

## Collective cellular movements and flocking transitions

Q1: The **Vicsek model (1995)** aims to describe at the collective movements observed in flocks of birds, schools of fish etc... Each particle/cell/animal  $i$  is described by its position  $\mathbf{r}_i$  and orientation  $\theta_i$  - which dictates in 2D the direction towards which it actively moves at speed  $v_o(\cos \theta, \sin \theta)$  in the plane. At each time point, each particle  $i$  looks at its neighbors  $j$  (within a radius  $\mathbf{R}_0$ ) and adjust its orientation to average of all neighbor orientation (with noise  $\eta$ ). Overall, the dynamics are described by the equations:

$$\begin{cases} \mathbf{r}_i(\mathbf{t} + d\mathbf{t}) = \mathbf{r}_i(\mathbf{t}) + \mathbf{v}_i(\mathbf{t})d\mathbf{t} \\ \theta_i(\mathbf{t} + d\mathbf{t}) = \langle \theta_j(\mathbf{t}) \rangle + \eta(\mathbf{t}) \end{cases}$$

where  $j$  is a neighbor list of  $i$ :  $|\mathbf{r}_i - \mathbf{r}_j| < \mathbf{R}_0$

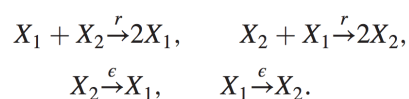
This model shows a phase transition towards an ordered/collective state when density of particles  $\rho$  compared to noise  $\eta$ .

*The question is to give a simple scaling for this dependency.*

*Hint: compare the typical time scales of loss of orientation for one particle to the time scale of encountering neighbors.*

Q2: Recent experiments have shown the reverse phenomenon, where smaller groups of animals can collectively move more easily than large ones. This is called noise-induced ordering. Imagine a population of  $N$  individual choosing between two choices/states  $X_1$  and  $X_2$ :

- Individual try to « convince » others of their choice (e.g. an orientation at rate  $r$ )...
- Individuals spontaneously change choice ( $\epsilon \ll r$ )



*Write kinetic/chemical equations for this process, in particular for the difference  $(X_1 - X_2)$  - what is the steady state?*

Simulations of this process for small numbers lead to bistability between high  $X_1$  and high  $X_2$ ...

*Is this what you expected from the noise-free steady-state? Can you intuitively see why?*

*Bonus: Write the noise terms in these equations to understand where the bistability come from*