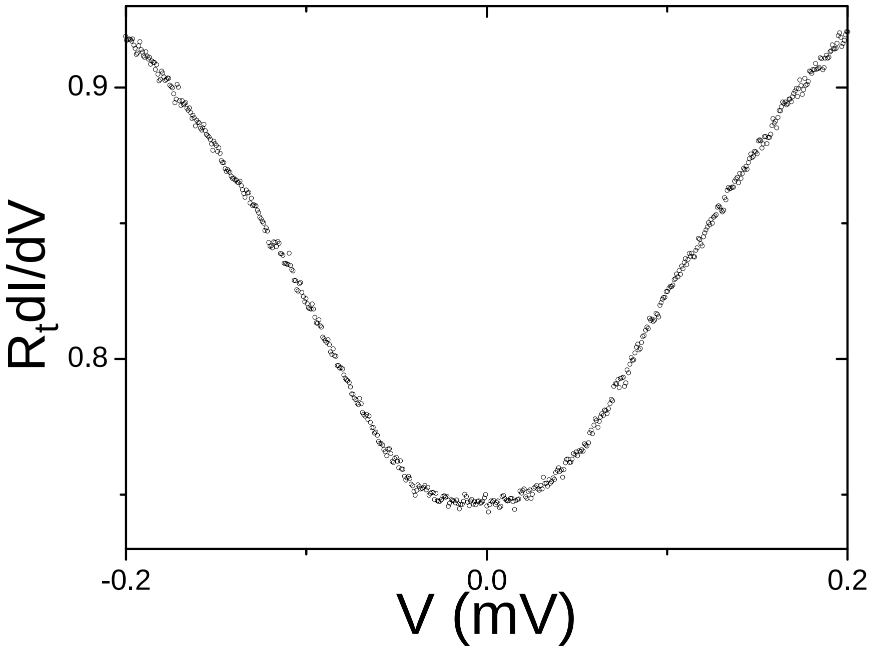
A controlled experiment

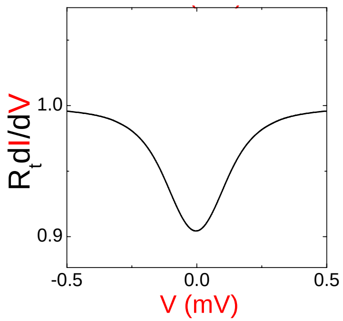


Effect of 1 ppm Mn on interactions ?

Experimental data at weak B

implanted

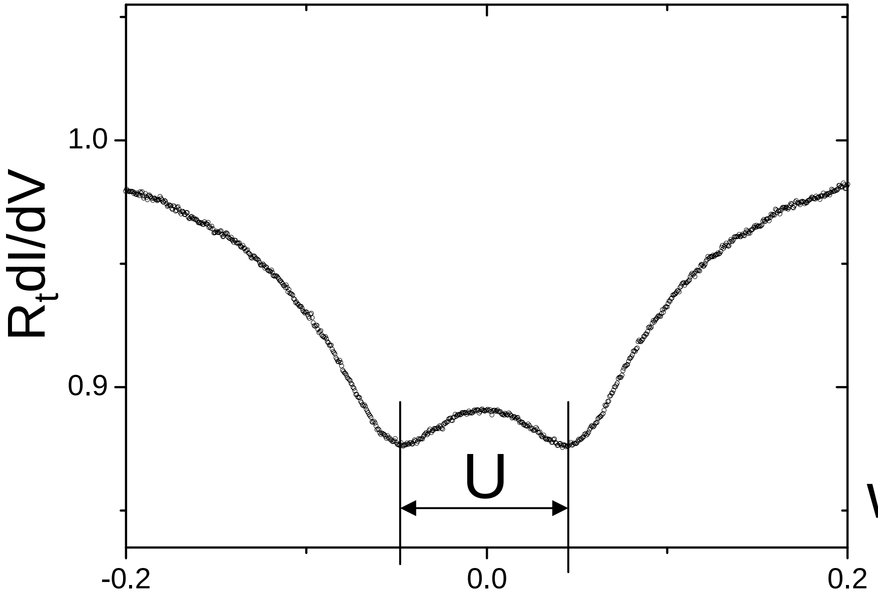


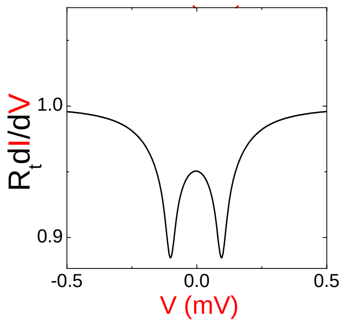


strong interaction

U = 0.1 mV
B = 0.3 T

bare





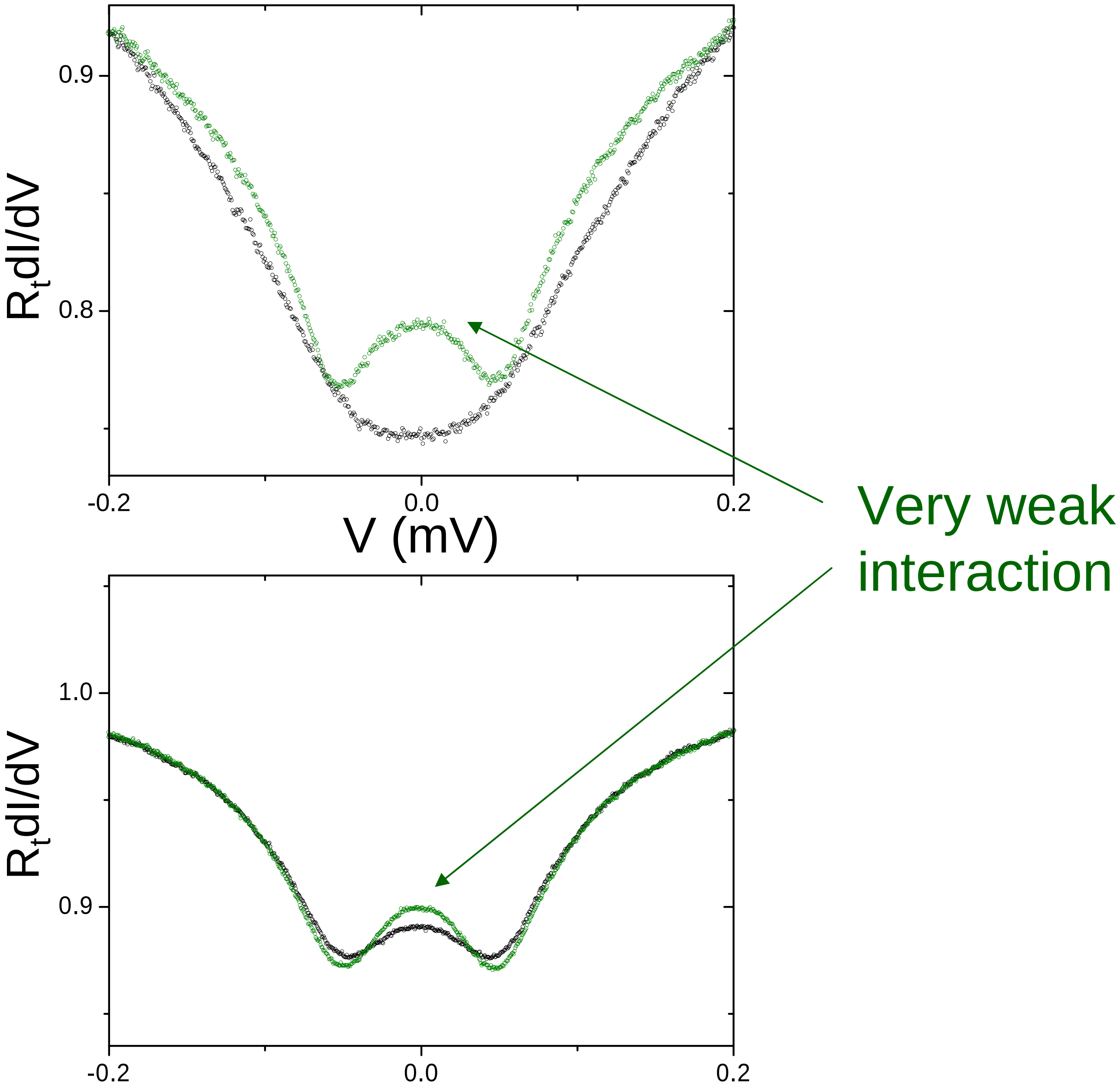
weak interaction

Experimental data at weak and at strong B

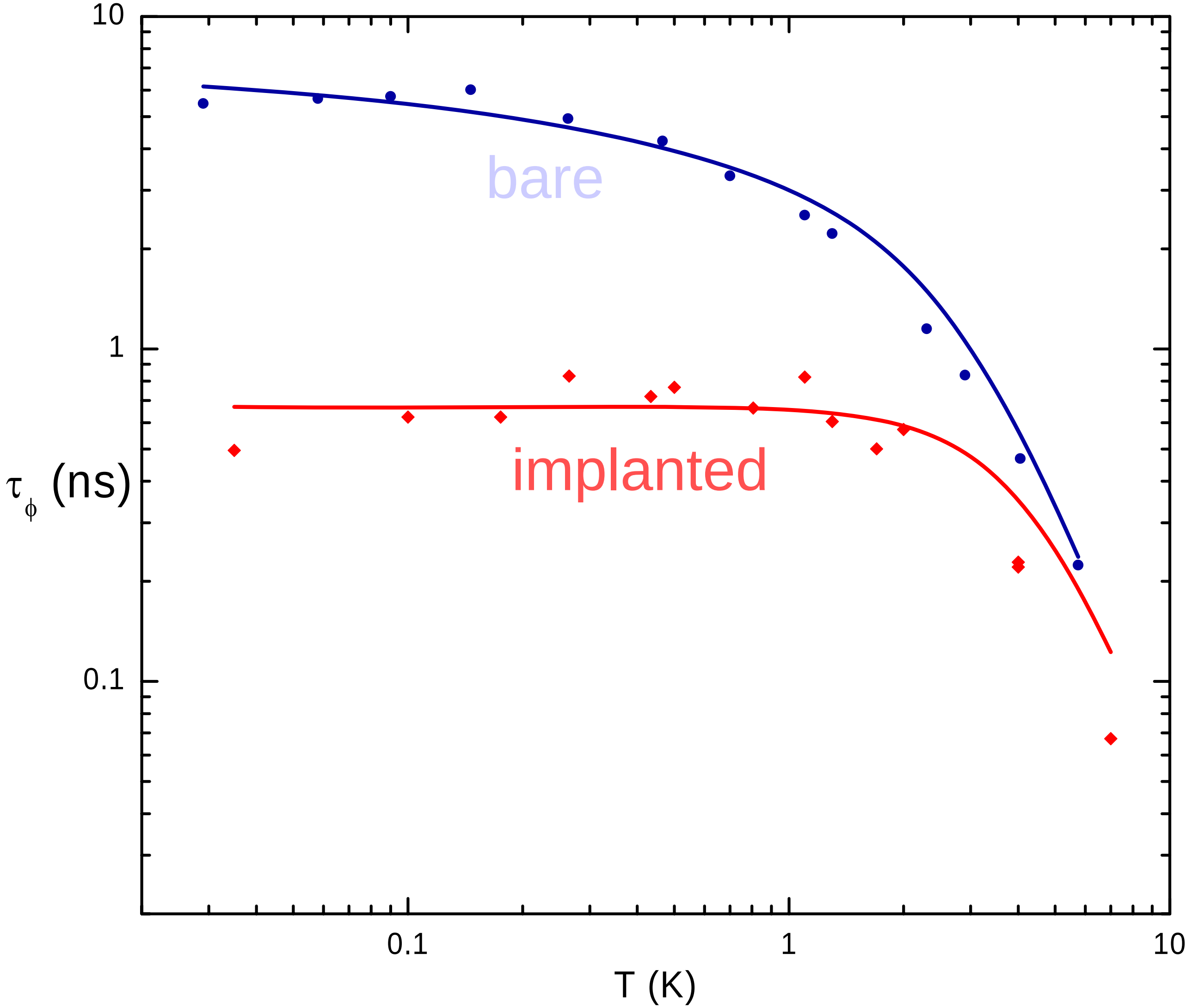
implanted

U = 0.1 mV
B = 0.3 T
B = 2.1 T

bare



Coherence time measurements on the same 2 samples

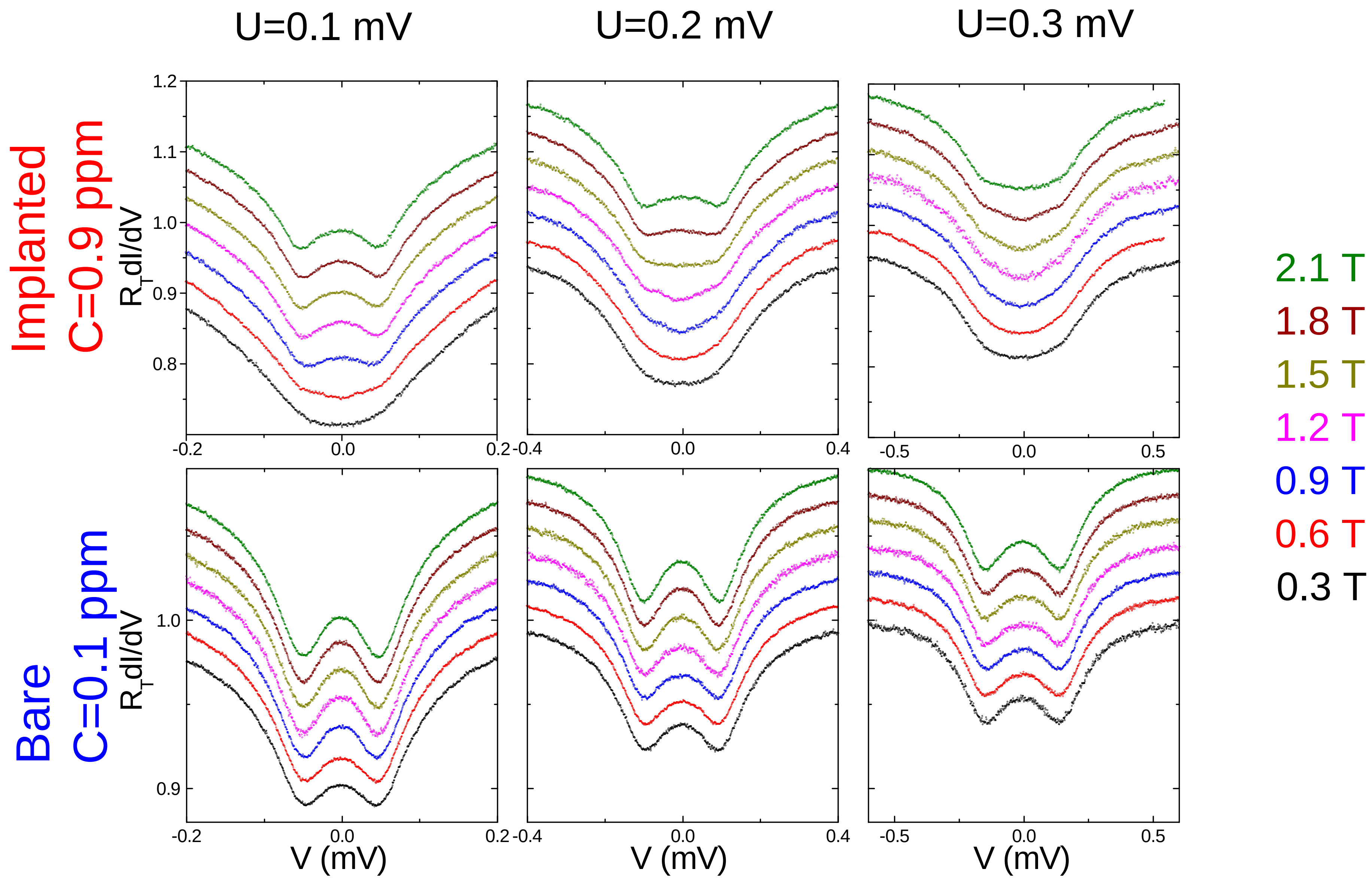


Fits:

C_{bare} =0.1 ppm

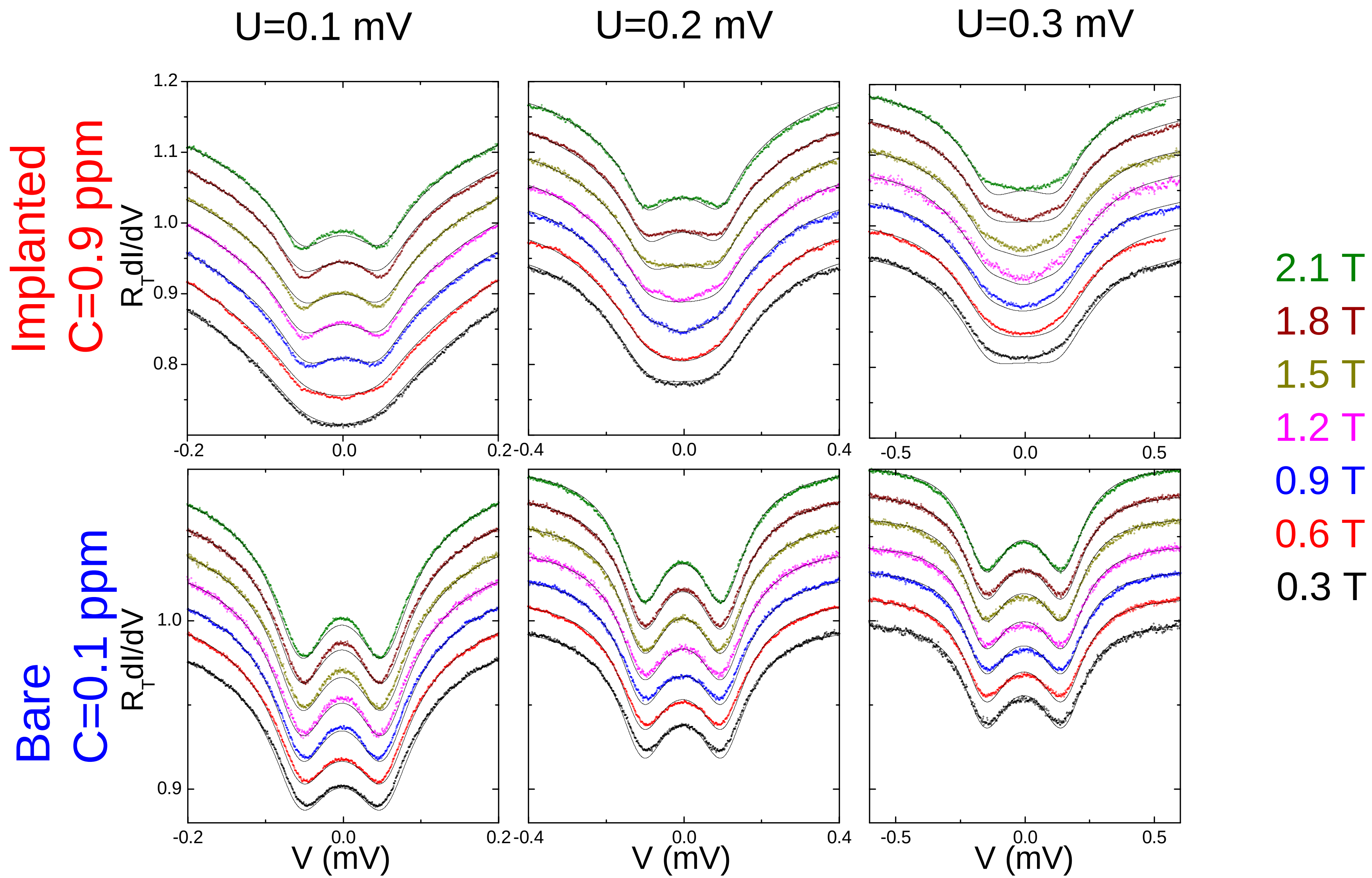
$C_{implanted}$=0.9 ppm

Full U,B dependence



Comparison with theory $\left(S = \frac{1}{2}\right)$

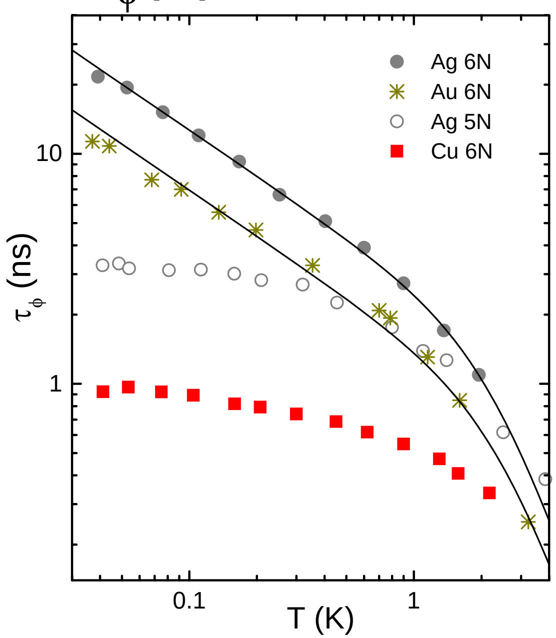
Goeppert, Galperin, Altshuler and Grabert, PRB 64, 033301 (2001)



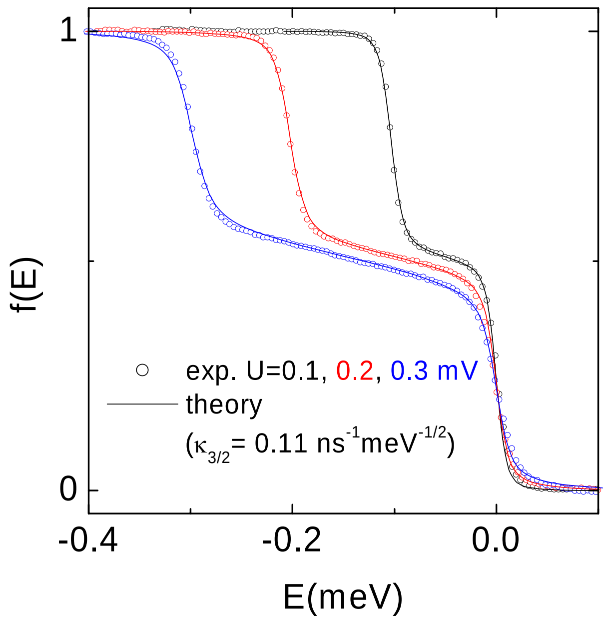
Conclusions – Lectures 2 & 3

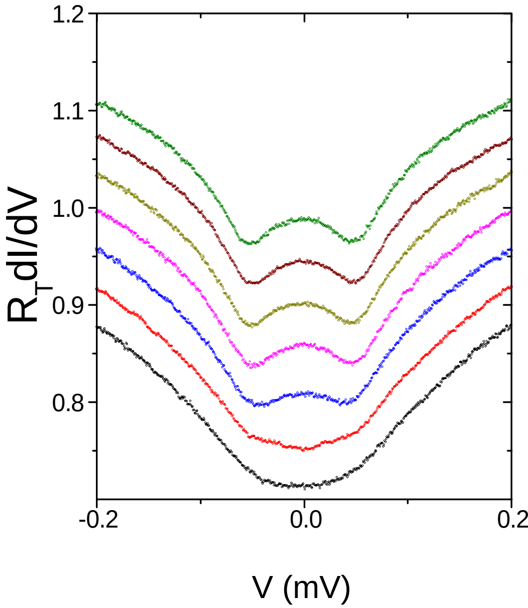
Two methods to investigate interactions in wires

$\tau_\phi(T)$



f(E)

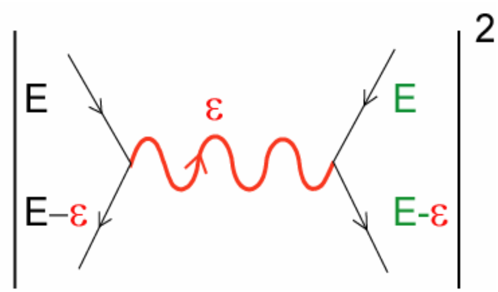




Moral of the story: even at concentrations as low as 1 ppm, magnetic impurities have a large influence on low-temperature electronic transport in metals.

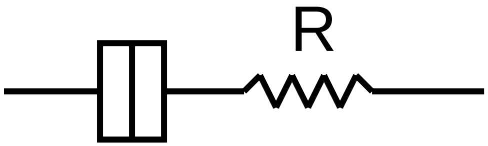
Four consequences of electron-electron interactions in quasi-1D diffusive wires (Altshuler & Aronov)

- loss of phase coherence: $\tau_\phi \sim T^{-2/3}$ (AA+Khmelnitskii)
- energy exchange between quasiparticles:



$\propto \varepsilon^{-3/2}$

- correction to resistance: $\delta R(T) \sim T^{-1/2}$
- correction to tunneling DOS, or dynamic Coulomb blockade: $dI/dV \sim V^{2R/R_Q}$



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- Energy Exchange
  - Pothier, Gueron, Birge, Esteve, Devoret, Phys. Rev. Lett. **79**, 3490 (1997).
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- Both
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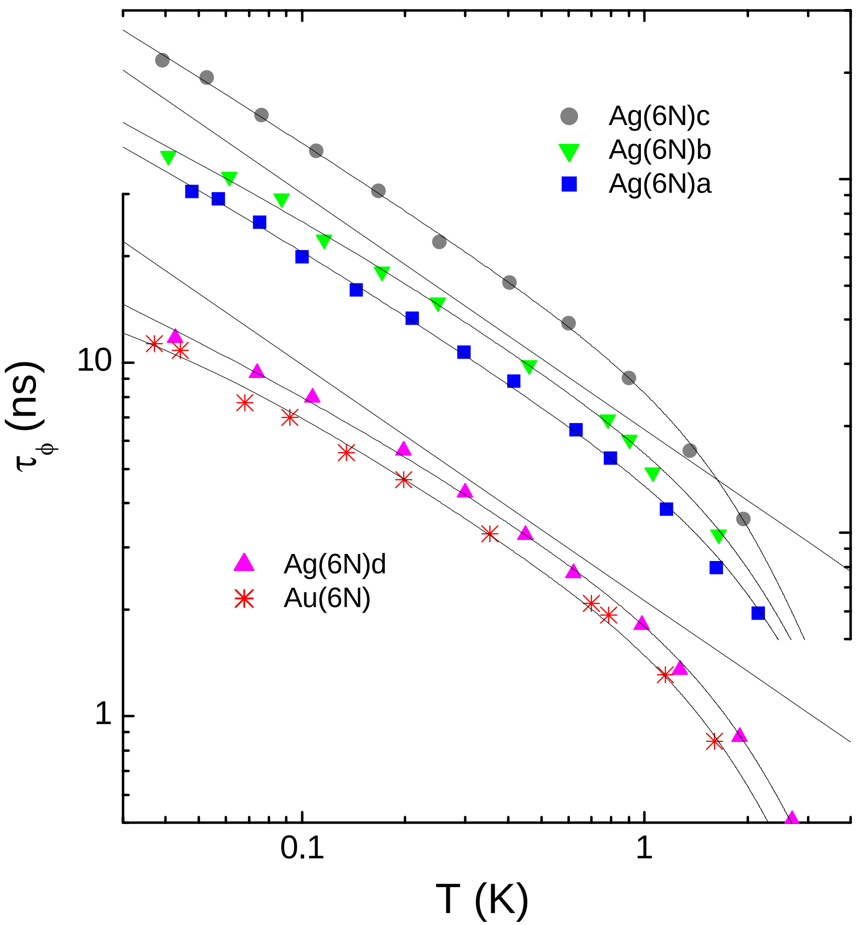
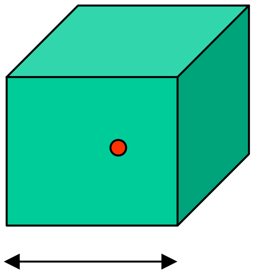
  - Pierre, Ann. Phys. Fr. **26**, N°4 (2001).
  - Huard, Anthore, Birge, Pothier, Esteve, to be published in PRL (2005).

Evidence for extremely dilute magnetic impurities even in purest samples



Sample	Imp.	T_K(K)	c (ppm)
Ag(6N)a	Mn	0.04	0.009
" b	"	"	0.011
" c	"	"	0.0024
" d	"	"	0.012
Au(6N)	Cr	0.01	0.02

1 ppm :



100 atoms ~ 25 nm

In the wire, 0.01 ppm = 3 impurities/µm

Compare τ_ϕ data with AAK and GZS theories