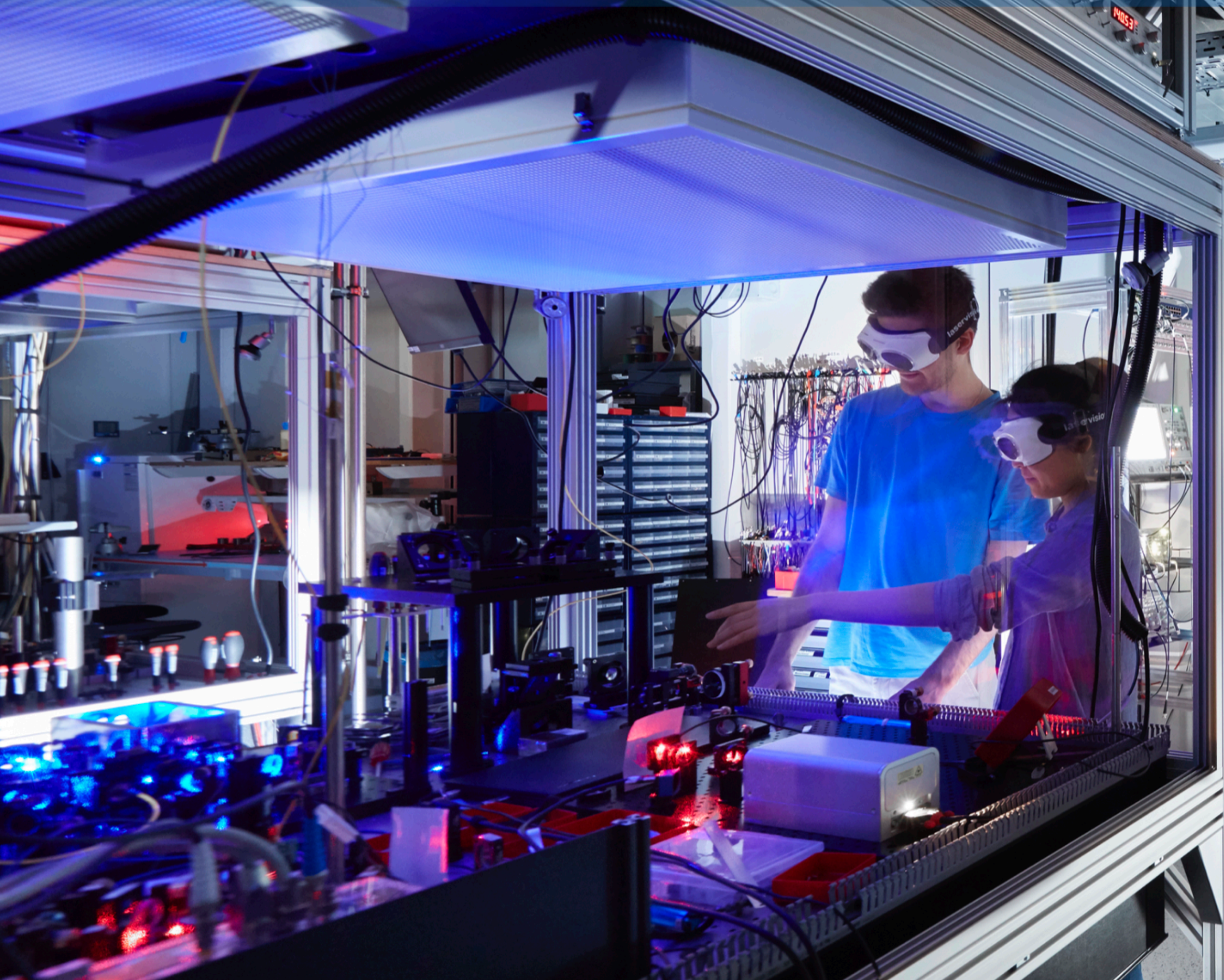


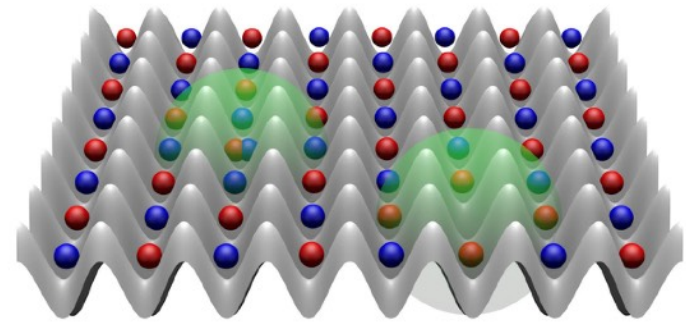
# Quantum Simulation using Ultracold Atoms



Boulder Summer School 2023

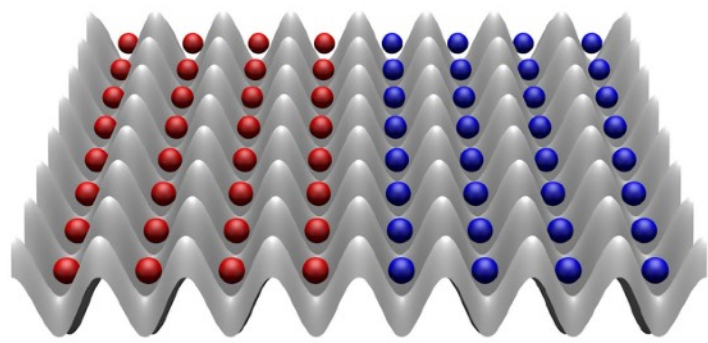






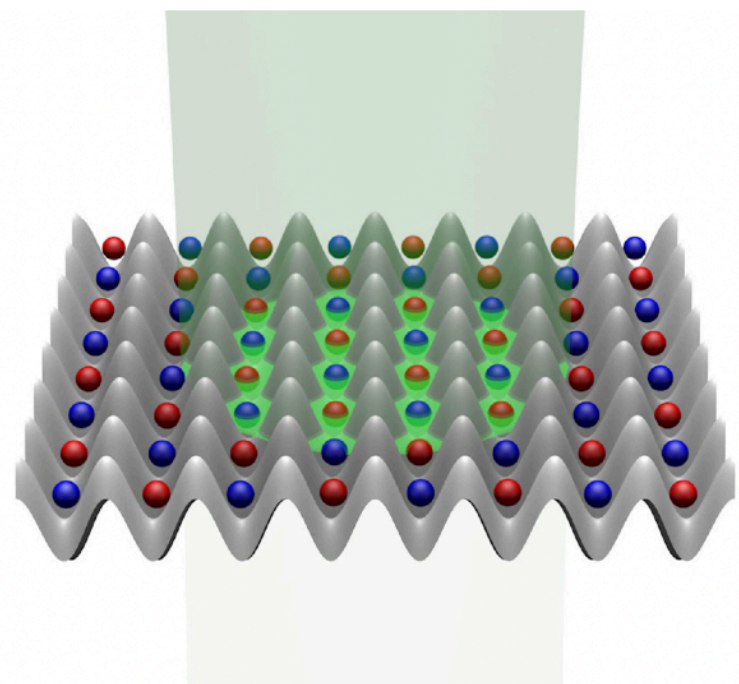
1

**Quantum Simulation & Computing using Ultracold Atoms**  
Potentials, Interactions, Gates, Models....



2

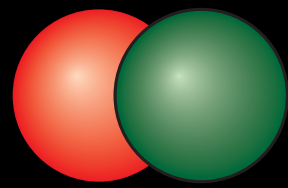
**Floquet Topological Quantum Matter & Quantum Gas Microscopy**  
Topological Matter, Floquet Systems, Anomalous Floquet Topological Systems, Quantum Gas Microscopy, Dynamical Spin-Charge Fractionalisation



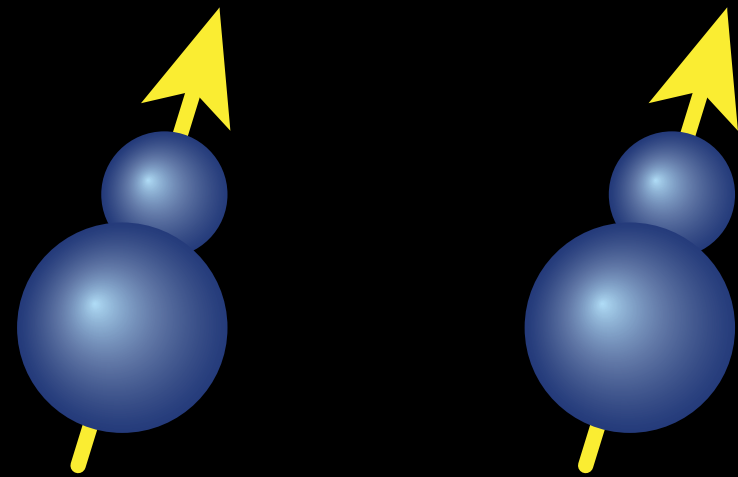
3

**Quantum Matter Out-Of-Equilibrium**  
Many Body Localisation, Thermalization, Measuring Entanglement Entropy, Fluctuation Hydrodynamics, Anomalous Spin Transport (KPZ)

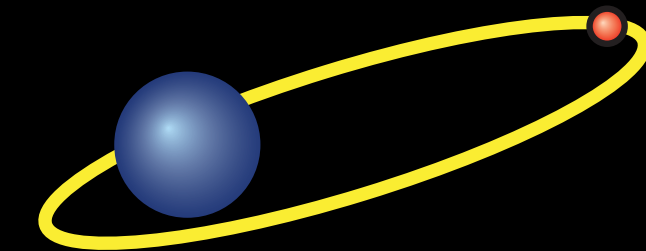
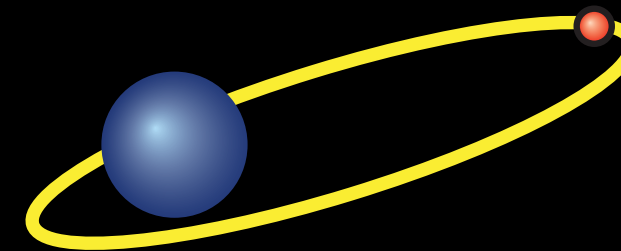
# Interaction Control



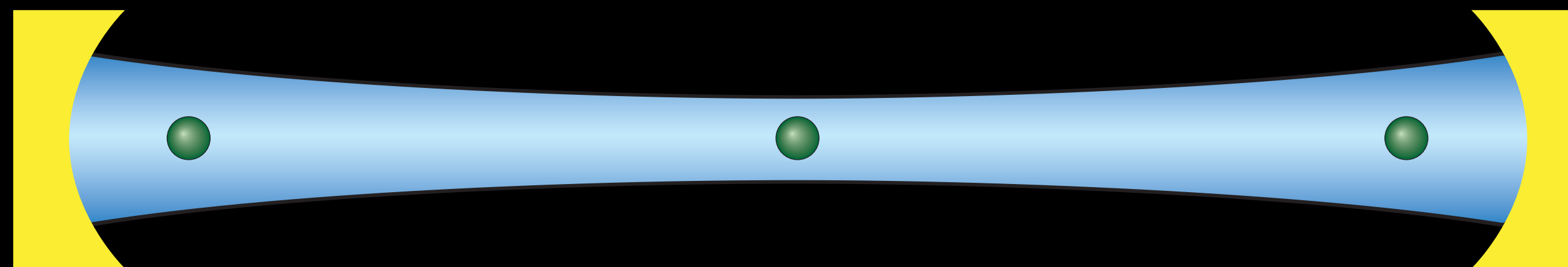
**Collisional**  
(onsite few kHz)



**Dipolar**  
**Magnetic or Electric**  
(several sites Hz to 10 kHz)



**Rydberg Dipole-Dipole**  
(several sites 1-500 MHz)

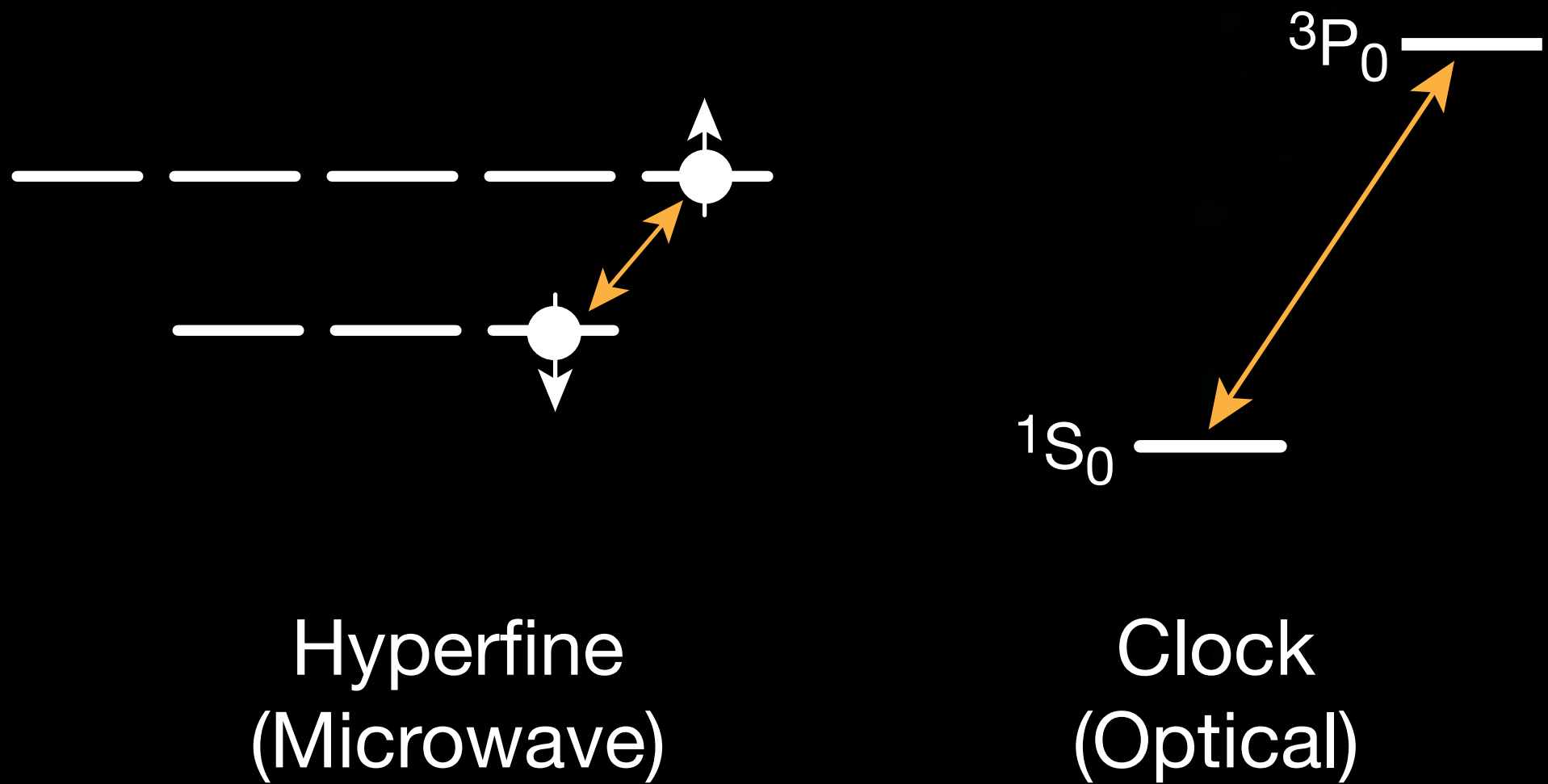


**Cavity Mediated**  
(All-to-All few Hz-kHz)



**Particles:** Fermions, Bosons, Mixtures

**Spin degree of freedom:**



**System size:** up to few thousands of particles

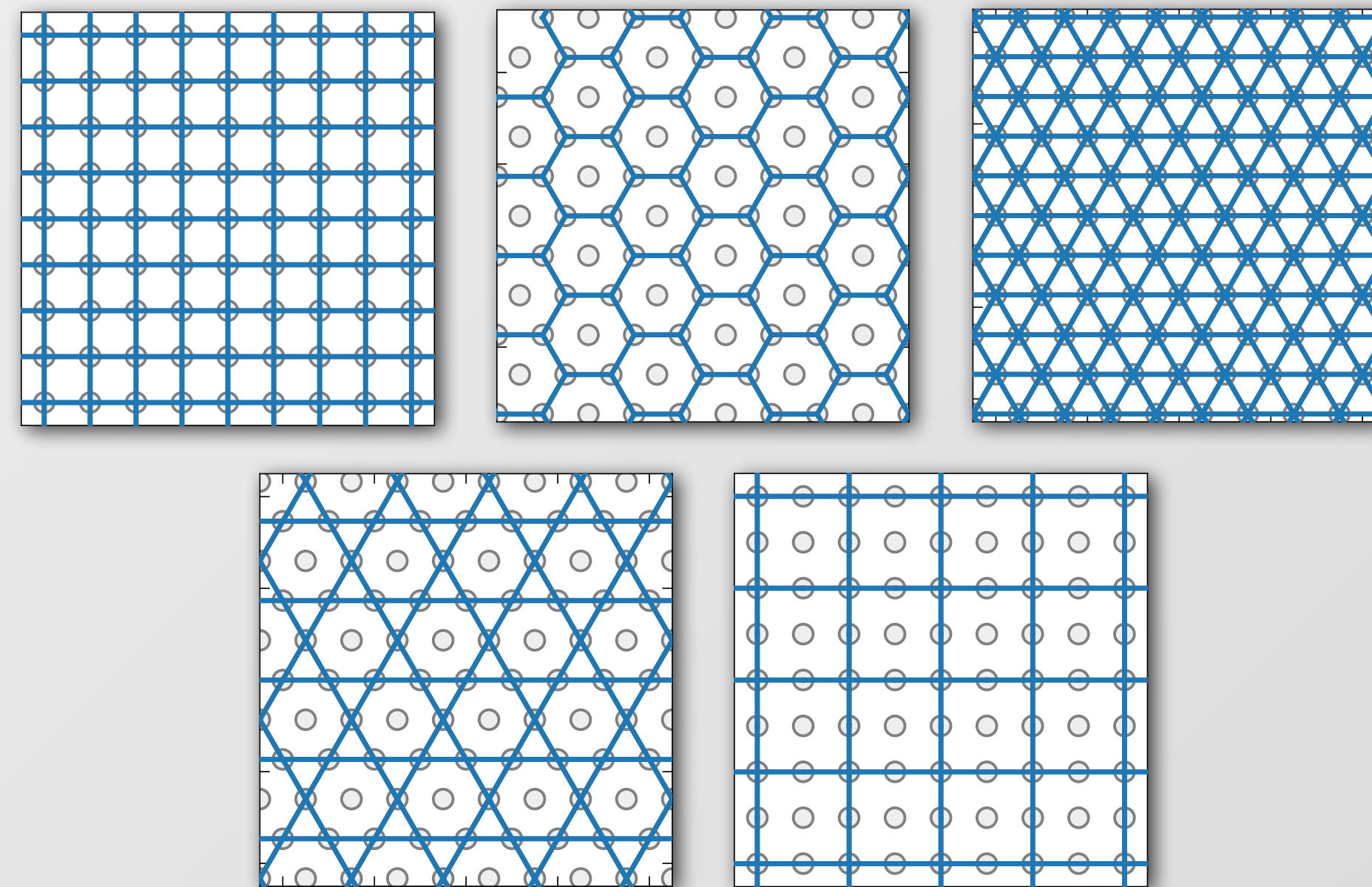
**Mobility:** itinerant or static





### Fourier synthesize arbitrary lattices:

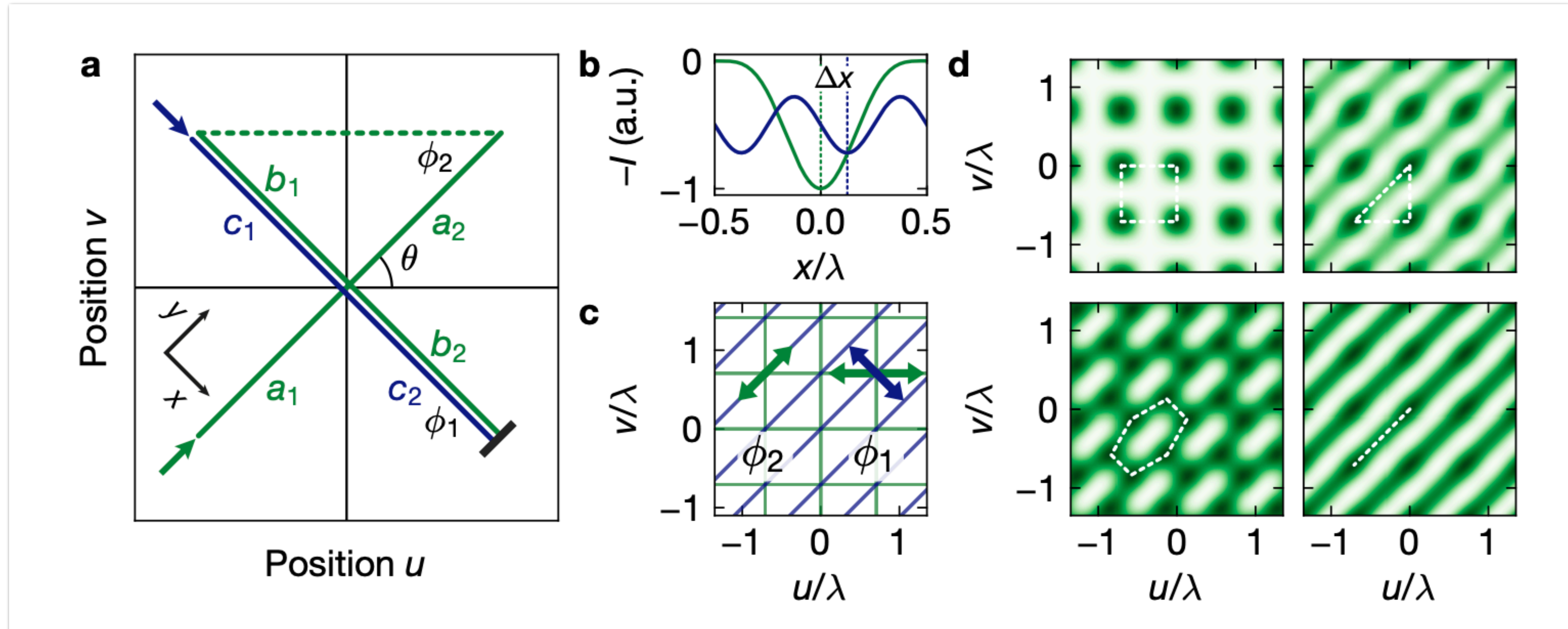
- Square
- Hexagonal/Triangular/Brick Wall
- Kagomé
- Superlattices
- *Spin dependent lattices*
- *Flux Lattices*
- ...



Full **dynamical** control over **lattice depth, geometry, dimensionality!**

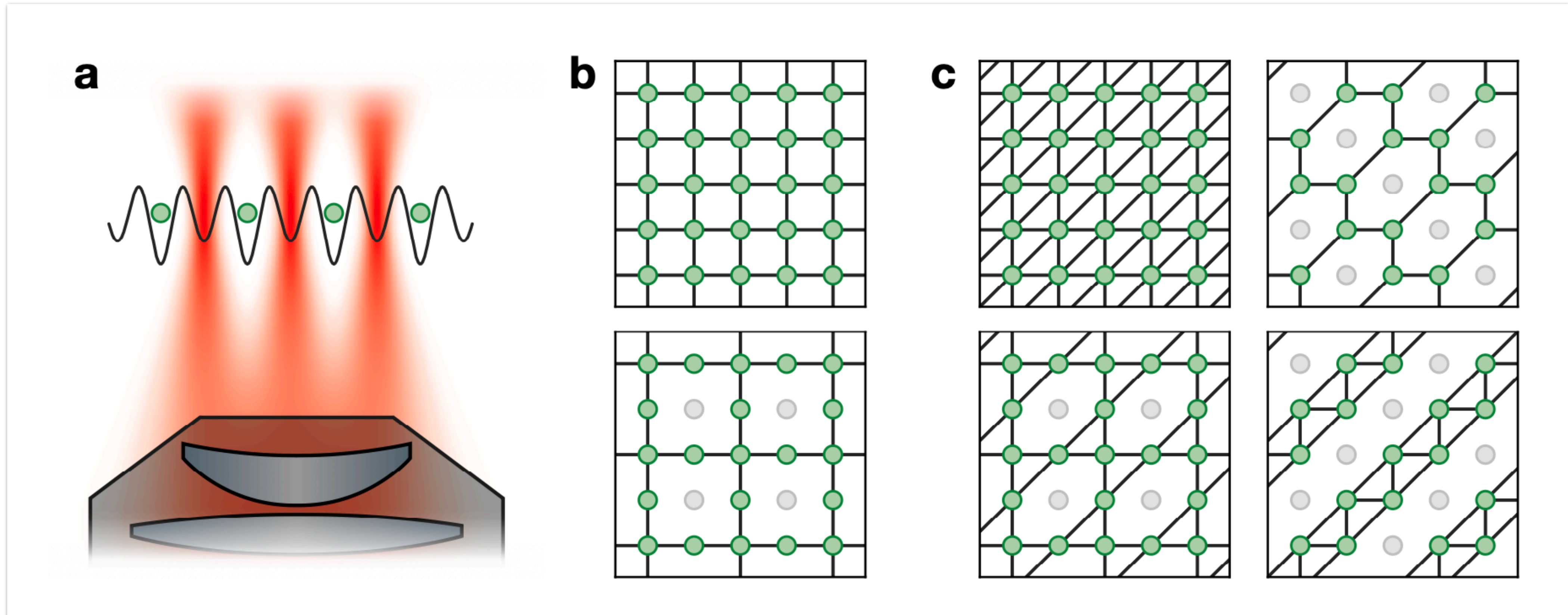






Phase stable setups yield  
**Square, Triangular, Hexagonal, Dimerised Lattices**



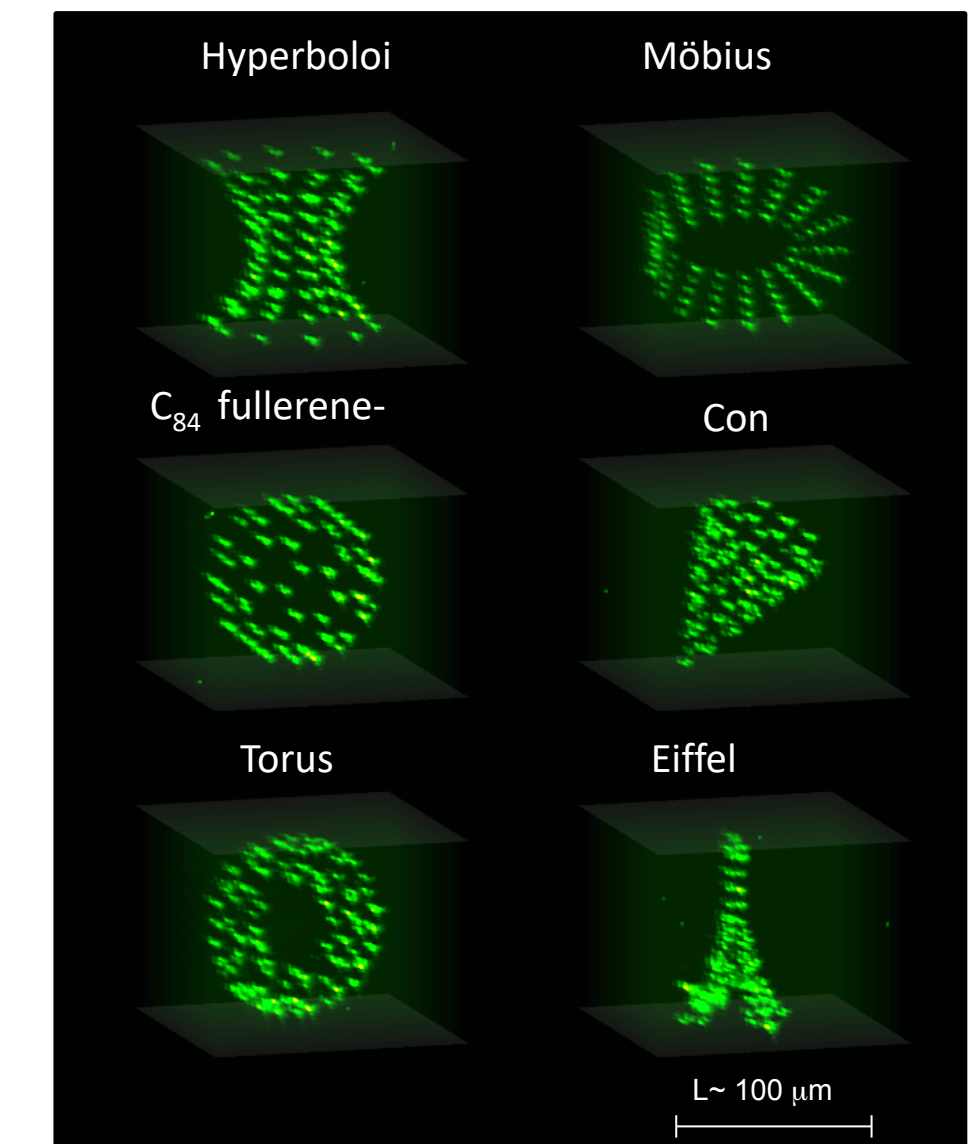
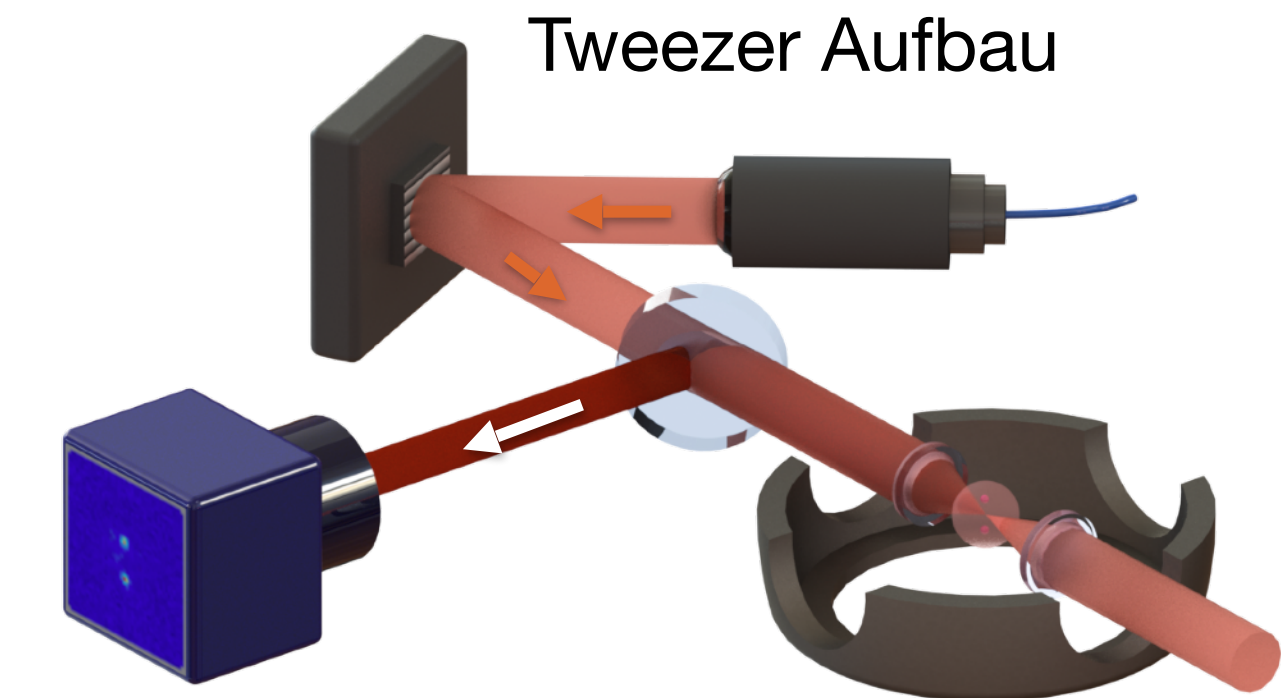
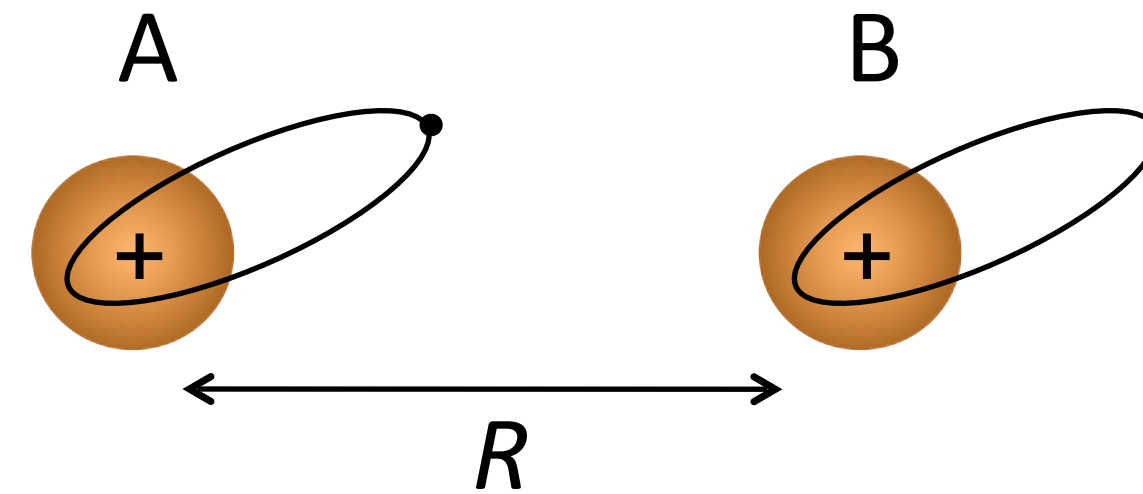


Blocking out sites gives additional flexibility:  
**Lieb lattice, Hexagonal, Kagome, Zig Zag,....**



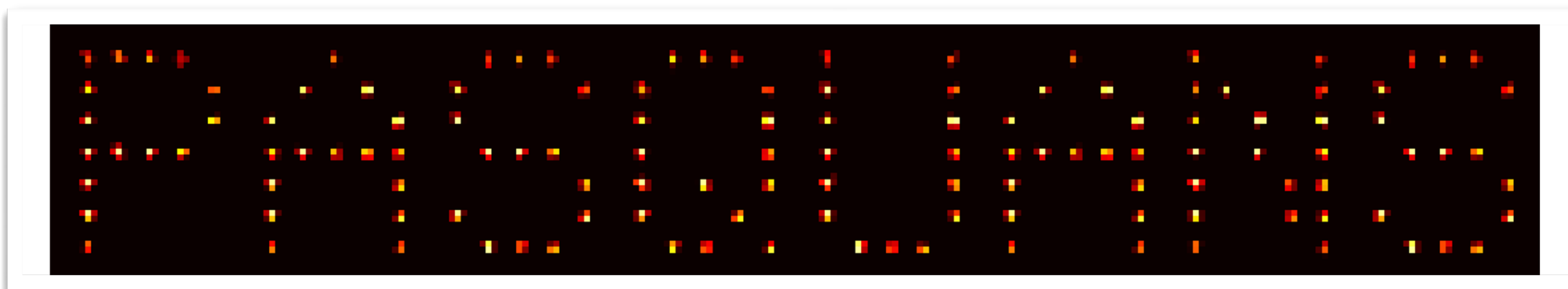
## Neutral Atoms in Optical Tweezers

- Several 100 atoms, **individually addressable** via focused laser beams
- **Entanglement** via Rydberg-Rydberg Interactions (see discussion on interactions)



Barredo *et al.*, Nature (2018)

## Flexible and dynamic trap geometries



See also: Lukin (Harvard), Endres (Caltech), Thompson (Princeton), Kaufman (JILA), Bernien (Chicago), Zeiher (MPQ)....



# Hubbard Regime Tweezer Arrays

Zoe Z. Yan, ..., W. Bakr, Phys. Rev. Lett. **129**, 123201 (2022)

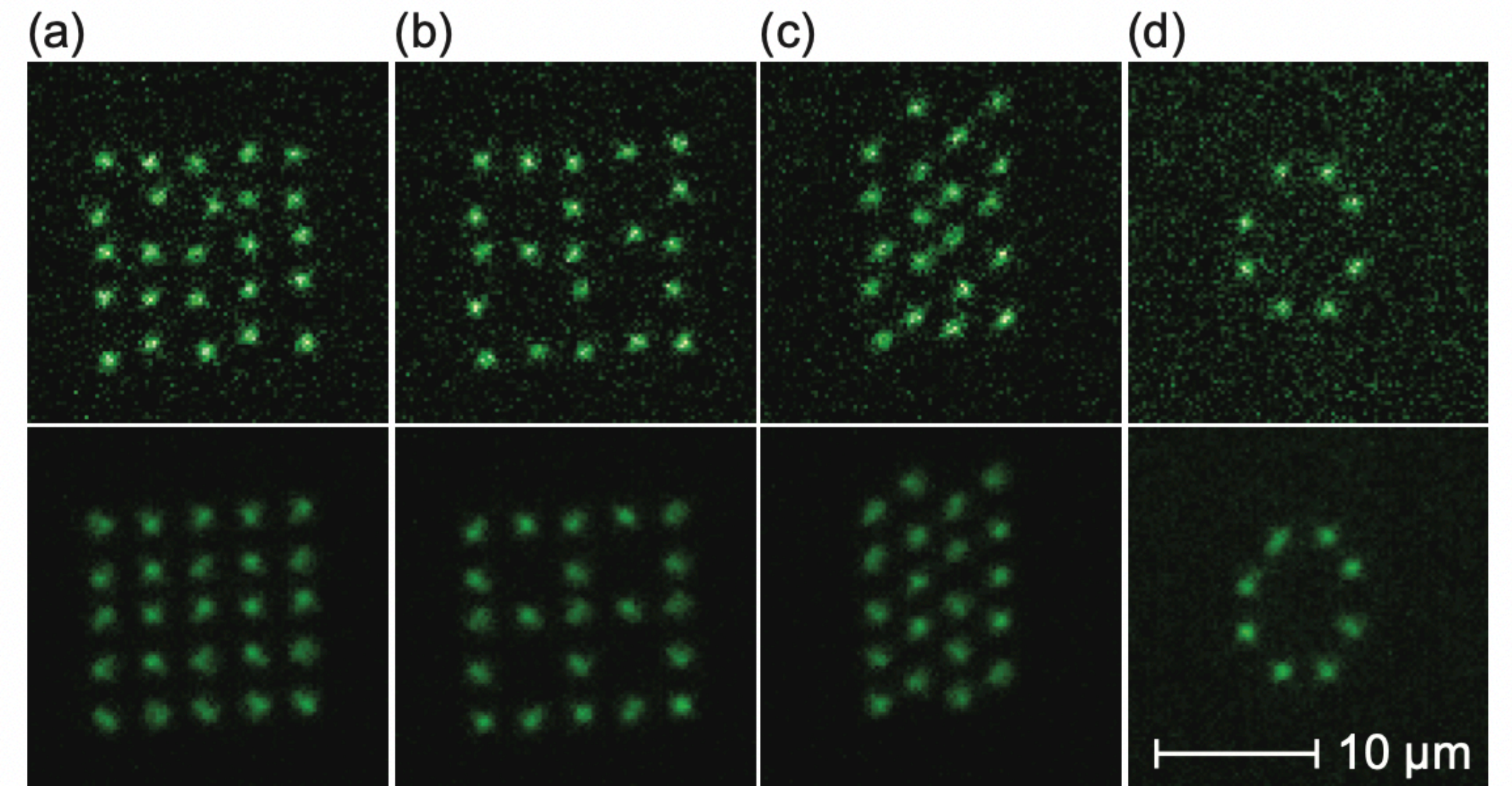
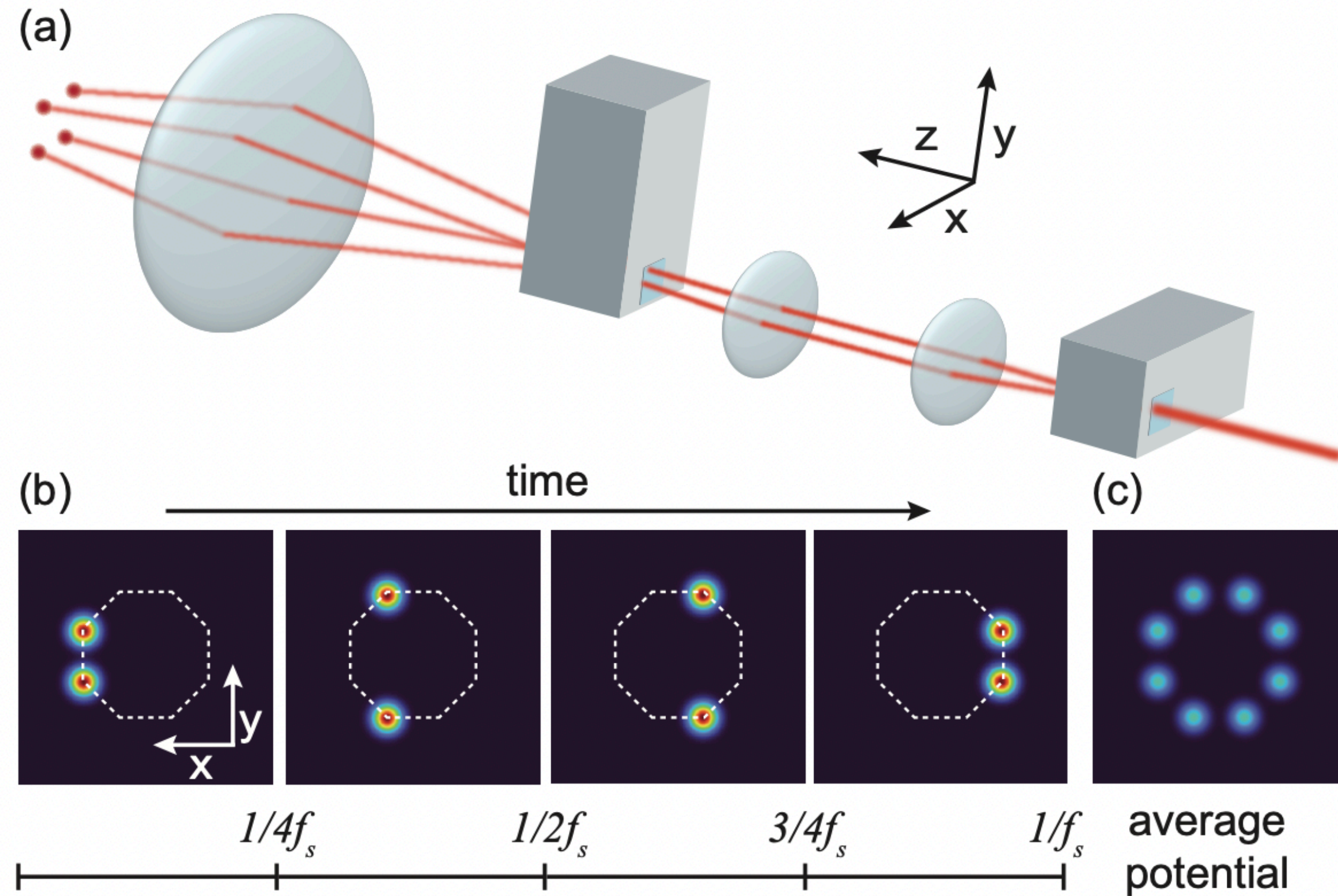


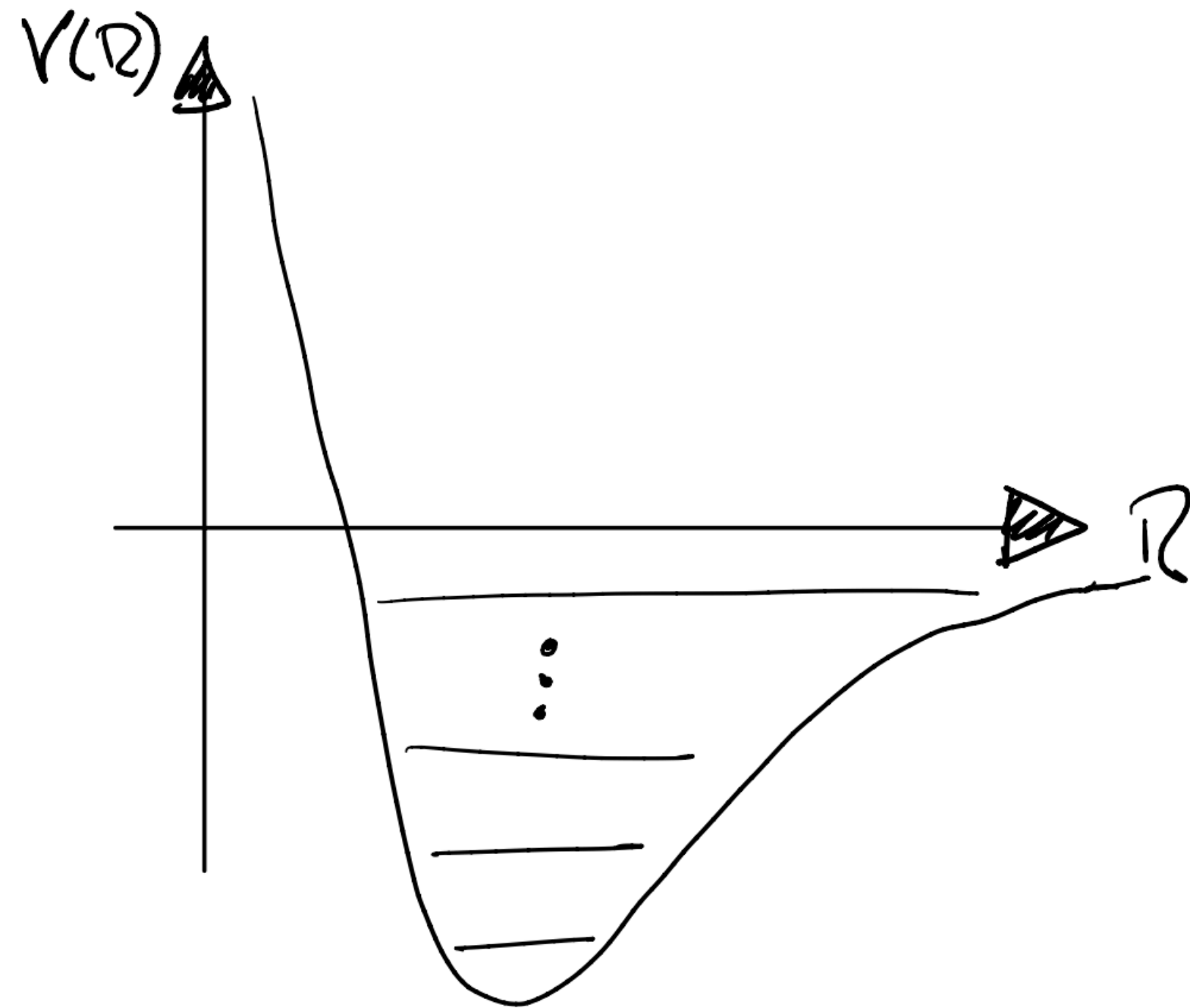
FIG. 2. Examples of band insulators of different geometries, showing (a) a 4x5 triangular lattice, (b) a circular ring, (c) a circular ring, and (d) a circular ring. The top row shows the initial state, and the bottom row shows the final state. The atom positions are shown in blue, and the average potential is shown in red. The atomization are (93, 92, 91, 89, 87, 85, 83, 81, 79, 77, 75, 73, 71, 69, 67, 65, 63, 61, 59, 57, 55, 53, 51, 49, 47, 45, 43, 41, 39, 37, 35, 33, 31, 29, 27, 25, 23, 21, 19, 17, 15, 13, 11, 9, 7, 5, 3, 1), out of (411, 254, 158, 100) sites.

- ▶ Flexible Geometry
- ▶ Dynamical
- ▶ Homogeneity
- ▶ Larger Spacing

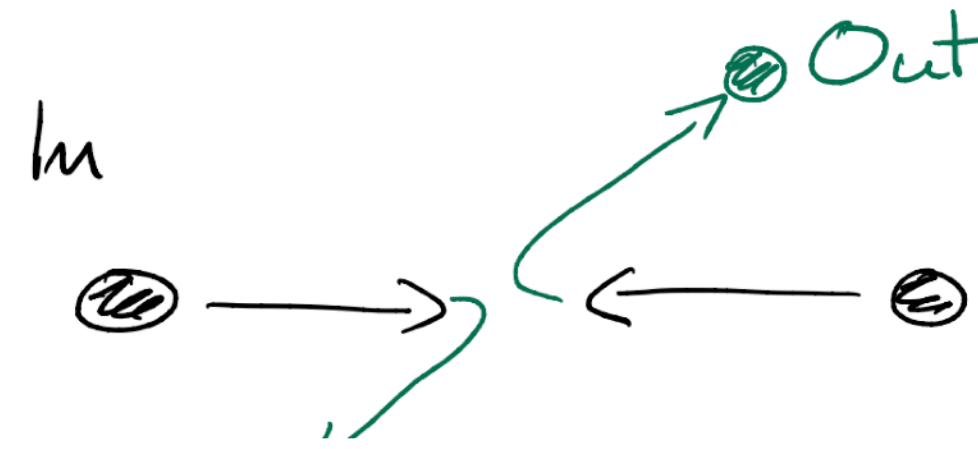


# Collisional Interactions

## Interaction Potential



Born-Oppenheimer Potential



Bosons: Symmetric WF  
Fermions: Anti-symmetric WF



$$\frac{4\pi\hbar^2 a}{m} S(\vec{k})$$

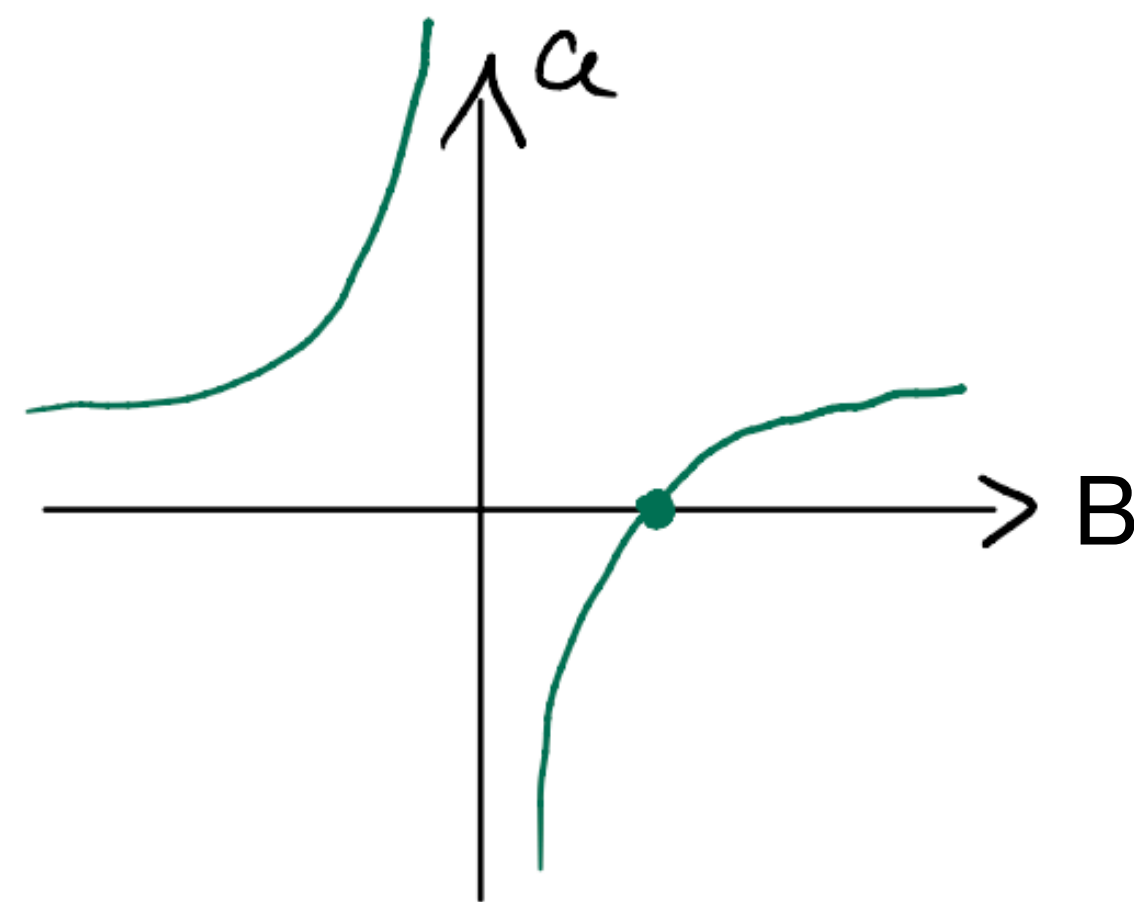
$a \hat{=} \text{scattering length}$

Pseudopotential

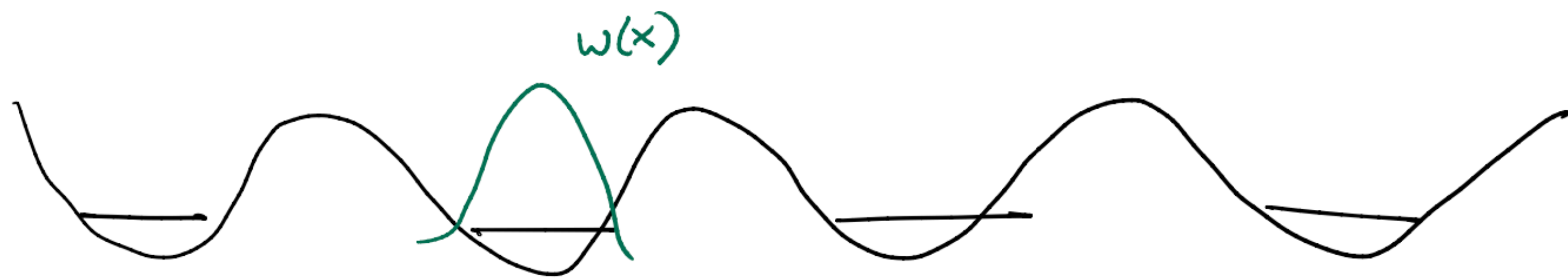


# Feshbach Resonances

$$a(B) = a_{bg} \left( 1 + \frac{\Delta}{B - B_0} \right)$$



Lattice (or tightly focused tweezer)



$$U = \frac{4\pi\hbar^2 a(B)}{m} \int |w(\vec{r})|^4 c^3 v$$



# Interacting Bosons & Fermions on a Lattice

$$H = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \sum_i \varepsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

**Bose-Hubbard**

$$\hat{H} = -J \sum_{\langle i,j \rangle, \sigma} \hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + U \sum_i \hat{n}_{i,\downarrow} \hat{n}_{i,\uparrow} + V_t \sum_{i,\sigma} i^2 \hat{n}_{i,\sigma}$$

**Fermi-Hubbard**

**to the Heisenberg spin-1/2 model (half filling, strong interactions)**

$$H = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$J = \frac{4t^2}{U}$$

(Or more generally XXZ Hamiltonian)



# Collisional Quantum Gates

## Functional building blocks for scalable multipartite entanglement in optical lattices

Wei-Yong Zhang,<sup>1,2,\*</sup> Ming-Gen He,<sup>1,2,\*</sup> Hui Sun,<sup>1,2,\*</sup> Yong-Guang Zheng,<sup>1,2</sup> Ying Liu,<sup>1,2</sup> An Luo,<sup>1,2</sup> Han-Yi Wang,<sup>1,2</sup> Zi-Hang Zhu,<sup>1,2</sup> Pei-Yue Qiu,<sup>1,2</sup> Ying-Chao Shen,<sup>1,2</sup> Xuan-Kai Wang,<sup>1,2</sup> Wan Lin,<sup>1,2</sup> Song-Tao Yu,<sup>1,2</sup> Bin-Chen Li,<sup>1,2</sup> Bo Xiao,<sup>1,2</sup> Meng-Da Li,<sup>1,2</sup> Yu-Meng Yang,<sup>1,2</sup> Xiao Jiang,<sup>1,2</sup> Han-Ning Dai,<sup>1,2</sup> You Zhou,<sup>1,3</sup> Xiongfeng Ma,<sup>4</sup> Zhen-Sheng Yuan,<sup>1,2,5</sup> and Jian-Wei Pan<sup>1,2,5</sup>

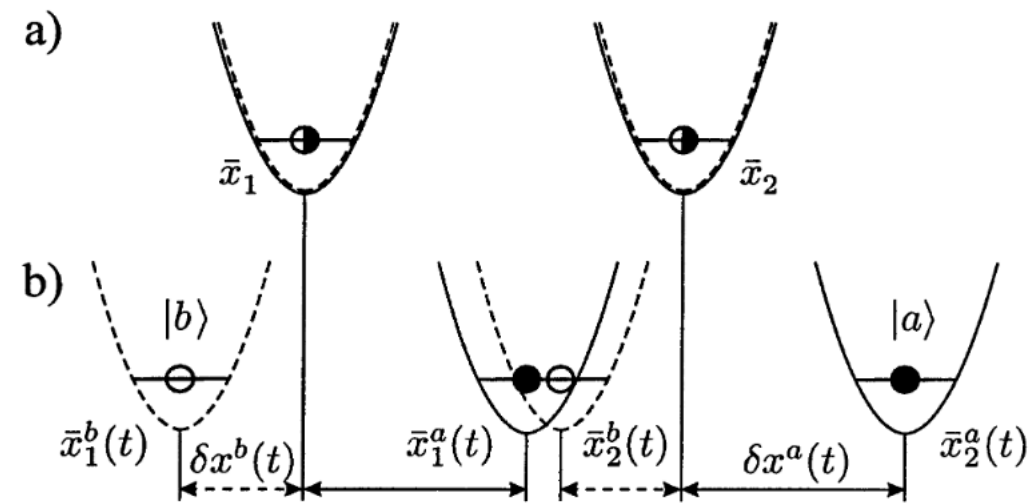
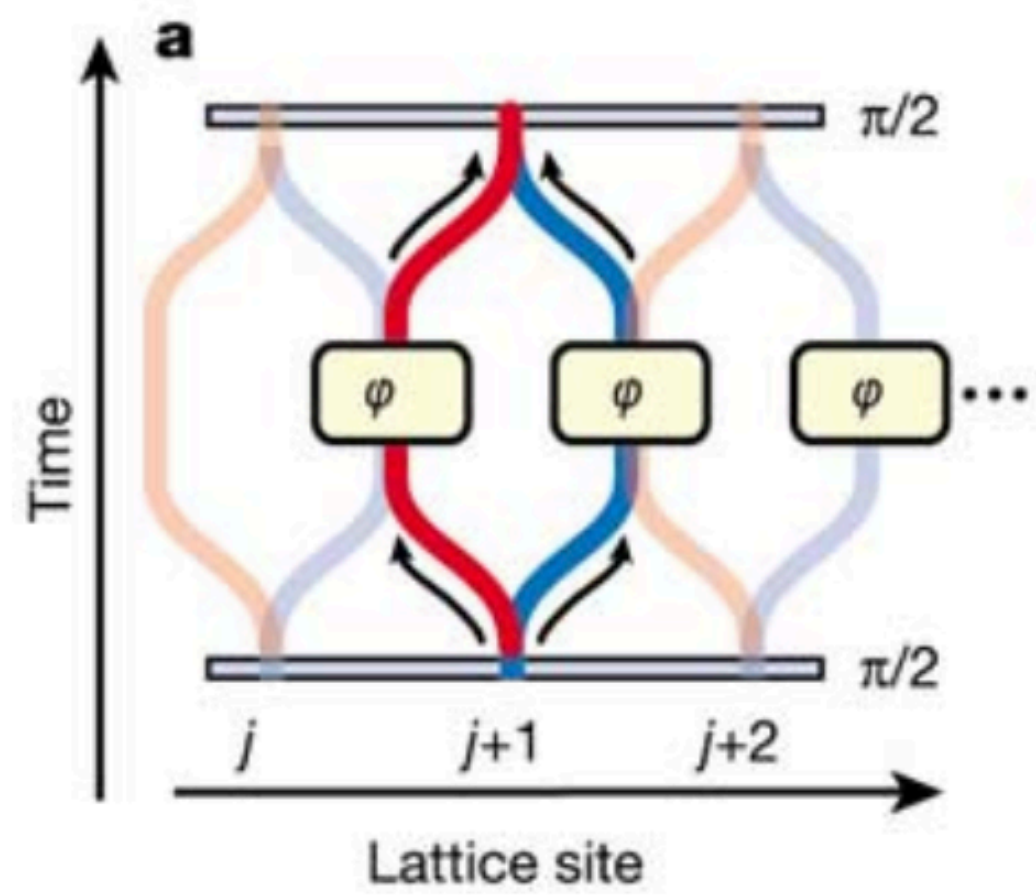
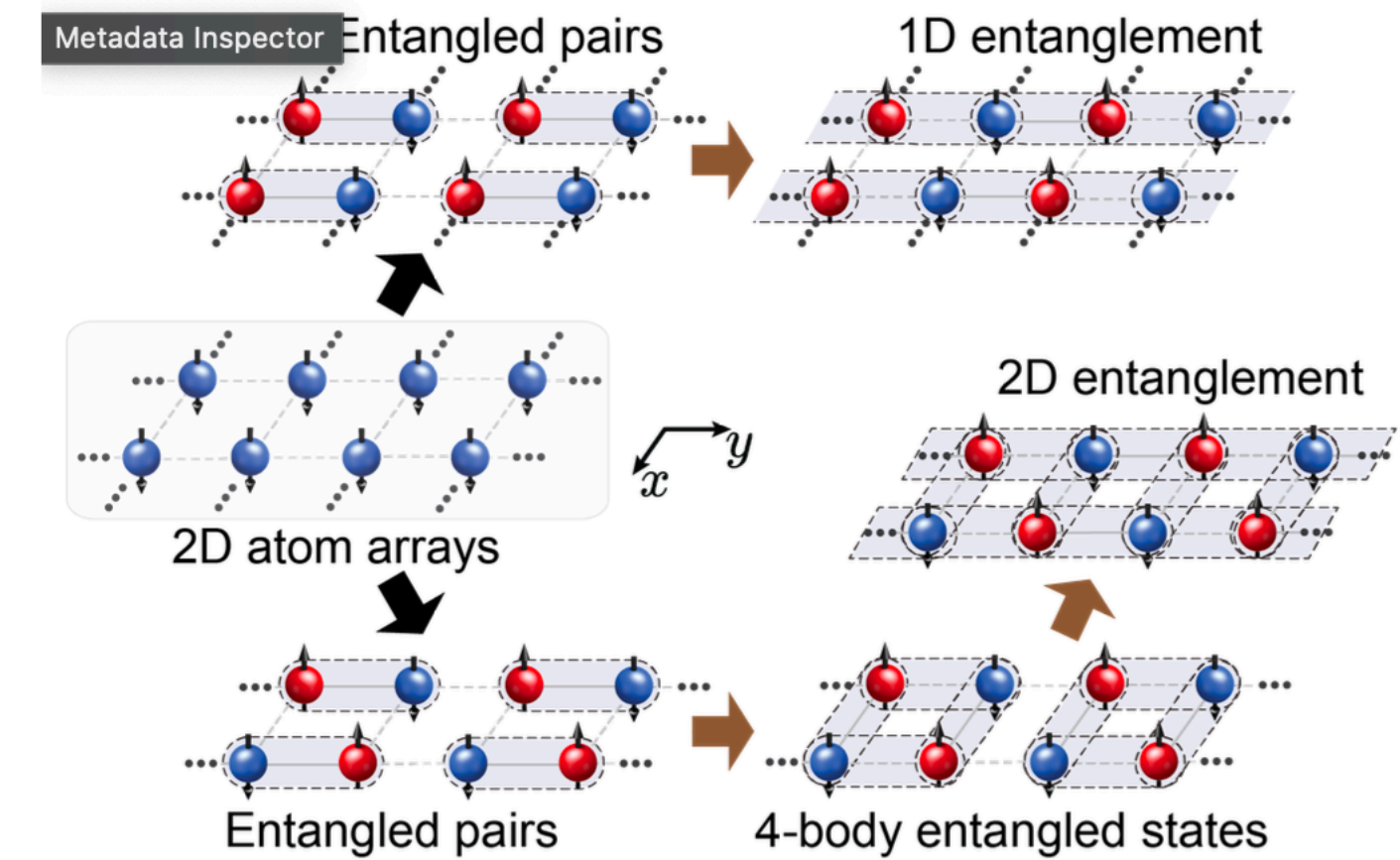
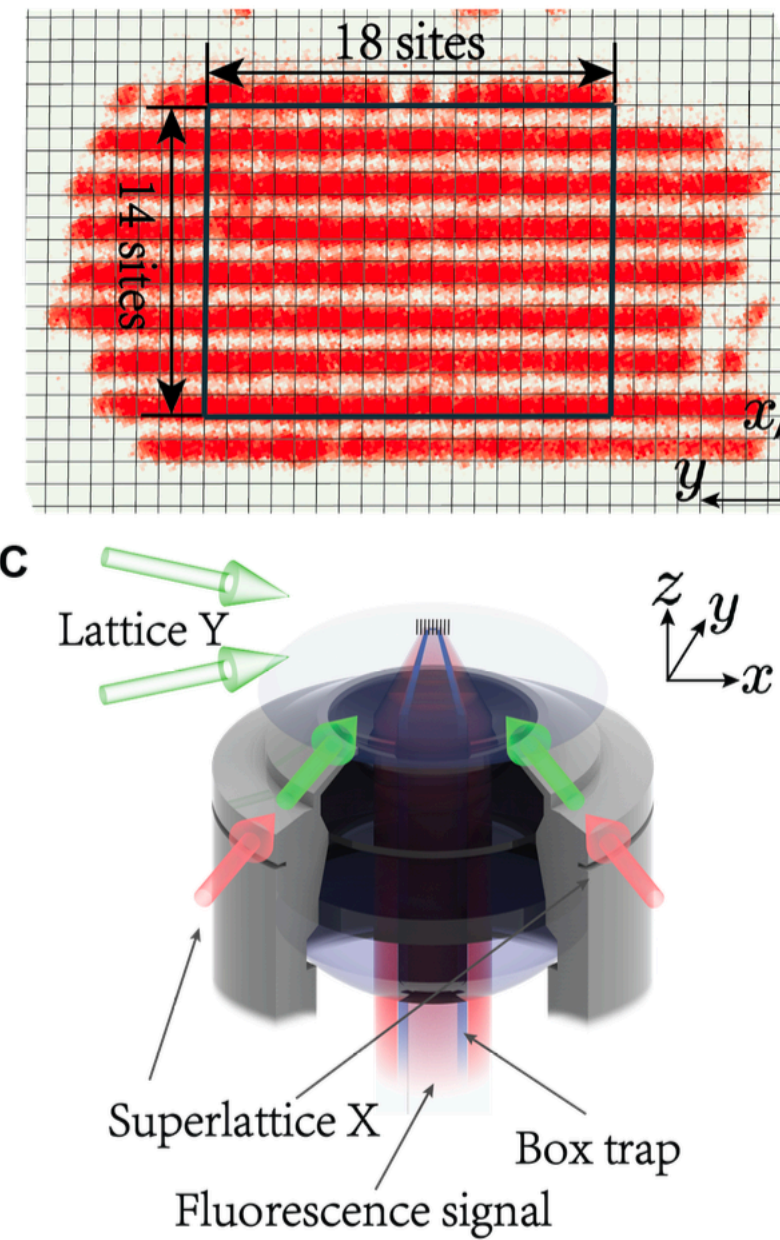


FIG. 1. Configurations at times  $\pm\tau$  (a) and at  $t$  (b). The solid (dashed) curves show the potentials for particles in the internal state  $|a\rangle$  ( $|b\rangle$ ), respectively. Center positions  $\bar{x}_j^\beta(t)$  and displacements  $\delta x^\beta(t)$  are as defined in the text.



D. Jaksch, et al. Phys. Rev. Lett. **82**, 1975 (1999)  
O. Mandel, et al. Nature **425**, 937 (2003)

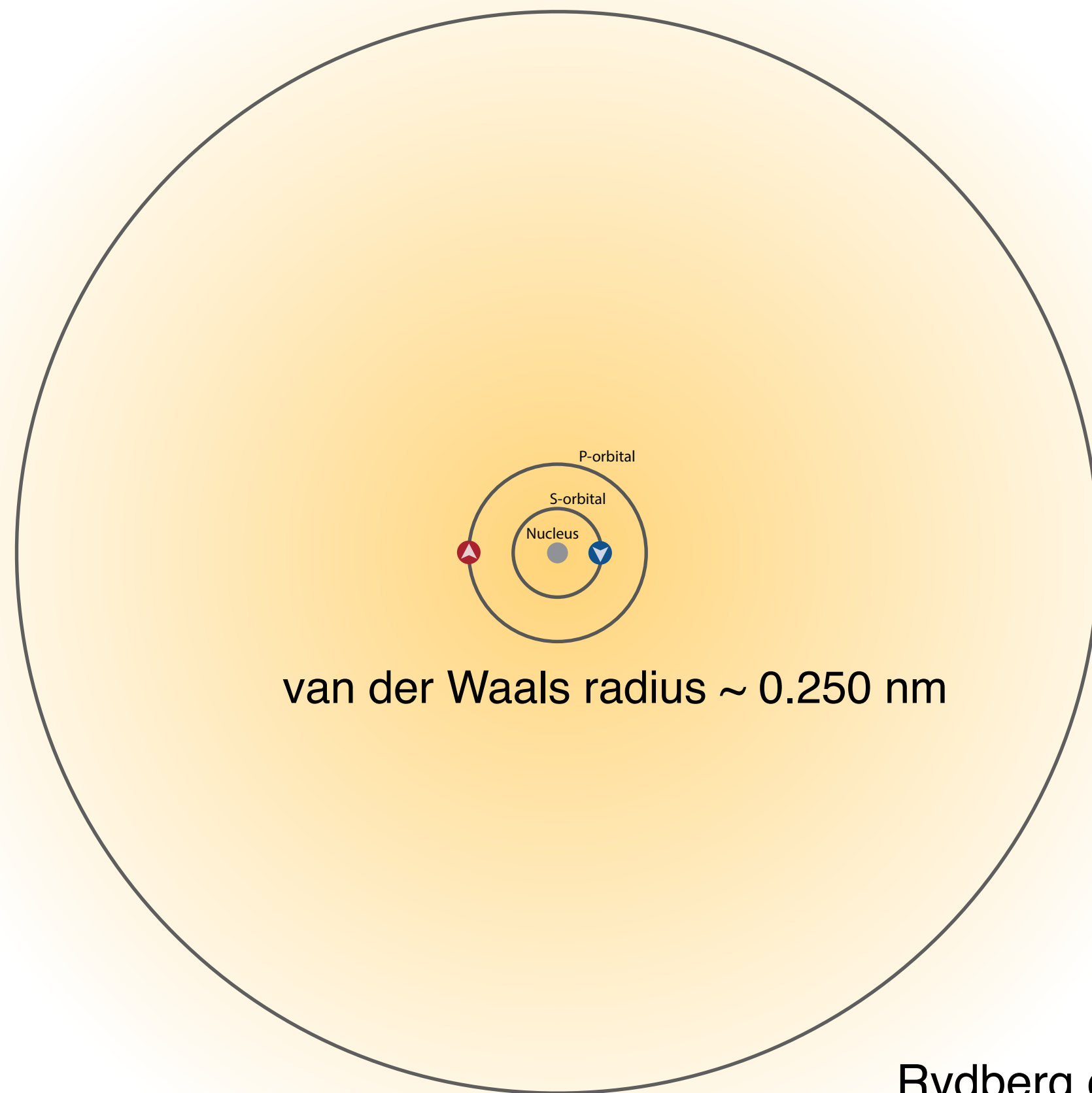


**Two-Qubit Gate Fidelity**  
SQRT(SWAP) = 99.3%

B. Yang, B. et al. Science **369**, 550 (2020)  
Zhang, W.-Y. et al. arXiv:2210.02936



# Giant Rydberg Atoms



Rydberg orbital  
radius  $\sim 300$  nm for  $n \sim 60$

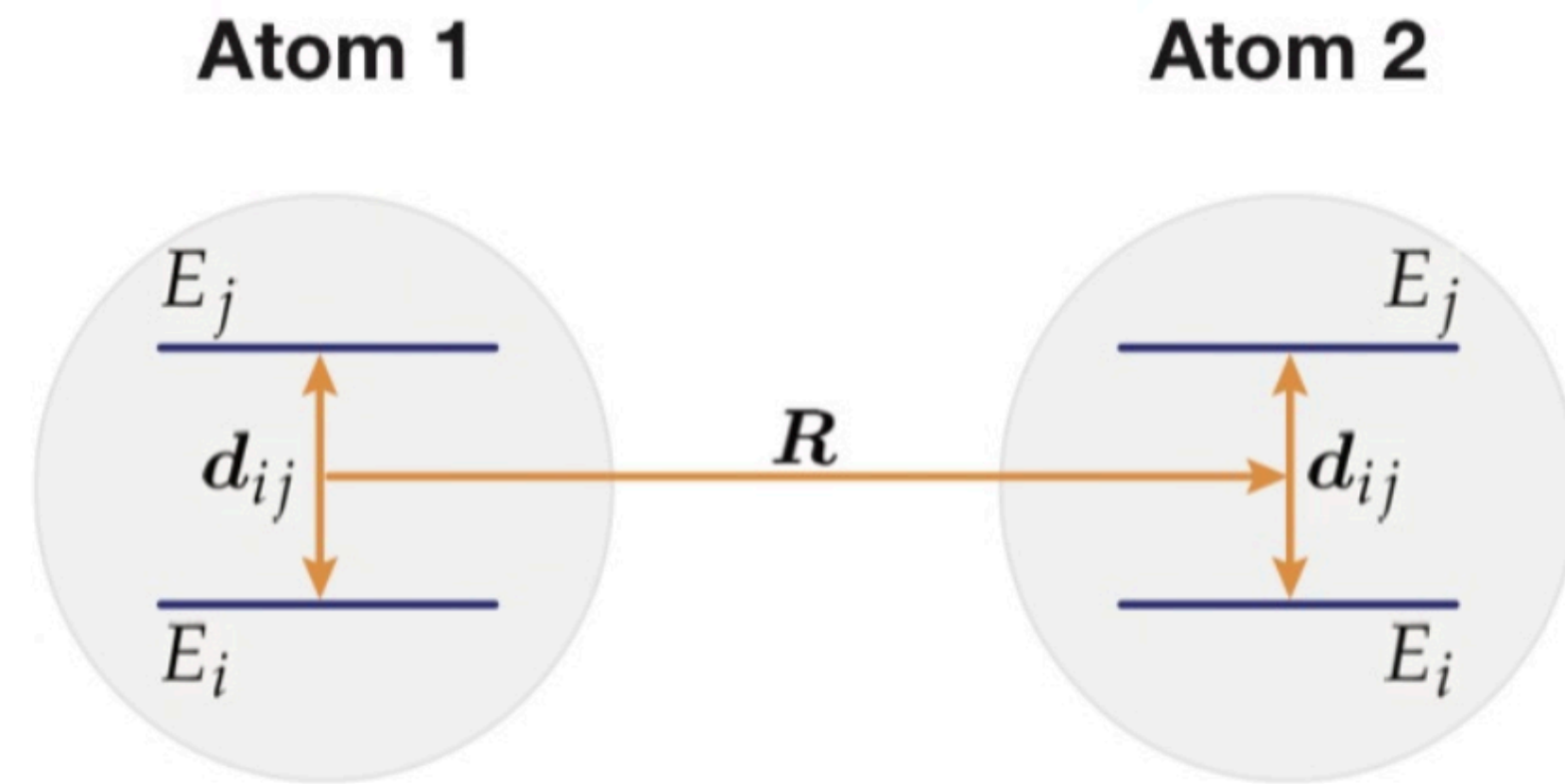
Quantity	Symbol	Scaling	Ref.	Values for $31P_{3/2}$
Energy	$E_{n,L,J}$	$n^{-2}$	[90, 91]	$-4.1$ THz
Orbital radius	$\langle r^2 \rangle^{1/2}$	$n^2$	[91]	$67$ nm
Coupling to $5S_{1/2}$	$\mathcal{R}_{5,0,1/2}^{n',1,J'}$	$n^{-3/2}$	[92]	$0.0036 ea_0$
Coupling to $nS_{1/2}$	$\mathcal{R}_{n,0,1/2}^{n,1,J'}$	$n^2$	[92]	$899 ea_0$ (to $31S_{1/2}$ )
Effective lifetime	$\tau$	$n^2$	[93]	$28 \mu\text{s}$
Radiative lifetime	$\tau_{\text{rad}}$	$n^3$	[91, 93]	$58 \mu\text{s}$
Black-body lifetime	$\tau_{\text{BB}}$	$n^2$	[91, 93, 94]	$53 \mu\text{s}$
Energy splitting	$\Delta E_{n,L,J}$	$n^{-3}$	[90, 91]	$-145$ GHz (to $31S_{1/2}$ )
$C_6$ -coefficient	$C_6$	$n^{11}$	[92, 95]	$-h 201$ MHz $\mu\text{m}^6$
Blockade radius	$R_b, R_c$	$\sim n^{1.8}$		$1.43 \mu\text{m}$ ( $\Delta/2\pi = 6$ MHz)



# Interactions in two-site model

For simplicity, we assume

$$V(R) = \frac{|\vec{d}|^2}{4\pi\epsilon_0 R^3} \quad (\text{dipoles} \perp \vec{e}_z)$$



$$H = \begin{matrix} & \langle i, i | & \langle i, j | & \langle j, i | & \langle j, j | \\ \begin{matrix} |i, i\rangle \\ |i, j\rangle \\ |j, i\rangle \\ |j, j\rangle \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & V \\ 0 & \hbar\Delta & V & 0 \\ 0 & V^* & \hbar\Delta & 0 \\ V^* & 0 & 0 & 2\hbar\Delta \end{pmatrix} \end{matrix}$$



# Rydberg - Direct Exchange Interactions

Start with atoms in different initial states  $|i, j\rangle \leftrightarrow |j, i\rangle$

Atoms initially in different states !

$$|\pm\rangle = \frac{1}{\sqrt{2}} (|i, j\rangle \pm |j, i\rangle)$$
$$E_{\pm} = \Delta \pm |V|$$

Resonant exchange

$\Rightarrow$  Resonant exchange oscillations

$$\omega_{ex} = 2|V|/\hbar$$



# Rydberg - van der Waals

Atoms initially in same state !

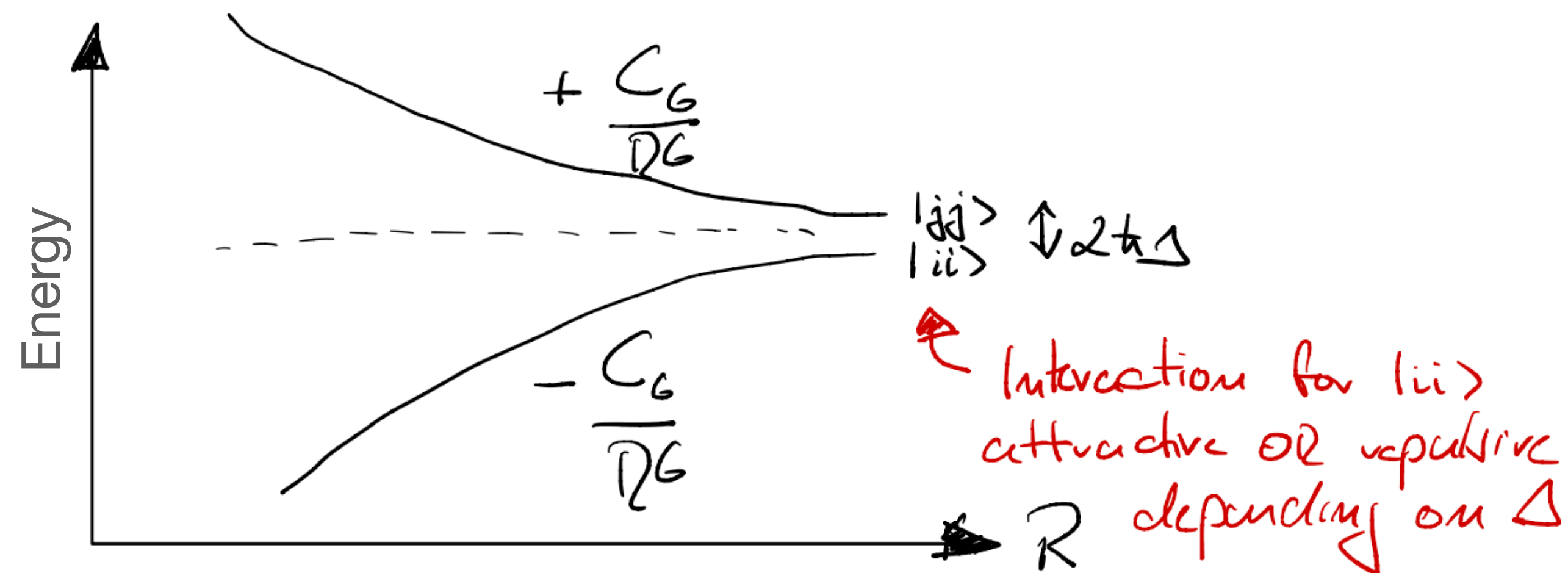
$$H_{\text{vdW}} = \begin{matrix} & \langle i, i | & \langle j, j | \\ |i, i\rangle & \begin{pmatrix} 0 & V \\ V^* & 2\hbar\Delta \end{pmatrix} \\ |j, j\rangle & \end{matrix}$$

$$E_- = \hbar\Delta - \sqrt{(\hbar\Delta)^2 + |V|^2}$$

$$\approx \hbar\Delta - \hbar|\Delta| - \frac{|V|^2}{2\hbar|\Delta|} = -\frac{|d|^4}{32\pi^2\epsilon_0^2\hbar|\Delta|R^6} \equiv -\frac{C_6}{R^6}$$

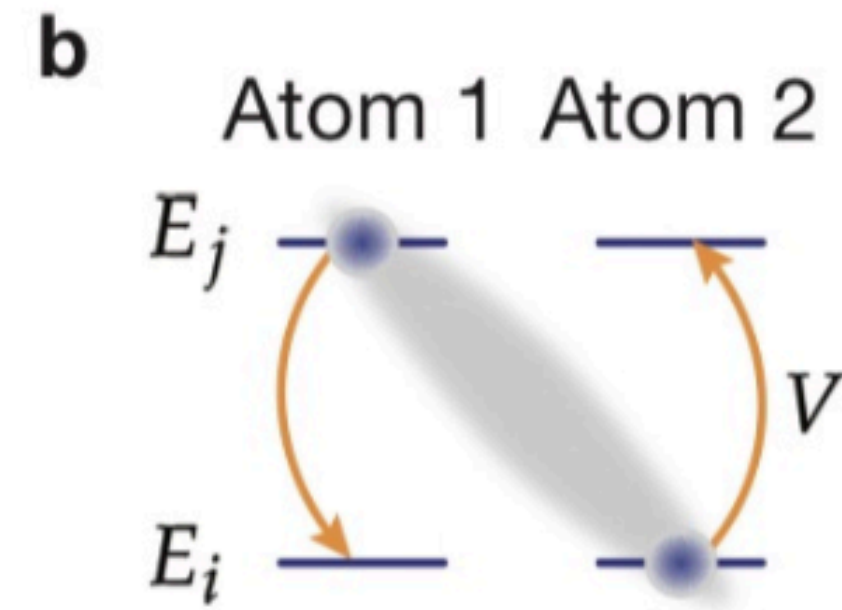
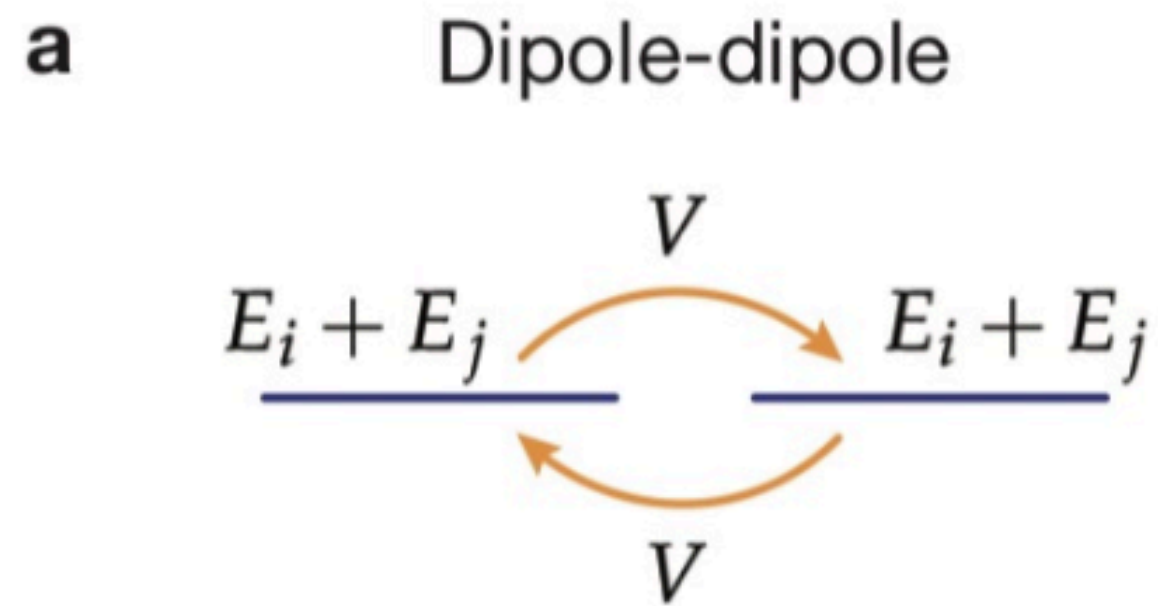
detuned coupling  $\Delta$

$$E_+ = + \frac{C_6}{R^6}$$

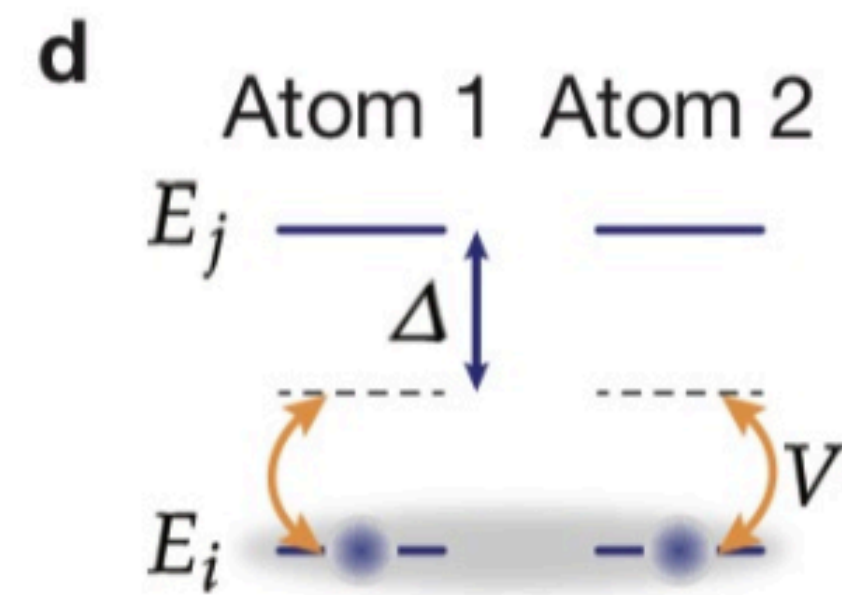
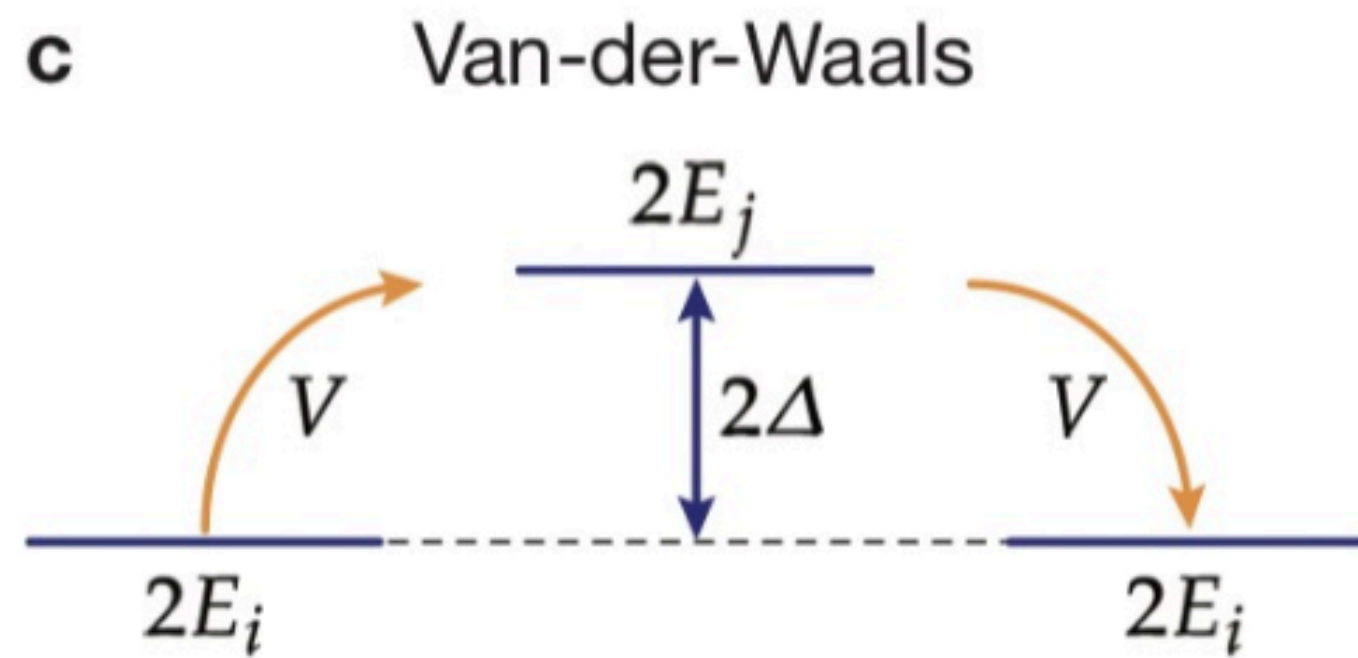




# Rydberg - Summary



Resonant DD

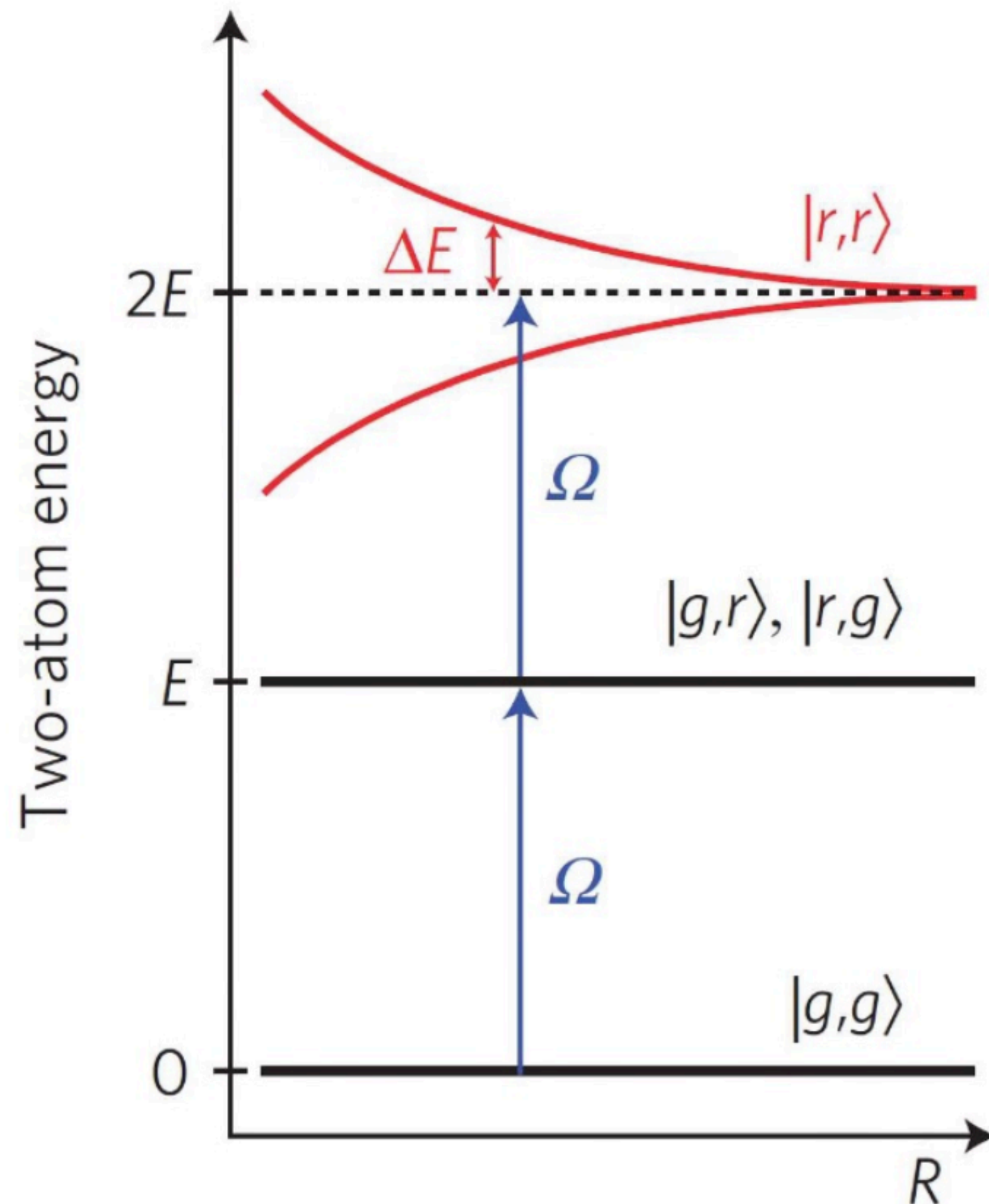


vdW

van der Waals



# Rydberg Blockade



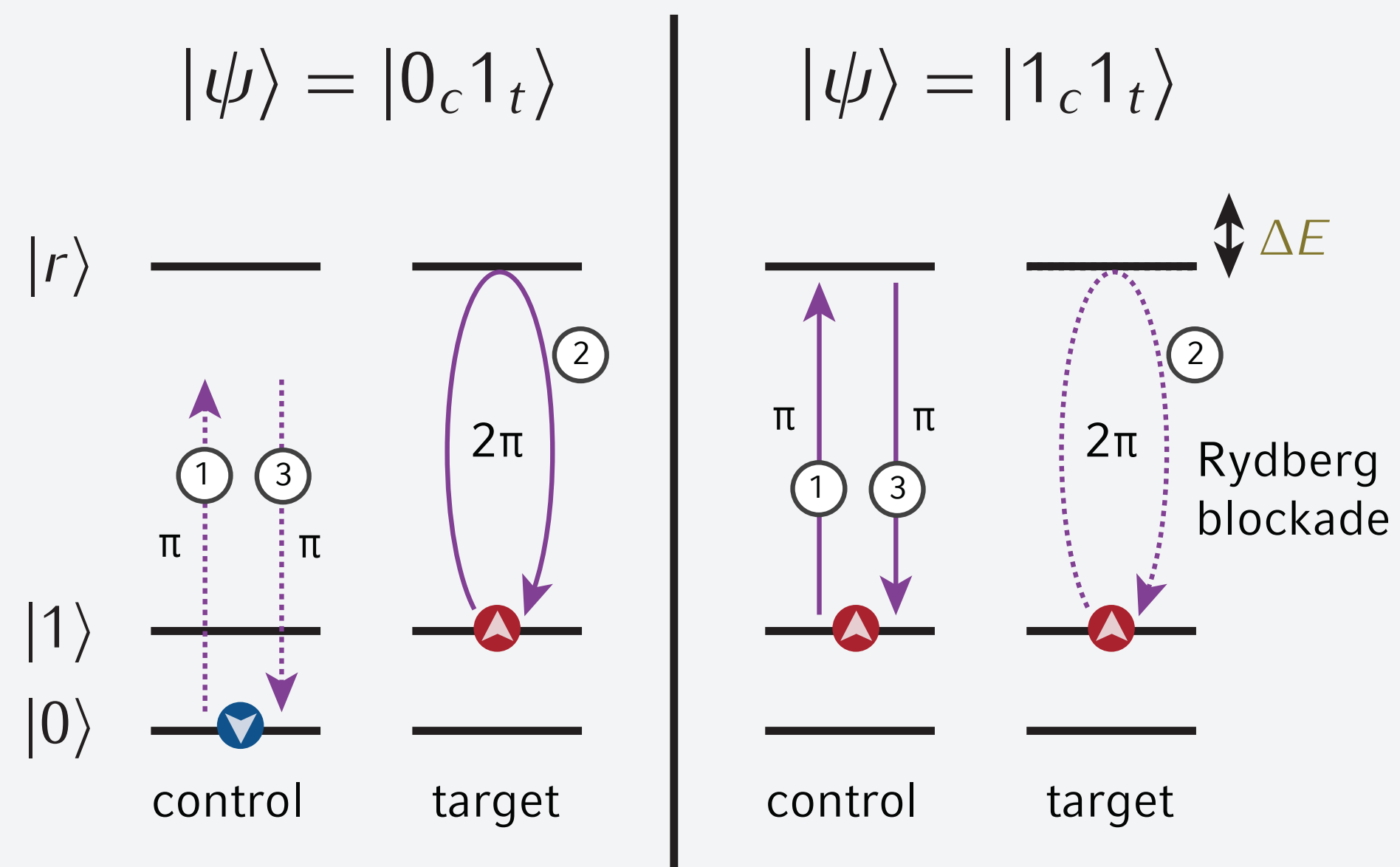
Two atom separation

Blockade Regime  $\Delta E \gg \Omega$

cannot excite two atoms at close distance



## Sequential pulses



E. Urban et al., Nat. Phys. 5, 110 (2009)