Collective cellular movements and flocking transitions

Q1: The **Vicsek model (1995)** aims to describe at the collective movements observed in flocks of birds, schools of fish etc… Each particle/cell/animal *i* is described by its position $\boldsymbol{r_i}$ and orientation $\boldsymbol{\theta_i}$ - which dictates in 2D the direction towards which it actively moves at speed $v_o(\cos\theta, sin\theta)$ in the plane. At each time point, each particle *i* looks at its neighbors *j* (within a radius $\boldsymbol{R_0}$) and adjust its orientation to average of all neighbor orientation (with noise $\boldsymbol{\eta}$). Overall, the dynamics are described by the equations:

$$\begin{cases} \boldsymbol{r_i(t+dt) = r_i(t) + v_i(t)dt} \\ \boldsymbol{\theta_i(t+dt) =< \theta_j(t) > +\eta(t)} \end{cases}$$

where *j* is a neighbor list of *i* : $\boldsymbol{|r_i - r_j| < R_0}$

This model shows a phase transition towards an ordered/collective state when density of particles $\boldsymbol{\rho}$ compared to noise $\boldsymbol{\eta}$.

The question is to give a simple scaling for this dependency.
Hint: compare the typical time scales of loss of orientation for one particle to the time scale of encountering neighbors.

Q2: Recent experiments have shown the reverse phenomenon, where smaller groups of animals can collectively move more easily than large ones. This is called noise-induced ordering. Imagine a population of N individual choosing between two choices/states X_1 and X_2:

- Individual try to « convince » others of their choice (e.g. an orientation at rate r)…
- Individuals spontaneously change choice ($\epsilon \ll r$)

$$X_1 + X_2 \xrightarrow{r} 2X_1, \qquad X_2 + X_1 \xrightarrow{r} 2X_2,$$
$$X_2 \xrightarrow{\epsilon} X_1, \qquad X_1 \xrightarrow{\epsilon} X_2.$$

Write kinetic/chemical equations for this process, in particular for the difference (X1-X2) - what is the steady state?

Simulations of this process for small numbers lead to bistability between high X1 and high X2…

Is this what you expected from the noise-free steady-state? Can you intuitively see why?

Bonus: Write the noise terms in these equations to understand where the bistability come from