Dynamics of Oceans: Pt 2: Cascades Across Scales

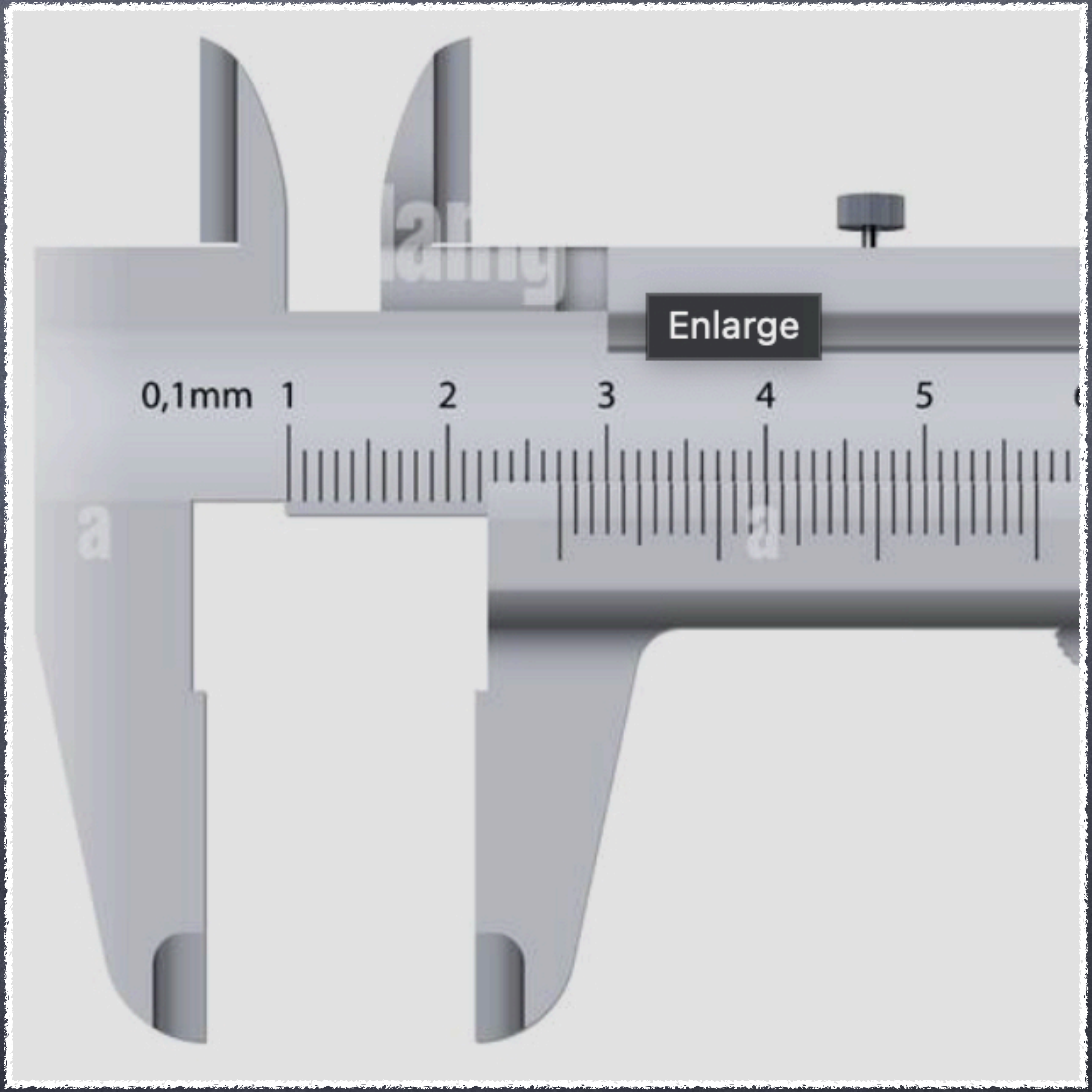
Professor Baylor Fox-Kemper

Brown University
Dept. of Earth, Environmental, and Planetary Sciences
(Formerly U. Colorado Atmospheres and Oceans)
Supported by NASA (NNX09AF38G), NSF (0934737, 1245944, 2220280), ONR (N00014-17-1-2963), NOAA (NA19OAR4310366), Gulf of Mexico Research Initiative, Schmidt Futures

Boulder School for Condensed Matter and Materials Physics
July 19, 2022

For Today—Scale Maths

- Diagnosis by Scale
  - Filtering & Reynolds-Averaging
  - Spectral/Fourier
  - Structure Functions
    (Drifters & Moorings!)
- Parameterizations & Augments
  - Energy, PV
- Lagrangian vs. Eulerian Structure Functions
- Patchy Injection through Sea Ice



Last Time, we saw that Boussinesq equations were good for all ocean motions (remove sound waves), and additionally hydrostatic approx can be used for mesoscale & submesoscale, but not Langmuir scale

- No more need for dimensionless equations, as we've sorted the transitions already
- Here I compress the non-conservative terms into one.
- I'm assuming Cartesian coordinates and Einstein summation.
- The hydrostatic approximation form is a bit messier, as vertical equation is different from horizontal.

$$
\begin{aligned}
\partial_t u_j + u_i \nabla_i u_j + \epsilon_{jkl} f_k u_l &= -\nabla_j \Phi + b\hat{z} + \dot{\mathcal{M}}_j \\
\partial_t S + u_i \nabla_i S &= \dot{\mathcal{S}} \\
\partial_t \Theta + u_i \nabla_i \Theta &= \dot{\mathcal{T}} \\
\nabla_i u_i &= 0 \\
b &= b(S, \Theta, z)
\end{aligned}
$$

Now, we'd like to examine some derived balances from this system

It's simpler to just use buoyancy rather than both salinity & temperature, and the buoyancy equation has the same form as T & S, and all concentration tracer equations.

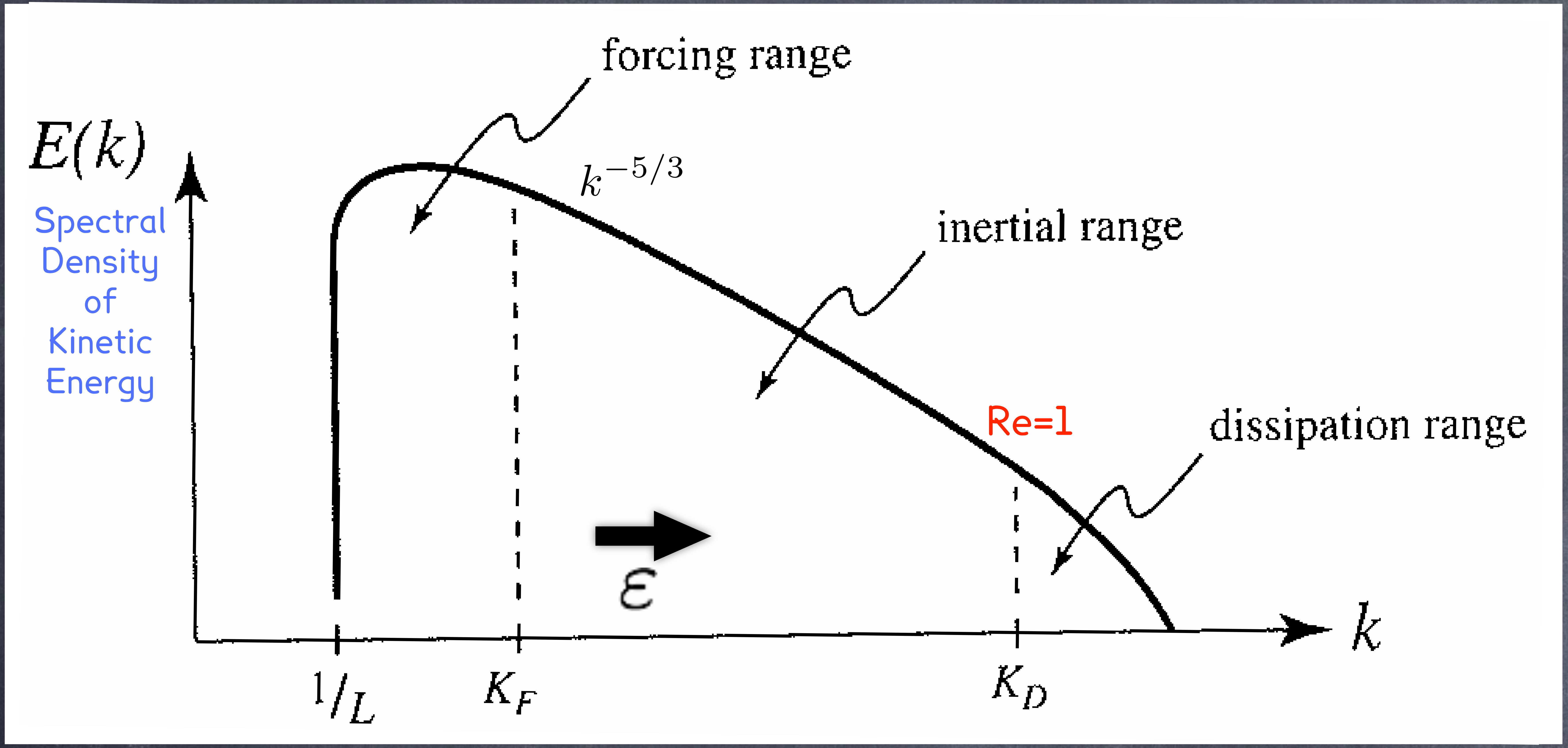
(Simplifyingly, in Boussinesq concentration by mass fraction & volume fraction have the same equation, not true in compressible fluids).

$$\begin{aligned} \partial_t u_j + u_i \nabla_i u_j + \epsilon_{jkl} f_k u_l &= -\nabla_j \Phi + b\hat{z} + \dot{\mathcal{M}}_j \\ \partial_t b + u_i \nabla_i b &= \dot{\mathcal{B}} \\ \nabla_i u_i &= 0 \end{aligned}$$

$$\begin{aligned} \partial_t u_j + u_i \nabla_i u_j + \epsilon_{jkl} f_k u_l &= -\nabla_j \Phi + b\hat{z} + \dot{\mathcal{M}}_j \\ \partial_t S + u_i \nabla_i S &= \dot{\mathcal{S}} \\ \partial_t \Theta + u_i \nabla_i \Theta &= \dot{\mathcal{T}} \\ \nabla_i u_i &= 0 \\ b &= b(S, \Theta, z) \end{aligned}$$

Now, we'd like to examine some derived balances from this system,
Let's motivate first with cascades

3D Turbulence Cascade



Kolmogorov Inertial Range

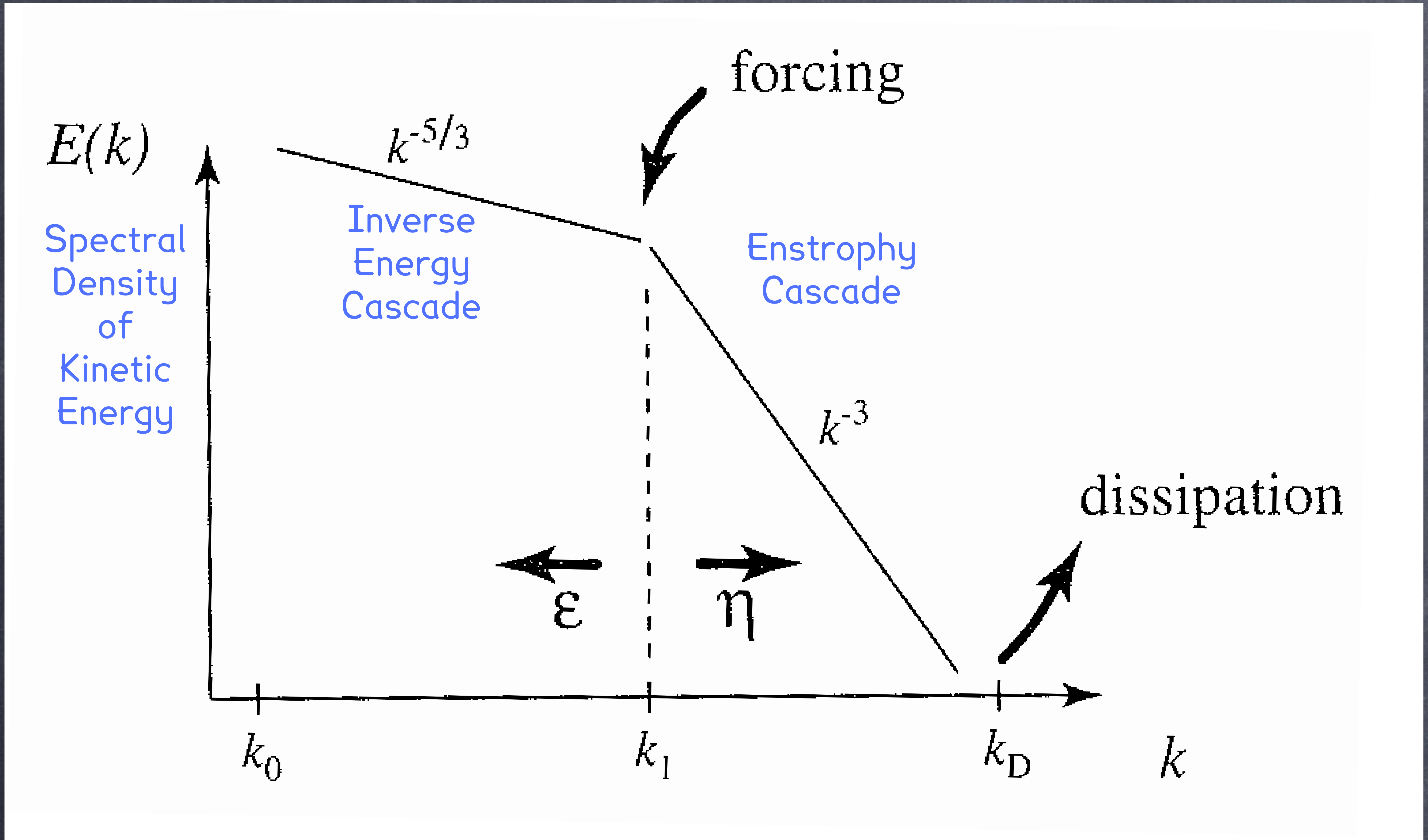
Suitable For Nonhydostatic Boussinesq; Wave-averaged

$$Re^* = UL/\nu_*$$

Another problem... Turbulence can't be 3D Turbulence at the >4km scale

- The ocean is wide (10,000km)
- But not deep (4km)
  - Motions mainly in upper 1km
- The layer of blue paint on a globe has roughly the right aspect ratio!
- Atmosphere is a little taller (30km), but eddies are bigger (1000km)
- Thus, A-O motions are largely 2d

2D Turbulence Differs



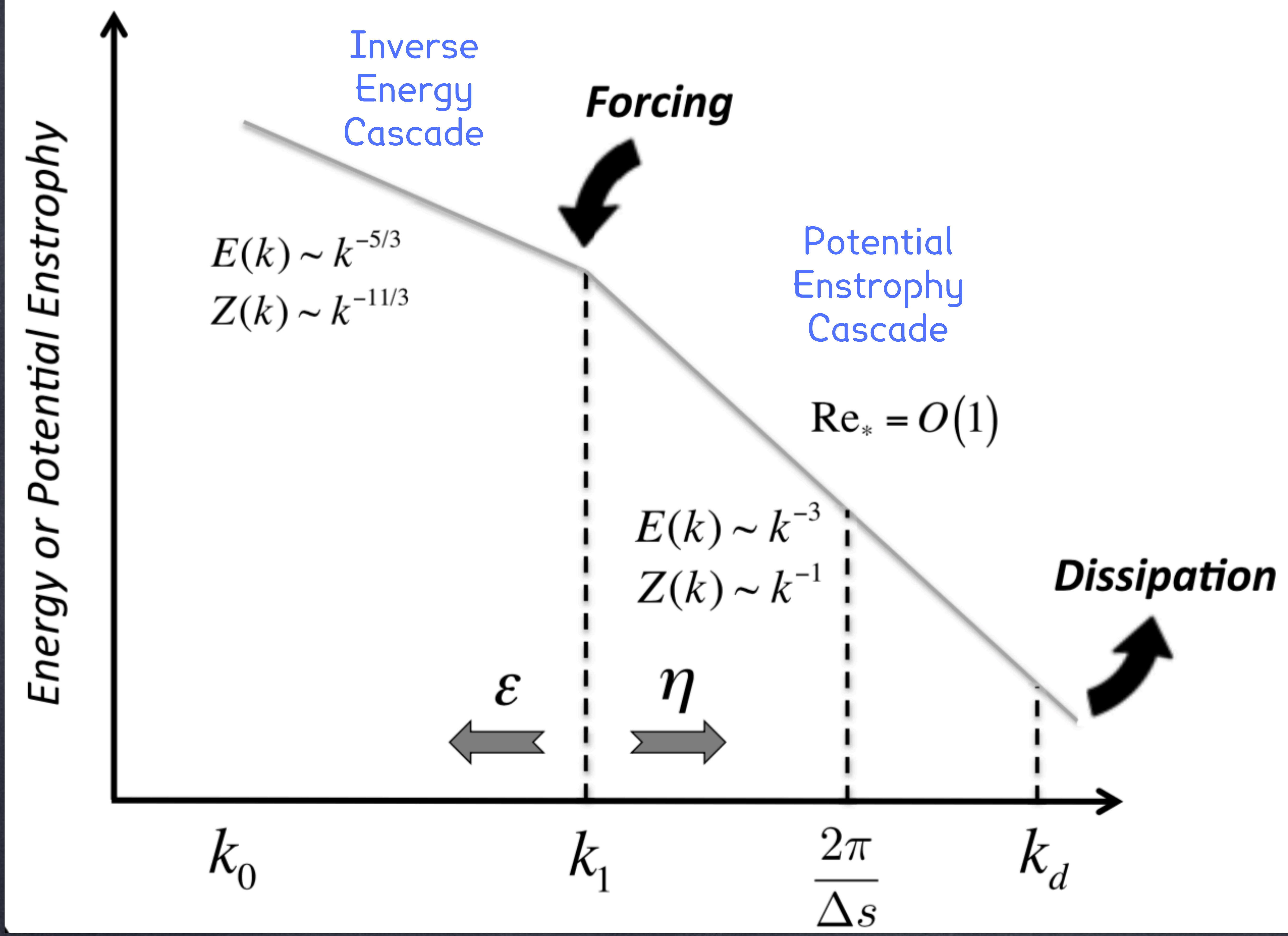
Suitable For 2D Oceans, E.g., Stommel & Munk Gyres

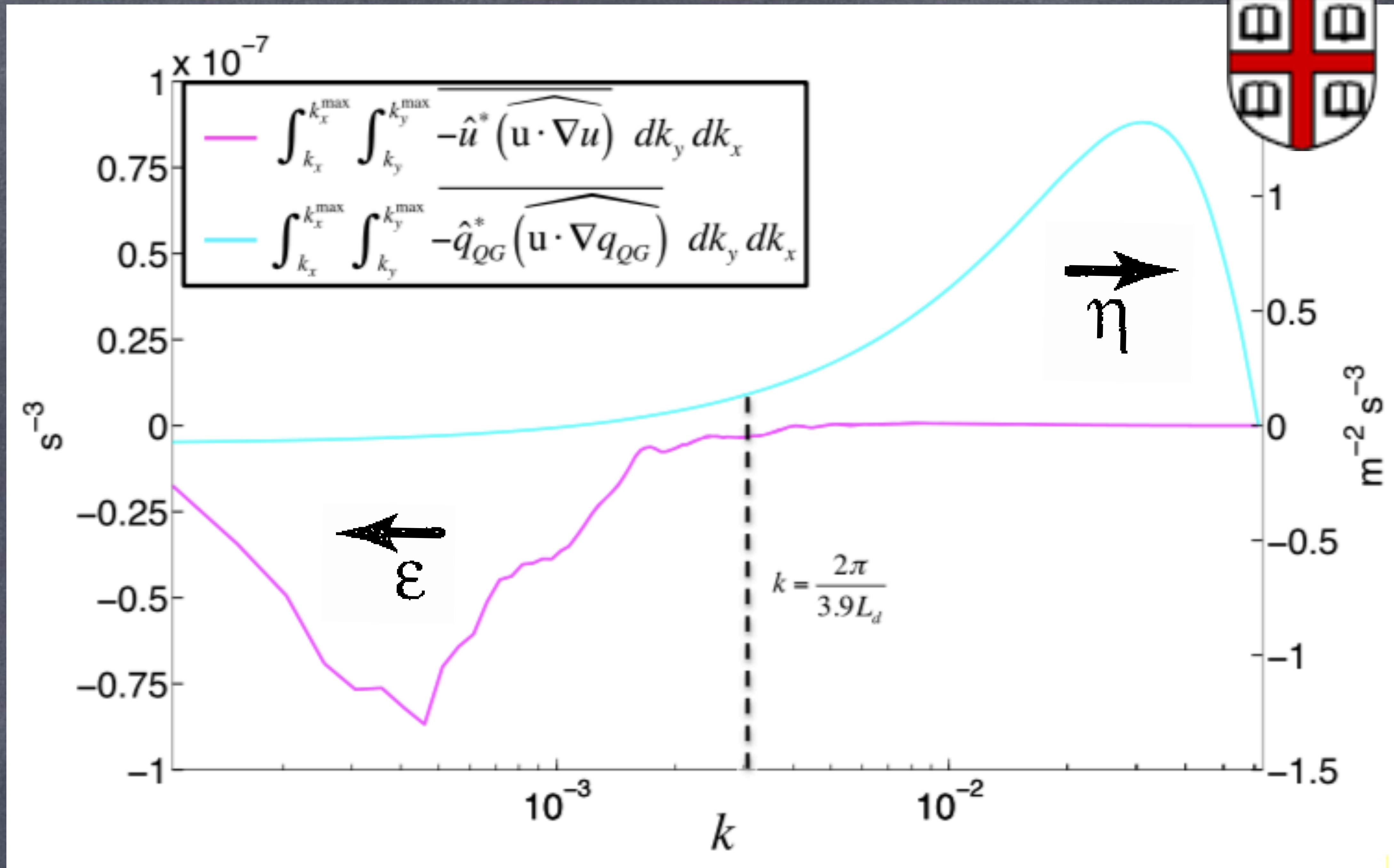
1967: Kraichnan Dual Cascades,
So either inverse energy or enstrophy (vorticity^2) cascades

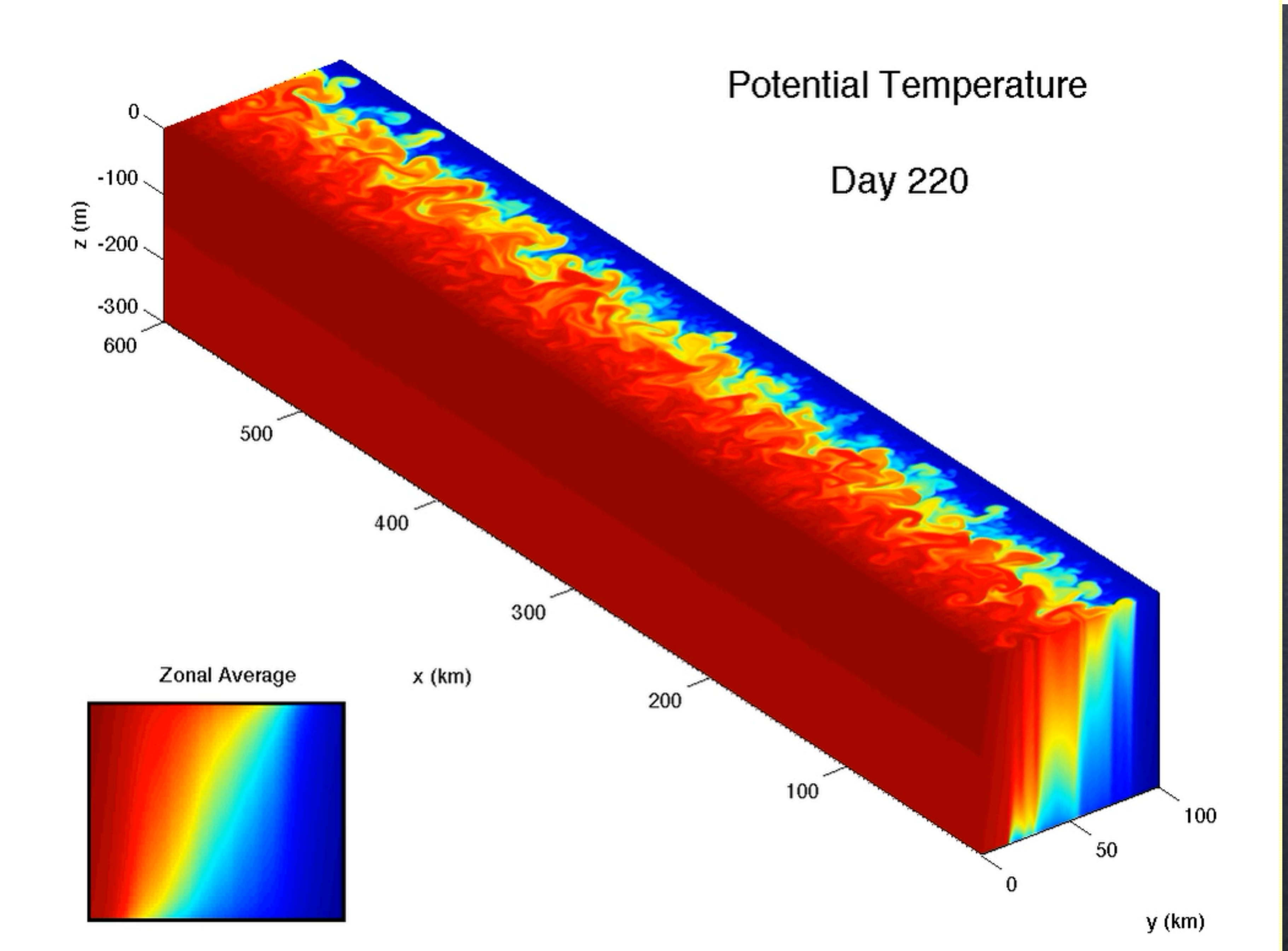
R. Kraichnan, 1967 JFM

Quasigeostrophic Theory (Charney 1971, Salmon 1978):

energy or potential enstrophy cascade



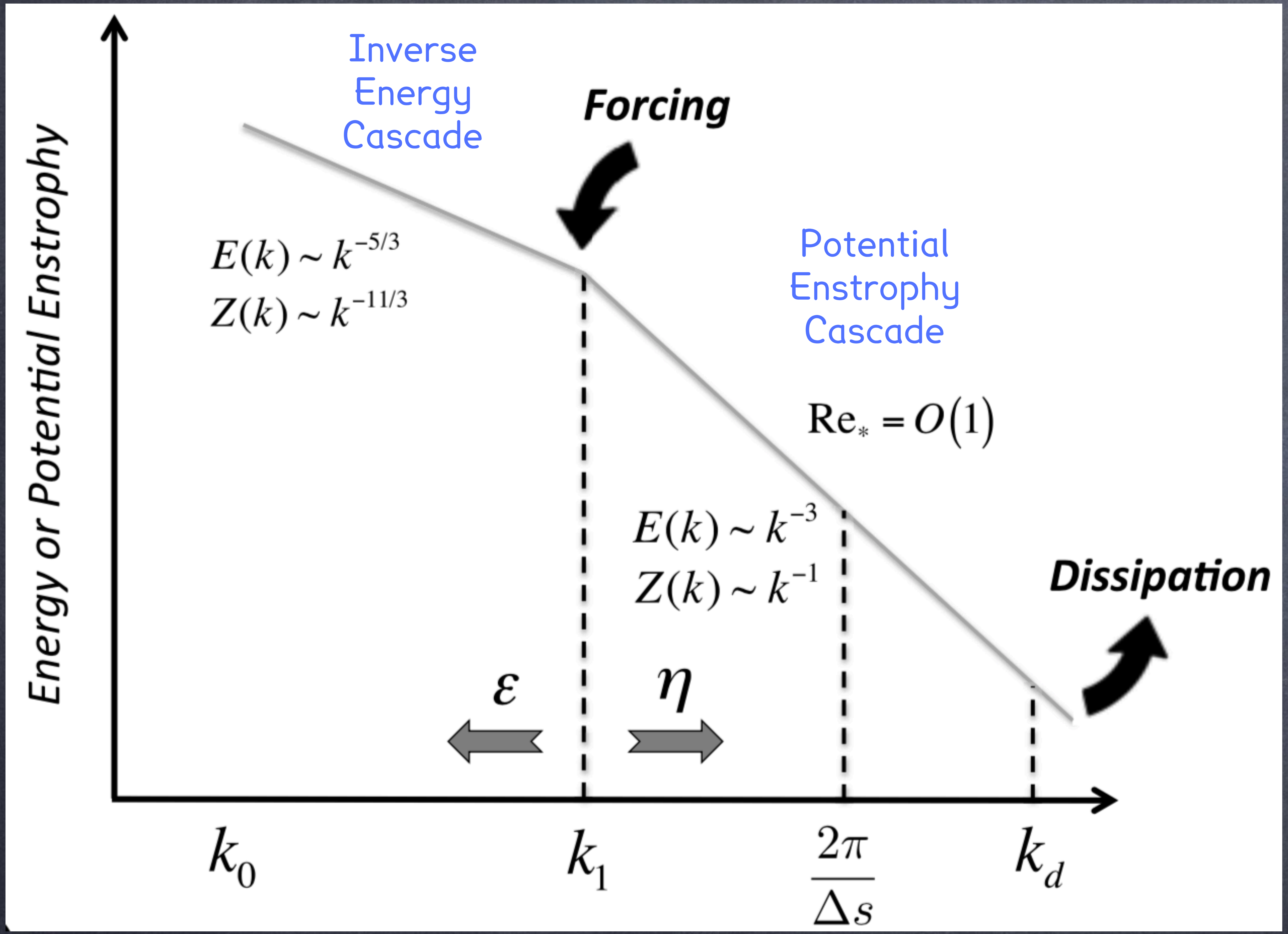


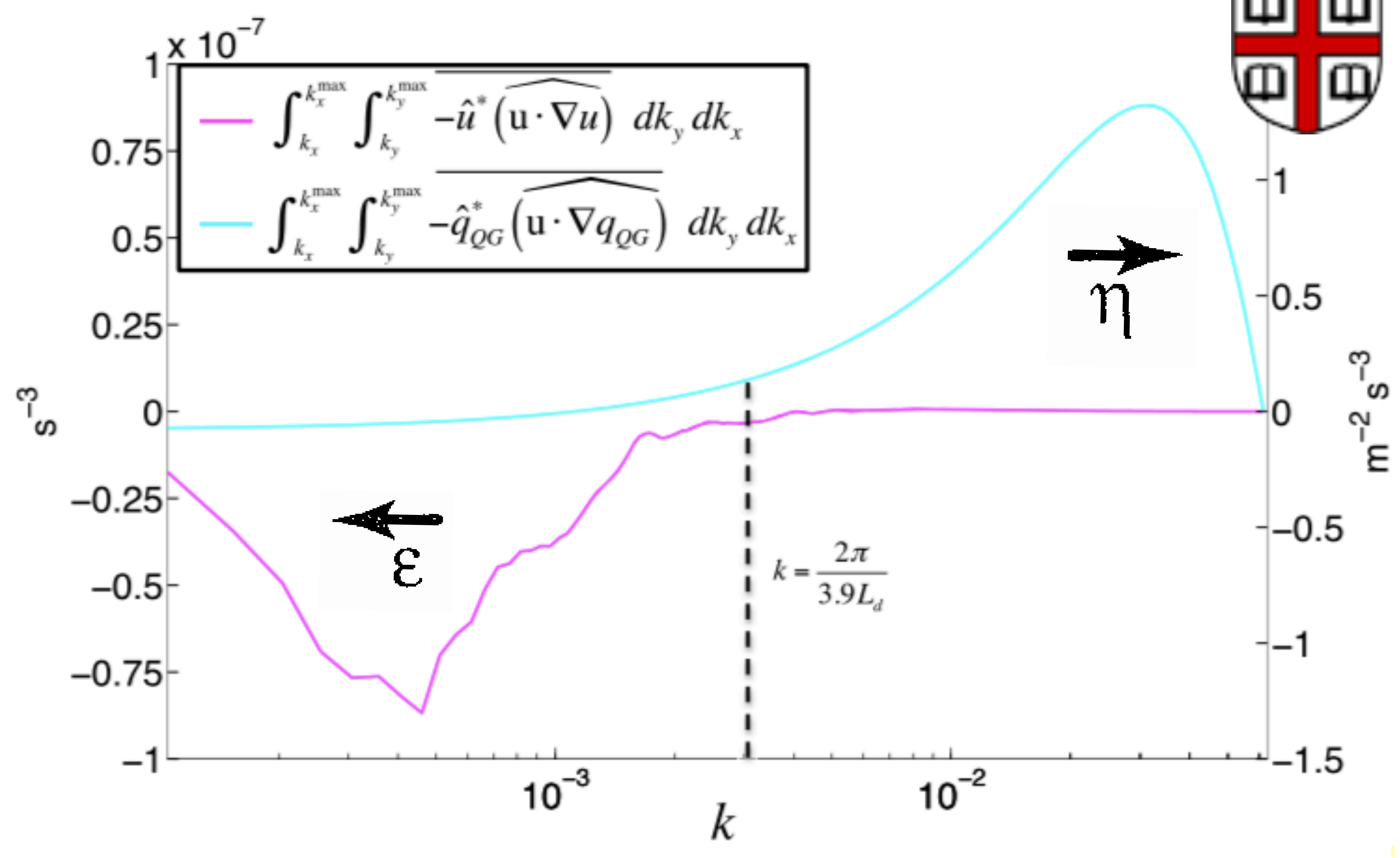


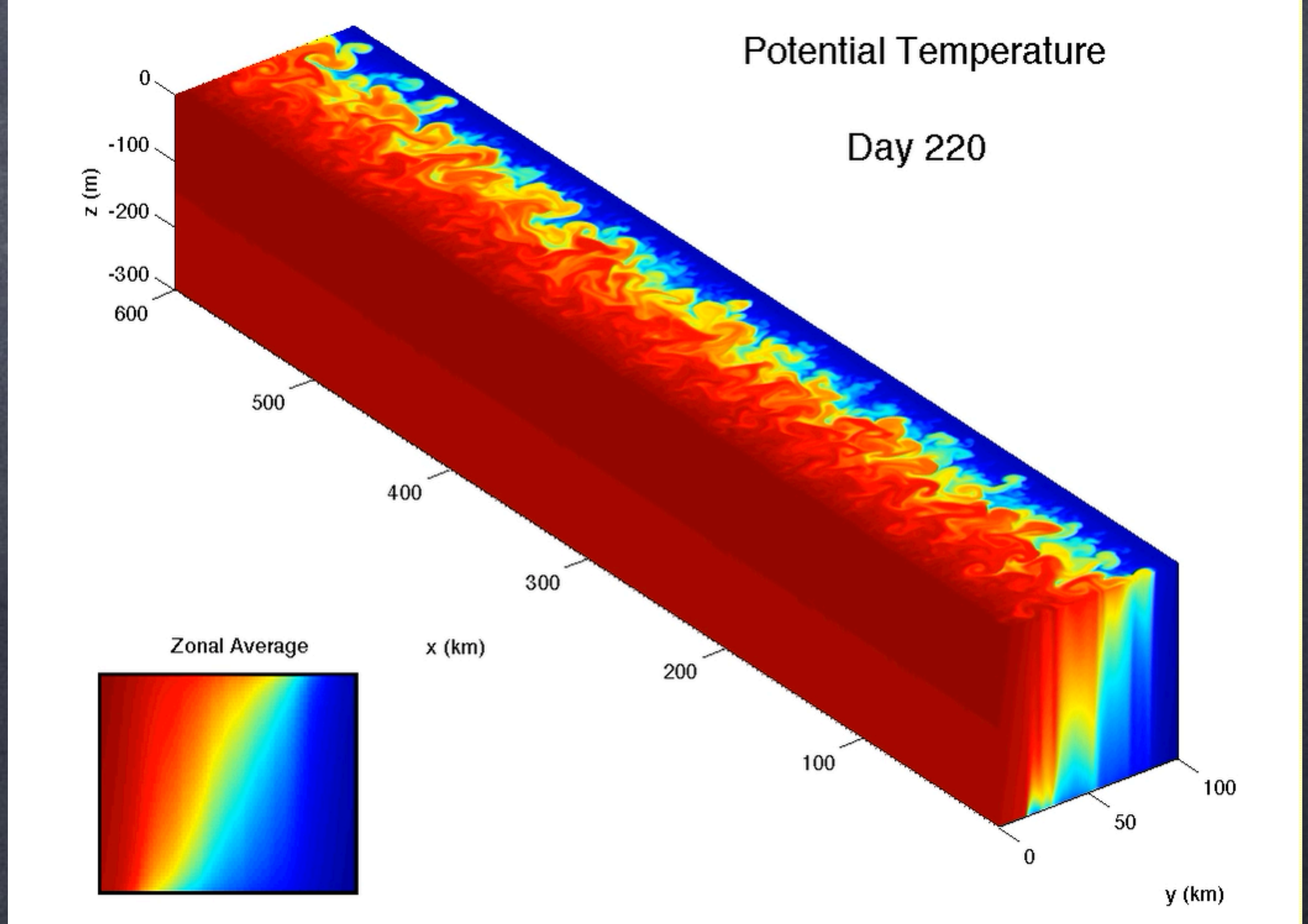
S. D. Bachman, BFK, and B. Pearson. A scale-aware subgrid model for quasigeostrophic turbulence. Journal of Geophysical Research-Oceans, 122:1529-1554, 2017.

B. Pearson, BFK, S. D. Bachman, and F. O. Bryan, 2017: Evaluation of scale-aware subgrid mesoscale eddy models in a global eddy-rich model. Ocean Modelling, 115:42–58.

Quasigeostrophic Theory (Charney 1971, Salmon 1978):
energy or potential enstrophy cascade







S. D. Bachman, BFK, and B. Pearson. A scale-aware subgrid model for quasigeostrophic turbulence. Journal of Geophysical Research-Oceans, 122:1529-1554, 2017.

B. Pearson, BFK, S. D. Bachman, and F. O. Bryan, 2017: Evaluation of scale-aware subgrid mesoscale eddy models in a global eddy-rich model. Ocean Modelling, 115:42–58.

First-Order Balances

- By first order, I mean only derivatives, not products.
- Vorticity & Divergence
- Buoyancy Gradient
- I don't show the (coupled) horizontal divergence and vertical vorticity equation, even though they are valuable (but ugly).

$$\begin{aligned}
\partial_t u_j + u_i \nabla_i u_j + \epsilon_{jkl} f_k u_l &= -\nabla_j \Phi + b\hat{z} + \dot{\mathcal{M}}_j \\
\partial_t b + u_i \nabla_i b &= \dot{\mathcal{B}} \\
\nabla_i u_i &= 0
\end{aligned}$$

$$\begin{aligned}
\partial_t \omega_i + u_j \nabla_j \omega_i &= \omega_j \nabla_j u_i + \nabla_j \left[\epsilon_{ijz} b + \epsilon_{ijk} \dot{\mathcal{M}}_k\right] \\
\nabla_i u_i &= 0,
\end{aligned}$$

$$\partial_t \nabla_i b + u_j \nabla_j \nabla_i b = -(\nabla_j b)(\nabla_i u_j) + \nabla_i \dot{\mathcal{B}}$$

(Vorticity ω includes both planetary (f) & relative)

Second-Order Balances

- Now we consider equations for products.
- Kinetic, Potential & Total Energy
- Tracer Variance
- Enstrophy
- All of these are formed by left multiplying by a factor & crunching

$$\begin{aligned}
\partial_t u_j + u_i \nabla_i u_j + \epsilon_{jkl} f_k u_l &= -\nabla_j \Phi + b\hat{z} + \dot{\mathcal{M}}_j \\
\partial_t b + u_i \nabla_i b &= \dot{\mathcal{B}} \\
\nabla_i u_i &= 0
\end{aligned}$$

Kinetic

$$\partial_t \frac{u_j u_j}{2} + u_i \nabla_i \frac{u_j u_j}{2} = -\nabla_j (\Phi u_j) + u_z b + u_j \dot{\mathcal{M}}_j$$

Second-Order Balances

- Now we consider equations for products.
- Kinetic, Potential & Total Energy
- Tracer Variance
- Enstrophy
- All of these are formed by left multiplying by a factor & crunching (or guessing and then cleaning up the mess!)

$$\begin{aligned} \partial_t u_j + u_i \nabla_i u_j + \epsilon_{jkl} f_k u_l &= -\nabla_j \Phi + b\hat{z} + \dot{\mathcal{M}}_j \\ \partial_t b + u_i \nabla_i b &= \dot{\mathcal{B}} \\ \nabla_i u_i &= 0 \end{aligned}$$

The partner to the KE equation is the PE equation, which is formed by noting that the potential energy per unit volume is just the first term, then that offsetting it by a constant doesn't change its meaning, so

$$\rho g z \rightarrow (\rho - \rho_0) g z = -\rho_0 b z \tag{22}$$

$$\partial_t(-zb) + u_i \nabla_i(-zb) = -u_z b - z\dot{\mathcal{B}}$$

Second-Order Balances

- Now we consider equations for products.
- Kinetic, Potential & Total Energy
- Tracer Variance
- Enstrophy
- All of these are formed by left multiplying by a factor & crunching

Kinetic

$$\partial_t \frac{u_j u_j}{2} + u_i \nabla_i \frac{u_j u_j}{2} = -\nabla_j(\Phi u_j) + u_z b + u_j \dot{\mathcal{M}}_j$$

Potential

$$\partial_t(-zb) + u_i \nabla_i(-zb) = -u_z b - z\dot{\mathcal{B}}$$

Total Mechanical Energy
(Incl. Bernoulli Function)

$$\partial_t\left(\frac{u_j u_j}{2} - zb\right) + u_i \nabla_i \left(\frac{u_j u_j}{2} - zb + \Phi\right) = u_j \dot{\mathcal{M}}_j - z\dot{\mathcal{B}}$$

Where's the internal energy? It's inaccessible in Boussinesq approx.

Second-Order Balances

- Now we consider equations for products.
- Kinetic, Potential & Total Energy
- Tracer Variance
- Enstrophy
- All of these are formed by left multiplying by a factor & crunching

$$
\begin{aligned}
\partial_t u_j + u_i \nabla_i u_j + \epsilon_{jkl} f_k u_l &= -\nabla_j \Phi + b\hat{z} + \dot{\mathcal{M}}_j \\
\partial_t b + u_i \nabla_i b &= \dot{\mathcal{B}} \\
\nabla_i u_i &= 0
\end{aligned}
$$

$$
\begin{aligned}
\partial_t \frac{b^2}{2} + u_i \nabla_i \frac{b^2}{2} &= b\dot{\mathcal{B}} \\
\partial_t \frac{S^2}{2} + u_i \nabla_i \frac{S^2}{2} &= S\dot{\mathcal{S}} \\
\partial_t \frac{\Theta^2}{2} + u_i \nabla_i \frac{\Theta^2}{2} &= \Theta\dot{\mathcal{T}}
\end{aligned}
$$

Second-Order Balances

- Now we consider equations for products.
- Kinetic, Potential & Total Energy
- Tracer Variance
- Enstrophy = (vert vorticity)^2
- All of these are formed by left multiplying by a factor & crunching

$$\begin{aligned} \partial_t \omega_i + u_j \nabla_j \omega_i &= \omega_j \nabla_j u_i + \nabla_j \left[\epsilon_{ijz} b + \epsilon_{ijk} \dot{\mathcal{M}}_k \right] \\ \nabla_i u_i &= 0, \end{aligned}$$

$$\begin{aligned} \partial_t \frac{\omega_z^2}{2} + u_\alpha \nabla_\alpha \frac{\omega_z^2}{2} &= \omega_z \nabla_j \left[\epsilon_{zjk} \dot{\mathcal{M}}_k \right] \\ &\quad + \omega_z \omega_j \nabla_j u_z - u_z \nabla_z \frac{\omega_z^2}{2} \end{aligned}$$

For 2D flows:

$$\partial_t \frac{\omega_z^2}{2} + u_\alpha \nabla_\alpha \frac{\omega_z^2}{2} = \omega_z \nabla_j \left[\epsilon_{zjk} \dot{\mathcal{M}}_k \right]$$

(Vorticity includes both planetary (f) & relative)

Second-Order Balances

- Potential Vorticity
- This is formed in a different way, by combining two first-order equations: the vorticity and buoyancy gradient equations.
- Also, potential enstrophy seems easy from PV, but actually it's 4th order!

$$\begin{aligned} \partial_t \omega_i + u_j \nabla_j \omega_i &= \omega_j \nabla_j u_i + \nabla_j \left[\epsilon_{ijz} b + \epsilon_{ijk} \dot{\mathcal{M}}_k \right] \\ \nabla_i u_i &= 0, \end{aligned}$$

$$\partial_t \nabla_i b + u_j \nabla_j \nabla_i b = -(\nabla_j b)(\nabla_i u_j) + \nabla_i \dot{\mathcal{B}}$$

Potential vorticity

$$\begin{aligned} \partial_t q + u_i \nabla_i q &= (\nabla_i b) \epsilon_{ijk} \nabla_j \dot{\mathcal{M}}_k + \omega_i \nabla_i \dot{\mathcal{B}} \\ q &\equiv \omega_i \nabla_i b \end{aligned}$$

Potential enstrophy

$$\partial_t \frac{q^2}{2} + u_i \nabla_i \frac{q^2}{2} = q (\nabla_i b) \epsilon_{ijk} \nabla_j \dot{\mathcal{M}}_k + q \omega_i \nabla_i \dot{\mathcal{B}}$$

(Vorticity includes both planetary (f) & relative)

Introducing Scale: Reynolds-Average and Filtered Terms

Example:

Following Sadek and Aluie (2018) and Salmon (2013), we define a generalized average tracer concentration—including time-, space-, and ensemble-averaging—as

$$\overline{\tau}(\mathbf{x}, t) = \int \mathrm{d}t' \iiint \mathrm{d}^3\mathbf{x}' \int \mathrm{d}\mu G(\mathbf{x} - \mathbf{x}', t - t', \mu)\tau(\mathbf{x}', t', \mu) \tag{2}$$

We will consider linear avgs/filters
And assume they commute with differentiation, e.g.,

$$\begin{aligned} \overline{\nabla_i u} &= \nabla_i \overline{u} \\ \overline{\partial_t u} &= \partial_t \overline{u} \\ \overline{c(u+v)} &= c\overline{u} + c\overline{v} \end{aligned}$$

Introducing Scale: Fourier transform

Example:

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{f}(k) e^{ikx} dk \qquad \tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

Note that these are also linear operators
And they commute with differentiation (becomes multiplication),
And by Plancherel's theorem, they have a meaningful quadratic.

$$\begin{aligned}
\widetilde{\nabla_i u} &= \nabla_i \tilde{u} \\
\widetilde{\partial_t u} &= \partial_t \tilde{u} \\
\widetilde{c(u+v)} &= c\tilde{u} + c\tilde{v} \\
\int_{-\infty}^{\infty} |\tilde{u}|^2 dk &= \int_{-\infty}^{\infty} |u|^2 dx
\end{aligned}$$

Introducing Scale: Structure Function

Increment Example: $\delta u_i(x_j,t;s_j,\tau) = u_i(x_j+s_j,t+\tau) - u_i(x_j,t)$

Structure Fct. Formed From increments:

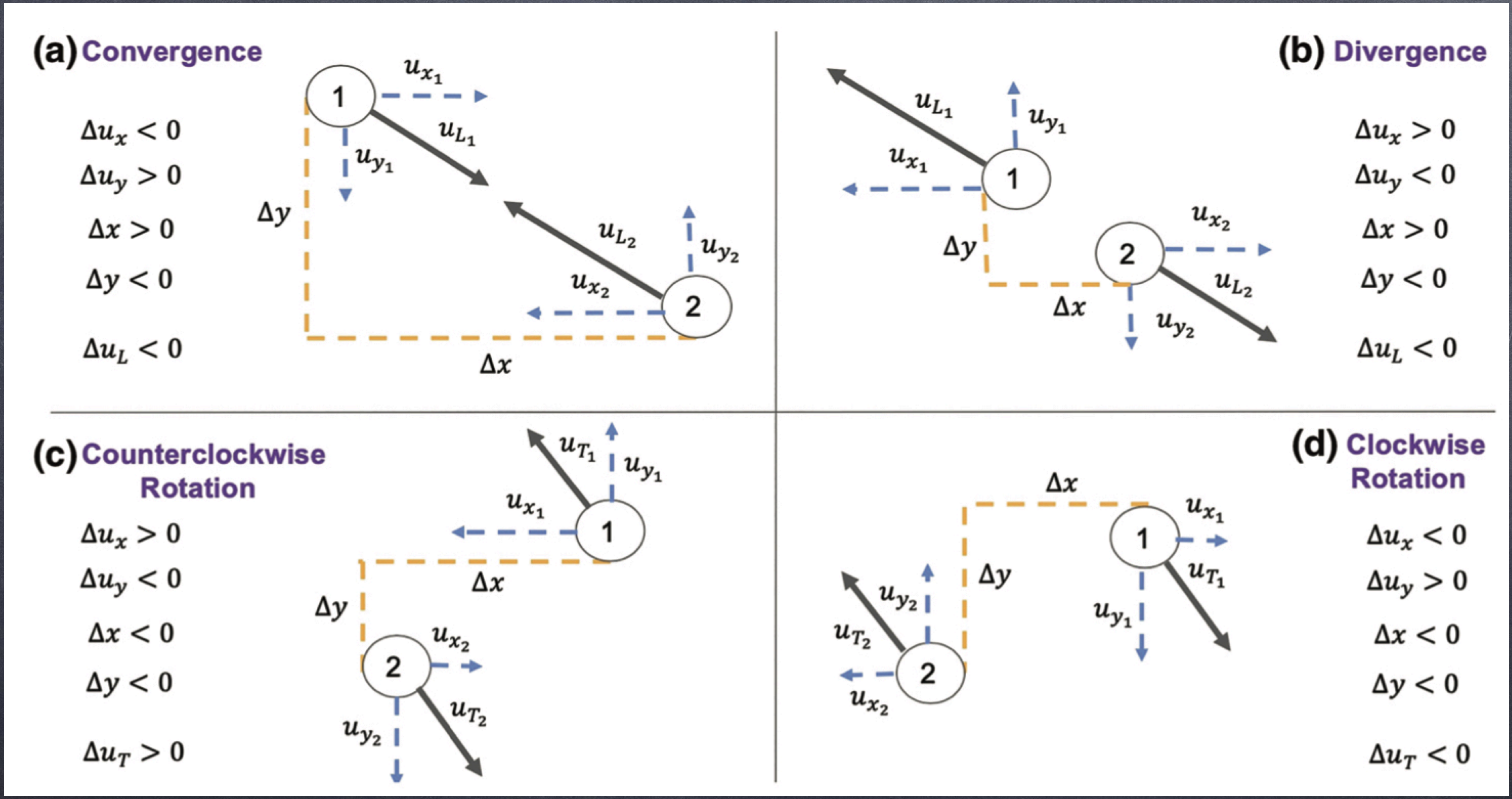
$$\begin{aligned} S^n_{ij}(u;x,t;s,\tau) &= \overline{\delta^n u_i(x_j,t;s_j,\tau)} \\ &= \overline{[u_i(x_j+s_j,t+\tau) - u_i(x_j,t)]^n} \end{aligned}$$

Note that increments are also linear operators
And they commute with differentiation (becomes multiplication)

Unlike spectra & Many averages, You must specify:

$$\begin{aligned} S^n(u;x,t;s,\tau) &= \overline{[u(x+s,t+\tau) - u(x,t)]^n} \text{ if isotropic} \\ S^n(u;x;s,\tau) &= \overline{[u(x+s,t+\tau) - u(x,t)]^n} \text{ if stationary} \\ S^n(u;s,\tau) &= \overline{[u(x+s,t+\tau) - u(x,t)]^n} \text{ if homogeneous} \end{aligned}$$

Introducing Scale:
Structure Function (velocity examples)



J. Pearson, BFK, B. Pearson, H. Chang, B. K. Haus, J. Horstmann, H. S. Huntley, A. D. Kirwan, Jr., B. Lund, and A. Poje. Biases in structure functions from observations of submesoscale flows. JGR-Oceans, 125:e2019JC015769, 2020.

Impress with these two weird tricks...

$$
\begin{aligned}
\partial_t \underline{u_j} + \underline{u_i} \nabla_i \underline{u_j} + \epsilon_{jkl} f_k \underline{u_l} &= -\nabla_j \underline{\Phi} + \underline{b}\hat{z} + \underline{\dot{\mathcal{M}}_j}^{+} \\
-\nabla_i \underline{u_i u_j} + \nabla_i \underline{u_i}\ \underline{u_j} + \underline{\dot{\mathcal{M}}_i} &\equiv \underline{\dot{\mathcal{M}}_i}^{+} \\
\partial_t \underline{b} + \underline{u_i} \nabla_i \underline{b} &= \underline{\dot{\mathcal{B}}}^{+} \\
-\nabla_i \underline{u_i b} + \nabla_i \underline{u_i}\ \underline{b} + \underline{\dot{\mathcal{B}}} &\equiv \underline{\dot{\mathcal{B}}}^{+} \\
\nabla_i \underline{u_i} &= 0
\end{aligned}
$$

1) Augment the non-conservative terms

2) Note that Reynolds, Filter, Fourier, Increment are isomorphic, represent any of them with an underline.

With this, it’s trivial to find the scale-selective equations...

$$
\begin{aligned}
\partial_t \underline{\omega_i} + \underline{u_j} \nabla_j \underline{\omega_i} &= \underline{\omega_j} \nabla_j \underline{u_i} + \nabla_j \left[\epsilon_{ijz} \underline{b} + \epsilon_{ijk} \underline{\dot{\mathcal{M}}_k}^{+} \right] \\
\partial_t \nabla_i \underline{b} + \underline{u_j} \nabla_j \nabla_i \underline{b} &= -(\nabla_j \underline{b})(\nabla_i \underline{u_j}) + \nabla_i \underline{\dot{\mathcal{B}}}^{+}
\end{aligned}
$$

$$
\partial_t \frac{\underline{\underline{u_j u_j}}}{2} + \underline{u_i} \nabla_i \frac{\underline{\underline{u_j u_j}}}{2} = -\nabla_j (\underline{\Phi} \underline{u_j}) + \underline{u_z} \underline{b} + \underline{u_j \dot{\mathcal{M}}_j}^{+}
$$

$$
\partial_t (-z\underline{b}) + \underline{u_i} \nabla_i (-z\underline{b}) = -\underline{u_z} \underline{b} - z \underline{\dot{\mathcal{B}}}^{+}
$$

$$
\partial_t \left(\frac{\underline{\underline{u_j u_j}}}{2} - z\underline{b} \right) + \underline{u_i} \nabla_i \left(\frac{\underline{\underline{u_j u_j}}}{2} - z\underline{b} + \underline{\Phi} \right) = \underline{u_j}\ \underline{\dot{\mathcal{M}}_j}^{+} - z \underline{\dot{\mathcal{B}}}^{+}
$$

$$
\begin{aligned}
\partial_t \frac{\underline{b}^2}{2} + \underline{u_i} \nabla_i \frac{\underline{b}^2}{2} &= \underline{b}\, \underline{\dot{\mathcal{B}}}^{+} \\
\partial_t \frac{\underline{S}^2}{2} + \underline{u_i} \nabla_i \frac{\underline{S}^2}{2} &= \underline{S}\, \underline{\dot{\mathcal{S}}}^{+} \\
\partial_t \frac{\underline{\Theta}^2}{2} + \underline{u_i} \nabla_i \frac{\underline{\Theta}^2}{2} &= \underline{\Theta}\, \underline{\dot{\mathcal{T}}}^{+}
\end{aligned}
$$

$$
\begin{aligned}
\partial_t \frac{\underline{\omega_z}^2}{2} + \underline{u_\alpha} \nabla_\alpha \frac{\underline{\omega_z}^2}{2} &= \underline{\omega_z} \nabla_j \left[\epsilon_{zjk} \underline{\dot{\mathcal{M}}_k}^{+} \right] \\
&\quad + \underline{\omega_z} \omega_j \nabla_j \underline{u_z} - \underline{u_z} \nabla_z \frac{\underline{\omega_z}^2}{2}
\end{aligned}
$$

$$
\begin{aligned}
\partial_t \tilde{q} + \underline{u_i} \nabla_i \tilde{q} &= (\nabla_i \underline{b}) \epsilon_{ijk} \nabla_j \underline{\dot{\mathcal{M}}_k}^{+} + \underline{\omega_i} \nabla_i \underline{\dot{\mathcal{B}}}^{+} \\
\tilde{q} &\equiv \underline{\omega_i} \nabla_i \underline{b}
\end{aligned}
$$

Covariances->cospectra or blended structure functions

All of the “interesting” terms: cross-scale transfer, shear production, etc., end up in realizing the augmented non-conservative terms.

For example… mean energy equation

$$\overline{D}_t \frac{\overline{u_j}\,\overline{u_j}}{2} = -\nabla_j(\overline{\Phi}\overline{u_j}) + \overline{u_z}\,\overline{b} + \overline{u_j}\,\overline{\dot{\mathcal{M}}_j}^{+}$$

$$\overline{D}_t(-z\overline{b}) = -\overline{u_z}\,\overline{b} - z\overline{\dot{\mathcal{B}}}^{+}$$

$$\underbrace{-\overline{u_j}\left[\nabla_i\,\overline{u_i u_j} - \nabla_i\overline{u_i}\,\overline{u_j}\right]}_{\text{subgrid shear production}} + \underbrace{\overline{u_i}\,\overline{\dot{\mathcal{M}}_i}}_{\text{wind \& diss.}} \equiv \overline{u_i}\,\overline{\dot{\mathcal{M}}_i}^{+} \quad (61)$$

$$\underbrace{z\left[\nabla_i\overline{u_i b} - \nabla_i\overline{u_i}\,\overline{b}\right]}_{\text{subgrid buoy prod.}} - \underbrace{z\overline{\dot{\mathcal{B}}}}_{\text{mix, rad, \& diss.}} \equiv -z\overline{\dot{\mathcal{B}}}^{+} \quad (62)$$

Or, spectral:

$$\partial_t \frac{\tilde{u_j}^{*}\,\tilde{u_j}}{2} = \tilde{u_z}^{*}\tilde{b} + \tilde{u_j}^{*}\,\widetilde{\dot{\mathcal{M}}_j}^{+}$$

Or, structure functions:
Gives Karman-Howarth-Monin eqtn.

And the interpretation of Ertel potential vorticity in Large Eddy Simulations (Bodner & BFK, 2021)

- The potential vorticity is a second-order derived conserved property
- Because it is second-order, it has complex nonlinear aspects
- It turns out that *it has a high-wavenumber divergence* in 3D turbulence
- Thus, you need to pre-filter the equations in order to use PV in 3D simulations permitting 3D turbulence, but the “augmented” terms capture this simply.

$$\partial_t \omega_i + u_j \nabla_j \omega_i = \omega_j \nabla_j u_i + \nabla_j \left[\epsilon_{ijz} b + \epsilon_{ijk} \dot{\mathcal{M}}_k\right]$$

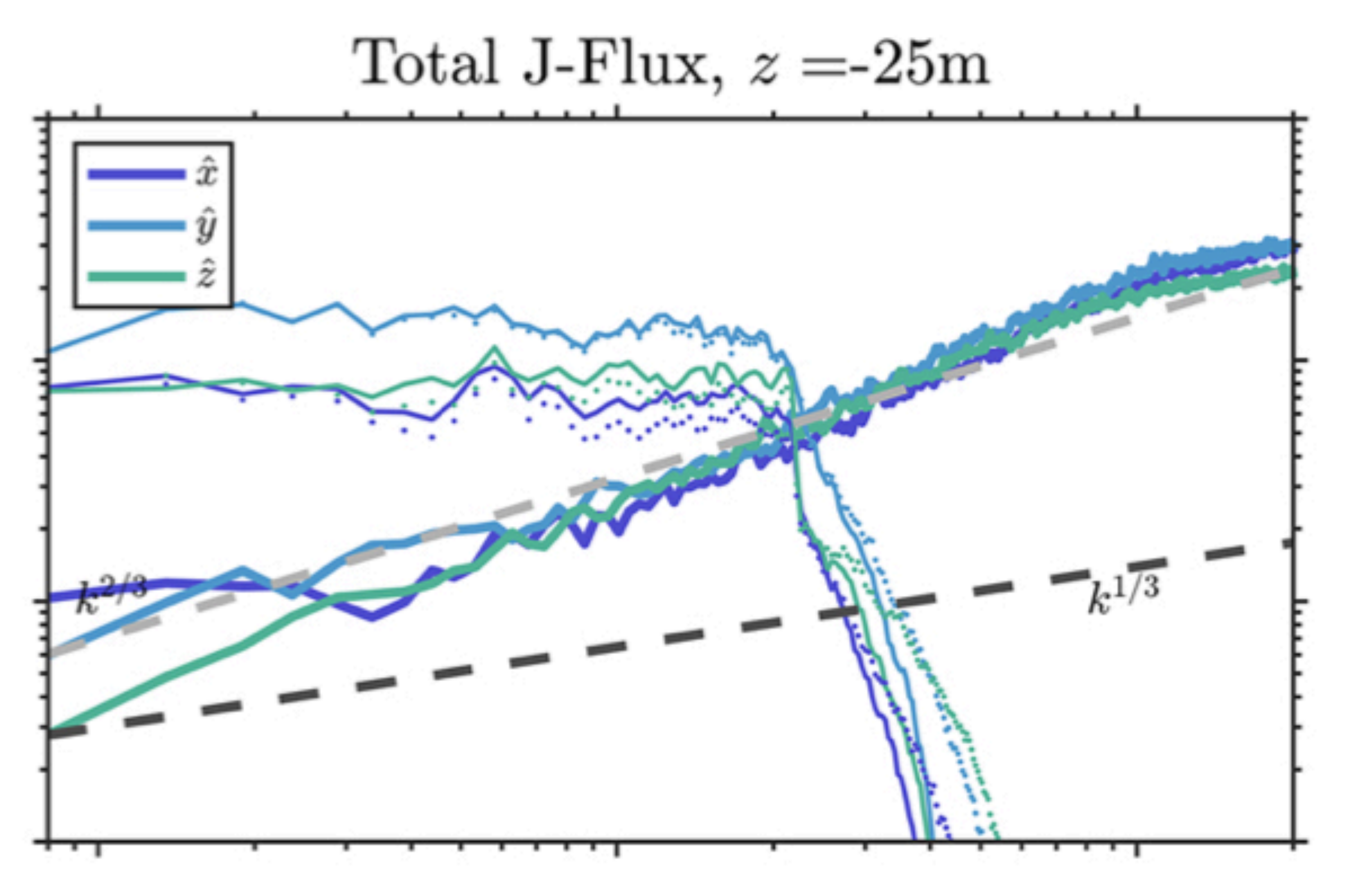
$$\partial_t \nabla_i b + u_j \nabla_j \nabla_i b = -(\nabla_j b)(\nabla_i u_j) + \nabla_i \dot{\mathcal{B}}$$

$$\begin{aligned} \partial_t q + u_i \nabla_i q &= (\nabla_i b)\epsilon_{ijk} \nabla_j \dot{\mathcal{M}}_k + \omega_i \nabla_i \dot{\mathcal{B}} \\ q &\equiv \omega_i \nabla_i b \end{aligned}$$

$$\begin{aligned} \partial_t \tilde{q} + \overline{u_i} \nabla_i \tilde{q} &= (\nabla_i \overline{b})\epsilon_{ijk} \nabla_j \overline{\dot{\mathcal{M}}_k}^{+} + \overline{\omega_i} \nabla_i \overline{\dot{\mathcal{B}}}^{+} \\ \tilde{q} &\equiv \overline{\omega_i} \nabla_i \overline{b} \end{aligned}$$

And the interpretation of Ertel potential vorticity in Large Eddy Simulations (Bodner & BFK, 2021)

- The potential vorticity is a second-order derived conserved property
- Because it is second-order, it has complex nonlinear aspects
- It turns out that *it has a high-wavenumber divergence* in 3D turbulence
- Thus, you need to pre-filter the equations in order to use PV in 3D simulations permitting 3D turbulence, but the "augmented" terms capture this simply.



$$\frac{\partial \tilde{q}}{\partial t} = -\nabla \cdot \left[\overline{\mathbf{u}}\,\tilde{q} - \overline{\mathbf{F}^{+}} \times \nabla \overline{b} - (\mathbf{f} + \omega)\overline{\mathscr{D}^{+}} \right].$$

-div(J-flux)

Second order structure functions are often equivalent to spectra & co-spectra

$$\overline{U^2} = \int_0^\infty E(k)\,dk \qquad \text{(A9)}$$

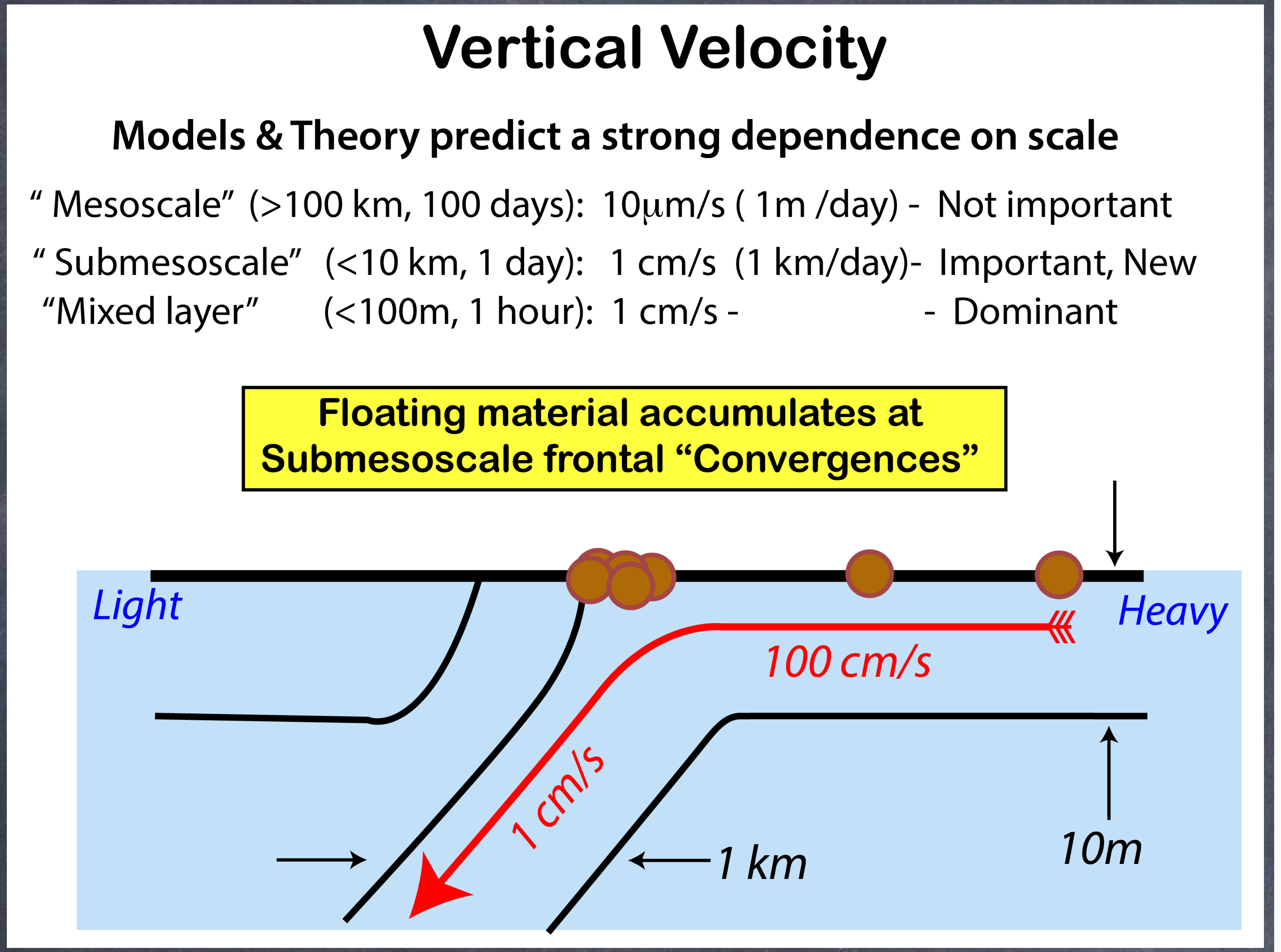
and

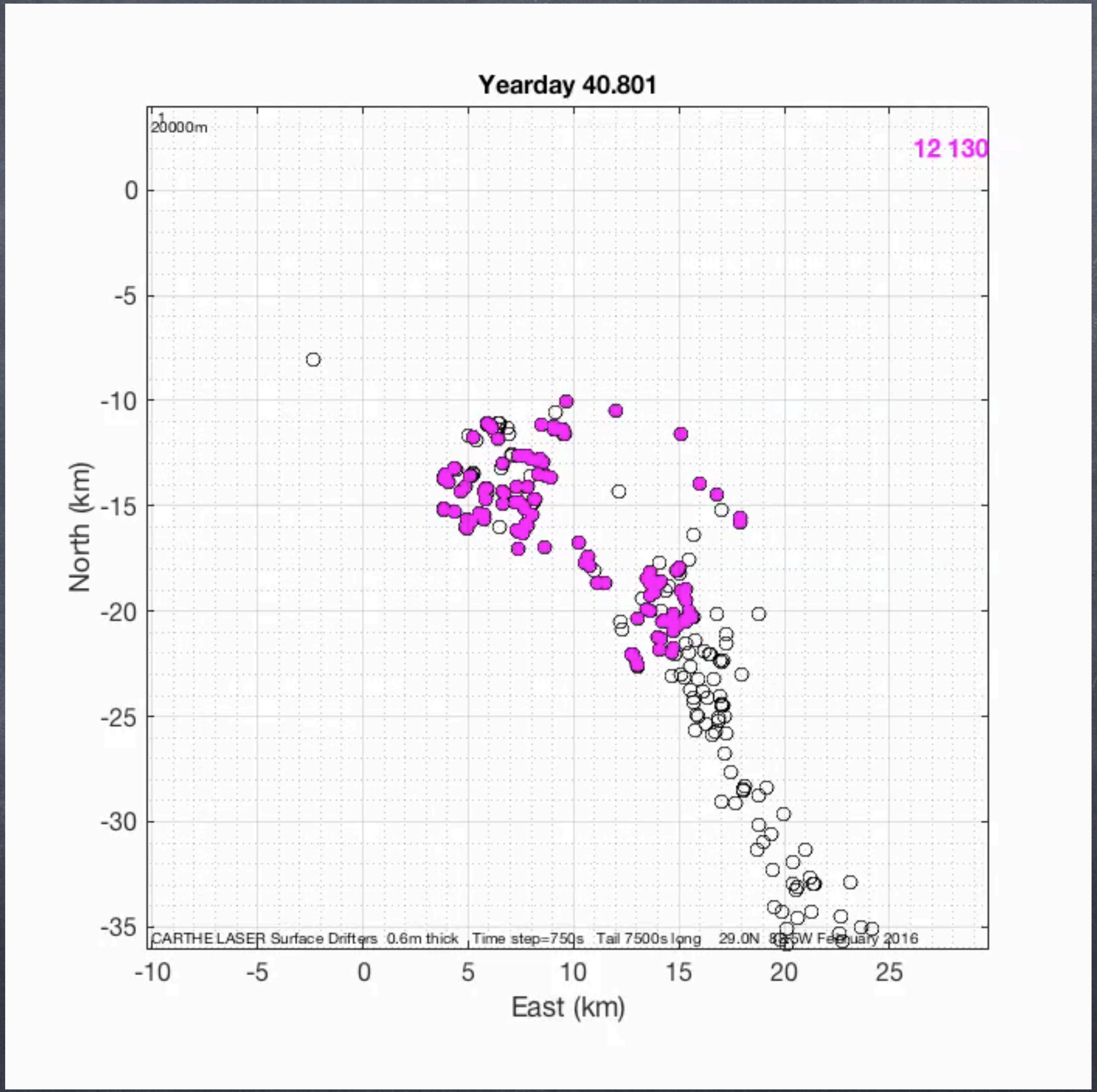
$$D_U(s) = \overline{[u(x) - u(x+s)]^2}. \qquad \text{(A10)}$$

Following the same method, $D_U(s) \propto s^{\beta_D}$ and $E(k) \propto k^{\beta_E}$ produce the same relationship: $\beta_D = -\beta_E - 1$.

K. McCaffrey, BFK, and G. Forget. Estimates of ocean macro-turbulence: Structure function and spectral slope from Argo profiling floats. Journal of Physical Oceanography, 45(7):1773-1793, July 2015.

Quasigeostrophy? Not for submesoscales…

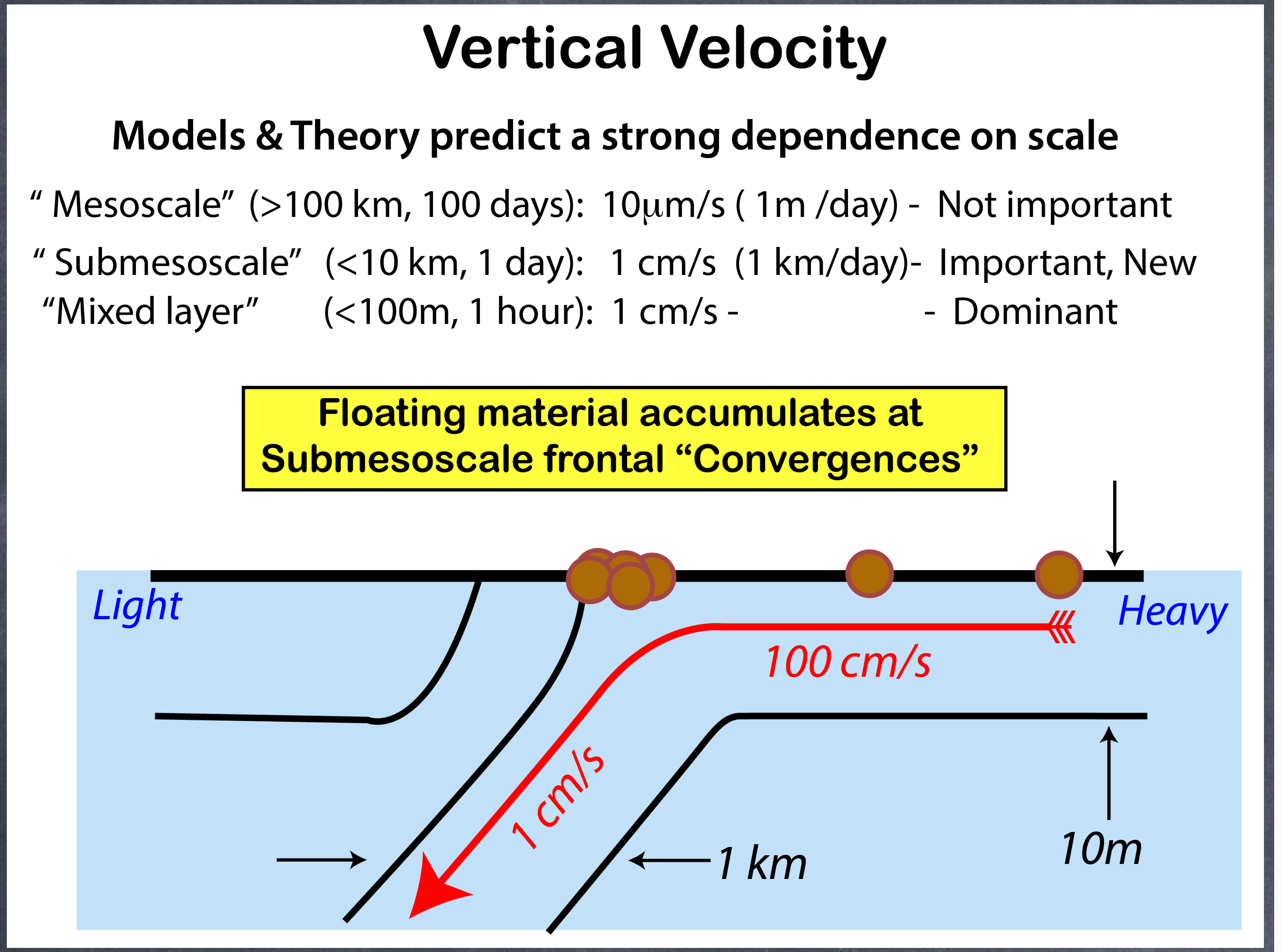


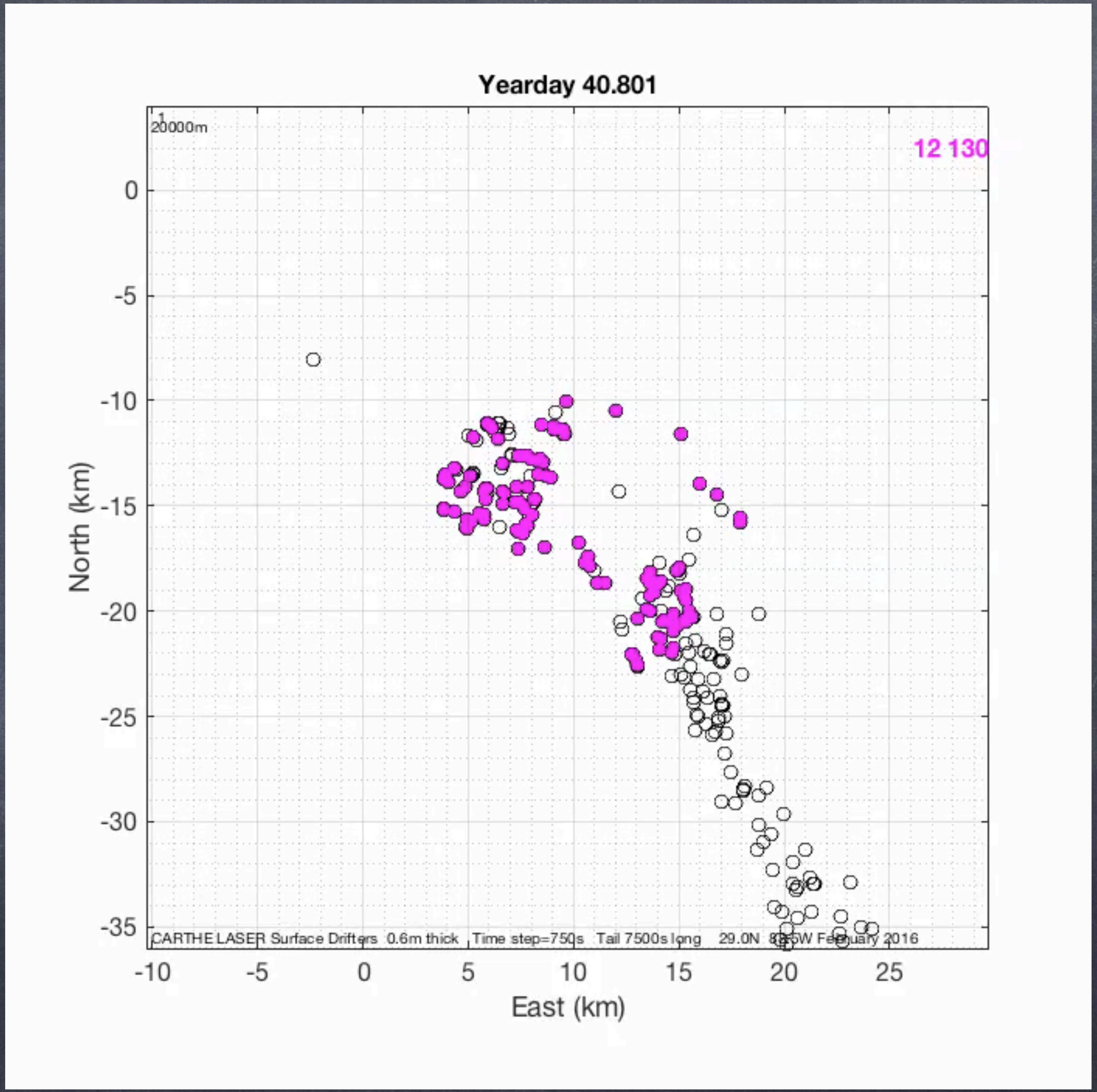


Movie & Slide Courtesy of Eric D'Asaro: See D'Asaro, E.A., Shcherbina, A.Y., Klymak, J.M., Molemaker, J., Novelli, G., Guigand, C.M., Haza, A.C., Haus, B.K., Ryan, E.H., Jacobs, G.A. and Huntley, H.S., 2018. Ocean convergence and the dispersion of flotsam. Proceedings of the National Academy of Sciences, 115(6), pp.1162-1167.

E. A. D'Asaro, D. F. Carlson, M. Chamecki, R. R. Harcourt, B. K. Haus, B. Fox-Kemper, M. J. Molemaker, A. C. Poje, and D. Yang. Advances in observing and understanding small-scale open ocean circulation during the Gulf of Mexico Research Initiative era. Frontiers in Marine Science, 7:349, May 2020.

Quasigeostrophy? Not for submesoscales…

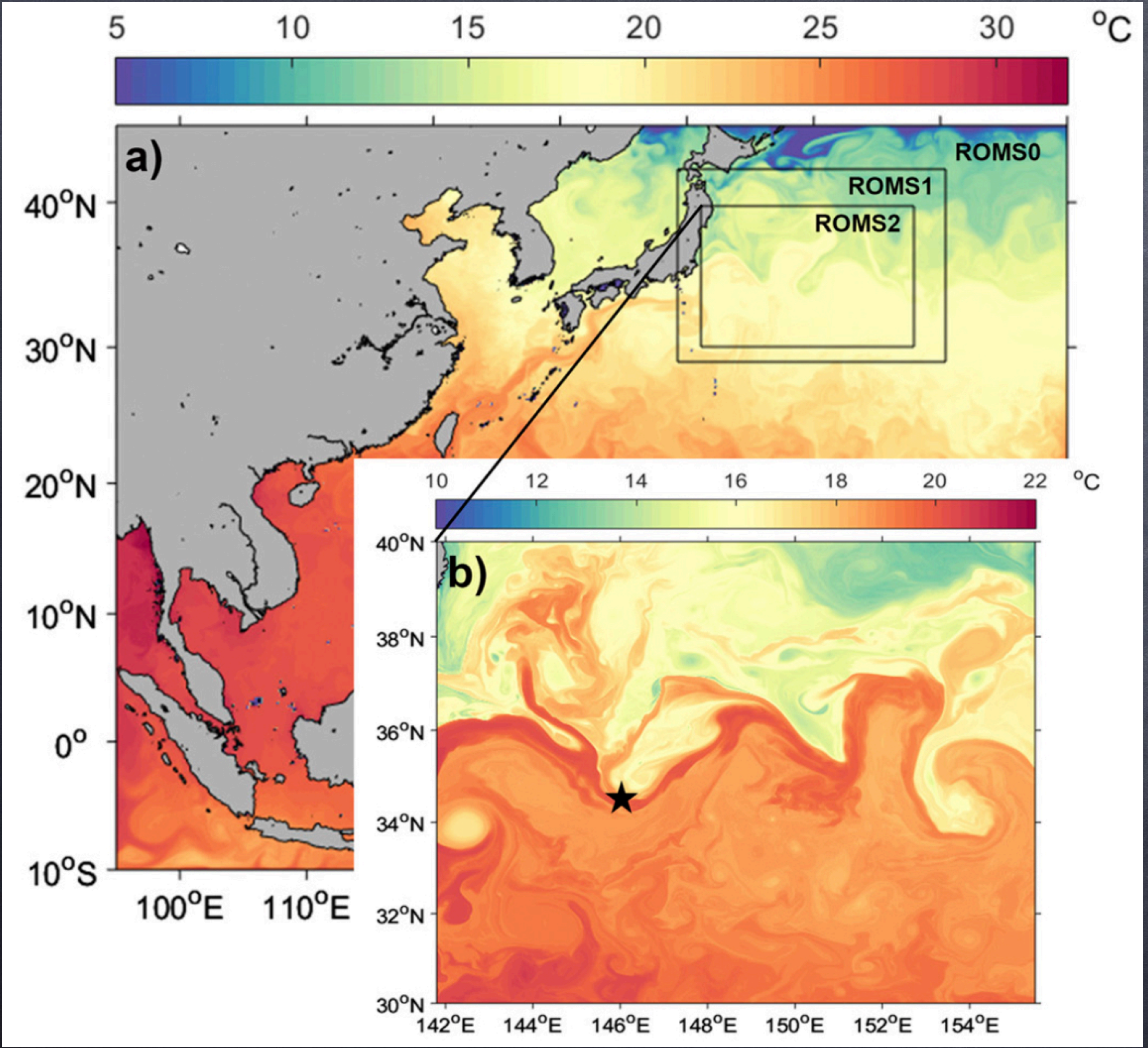


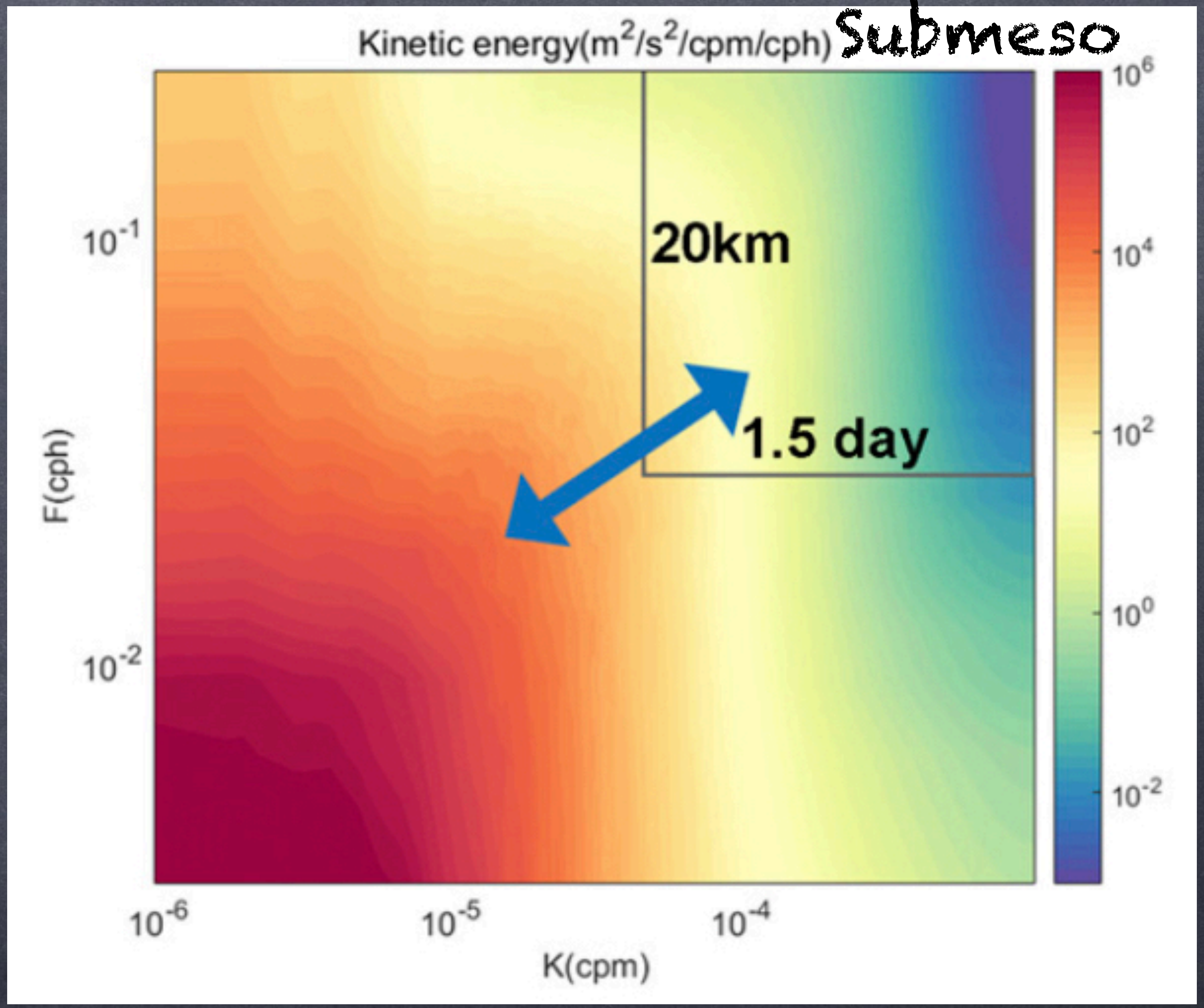


Movie & Slide Courtesy of Eric D'Asaro: See D'Asaro, E.A., Shcherbina, A.Y., Klymak, J.M., Molemaker, J., Novelli, G., Guigand, C.M., Haza, A.C., Haus, B.K., Ryan, E.H., Jacobs, G.A. and Huntley, H.S., 2018. Ocean convergence and the dispersion of flotsam. Proceedings of the National Academy of Sciences, 115(6), pp.1162-1167.

E. A. D'Asaro, D. F. Carlson, M. Chamecki, R. R. Harcourt, B. K. Haus, B. Fox-Kemper, M. J. Molemaker, A. C. Poje, and D. Yang. Advances in observing and understanding small-scale open ocean circulation during the Gulf of Mexico Research Initiative era. Frontiers in Marine Science, 7:349, May 2020.

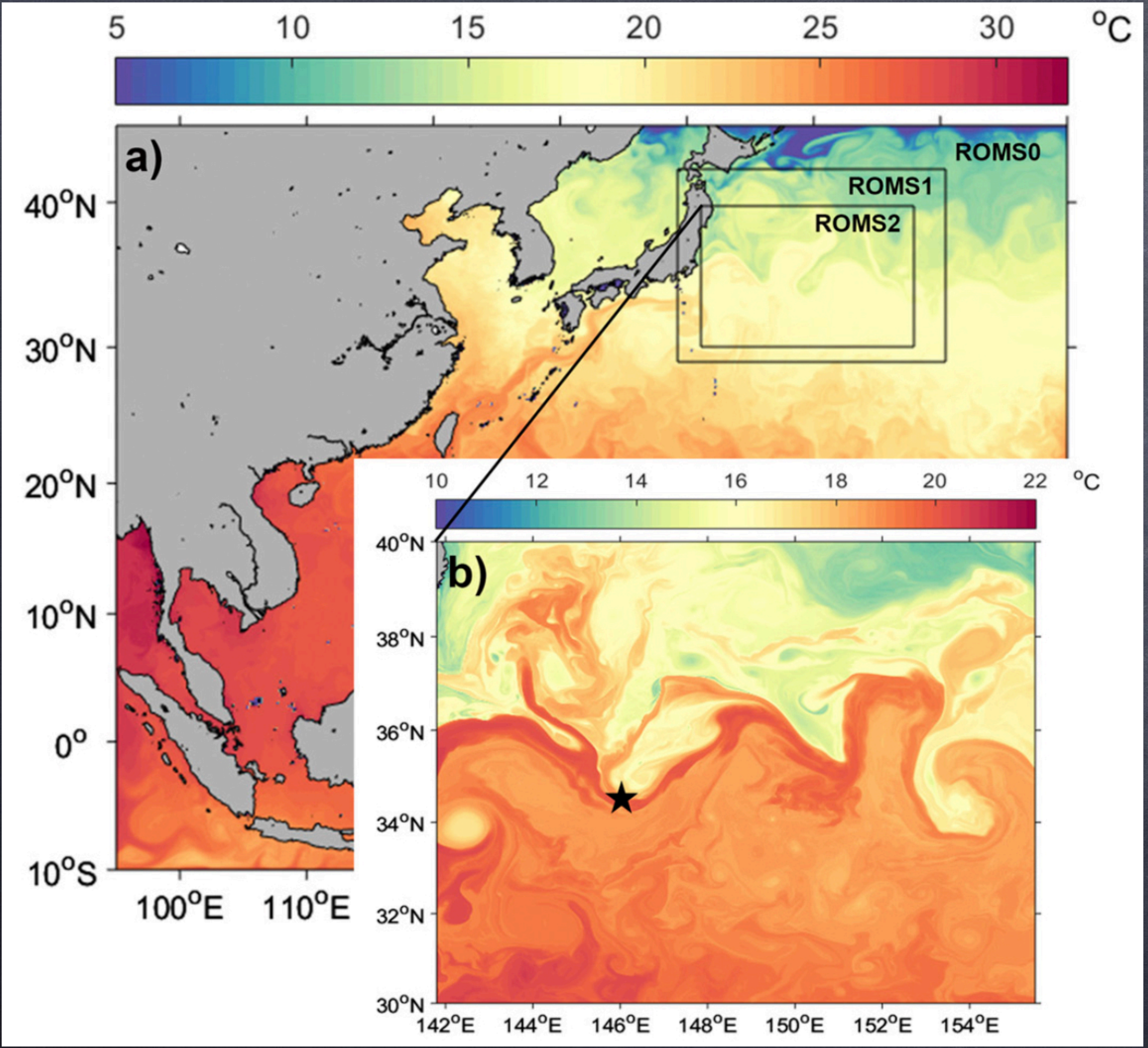
Pacific Modeling for a Frequency-Wavenumber perspective…

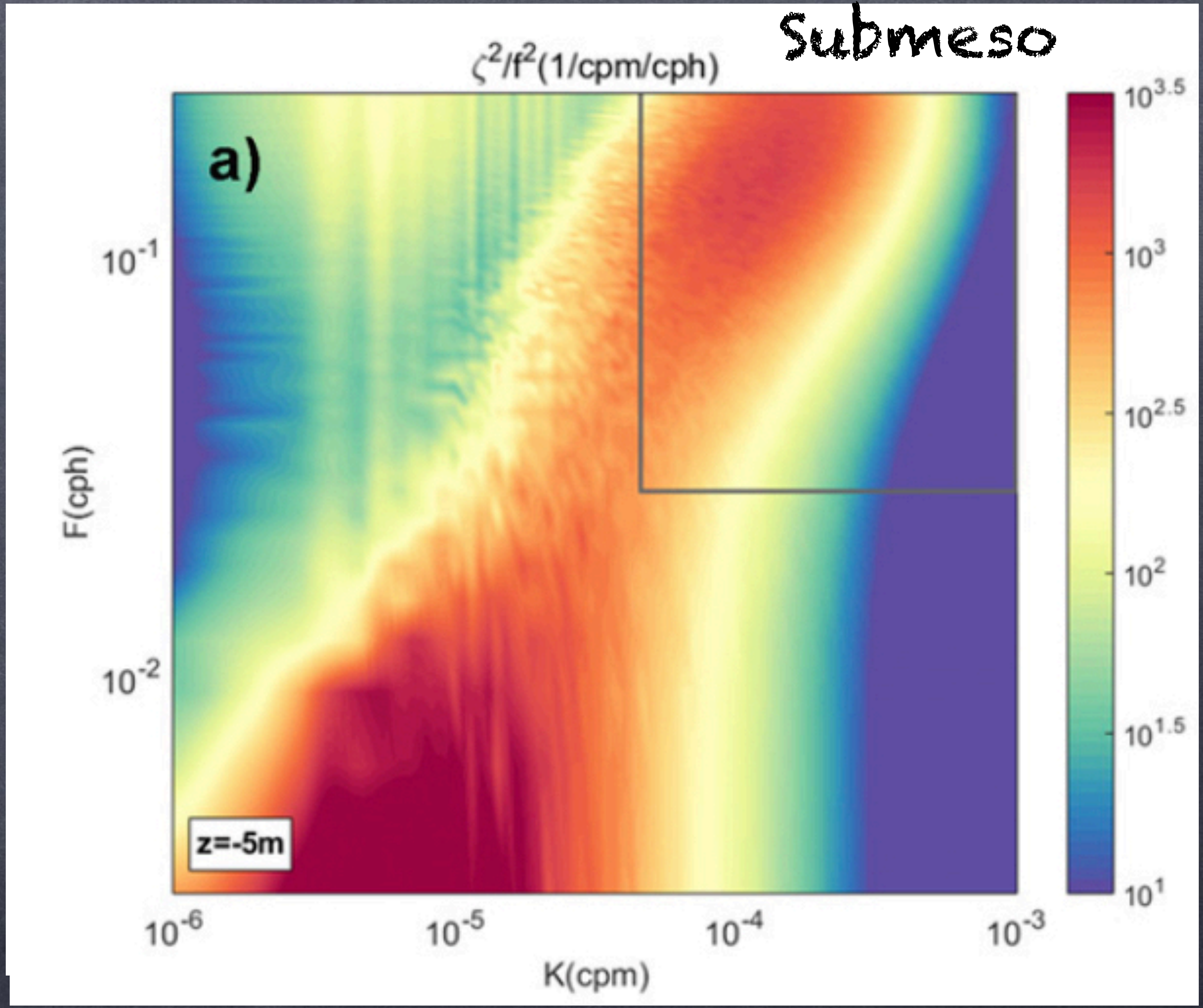




H. Cao, BFK, and Z. Jing. Submesoscale eddies in the upper ocean of the Kuroshio Extension from high-resolution simulation: Energy budget. Journal of Physical Oceanography, 51(7):2181-2201, July 2021.

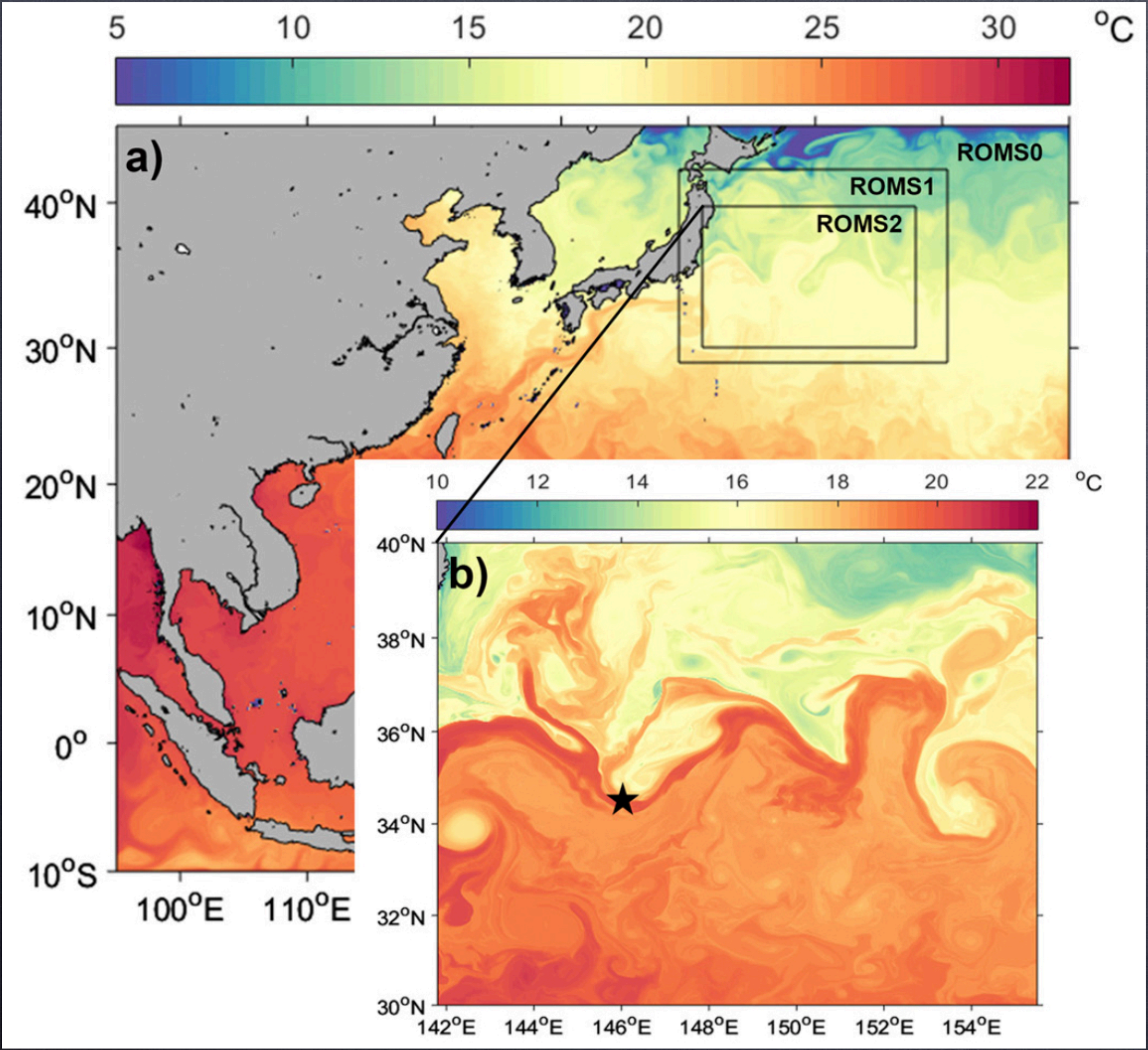
Pacific Modeling for a Frequency-Wavenumber perspective…

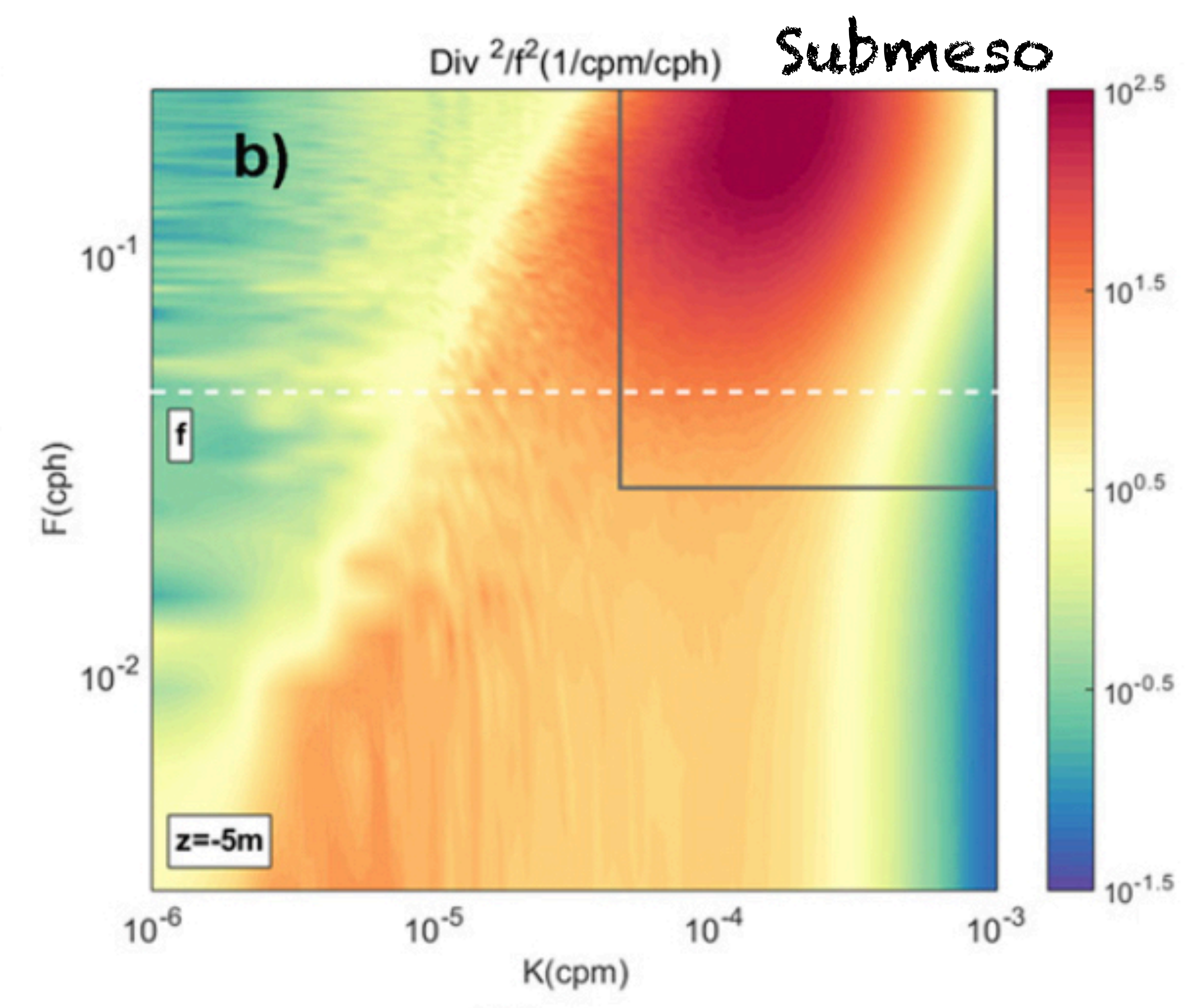




H. Cao, BFK, and Z. Jing. Submesoscale eddies in the upper ocean of the Kuroshio Extension from high-resolution simulation: Energy budget. Journal of Physical Oceanography, 51(7):2181-2201, July 2021.

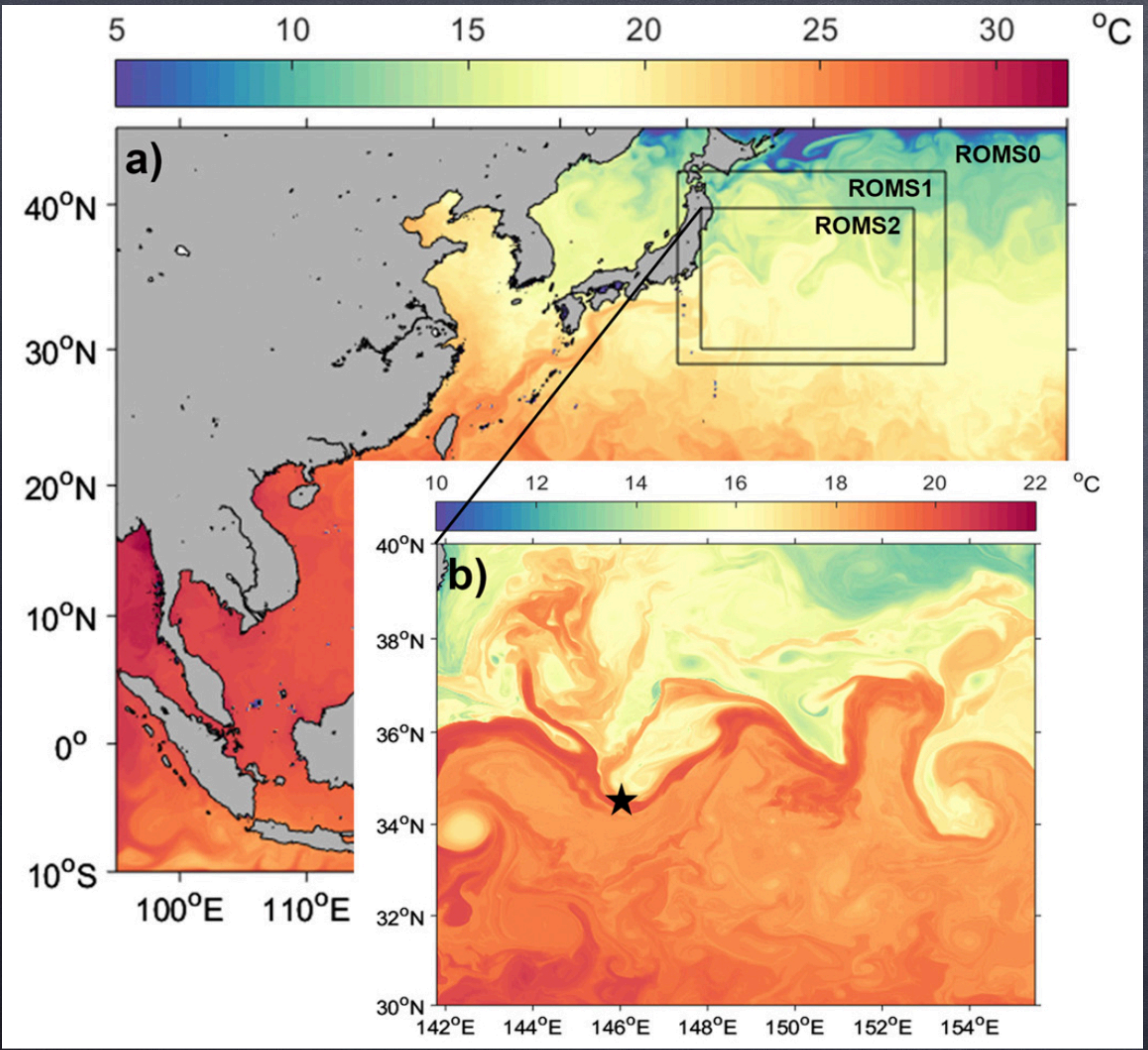
Pacific Modeling for a Frequency-Wavenumber perspective...

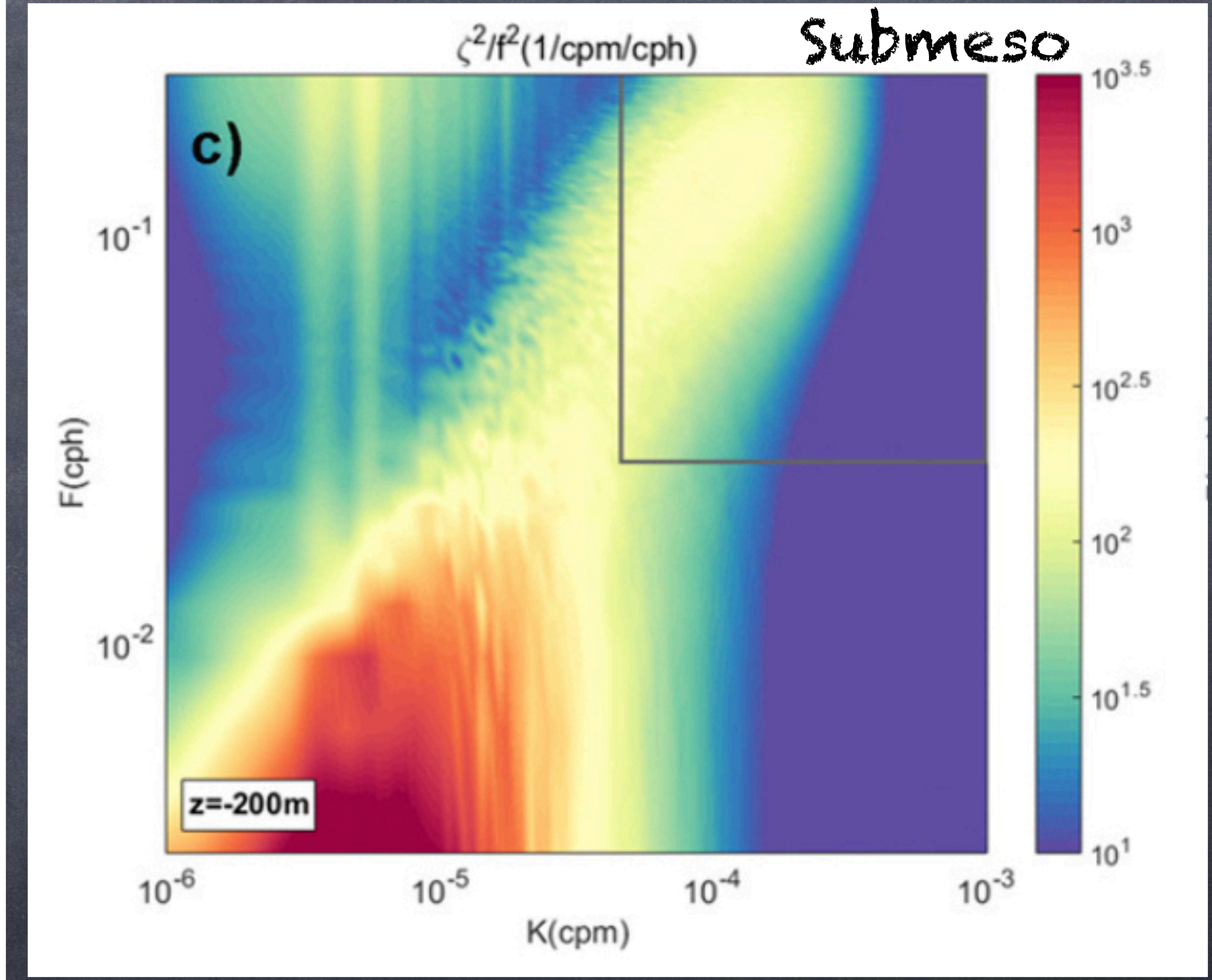




H. Cao, BFK, and Z. Jing. Submesoscale eddies in the upper ocean of the Kuroshio Extension from high-resolution simulation: Energy budget. Journal of Physical Oceanography, 51(7):2181-2201, July 2021.

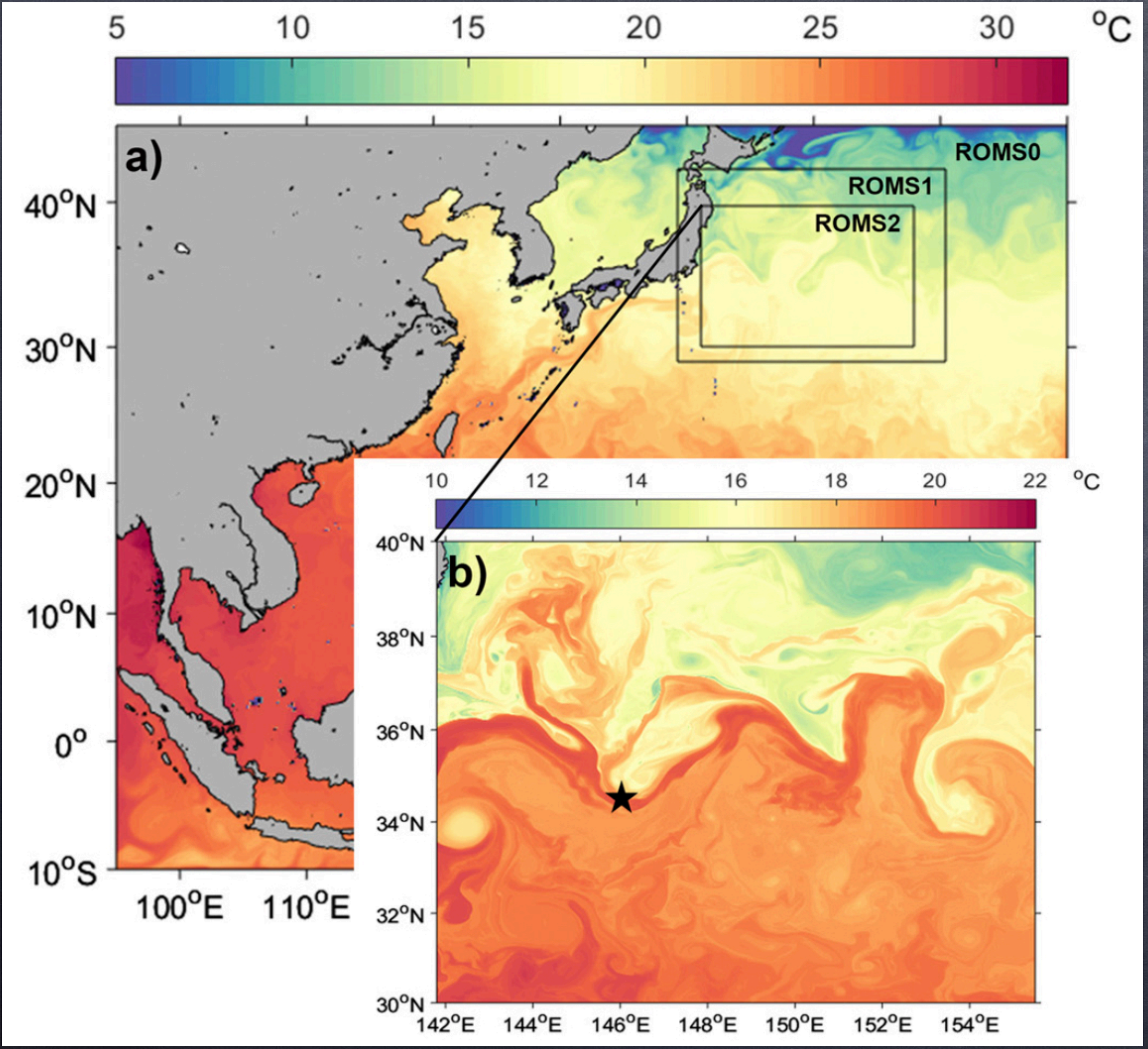
Pacific Modeling for a Frequency-Wavenumber perspective...

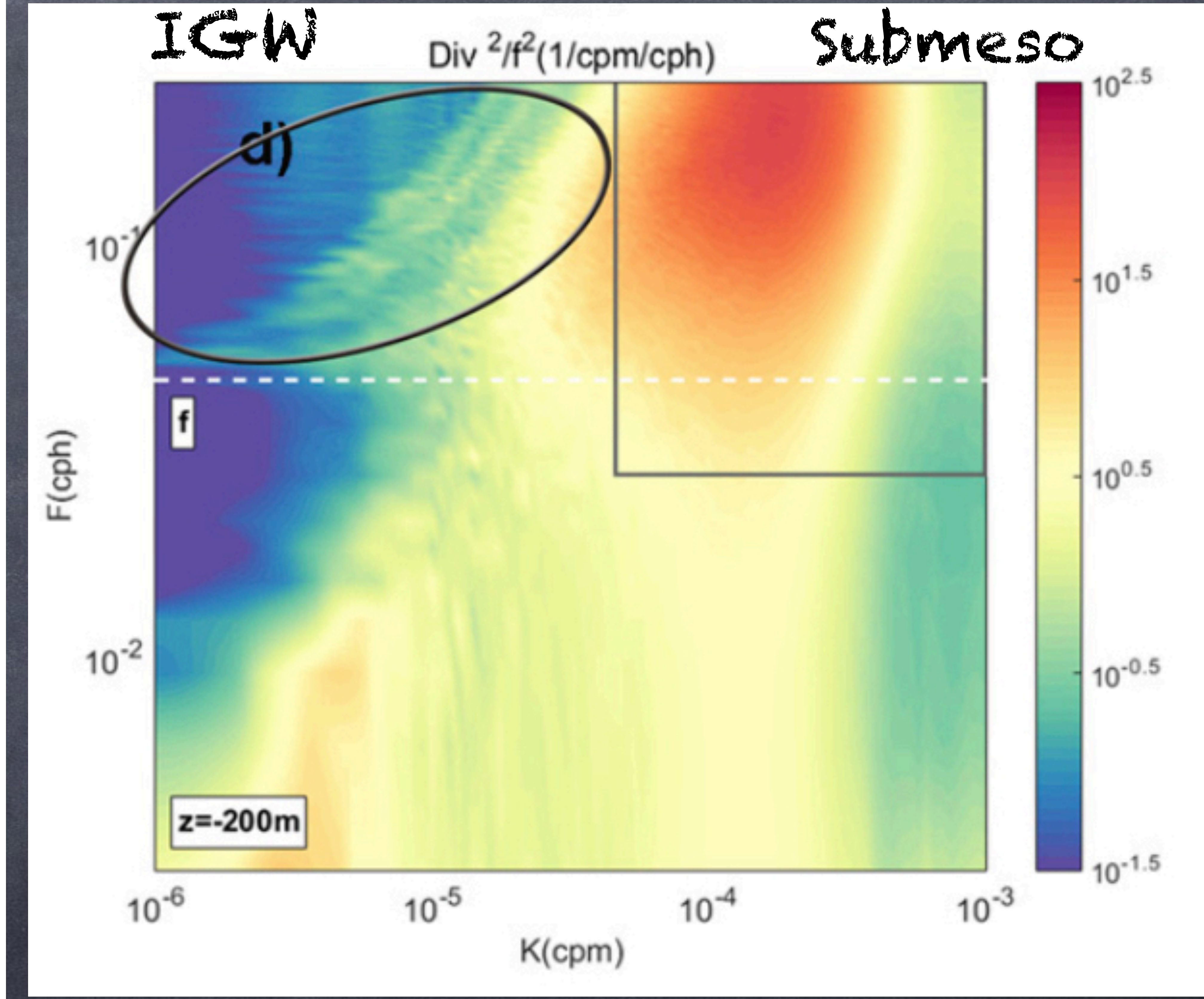




H. Cao, BFK, and Z. Jing. Submesoscale eddies in the upper ocean of the Kuroshio Extension from high-resolution simulation: Energy budget. Journal of Physical Oceanography, 51(7):2181-2201, July 2021.

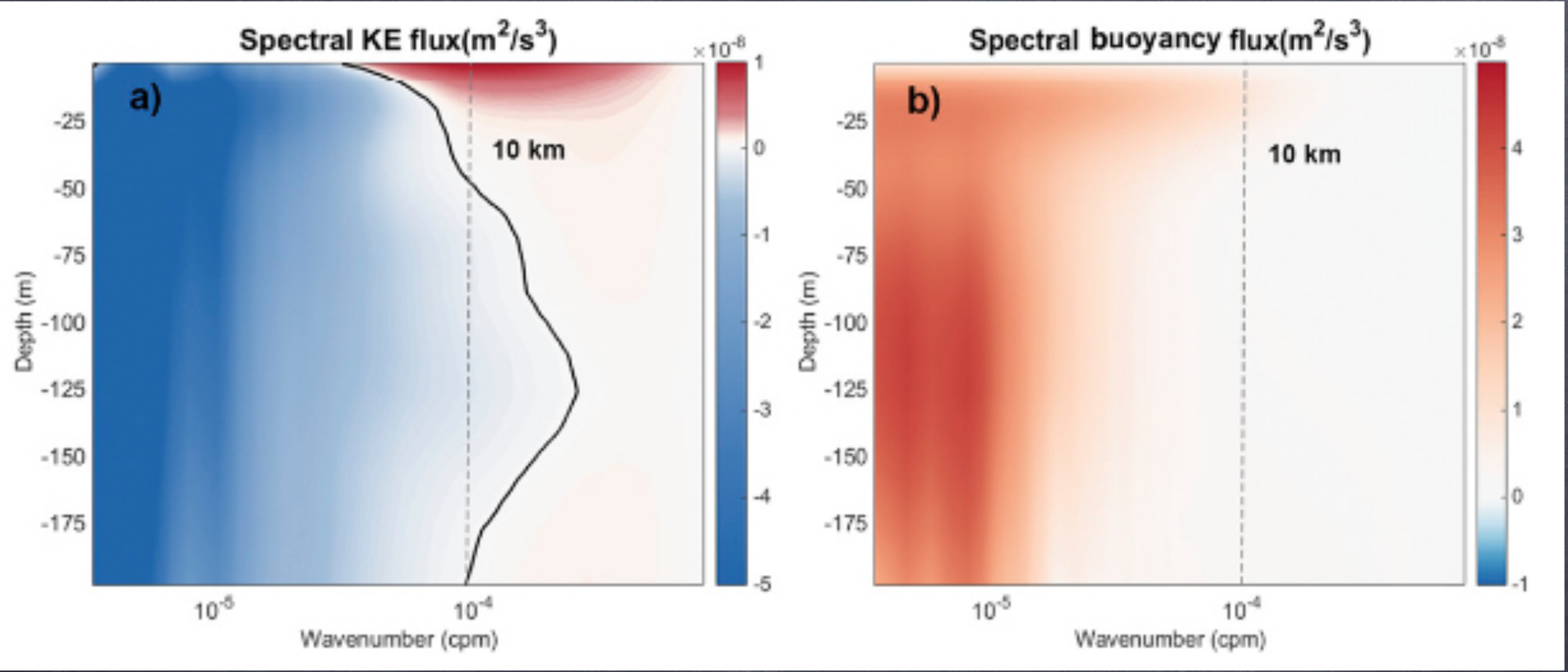
Pacific Modeling for a Frequency-Wavenumber perspective...





H. Cao, BFK, and Z. Jing. Submesoscale eddies in the upper ocean of the Kuroshio Extension from high-resolution simulation: Energy budget. Journal of Physical Oceanography, 51(7):2181-2201, July 2021.

MLI & SI: Upper Ocean MLD, Energetics, Entrainment, Dispersion, and Frontal Strength



Surface submesoscales (MLI) play a major role in cross-scale energy fluxes

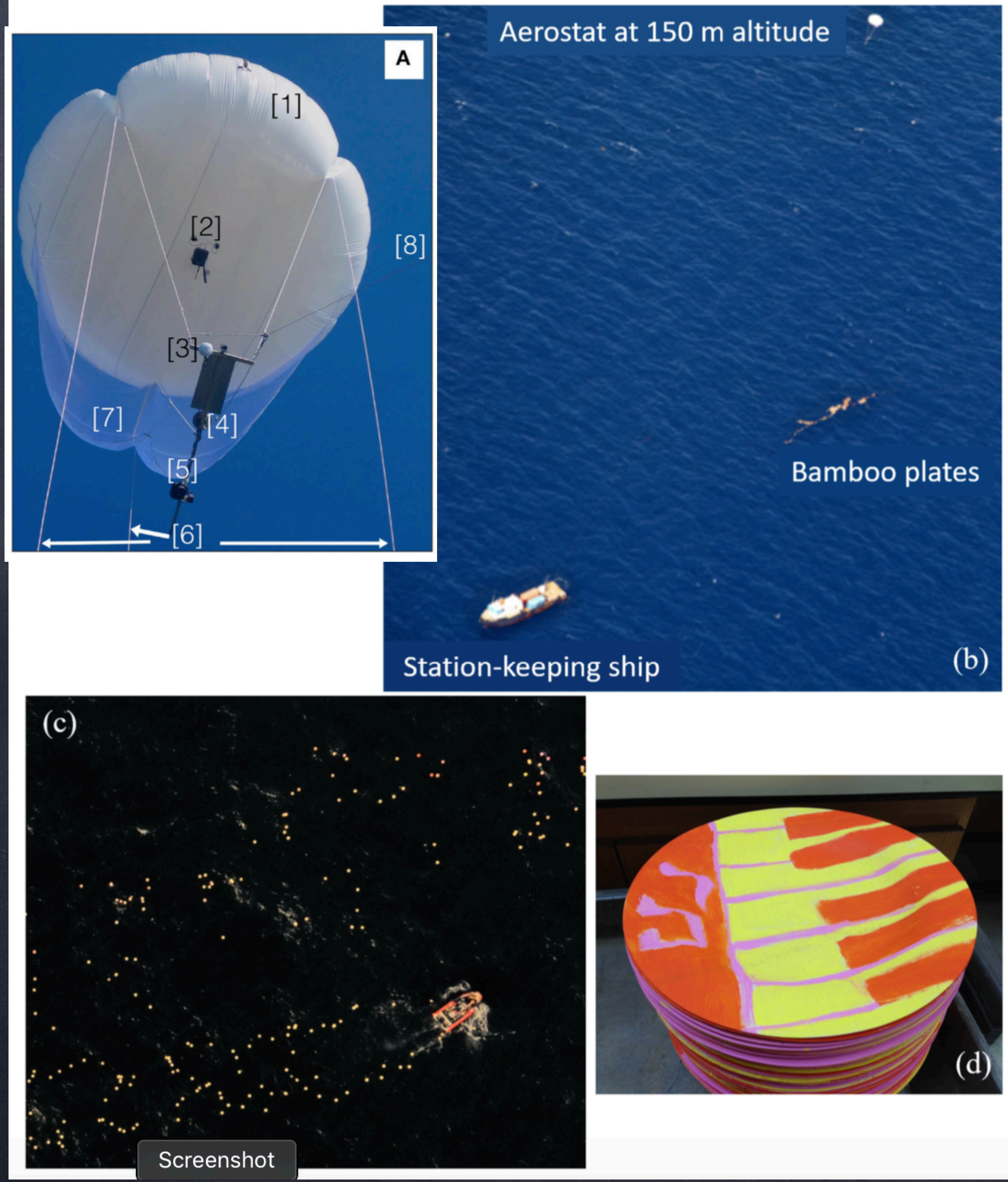
G. Hall and B. Fox-Kemper, 2021: Regional mixed layer depth as a climate diagnostic and emergent constraint. In prep.

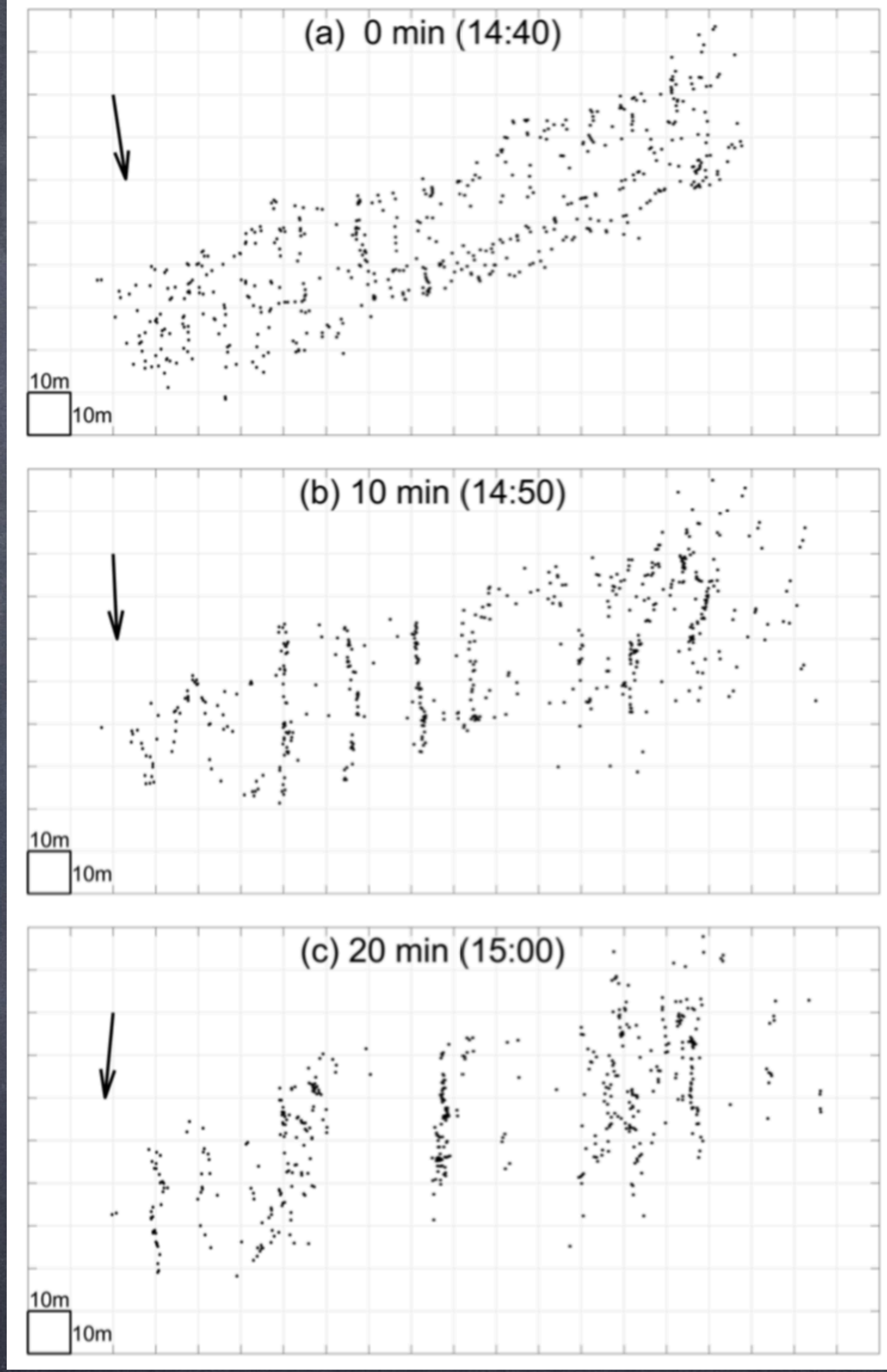
J. Pearson, BFK, R. Barkan, J. Choi, A. Bracco, and J. C. McWilliams, 2019: Impacts of convergence on Lagrangian statistics in the Gulf of Mexico. Journal of Physical Oceanography, 49(3):675–690. URL http://dx.doi.org/10.1175/JPO-D-18-0029.1.

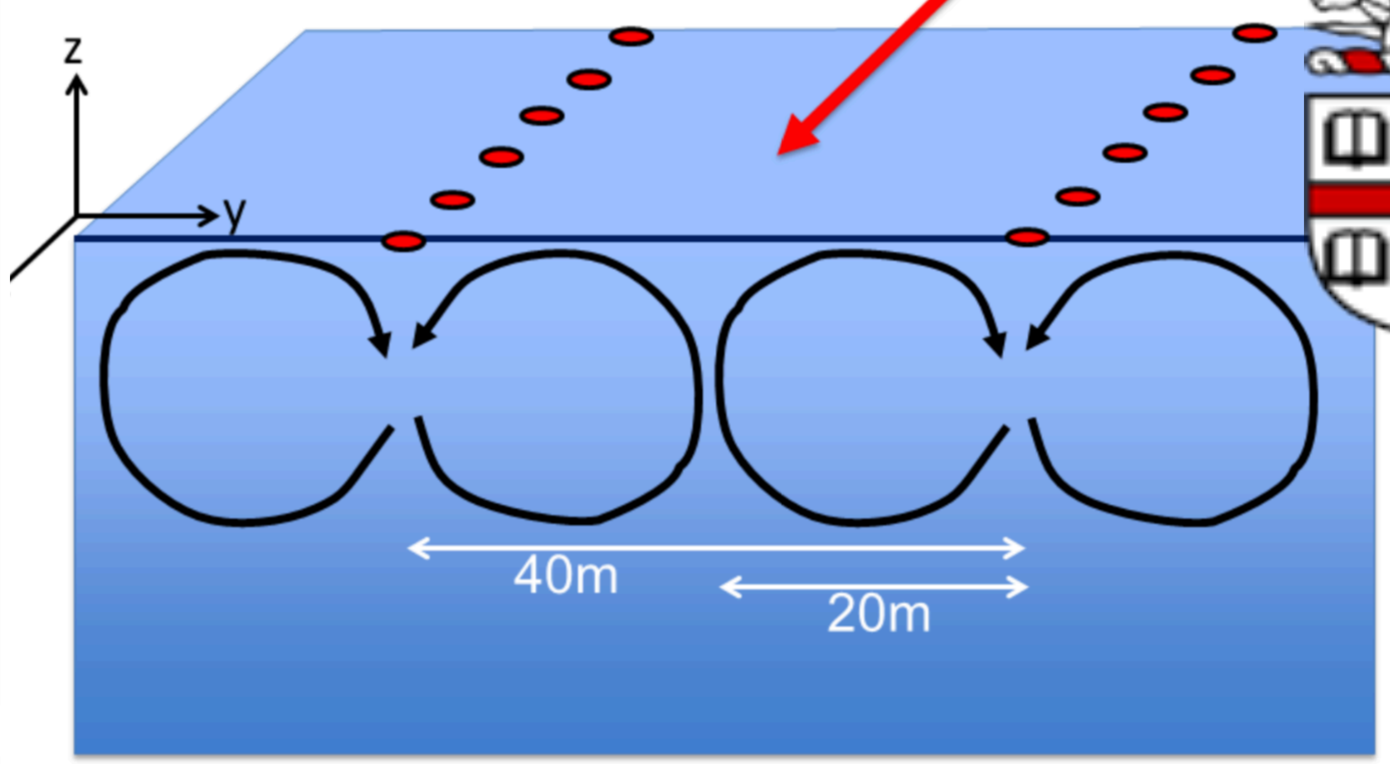
K. M. Smith, P. E. Hamlington, and BFK, 2016: Effects of submesoscale turbulence on ocean tracers. Journal of Geophysical Research-Oceans, 121(1):908–933. URL http://dx.doi.org/ 10.1002/2015JC011089.

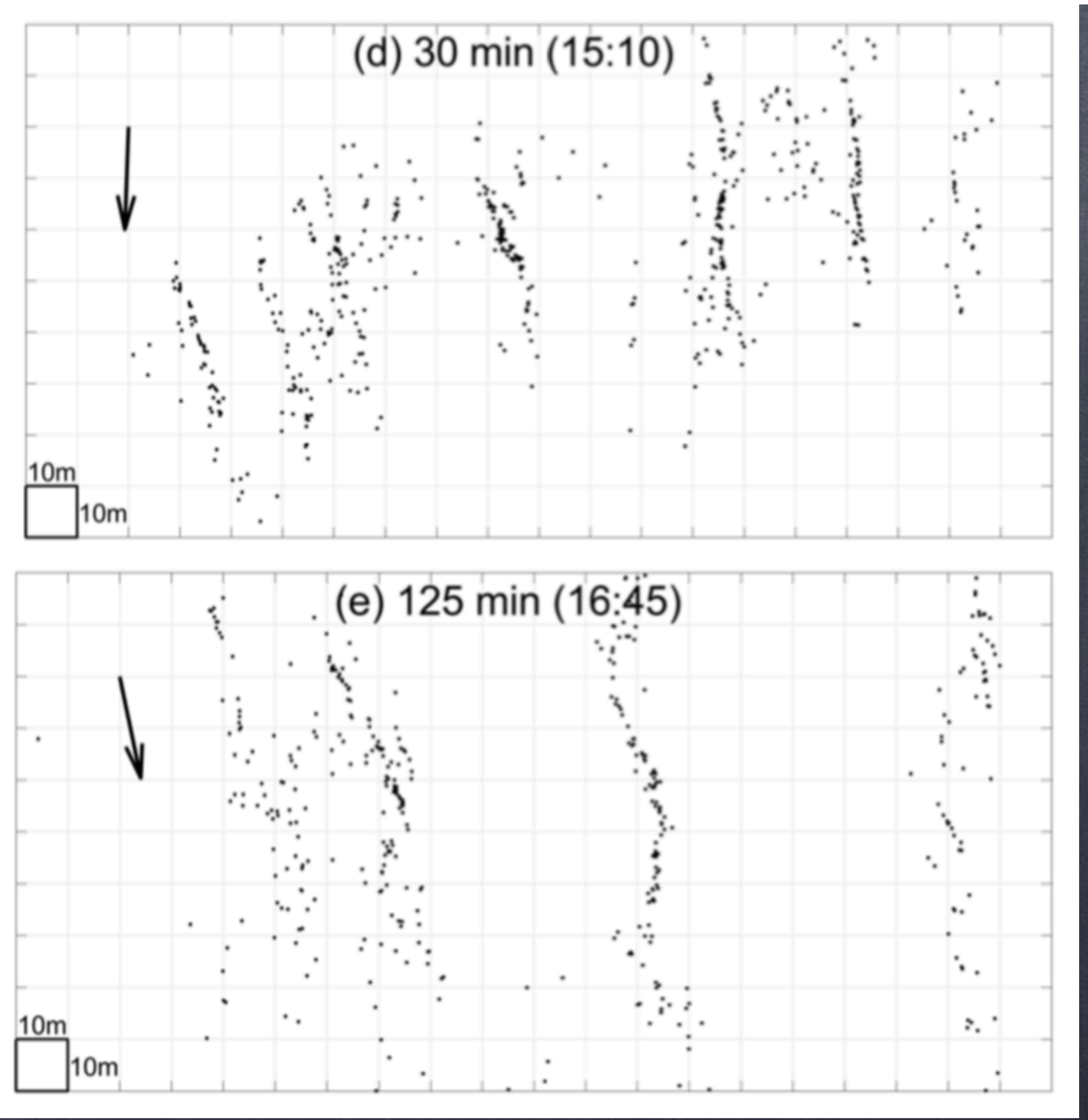
H. Cao, BFK, and Z. Jing, 2021: Submesoscale eddies in the upper ocean of the Kuroshio Extension from high-resolution simulation: Energy budget. Journal of Physical Oceanography, 51(7):2181–2201. https://doi.org/10.1175/JPO-D-20-0267.1.

J. Dong, B. Fox-Kemper, J. Zhu, and C. Dong, 2021: Application of symmetric instability parameterization in the Coastal and Regional Ocean Community Model (CROCO). Journal of Advances in Modeling Earth Systems (JAMES), 13(3):e2020MS002302. URL https://doi.org/10. 1029/2020MS002302.



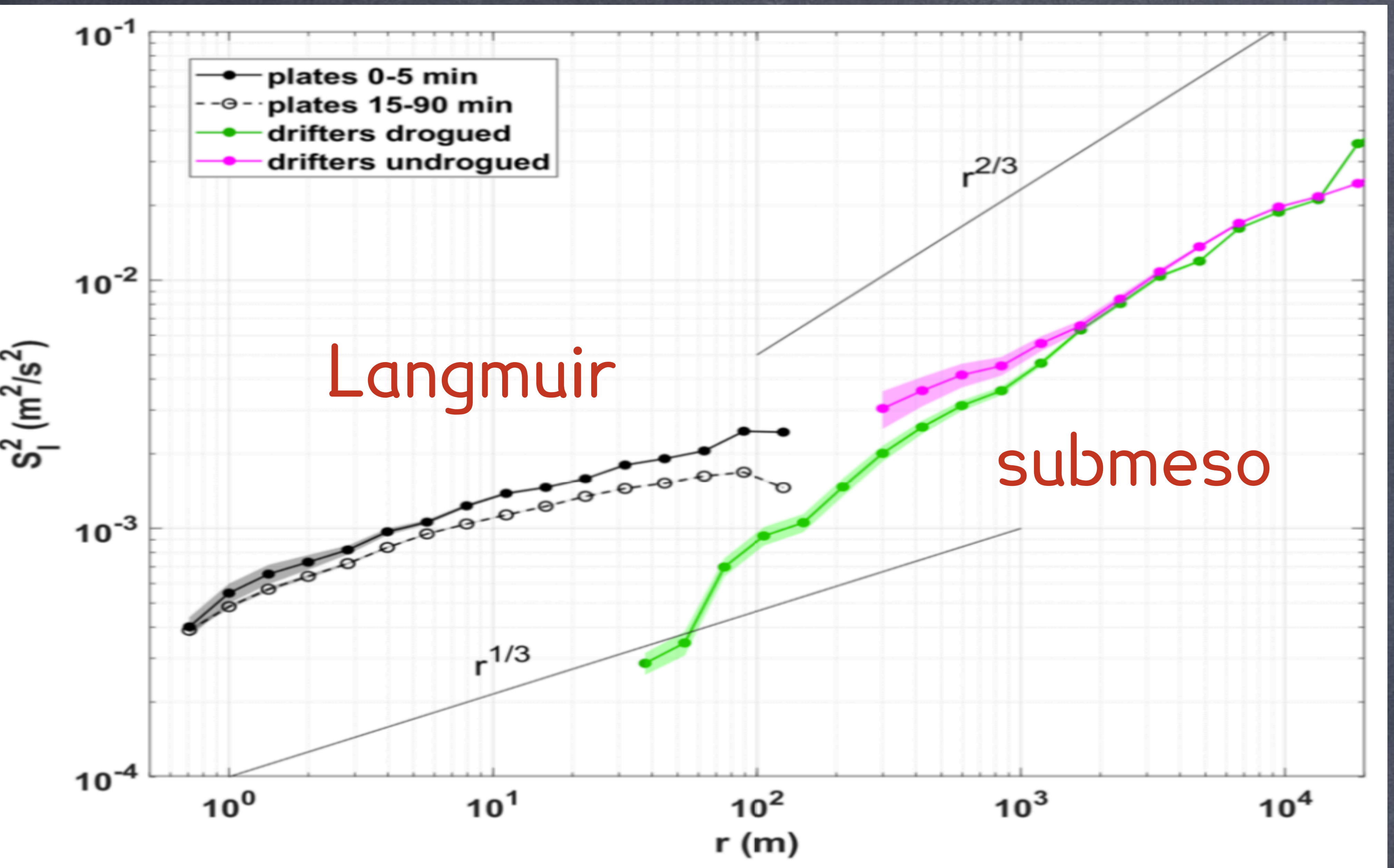




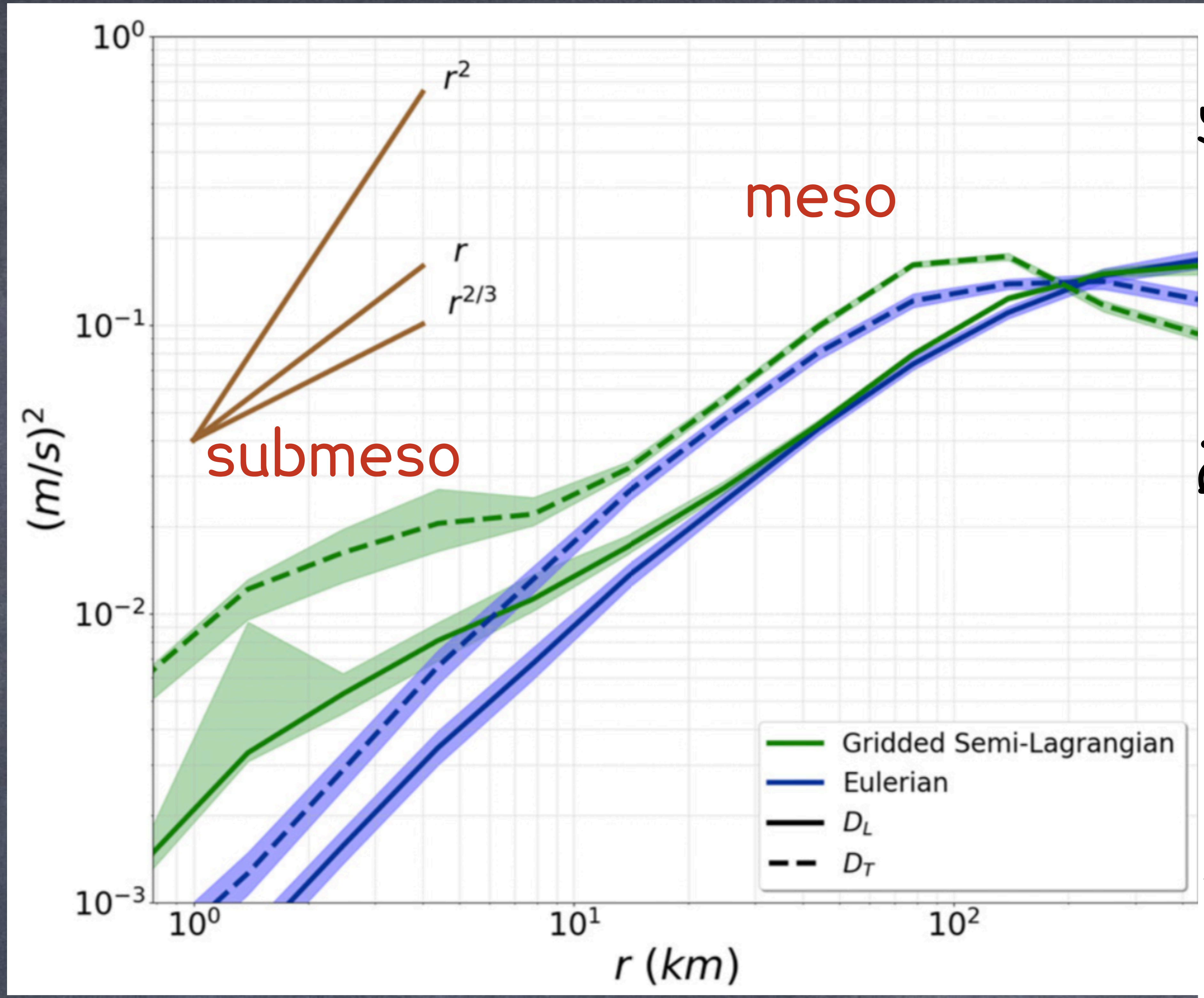


Chang, H., Huntley, H.S., Kirwan Jr, A.D., Carlson, D.F., Mensa, J.A., Mehta, S., Novelli, G., Özgökmen, T.M., Fox-Kemper, B., Pearson, B. and Pearson, J., 2019. Small-scale dispersion in the presence of Langmuir circulation. Journal of Physical Oceanography, (2019).

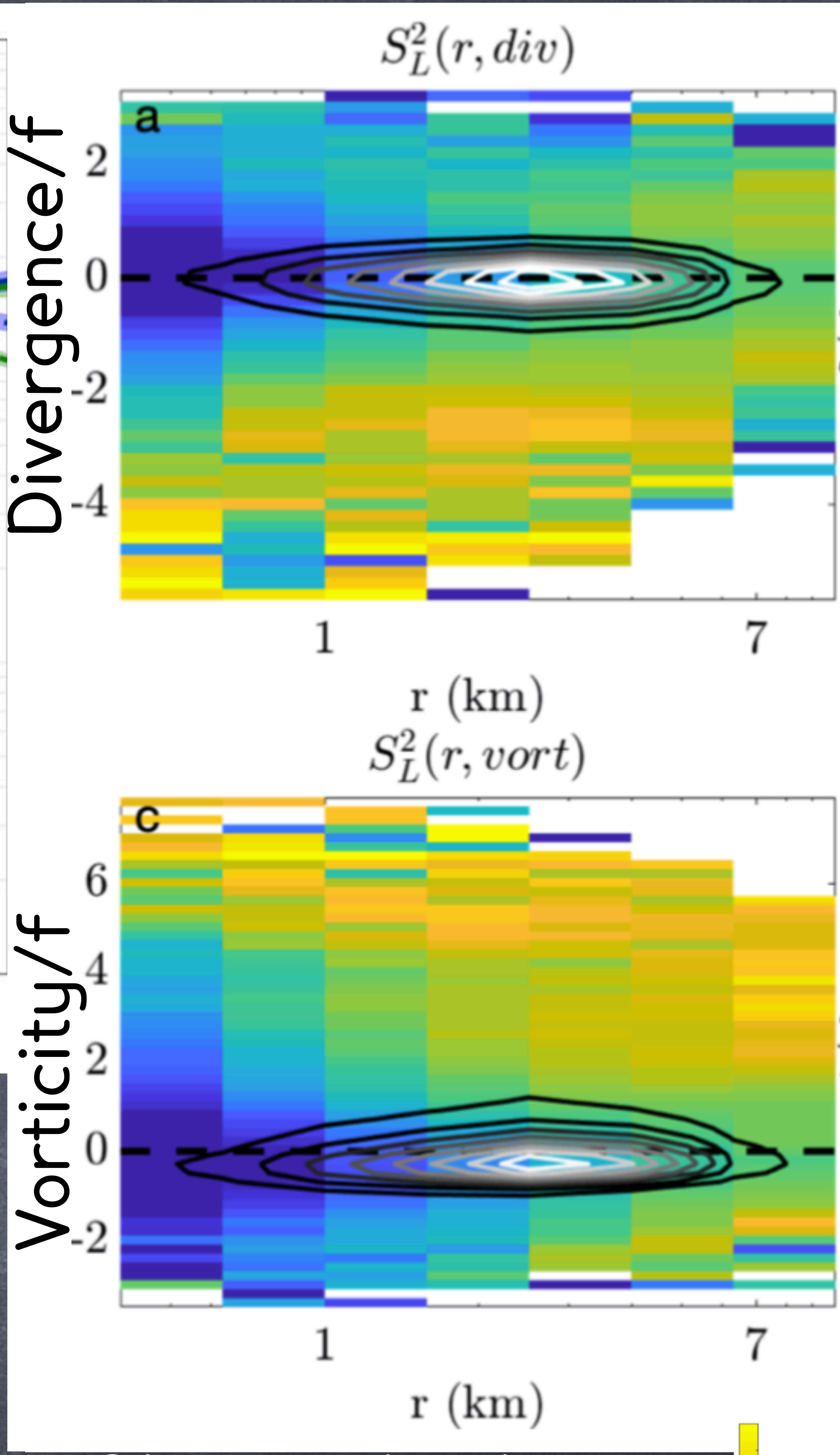
Carlson, D.F., Özgökmen, T., Novelli, G., Guigand, C., Chang, H., Fox-Kemper, B., Mensa, J., Mehta, S., Fredj, E., Huntley, H. and Kirwan Jr, A.D., 2018. Surface Ocean Dispersion Observations From the Ship-Tethered Aerostat Remote Sensing System. Frontiers in Marine Science, 5, p.479.

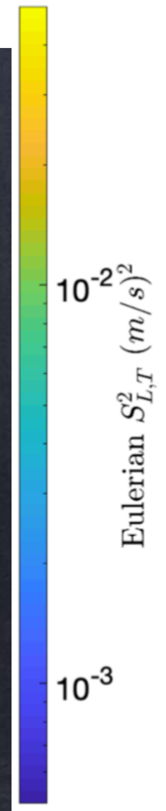


Observed Plates & Drifters



Simulated Drifters and Eulerian Grid





Observed Eulerian Currents

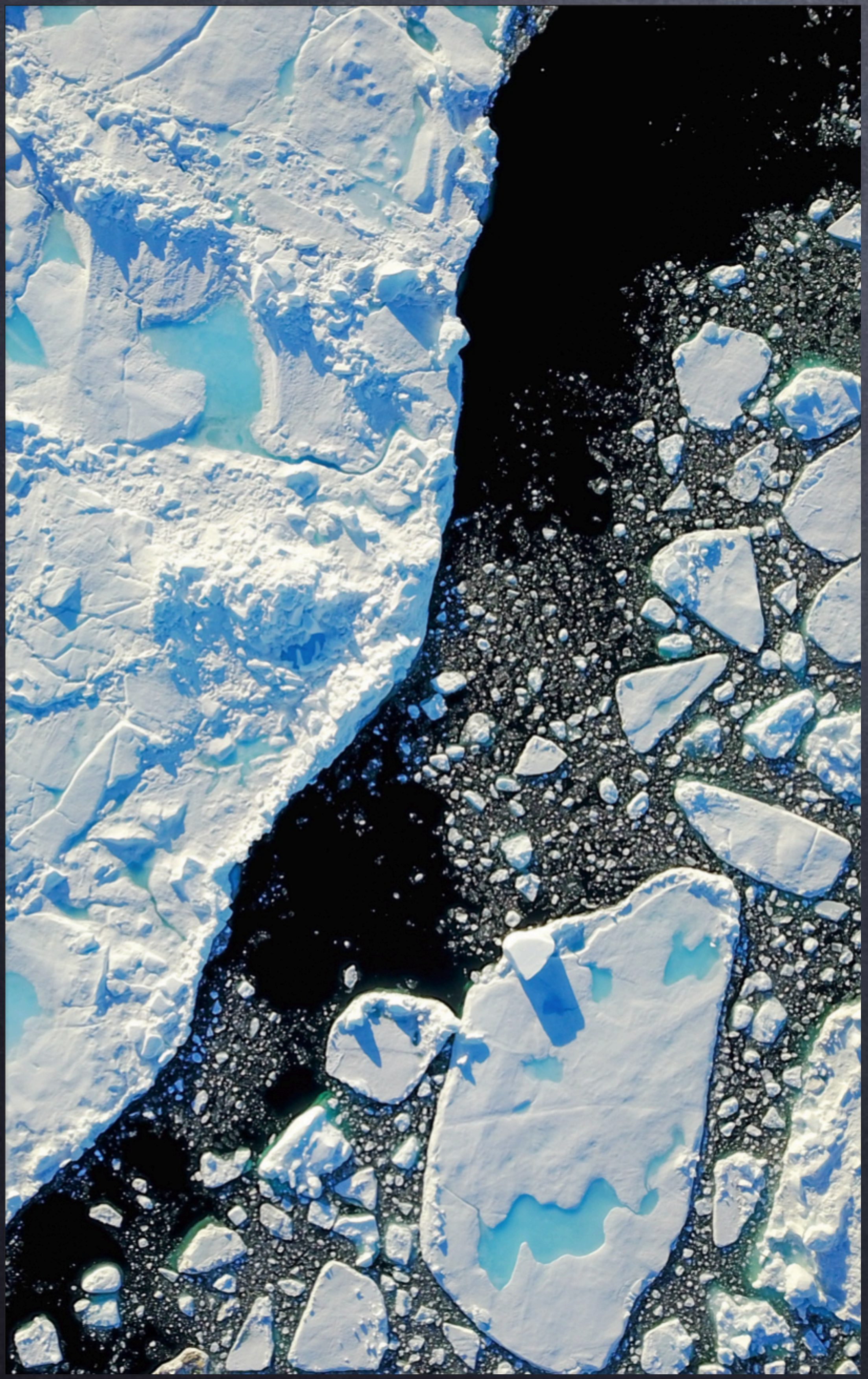
$$S_L^2 = \langle \{[\vec{\mathbf{u}}(\vec{\mathbf{x}} + \vec{\mathbf{r}}) - \vec{\mathbf{u}}(\vec{\mathbf{x}})] \cdot \hat{\mathbf{r}}\}^2 \rangle$$

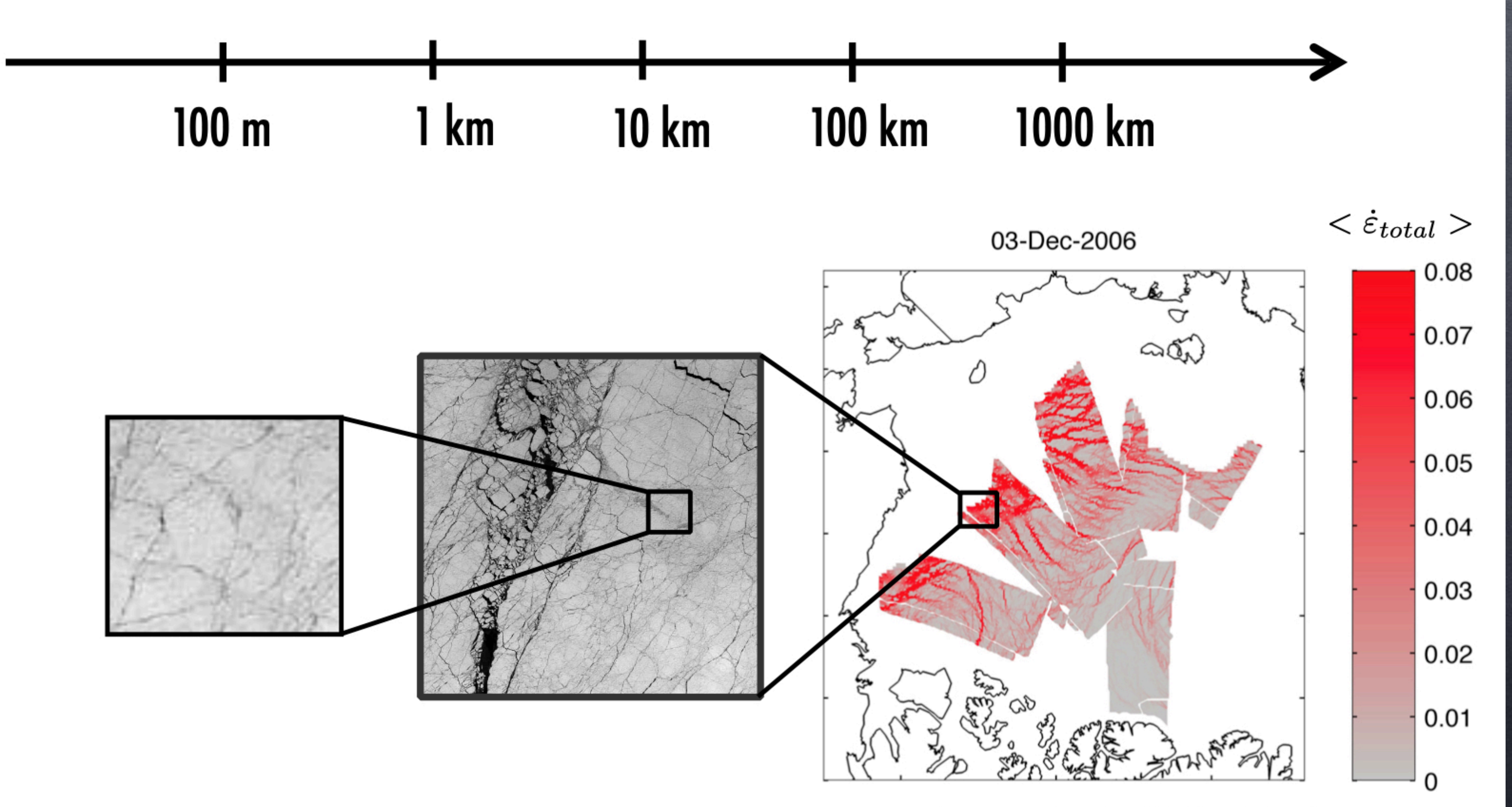
J. Pearson, BFK, R. Barkan, J. Choi, A. Bracco, and J. C. McWilliams. Impacts of convergence on Lagrangian statistics in the Gulf of Mexico. JPO, 49(3):675-690, 2019.

J. Pearson, BFK, B. Pearson, H. Chang, B. K. Haus, J. Horstmann, H. S. Huntley, A. D. Kirwan, Jr., B. Lund, and A. Poje. Biases in structure functions from observations of submesoscale flows. JGR-Oceans, 2019. Submitted.

H. Chang, H. S. Huntley, J. A. D. Kirwan, D. F. Carlson, J. A. Mensa, S. Mehta, G. Novelli, T. Ozgokmen, BFK, B. Pearson, J. Pearson, R. Harcourt, and A. J. Poje. Small-scale dispersion in the presence of Langmuir circulation. JPO, 2019. Accepted.

What happens when surface fluxes filter through sea ice?

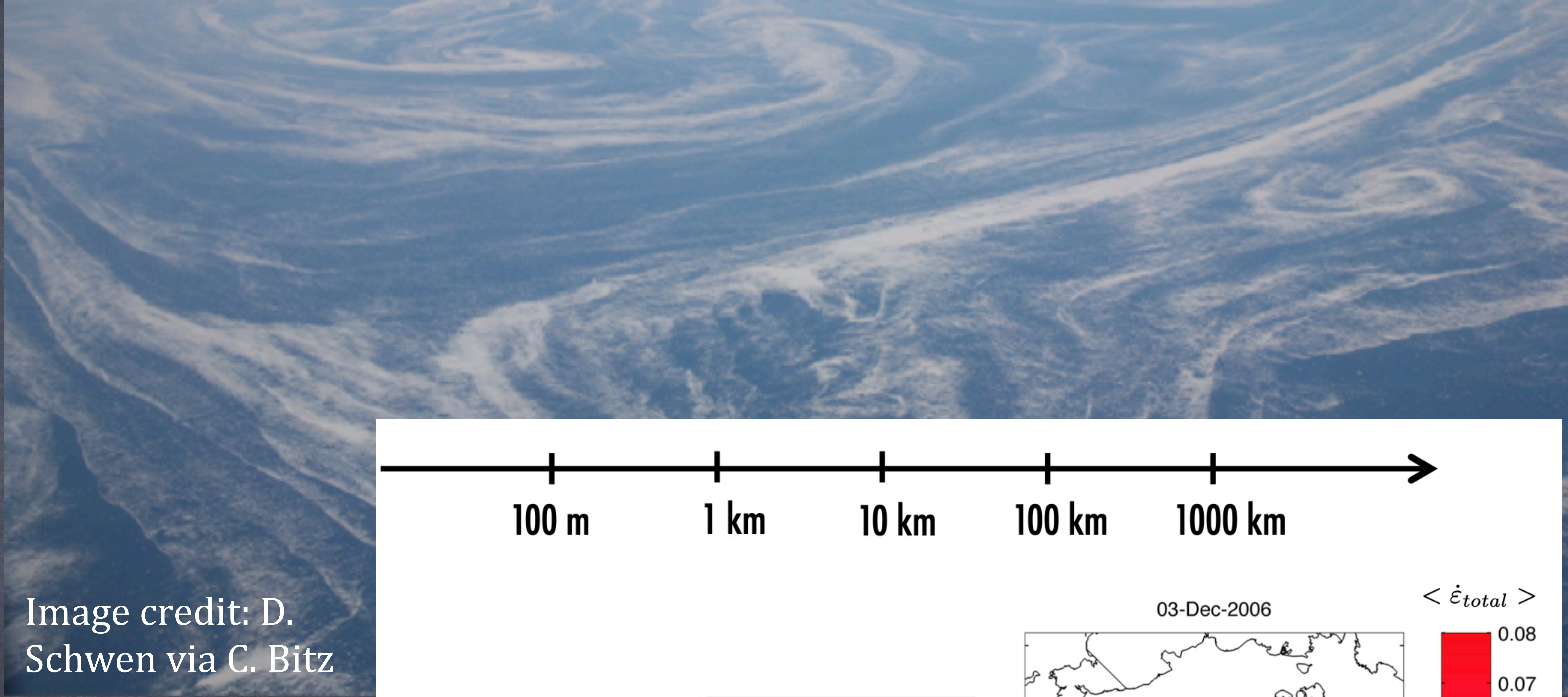




Deformation rate, from RGPS data (day^{-1})

Credit: S. Bouillon

What happens when surface fluxes filter through sea ice?



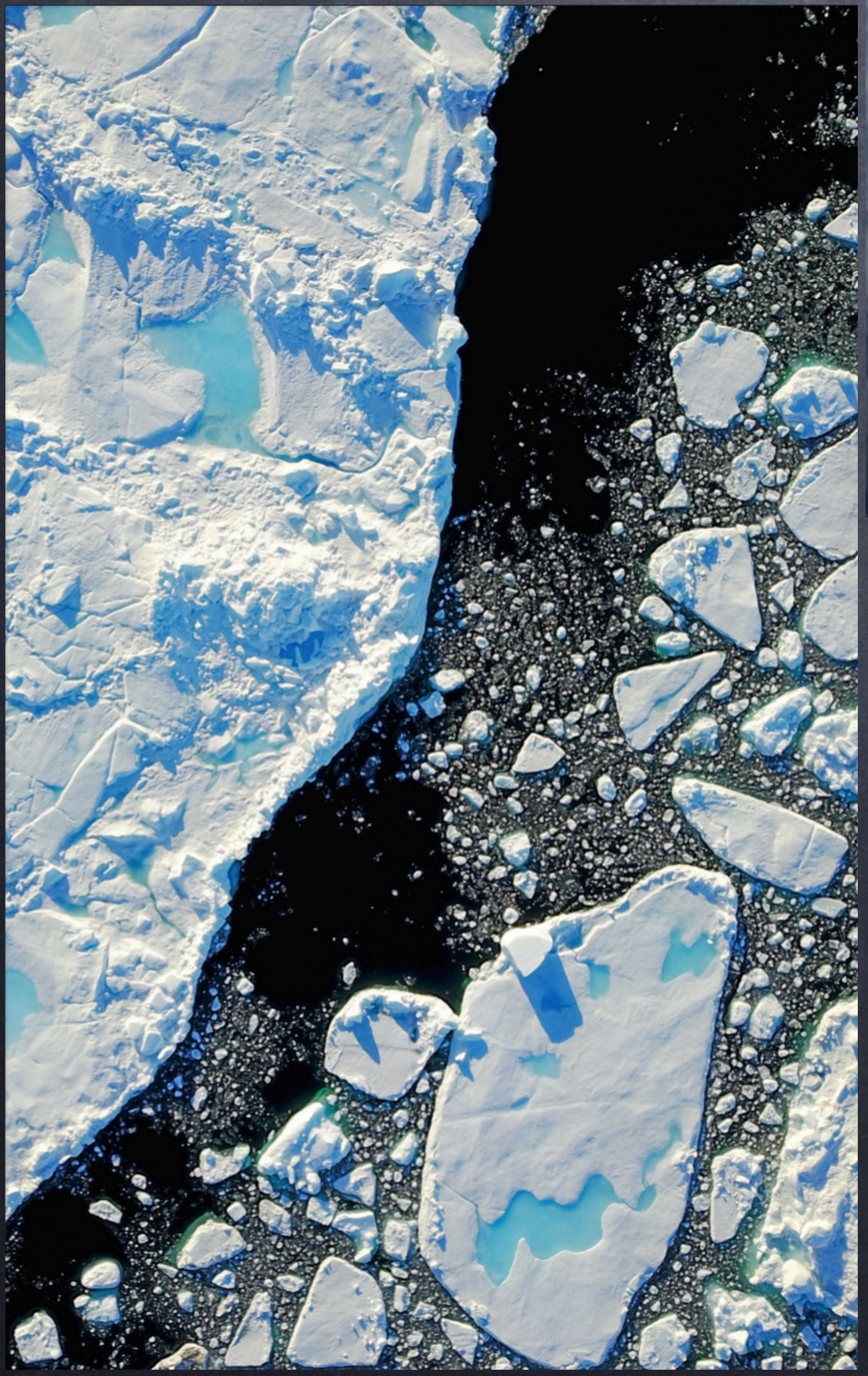
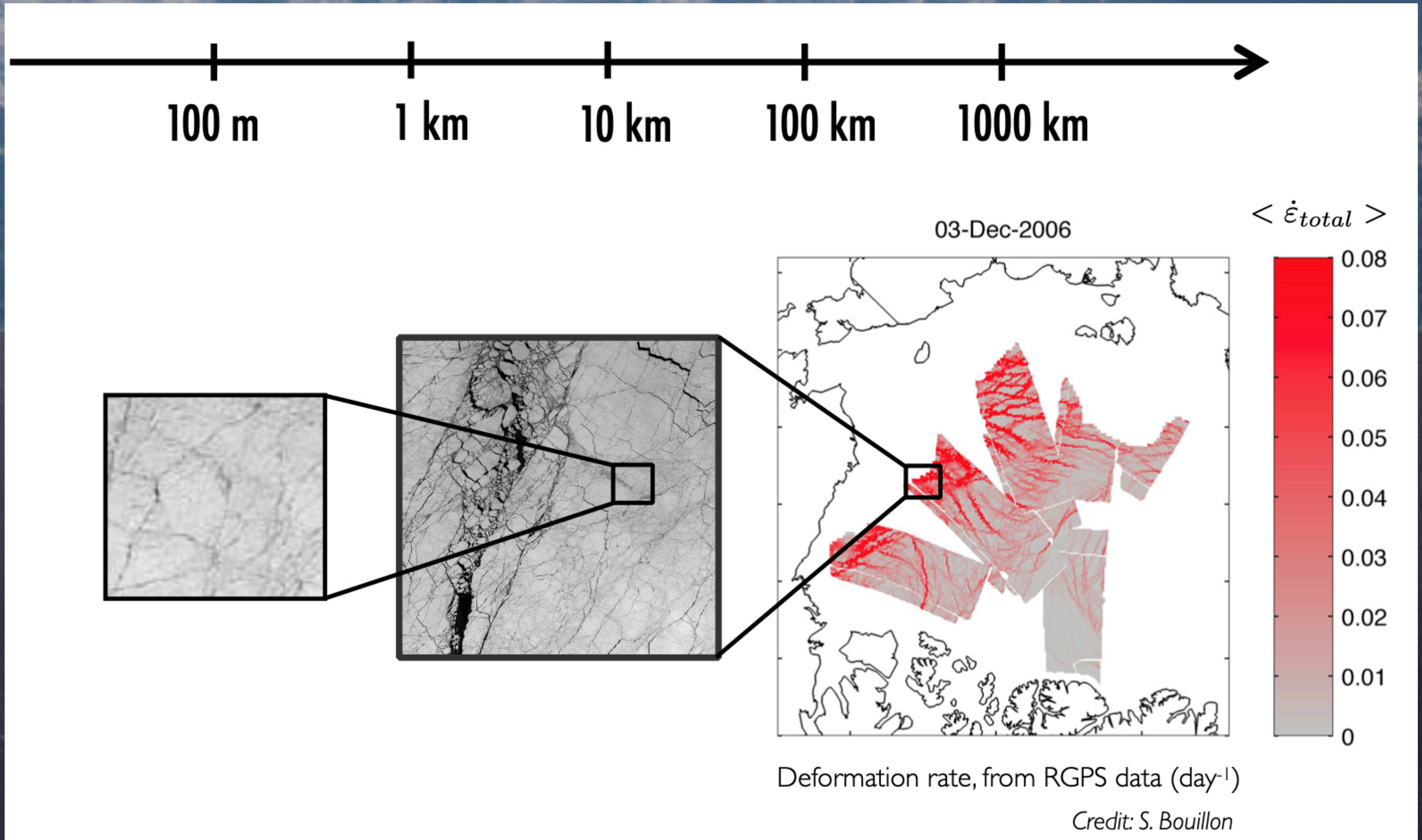


Image credit: D. Schwen via C. Bitz



For Today—Scale Maths

- Diagnosis by Scale
  - Filtering & Reynolds-Averaging
  - Spectral/Fourier
  - Structure Functions
    (Drifters & Moorings!)
- Parameterizations & Augments
  - Energy, PV
- Lagrangian vs. Eulerian Structure Functions
- Patchy Injection through Sea Ice