a) ① Paper is thin, no shear force.

② No stretching

③ Small sag → Curvature in Monge gauge is $\frac{f''}{\sqrt{1-f'^2}}$

for $|f'|$ small, we approx as f''

b)

$$\int_0^L dx \left\{ \frac{1}{2}\kappa \left[f''(x) + \delta f''\right]^2 - \frac{1}{2}\kappa (f'')^2 + \rho g (f + \delta f) - \rho g f \right\}$$

$$= \int_0^L dx \left\{ \kappa f''(x)\, \delta f'' + \rho g \delta f \right\}$$

$$= \kappa f''(x) \delta f' \Big|_0^L - \int_0^L \kappa\, \delta f' f''' dx + \int_0^L dx\, \rho g \cdot \delta f$$

$$= \kappa f''(x) \delta f' \Big|_0^L - \kappa f''' \delta f \Big|_0^L + \int_0^L \kappa f'''' \delta f\, dx + \int_0^L \rho g\, \delta f \cdot dx$$

c) B.C. fixed @ $x=0$ ⇒ $\delta f(x=0) = 0$ $\delta f'(x=0) = 0$

$f(x=0) = 0$ $f'(x=0) = 0$

And we have $\kappa f''(x) \delta f' \Big|_0^L - \kappa f''' \delta f \Big|_0^L = 0$

thus. $f''(L)\delta f'(L) - f''' \delta f(L) = 0 \Rightarrow \begin{cases} f''(L) = 0 \\ f'''(L) = 0 \end{cases}$

d) Obvious from c).

e) $\frac{f(x=\ell)}{L} = -\frac{1-4+6}{24\,\tilde{\ell}^3} = -\frac{1}{8}\left(\frac{L}{\ell}\right)^3$

$S = \frac{1}{8}\frac{\rho g}{K}L^4$

Measurement: $\begin{cases} S = 2 \text{ cm} \\ L = 8.3 \text{ cm} \end{cases}$

$K = 2 \times 10^{-4}$ J

f) $\bar{K}_A = 3 \times 10^5$ J/m^2

No surprising. This coorespends to Young's modulus of GPa, agree with ChatGPT.

2cm

8.4 cm.

$$\frac{f(x=L)}{L} = -\frac{1-4+6}{24\,\tilde{l}^{3}} = -\frac{1}{8}\left(\frac{L}{l}\right)^{3}$$

$$S = \frac{\rho g L^{4}}{8K}$$

$$K = \frac{1}{8}\rho g \cdot \frac{L^{4}}{S} = \frac{1}{8} \cdot 75\ g/m^{2} \cdot 9.8\ m/s^{2} \cdot \frac{(8.4\,cm)^{4}}{2cm}$$

$$= \frac{1}{8} \cdot 75 \cdot 9.8 \cdot \frac{(8.4)^{4}}{2} \cdot \frac{kg\,10^{-3}}{m} \cdot \frac{1}{s^{2}} \cdot \cancel{cm^{3}}\ m^{3} \cdot 10^{-6}$$

$$= 2.3 \times 10^{-4} \cdot kg \cdot m/s^{2} \cdot m$$

$$(f''\delta f)'' = (f'''\delta f + f''\delta f')' = f''''\delta f + 2f'''\delta f' + f''\delta f''$$

$$(f'''\delta f)' = f'''\delta f' + f''''\delta f$$

$$\rightarrow \kappa f'''' + \rho g = 0$$

$$(f''\delta f)' = f'''\delta f + f''\delta f'$$

$$\Rightarrow \quad (f''\delta f)'\Big|_0^L - 2f'''\delta f\Big|_0^L$$