Problem 9.8 ***Odd elasticity: Matrix relations***

Using the basis matrices τ^{α}_{ij}, we can write the elastic stress and strain tensors as column vectors σ^{α}, u^{α} defined by

$$\sigma_{ij} = \sigma_{\alpha}(\tau_{\alpha})_{ij}, \qquad u_{ij} = u_{\alpha}(\tau_{\alpha})_{ij}. \tag{9.155}$$

a. Prove the identity (9.41), $(\tau_{\alpha})_{ij}(\tau_{\beta})_{ij} = 2\delta_{\alpha\beta}$.

b. Show that the stress-strain relation $\sigma_{ij} = K_{ijkl}u_{kl}$ can be written as a matrix equation $\sigma_{\alpha} = K_{\alpha\beta}u_{\beta}$ and write the definition of $K_{\alpha\beta}$ that allows this.

Hint: Make use of the orthogonality condition derived in step a.

c. The basis matrices define four modes of deformation or stress that can occur in a two-dimensional solid. We will obtain a pictorial understanding of these modes. Start by drawing an arbitrary shape such as a square or rectangle.

c1. Draw the same shape after a small deformation by each basis matrix.

c2. Draw arrows on the shape corresponding to each stress mode.

c3. Check the consistency of your result with equation (9.44).

Problem 9.9 ***The work done by an odd-elastic network during a cycle***

We showed in section 9.6.2 that one can use odd-elastic energy cycles to extract work from an active solid. In this problem we show this explicitly for the microscopic model with force law (9.49).

a. Consider moving the end of the spring through the cyclic path indicated in figure 9.15. Show that the energy needed to compress the spring during the first phase of the cycle is released again during the third phase when the spring extends.

c. Compare your results to the experimental findings reported in the paper "Odd dynamics of living chiral crystals", Tan et. al. Nat. 2022 and identify the physical origin of transverse forces among the spinning embryos.