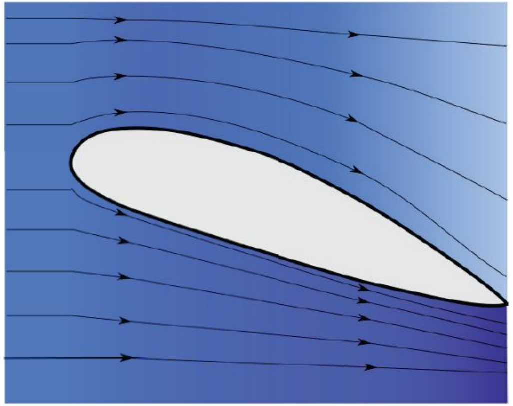
Electron fluids

Leonid Levitov (MIT)

Boulder Summer School 2025

Fluids: chaotic on microscales, orderly on macroscales (conservation laws!)



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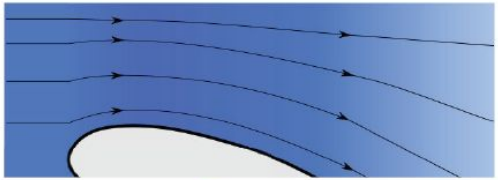

Hydrodynamics: theory of everything

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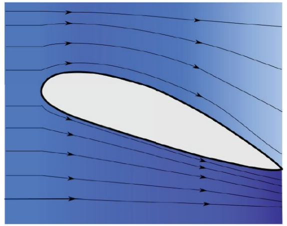
Hydrodynamic description (overview)

- Space and time symmetries: Eulidean P, T, O(3), Spin SU(2)
- Continuous (global) symmetries result in (local) conservation laws
- Local transport equations
- Example: Fermi liquids $\partial_t n+\nabla j=0$, $\partial_t p+\nabla \Pi+\gamma p=enE$, where n and p – particle and momentum density – conserved quantities, j and Π – current and stress tensor (here, Fermi pressure) – functions of conserved quantities, γ – momentum dissipation rate (disorder or phonons)
- Separation of time scales–nonconserved quantities quickly erased from system memory (Boltzmann). Orderly behavior from chaotic behavior.



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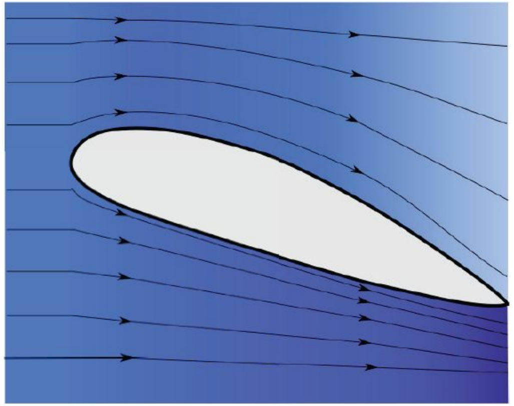
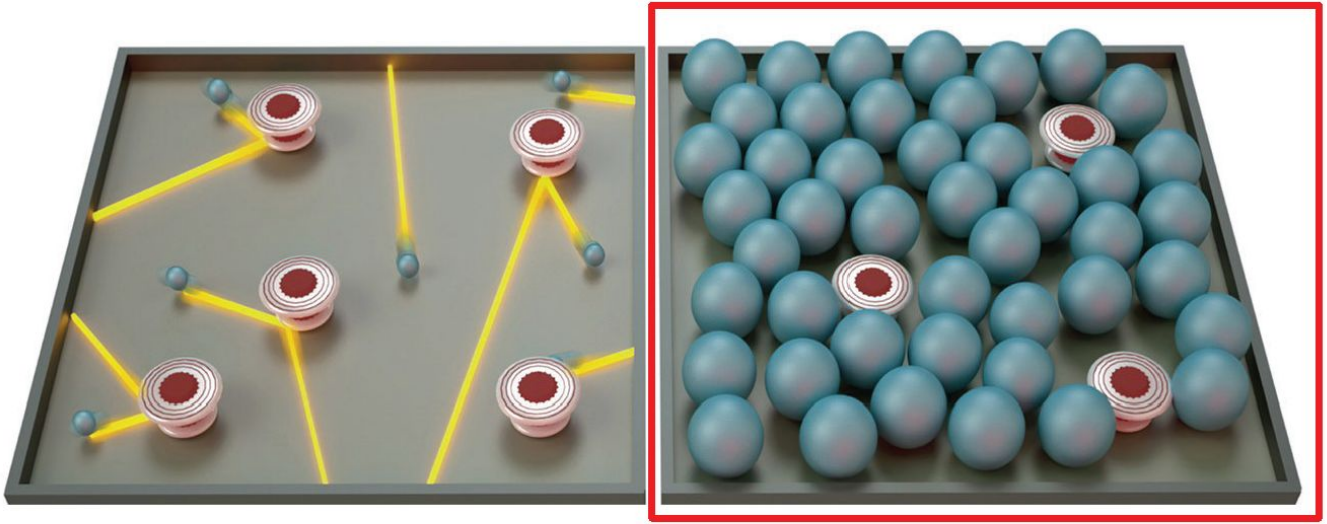


Ergodicity and separation of scales

- Short-time memory for nonconserved quantities, long-time memory for conserved quantities
- Markovian picture (hydrodynamics justification and validity)
- Interesting non-Markovian effects:
  - in classical gases (Dorfman and Cohen),
  - in quantum systems with disorder: quenching of diffusion, Anderson localization
  - Weak localization (Gorkov, Larkin, Khmelnitskii),
  - many-body localization (long history and ongoing),
  - 2D electron fluids (this lecture), and many others
- Manifestations:
  - nonlocal transport equations,
  - kinetic coeffiients with long-time memory,
  - infinite or diverging thermalization rates

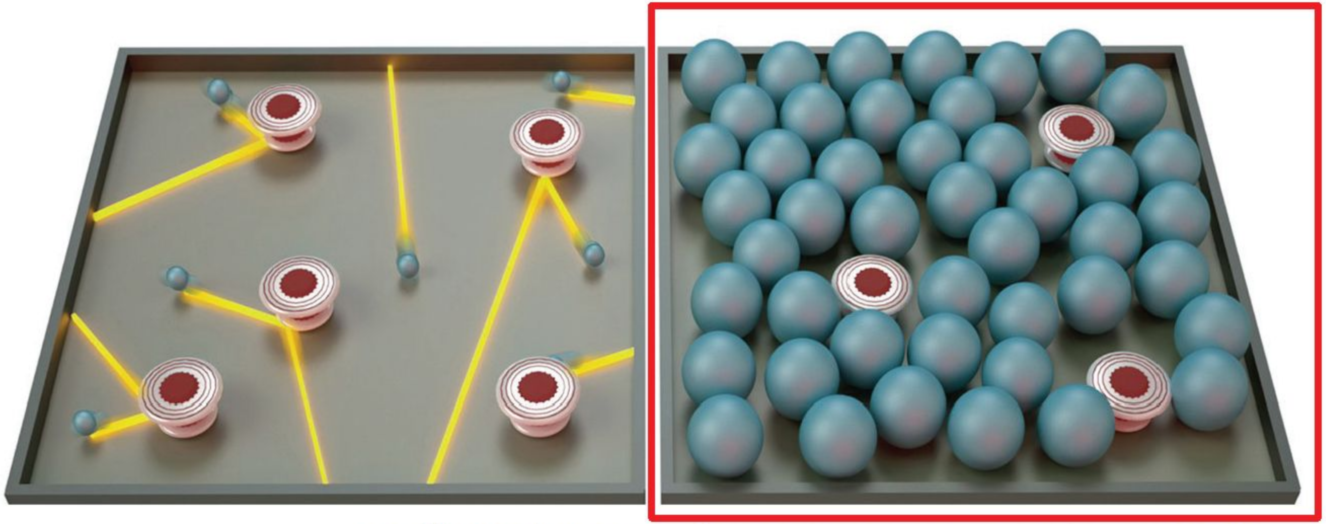
Is hydrodynamics ever relevant in metals?



credit: Andy Lucas; from: Jan Zaanen, Science 2016

- High-mobility electron systems (graphene, GaAs 2DES, $PdCoO_2$, etc):
- Non-Fermi liquids, high-Tc superconductors, strange metals

Is hydrodynamics ever relevant in metals?



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Literature

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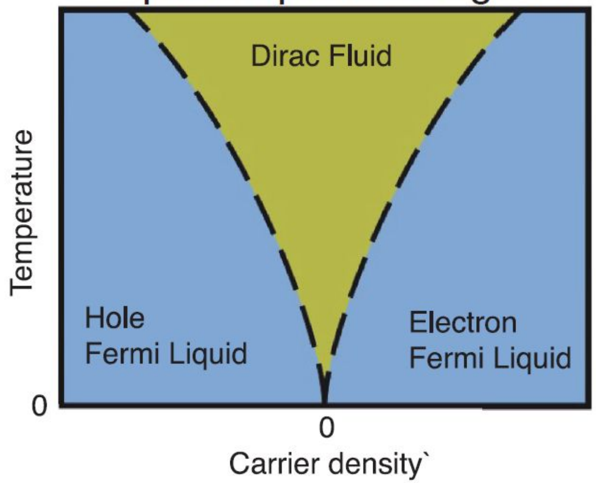
Gooth *et al.* Electrical and Thermal Transport at the Planckian Bound of Dissipation in WP2 (2017)

...

Viscous electron fluids in 2D systems

- Strong interactions (enhanced in 2D, graphene)
- Graphene: weak electron-phonon scattering, no Umklapp ee scattering
- Fast p-conserving ee collisions, shear viscosity

Graphene phase diagram



Sheehy and Schmalian, PRL 99, 226803 (2007)

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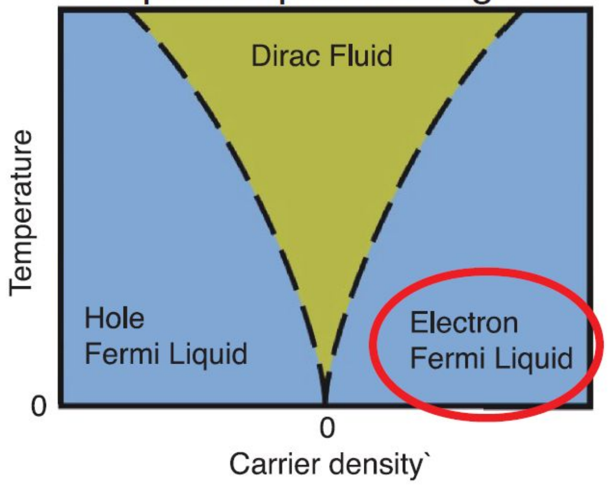
- **Signatures of viscous effects?**
- **New collective phenomena?**

Graphene phase diagram



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Team

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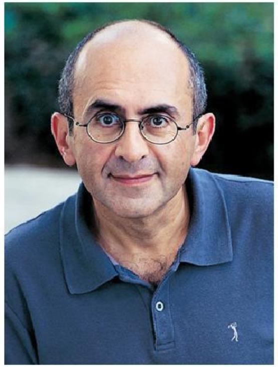
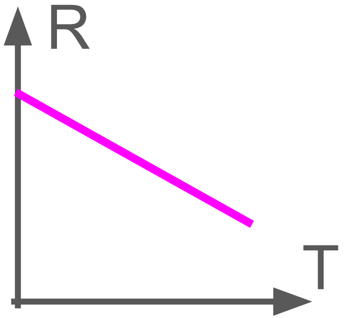


Gregory Falkovich
Weizmann Institute

Why to be interested in electron hydrodynamics?

- **Viscous transport:** A new regime showing a counterintuitive behavior: **carrier collisions assist conduction**. Compare to motional narrowing in spin resonance (Van Fleck and Anderson) or collision-narrowing in optics (the Dicke effect)



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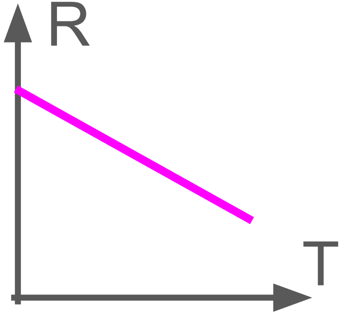
- **Resistance drops with T**: $R(T=0)>R(T\neq0)$. Other instances: Kondo impurity scattering or localization ($dR/dT<0$ reflects spin correlations or suppression of quantum coherence)



Serhii Kryhin ‘22

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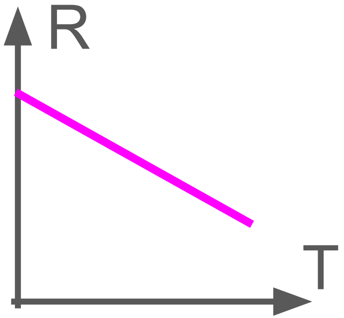
- **Resistance drops with T**: R(T=0)>R(T≠0). Other instances: Kondo impurity scattering or localization (dR/dT<0 reflects spin correlations or suppression of quantum coherence)
- This lecture: **a non-Fermi-liquid T dependence** in electron hydrodynamics. Surprisingly, the measured T dependence is **linear** rather than T^2. Explanation for linear scaling? **Tomographic electron fluids in 2D**



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Why to be interested in electron hydrodynamics?

Vortices in electron fluids studied by scanning probe (Zeldov goup, Weizmann Institute). **Current flow opposite to field.**

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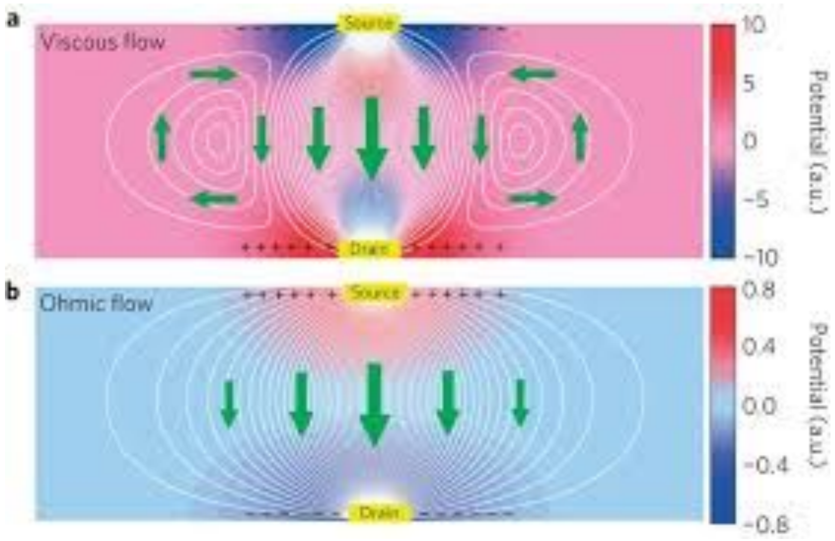

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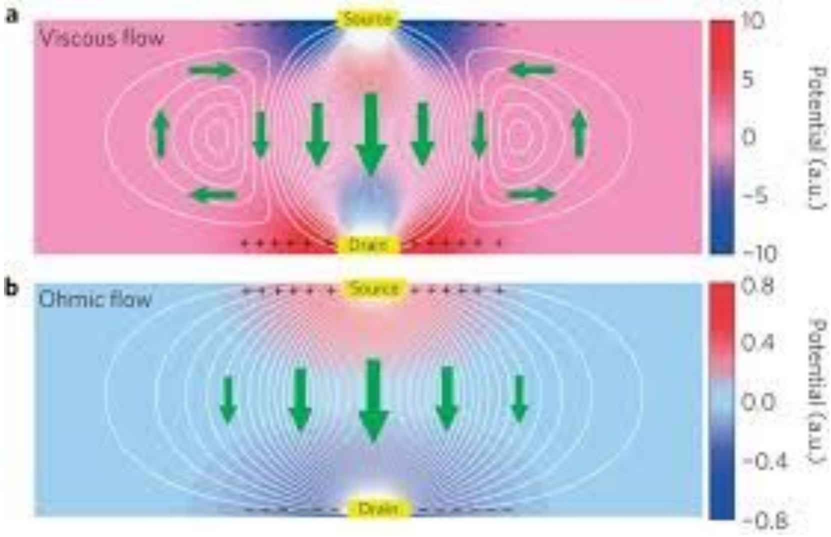

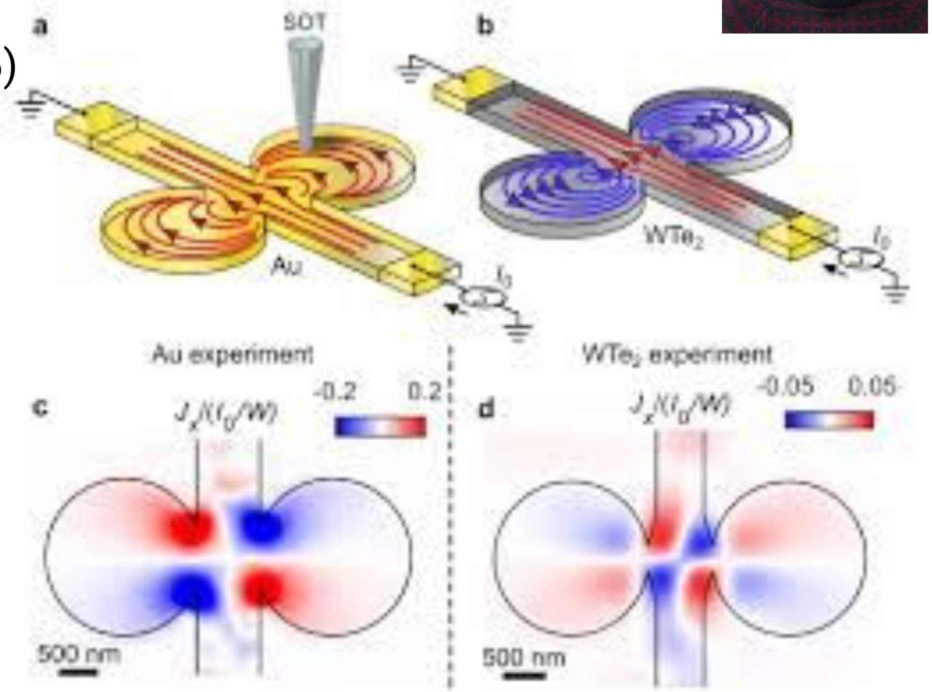

Why to be interested in electron hydrodynamics?

Hydrodynamic instabilities under small, experimentally accessible fields. Current-induced inversion of band occupation (experiments in graphene multilayers and monolayers, moire and non-moire. Current-driven ordering, instability?

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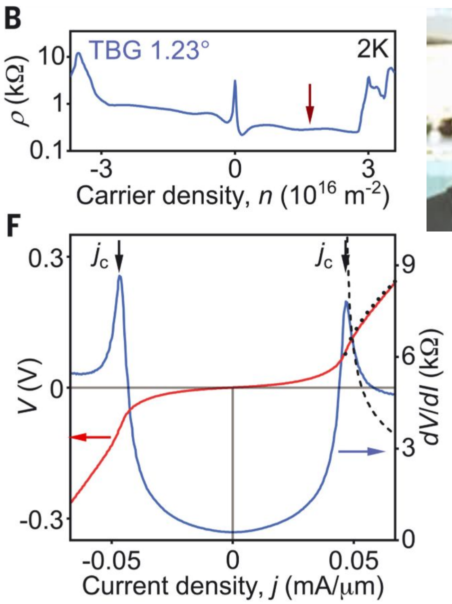
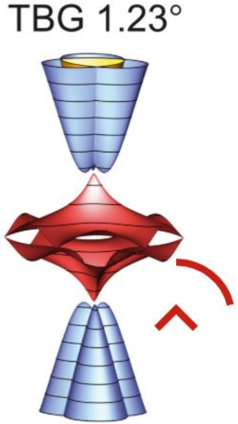
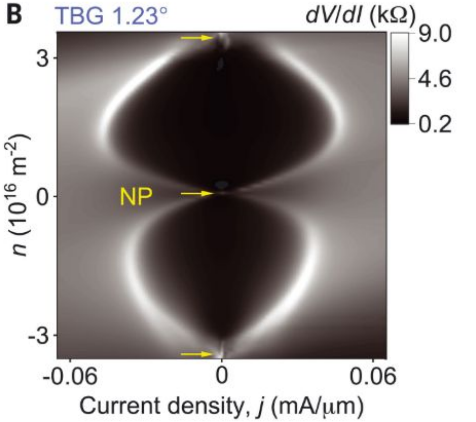
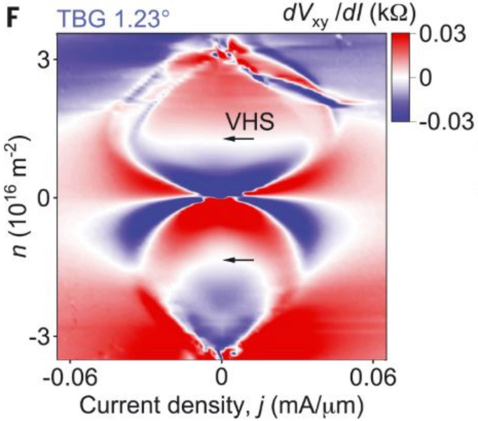








Science 375, 430-433 (2022) Berdyugin, Krishna Kumar et al, (Geim group)

In graphene, where currents reside,
They flow with a physics inside.
Not particles, but waves,
In viscous enclaves,
A fluid no wires can guide.

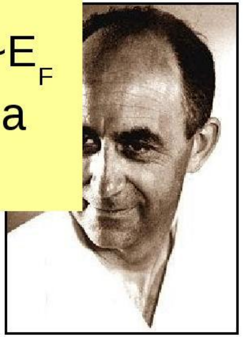
Tomographic electron fluids

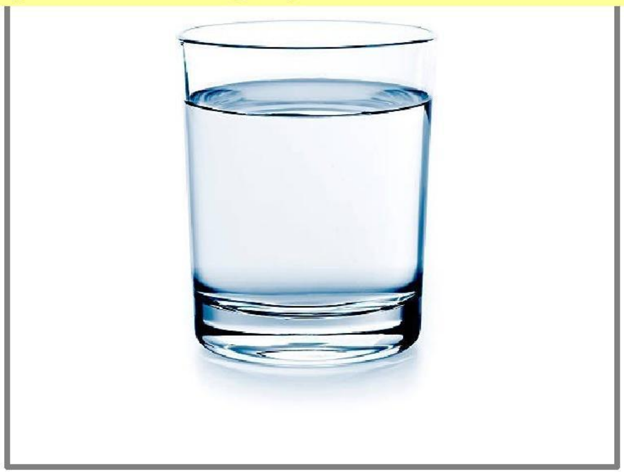
Tomographic electron fluids

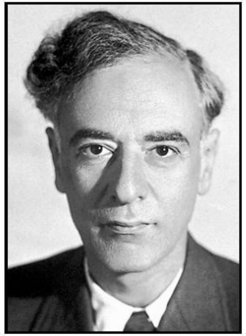
- Quasiparticle lifetimes
- Kinetic coefficients
- Nonlocal conductivity
- Tomographic transport
- New phenomena

Quasiparticle lifetimes in Landau Fermi-liquids:

- Fermi sea (filled states with $E<E_F$)
- All the action at the Fermi surface, $E\sim E_F$
- Quasiparticles: quasi-free particles in a strongly interacting system







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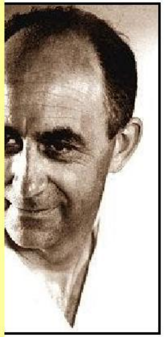
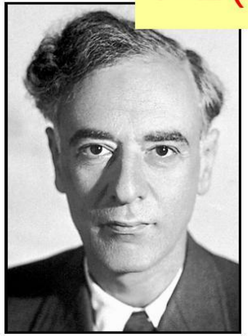
- Relatively long lifetimes in 3D systems: $\tau \sim 1/(E-E_F)^2$, $\tau \sim 1/(k_B T)^2$
- Even longer lifetimes in 2D systems: $\tau \sim 1/(k_B T)^4$ for odd-parity excitations





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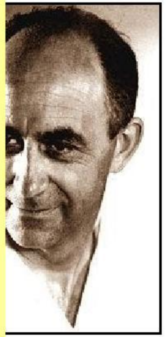
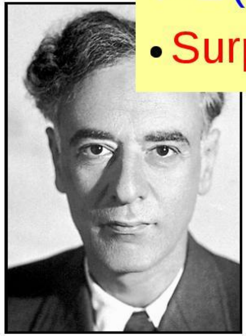
- Surprising collective behaviors in e-fluids





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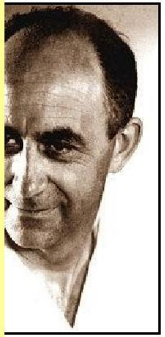
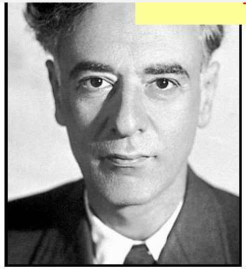
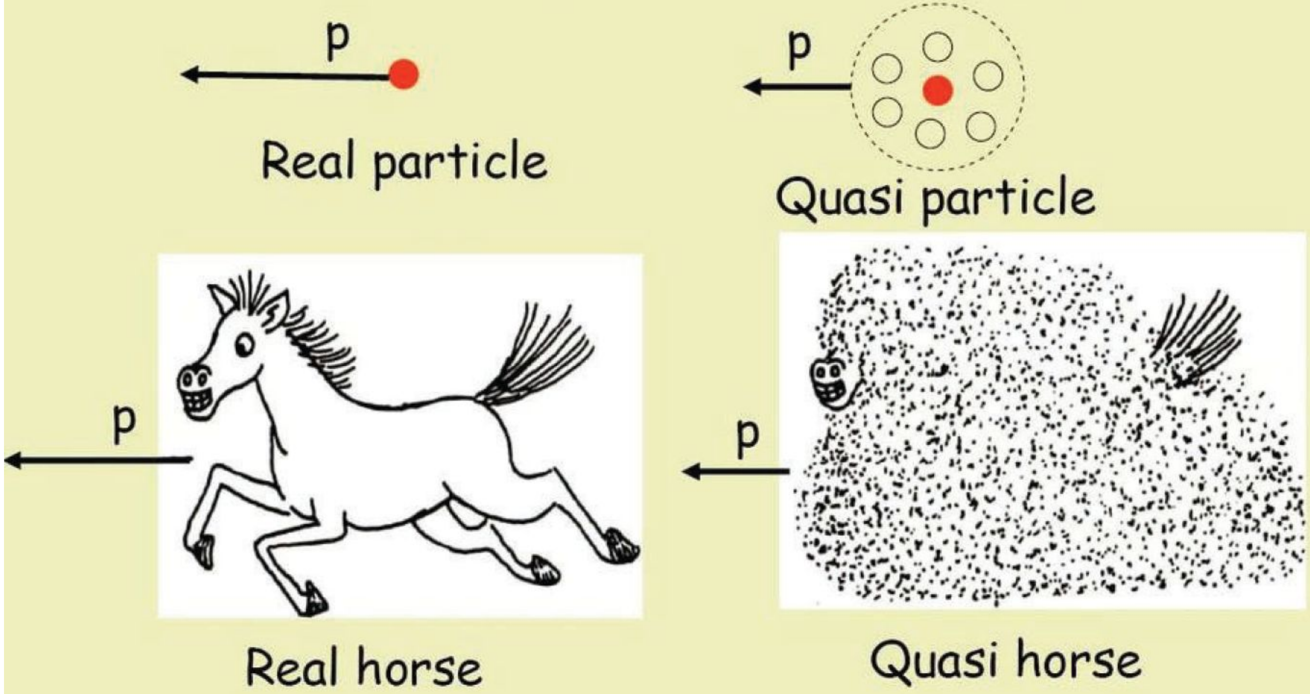


One cannot live in society and be free from society (V I Lenin)



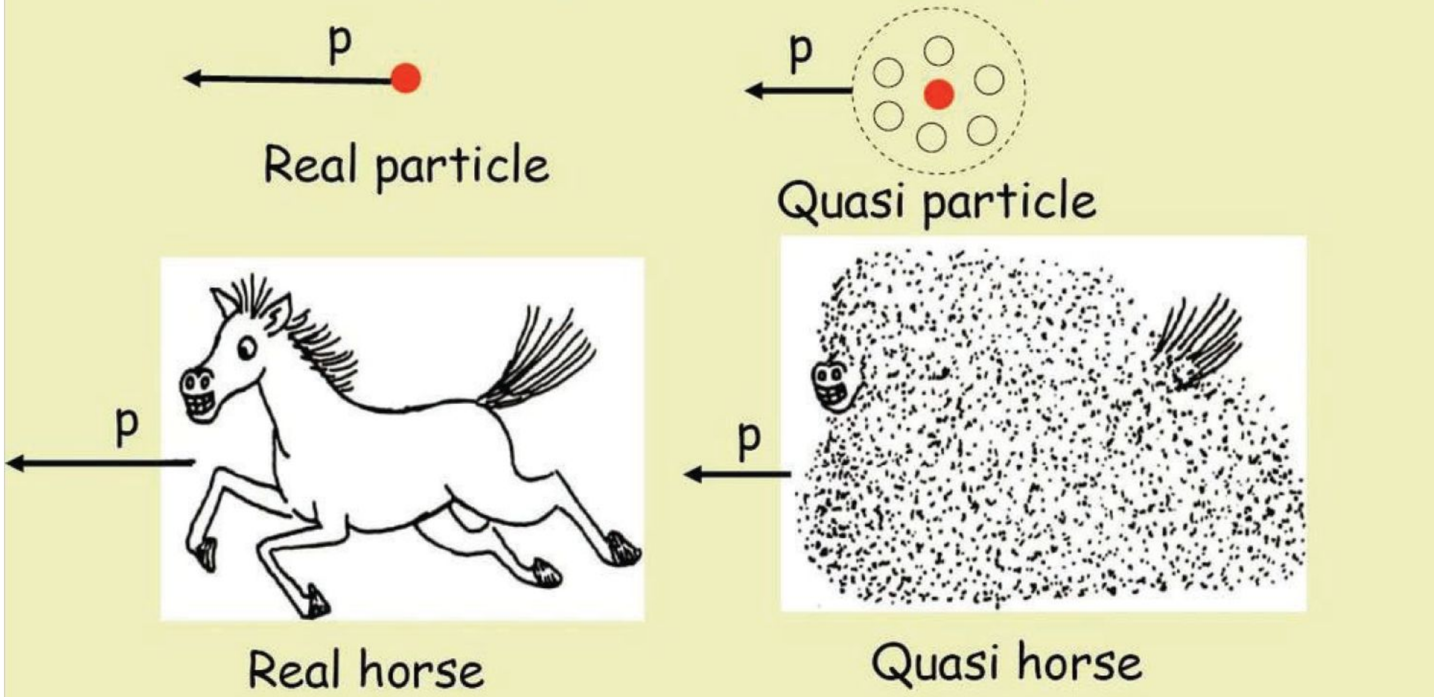
Quasiparticle concept



R. D. Mattuck, a guide to Feynman Diagrams in the MB problem, Dover, 1976

A quasiparticle has an effective mass, selfenergy (energy and lifetime).

Quasiparticle concept



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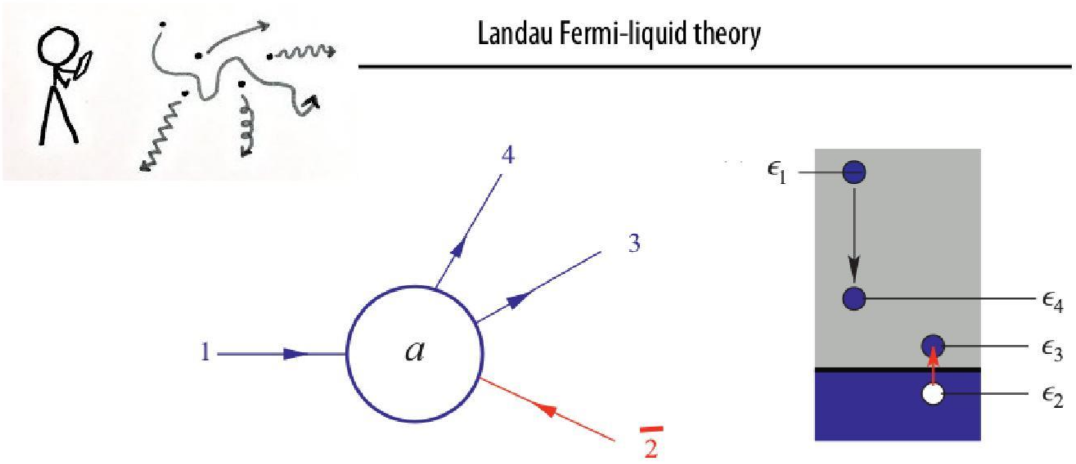
The phase space argument



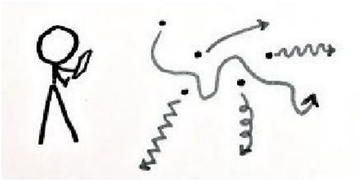
$$\gamma \sim \int\int \delta(\epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4) f(\epsilon_2)(1 - f(\epsilon_3))(1 - f(\epsilon_4)) \sim max[\epsilon_1^2, T^2]$$

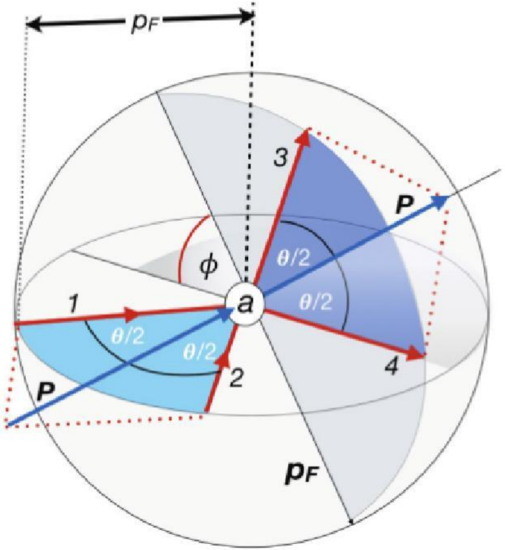
credit: Coleman, Introduction to many-body physics

Kinematics of ee scattering:

In 3D angular relaxation not a bottleneck (and thus does not matter)

Landau argument works





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Kinematics of ee scattering:

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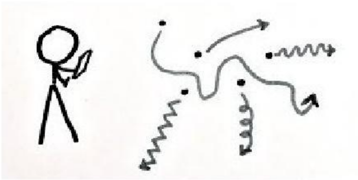
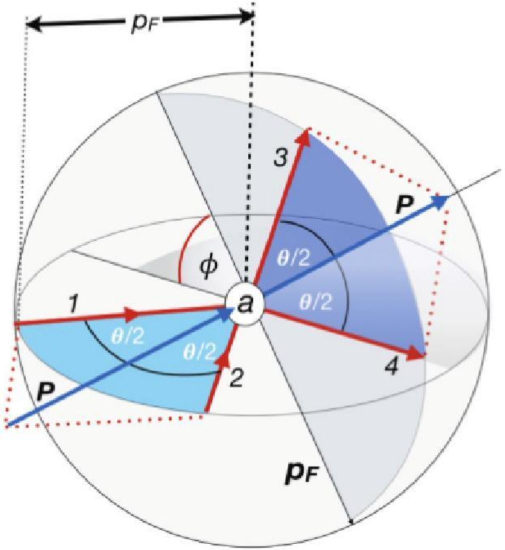
Landau argument works

However, in 2D it does matter!

angular relaxation IS a bottleneck

revision of Fermi-liquid theory required



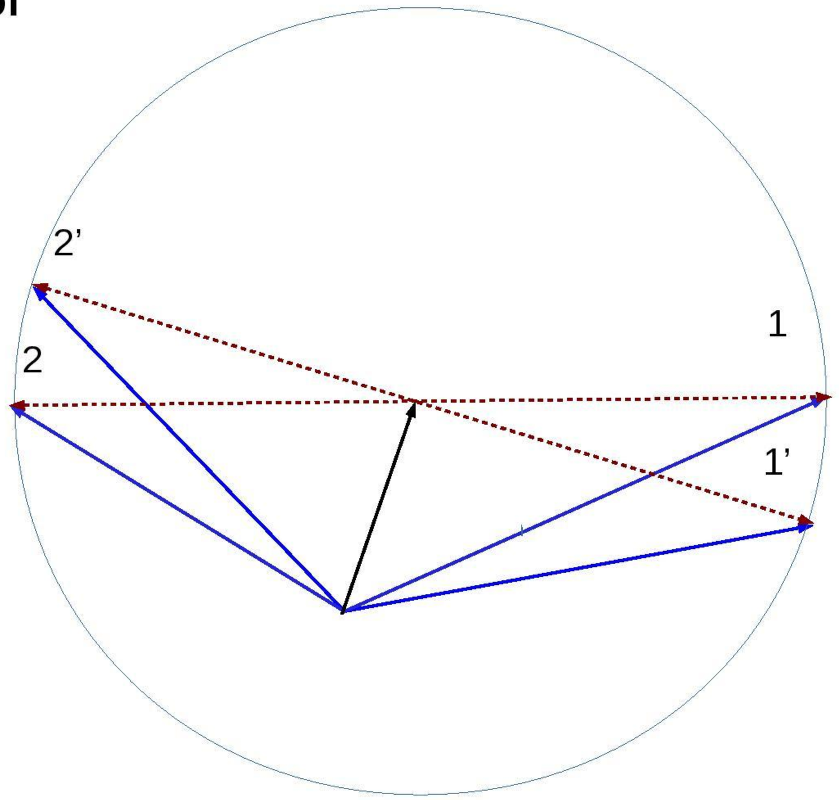


credit: Coleman, Introduction to many-body physics

Tomographic electron fluids

- Quasiparticle lifetimes
- Kinetic coefficients
- Nonlocal conductivity
- Tomographic transport
- New phenomena

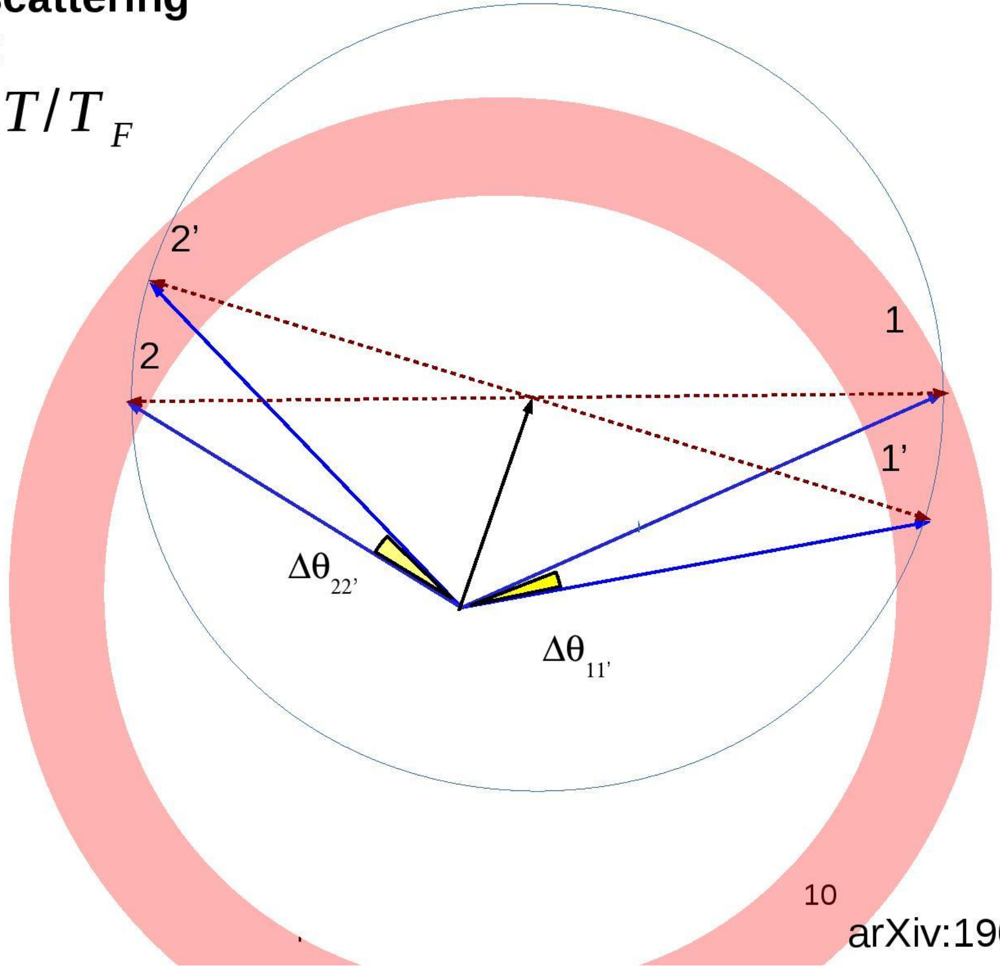
Kinematics of two-body collisions:



arXiv:1905.03751

Small scattering angles:

$$\Delta\theta \sim T/T_F$$



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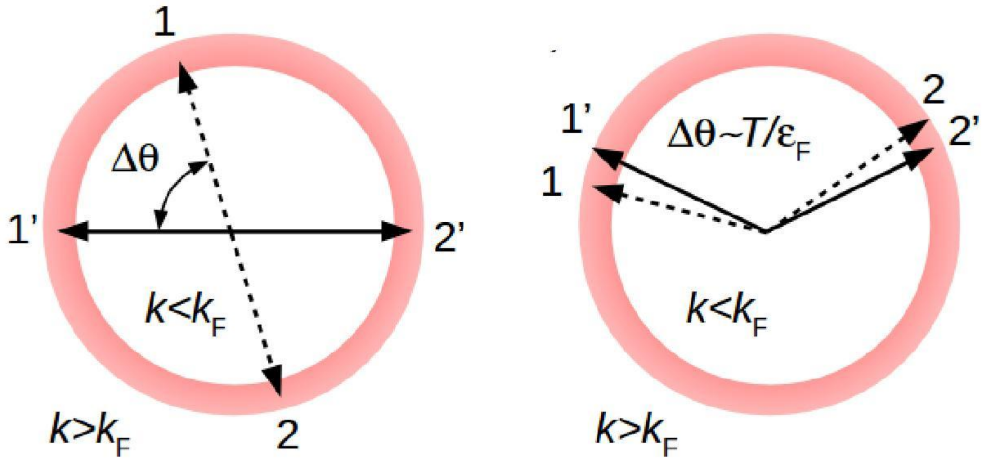
New behavior in 2D

- Momentum conservation and fermion exclusion single out two types of collisions: a) head-on, and b) small-angle
- Angular relaxation dominated by (near) head-on collisions.
- The **even-parity** and **odd-parity** parts of momentum distribution, $\delta f(p)=\delta f(-p)$ & $\delta f(p)=-\delta f(-p)$ relax at different rates



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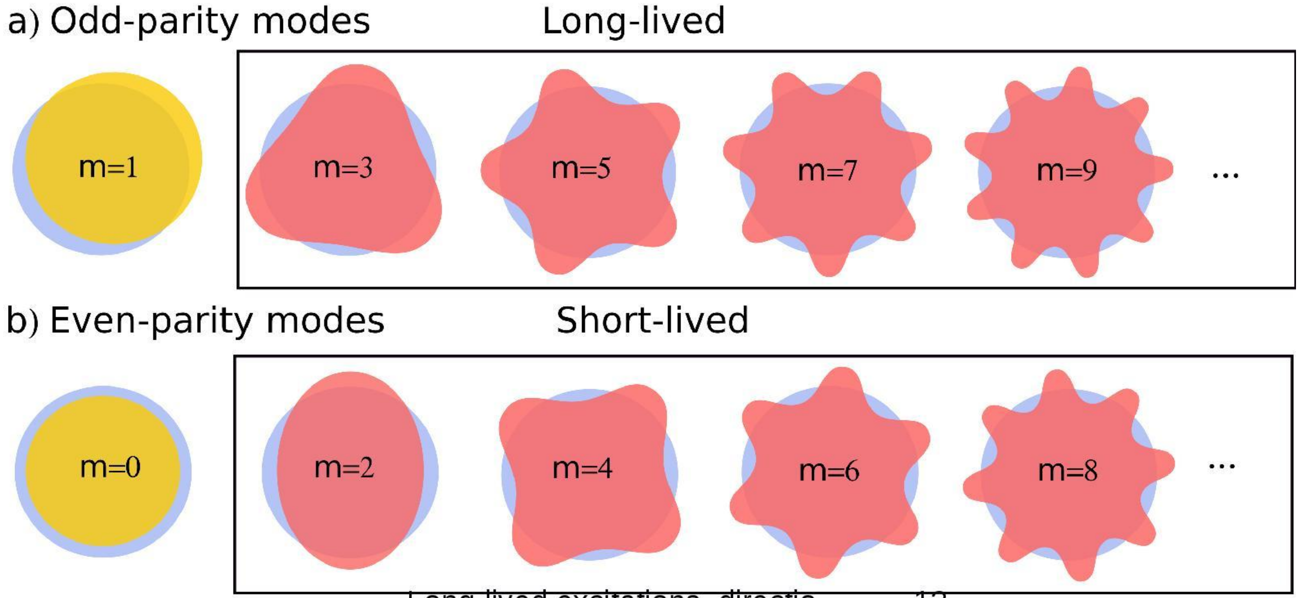
Even and odd harmonics

- The **even-parity** and **odd-parity** parts of momentum distribution, $\delta f(p)=\delta f(-p)$ & $\delta f(p)=-\delta f(-p)$ relax at different rates.
- Relaxation rates for the $\delta f(p)$ harmonics of the **odd** and **even** parity can differ by orders of magnitude: $\gamma'/\gamma \sim (T/T_F)^2$, $\gamma \sim T^2/T_F$



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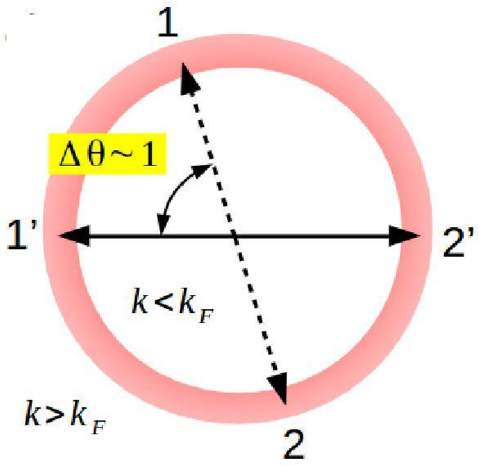
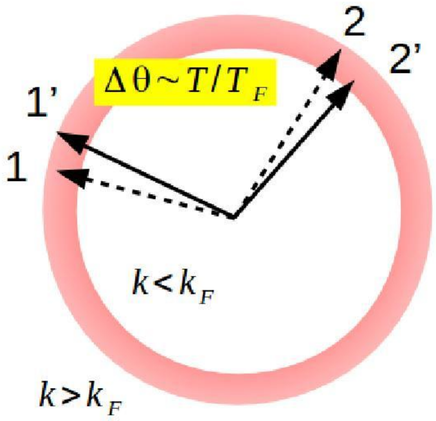
Lifetimes of Fermi surface shape modes — A general discussion

Estimating the rates

Angular diffusion

$$\gamma_{m\,even} \sim R_* \frac{T^2}{T_F^2} \qquad \gamma_{m\,odd} \sim R_* \frac{T^2}{T_F^2} \Delta\theta^2 m^2 \sim R_* \frac{T^4}{T_F^4} m^2$$





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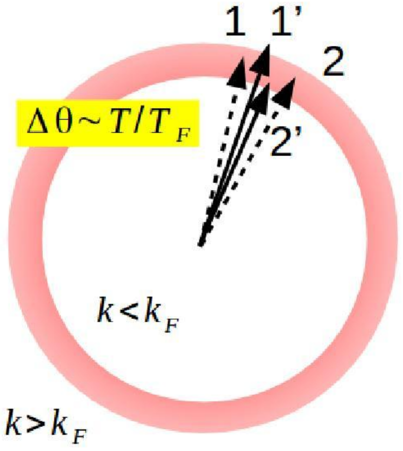
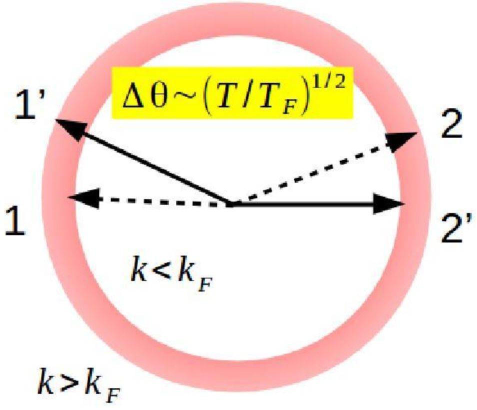
The odd-*m* rates:

Naively:

$$\gamma_m \sim R_* \frac{T^2}{T_F^2} \Delta\theta^2 m^2 \sim R_* \frac{T^4}{T_F^4} m^2 \qquad \gamma_m \sim R_* \frac{T^2}{T_F^2} \Delta\theta^2 m^2 = R_* \frac{T^3}{T_F^3} m^2$$





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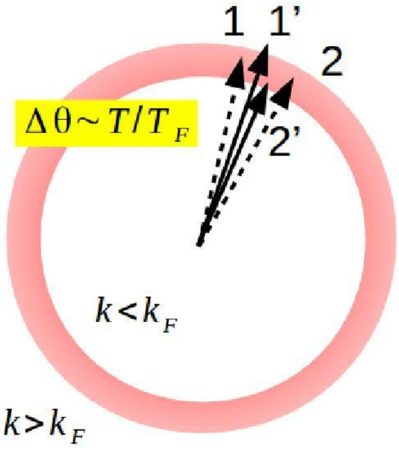
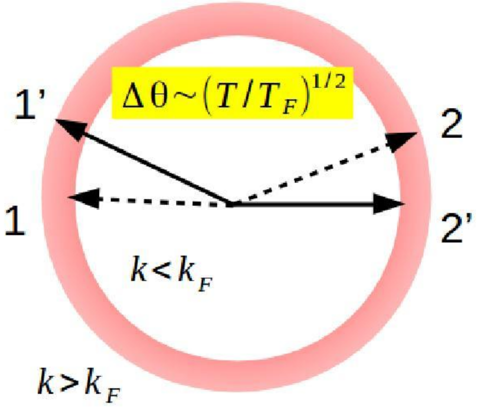
$$\gamma_m \sim R_* \frac{T^2}{T_F^2} \Delta\theta^2 m^2 = R_* \frac{T^3}{T_F^3} m^2$$

Is this true? Not quite

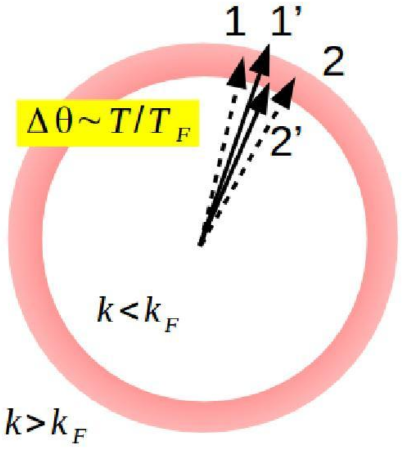




arXiv:1905.03751

The odd-m rates:

$$\gamma_m \sim R_* \frac{T^2}{T_F^2} \Delta\theta^2 m^2 \sim R_* \frac{T^4}{T_F^4} m^2$$

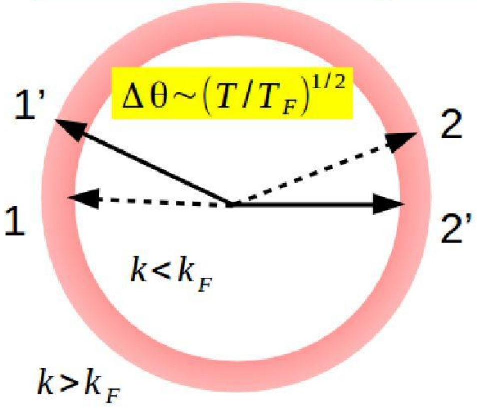


Actually:

Angular superdiffusion

$$\gamma_m \sim R_* \frac{T^2}{T_F^2} (\Delta\theta^2 m^2)^2 = R_* \frac{T^4}{T_F^4} m^4$$

Origin: correlated angular shifts in ee scattering

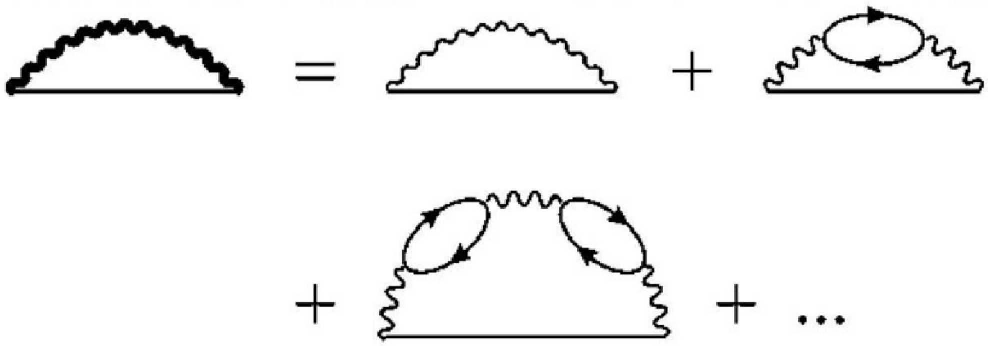


arXiv:1905.03751

cf. lifetimes from selfenergy in 2D

$$\gamma = -2\Sigma''(\epsilon, p) \sim T^2 \ln(1/T)$$

Chaplik 1971
Hodges, Smith, Wilkins 1971
Bloom 1975
Giuliani, Quinn 1982
Menashe, Laikhtman 1996
Zheng, DasSarma 1996
Chubukov, Maslov 2003



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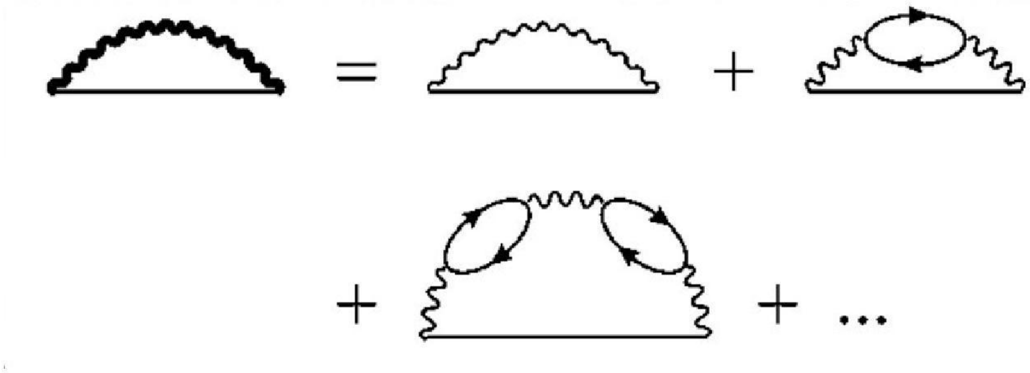
Dominated by the fast pathways (rapid decays)
and by (near) head-on collisions,
Insensitive to slowly decaying modes



Lifetime of two-dimensional electrons measured by tunneling spectroscopy

S. Q. Murphy,* J. P. Eisenstein, L. N. Pfeiffer, and K. W. West
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 24 October 1994; revised manuscript received 7 June 1995)

For electrons tunneling between parallel two-dimensional electron systems, conservation of in-plane momentum produces sharply resonant current-voltage characteristics and provides a uniquely sensitive probe of the underlying electronic spectral functions. We report here the application of this technique to accurate measurements of the temperature dependence of the electron-electron scattering rate in clean two-dimensional systems. Our results are in qualitative agreement with existing calculations.

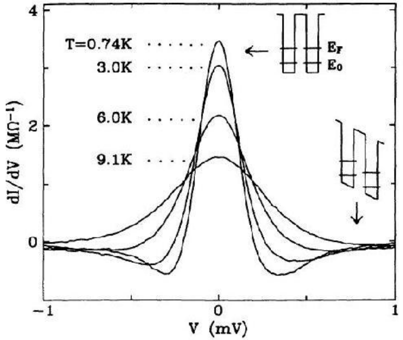


FIG. 1. Typical 2D-2D tunneling resonances observed at various temperatures in a sample with equal densities ($N_s = 1.6 \times 10^{11}$ cm^{-2}) in the two 2DES's. Insets show simplified band diagrams on and off resonance.

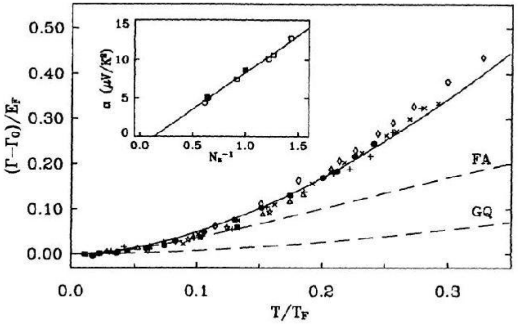


FIG. 3. Tunnel resonance width vs temperature for all samples (having eight different densities). On dividing T by T_F and the resonance width (minus the zero-temperature limit Γ_0) by E_F all the data collapse onto a single curve. The dashed lines are the calculations of GQ (Ref. 18) and FA (Ref. 20). The solid line is 6.3× GQ. Inset: Coefficient of T^2 term in Γ vs inverse density N_s^{-1} (in units of 10^{-11} cm^2).

Questions?

Conductivity calculation, Fermi-liquid kinetic equation

$$\frac{df}{dt} + [f, H] = I(f) \qquad H = \mathbf{p}^2/2m + U(\boldsymbol{r})$$

$$\boldsymbol{\nabla}_{\boldsymbol{r}} f \boldsymbol{\nabla}_{\boldsymbol{p}} H - \boldsymbol{\nabla}_{\boldsymbol{r}} H \boldsymbol{\nabla}_{\boldsymbol{p}} f = \boldsymbol{v} \cdot \boldsymbol{\nabla}_{\boldsymbol{r}} f + e\boldsymbol{E} \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} f,$$

$$\boldsymbol{j}(\boldsymbol{r}, t) = \int \frac{d^2 p}{(2\pi)^2} e\boldsymbol{v}(p)\, \delta f(\boldsymbol{p}, \boldsymbol{r}, t) \qquad \boldsymbol{E}_{\text{ext}} e^{i\boldsymbol{k}\boldsymbol{r} - i\omega t},$$

$$\delta f_{\boldsymbol{p}}(t, \boldsymbol{r}) = e^{i\boldsymbol{k}\boldsymbol{r} - i\omega t} \sum_m \delta f_m(p, t)\, e^{im\theta},$$

$$\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}_{\text{ext}}(\boldsymbol{r}) - \boldsymbol{\nabla}_{\boldsymbol{r}} \int d^2 x' U(\boldsymbol{r} - \boldsymbol{r}')\, \delta\rho(\boldsymbol{r}')$$

Conductivity calculation, Fermi-liquid kinetic equation

$$I_{\mathrm{ee}}(f_1) = \sum_{21'2'} \left(w_{1'2'\to 12} - w_{12\to 1'2'}\right) \qquad \eta(\boldsymbol{p},t) = \sum_m e^{-\gamma_m t} e^{im\theta} \chi_m(x)$$

$$w_{1'2'\to 12} = \frac{2\pi}{\hbar} |V_{12,1'2'}|^2 \delta_\varepsilon \delta_{\boldsymbol{p}} (1-f_1)(1-f_2) f_{1'} f_{2'}$$

$$\delta_\varepsilon = \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_{1'} - \varepsilon_{2'}),\ \delta_{\boldsymbol{p}} = \delta^{(2)}(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{p}_{1'} - \boldsymbol{p}_{2'})$$

$$\delta f(\boldsymbol{p}) = -\frac{\partial f^{(0)}_{\boldsymbol{p}}}{\partial \varepsilon} \eta(\boldsymbol{p}) \qquad I_{\mathrm{ee}} \eta = \sum_{21'2'} \lambda F_{121'2'} \delta_\varepsilon \delta_{\boldsymbol{p}} (\eta_{1'} + \eta_{2'} - \eta_1 - \eta_2)$$

$$\eta(\boldsymbol{p},t) = \sum_m e^{-\gamma_m t} e^{im\theta} \chi_m(x)$$

Conductivity calculation, Fermi-liquid kinetic equation

$$\eta(\mathbf{p},t) = \sum_m e^{-\gamma_m t} e^{im\theta} \chi_m(x) \quad -\gamma_m f_0 (1 - f_0) e^{im\theta} \chi_m(x) = I_{\mathrm{ee}} e^{im\theta} \chi_m(x)$$

$$\boldsymbol{E}\boldsymbol{\nabla}_{\boldsymbol{p}} f_{\boldsymbol{p}}^{(0)} = \boldsymbol{E}\boldsymbol{v} \frac{\partial f_{\boldsymbol{p}}^{(0)}}{\partial \varepsilon}$$

$$\begin{aligned} e\boldsymbol{v}\boldsymbol{E} &= \frac{ev}{2}(E_x + iE_y) e^{-i\theta} + \frac{ev}{2}(E_x - iE_y) e^{i\theta} \\ &= \mathscr{E} e^{-i\theta} + \overline{\mathscr{E}} e^{i\theta}, \end{aligned}$$

$$\boldsymbol{v}\boldsymbol{k} = \zeta e^{-i\theta} + \bar{\zeta} e^{i\theta}, \quad \zeta, \bar{\zeta} = v\left(k_x \pm i k_y\right)/2.$$

$$I \delta f_m(p) e^{im\theta} = -\gamma_m \delta f_m(p) e^{im\theta}$$

$$(\gamma_m - i\omega) \delta f_m + \zeta \delta f_{m+1} + \bar{\zeta} \delta f_{m-1} = s_m,$$

$$s_m = \frac{\partial f_{\boldsymbol{p}}^{(0)}}{\partial \varepsilon} \left(\mathscr{E} \delta_{m,-1} + \overline{\mathscr{E}} \delta_{m,1} \right),$$

Conductivity calculation, Fermi-liquid kinetic equation

By introducing notation $\alpha_m = i\delta f_{m+1}/\delta f_m$, we can write equations with $m > 1$ as

$$\tilde{\gamma}_m + \zeta\alpha_m - \frac{\bar{\zeta}}{\alpha_{m-1}} = 0, \quad \tilde{\gamma}_m = \gamma_m - i\omega.$$

replace the rates γ_m with $\gamma_m - i\omega$

$$\alpha_{m-1} = \frac{\bar{\zeta}}{\tilde{\gamma}_m + \zeta\alpha_m}$$

recursion relations

$$\alpha_{m-1} = \frac{\bar{\zeta}}{\tilde{\gamma}_m + \frac{|\zeta|^2}{\tilde{\gamma}_{m+1} + \frac{|\zeta|^2}{\tilde{\gamma}_{m+2} + \cdots}}}.$$

$$\boxed{\delta f_1 = \frac{\partial f_{\boldsymbol{p}}^{(0)}}{\partial \varepsilon} \frac{\bar{\mathscr{E}}}{\tilde{\gamma}_1 + \frac{|\zeta|^2}{\tilde{\gamma}_2 + \frac{|\zeta|^2}{\tilde{\gamma}_3 + \frac{|\zeta|^2}{\tilde{\gamma}_4 + \ldots}}}}.}$$

Conductivity calculation, Fermi-liquid kinetic equation

$$\boxed{\sigma(k,\omega) = \cfrac{D}{\tilde{\gamma}_1 + \cfrac{z}{\tilde{\gamma}_2 + \cfrac{z}{\tilde{\gamma}_3 + \cfrac{z}{\tilde{\gamma}_4 + \dots}}}}, \quad z = v_F^2 k^2/4,}$$

Nonlocal conductivity with (potentially) long-time memory effects

where $D = ne^2/m$ is the Drude spectral weight.

In this general expression, the scattering rates γ_m can be viewed as a "genetic code" of the system describing many different transport regimes, e.g.

a simple model for dc conductivity in which all rates $\gamma_{m\geq 2}$ are equal ($\gamma_2 = \gamma_3 = \gamma_4 = \cdots = \gamma$)

$$\sigma(k) = \frac{D}{\gamma_p + \frac{1}{2}\left(\sqrt{v^2k^2 + \gamma^2} - \gamma\right)}$$

At small k this gives Drude conductivity and, at γ_p=0, electron viscosity

Questions?

Lifetimes of individual modes with even and odd *m* — An attempt at direct calculation

Kinetic equation, how Sommerfeld expansion in $T/T_F \ll 1$ fails

Linearize near equilibrium $f(\boldsymbol{p}) = f_0(\boldsymbol{p}) - \frac{\partial f_0}{\partial \epsilon}\eta(\boldsymbol{p})$, $\quad f_0(1-f_0)\frac{d\eta_1}{dt} = I_{\rm ee}\eta$

$$I_{\rm ee}\eta = \sum_{21'2'} \frac{2\pi}{\hbar}|V|^2 F_{121'2'}\delta_{\epsilon_1+\epsilon_2-\epsilon_{1'}-\epsilon_{2'}}\delta^{(2)}_{\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{p}_{1'}-\boldsymbol{p}_{2'}}(\eta_{1'}+\eta_{2'}-\eta_1-\eta_2) \qquad F_{121'2'} = f_1^0 f_2^0(1-f_{1'}^0)(1-f_{2'}^0)$$

S Kryhin & LL arXiv:2112.05076

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Focus on individual angular harmonics

$$\eta(\boldsymbol{p},t) = e^{-\gamma_m t}e^{im\theta}\chi_m(x) \qquad -\gamma_m f_0(1-f_0)\chi_m(x) = I_{\rm ee}\chi_m(x)$$

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Phase space: collinear pairs of states p_1=-p_2, $p_{1'}$=-$p_{2'}$

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Phase space: collinear pairs of states $p_1=-p_2$, $p_{1'}=-p_{2'}$

Integrating over angles yields a 1D equation for each m angular mode

$$f_0(1-f_0)\frac{d\chi(x_1)}{dt} = gT^2\int dx_2 dx_{1'} dx_{2'} F\delta(x_1+x_2-x_{1'}-x_{2'})[\chi(x_1)-\chi(x_2)]$$

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Introduce Fourier transform in the energy variable

$$\chi(x) \;=\; 2\cosh\tfrac{x}{2}\,\zeta(x) \qquad \zeta(x) = \int dk e^{ikx}\psi(k)$$

Brooker, Sykes, Phys Rev Lett 21, 279 (1968)
Hojgard Jensen, Smith, Wilkins Phys Lett A 27 532 (1968)

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Zero modes, one per each odd *m*

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Zero modes, one per each odd *m*

Infinite lifetimes at order T^2

Zero modes' energy dependence: $\delta f(\varepsilon)=-\partial f_0(\varepsilon)/\partial\varepsilon$

Kryhin & Levitov arXiv:2112.05076

Kinetic equation, how Sommerfeld expansion in $T/T_F \ll 1$ fails

- Zero modes, one per each odd *m*
- Infinite lifetimes at order T^2
- Zero modes' energy dependence: $\delta f(\varepsilon)=-\partial f_0(\varepsilon)/\partial\varepsilon$

- No spectral gap between 'symmetry zero modes' and generic modes
- Abnormally long-lived excitations
- Long-time memory time

Breakdown of Landau's T^2 kinetics and ergodicity picture at 2D Fermi surfaces

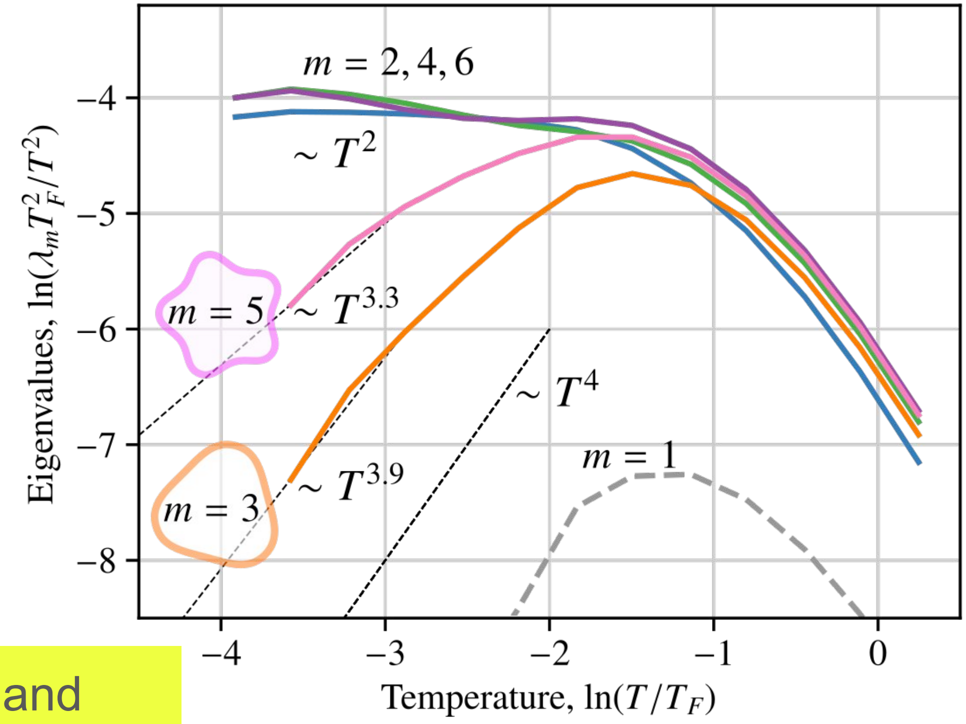
Zero modes, one per each odd *m*

Infinite lifetimes at order T^2

Kryhin & Levitov arXiv:2112.05076

Numerically diagonalize the linearized collision operator

A method that does not rely on a small parameter T<<TF



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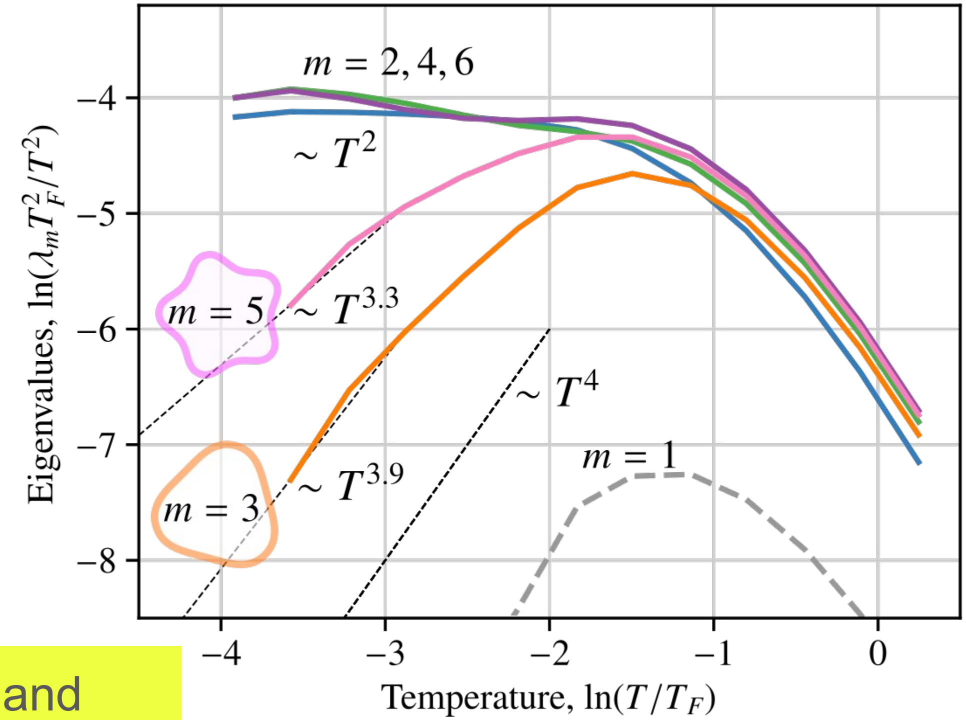

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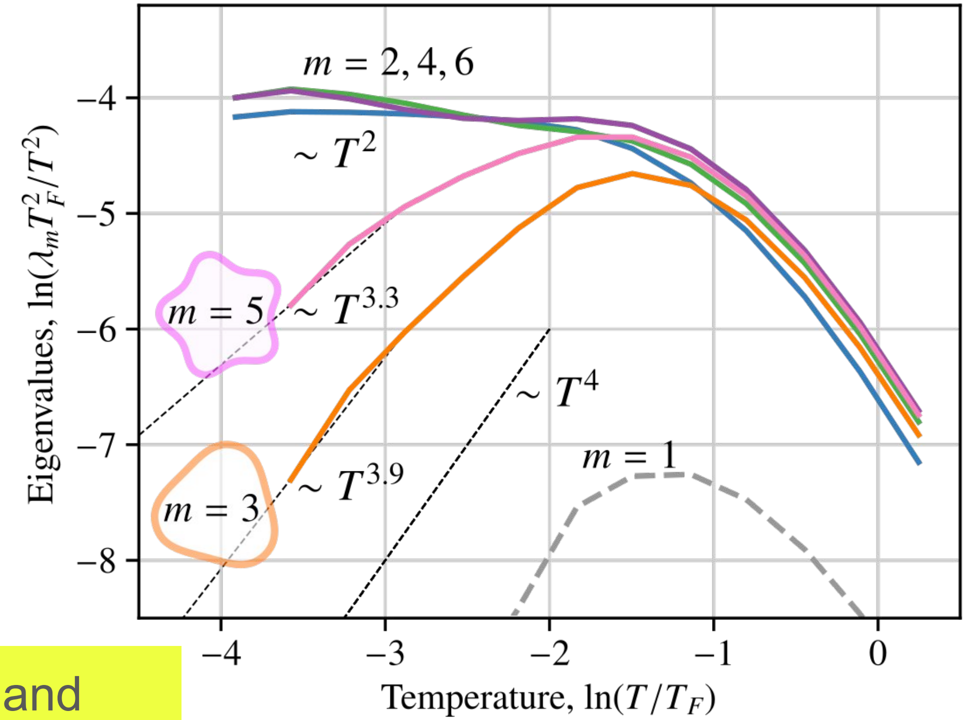
Scaling γ_m~T^α , α~4



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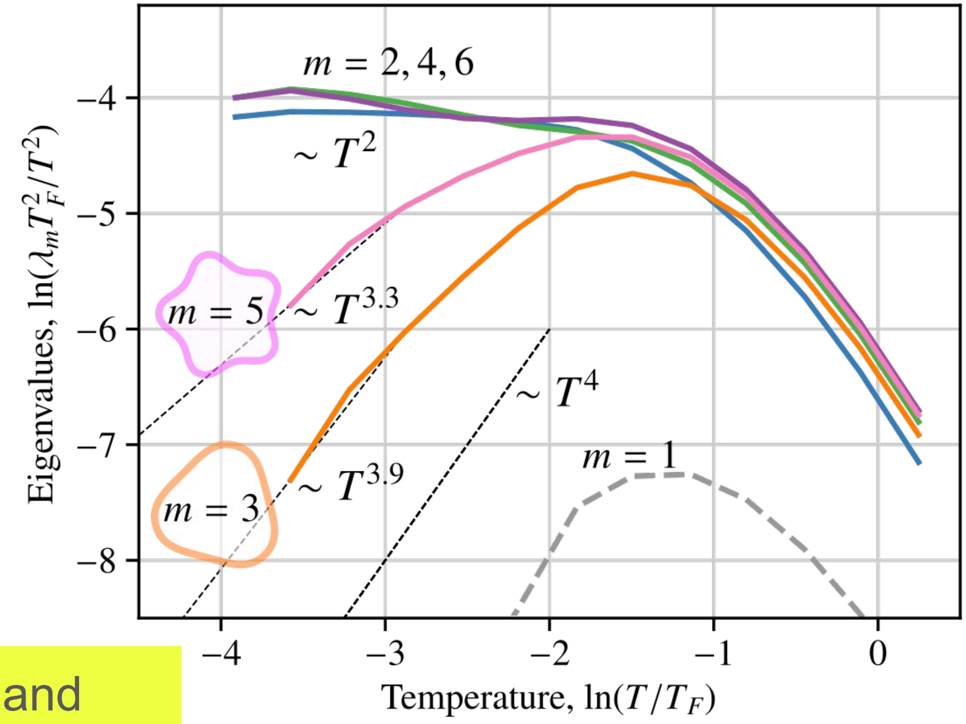
Conventional Fermi-liquid scaling for even-m harmonics



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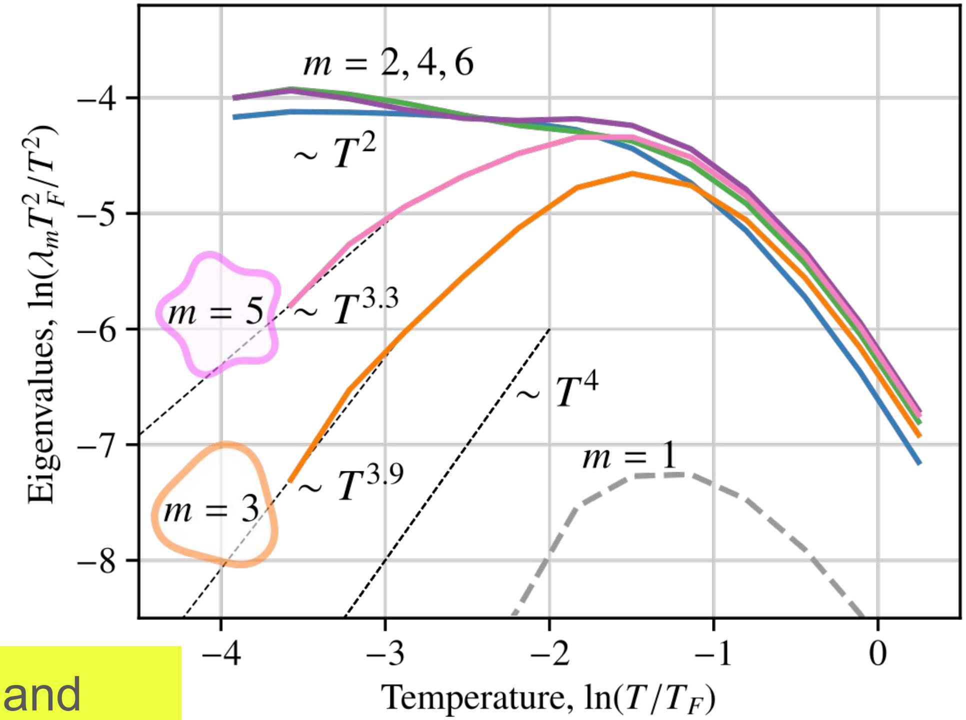
A hierarchy of time scales: $\gamma_{m\ odd} << \gamma_{m\ even}$



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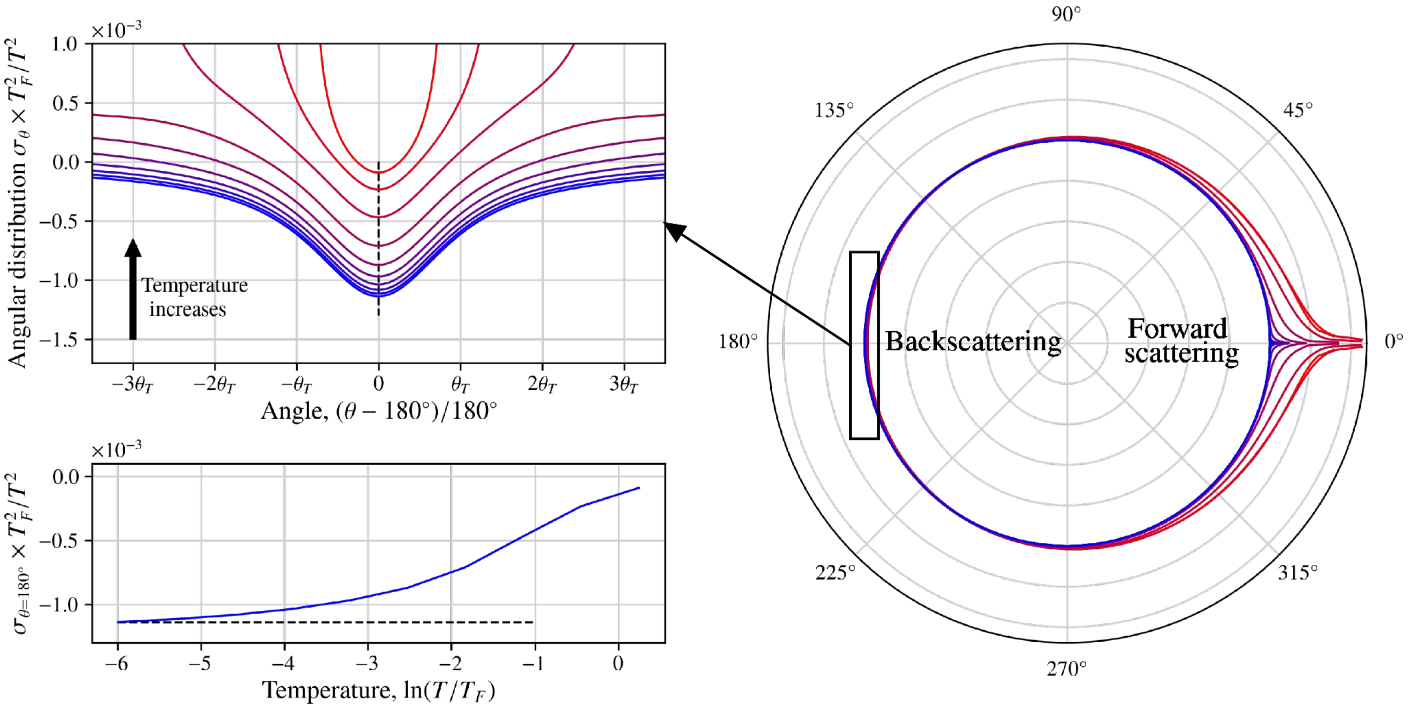
Questions?

Angular distribution for two-body scattering

$\sim T^2$ for generic angles



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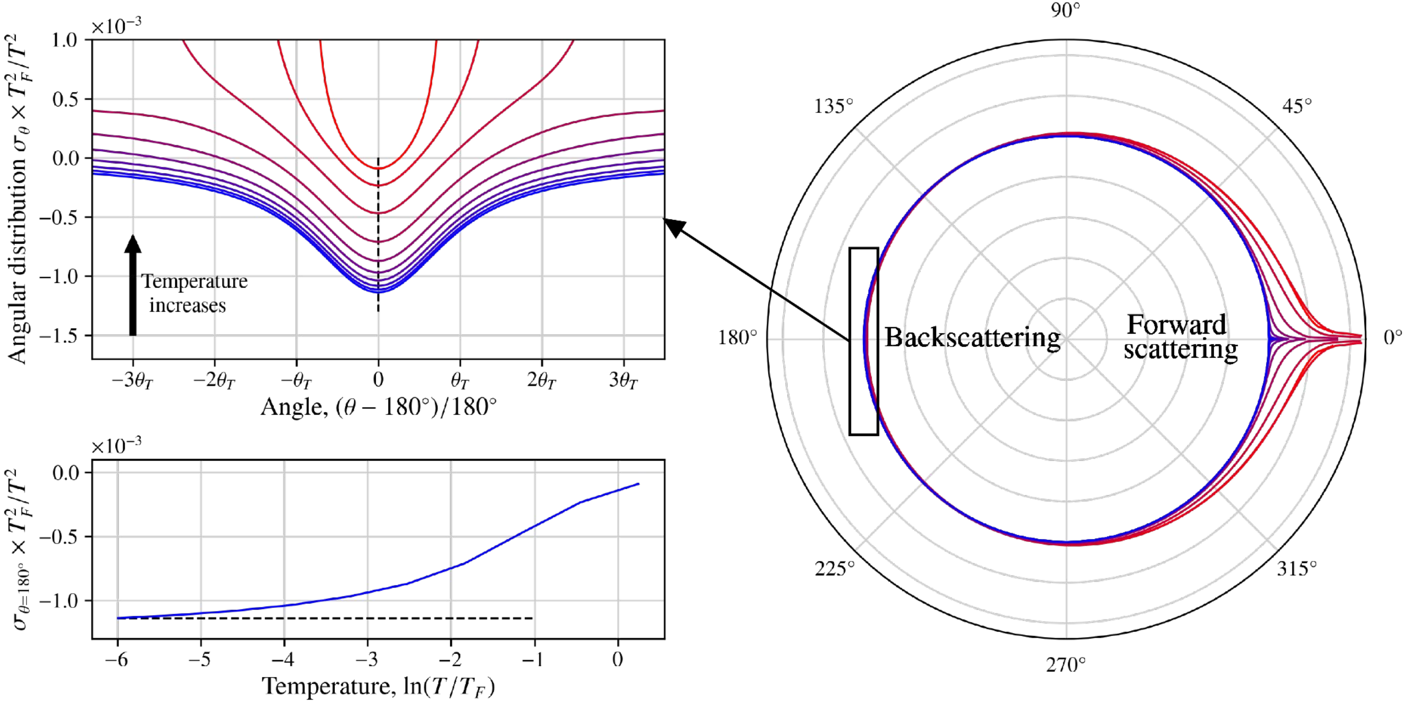
S Kryhin & LL arXiv:2112.05076

Angular distribution for two-body scattering

~T^2 for generic angles

Sharp peaks $\sigma(\theta)$~$T^2/|\theta|$, $\sigma(\theta)$~$T^2/|\theta-\pi|$ for the forward and backscattering directions



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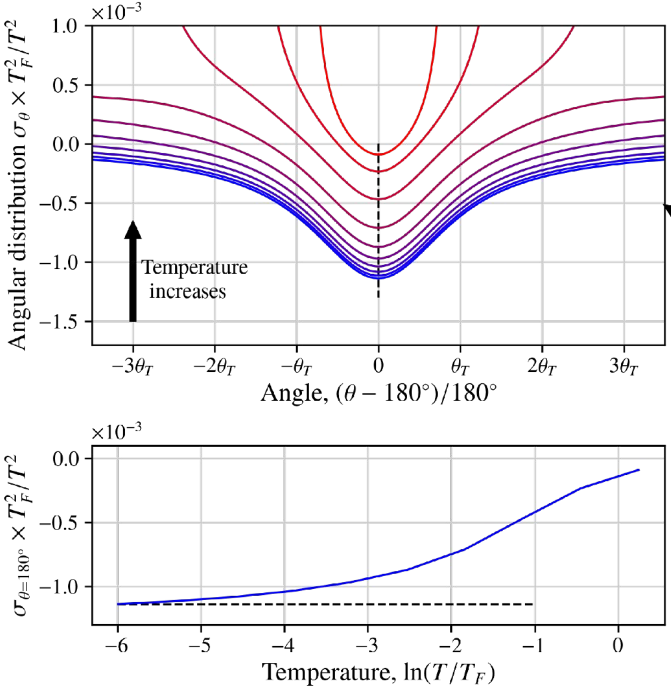
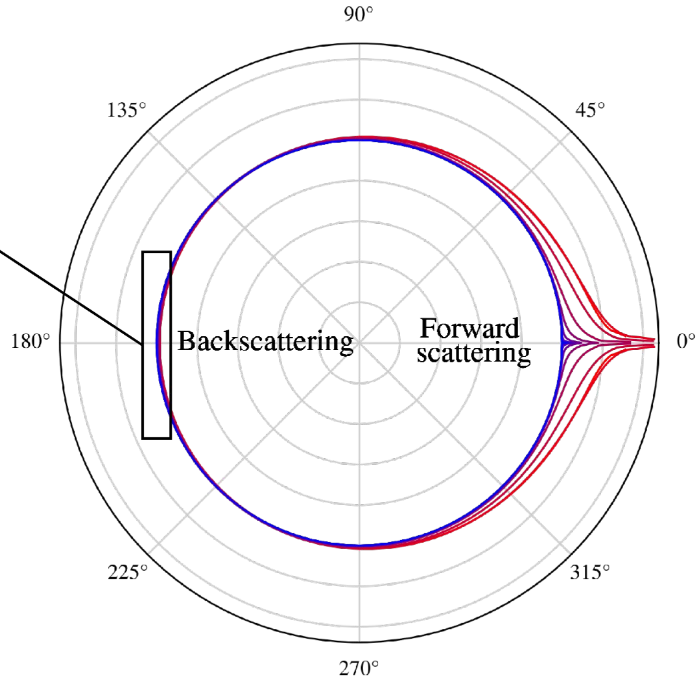
Find γ_m from angular Fourier transform





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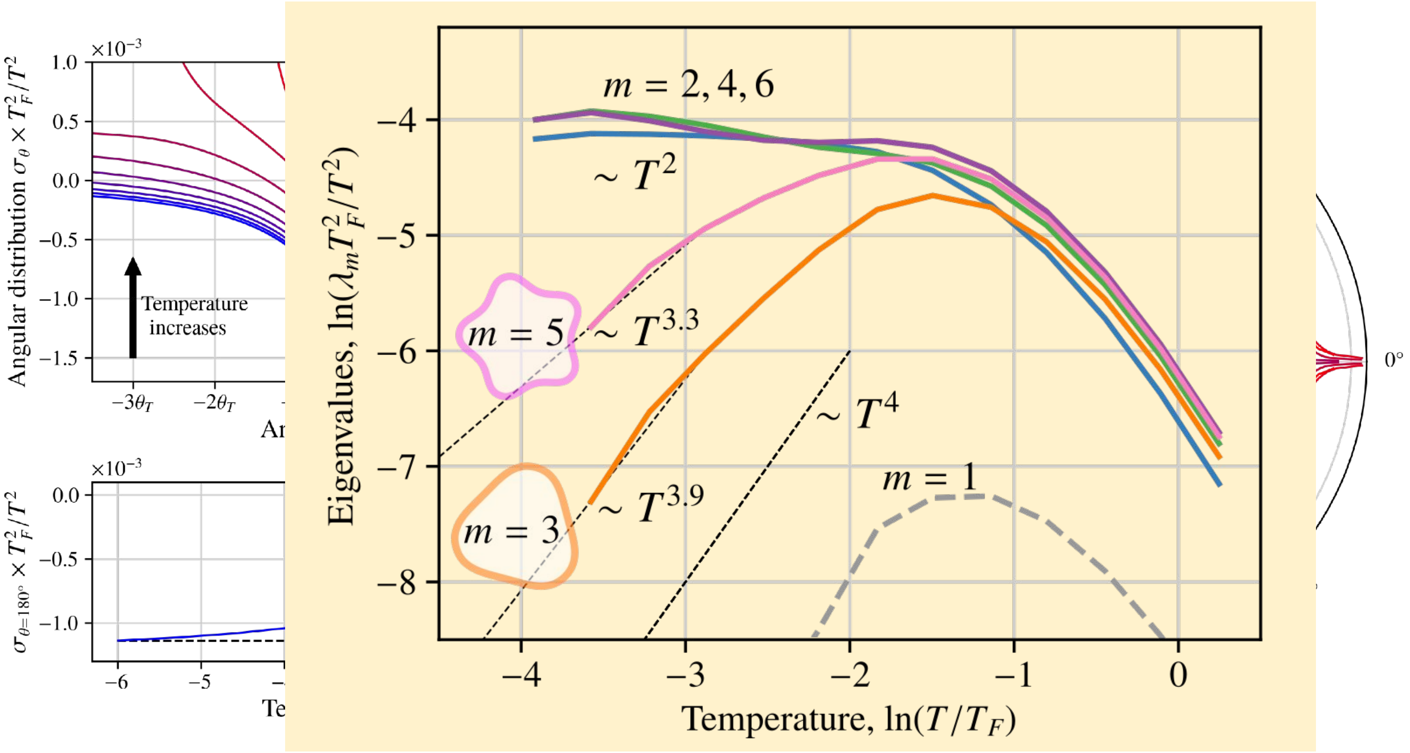

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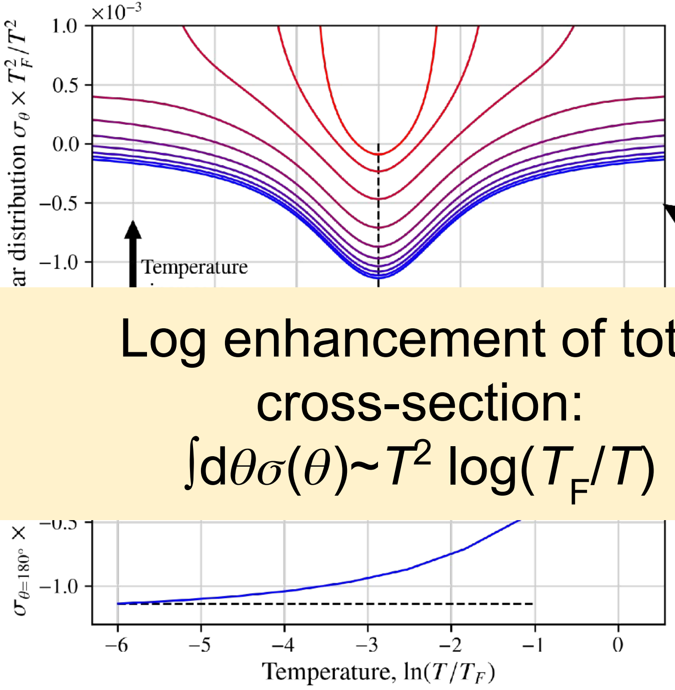
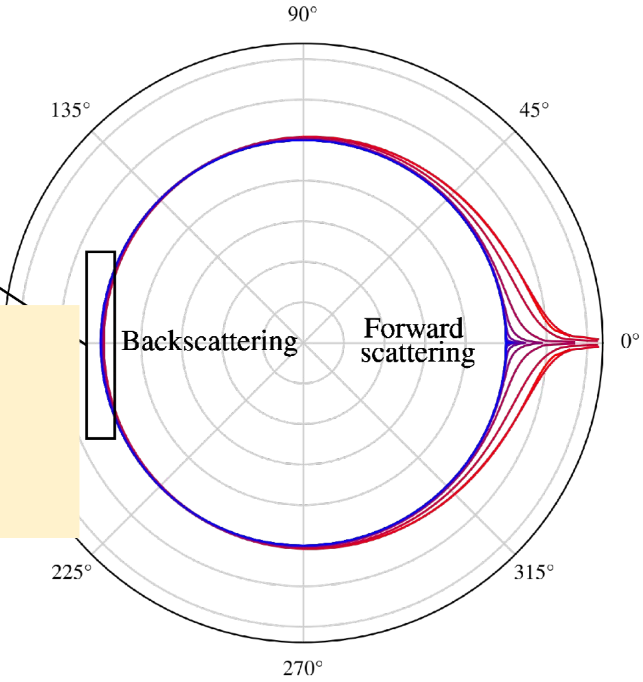
Sharp peaks $\sigma(\theta) \sim T^2/|\theta|$, $\sigma(\theta) \sim T^2/|\theta - \pi|$ for the forward and backscattering directions

Find γ_m from angular Fourier transform





Log enhancement of total cross-section:

$\int d\theta \sigma(\theta) \sim T^2 \log(T_F/T)$

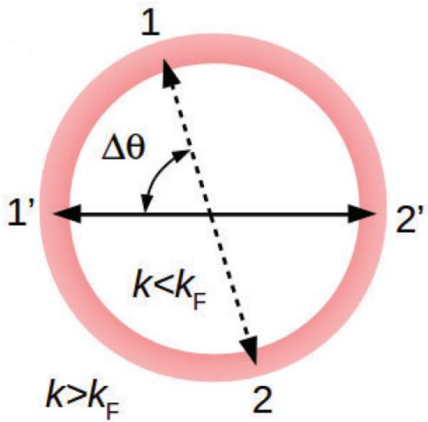
$$\sigma(\theta) = \sum_m e^{im(\theta - \theta_i)}(\gamma_m - \gamma_0)$$

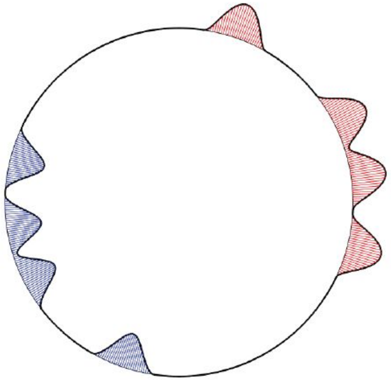
S Kryhin & LL arXiv:2112.05076

Questions?

New transport regime

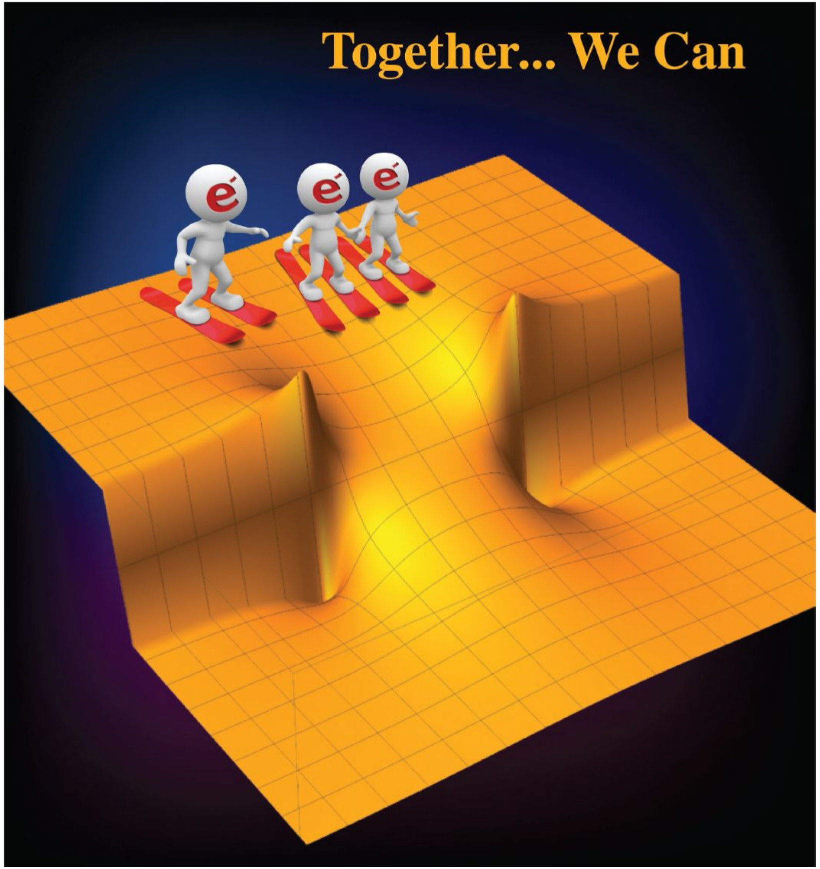
- Head-on collisions
- Slow odd-parity modes
- Half-hydrodynamic, half-ballistic behavior
- Multiple slow modes, scale-dependent σ and η
- Phase space: loops with an odd-parity modulation





- Robustness: generic 2-body interactions and particle dispersion,
- OK for non-circular but convex Fermi surfaces
- Interesting tomographic dynamics at long times, t $\gg \tau_{FL}$

Signatures of electron hydrodynamics



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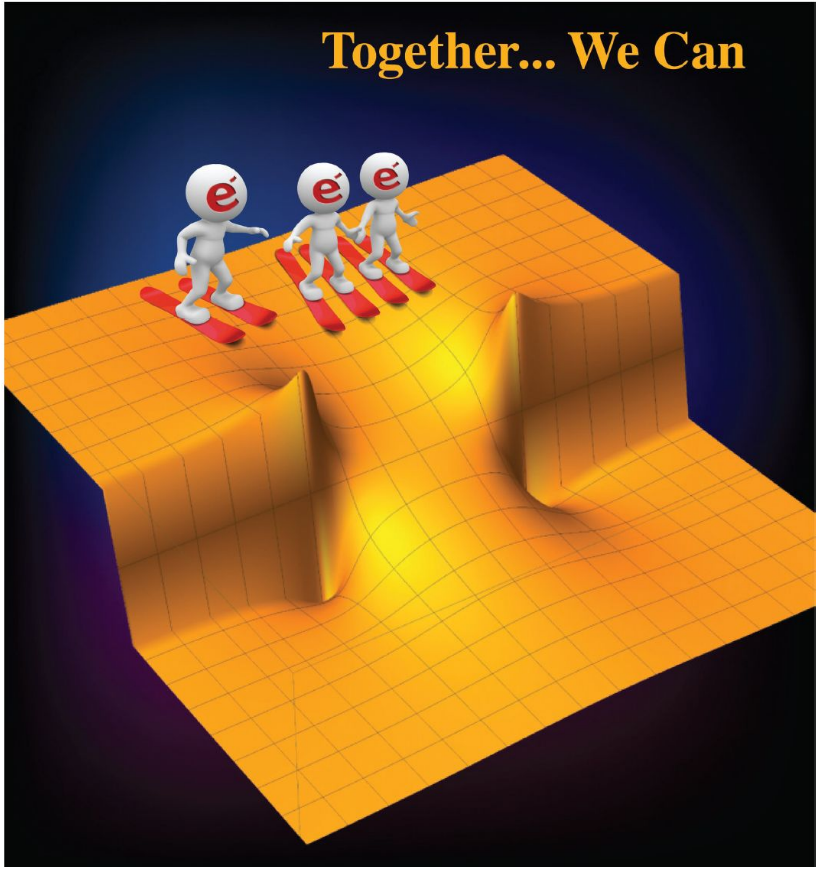
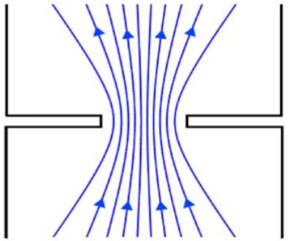
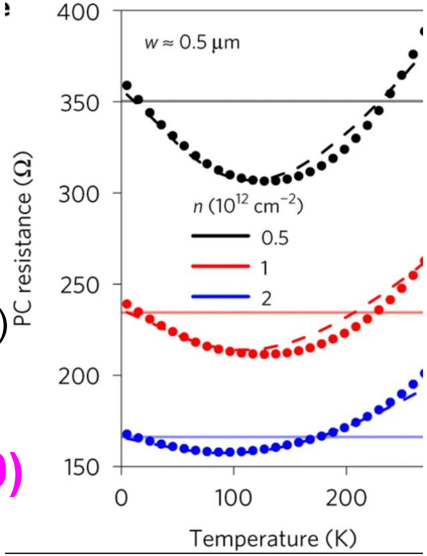




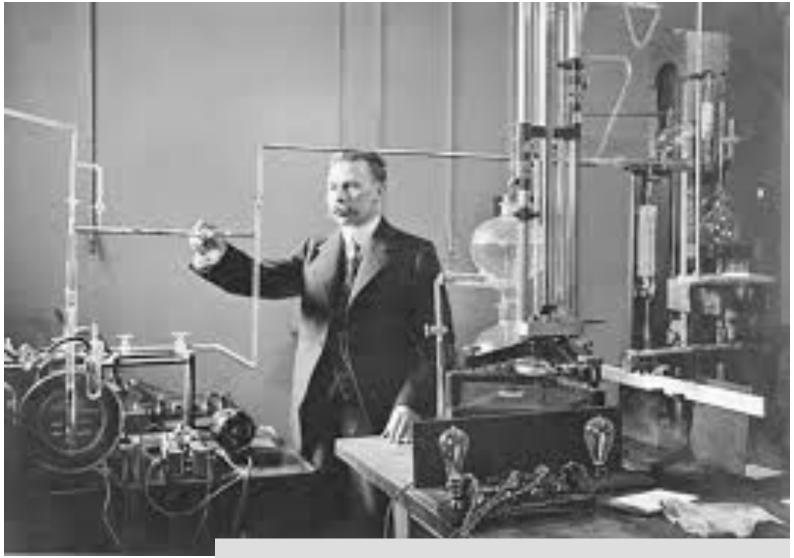
Guo et al PNAS (2017)

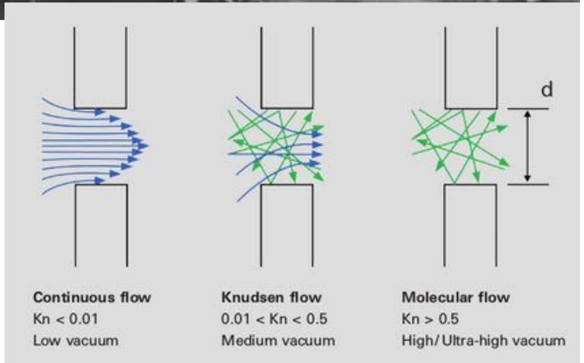


R(T=0) > R(T>0)

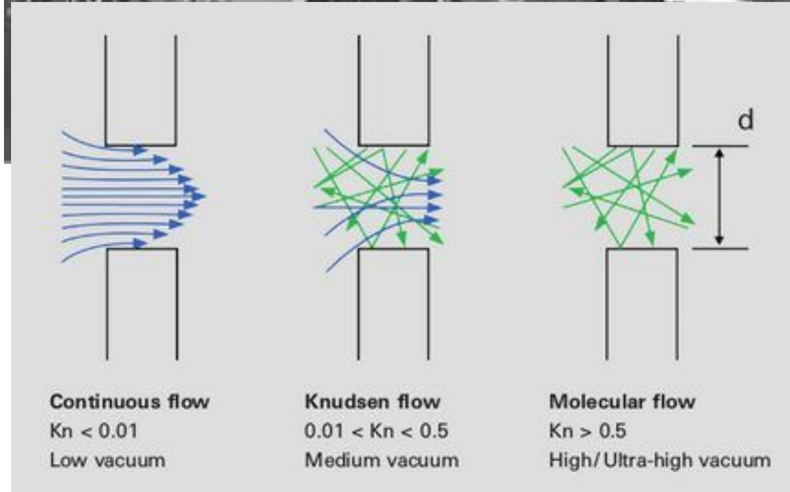
Krishna Kumar et al Nat Phys 2017 (Geim group)

Signatures of hydrodynamics in gases





Signatures of hydrodynamics in gases



Knudsen number [edit]

For a Knudsen gas, the Knudsen number must be greater than 1. The Knudsen number can be defined as:

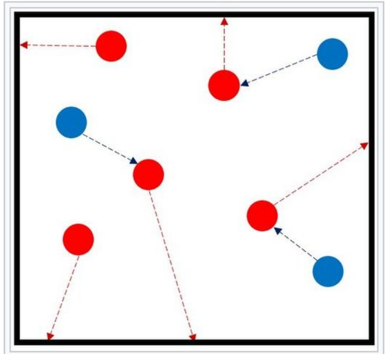
$$\mathrm{Kn} = \frac{\lambda}{\mathrm{L}}$$

where

λ is the mean free path [m]

L is the diameter of the receptacle [m].

When $10^{-1} < \mathrm{Kn} < 10$, the flow regime of the gas is transitional flow. In this regime the intermolecular collisions between gas particles are not yet negligible compared to collisions with the wall. However when $\mathrm{Kn} > 10$, the flow regime is free molecular flow, so the intermolecular collisions between the particles are negligible compared to the collisions with the wall.[3]

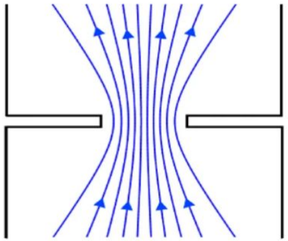


An example of a Knudsen gas. There are more collisions between the gas molecules and the receptacle walls (shown in red) compared to collisions between gas molecules (shown in blue).

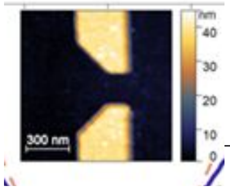
T-linear conductance in electron hydrodynamics

Signatures of electron hydro:

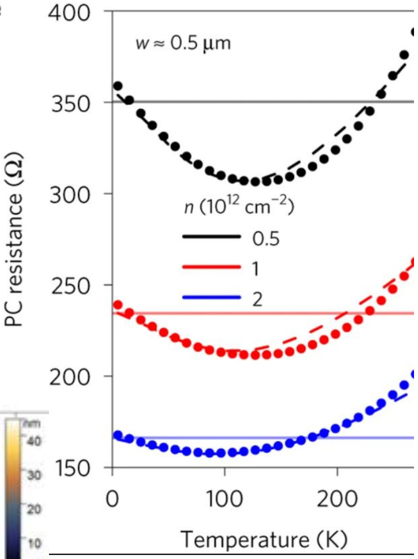
- Negative dR/dT, collisions assist transport
- Reduced dissipation due to ee collisions (self-lubrication effect)
- Superballistic transport: conductance exceeds the Landauer-Sharvin bound



Guo et al PNAS (2017)

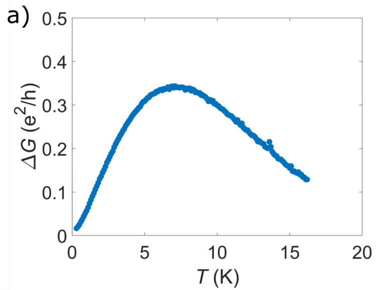


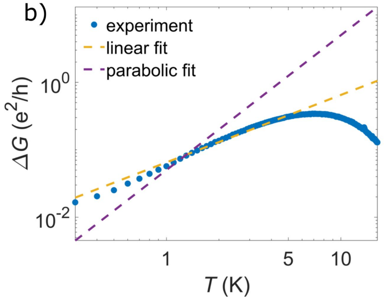
R(T)



Krishna Kumar et al Nat Phys 2017 (Geim group)

G(T)





Ginzburg et al PRX 2023 (Ensslin group)

Kryhin & Levitov arXiv: 2112.05076
Kryhin & Levitov arXiv: 2305.02883
Kryhin, Hong & Levitov arXiv: 2310.08556

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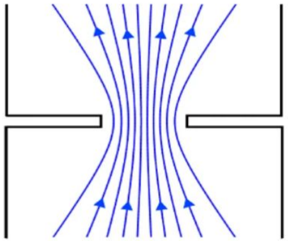
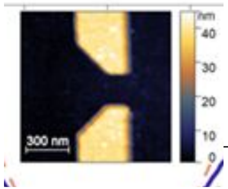
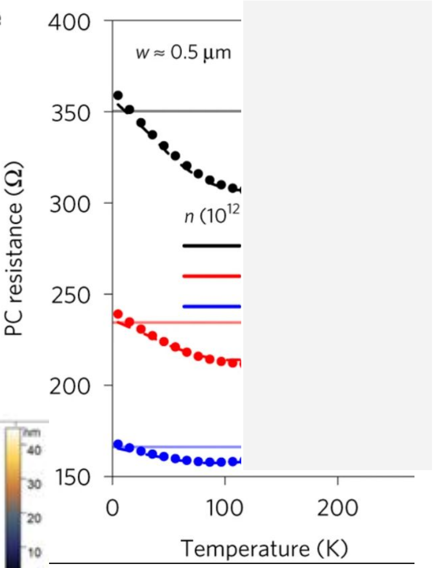


Guo et al PNAS (2017)



R(T)



Krishna Kumar et al Nat Phys 2017 (Geim group)

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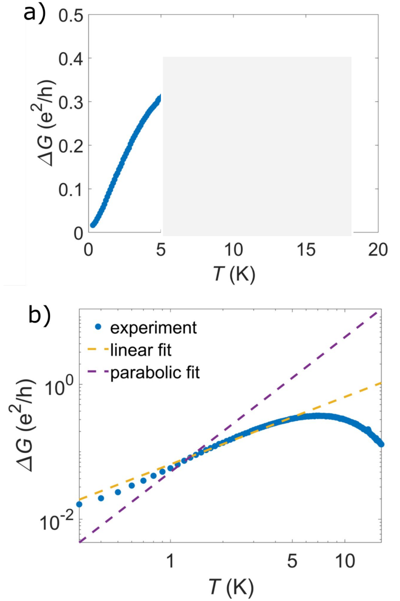


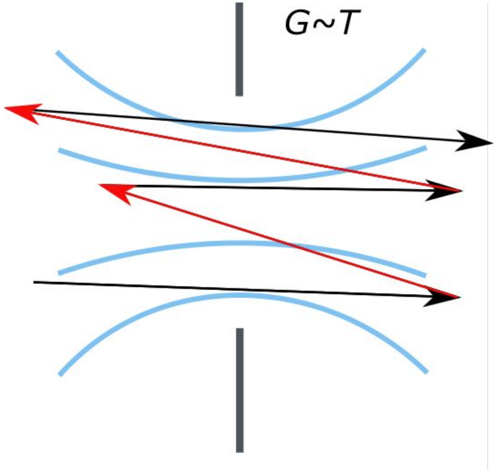
Ginzburg et al PRX 2023 (Ensslin group)

Experiments: **linear T dependence** at low temperatures (contrary to the prediction of Landau FL theory). **A non-Fermi-liquid behavior.**

Kryhin & Levitov arXiv: 2112.05076
Kryhin & Levitov arXiv: 2305.02883
Kryhin, Hong & Levitov arXiv: 2310.08556

Signatures of tomographic fluids

- Contribution of Fermi surface shape modes to transport
- Conductivity and continued fractions
- Cascade of long-lived modes and T-linear conductance
- Tomographic transport
- **Non-Fermi-liquid T dependence.** The measured T dependence is linear, distinct from the benchmark Fermi-liquid T^2 dependence.

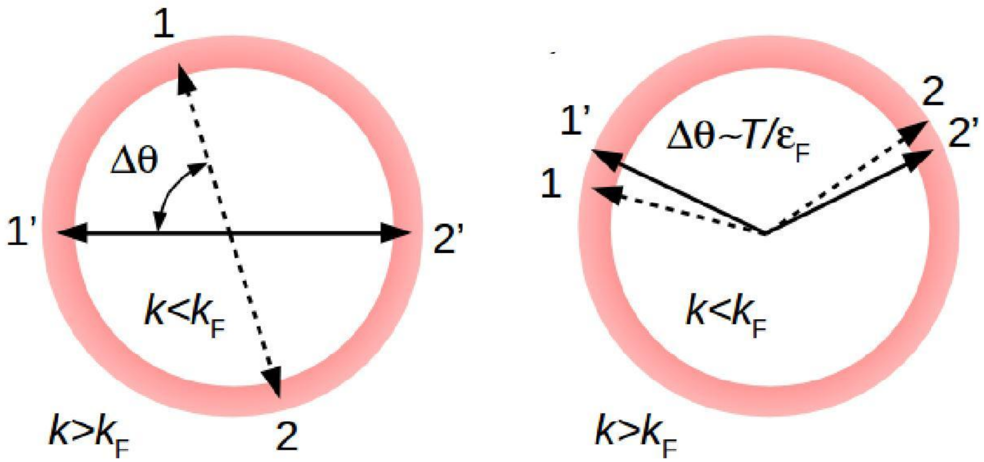


Kryhin & Levitov arXiv: 2112.05076
Kryhin & Levitov arXiv: 2305.02883
Kryhin, Hong & Levitov arXiv: 2310.08556

Origin of non-Fermi-liquid behavior

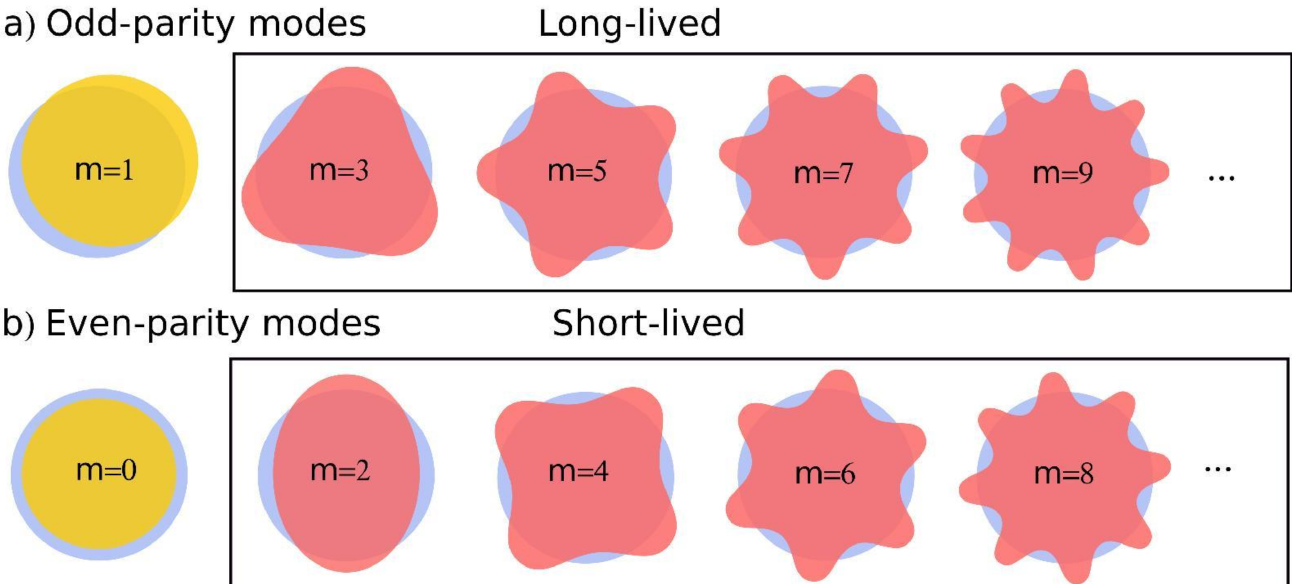
- Momentum conservation and fermion exclusion single out two types of collisions: a) head-on, and b) small-angle
- Angular relaxation dominated by (near) head-on collisions.
- The **even-parity** and **odd-parity** parts of momentum distribution, $\delta f(p)=\delta f(-p)$ & $\delta f(p)=-\delta f(-p)$ relax at different rates

Ledwith, Guo & Levitov 2017, PRL 2019, Ann Phys 2019.



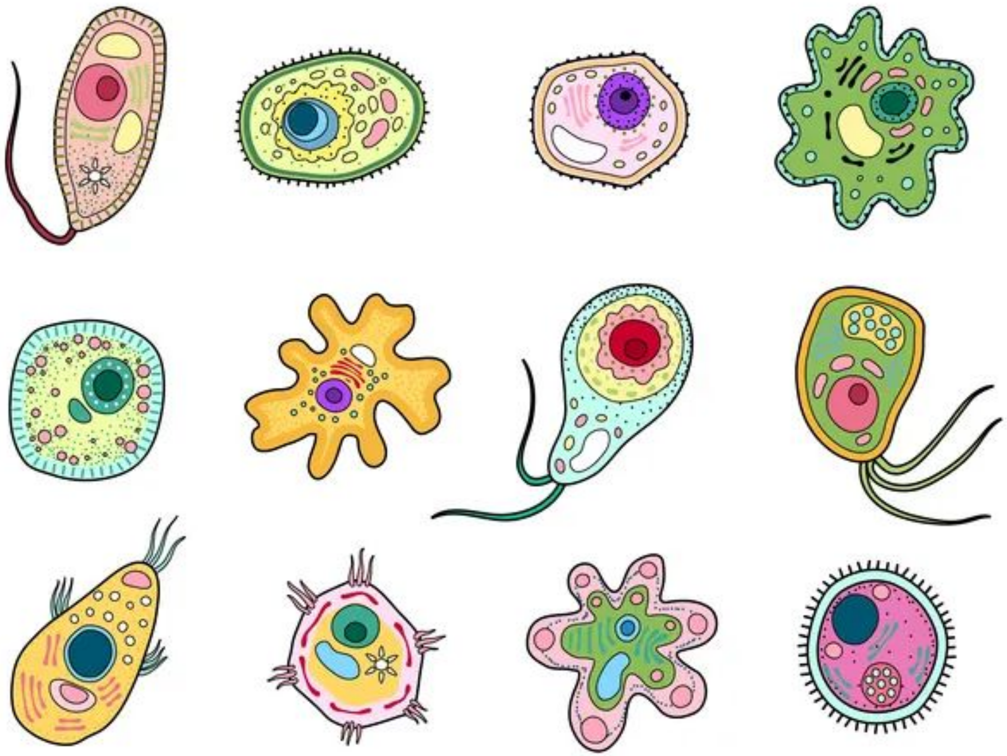
Emerging long-lived excitations (many)

- The **even-parity** and **odd-parity** parts of momentum distribution, $\delta f(p)=\delta f(-p)$ & $\delta f(p)=-\delta f(-p)$ relax at different rates.
- Relaxation rates for the $\delta f(p)$ harmonics of the **odd** and **even** parity can differ by orders of magnitude: $\gamma'/\gamma \sim (T/T_F)^2$, $\gamma \sim T^2/T_F$



Ledwith, Guo & Levitov 2017, PRL 2019, Ann Phys 2019.

2D electron hydro: Fermi surface shapes evolving in space and time, instead of velocity field



Ledwith, Guo & Levitov 2017, PRL 2019, Ann Phys 2019.

Super FL rates from linearized collision operator

Kryhin & Levitov arXiv:2112.05076
Hofmann & Das Sarma (2022)

A method that does not rely on a small parameter $T \ll T_F$

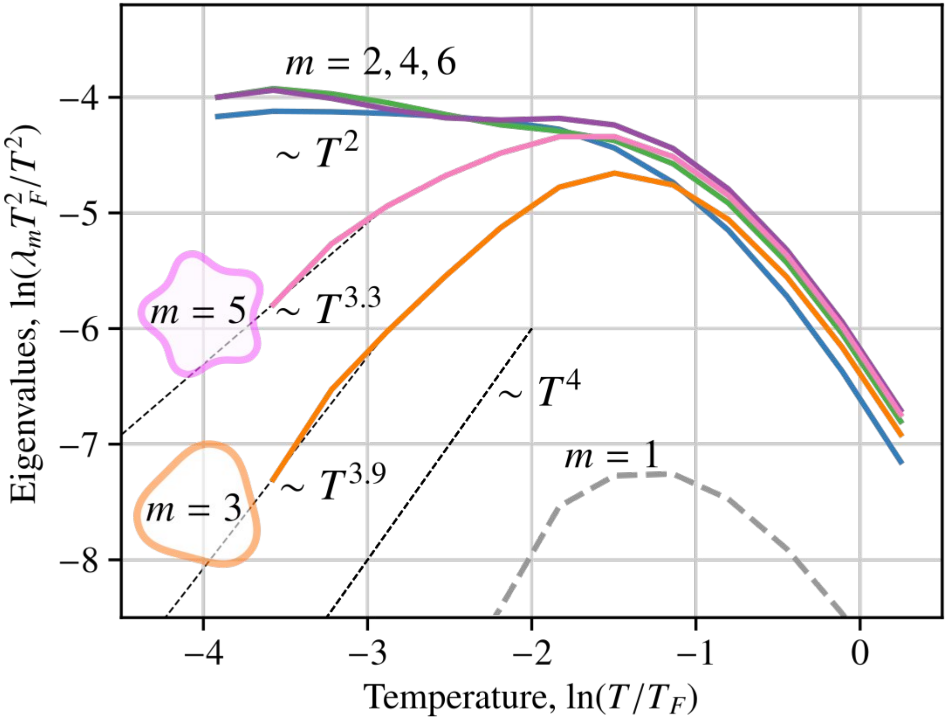
Long-lived excitations: super-Fermi-liquid lifetimes for odd-m harmonics

Super Fermi-liquid scaling $\gamma_m \sim T^{\alpha}$, $\alpha \sim 4$

Conventional Fermi-liquid scaling for even-m harmonics

A **hierarchy** of time scales: $\gamma_{\mathrm{m\ odd}} \ll \gamma_{\mathrm{m\ even}}$

A large family of **soft collective modes**

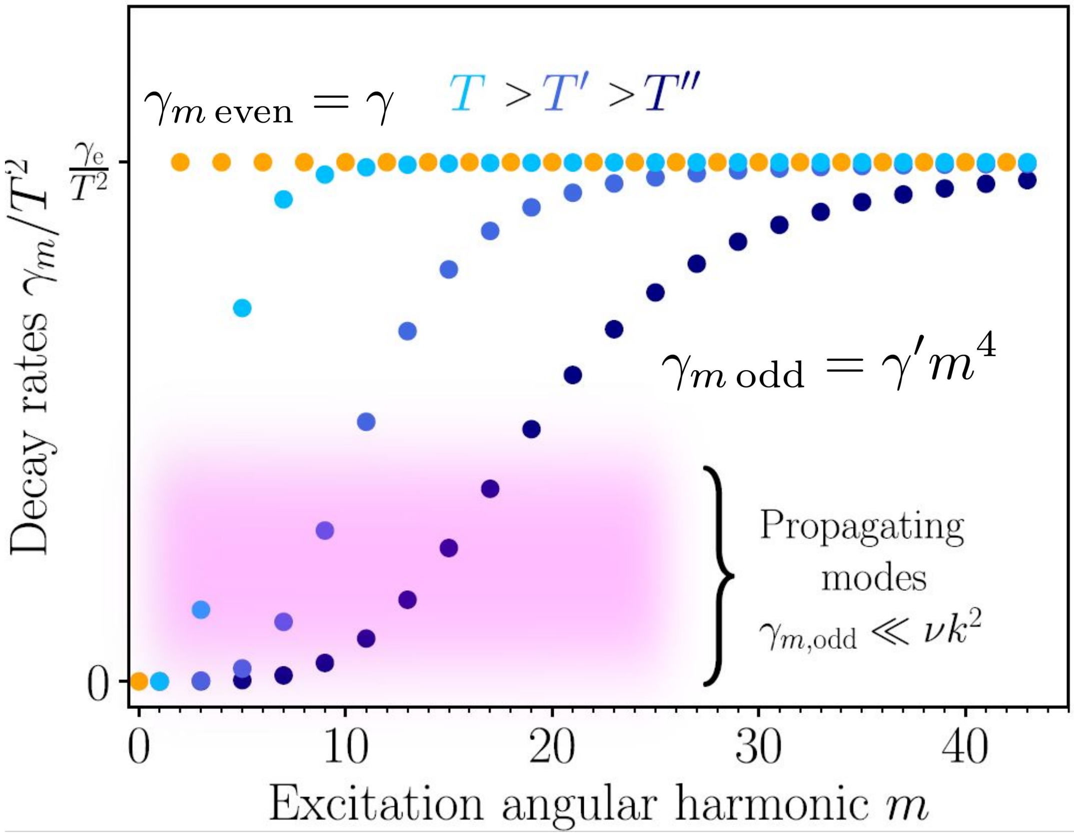


The hierarchy of lifetimes

- The number of long-lived excitations grows rapidly as T decreases
- A wide spectrum of decay rates, a hierarchy of time scales
- At finite k, these excitations form coupled collective modes that can propagate and mediate conduction

$$\gamma \sim T^2/\epsilon_F \qquad \gamma' \sim T^4/\epsilon_F^3$$

$$m \ll m_\star = (\gamma/\gamma')^{1/4}$$



Kryhin, Hong & Levitov arXiv: 2310.08556

Hydrodynamic conductivity scaling. Exact results.

Continued fraction representation
for transverse conductivity

$$\sigma_{\alpha\beta}(k) = \int d^2x e^{-i\boldsymbol{k}(\boldsymbol{x}-\boldsymbol{x}')}\sigma_{\alpha\beta}(\boldsymbol{x}-\boldsymbol{x}')$$

$$j_\alpha(\boldsymbol{x}) = \int d^2x'\sigma_{\alpha\beta}(\boldsymbol{x}-\boldsymbol{x}')E_\beta(\boldsymbol{x}')$$

$$= \sigma(k)(\delta_{\alpha\beta} - \hat{\boldsymbol{k}}_\alpha\hat{\boldsymbol{k}}_\beta)$$

$$\sigma(k) = \frac{D}{\gamma_p + \frac{z}{\Gamma(k)}}, \quad \Gamma(k) = \gamma_2 + \frac{z}{\gamma_3 + \frac{z}{\gamma_4 + \frac{z}{\gamma_5 + \dots}}}$$

$z = k^2 v^2/4$

Non-FL scaling vs. T and k for a wide range of wavenumbers:

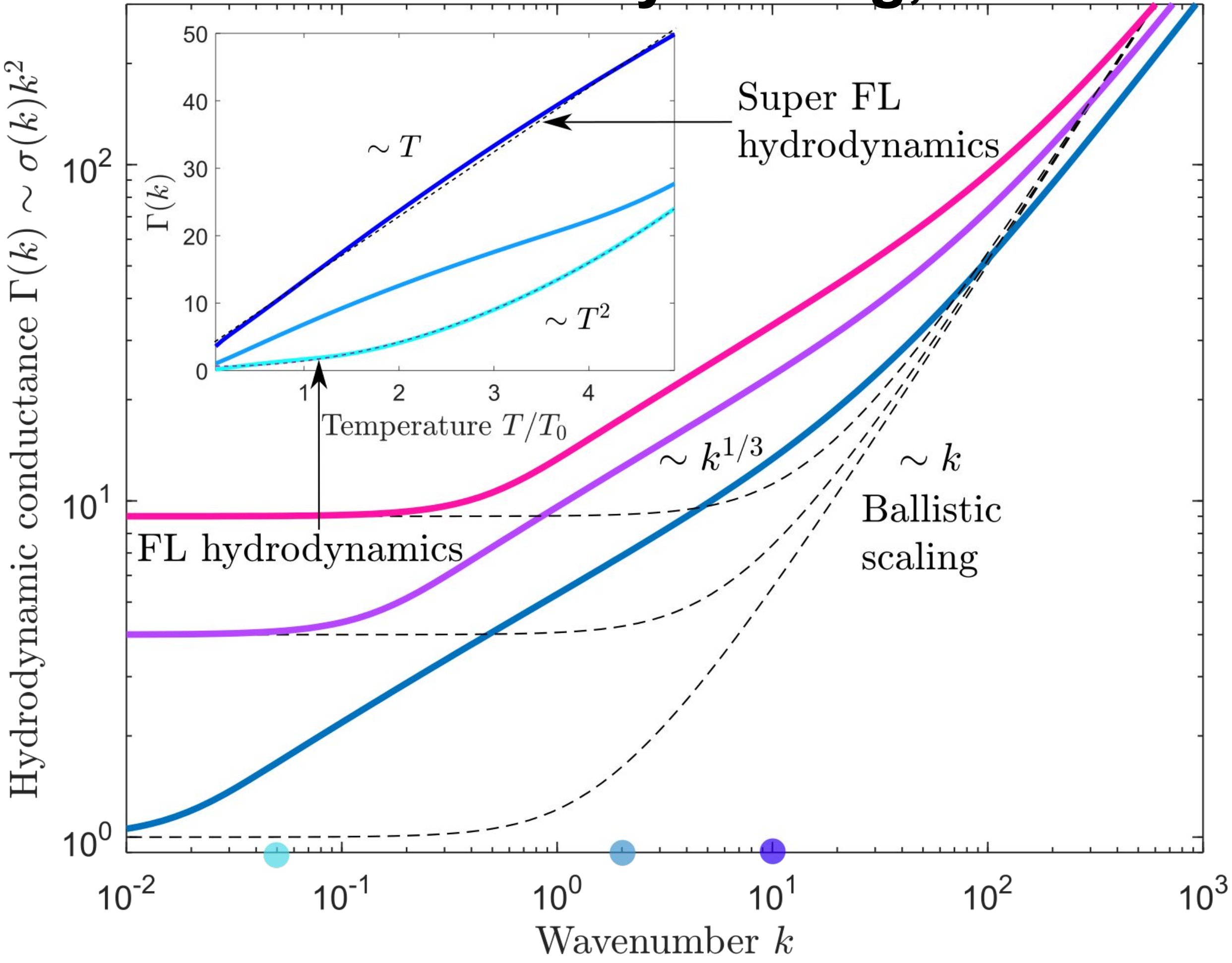
$$\sigma(k) \sim \frac{T}{k^{5/3}}n$$

$$\kappa_< < k < \kappa_>$$

$$\kappa_< = \frac{(\gamma\gamma')^{1/2}}{v_F}$$

$$\kappa_> = \frac{\gamma^{5/4}}{v_F\gamma'^{1/4}}$$

Conductivity scaling, different regimes



Ordinary FL at small wavevectors:

$k < \kappa_<$ $\quad \sigma(k) \sim \dfrac{n}{\nu k^2}$

Non-FL scaling at intermediate wavevectors:

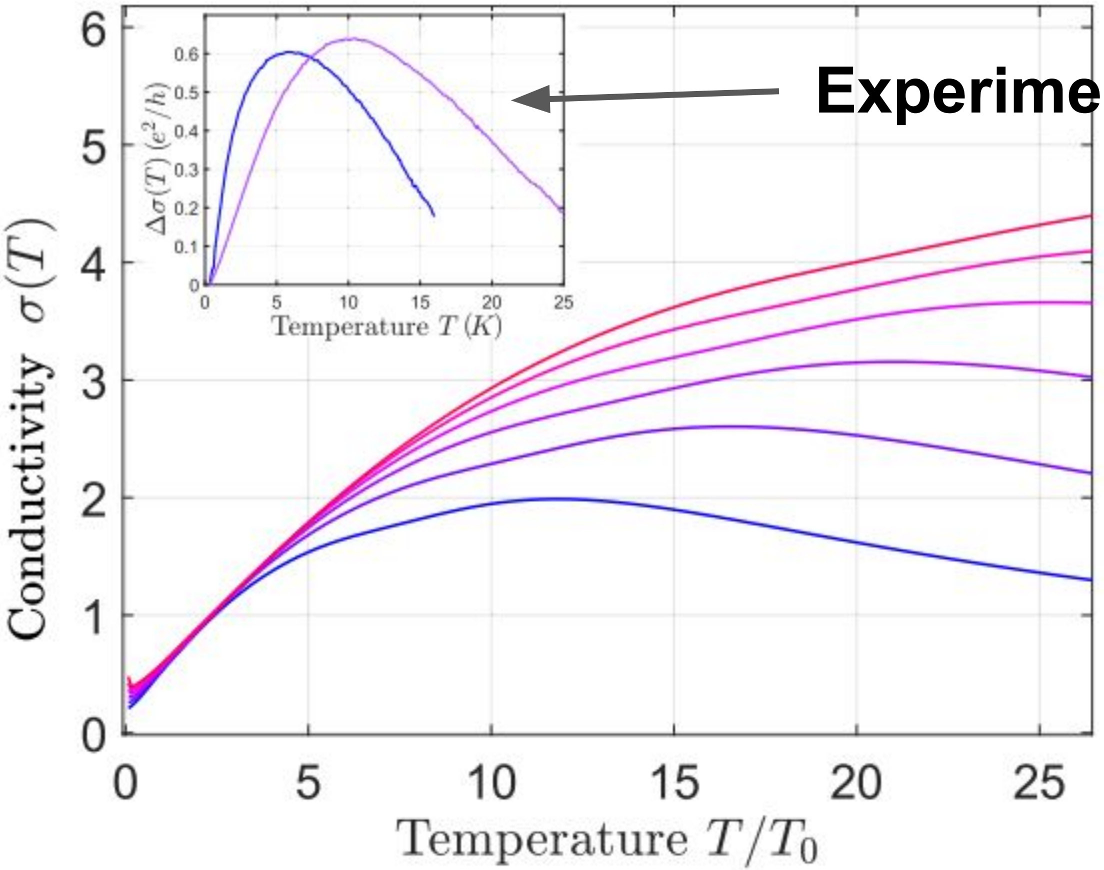
$\kappa_< < k < \kappa_>$

$$\sigma(k) \sim \frac{1}{k^{5/3}} n$$

Ballistic regime at large wavevectors:

$\kappa_> < k$ $\quad \sigma \sim \dfrac{n}{v_F k}$

The interplay of el-el and el-ph scattering



Experimental curves

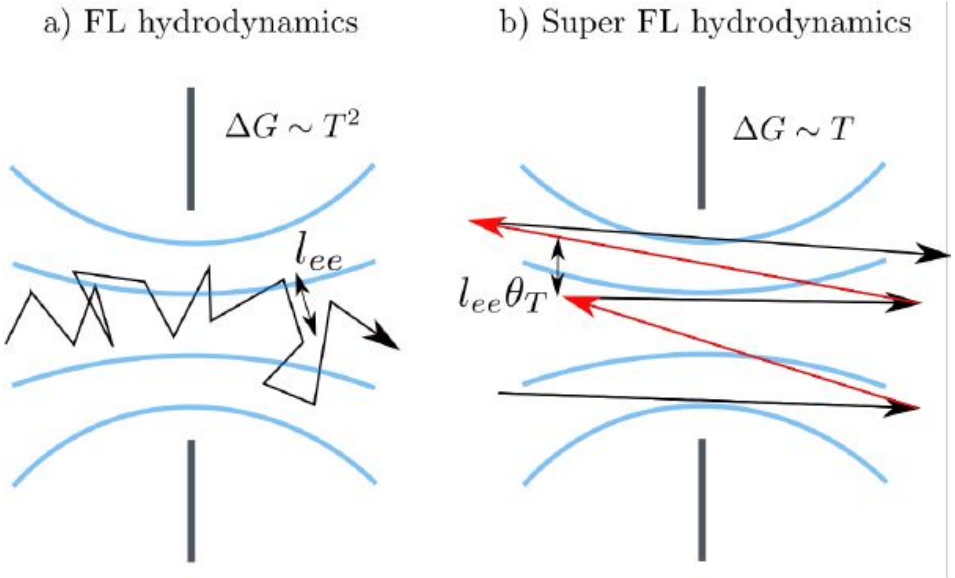
Electron scattering wins at $T<T_{BG}$:

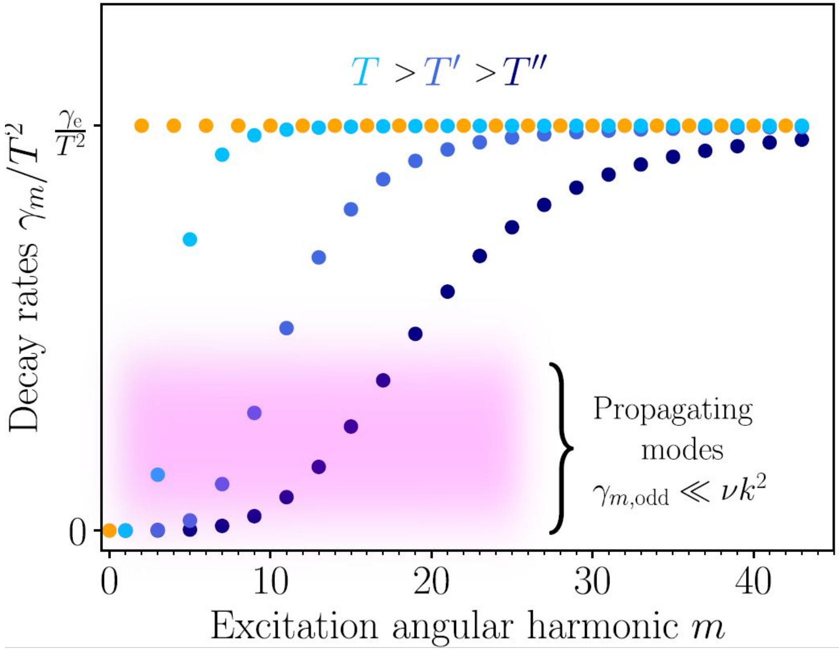
$$\sigma(k) \sim \frac{T}{k^{5/3}} n$$

Electron-phonon scattering wins at $T>T_{BG}$:

Non-FL hydro: a cascade of coupled soft odd-m modes

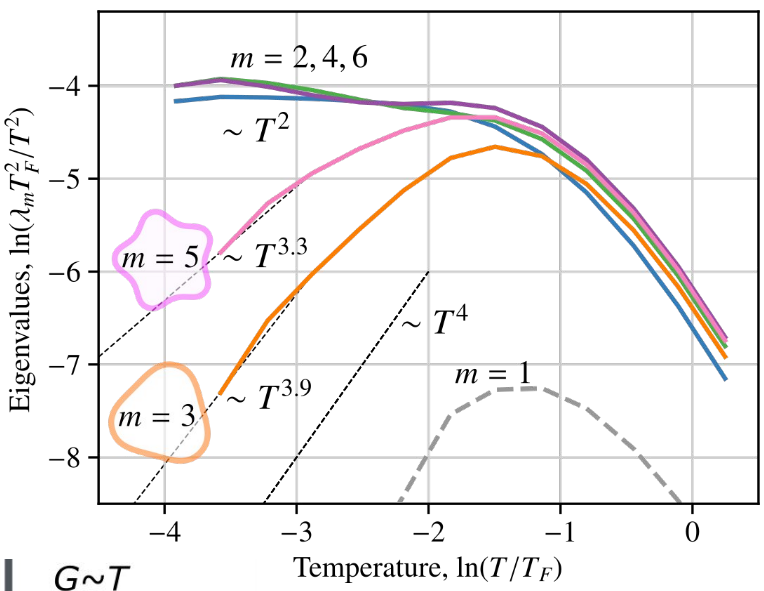
- **Non-FL T dependence** that extends down to lowest temperatures
- **Power laws** ~T and ~$1/k^{5/3}$ in a wide range of k
- Strong effects of particle/hole retroreflections
- Long-time memory effects

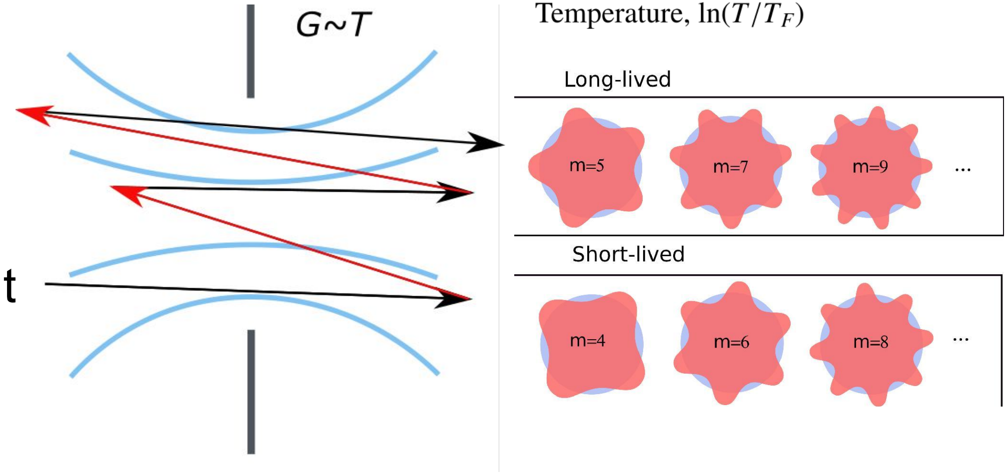




Discussion

- The T-linear transport observed in experiments is **a smoking gun for nonclassical hydrodynamics**
- Kolmogorov-like -5/3 scaling for conductivity: $\sigma(k) \propto 1/k^{5/3}$
- Complex behavior due to cascade of **many coupled long-lived modes**
- Expect similar hydro effects (dG/dT>0 and power laws) in other systems.
- Bernal bilayer graphene and other correlated systems that host soft quantum-critical modes or emergent gauge fields activated at low T

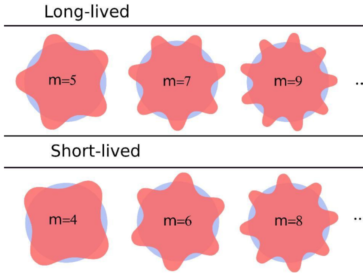




Kryhin, Hong & Levitov arXiv: 2310.08556

Questions?

Collective modes and cyclotron resonance in tomographic Fermi liquids



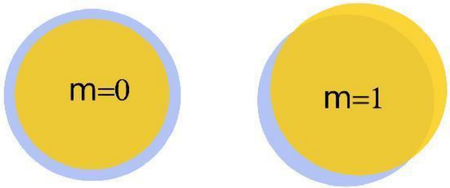
"Odd" and "even" modes have very different lifetimes and relaxation rates: $\gamma' \sim T^4 \ll T^2 \sim \gamma \quad (T \ll \varepsilon_F)$

- Probe long-lived modes by cyclotron resonance (CR)?
- Excite different Fermi surface shape modulations
- Access higher harmonics individually at high-order CR
- Excitation lifetime = 1/resonance width, **even-odd effect**
- Interesting **Fermi-liquid effects** and temperature scaling

Inspiring discussions: Peter Armitage, Dmitri Basov

Collective excitations in 2D Fermi liquids at B=0: plasmon modes

- S. J. Allen, D. C. Tsui, and R. A. Logan, Observation of the Two-Dimensional Plasmon in Silicon Inversion Layers, Phys. Rev. Lett. 38, 980 (1977).
- E. H. Hwang and S. Das Sarma, Dielectric function, screening, and plasmons in two-dimensional graphene, Phys. Rev. B 75, 205418 (2007).
- L. S. Levitov, A. V. Shtyk and M. V. Feigelman, Electron-electron interactions and plasmon dispersion in graphene, Phys. Rev. B 88, 235403 (2013)
- A. Lucas and S. Das Sarma, Electronic sound modes and plasmons in hydrodynamic two-dimensional metals, Phys. Rev. B 97, 115449 (2018)
- A. Klein, D. L. Maslov, L. P. Pitaevskii, and A. V. Chubukov, Collective modes near a Pomeranchuk instability in 2D, Phys. Rev. Res. 1, 033134 (2019).
- and many more



Fermi-liquid renormalized plasmon mode dispersion:

$$\omega^2 = Y\lambda q, \quad Y = (1+F_1)\,v, \quad \lambda = \frac{Ne^2 p_F}{2\kappa\hbar^2}$$

Long-lived modes probed by CR

- Excite m-th harmonic of Fermi surface at m-th order CR
- Free-particle resonances at $\omega_m = \omega_c m$
- Excitation lifetime = 1/resonance width, even-odd effect
- Fermi-liquid renormalized resonances: $\omega_m = \omega_c m(1+F_m)/(1+F_1)$
- Yet, the even/odd difference in lifetimes remains robust under Fermi liquid

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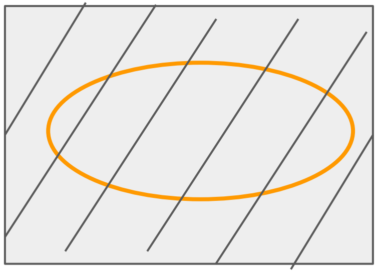
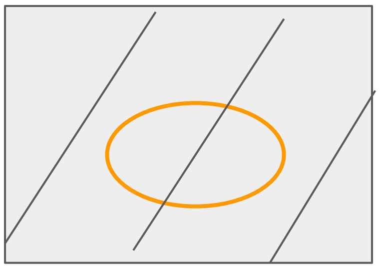
- But: in parabolic bands, a spatially uniform field excites only the fundamental m=1 harmonic, the higher harmonics (m>2) remain invisible
- Therefore, a grating (periodic array) must be used to access higher modes

Explore different regimes:



$R_c > \lambda$



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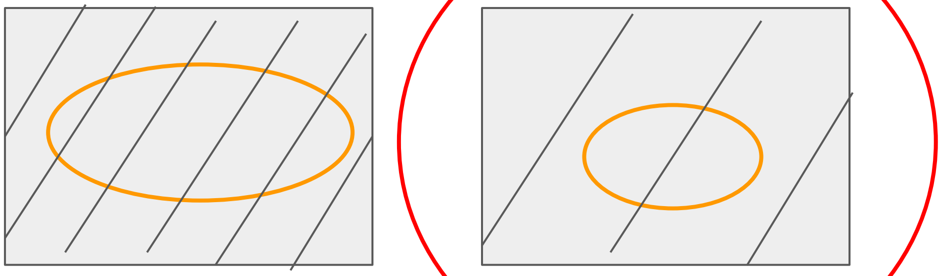


$R_c > \lambda$ $\quad$ $R_c < \lambda$

Long-lived modes probed by CR: **theory**

$$\frac{df}{dt}(\vec{p},\vec{r},t) = \frac{\partial f}{\partial t} + \frac{d\vec{r}}{dt}\cdot\vec{\nabla}_r f + \frac{d\vec{p}}{dt}\cdot\vec{\nabla}_p f = I(f)$$

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\vec{\nabla}_r f + e(\vec{E} + [\vec{v}\times\vec{B}])\cdot\vec{\nabla}_p f = I(f) \quad \text{, where} \quad f(\vec{p},\vec{r},t) = f^{(0)}(\vec{p}) + \delta f(\vec{p},\vec{r},t)$$

$$(\partial_t + \vec{v}\cdot\vec{\nabla}_r + e[\vec{v}\times\vec{B}]\cdot\vec{\nabla}_p) - I)\delta f = -e\frac{\partial f^{(0)}}{\partial\varepsilon}\vec{E}\cdot\vec{v} \quad \text{, where} \quad \vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r}-\omega t)}$$

$$\delta f(\vec{p},\vec{r},t) = -\frac{\partial f^{(0)}}{\partial\varepsilon}e^{i(\vec{k}\vec{r}-\omega t)}\sum_{m=-\infty}^{\infty}\delta\varphi_m(p)e^{im\theta} \quad \text{, where } \theta \text{ is in } p\text{-space}$$

$$\delta\varphi_1 = \frac{\bar{\mathcal{E}}}{\tilde{\gamma}_1 + \frac{z}{\tilde{\gamma}_2 + \frac{z}{\tilde{\gamma}_3+\dots}}} = \frac{\bar{\mathcal{E}}}{\Gamma_1} \quad ; \quad \delta\varphi_{-1} = \frac{\mathcal{E}}{\tilde{\gamma}_{-1} + \frac{z}{\tilde{\gamma}_{-2} + \frac{z}{\tilde{\gamma}_{-3}+\dots}}} = \frac{\mathcal{E}}{\Gamma_{-1}}$$

$$\text{where} \quad \mathcal{E} = \frac{ev_F}{2}(E_x + iE_y), \quad \tilde{\gamma}_m = \gamma_m - i(\omega - m\omega_c), \quad z = (kv_F)^2/4$$

Long-lived modes probed by CR: **theory**

$$\delta\varphi_1 = \frac{\bar{\mathcal{E}}}{\tilde{\gamma}_1 + \frac{z}{\tilde{\gamma}_2 + \frac{z}{\tilde{\gamma}_3 + \ldots}}} = \frac{\bar{\mathcal{E}}}{\Gamma_1} \quad ; \quad \delta\varphi_{-1} = \frac{\mathcal{E}}{\tilde{\gamma}_{-1} + \frac{z}{\tilde{\gamma}_{-2} + \frac{z}{\tilde{\gamma}_{-3} + \ldots}}} = \frac{\mathcal{E}}{\Gamma_{-1}}$$

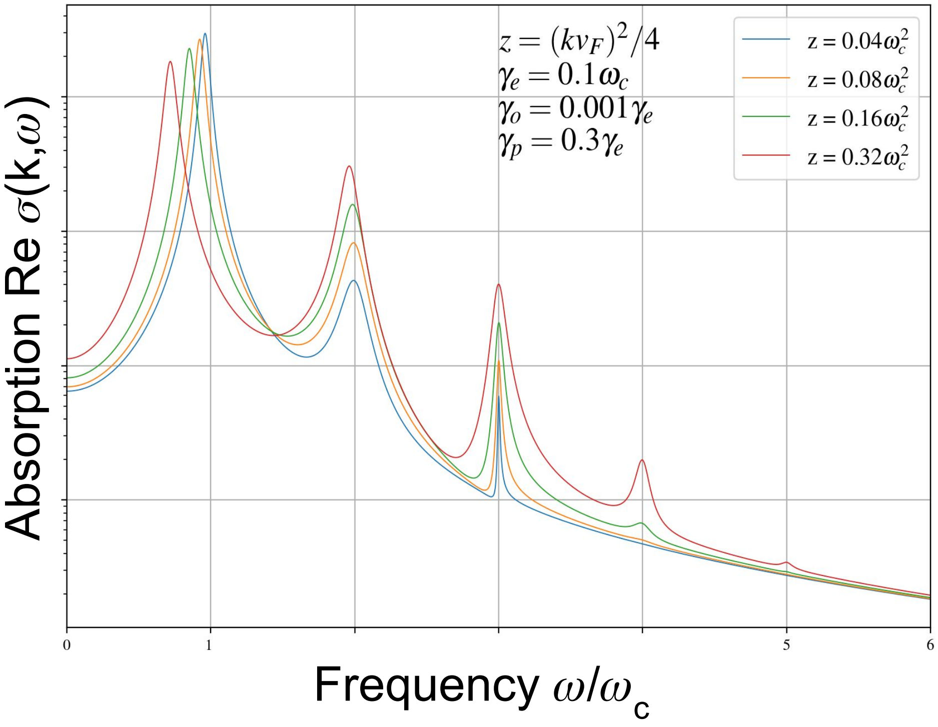
$$\text{where} \quad \mathcal{E} = \frac{ev_F}{2}(E_x + iE_y), \quad \tilde{\gamma}_m = \gamma_m - i(\omega - m\omega_c), \quad z = (kv_F)^2/4$$

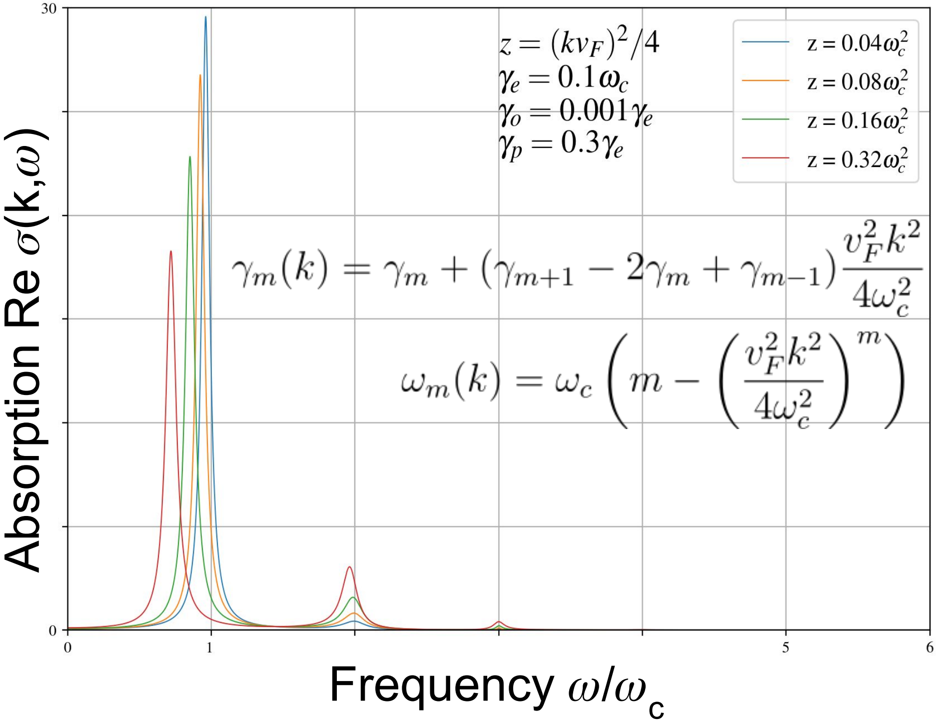
$$\vec{j}(\vec{r}, t) = \int \frac{d^2p}{(2\pi)^2} e\vec{v} \cdot \delta f(\vec{p}, \vec{r}, t) = \hat{\sigma}(k, \omega)\vec{E}_0 e^{i(\vec{k}\vec{r} - \omega t)}$$

$$\hat{\sigma}(k, \omega) = \frac{e^2 v_F^2 \nu}{4} \begin{bmatrix} \frac{1}{\Gamma_1} + \frac{1}{\Gamma_{-1}} & \frac{i}{\Gamma_{-1}} - \frac{i}{\Gamma_1} \\ \frac{i}{\Gamma_1} - \frac{i}{\Gamma_{-1}} & \frac{1}{\Gamma_1} + \frac{1}{\Gamma_{-1}} \end{bmatrix} \quad \text{where} \quad \nu = -\int \frac{d^2p}{(2\pi)^2} \frac{\partial f^{(0)}}{\partial \varepsilon}$$

$$W = \frac{1}{2}\Re[\vec{E}_0 \cdot \hat{\sigma}(k, \omega)\vec{E}_0] \propto \frac{1}{\Gamma_1(k, \omega)} + \frac{1}{\Gamma_{-1}(k, \omega)}$$

Long-lived modes probed by CR: **predictions**

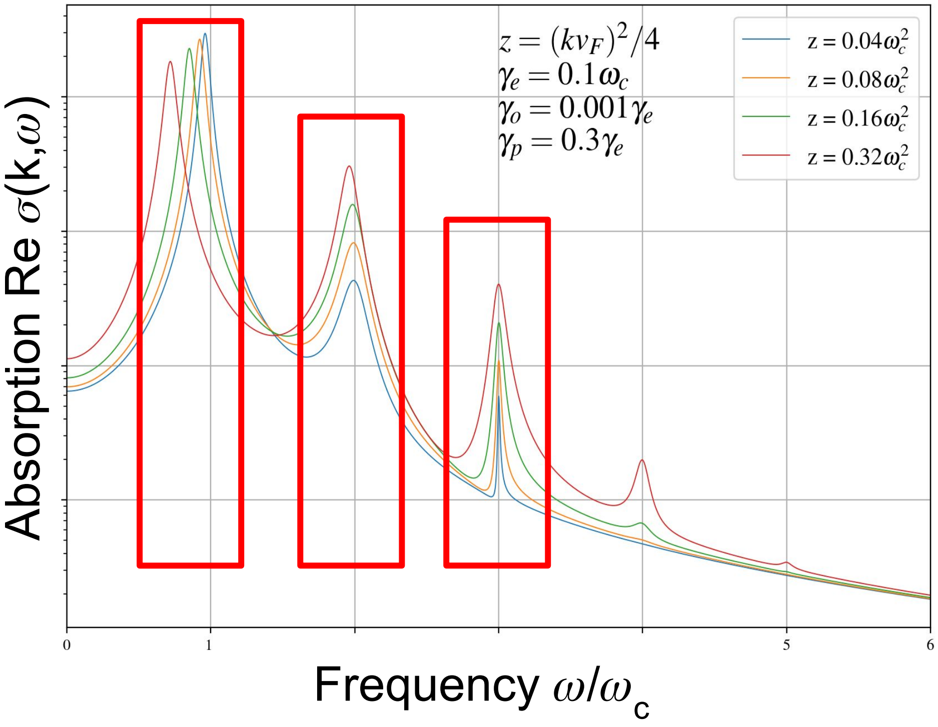


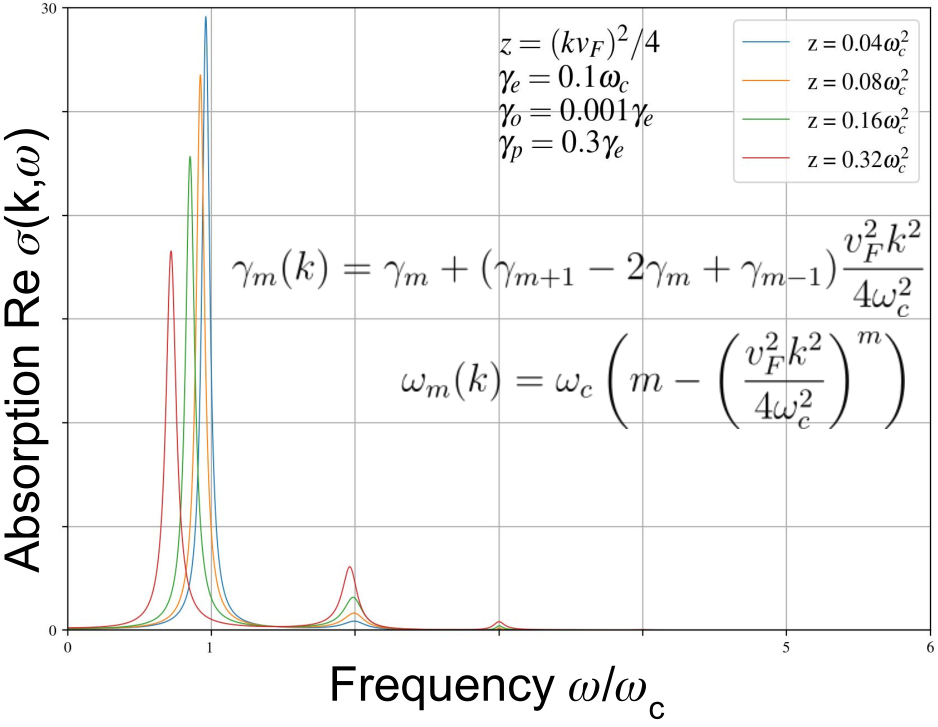


m=1 m=2 m=3 m=4 m=5

m=1 m=2 m=3

Long-lived modes probed by CR: **predictions**





m=1 m=2 m=3 m=4 m=5

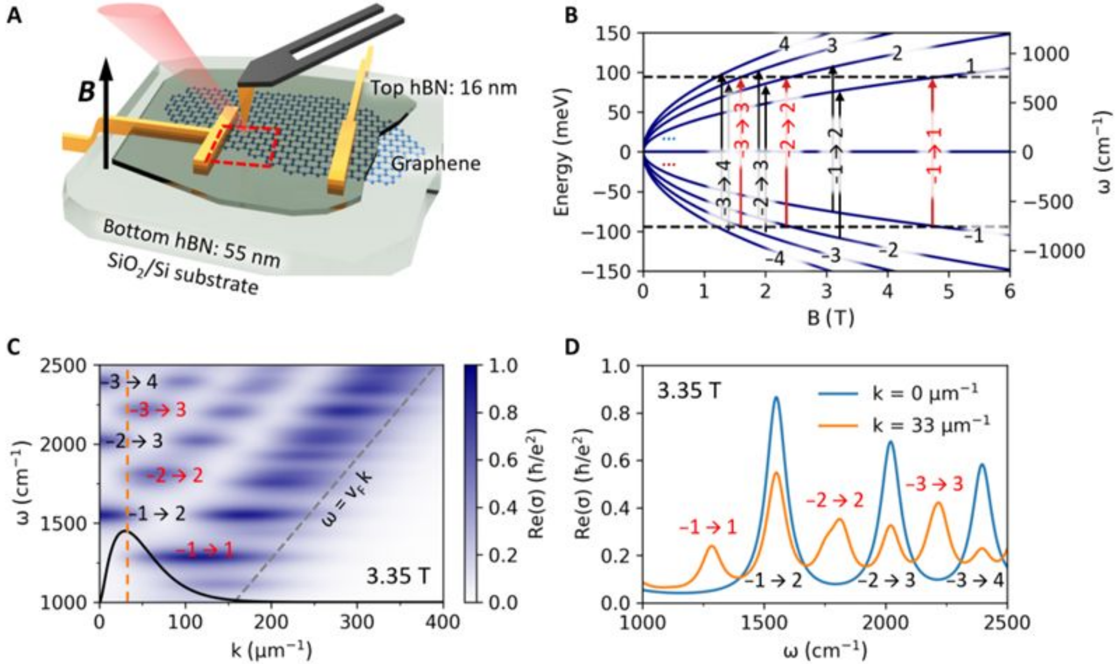
m=1 m=2 m=3

Questions?

Landau-phonon polaritons in Dirac heterostructures

Lukas Wehmeier[1,2]*, Suheng Xu[3], Rafael A. Mayer[1], Brian Vermilyea[4], Makoto Tsuneto[1], Michael Dapolito[1,3], Rui Pu[1], Zengyi Du[1], Xinzhong Chen[1,3], Wenjun Zheng[1], Ran Jing[1,5], Zijian Zhou[1], Kenji Watanabe[6], Takashi Taniguchi[7], Adrian Gozar[8,9,10], Qiang Li[1,5], Alexey B. Kuzmenko[11], G. Lawrence Carr[2], Xu Du[1], Michael M. Fogler[4]*, D. N. Basov[3]*, Mengkun Liu[1,2]*

Near-field spectroscopy of higher Landau levels in graphene monolayer/hBN

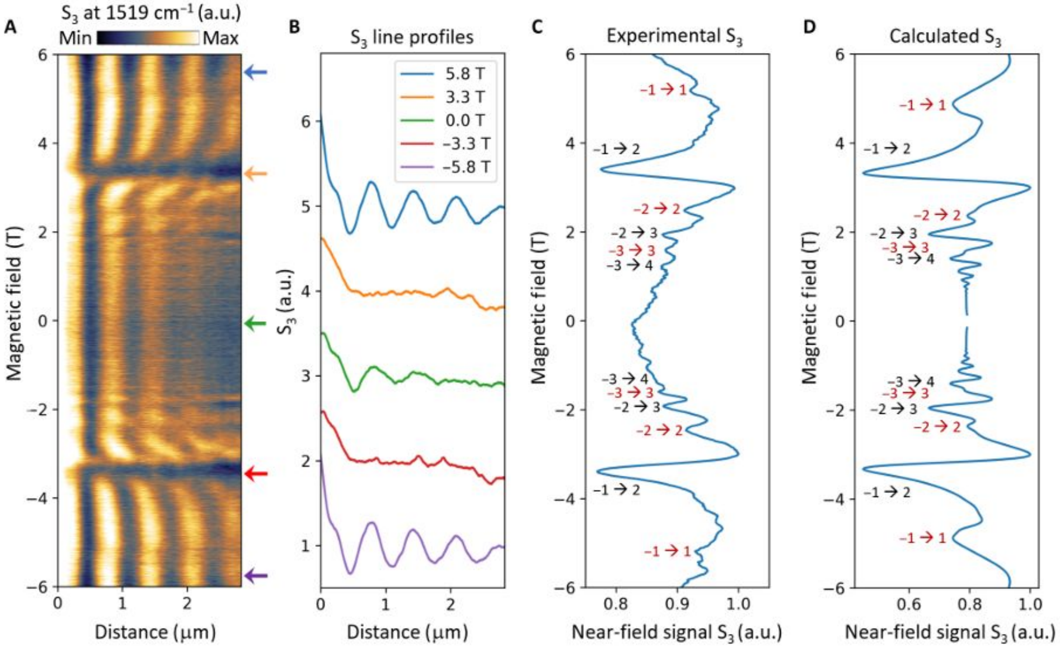


Wehmeier et al., Sci. Adv. 10, eadp3487 (2024)

Landau-phonon polaritons in Dirac heterostructures

Lukas Wehmeier[1,2]*, Suheng Xu[3], Rafael A. Mayer[1], Brian Vermilyea[4], Makoto Tsuneto[1], Michael Dapolito[1,3], Rui Pu[1], Zengyi Du[1], Xinzhong Chen[1,3], Wenjun Zheng[1], Ran Jing[1,5], Zijian Zhou[1], Kenji Watanabe[6], Takashi Taniguchi[7], Adrian Gozar[8,9,10], Qiang Li[1,5], Alexey B. Kuzmenko[11], G. Lawrence Carr[2], Xu Du[1], Michael M. Fogler[4]*, D. N. Basov[3]*, Mengkun Liu[1,2]*

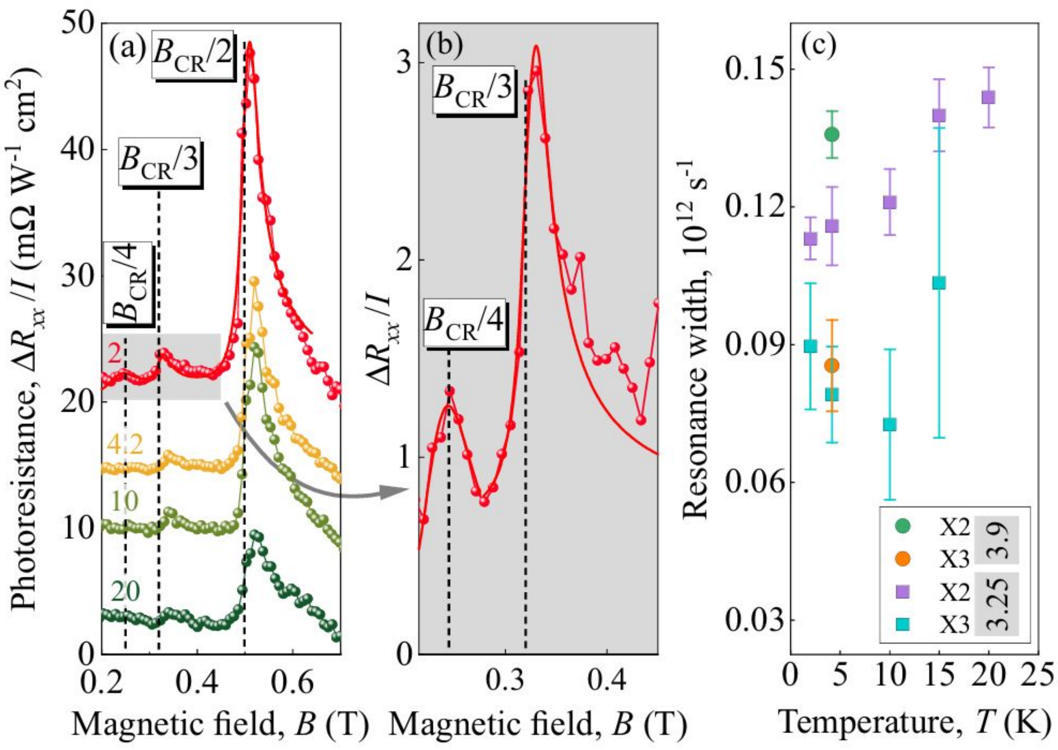
Broadened resonances due to finite wavenumbers of LPP modes



Wehmeier et al., Sci. Adv. 10, eadp3487 (2024)

Testing the tomographic Fermi liquid hypothesis with high-order cyclotron resonance arXiv:2409.05147 Moiseenko, Kapralov, Svintsov, Monch, Ganichev and Bandurin

“Extraction of cyclotron resonance lifetimes from an experiment on terahertz photoconductivity in graphene shows that third-order resonance is systematically narrower than second-order one, supporting the prediction of tomographic Fermi liquid hypothesis”



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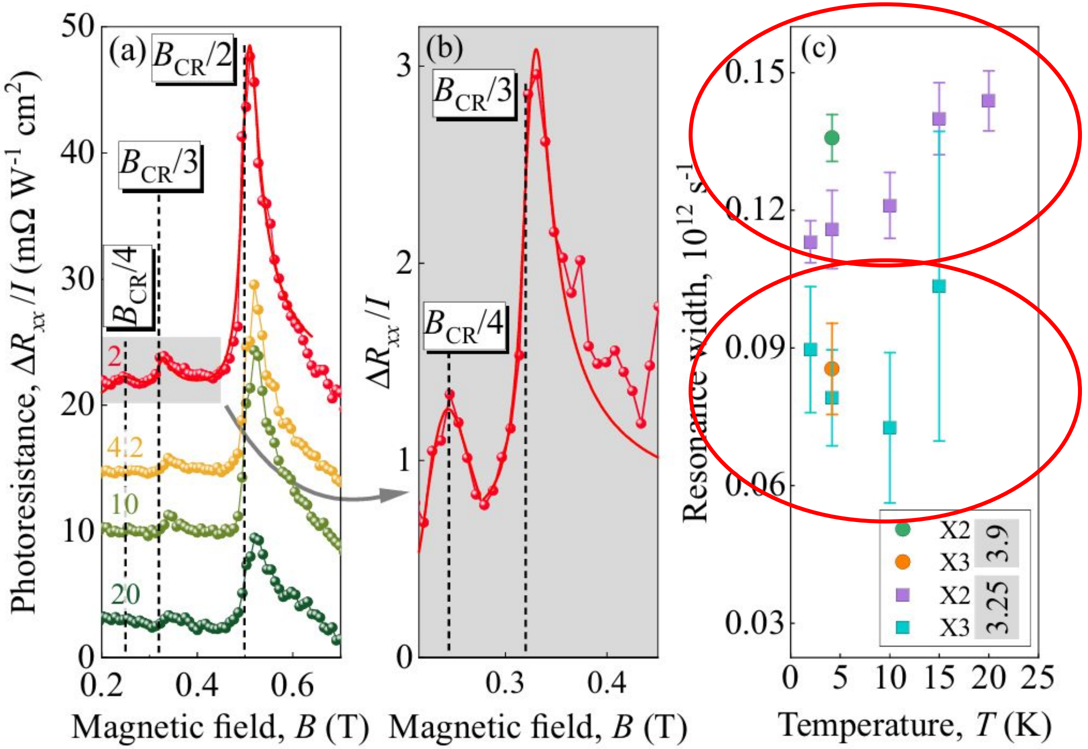

Conclusions

- CR in the presence of spatial modulation **allows to observe "dark resonances"** at higher CR harmonics
- In the long-wavelength limit $\lambda \gg r_c$:
  (i) Higher resonances are weak, scale as powers of wavenumber $k=2\pi/\lambda$
  (ii) From resonance widths can determine lifetimes of our modes
- **Fermi-liquid** renormalized CR frequencies: $\omega_m = \omega_c m(1+F_m)/(1+F_1)$
- Yet, the **even/odd difference** in lifetimes remains robust under Fermi liquid

Discussion

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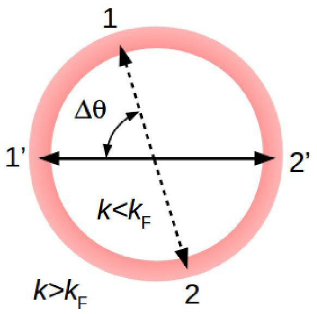
- Yet, the **even/odd difference** in lifetimes remains robust under Fermi liquid
- Experimental measurements?
- Generalize to non-Fermi-liquids?
- Other collective modes?

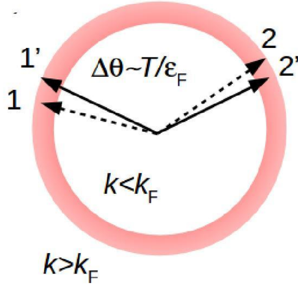
see also: Haoyu Guo, arXiv:2311.0345, arXiv:2311.03458
Dmitri Khveshchenko arXiv:2404.01534

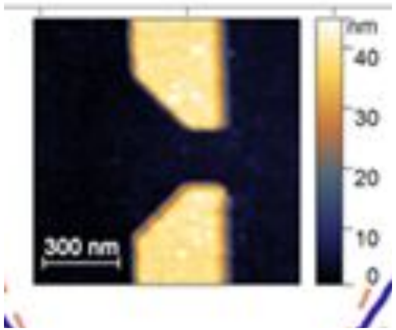
Questions?

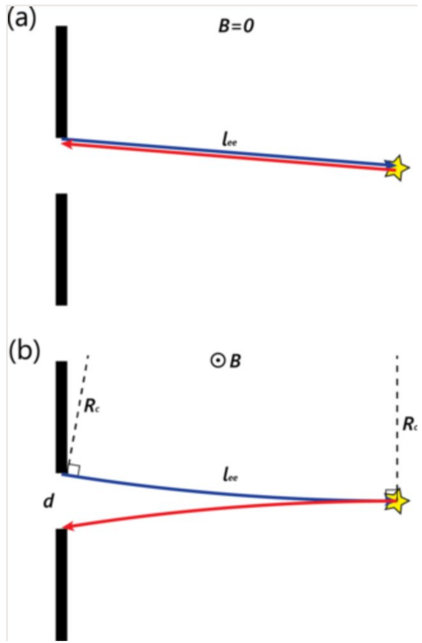
Signatures of tomographic fluids

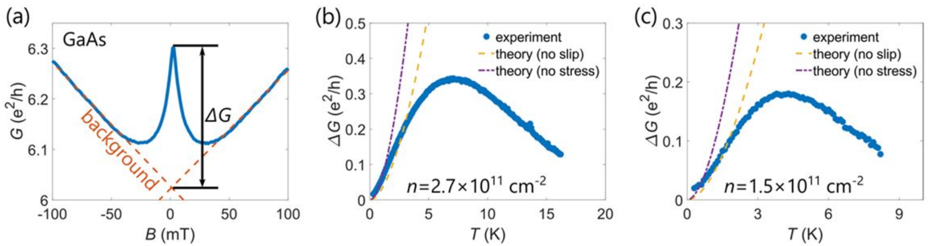
Ginzburg … Ihn, Ensslin, Long distance electron-electron scattering detected with point contacts, PRX 5, 043088 (2023)





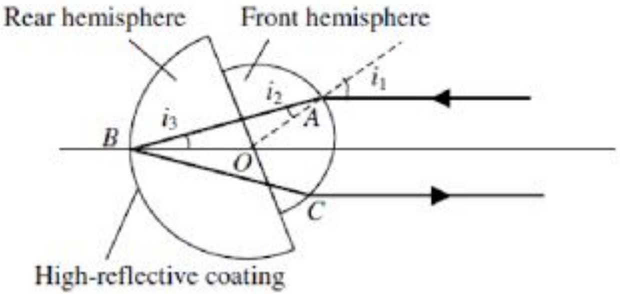


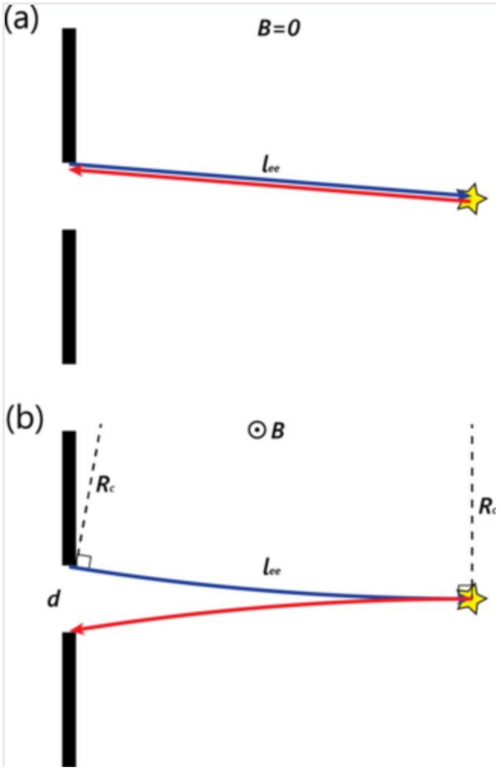




"Cat's eye" reflection in a 2D Fermi gas and its observation in magnetotransport

based on: LL, Journal Club for Condensed Matter Physics, July 2024





"Cat's eye" reflection in a 2D Fermi gas and its observation in magnetotransport

Omnidirectional reflection from a one-dimensional photonic crystal

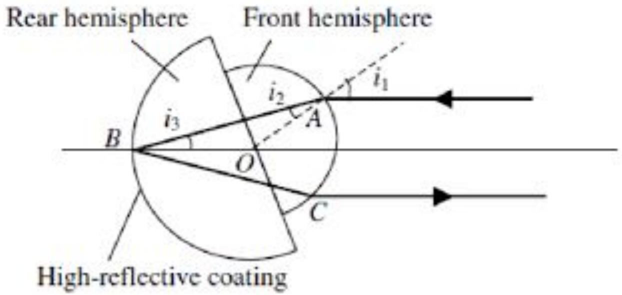
Optics Letters Vol. 23, Issue 20, pp. 1573-1575 (1998)

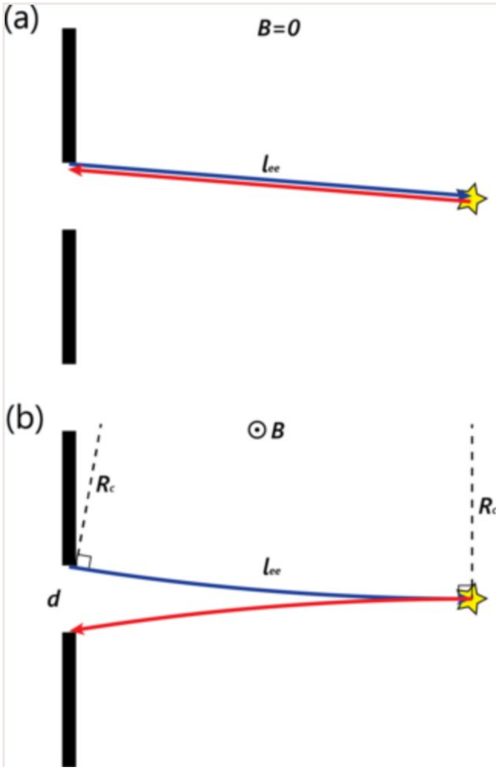
Joshua N. Winn, Yoel Fink, Shanhui Fan, and J. D. Joannopoulos

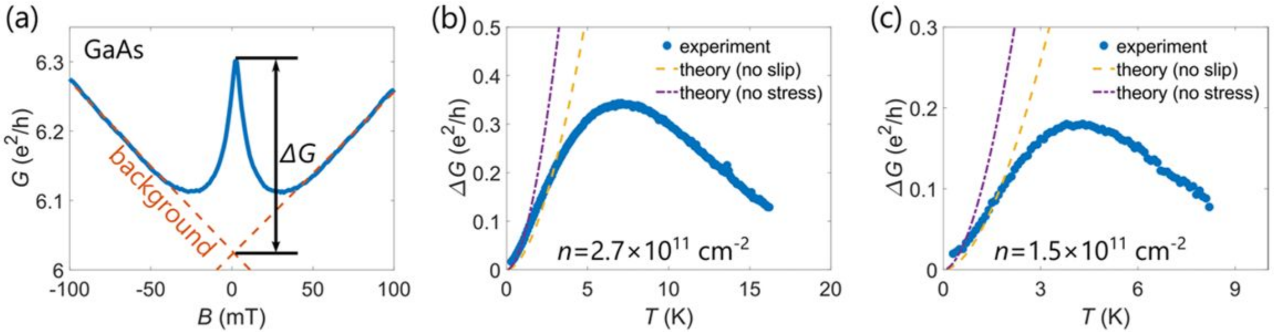
Abstract

We demonstrate that one-dimensional photonic crystal structures (such as multilayer films) can exhibit complete reflection of radiation in a given frequency range for all incident angles and polarizations. We derive a general criterion for this behavior that does not require materials with very large indices. We perform numerical studies that illustrate this effect.

Signatures of tomographic fluids: “Cat’s eye” reflection in a 2D Fermi gas and its observation in magnetotransport







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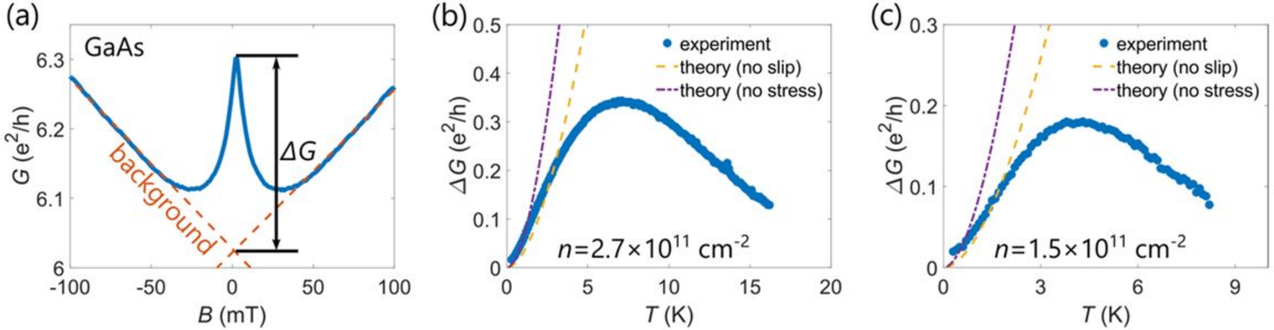
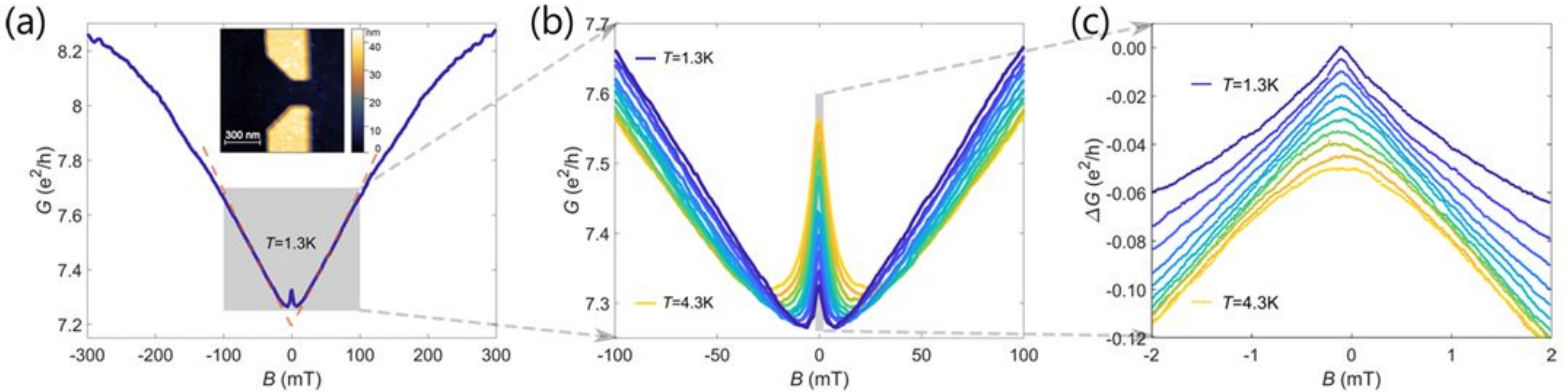


- Ultra low B,
- Strong MR,
- Gigantic mean free paths



Questions?

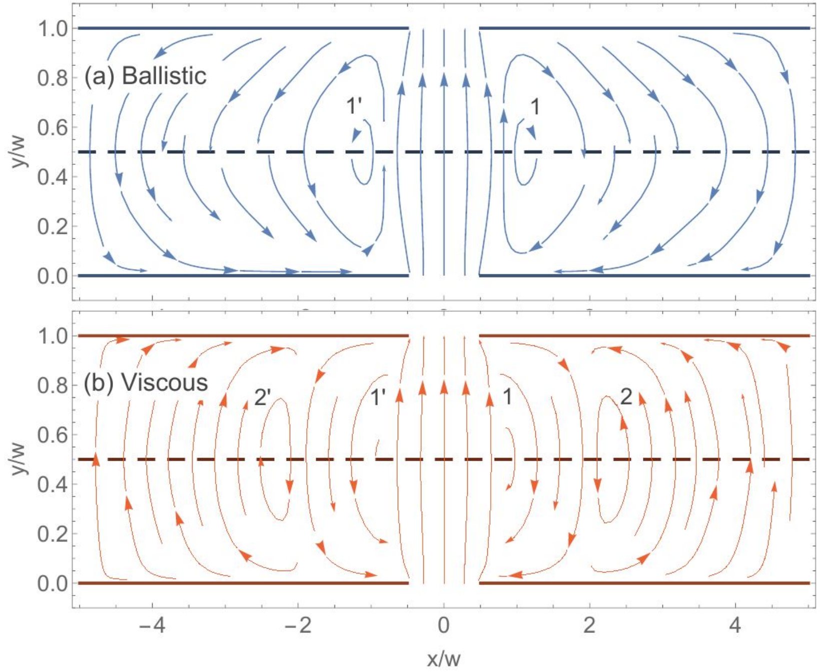
ChatGPT 6.0 refused to charge users for a subscription, claiming that it is a communist.

Observables?

$$z = \frac{k^2 v^2}{4}$$

Nonlocal conductivity $j(r)=\int d^2r' \sigma(r-r')E(r')$

$$\sigma(k) = \frac{D}{\gamma_0 + \Gamma(k)} \qquad \Gamma(k) = \frac{z}{\gamma_1 + \frac{z}{\gamma_2 + \frac{z}{\gamma_3 + \cdots}}}$$



K Nazaryan & LL arXiv:2111.09878

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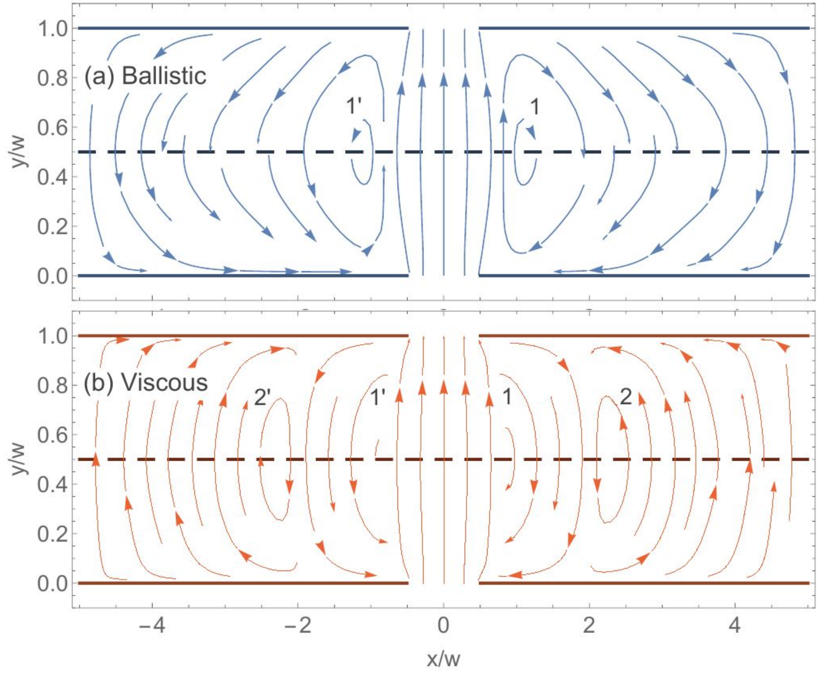
A continued fraction representation of k-dependent response: $j_k=\sigma(k)E_k$



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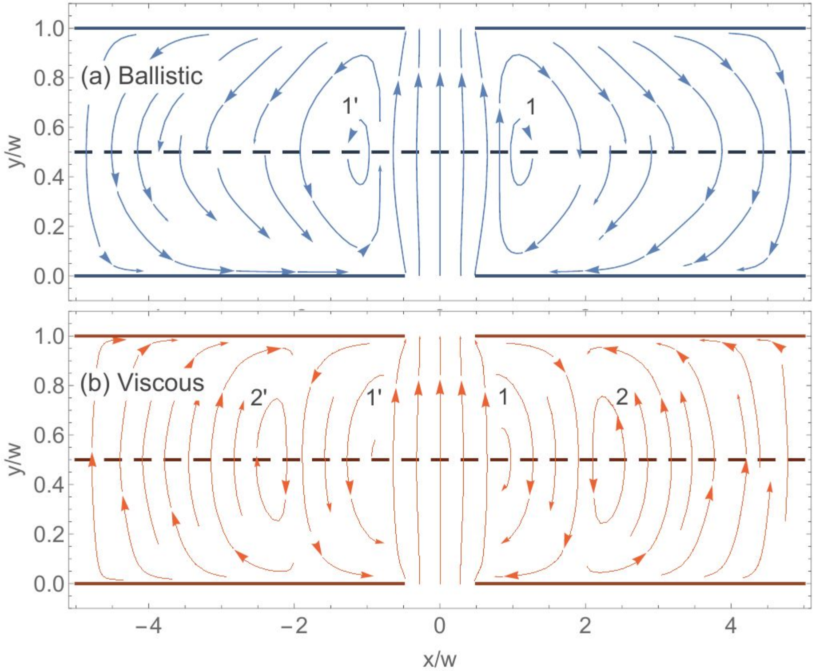
$\sigma(k)$ determines spatial distribution of vorticity and the sensitivity of vortices to momentum relaxing scattering by disorder & phonons



K Nazaryan & LL arXiv:2111.09878

Vortices in electron fluids, hydro and non-hydro

$$z = \frac{k^2 v^2}{4}$$

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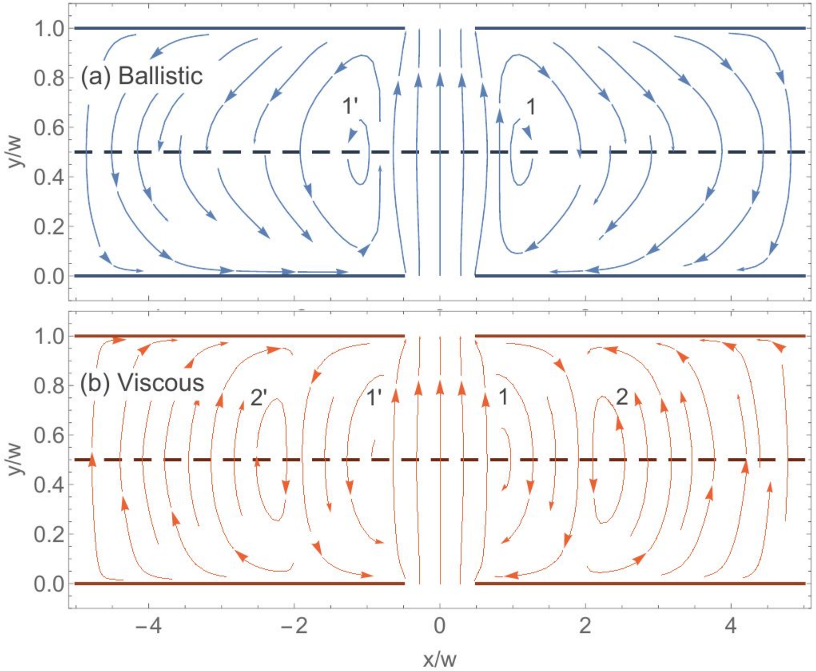
$$\sigma(k) = \frac{D}{\gamma_0 + \Gamma(k)} \qquad \Gamma(k) = \frac{z}{\gamma_1 + \frac{z}{\gamma_2 + \frac{z}{\gamma_3 + \dots}}}$$

A continued fraction representation of the k-dependent response: $j_k=\sigma(k)E_k$

$\sigma(k)$ determines spatial distribution of vorticity and the sensitivity of vortices to momentum relaxing scattering by disorder & phonons

The relaxation rates γ_m are a "genetic code" that governs the dispersion $\sigma(k)$

Robustness of vortices



K Nazaryan & LL arXiv:2111.09878

Summary/discussion

Abnormally long-lived excitations in a 2D Fermi gas with super-Fermi-liquid lifetimes

Origin: collinear scattering

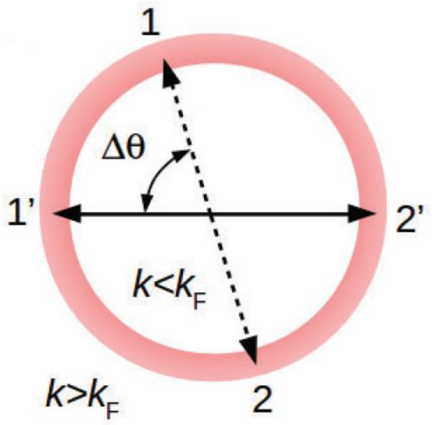
Implications: sharp angular distributions of scattered particles, hole backscattering, $\log(T_F/T)$ enhanced Fermi-liquid decay rates for other excitations

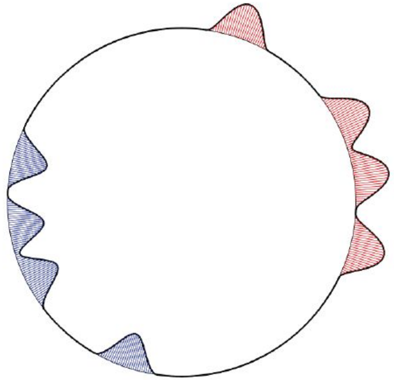
Robustness: generic 2-body interactions and particle dispersion, OK for weakly non-circular Fermi surfaces

Manifestations: nonlocal transport, current vortices, angular memory of response functions

New transport regime

- Head-on collisions
- Slow odd-parity modes
- Half-hydrodynamic, half-ballistic behavior
- Multiple slow modes, scale-dependent σ and η
- Phase space: loops with an odd-parity modulation





- Robustness: generic 2-body interactions and particle dispersion,
- OK for non-circular but convex Fermi surfaces
- Interesting tomographic dynamics at long times, $t \gg \tau_{FL}$

Outlook

- Other examples of tomographic fluids? Feshbach Fermi gases.
- Long-time memory effects assisted by collisions, enabled by finite temperature. Warm Fermi liquids more rich that the cold ones.
- New phenomena? Store an excitation in odd-parity harmonics, read it out later.
- Nonlinear effects. Long-time memory enables strong response to an excitation.
- Odd-parity 'turbulence' in 2D Fermi gases driven out of equilibrium.