Exercises on continuum mechanics and adhesion

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1 Exercise 1: Relaxation function for standard linear solid

Consider the standard linear solid, which is defined as a Maxwell model (1d moduli C_1 and C_η) in parallel with a spring (1d modulus C_2). Derive the constitutive equation and analytically calculate the relaxation function $G(t)$ (that is force $F(t)$ after a strain jump to ϵ_0).

2 Exercise 2: Relations between relaxation and creep functions

In the lecture, we have discussed the Maxwell and Kelvin-Voigt models as the two most basic models for viscoelasticity. For each of the two models, (re)derive the relaxation function $G(t)$ and the creep function $J(t)$. Show by direct calculation that in each of the two cases, they are related by the integral equation

$$\int_0^t dt' J(t-t') G(t') = t \; . \tag{1}$$

Alternatively, one can show in Laplace space that $\hat{G}(s)\hat{J}(s) = \frac{1}{s^2}$.

3 Exercise 3: Stability of adhesion clusters

We consider a cluster of N_t parallel adhesion bonds, each of which dynamically open and close. While association occurs with a constant rate, dissociation is exponentially increased by force. At a given time t, we denote with $N(t)$ the number of closed bonds. The corresponding dynamical equation reads

$$\frac{dN}{dt} = -N e^{f/N} + \gamma(N - N_t) \tag{2}$$

where f and γ are dimensionless force and rebinding rate, respectively, and where we assume that force is shared equally between all closed bonds. Analyze the steady state and show that there is a bifurcation in which stability is lost. Calculate the stability threshold, which should come out to be

$$f_c = N_t plog(\gamma/e) \tag{3}$$

where the product logarithm $plog(a)$ is the solution x of $xe^x = a$. Discuss the meaning of this equation.