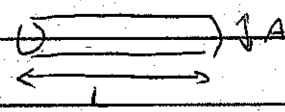
Questions from last time:

1. Efficiency of fish swimming
   - Data from Fish + Lauder
   - Data I have: <u>Energy</u>
     mass. dist.

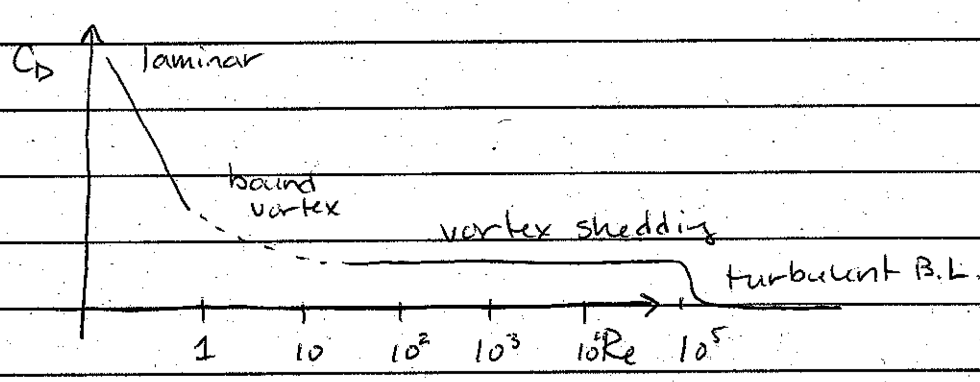
Cylindrical fish:  $m \sim \rho A L$

Sardines: $L \sim 200$ mm
$m \sim 100$ g
$V \sim 400$ mm/s
$E \sim 1$ J/kg-m

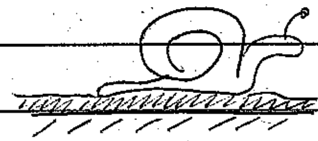
$$\eta = \frac{F \cdot V}{\phi} = \frac{\frac{1}{2} C_D \rho V^2 A \cdot V}{E m V} = \frac{\frac{1}{2} \overbrace{C_D}^{\approx 1} \cancel{\rho} V^2 \cancel{A} \cancel{V}}{E \cancel{\rho} \cancel{A} L \cancel{V}} = \boxed{\frac{V^2}{2EL}}$$

$$\boxed{\eta_{sardines} \approx 40\%}$$

2. Transition from laminar to turbulent drag

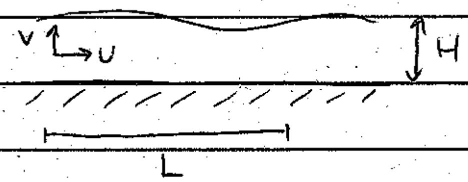


Crawling



Locomotion on thin films of fluid

Again start with Re.



$$\rho\left[\frac{\partial \underline{u}}{\partial t} + (\underline{u}\cdot\nabla)\underline{u}\right] = -\nabla p + \mu\nabla^2\underline{u}$$

compare

$$\rho\frac{U^2}{L} \overset{?}{\ll} \mu\frac{U}{H^2} \Rightarrow \frac{\rho U H^2}{\mu L} \ll 1$$

$$\Rightarrow \left(\frac{H}{L}\right)^2 Re \ll 1$$ can neglect inertial terms if this is true.

In snails, $H \sim 10\text{-}20\,\mu m$

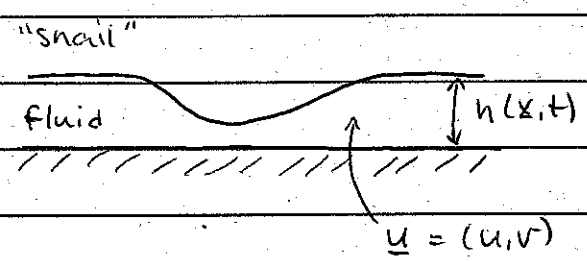
$L \sim 1\,mm \Rightarrow \left(\frac{H}{L}\right)^2 \sim 10^{-4}$

Waving sheet (analogous to Taylor's sheet)

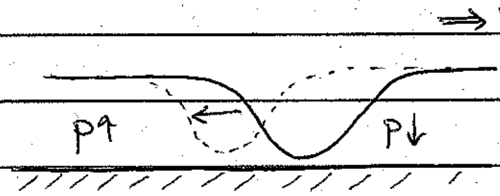
→ this is not how snails crawl

but it's still interesting...

- amplitude limited by film thickness
- inextensible

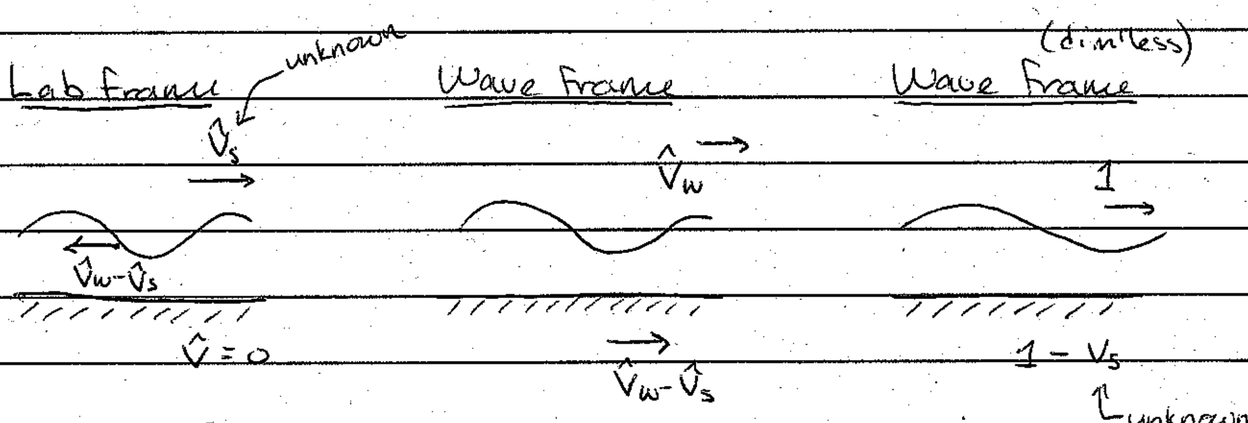


Suppose h is a traveling wave



Find V_s (snail velocity):

- Pick shape of foot $h(x,t)$ (selected by snail)
- Work in frame moving w. wave to eliminate time dep.
- "hat" = dimensional quantity



Rescale: $\hat{u} = u\,\hat{V}_w$, $\hat{v} = v\,\frac{\hat{H}}{\hat{L}}\,\hat{V}_w$, $\hat{y} = y\,\hat{H}$, $\hat{x} = x\,\hat{L}$

$$\hat{p} = p\,\frac{\mu \hat{V}_w \hat{L}}{\hat{H}^2}$$

Dim'less lubrication equation: $\boxed{\frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial y^2}, \quad \frac{\partial p}{\partial y} = 0}$

$$\Rightarrow \boxed{u = \frac{\partial p}{\partial x}\,\frac{1}{2}\,y(y-h) + V_s\left(\frac{y}{h} - 1\right) + 1}$$

(x) cons. of mom.

$$\text{st. st.} \Rightarrow \text{Flux} = Q = \int_0^h u\,dy = \text{const}$$

$$Q = -\frac{1}{12}\frac{\partial p}{\partial x}h^3 - \frac{1}{2}hV_s + h \Rightarrow \boxed{\frac{\partial p}{\partial x} = \frac{12}{h^3}\left[h\left(1-\frac{V_s}{2}\right) - Q\right]}$$

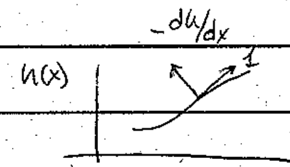
(**) cons of mass

Periodicity $\Rightarrow p(0) = p(1)$

$$\Rightarrow \int_0^1 \frac{\partial p}{\partial x}\,dx = 0 = \int_0^1 \frac{12}{h^3}\left[h\left(1-\frac{V_s}{2}\right) - Q\right]dx$$

$$= \left(1-\frac{V_s}{2}\right)\underbrace{\int_0^1 \frac{dx}{h^2}}_{I_2} - Q\underbrace{\int_0^1 \frac{dx}{h^3}}_{I_3}$$

$$\Rightarrow \boxed{Q = \left(1-\frac{V_s}{2}\right)\frac{I_2}{I_3}}$$



Force balance: $\underline{F} = 0$

$$\text{traction} = \underline{f} = \underline{\underline{\sigma}}\cdot\underline{n} \Rightarrow f_x = p\frac{dh}{dx} + \frac{\partial u}{\partial y}$$

$$F_x = \int_0^1 \left(p\frac{dh}{dx} + \frac{\partial u}{\partial y}\right)\Big|_{y=h} dx = 0$$

$$= \cancel{ph\Big|_0^1}^{\,0} - \int_0^1 \underbrace{h\frac{\partial p}{\partial x}}_{(**)}\Big|_{y=h} dx + \int_0^1 \underbrace{\frac{\partial u}{\partial y}}_{(*)}\Big|_{y=h} dx = 0$$

$$\boxed{V_s = \frac{6(1-A)}{4-3A}} \quad \text{where } A = \frac{I_2^2}{I_1 I_3} \quad I_n = \int_0^1 \frac{dx}{h^n}$$

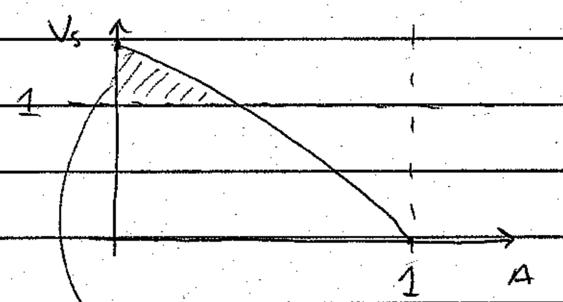
Given shape of foot $\Rightarrow$ speed of "snail"

Properties of A:

Cauchy-Schwartz inequality:

$$I_2^2 = \left(\int h^{-1/2} h^{-3/2} dx\right)^2 \leq \int h^{-3} dx \int h^{-1} dx = I_3 I_1$$

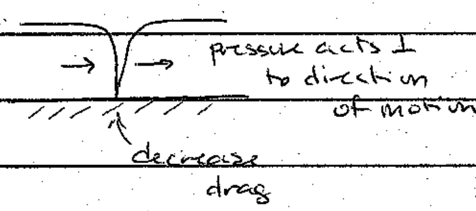
$$\Rightarrow \quad 0 < A < 1$$



snail moves faster than the wave!

small A ⇒ faster snail

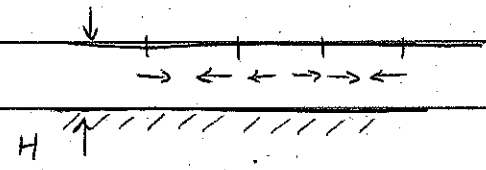
"sharp" profiles ↓ A



[Shelley/ Wilkening movie]

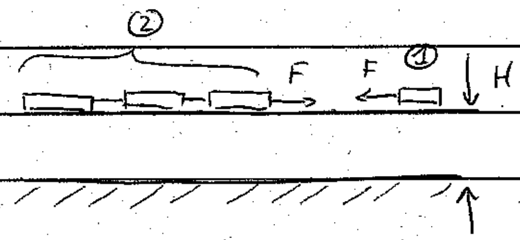
Comment on lubrication.

But real snails don't move this way



waves of compression + expansion

Back-of-the-envelope calc



A = area of each pad

N = # of pads

Force balance on 1: $-F = A \frac{V_1 \mu_1}{H}$

Force balance on 2: $F = (N-1) A \frac{V_2 \mu_2}{H}$

Velocity of center of mass: $[V_1 + (N-1)V_2]/N = V_{cm}$

$$\Rightarrow \boxed{V_{cm} = \frac{FH}{AN}\left(\frac{1}{\mu_2} - \frac{1}{\mu_1}\right)}$$