Boulder Summer School 2005 – Lecture #4

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UCF and 1/f noise in Diffusive Metals

Collaborators:

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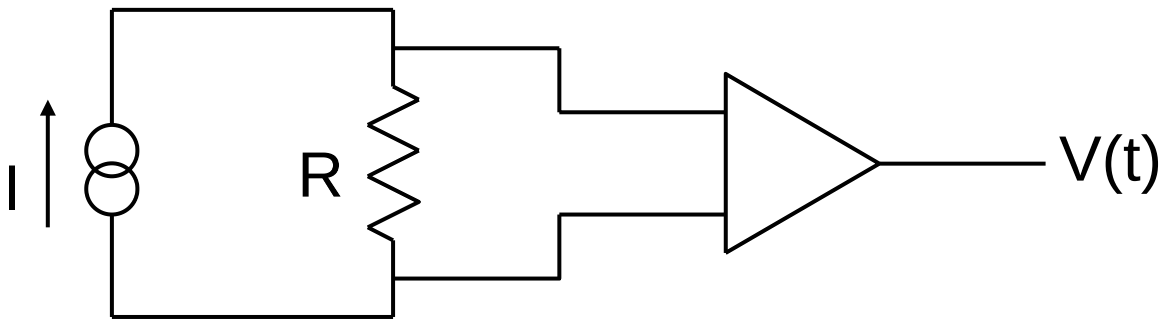
With thanks to A.D. Stone and S. Feng

Supported by NSF DMR

Outline

- 1/f noise in metals - background
- Low-temperature 1/f noise and UCF
- Symmetries and Random Matrix Theory
- 1/f noise vs. B in Bi, Li, and Ag
- Comparison of L_ϕ from WL and UCF
- Appendix: Tunneling systems in disordered solids

1/f noise in metals



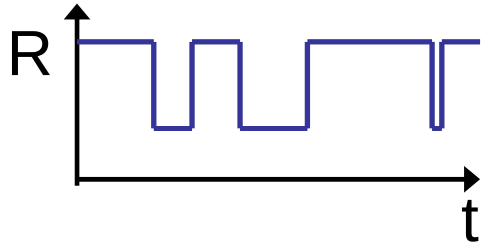
Power spectral density: $$S_V(f) \equiv 4\int_0^\infty \langle V(t)V(0)\rangle \cos(2\pi f t)\,dt$$

Experimental observations: $$S_V(f) = 4k_B TR + I^2 S_R(f)$$

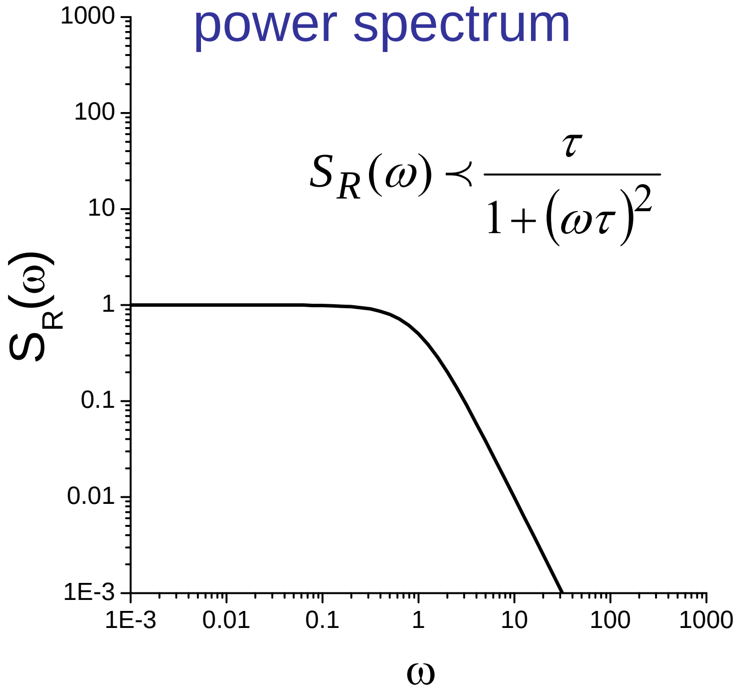
with $S_R(f) \propto \dfrac{1}{f^\alpha}$ and $S_R(f) \propto \dfrac{1}{\text{volume}}$

How to produce a 1/f spectrum

Single two-level fluctuator:

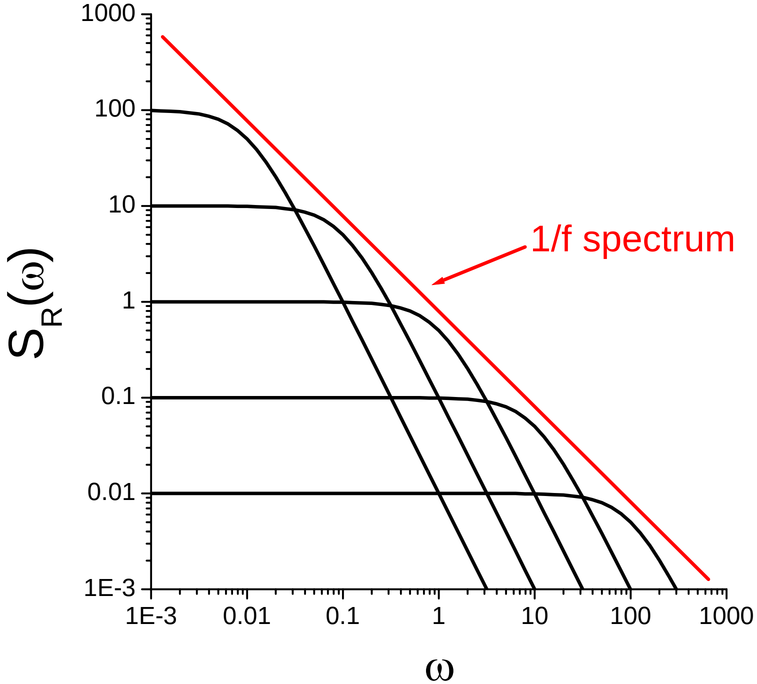


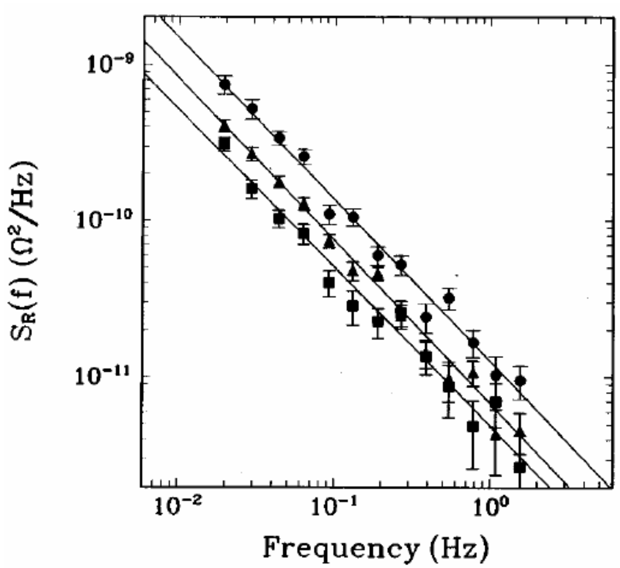
power spectrum



$$S_R(\omega) \prec \frac{\tau}{1+(\omega\tau)^2}$$

Many fluctuators, P(log(τ))~const.

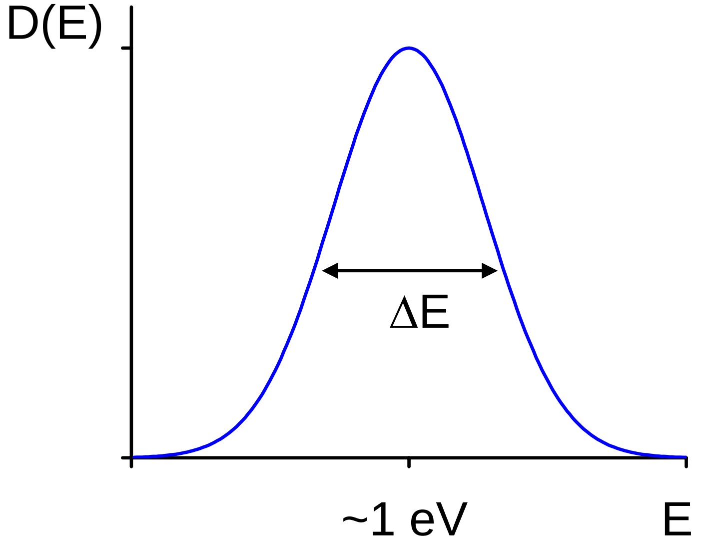




The Dutta-Horn model

Broad distribution of thermally activated processes produces nearly 1/f spectrum:

$$S_R(\omega) = \delta R^2 \int_0^\infty \frac{2\tau}{1+\omega^2\tau^2} D(\tau)\,\mathrm{d}\tau$$



$\Delta E \gg kT \Rightarrow$ distribution is nearly flat on scale of kT

Mobile defects $\Rightarrow$ 1/f noise

Additional evidence:

annealing reduces noise

symmetry properties of noise

correlates with low resistivity ratio

Limitations of Dutta-Horn model:

assumes T-independent coupling of defect motion to resistance

“local interference” model

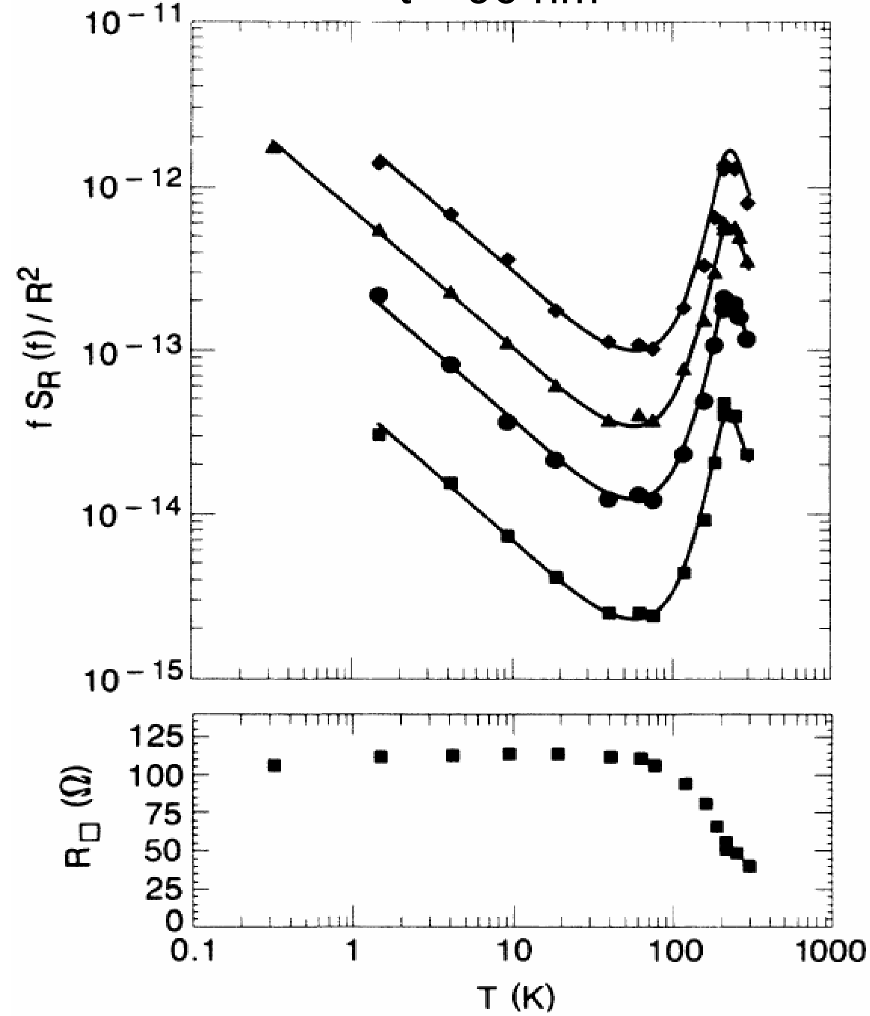
1/f noise vs. T in Bi films

Birge, Golding, Haemmerle, PRL 62, 195 (1989) & PRB 42, 2735 (1990).

Bi low carrier density ⇒ high resistivity

1/f noise grows
as T decreases!

t = 90 nm



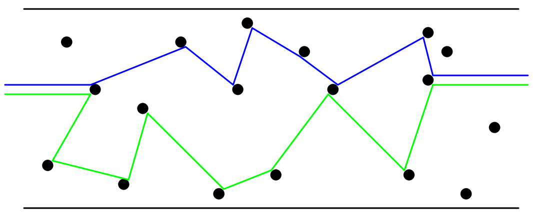
Why does 1/f noise grow at low T?

density of mobile defects decreases as T drops

$\Longrightarrow$ coupling of defect motion to resistance must increase

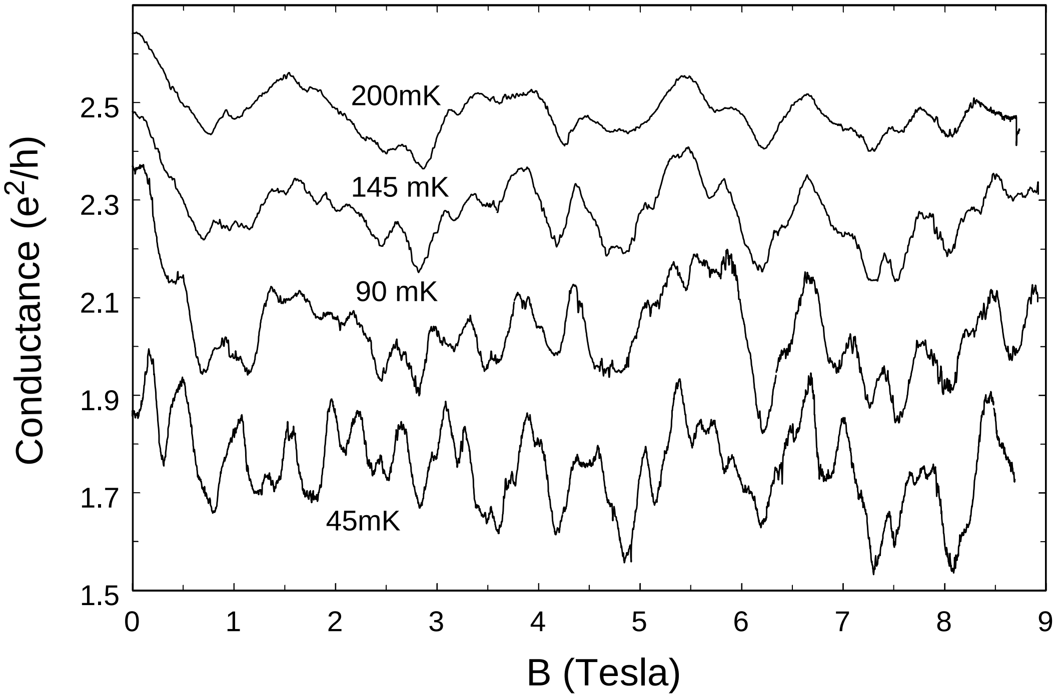
not local interference

Recall from previous lectures: Universal conductance fluctuations (UCF) in wires

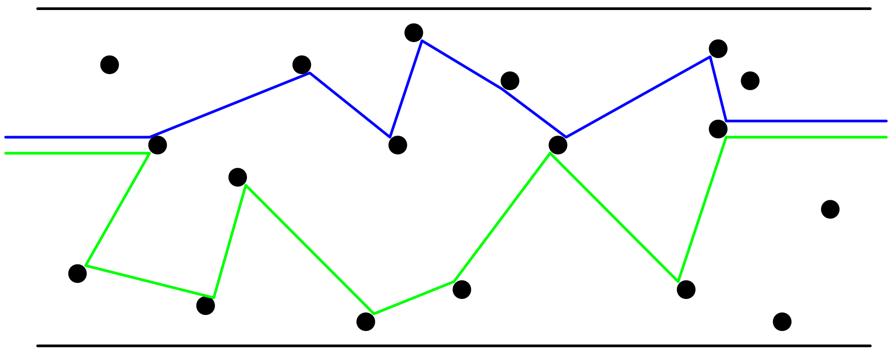


G varies randomly with B or E_F

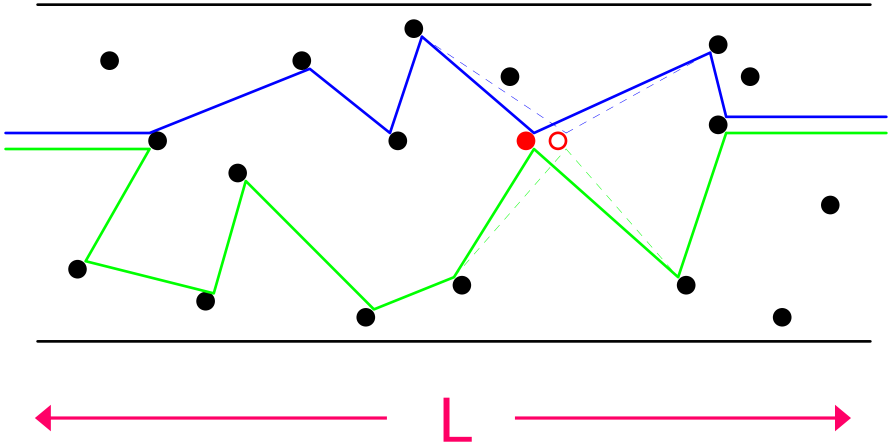
$\delta G \approx e^2/h$ for $L < L_\phi$



UCF “magnetofingerprint” depends on exact positions of scatterers



UCF "magnetofingerprint" depends on exact positions of scatterers



Conductance change $\delta G \approx \frac{e^2}{h}$ for phase-coherent sample

with L < L_ϕ (phase coherence length)

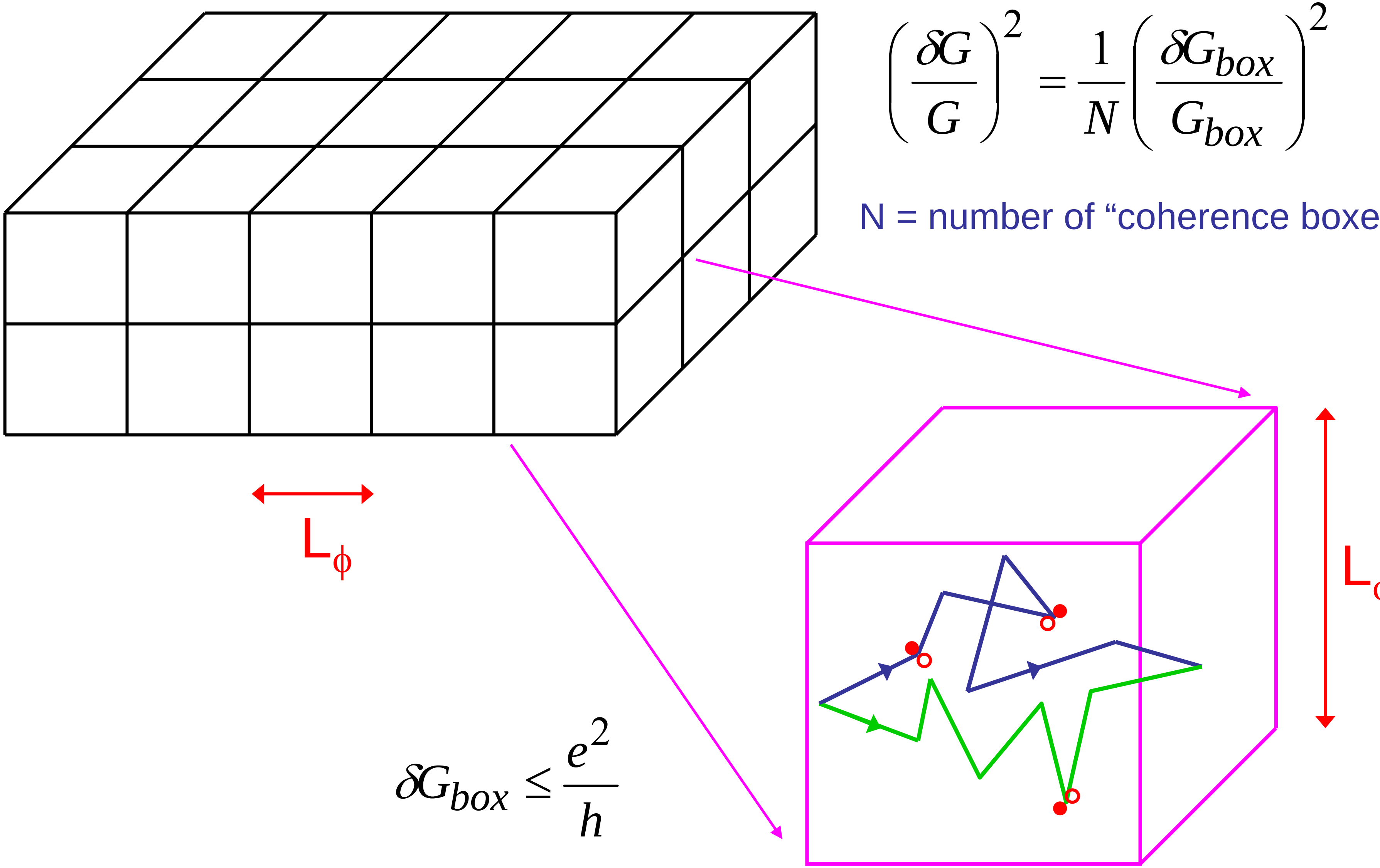
Altshuler & Spivak, JETP Lett. 42, 447 (1985)

Feng, Lee, & Stone, PRL 56, 1960 (1988)

UCF in macroscopic samples ($L>L_\phi$):

$$\left(\frac{\delta G}{G}\right)^2 = \frac{1}{N}\left(\frac{\delta G_{box}}{G_{box}}\right)^2$$

N = number of “coherence boxes”

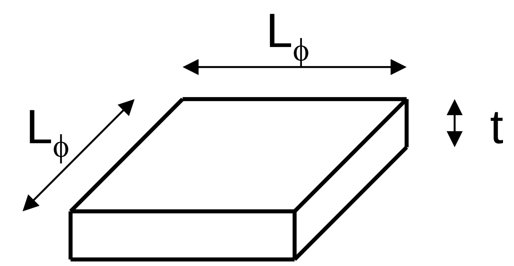


$$\delta G_{box} \le \frac{e^2}{h}$$

Temperature dependence of “UCF noise”

Feng, Lee & Stone, PRL 56, 1960 (1986)

(quasi-2D case)



1. Conductance change in “coherence volume”

motion of single defect: $\delta G_1^2 \approx \frac{1}{\beta}\left(\frac{e^2}{h}\right)^2 \frac{1}{(k_F l_e)^2}\alpha(k_F\delta r)\left(\frac{L}{l_e}\right)^{2-d}$ ← $\frac{l_e}{t}$

$\alpha(x) = 1 - \frac{\sin^2(x/2)}{(x/2)^2}$

multiply by number of defects in box: $n_S(T)\times L_\phi^2\, t$

energy averaging: If $k_B T > \frac{\hbar}{\tau_\phi}$, then $\times\frac{\hbar\tau_\phi^{-1}}{k_B T} \equiv \frac{L_T^2}{L_\phi^2}$

$L_{\min} \equiv \min\{L_\phi, L_T\}$

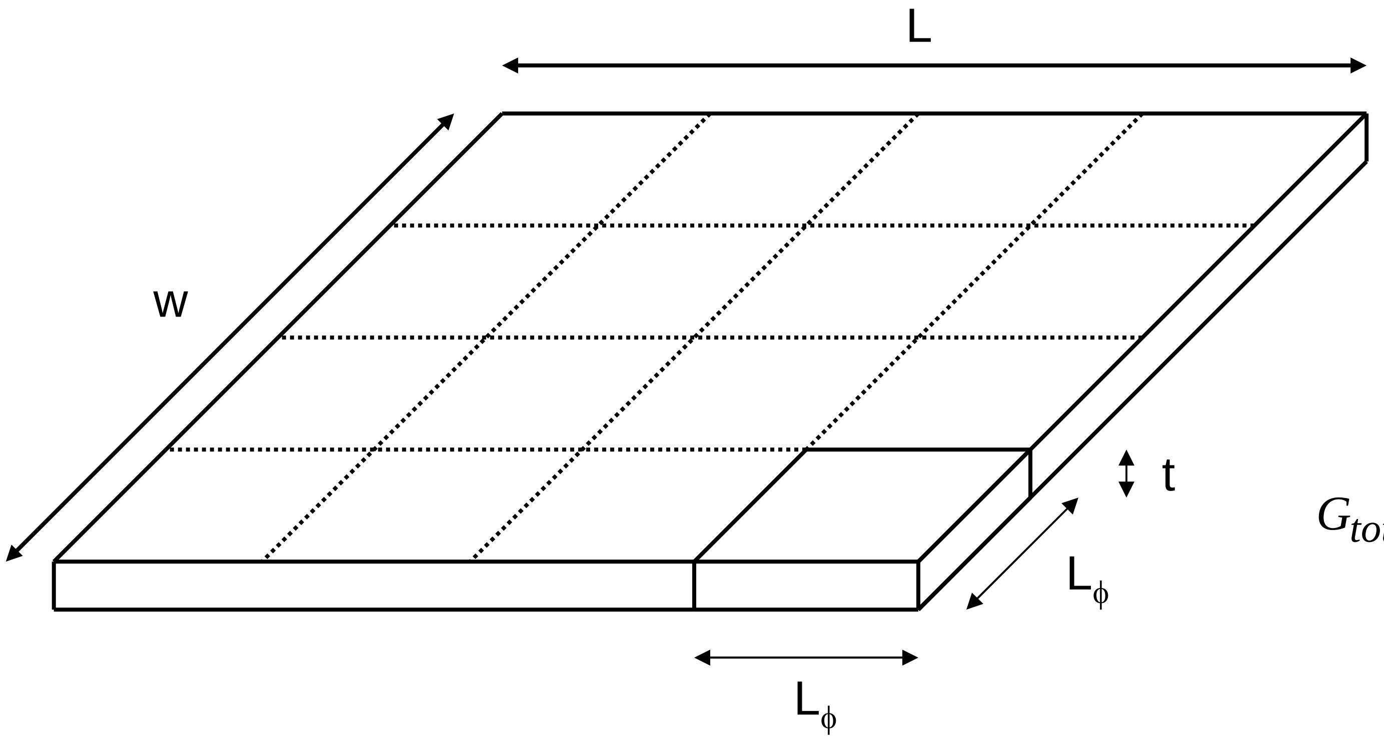
Total conductance fluctuation in box : $\delta G_{box}^2 \approx \frac{1}{\beta}\left(\frac{e^2}{h}\right)^2 \frac{1}{(k_F l_e)^2}\alpha(k_F\delta r)\, l_e\, L_{\min}^2\, n_S(T)$

Temperature dependence of “UCF noise”

Feng, Lee & Stone, PRL 56, 1960 (1986)

2. Conductance fluctuation in entire sample

$$\left(\frac{\delta G_{total}}{G_{total}}\right)^2 = \frac{1}{N_{boxes}}\left(\frac{\delta G_{box}}{G_{box}}\right)^2$$



$$N_{boxes} = \frac{L_\phi^2}{Lw}$$

$$G_{total} = \frac{w}{L} G_{box}$$

$$\delta G_{total}^2 \approx \frac{1}{\beta}\left(\frac{e^2}{h}\right)^2 \frac{1}{(k_F l_e)^2}\alpha(k_F \delta r)\frac{L_\phi^2 w}{L^3} l_e L_{\min}^2 n_s(T)$$

Temperature dependence of “UCF noise”

Feng, Lee & Stone, PRL 56, 1960 (1986)

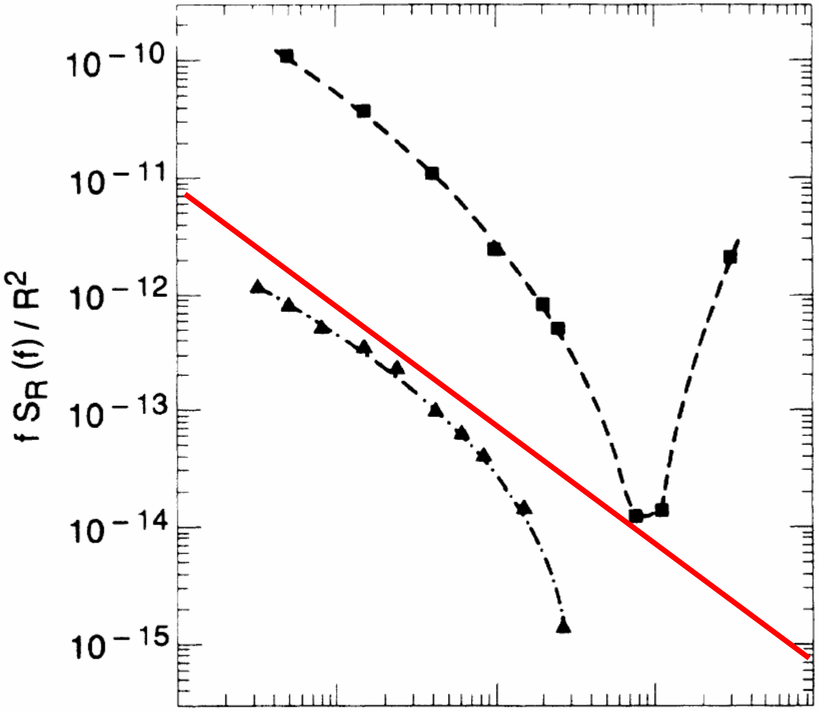
$$\delta G_{total}^2 \approx \frac{1}{\beta}\left(\frac{e^2}{h}\right)^2 \frac{1}{(k_F l_e)^2}\alpha(k_F \delta r)\frac{L_\phi^2 w}{L^3} l_e L_{\min}^2 n_s(T)$$

3. Tunneling model of disordered solids: $n_s(T) \propto T$

4. Dephasing length in quasi-2D: $L_\phi \propto T^{-1/2}$

Final answer: $\delta G_{total}^2 \propto L_\phi^2 L_{\min}^2 n_s(T) \propto \frac{1}{T}$

Bi, t=11 nm



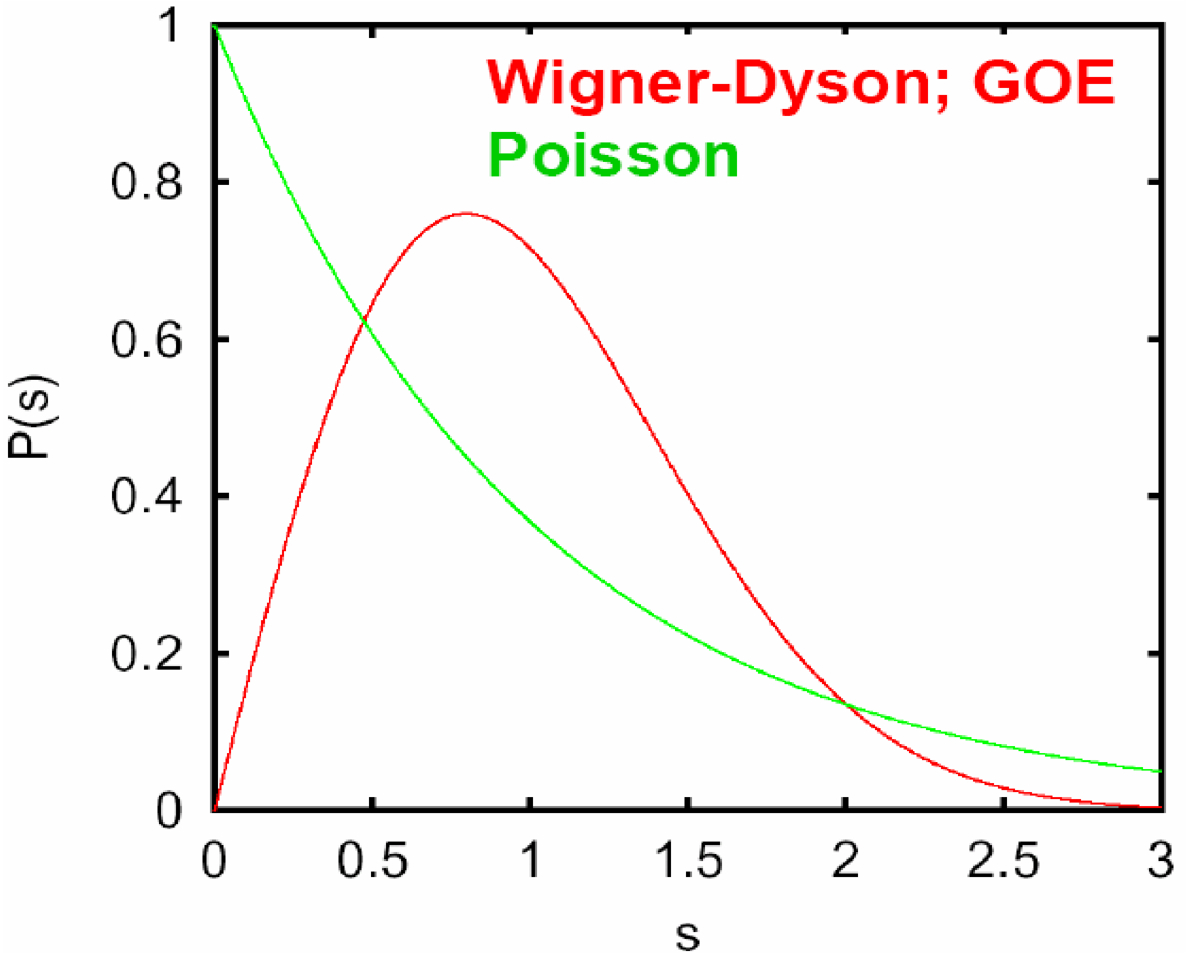
Symmetries and UCF: Random Matrix Theory

-- eigenvalues of random matrices do not obey Poisson statistics

-- eigenvalues exhibit level repulsion

Poisson: uncorrelated levels

Gaussian Orthogonal Ensemble



Wigner-Dyson ensembles
(table courtesy of Boris Altshuler):

Matrix elements	Ensemble	β	Realization
Real	Orthogonal	1	Time-reversal invariant
Complex	Unitary	2	Broken time-reversal invariance (e.g. by B field)
2x2 matrices??	Symplectic	4	T-invariant but with spin-orbit scattering

RMT and the UCF variance

Altshuler & Shklovskii, Soviet Physics JETP 64, 127 (1986).

$$\delta G_1^2 \propto \left(\frac{e^2}{h}\right)^2 \frac{ks^2}{\beta}$$

k = number of independent eigenvalue sequences; s = level degeneracy

	B=0	B > B_c
Low spin-orbit scattering	β=1, k=1, s=2 (GOE)	β=2, k=1, s=2 (GUE)
High spin-orbit scattering	β=4, k=1, s=2 (GSE)	β=2, k=1, s=1 (GUE + spin)

Noise vs. B in Bi

Birge, Golding, Haemmerle, PRL 62, 195 (1989) & PRB 42, 2735 (1990).

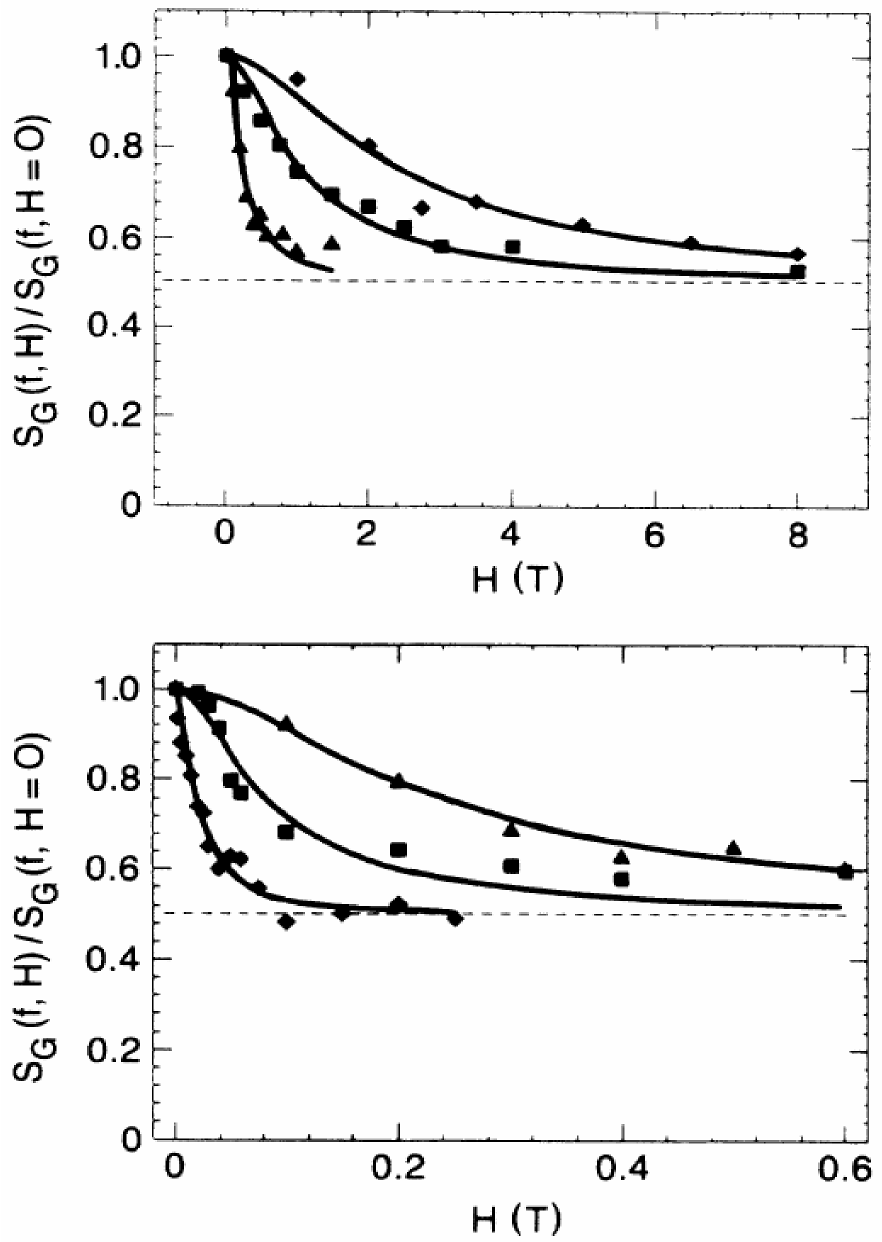
Noise drops by a factor of 2 in applied field $B > B_c = (h/e)/L_\phi^2$

Confirms connection to UCF:

$$\delta G_1^2 \propto \left(\frac{e^2}{h}\right)^2 \frac{ks^2}{\beta}$$

Noise crossover function (Stone):

$$\frac{S_G(f,B)}{S_G(f,0)} = \frac{-1}{2b^2}\Psi''\left[\frac{1}{2}+\frac{1}{b}\right] \qquad b = \frac{8\pi B L_\phi^2}{\Phi_0}$$

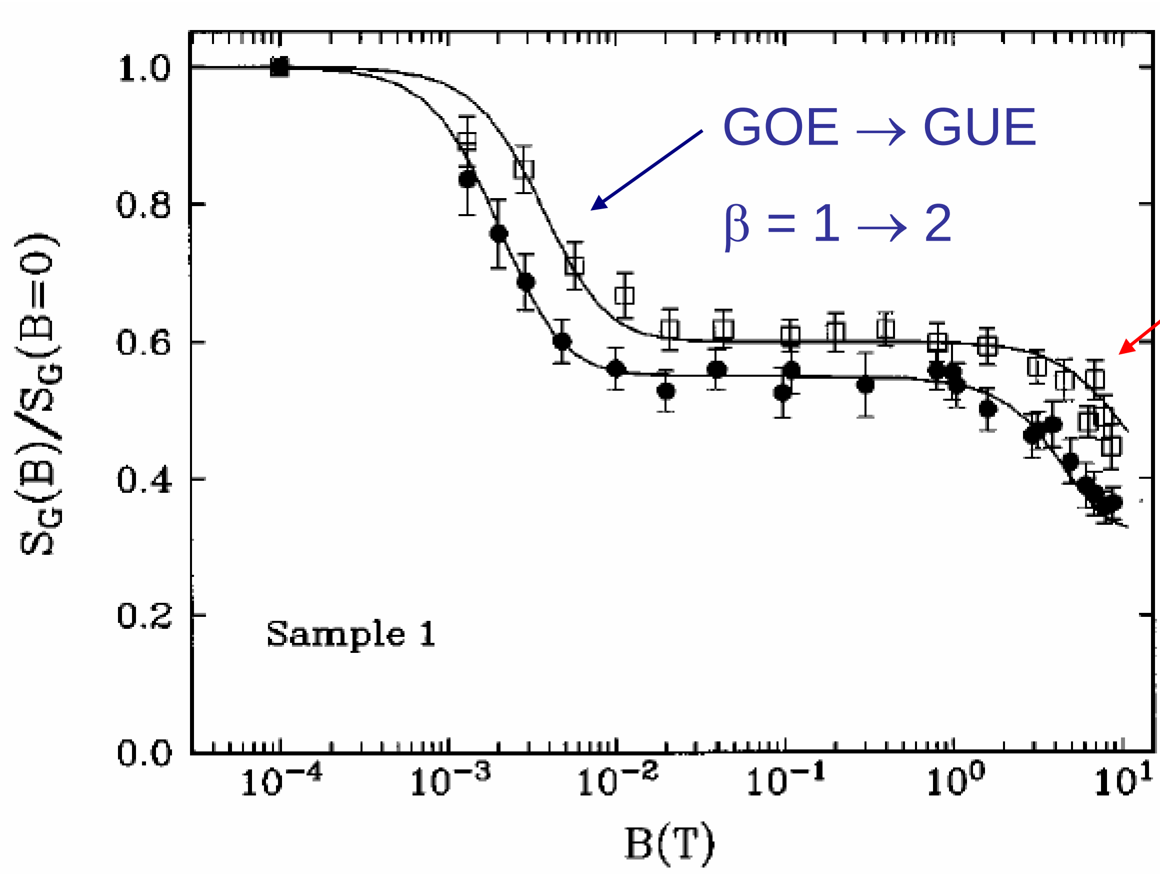


Other fun games with UCF and 1/f noise: Li

Moon, Birge & Golding, PRB 53, R4193 (1996) & 56, 15124 (1997).

$$\delta G_1^2 \propto \left(\frac{e^2}{h}\right)^2 \frac{ks^2}{\beta}$$

Li has low spin-orbit scattering ⇒ β=1 at B=0



GOE → GUE

β = 1 → 2

Break spin-degeneracy when $g\mu_B B > kT$

s = 2 → 1

k = 1 → 2

β = 2

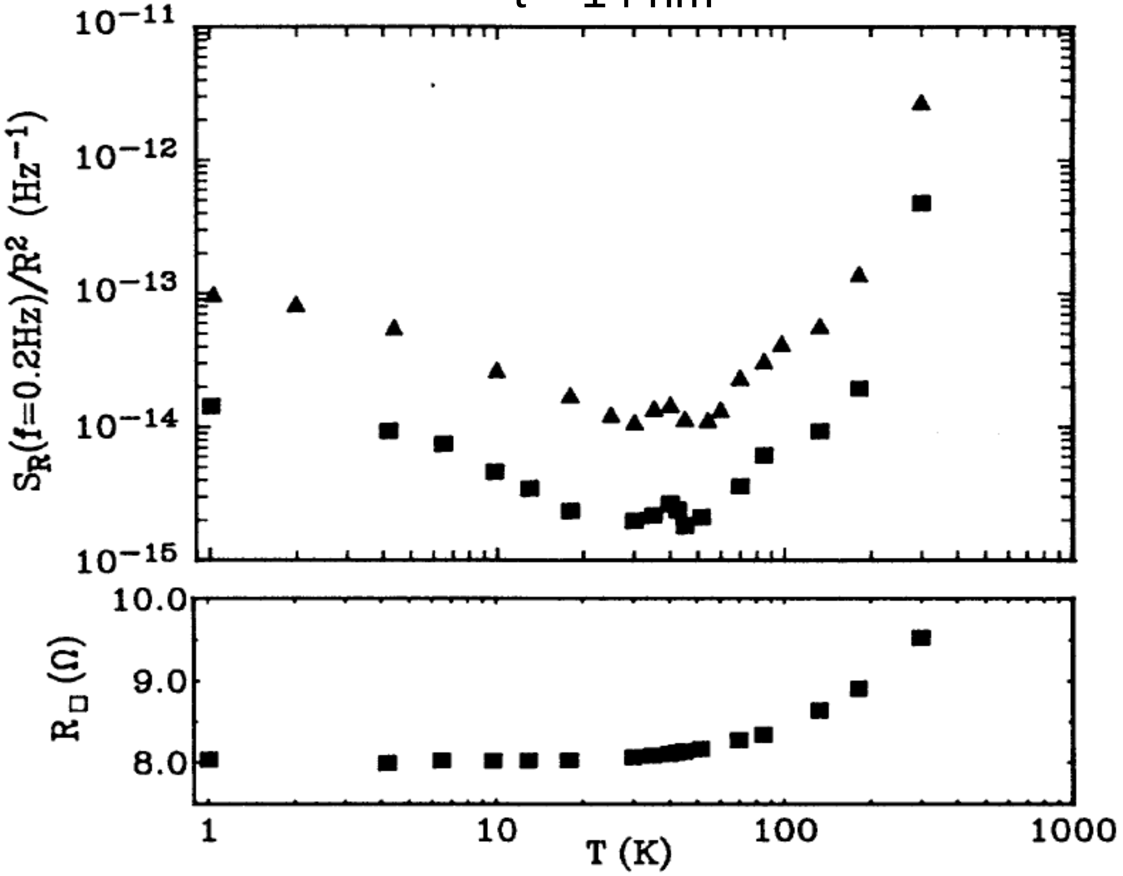
Can we obtain quantitative estimate of L_ϕ from 1/f noise vs. B?

Bi exhibits superconducting fluctuations at low T;

try Ag

Noise vs. T in Ag

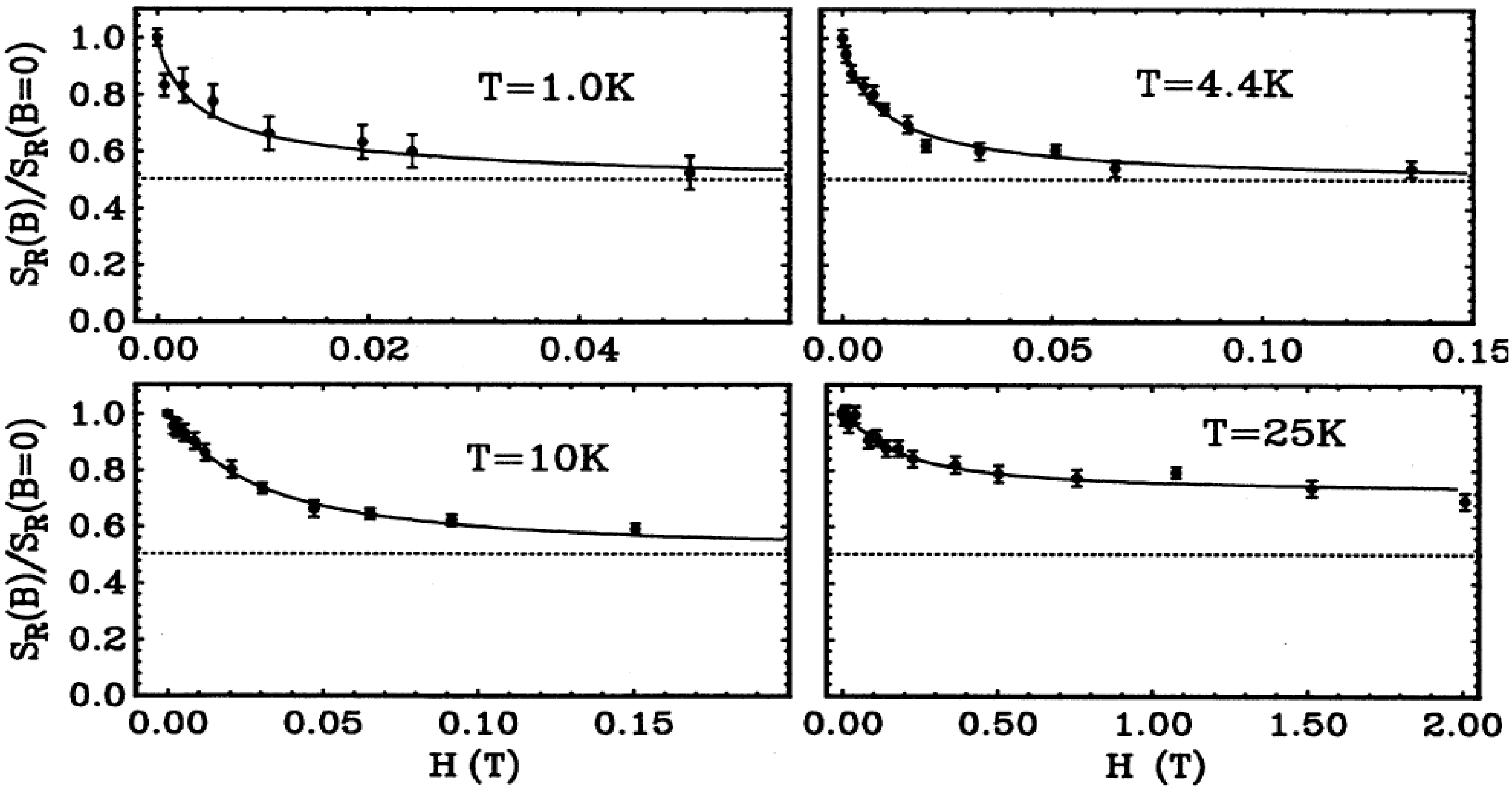
t = 14 nm



Noise vs. B in Ag

McConville and Birge, PRB 47, 16667 (1993).

(crossover function with help from D. Stone)



Noise crossover function with spin & SO

Altshuler & Spivak, JETP Lett. 42, 447(1985); Stone, PRB 39, 10736 (1989).

UCF correlation function, T=0

$$F_0(\Delta E,\Delta B,B)\equiv\langle\delta G(E_F,B)\delta G(E_F+\Delta E,B+\Delta B)\rangle$$

Response to small chance in impurity potential, T>0

$$\delta G'^2(B,T)=-\frac{4s^2}{\pi^2}\int\frac{d(\Delta E)}{2k_BT}K\left(\frac{\Delta E}{2k_BT}\right)\frac{d}{d(1/\tau_\phi)}\left(F_0(\Delta E,B)\right)$$

$$K(x)=(x\coth x-1)/\sinh^2 x$$

Incorporate spin effects: spin-orbit scattering, Zeeman splitting

$$\delta G'^2=\left[\tfrac{1}{4}\left(\delta G_s'(B)\right)^2+\tfrac{3}{4}\left(\delta G_t'(B,L_{so})\right)^2\right]_{Cooperon}+\left[\tfrac{1}{4}\left(\delta G_s'\right)^2+\tfrac{1}{4}\sum_{M_z}\left(\delta G_t'(M_z g\mu_B B,L_{so})\right)^2\right]_{Diffuson}$$

Noise crossover function:

$$\nu(B,T)=\frac{\delta G'^2(B,T)}{\delta G'^2(B=0,T)}$$

Weak Localization magnetoresistance

McConville and Birge, PRB 47, 16667 (1993).

$$\sigma(B) = -\frac{e^2}{2\pi^2\hbar}\left[\Psi\left(\frac{1}{2}+\frac{B_1}{B}\right)+\frac{1}{2}\Psi\left(\frac{1}{2}+\frac{B_2}{B}\right)-\frac{3}{2}\Psi\left(\frac{1}{2}+\frac{B_3}{B}\right)\right]$$

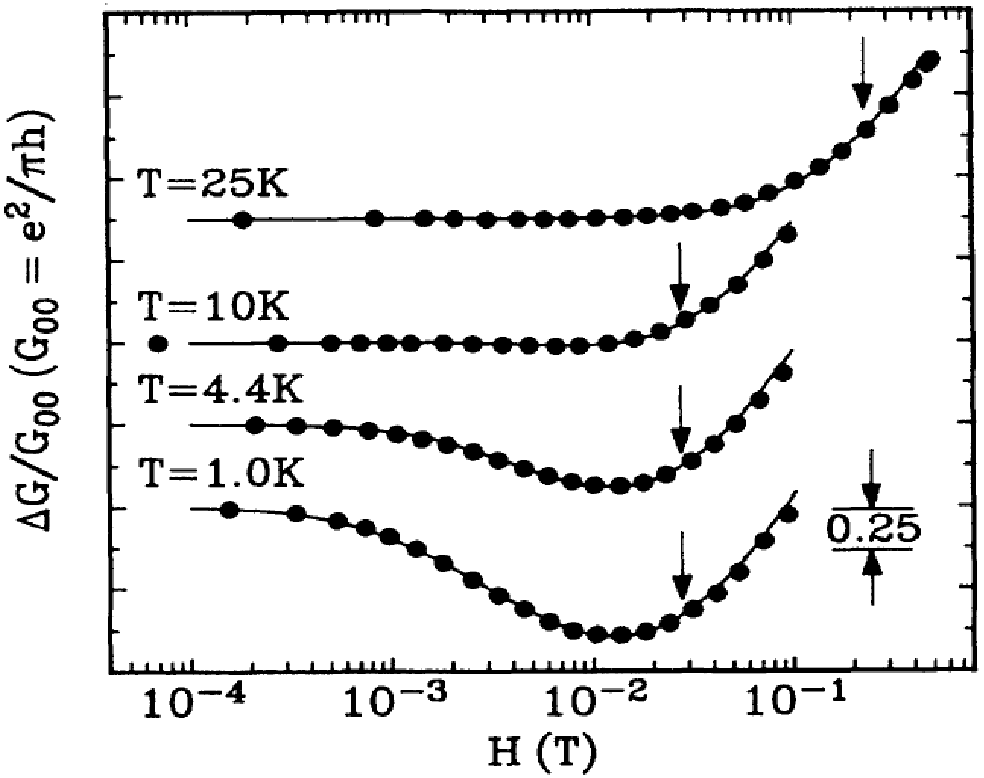
$$B_1 = B_0 + B_{SO}$$

$$B_2 = B_\phi$$

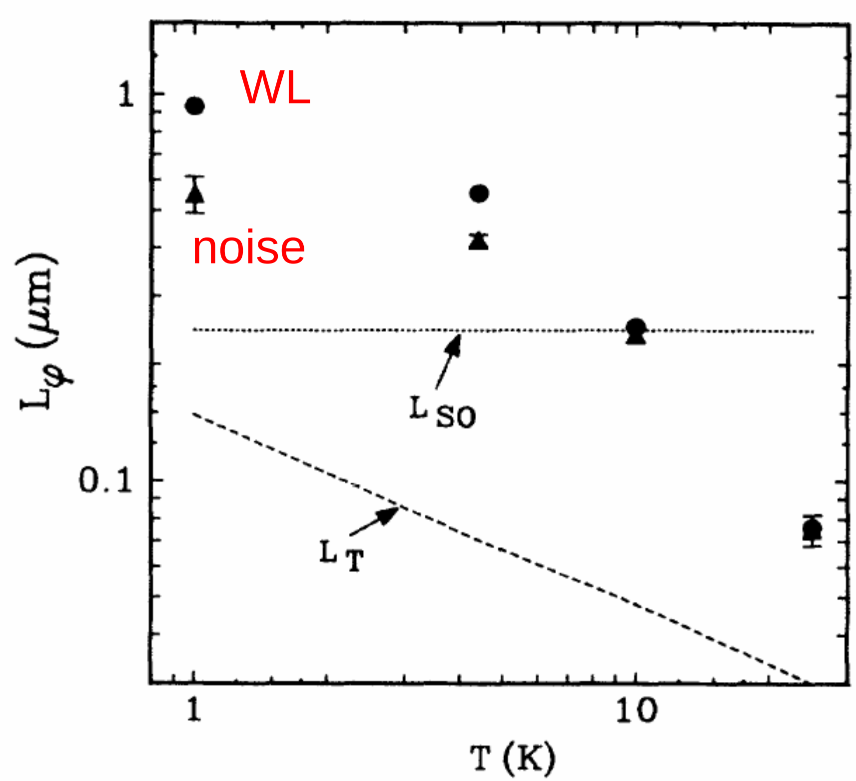
$$B_3 = B_\phi + \frac{4}{3}B_{SO}$$

$$B_0 = \frac{3\hbar}{4eD\tau_e}$$

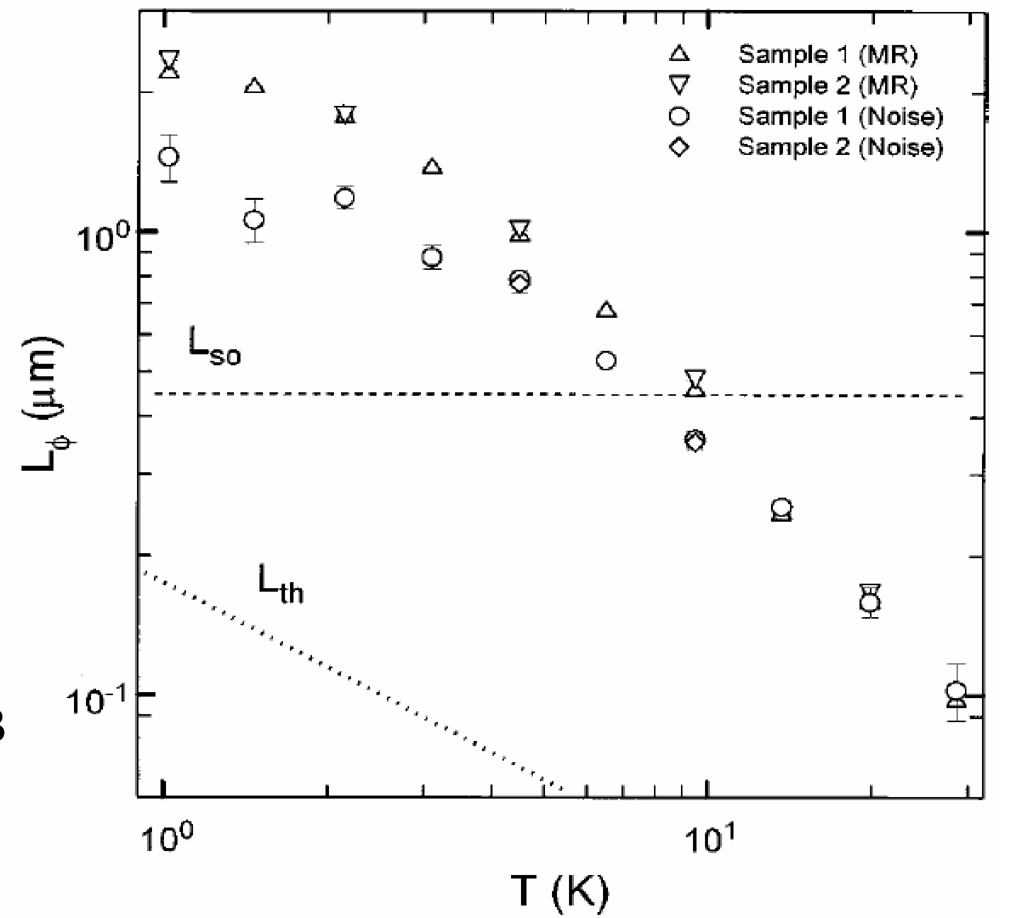
$$B_x = \frac{\hbar}{4eD\tau_x} = \frac{h/e}{8\pi L_x^2}$$



Comparison of L_ϕ from WL and UCF



McConville and Birge, PRB 47, 16667 (1993).



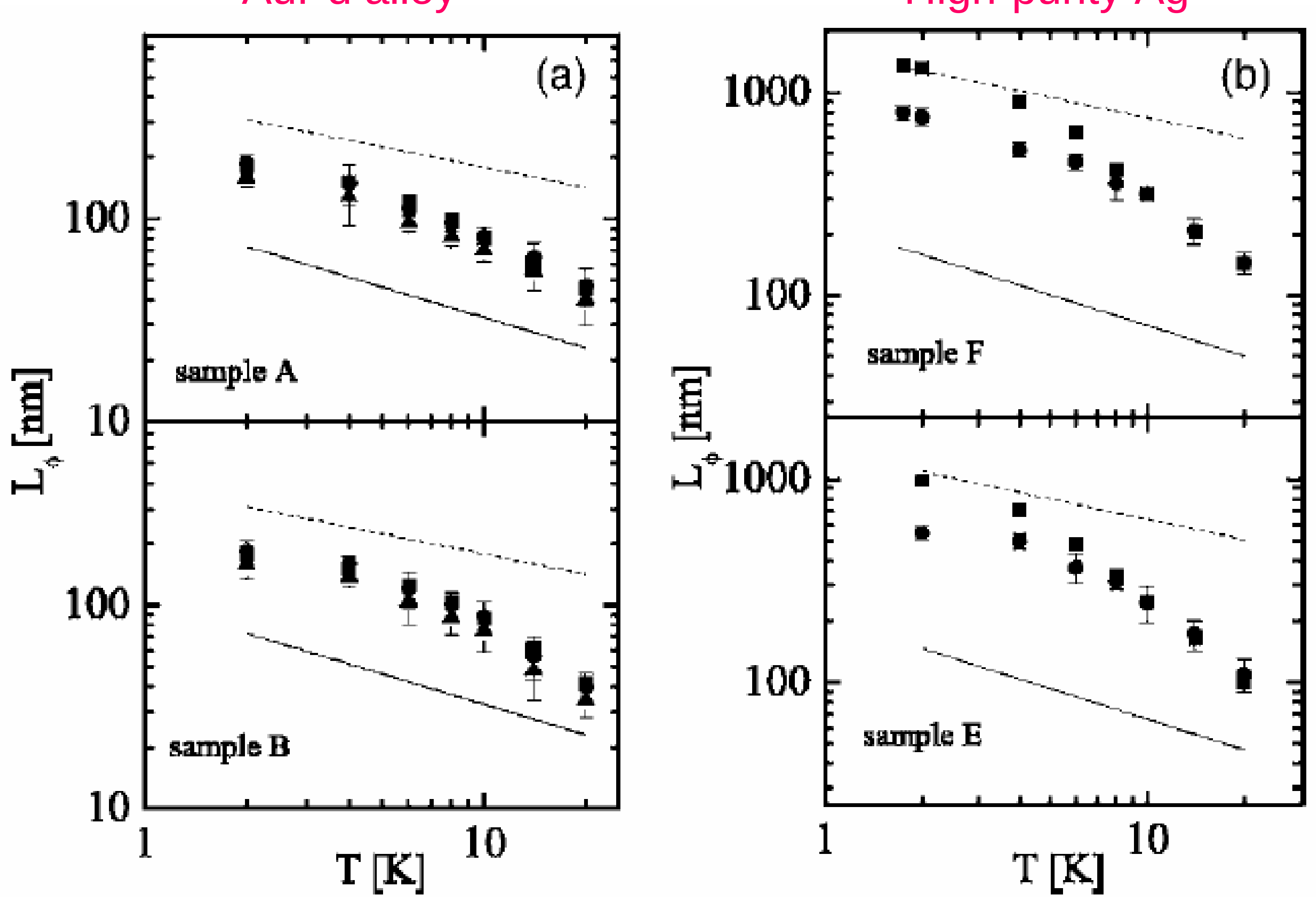
Hoadley, McConville and Birge, PRB 60, 5617 (1999).

Epilogue

Trionfi, Lee, and Natelson, PRB 72, 035407 (2005).

AuPd alloy

High-purity Ag

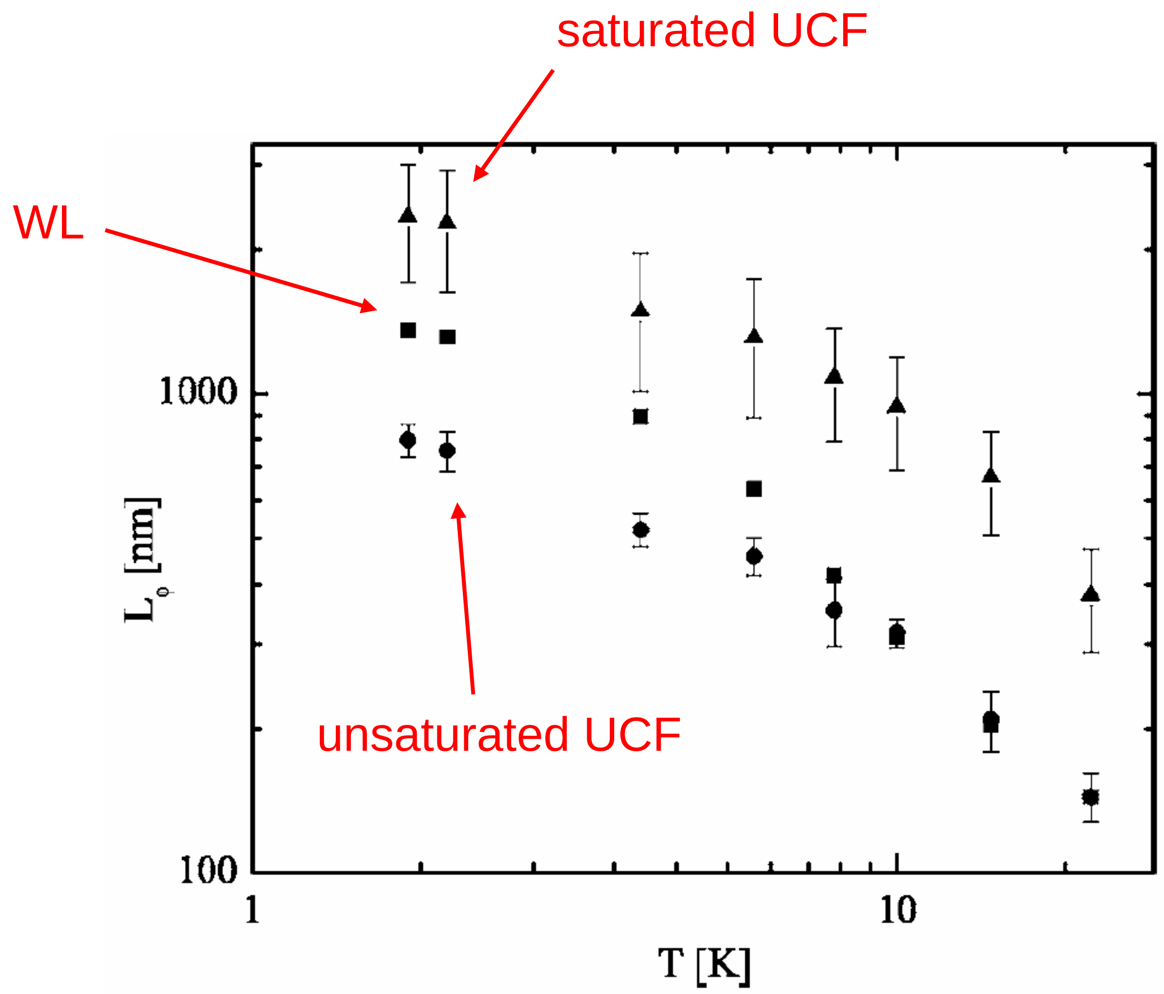


Why don't L_ϕ^{WL} and L_ϕ^{UCF} agree at low T in Ag?

- Crossover to strong spin-orbit scattering
  - But AuPd data is even stronger!
- Noise measurements are out of equilibrium?
  - Noise vs. B unchanged with drive current

A proposal to explain the discrepancy in Ag

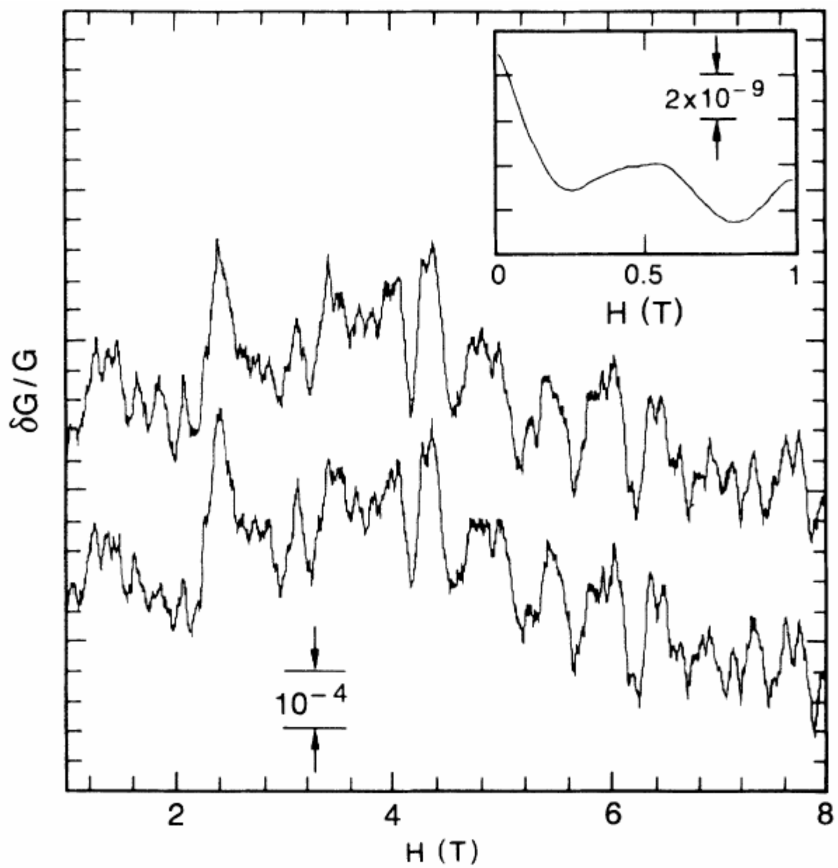
Trionfi, Lee, and Natelson, PRB 72, 035407 (2005).



Does the 1/f noise saturate UCF?

g(ln(t) = const. for $\tau_{min} < \tau < \tau_{max} \Rightarrow \quad \delta G^2 \approx \omega S_G(\omega) \ln\left(\frac{\tau_{max}}{\tau_{min}}\right)$

UCF "magnetofingerprint"



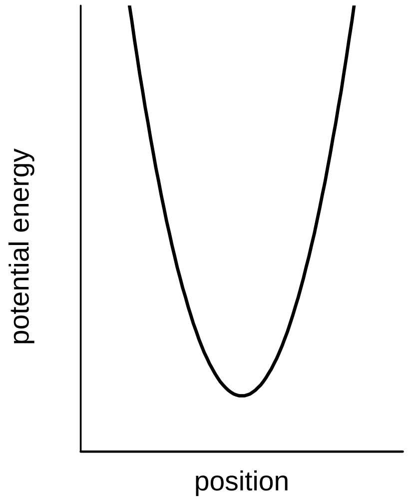
It would take ~ 200 decades of 1/f noise at the level measured to saturate the UCF!

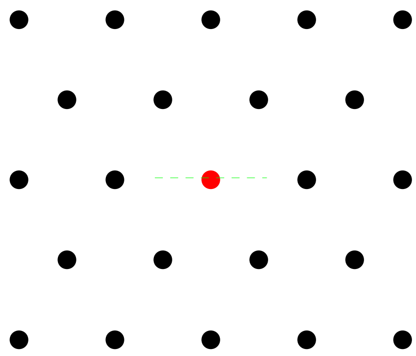
Summary

- 1/f noise in metals comes from defect motion
- 1/f noise is enhanced at low T due to long-range quantum interference (UCF)
- Noise vs. B reveals RMT crossovers (GOE$\rightarrow$GUE, Zeeman splitting, etc.)
- Discrepancy in L_ϕ determined from WL and UCF
  - Maybe due to crossover from unsaturated to saturated UCF

Appendix: Tunneling systems in disordered (insulating) solids

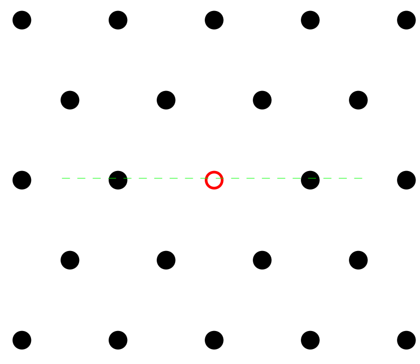
I. Crystalline Solids



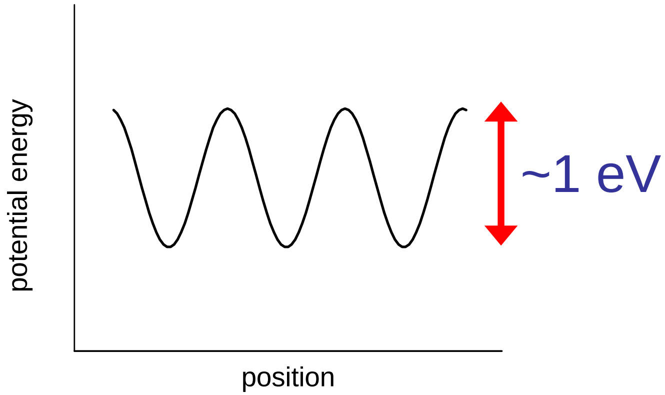


“Every atom knows its place”

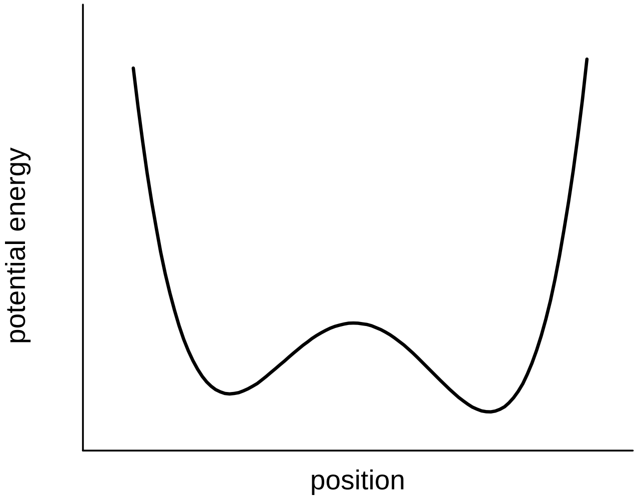
vacancy

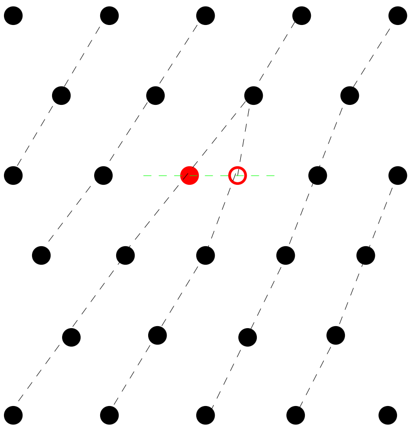


Point defects are not mobile at low temperature (1eV≅10,000K)



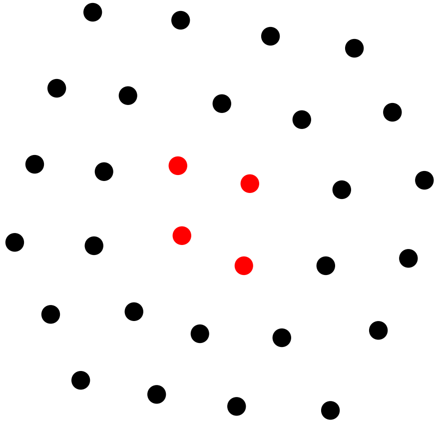
II. Disordered Solids



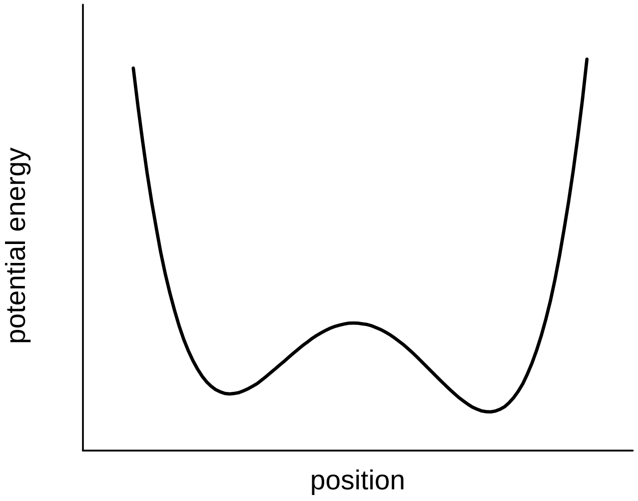


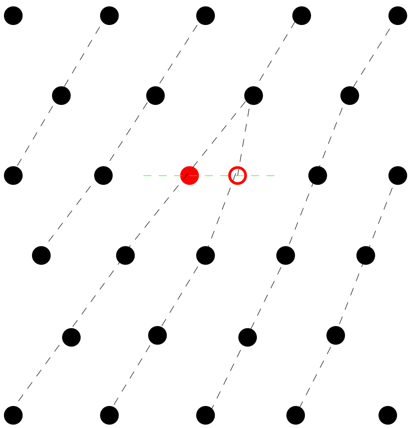
dislocation

More disorder $\Rightarrow$ more low barriers



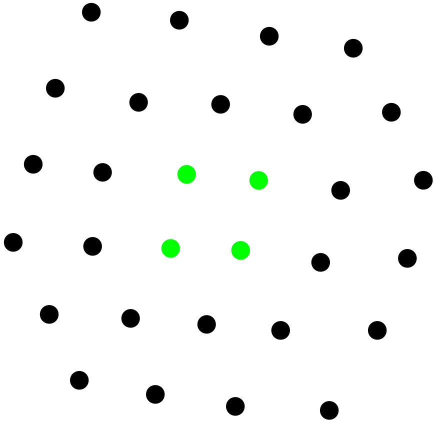
II. Disordered Solids



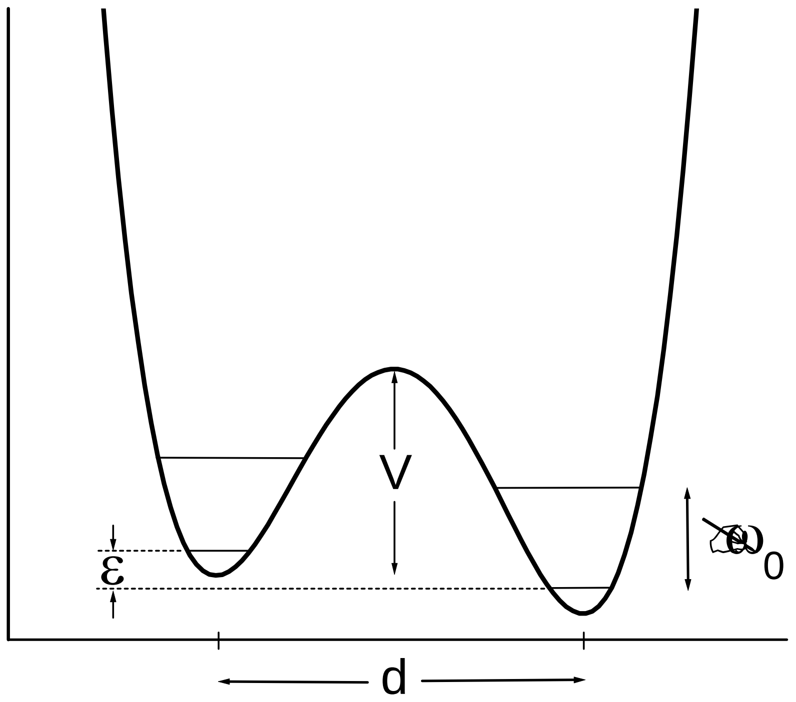


dislocation

More disorder ⇒ more low barriers



Dynamics in a double-well potential



High T: over the barrier
(thermal activation)

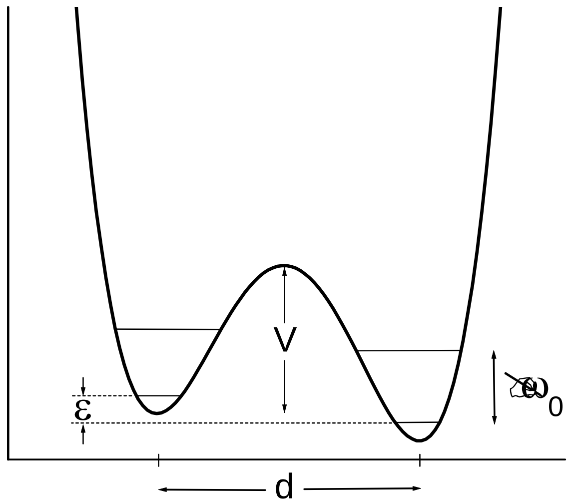
Low T: through the barrier
(tunneling)

Asymmetry energy: ε

Tunneling energy: $\Delta = \hbar\omega_0 e^{-\lambda} \approx \hbar\omega_0 e^{-\frac{\sqrt{2mV}}{\hbar}d}$

$$\varepsilon,\ \Delta,\ k_B T \ll \hbar\omega_0 < V$$

$\Rightarrow$ two-level tunneling system (TLS)



Hamiltonian in left-right basis

$$H = \frac{1}{2}\begin{pmatrix} \varepsilon & -\Delta \\ -\Delta & -\varepsilon \end{pmatrix} \qquad \Delta = \hbar\omega_0 e^{-\lambda} \approx \hbar\omega_0 e^{-\frac{\sqrt{2mV}}{\hbar}d}$$

Eigenvalues: $$E_{0,1} = \pm\frac{1}{2}\sqrt{\varepsilon^2 + \Delta^2}$$

Eigenstates: $$\left|\Psi_0\right\rangle = \sin\frac{\theta}{2}\left|L\right\rangle + \cos\frac{\theta}{2}\left|R\right\rangle$$

$$\left|\Psi_1\right\rangle = \cos\frac{\theta}{2}\left|L\right\rangle - \sin\frac{\theta}{2}\left|R\right\rangle$$

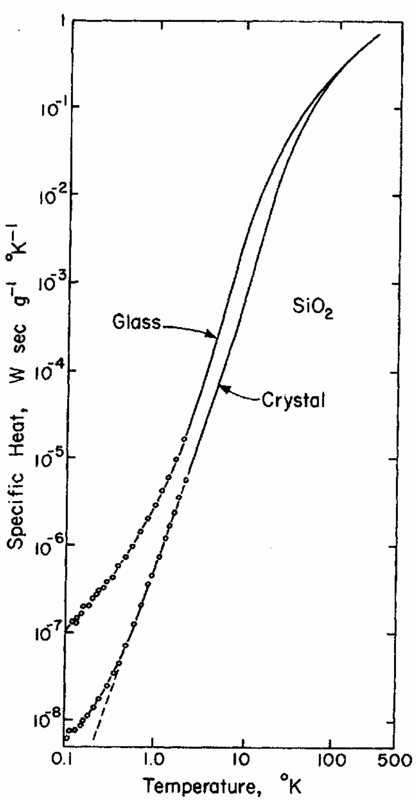
where $\tan\theta = \dfrac{\Delta}{\varepsilon}$

The standard "tunneling model"

Anderson, Halperin and Varma; Phillips (1972)

Hypotheses: 1) $P(\varepsilon,\lambda)=P_0 \Rightarrow c(T) \sim T$

2) TLS scatter phonons $\Rightarrow \kappa(T) \sim T^2$



Experimental Data:

Zeller and Pohl (1971)