BSS 2023 – Lectures on Quantum Simulations using Ultracold Atoms

July 26–28 2023

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1. Trapping, Interactions, Hamiltonians
   Collis., DD, Superexchange

2. Quantum Gas Microscopy and Dynamically driven Systems
   Floquet, Anomalous Floquet Phases

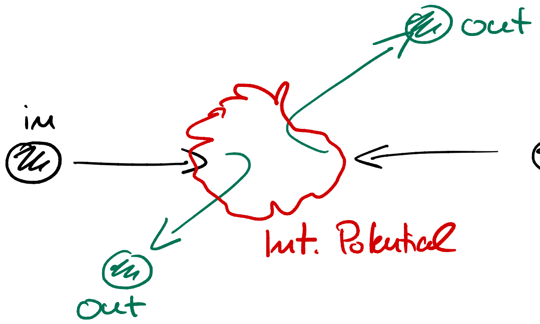
3. Out-of-Equilibrium Dynamics
   MBL, Thermalization, Entanglement Entropy Measurement

   Anomalous Spin Transport and KPZ

Questions? Write me: immanuel.bloch@mpq.mpg.de

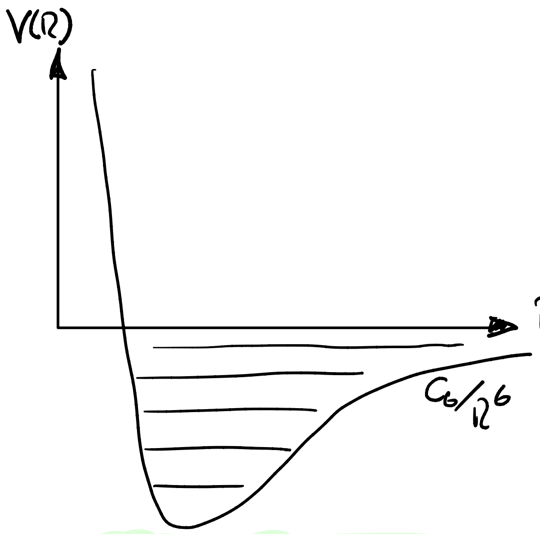
1 Interactions + Potentials

1.1 Collisional Interactions



bosons: symmetric wf

fermions: antisymm. wf.



Pseudopotential

$$\Longleftrightarrow \quad \frac{4\pi\hbar^2 a}{m}\,\delta(\vec{r})$$

$a \mathrel{\hat{=}}$ scattering length

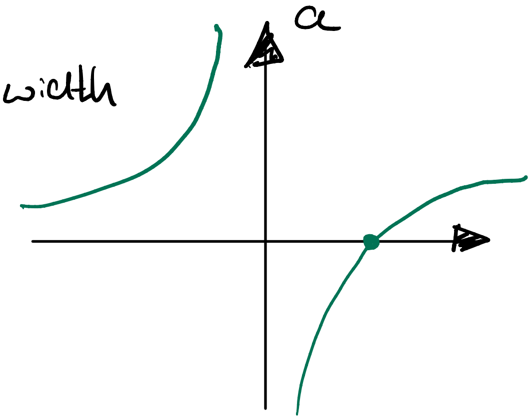
Born-Oppenheimer Potential

Pseudopotential

Feshbach Resonances

resonance width

$$\boxed{a(B) = a_{bg}\left(1 + \frac{\Delta}{B - B_0}\right)}$$



„Scattering Resonance"

See C. Chin, R. Grimm, P. Julienne, E. Tiesinga
Rev Mod Phys 82, 1225–1286 (2010)

1.2 Optical Trapping Potentials

Light-Atom interaction

$$\boxed{\hat{H} = -\hat{d}\cdot E(t)}$$

Laser light

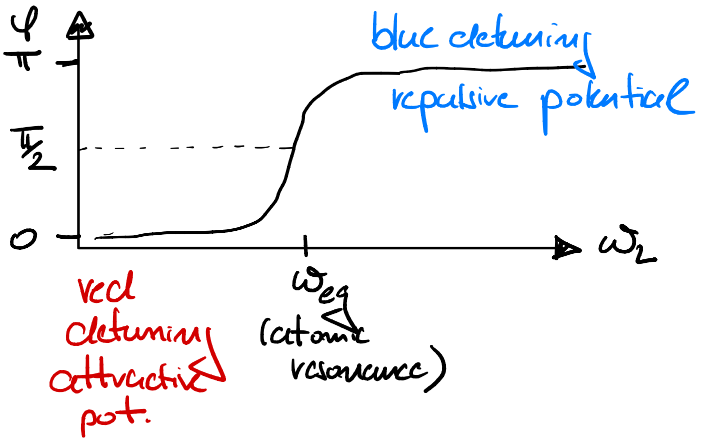
$$\sim -\vec{d}(t)\vec{E}(t)$$

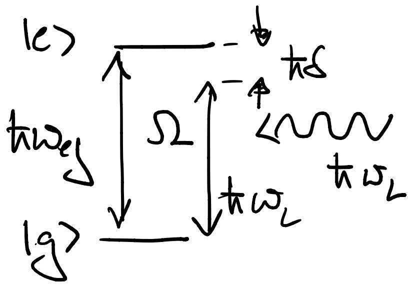
Light induces dipole in atom $\quad \vec{d}(t) = \alpha(\omega)\vec{E}(t)$

atomic polarizability

$$\rightsquigarrow \quad H \sim -\alpha(\omega)\,|E(t)|^2 \sim -\alpha(\omega)\, I$$

When is dipole in and out of phase with atom?





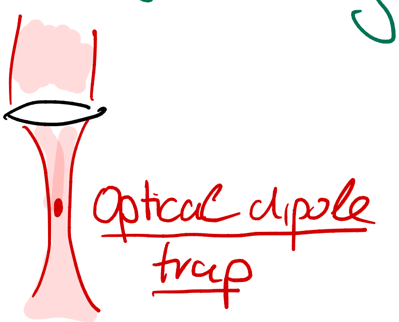
$$\boxed{H_{dip} = \frac{\hbar|\Omega|^2}{4\delta}}$$

$\delta < 0$ attractive

$\delta > 0$ repulsive

Conservative trapping potential
(no cooling possible)

$\hookrightarrow$



Optical Lattice

$$\hat{\Psi}(\vec{r}) = \sum \hat{a}_i\, W(\vec{r}-\vec{r}_i)$$

Notes:

* Potentials can be spin-dependent!

* Rapid dynamic control over potentials, spatial control (laser intensity or frequency switching)
* States can be „frozen" by suppressing dynamics
* Optical potentials useful for adding controlled disorder

1.3 Hubbard models

Many-body Hamiltonian in second quantization:

$$\boxed{\hat{H} = \int \hat{\psi}^+(\vec{r})\left[-\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(\vec{r})\right]\hat{\psi}(\vec{r})\,d^3r + \frac{1}{2}\iint \hat{\psi}^+(\vec{r})\,\hat{\psi}^+(\vec{r}\,')\,V_{int}(\vec{r}-\vec{r}\,')\,\hat{\psi}(\vec{r})\,\hat{\psi}(\vec{r}\,')\,d^3r\,d^3r'}$$

Lattice $\hat{\psi}(\vec{r}) = \sum \hat{a}_i\, w(\vec{r}-\vec{r}_i)$

Interactions $V_{int} \sim \frac{4\pi\hbar^2 a_s}{m}\delta(\vec{r}-\vec{r}\,')$

↳

$$H_{BHM} = -t\sum_{\langle i,j\rangle}\hat{b}_i^+\hat{b}_j + \frac{1}{2}U\sum \hat{n}_i(\hat{n}_i-1) + \sum_i \varepsilon_i \hat{n}_i$$

Bose-Hubbard

$$H_{FHM} = -t\sum_{\langle i,j\rangle,\sigma}\hat{c}_{i\sigma}^+\hat{c}_{j\sigma} + U_{\uparrow\downarrow}\sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow} + \sum_i \varepsilon_i\hat{n}_i$$

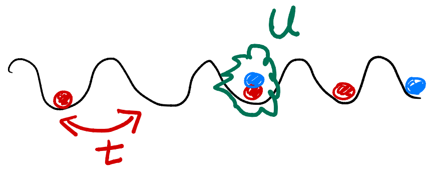
Fermi-Hubbard

For strong interactions $U/t \gg 0$ and $n = 1$

a) charge sector $T \lesssim U/k_B$ Mott-Insulator

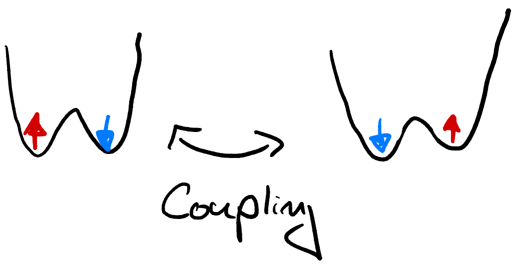
- one fermion per site, random spin
- finite gap $\Delta_c \sim U$ ⇝ incompressible
- suppressed density fluctuations

↳ low energy exc.: particle-hole pairs

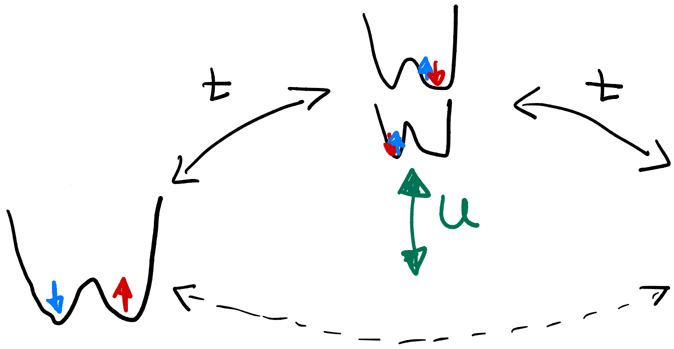


b) Spin sector $U/t \gg 1$ Spin configurations coupled by correlated spin flips

Example: Double Well



Compute effective coupling via 2nd order pert. theory



Note: Tuning via tilt ⚠

$\Rightarrow$ eff coupling $\boxed{J_{ex} = \frac{4t^2}{U}} > 0$

For spin mixtures at half filling H reduces to an effective spin Hamiltonian

$$\boxed{H_{eff} = \pm J_{ex} \sum_{\langle i,j \rangle} \vec{S}_i \vec{S}_j}$$

+ fermions AF

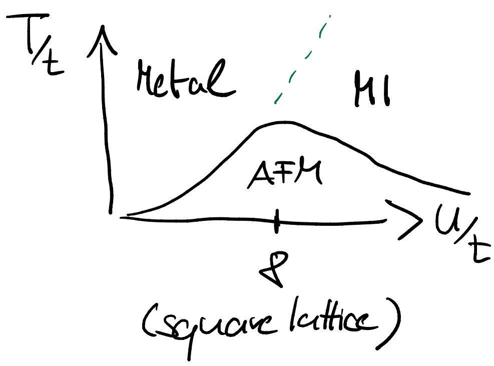
− bosons Ferro

for $T \lesssim J_{ex}$ (fermions) Heisenberg AFM ⚠

Summary:

$T_{MI} \sim U$ Mott insulator

$T_{AFM} \sim 4t^2/U$ $\left(\frac{T}{t}\right)_{max}$ around $U/t \sim 8$



Note: for two component bosonic mixtures we can have different int. strengths $U_{\uparrow\uparrow}, U_{\downarrow\downarrow}, U_{\uparrow\downarrow}$!

(so far we assumed them all to be the same)

AND can have spin dependent tunneling ! ($t_\uparrow, t_\downarrow$)

$$\boxed{H_{BB} = -\sum_{\langle i,j\rangle}\left[J\,\vec{S}_i\vec{S}_j + J'\,\hat{S}_i^z\hat{S}_j^z\right]}$$

$$J = 2t_\uparrow t_\downarrow / U_{\uparrow\downarrow} \quad ; \quad J' = -(t_\uparrow + t_\downarrow)^2/U_{\uparrow\downarrow} + 2t_\uparrow^2/U_{\uparrow\uparrow} + 2t_{\downarrow}^2/U_{\downarrow\downarrow}$$

for $t_\uparrow = t_\downarrow = t$

$$J = \frac{2t^2}{U_{\uparrow\downarrow}}, \quad J' = -\frac{4t^2}{U_{\uparrow\downarrow}} + \frac{2t^2}{U_{\uparrow\uparrow}} + \frac{2t^2}{U_{\downarrow\downarrow}}, \quad J'' = J + J' = -2t^2 \times\left(-\frac{1}{U_{\uparrow\downarrow}} + \frac{1}{U_{\uparrow\uparrow}} + \frac{1}{U_{\downarrow\downarrow}}\right)$$

$$\leadsto \boxed{H_{BB} = -J\sum_{\langle i,j\rangle}\left(\hat{S}_i^x\hat{S}_j^x + \hat{S}_i^y\hat{S}_j^y\right) - J''\sum_{\langle i,j\rangle}\hat{S}_i^z\hat{S}_j^z}$$

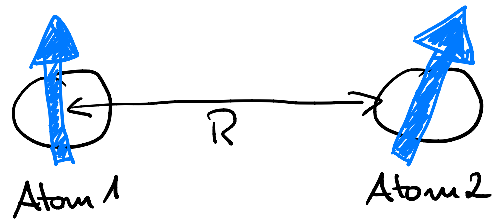
tunable XXZ !

(see spin models later)

see: A.B. Kuklov + B.V. Svistunov, PRL 90, 100401 (2003)
Counterflow Superfluidity...

SHOW COLLISIONAL GATES

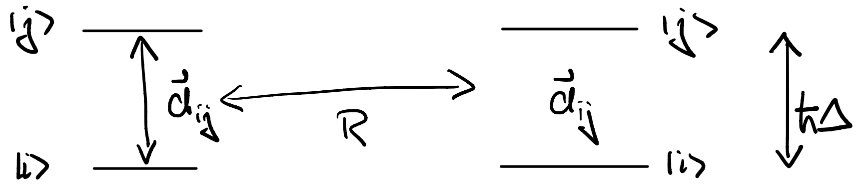
1.2 Rydberg Interactions



Dipole-Dipole Interaction

$$\hat{V}_{dd} = \frac{e^2 \hat{\vec{r}}_1 \cdot \hat{\vec{r}}_2 - 3e^2(\hat{\vec{r}}_1 \cdot \vec{e}_R)(\hat{\vec{r}}_2 \cdot \vec{e}_R)}{4\pi\varepsilon_0 |R|^3} = \frac{\hat{d}_1 \hat{d}_2 - 3(\hat{d}_1 \cdot \vec{e}_R)(\hat{d}_2 \cdot \vec{e}_R)}{4\pi\varepsilon_0 R^3}$$

Interactions in two-state model



For simplicity assume: (easier calculation, same physics)

$$\boxed{V(R) = \frac{|d|^2}{4\pi\varepsilon_0 R^3}}$$

(dipoles aligned and $\perp \vec{e}_R$)

$$H = \begin{array}{c|c} & \begin{array}{cccc} |i,i\rangle & |i,j\rangle & |j,i\rangle & |j,j\rangle \end{array} \\ \hline & \begin{pmatrix} 0 & 0 & 0 & V \\ 0 & \hbar\Delta & V & 0 \\ 0 & V & \hbar\Delta & 0 \\ V & 0 & 0 & 2\hbar\Delta \end{pmatrix} \end{array} \begin{array}{c} |i,i\rangle \\ |i,j\rangle \\ |j,i\rangle \\ |j,j\rangle \end{array}$$

"Two-Atom Hamiltonian"

Direct Exchange Rydberg Interaction

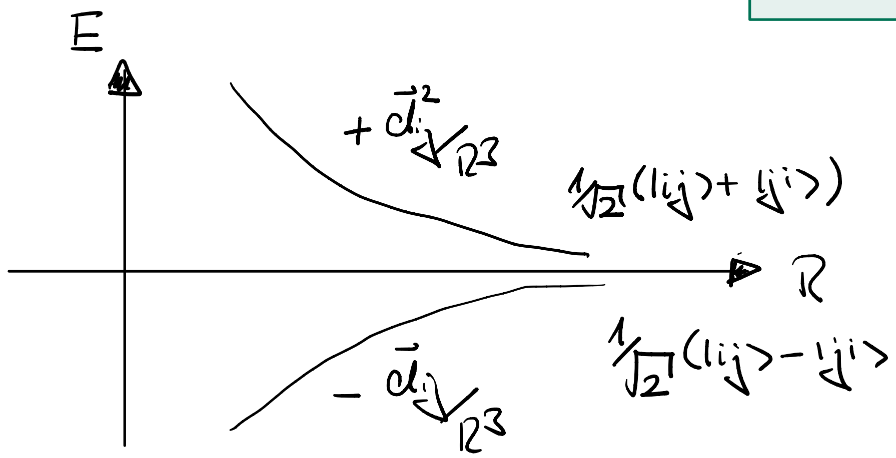
Start with atoms in <u>different initial states</u> $|i,j\rangle \leftrightarrow |j,i\rangle$

$$|\pm\rangle = \tfrac{1}{\sqrt{2}}\left(|i,j\rangle \pm |j,i\rangle\right)$$

$$E_\pm = \Delta \pm V$$

→ Resonant exchange oscillations

$$\boxed{\omega_{ex} = \frac{2|V|}{\hbar}}$$



Van der Waals interaction

Atoms initially in same state

$$H_{VdW} = \begin{array}{c|cc} & |ii\rangle & |jj\rangle \\ \hline |ii\rangle & \multicolumn{2}{c}{\multirow{2}{*}{$\begin{pmatrix} 0 & V \\ V & 2\hbar\Delta \end{pmatrix}$}} \\ |jj\rangle & & \end{array}$$

$$H_{VdW} = \begin{pmatrix} 0 & V \\ V & 2\hbar\Delta \end{pmatrix} \quad \text{(rows/columns: } |ii\rangle, |jj\rangle)$$

for $V \ll 2\hbar\Delta$

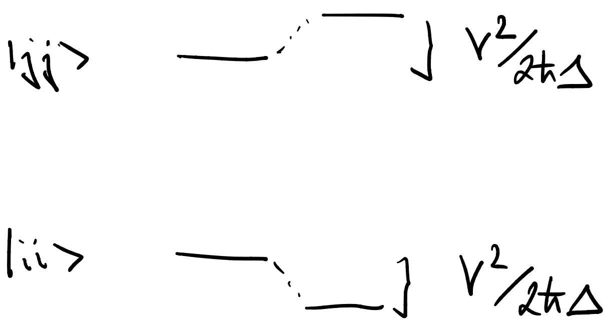
Can diagonalize H_{VdW}, but more intuition via 2nd order perturbation theory

Reminder:

$$E_i^{(2)} = E_i + \langle i|\hat{V}|i\rangle + \sum_{i\neq j} \frac{|\langle j|\hat{V}|i\rangle|^2}{E_i - E_j}$$

$$|\tilde{i}\rangle = |i\rangle + \sum_{i\neq j} \frac{\langle j|\hat{V}|i\rangle}{E_i - E_j} |j\rangle$$

here:

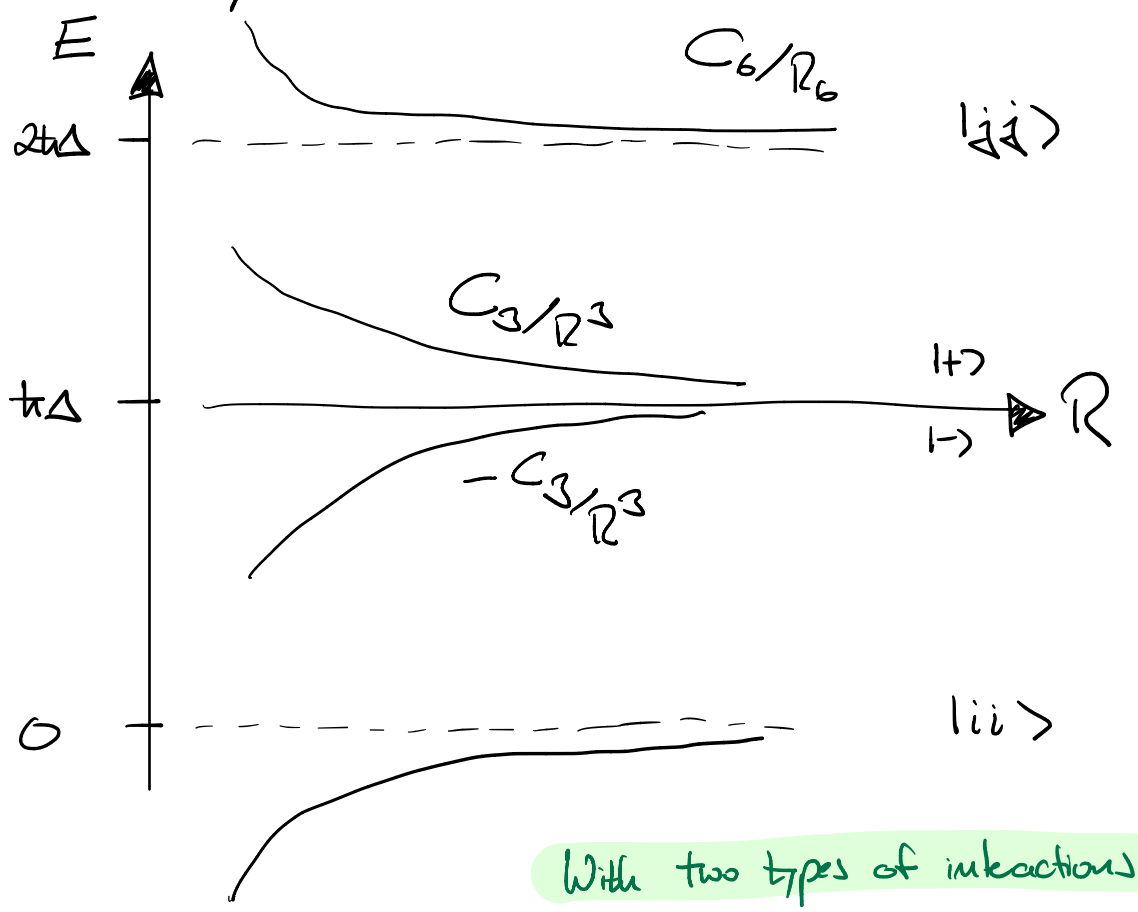


$$|\widetilde{jj}\rangle = |jj\rangle + \frac{V}{2\hbar\Delta}|ii\rangle$$

$$|\widetilde{ii}\rangle = |ii\rangle - \frac{V}{2\hbar\Delta}|jj\rangle$$

with $\frac{V^2}{2\hbar\Delta} = \left(\frac{d_{ij}^2}{R^3}\right)^2 / 2\hbar\Delta = \frac{C_6}{R^6}$ van der Waals Interaction Energy

Summary:



With two types of interactions!

* van der Waals
* resonant dipole-dipole

Can we identify spin models these correspond to?

Spin models for Spin-$\frac{1}{2}$ models on a lattice:

Heisenberg: $\hat{H} = \sum J_{ij} \vec{S}_i \vec{S}_j$ $\quad \vec{S} = \frac{\hbar}{2}\vec{\sigma}$

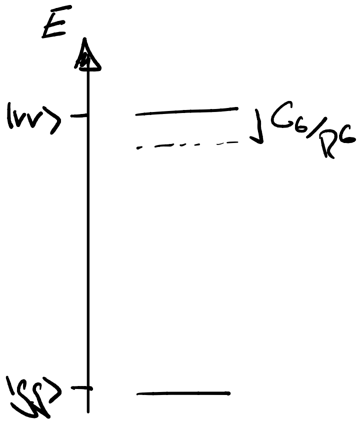
XXZ: $\hat{H} = J_{xy} \sum (\hat{S}_i^x \hat{S}_j^x + \hat{S}_i^y \hat{S}_j^y) + J_z \sum \hat{S}_i^z \hat{S}_j^z$

XY: $\hat{H} = J_{xy} \sum (\hat{S}_i^x \hat{S}_j^x + \hat{S}_i^y \hat{S}_j^y) \sim J_{xy} \sum (\hat{S}_i^+ \hat{S}_j^- + \hat{S}_i^- \hat{S}_j^+)$

exchange int. (flip-flop, like kinetic energy)

ZZ Ising: $\hat{H} = J \sum \hat{S}_i^z \hat{S}_j^z$

Implementation of Ising Model



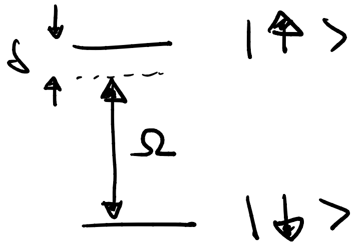
Rydberg occupation

$$\boxed{H_{int} = \frac{C_6}{R^6}\hat{n}_1\hat{n}_2} \quad \leftarrow \quad \hat{n} = \frac{(\hat{\mathbb{1}} + \hat{\sigma}_z)}{2}$$

$$\rightsquigarrow \boxed{H_{int} \sim J\,\hat{\sigma}^z_1\,\hat{\sigma}^z_2}$$

and adding coupling with laser (in RWA)

Single atom



$$\hat{H}_{LA} = \frac{\hbar\Omega}{2}\hat{\sigma}_x + \hbar\delta\hat{\sigma}_z$$

↳ Transverse field Ising Model

$$\boxed{\hat{H} = \underbrace{\frac{\hbar\Omega}{2}\sum\hat{\sigma}^x_i}_{\text{transverse field}} + \underbrace{\hbar\delta\sum_i\hat{n}_i}_{\text{longitudinal field}} + \underbrace{\sum_{i<j}\frac{C_6}{R^6}\hat{n}_i\hat{n}_j}_{\text{spin-spin Ising interaction}}}$$

Implementation of XY model

Resonant DD interaction (exchange dynamics)

$$\hookrightarrow \boxed{\hat{H} \sim \frac{C_3}{R^3}\sum\left(\hat{S}^+_i\hat{S}^-_j + \hat{S}^-_i\hat{S}^+_j\right)}$$

Like hardcore bosons hopping on a lattice $\hat{=}$ $S^+_iS^-_j + S^-_jS^+_i$

$$= a^+_i\hat{a}_j + \hat{a}^+_j\hat{a}_i$$