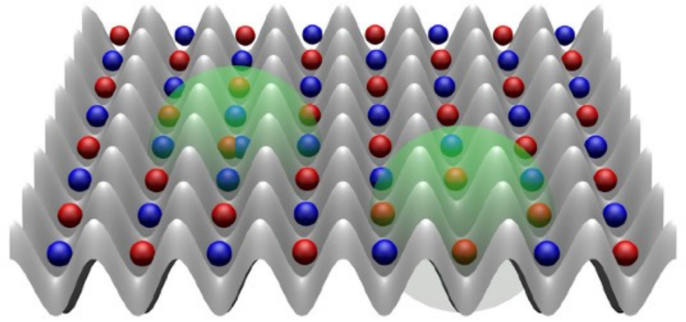
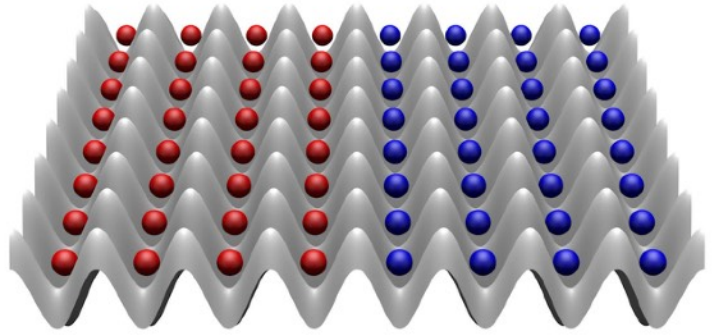
Lecture Outline



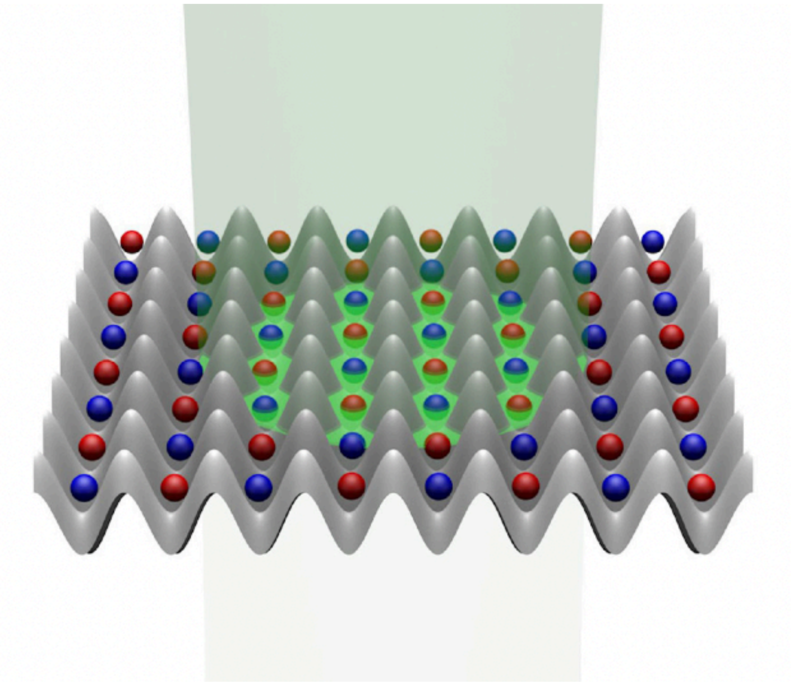
1

Quantum Simulation & Computing using Ultracold Atoms
Potentials, Interactions, Gates, Models….



2

Floquet Topological Quantum Matter & Quantum Gas Microscopy
Topological Matter, Floquet Systems, Anomalous Floquet Topological Systems, Quantum Gas Microscopy, Fractional Charge & Spin-Charge Separation

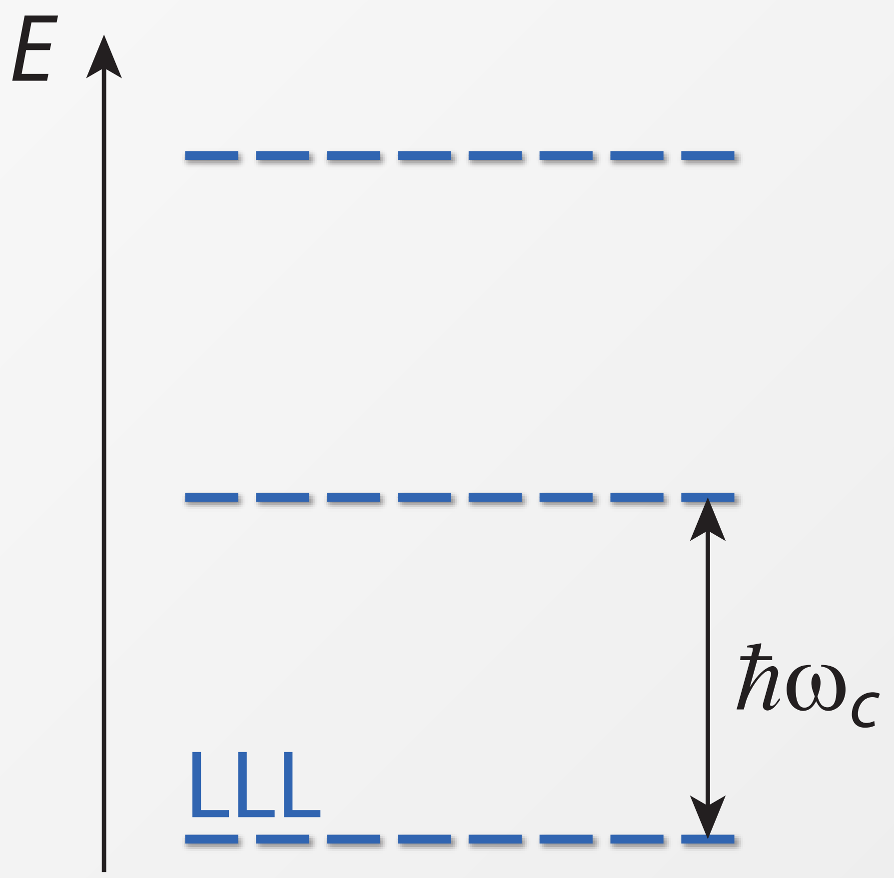


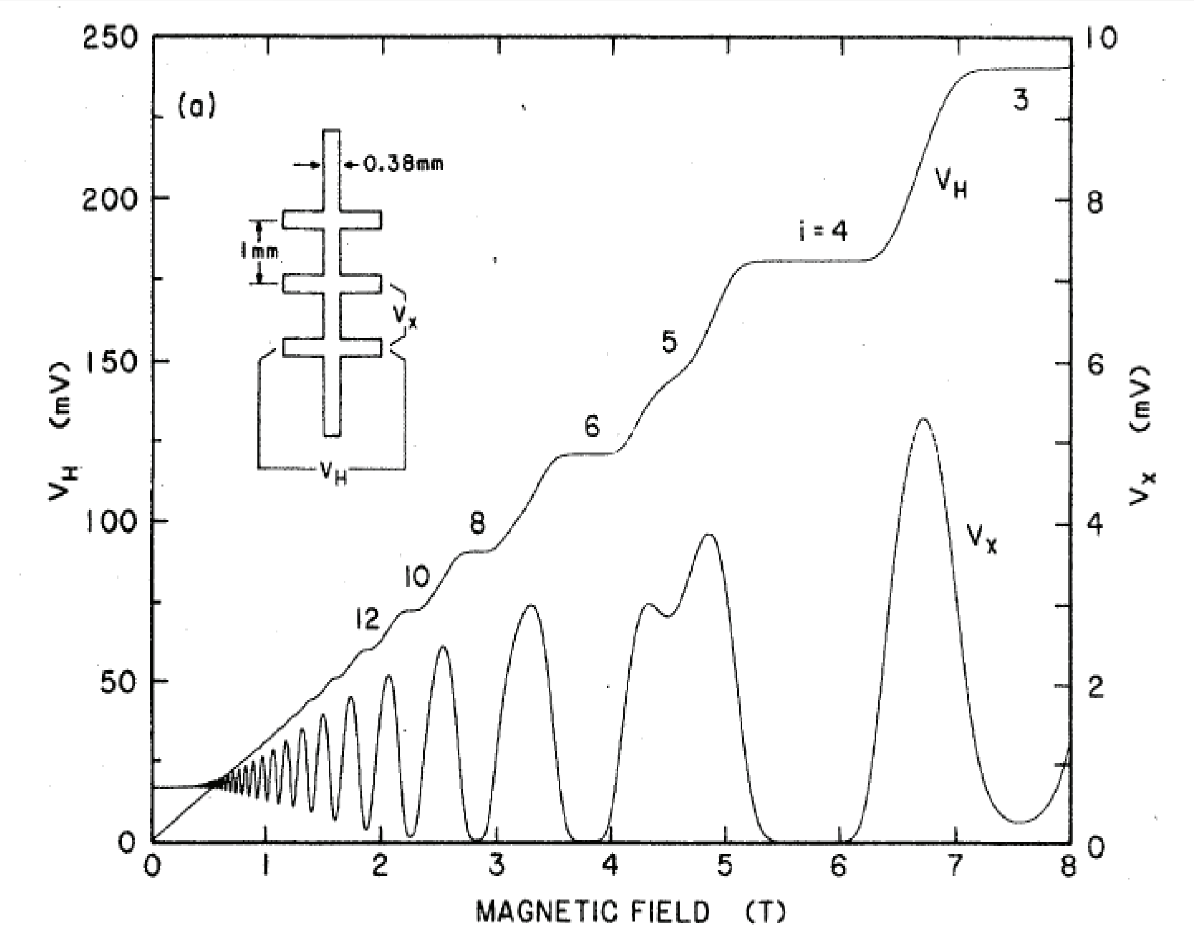
3

Quantum Matter Out-Of-Equilibrium
Many Body Localisation, Thermalization, Measuring Entanglement Entropy, Fluctuation Hydrodynamics, Anomalous Spin Transport (KPZ)

Quantum Hall Effect in 2D Electron Gases

Integer Quantum Hall Effect



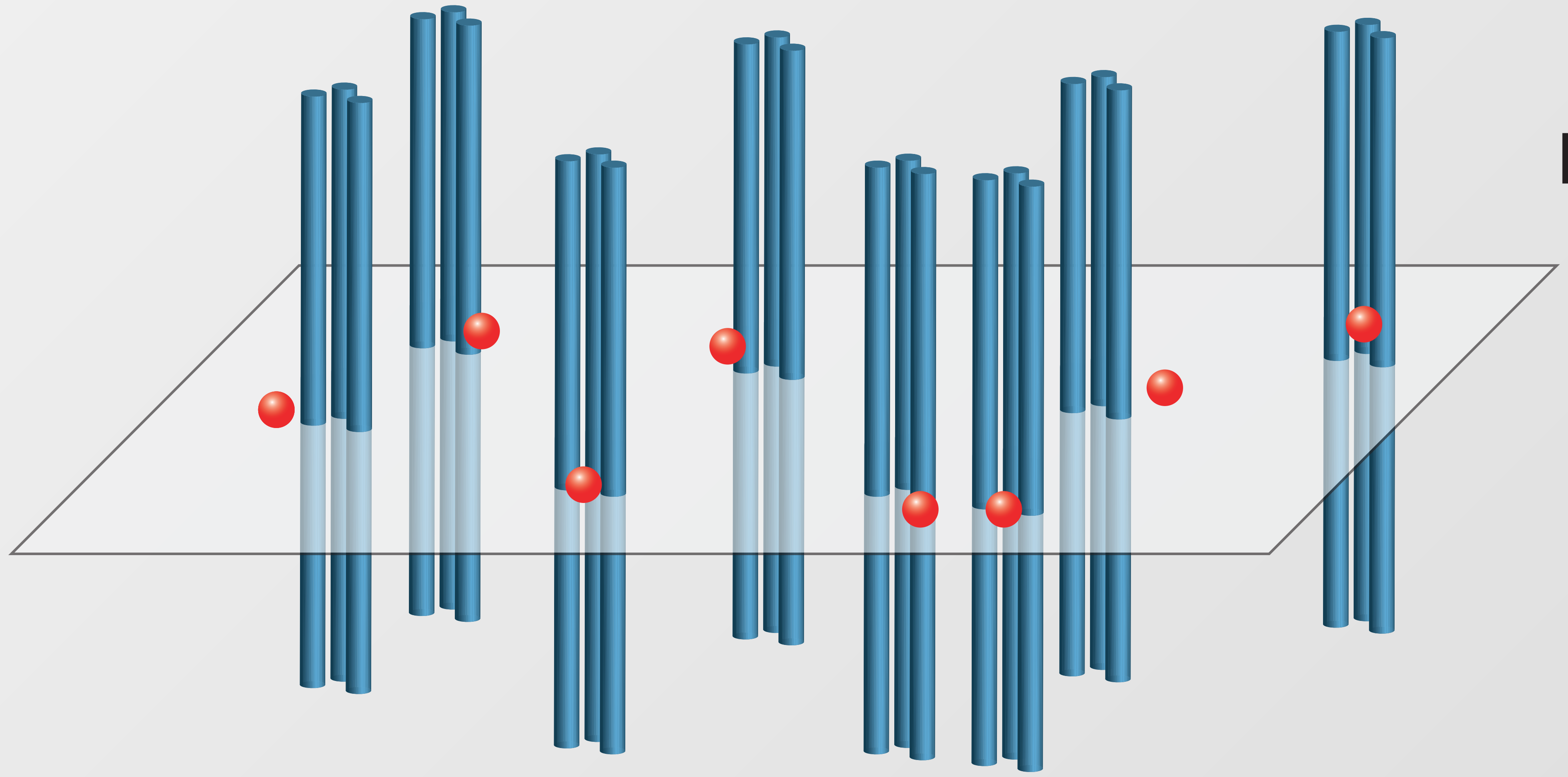


$$\sigma_{xy} = \nu e^2/h$$

ν Integer

Chern Insulators
(w/o magnetic field
see D. Haldane 1988)

Fractional Quantum Hall Effect



Laughlin state at $\nu = 1/3$

flux quantum ϕ_0=h/ec

electron

Berry Phase in Quantum Mechanics

$$\Psi(R) \rightarrow e^{i(\varphi_{\text{Berry}}+\varphi_{\text{dyn}})}\ \Psi(R)$$

Adiabatic evolution through closed loop

$$\varphi_{\text{Berry}} = \oint_{\mathcal{C}} A_n(R)dR = i\oint_{\mathcal{C}} \langle n(R)|\nabla_R|n(R)\rangle\, dR$$

$$\varphi_{\text{Berry}} = \oint_{\mathcal{A}} \Omega_n(R)\, dA$$

Berry Phase

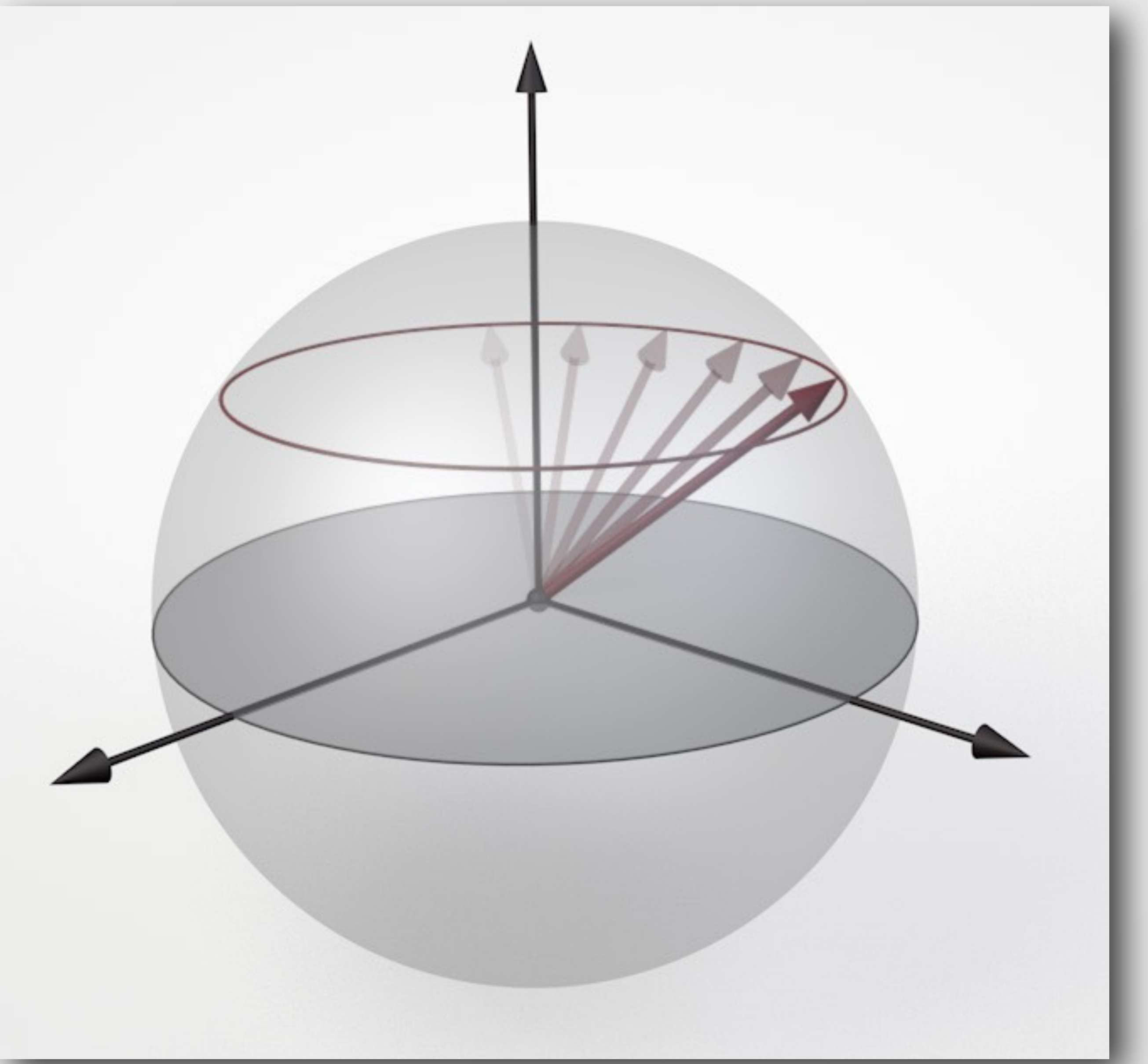
M.V. Berry, Proc. R. Soc. A (1984)

Berry connection

$$A_n(R) = i\langle n(R)|\nabla_R|n(R)\rangle$$

Berry curvature

$$\Omega_{n,\mu\nu}(R) = \frac{\partial}{\partial R^\mu} A_{n,\nu} - \frac{\partial}{\partial R^\nu} A_{n,\mu}$$

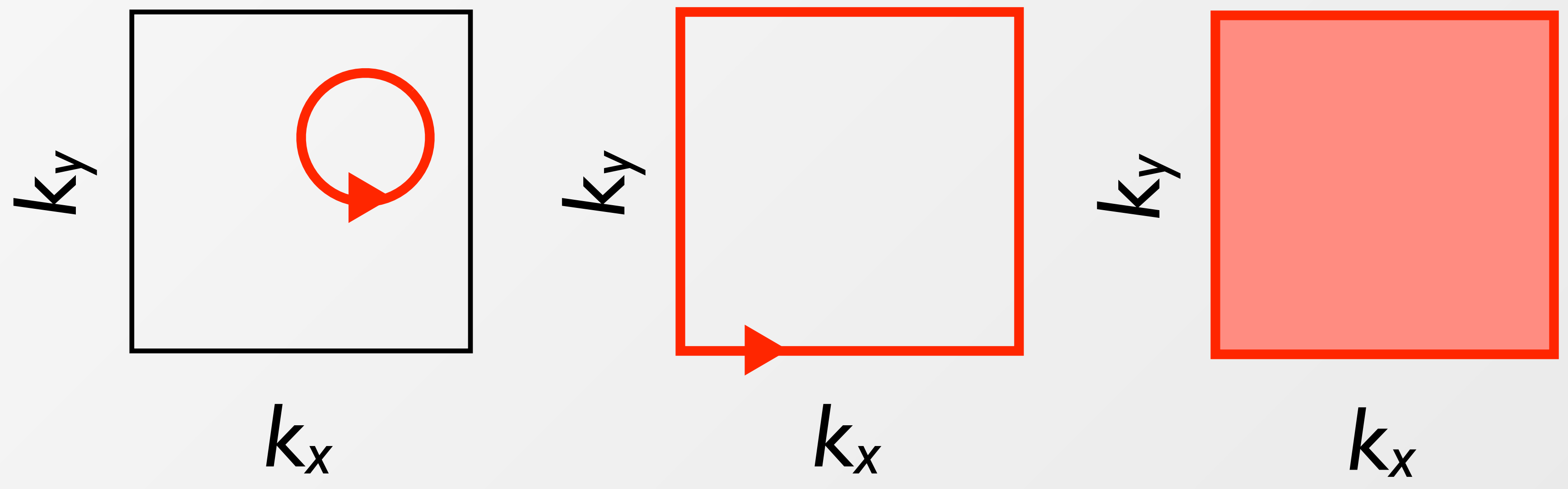


Example: Spin-1/2 particle in magnetic field

Berry Phase for Periodic Potentials

$$\Psi_k(\mathbf{r}) = e^{i\mathbf{k}\mathbf{r}} u_k(\mathbf{r})$$ Bloch wave in periodic potential

Adiabatic motion in momentum space generates Berry phase!



Berry phase is fundamental to characterize topology of energy bands

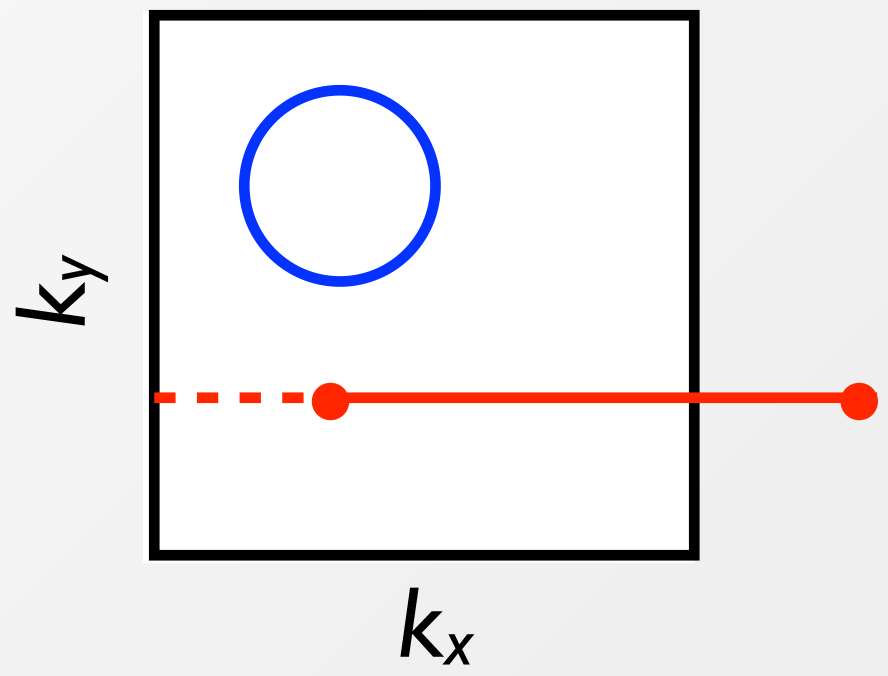
$$n_{\text{Chern}} = \frac{1}{2\pi} \oint_{BZ} A_k dk = \frac{1}{2\pi} \int_{BZ} \Omega_k \, d^2k \quad \Longleftrightarrow \quad \sigma_{xy} = n_{\text{Chern}} \, e^2/h$$

Chern Number (Topological Invariant) **Quantized Hall Conductance**

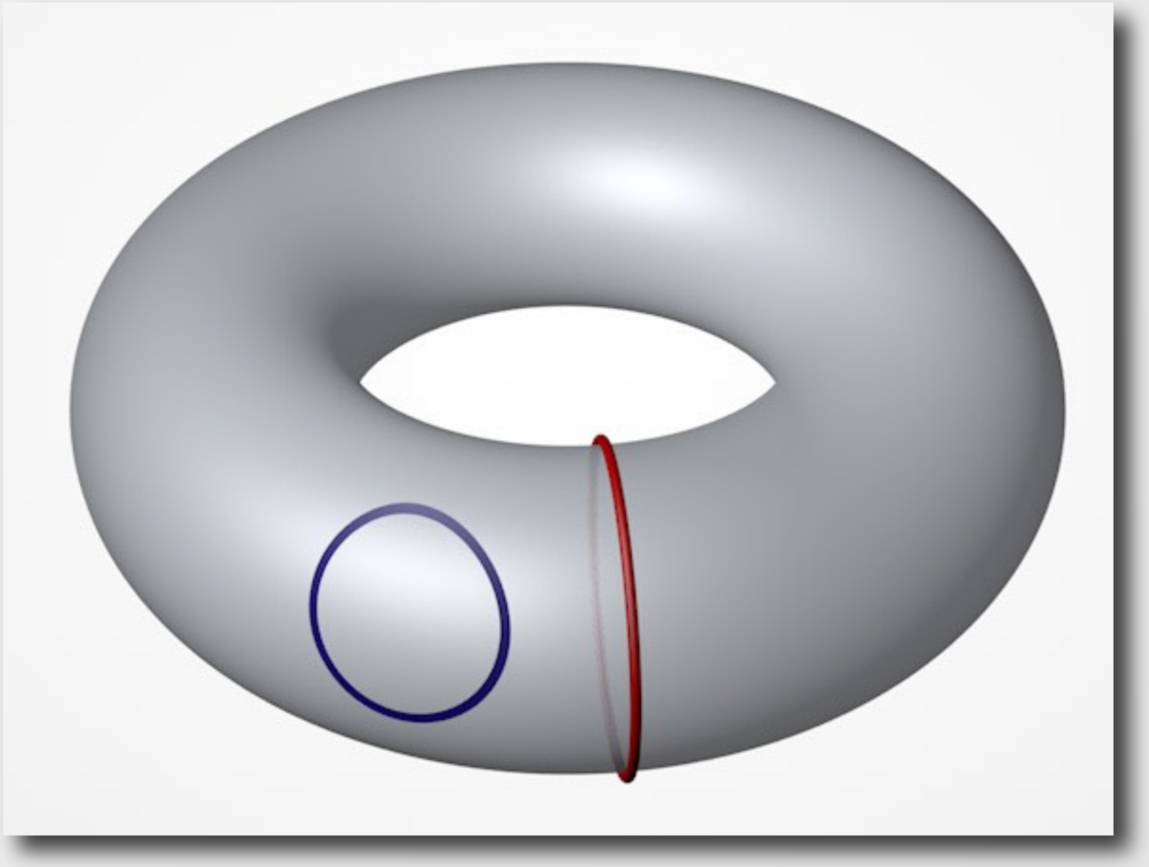
Thouless, Kohmoto, den Nijs, and Nightingale (TKNN), PRL 1982
Kohmoto Ann. of Phys. 1985

Zak Phase

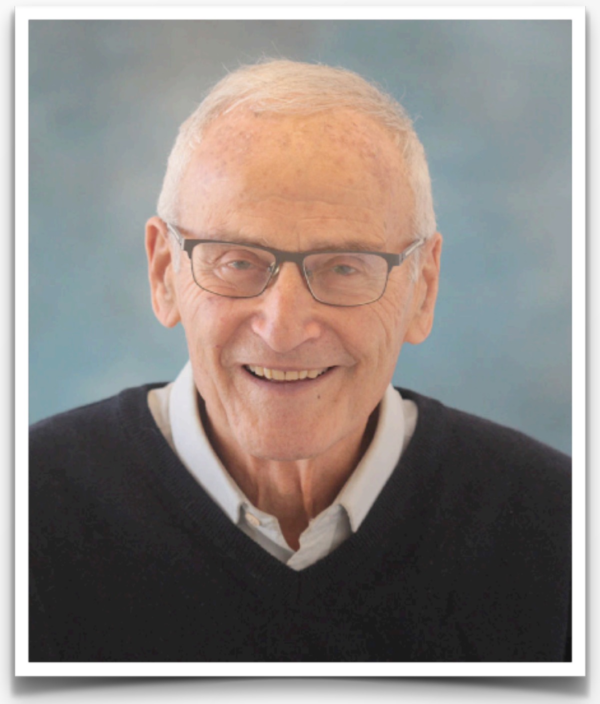
2D Brillouin Zone



going straight means going around!



Band structure has torus topology!



Joshua Zak

$$\varphi_{Zak} = i \int_{k_0}^{k_0+G} \langle u_k | \partial_k | u_k \rangle \ dk$$

Zak Phase - the 1D Berry Phase

J. Zak, Phys. Rev. Lett. **62**, 2747 (1989)
R. King-Smith & D. Vanderbilt Phys. Rev. B **47**, 1651 (1993)

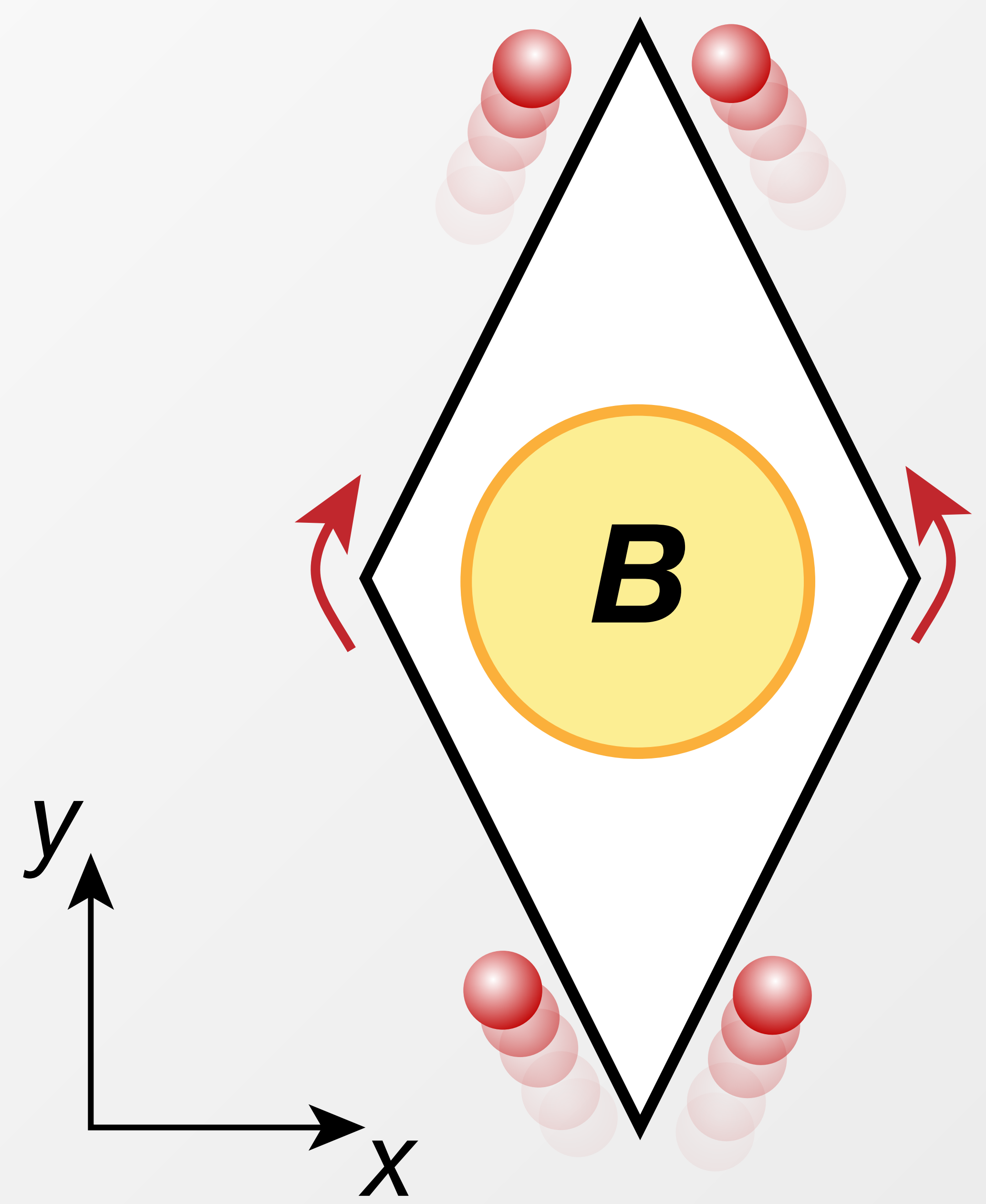
Non-trivial Zak phase:

- Topological Band
- Edge States (for finite system)
- Domain walls with fractional quantum numbers

R. Jackiw and C. Rebbi, Phys. Rev. D **13**, 3398 (1976)
J. Goldstone and F. Wilczek, Phys. Rev. Lett. **47**, 986 (1981)

Aharonov-Bohm Effect

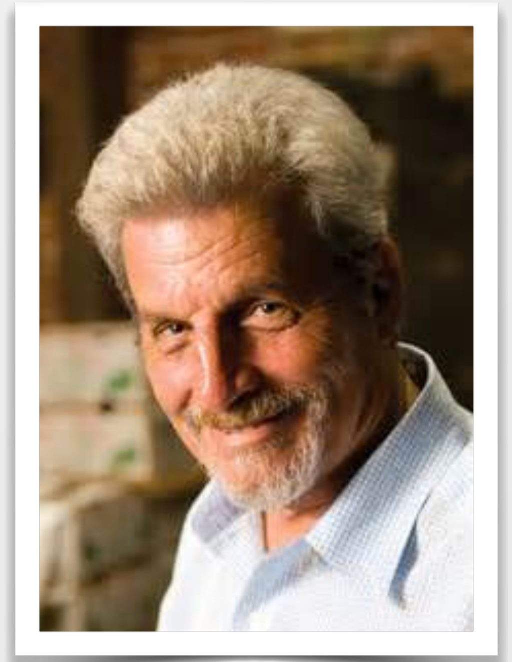
Real Space



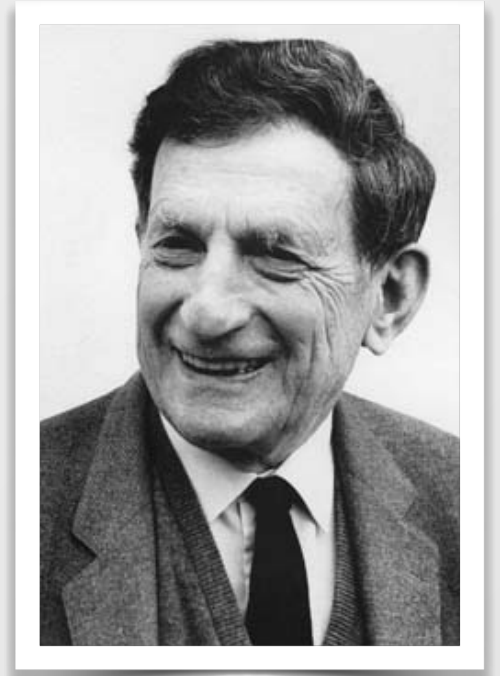
$$\varphi_{AB} = \frac{q}{\hbar}\oint_C \mathbf{A}(\mathbf{r})\, d\mathbf{r} = \frac{q}{\hbar}\int_S \nabla \times \mathbf{A}(\mathbf{r})\; d^2r$$

$$\varphi_{AB} = \frac{q}{\hbar}\int \mathbf{B}\, d\mathbf{S} = 2\pi\Phi/\Phi_0$$

Aharonov-Bohm Phase



Y. Aharonov

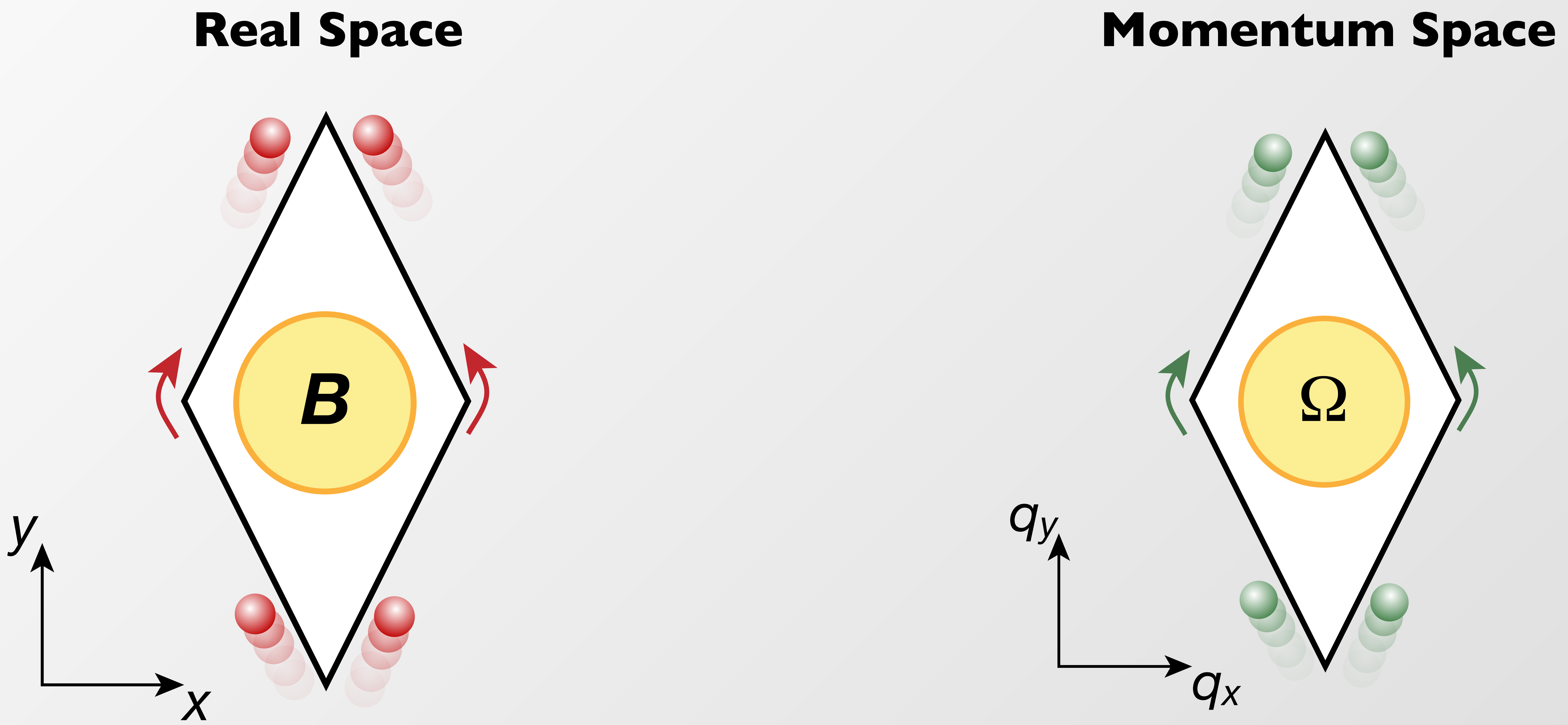


D. Bohm

...., contrary to the conclusions of classical mechanics, **there exist effects of potentials on charged particles, even in the region where all the fields (and therefore the forces on the particles) vanish.**

Y. Aharonov & D. Bohm Phys. Rev. (1959)
W. Ehrenberg & R. Siday Proc. Phys. Soc B (1949)
Exp: A. Tonomura, et al. Phys. Rev. Lett. (1986)

'Aharonov Bohm' Interferometer in Momentum Space



$$\varphi_{AB} = \frac{q}{\hbar}\oint_C \mathbf{A}(\mathbf{r})\,d\mathbf{r} = \frac{q}{\hbar}\int_S \nabla\times\mathbf{A}(\mathbf{r})\;d^2r$$

$$\varphi_{AB} = \frac{q}{\hbar}\int \mathbf{B}\,d\mathbf{S} = 2\pi\Phi/\Phi_0$$

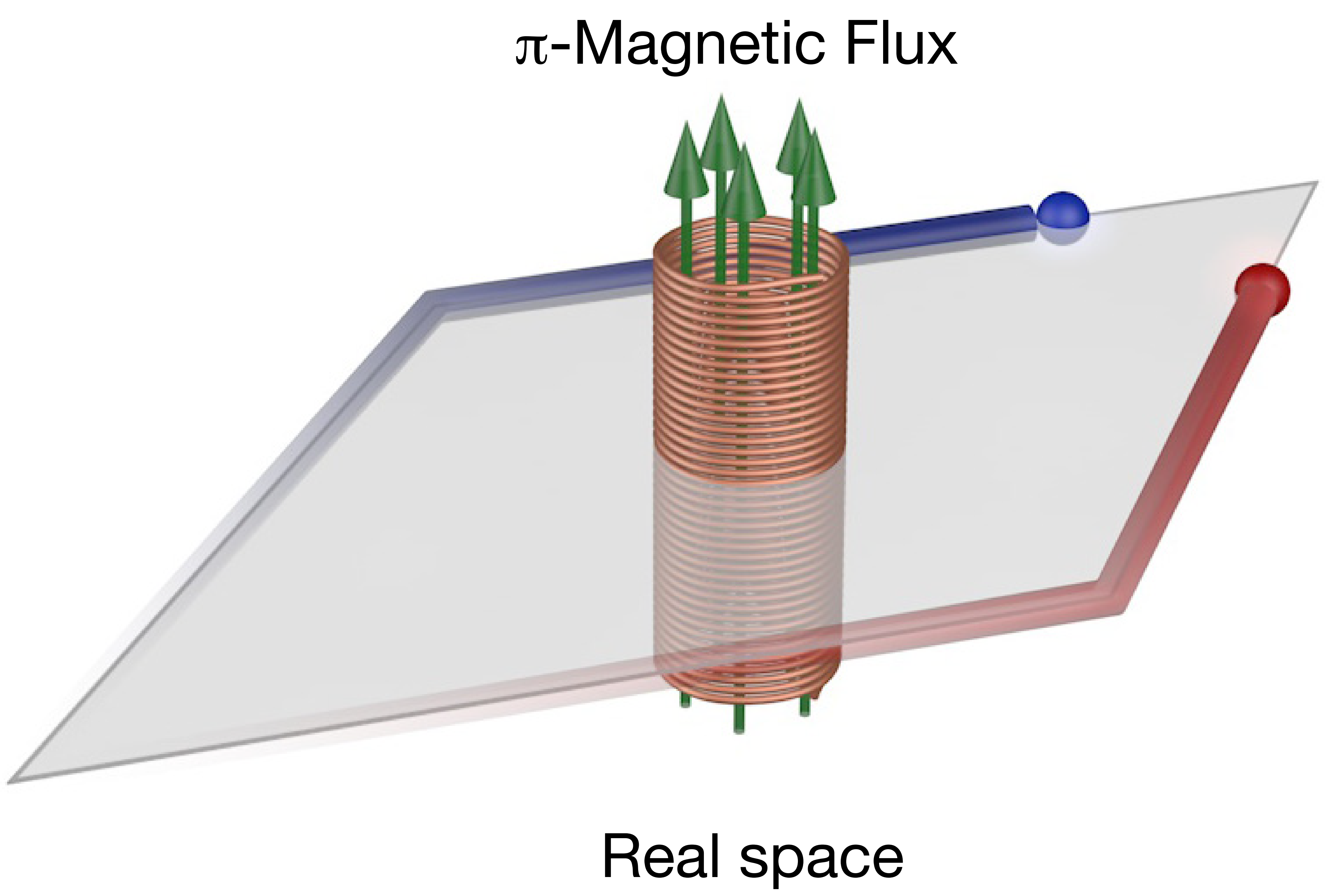
Aharonov-Bohm Phase

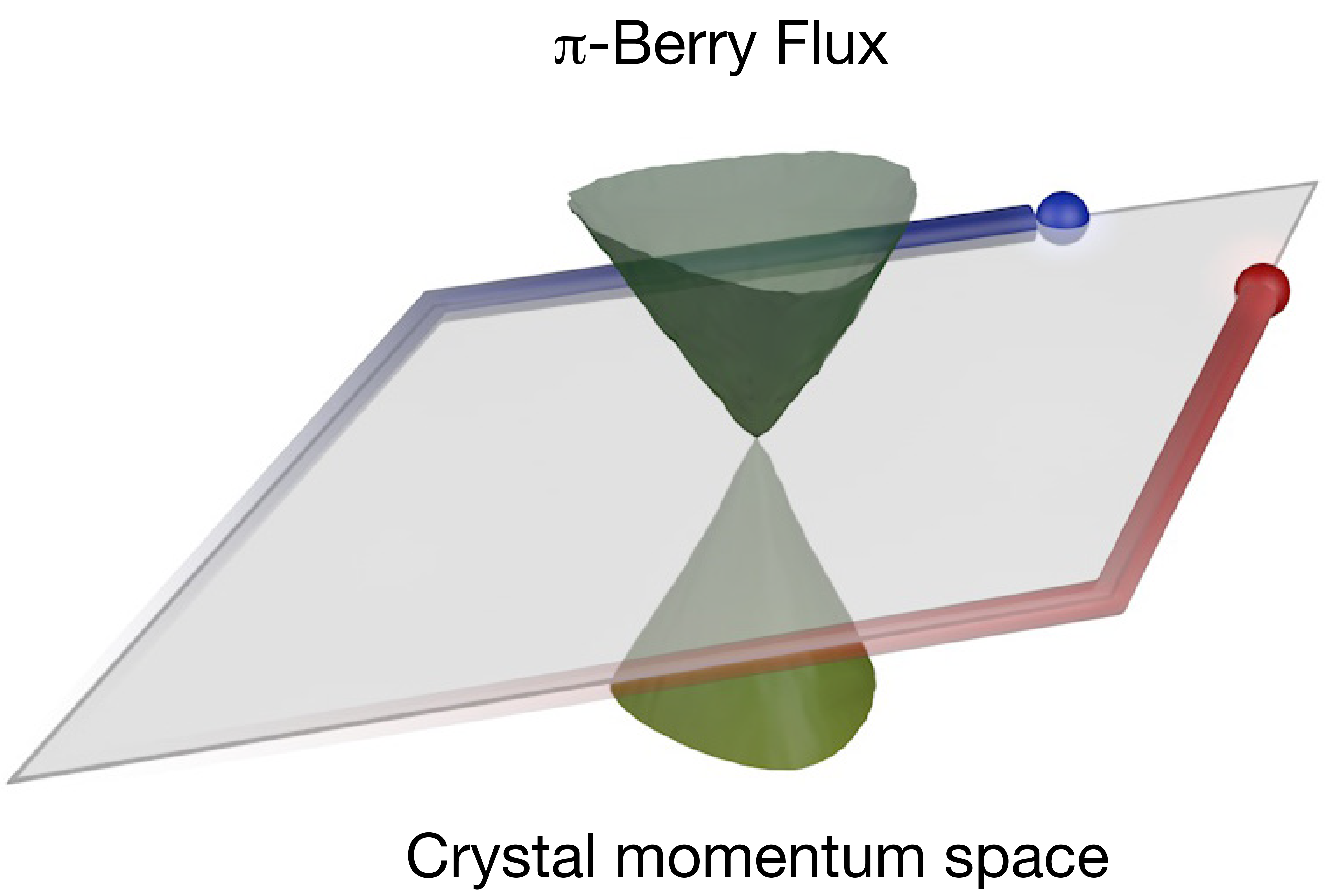
$$\varphi_{\text{Berry}} = \oint_C \mathbf{A}_n(\mathbf{q})\,d\mathbf{q} = \int_{S_q} \nabla\times\mathbf{A}_n(\mathbf{r})\;d\mathbf{S}_q$$

$$\varphi_{\text{Berry}} = \int \Omega_n(\mathbf{q})\,d\mathbf{S}_q$$

Berry Phase

Extensions to 2D and Multi-Bands: An Aharonov Bohm and Wilson Line Interferometer for Determining Bloch Band Topology





L. Duca et al. Science **347**, 288 (2015)
T. Li et al. Science **352**, 1094–1097 (2016)
D. Abanin et al. PRL **110**, 165304 (2013)

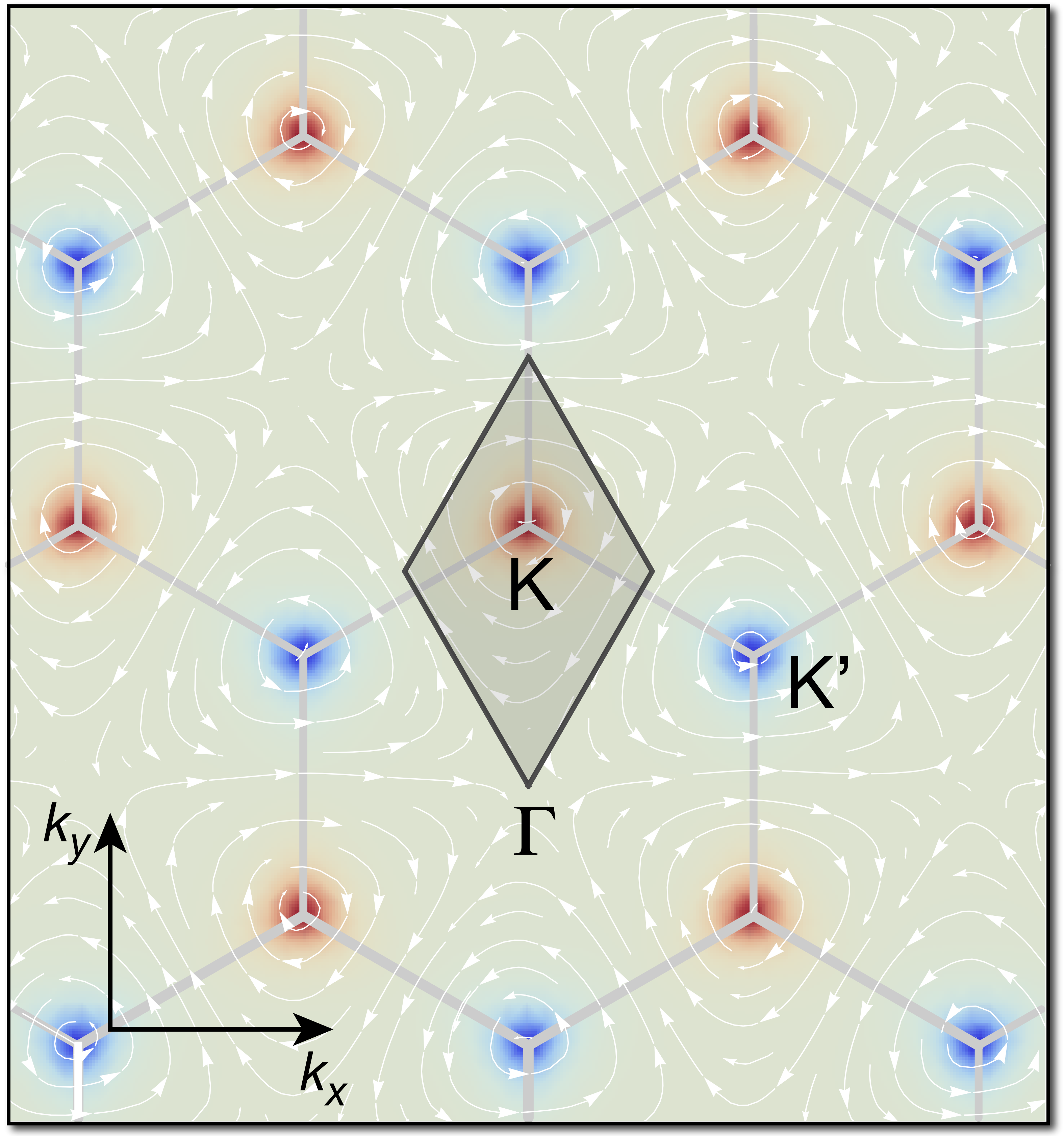
Berry Phases in Graphene

Berry Phase around K-Dirac cone

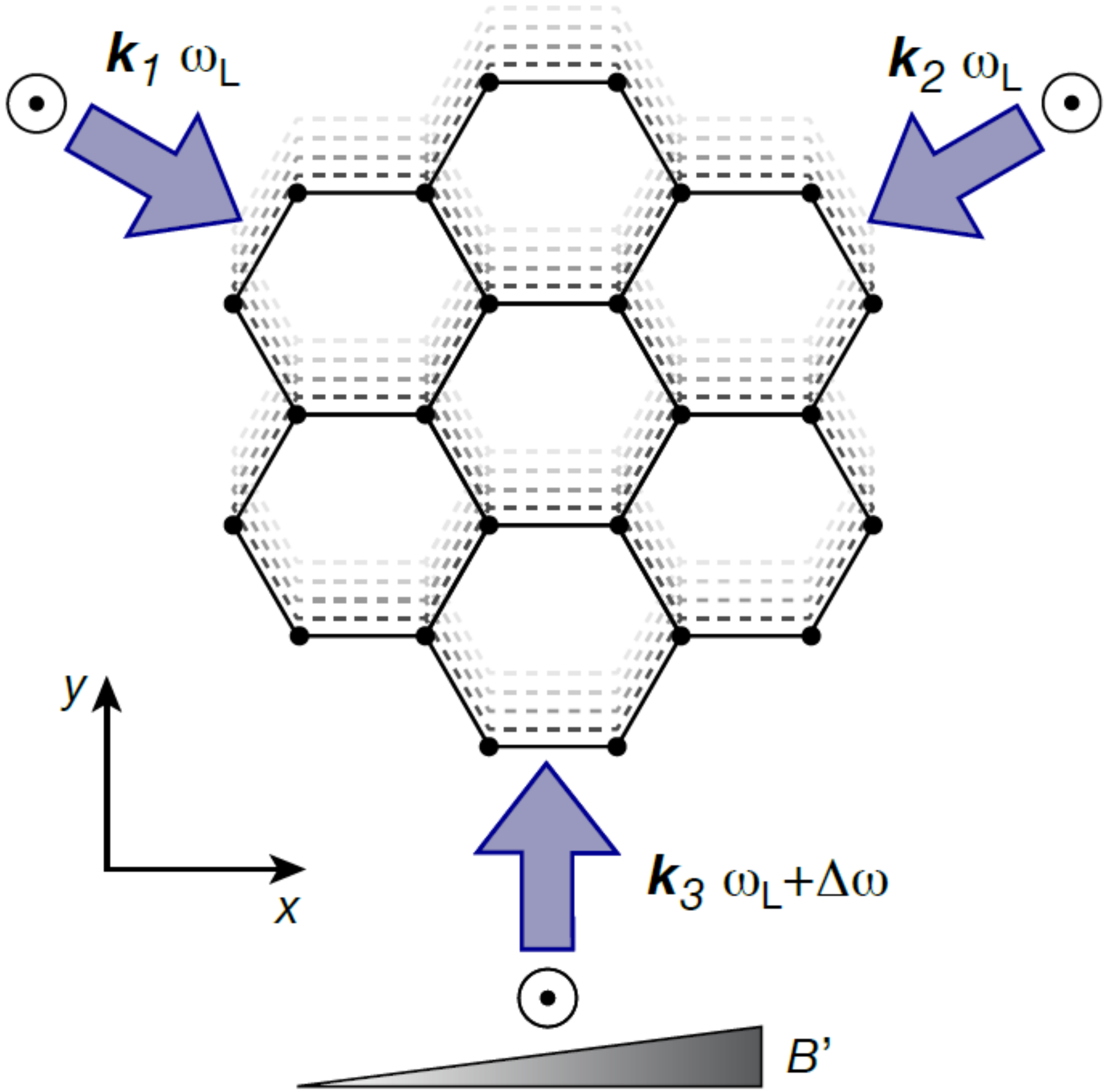
$$\varphi_{\mathrm{Berry},\mathbf{K}} = \oint_C \mathbf{A}(\mathbf{q})\, d\mathbf{q} = \pi$$

Berry Phase around K'-Dirac cone

$$\varphi_{\mathrm{Berry},\mathbf{K}'} = -\pi$$

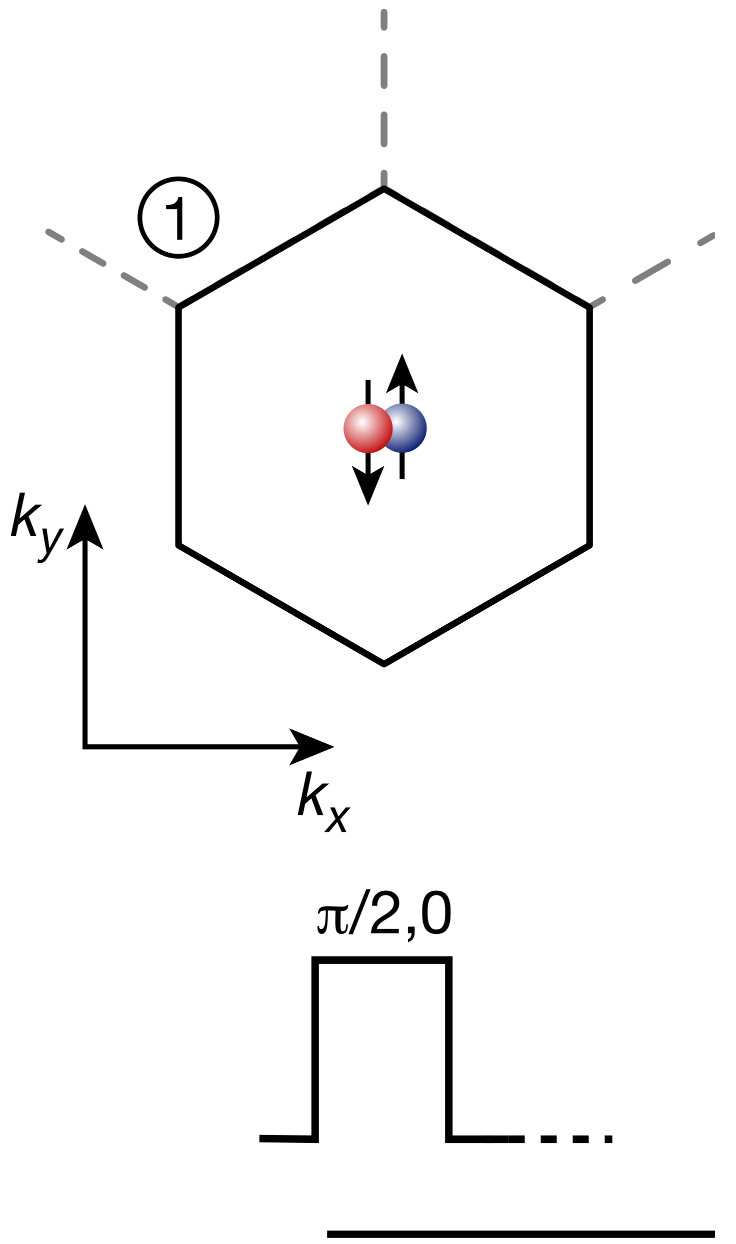


Accelerating the Lattice

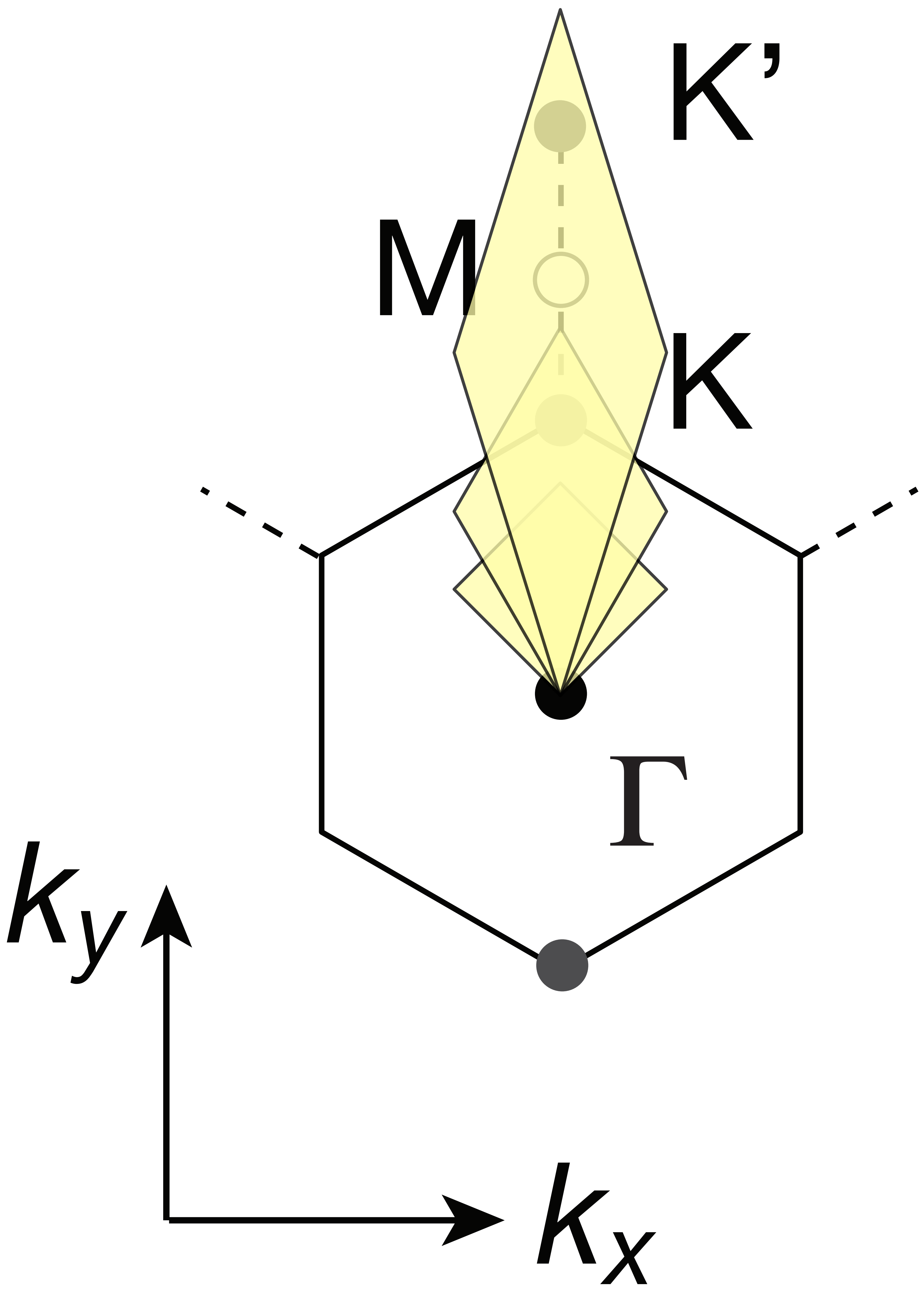


Arbitrary accelerations in any direction can be applied!

The Interferometer

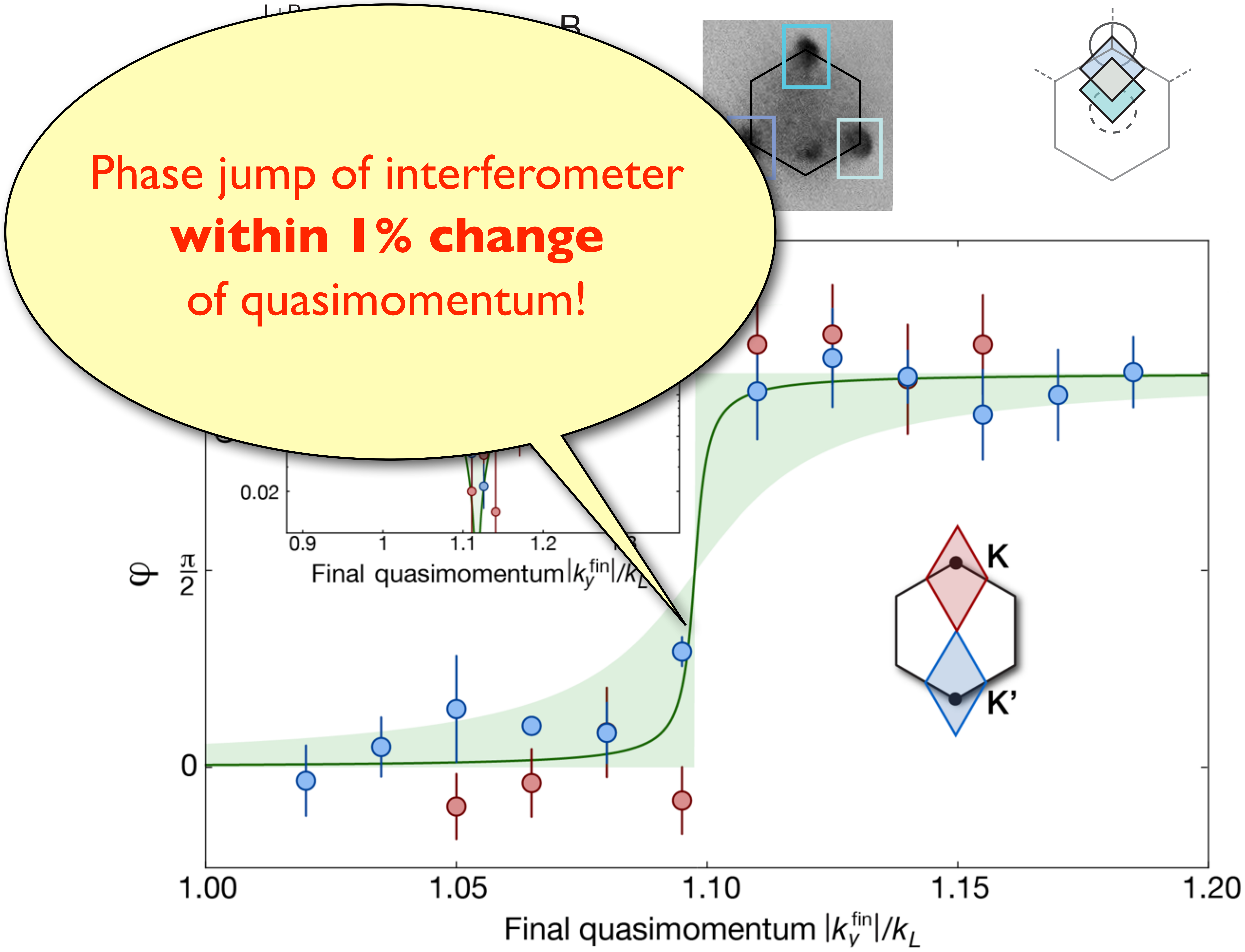


Forces applied by **lattice acceleration** and **magnetic gradients**!

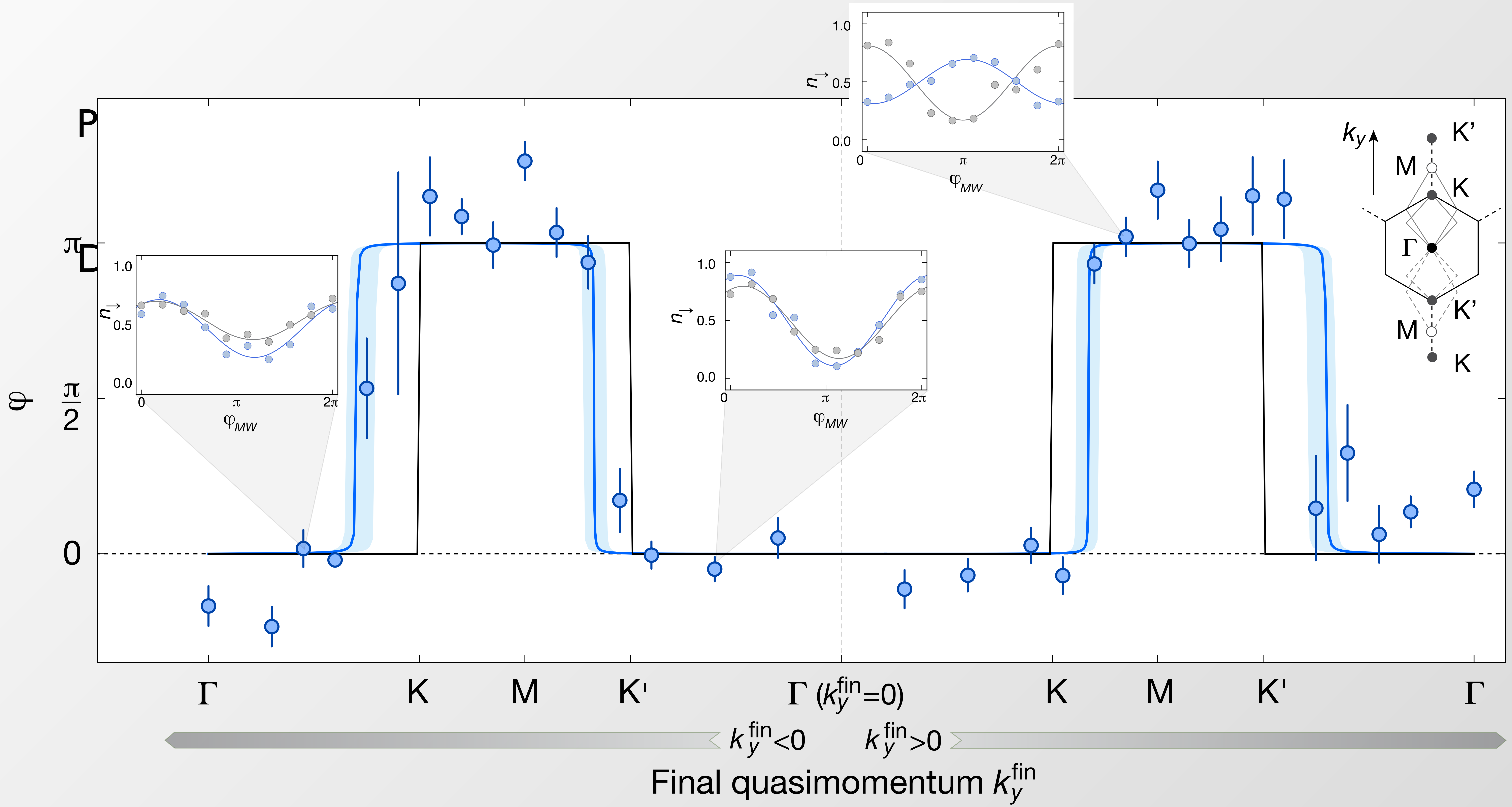

K'
M
K
Γ
k_y
k_x

Zooming into the Edge

Interferometer path | Location after band-mapping | Differential area

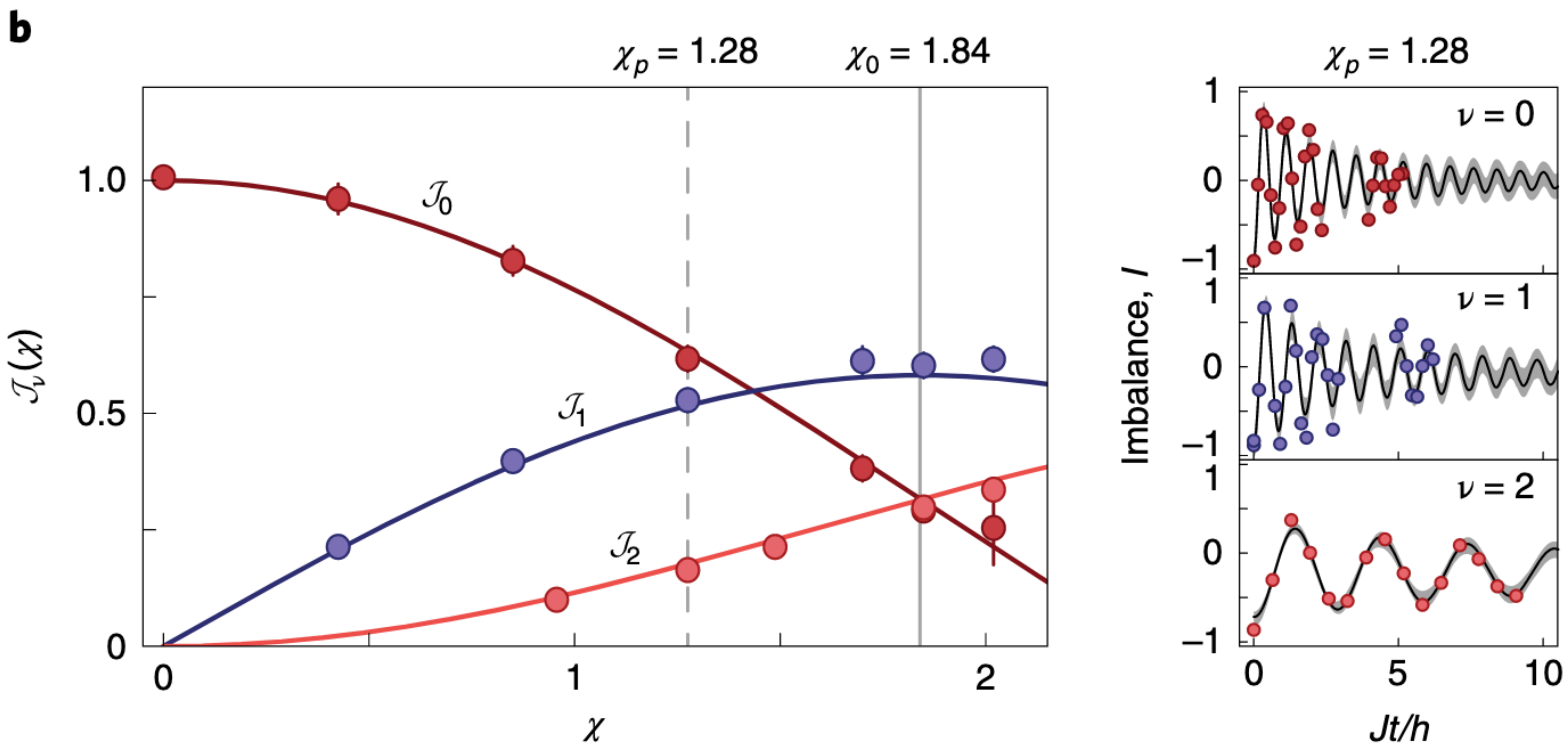


Interferometry Results



Examples for Floquet Hamiltonians

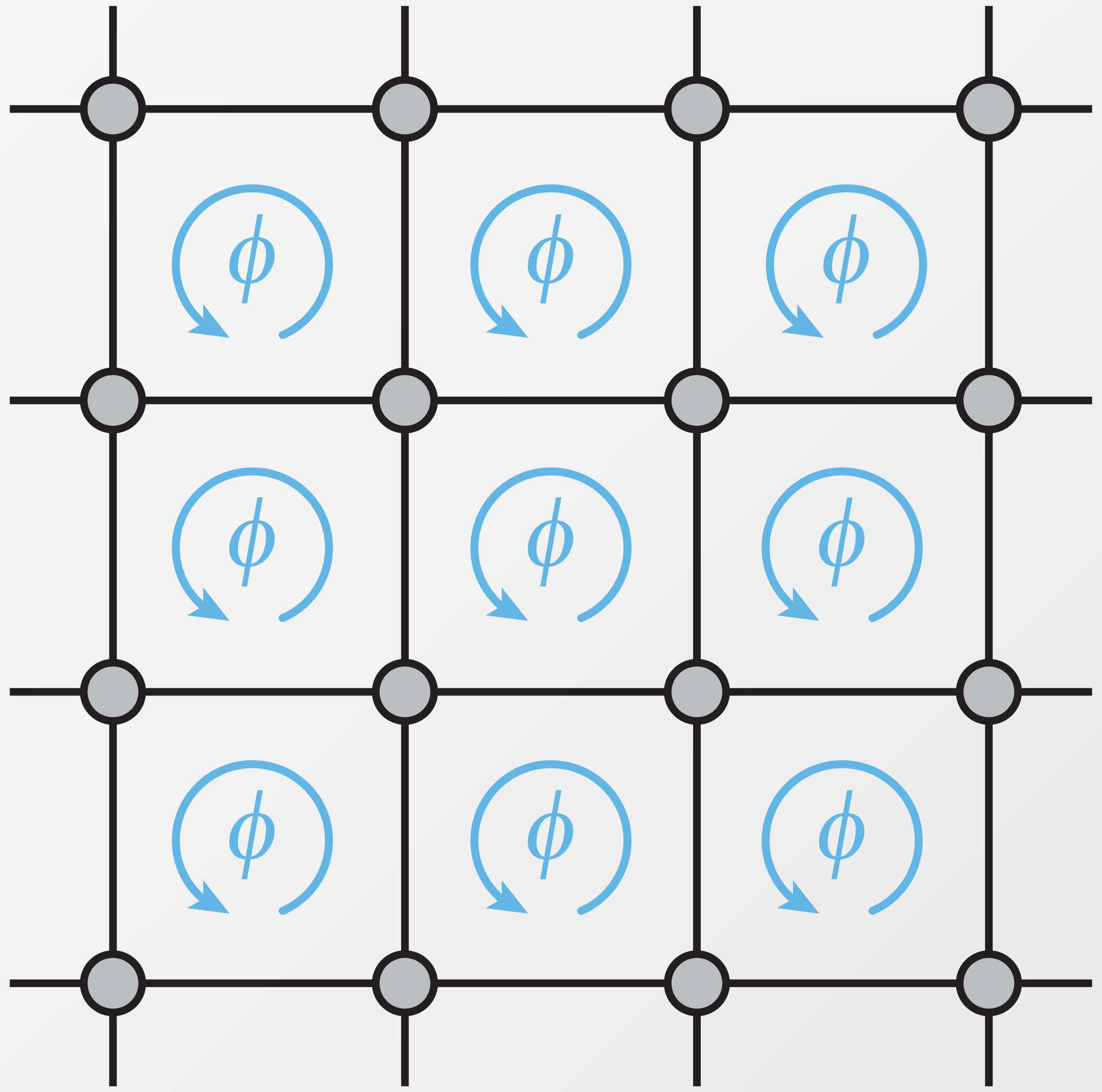
Double Well Tunneling

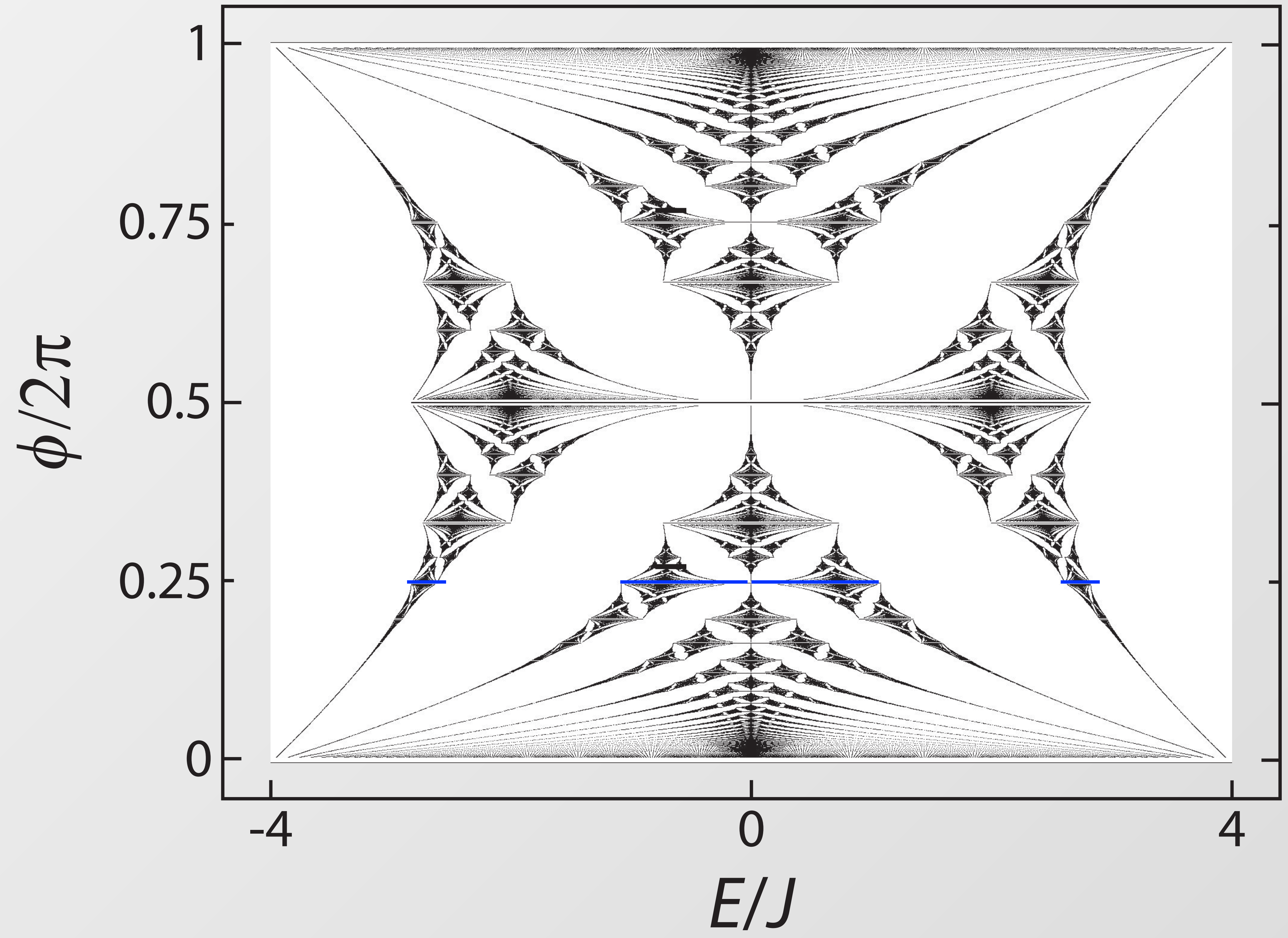


C. Schweizer *et al.* Floquet approach to ℤ2 lattice gauge theories with ultracold atoms in optical lattices. *Nat. Phys.* **15**, 1168–1173 (2019)

Harper Hamiltonian and Hofstadter Butterfly

Harper Hamiltonian: J=K and ϕ uniform.



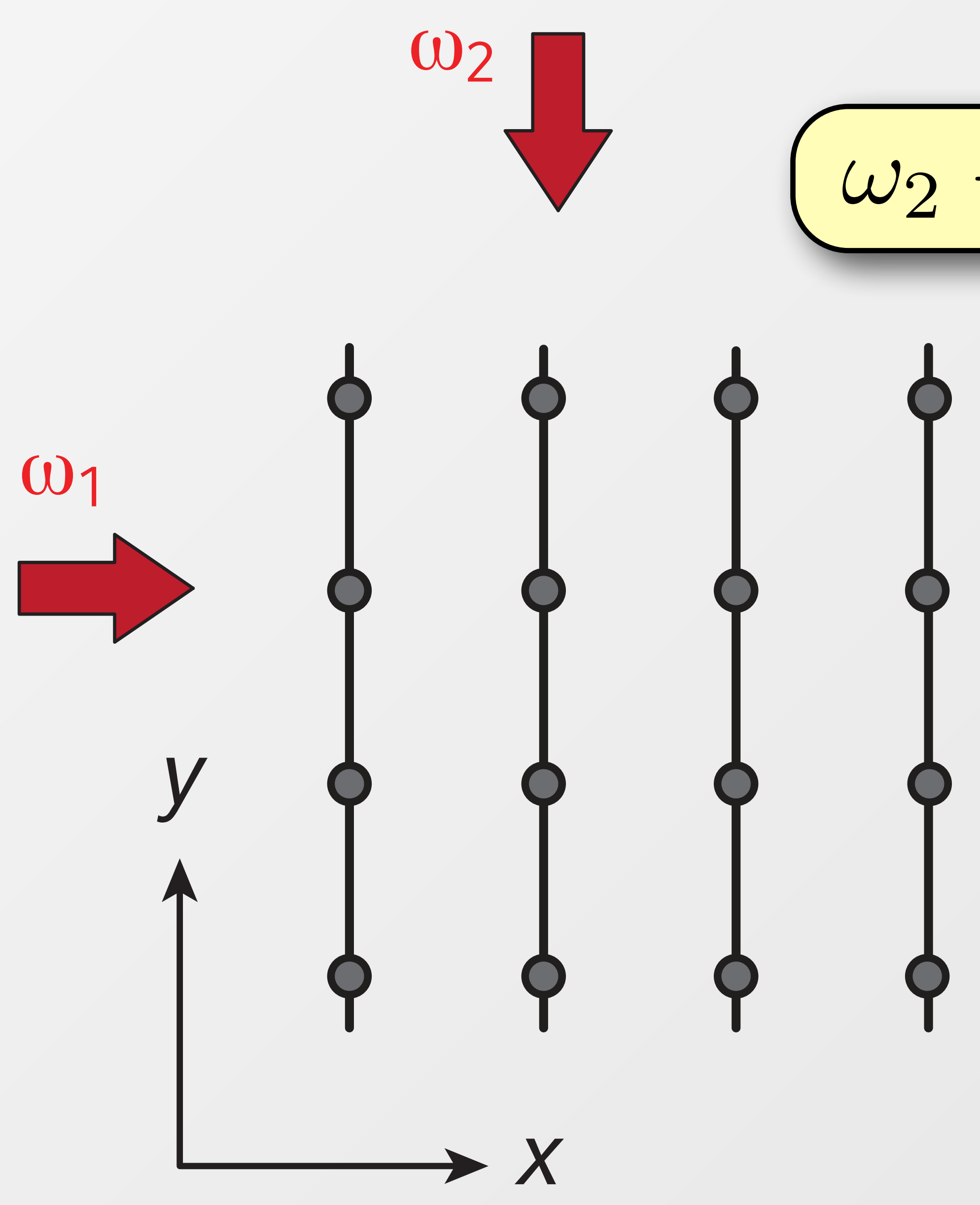


The lowest band is topologically equivalent to the lowest Landau level.

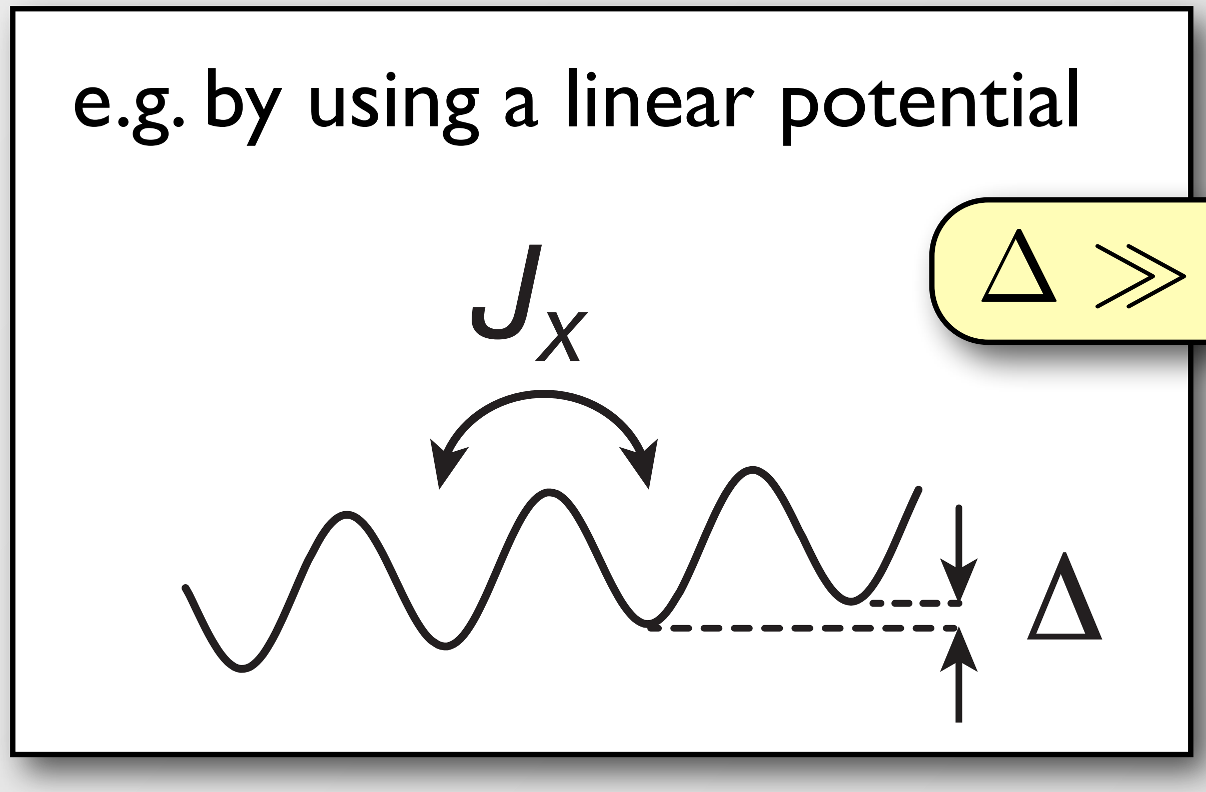
D.R. Hofstadter, Phys. Rev. B**14**, 2239 (1976)
see alo Y. Avron, D. Osadchy, R. Seiler, Physics Today 38, 2003

Experimental method

- Atoms in a 2D lattice
- Tunneling inhibited along one direction using energy offsets



$$\omega_2 - \omega_1 = \Delta/\hbar$$



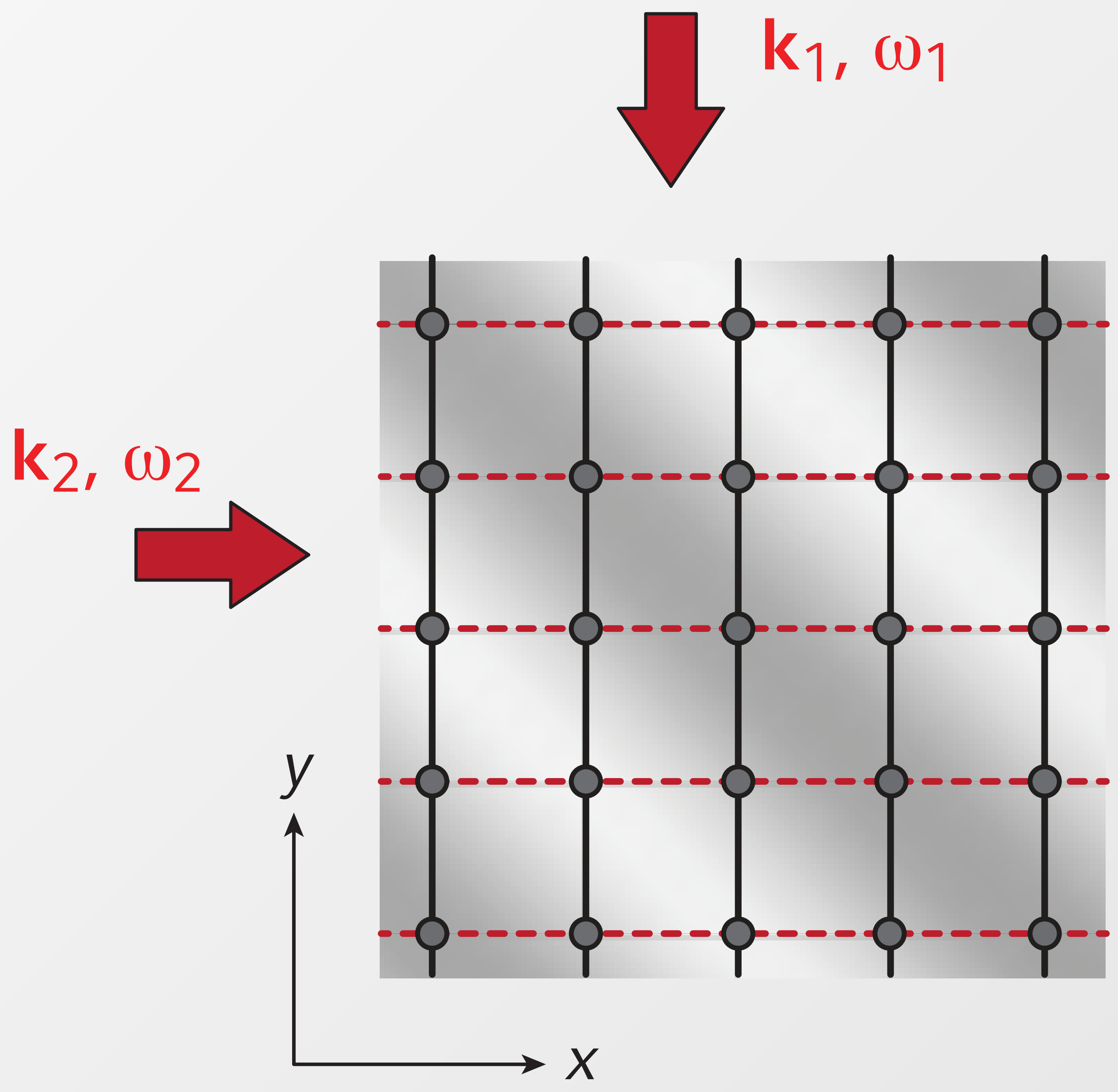
$$\Delta \gg J_x$$

- Induce resonant tunneling with a pair of ***far-detuned*** running-wave beams
  → ***Reduced heating*** due to spontaneous emission compared to Raman-assisted tunneling!
  → ***Independent*** of the internal structure of the atom

M. Aidelsburger *et al.*, PRL (2011); M. Aidelsburger *et al.*, Appl. Phys. B (2013)

Experimental method

- Interference creates a running-wave that ***modulates*** the lattice
- The ***phase of the modulation*** depends on the position in the lattice



Lattice modulation:

$$V_K^0 \cos(\omega t + \phi(\mathbf{r}))$$

with ***spatial-dependent*** phase

$$\phi(\mathbf{r}) = \delta\mathbf{k} \cdot \mathbf{r}$$

$$\delta\mathbf{k} = \mathbf{k}_2 - \mathbf{k}_1$$

$$\omega = \omega_2 - \omega_1$$

- Realization of ***time-dependent*** Hamiltonian, where tunneling is restored
- Discretization of the phase due to underlying lattice → $\phi_{m,n}$

M. Aidelsburger *et al.*, PRL (2011); M. Aidelsburger *et al.*, Appl. Phys. B (2013)

Experimental method

- Time-dependent Hamiltonian:

$$\hat{H}(t) = \sum_{m,n} \left(-J_x \; \hat{a}^\dagger_{m+1,n} \hat{a}_{m,n} - J_y \; \hat{a}^\dagger_{m,n+1} \hat{a}_{m,n} + \text{h.c.} \right)$$

$$+ \sum_{m,n} \left[m\Delta + V_K^0 \cos(\omega t + \phi_{m,n}) \right] \hat{n}_{m,n}$$

- Can be map
  for $\hbar\omega \gg$

Note: Corrections could be important!
see e.g. N. Goldman & J. Dalibard PRX 4, 031027 (2014)
& related work M. Bukov, L. D'Alessio & A. Polkovnikov arXiv:1407.4803

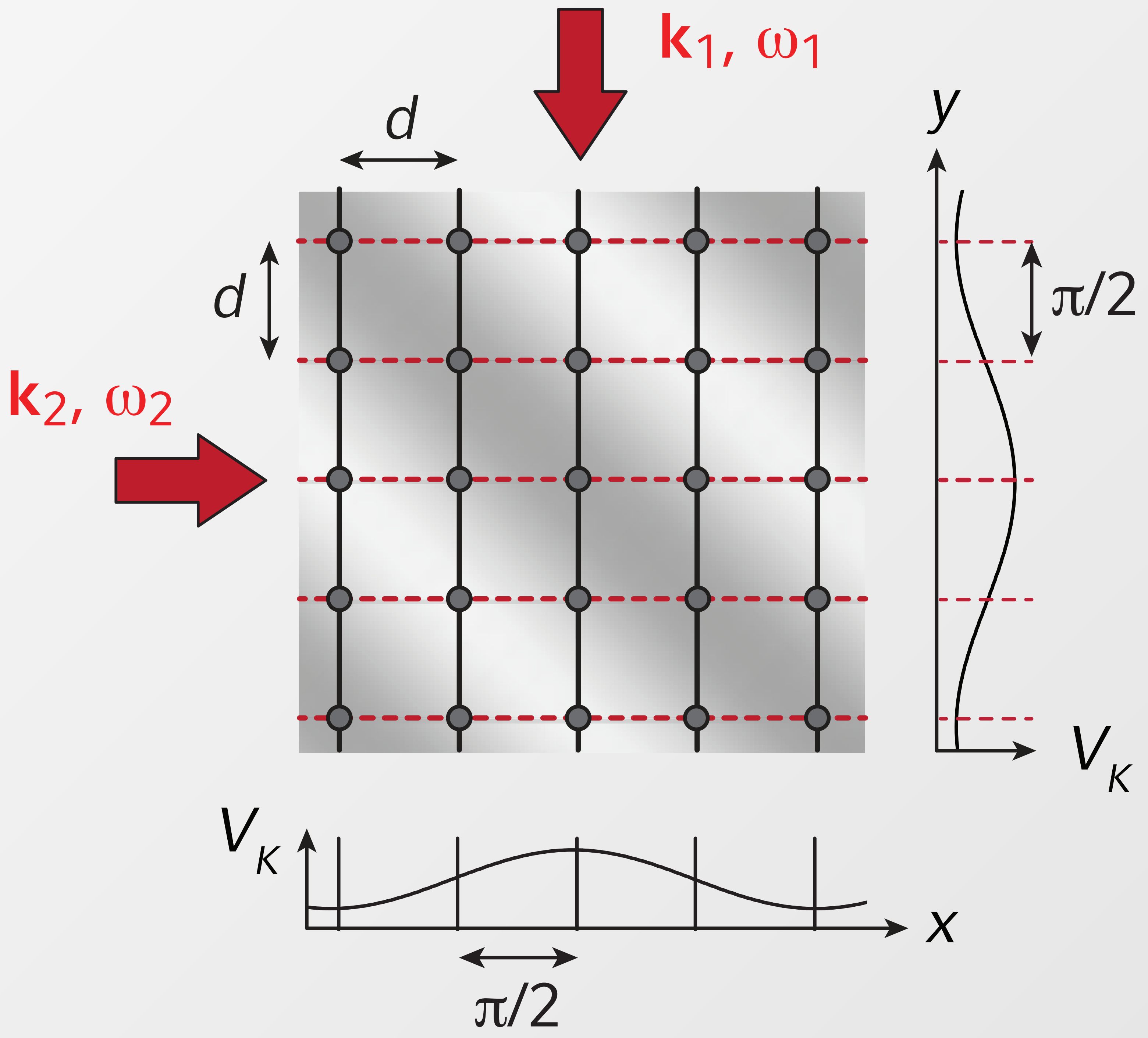
$$\hat{H}_{eff} = \sum_{m,n} \left(-K e^{i\phi_{m,n}} \; \hat{a}^\dagger_{m+1,n} \hat{a}_{m,n} - J \; \hat{a}^\dagger_{m,n+1} \hat{a}_{m,n} + \text{h.c.} \right)$$

- To avoid excitations to higher bands
  $\hbar\omega$ has to be smaller than the band gap

F. Grossmann and P. Hänggi, EPL (1992)
M. Holthaus, PRL (1992)
A. Kolovsky, EPL (2011); A. Eckardt, PRL (2005)
A. Eckardt, EPL (2007); P. Hauke, PRL (2012)
A. Bermudez, PRL (2011); A. Bermudez, NJP (2012)

Experimental setup

Experimental parameters:



$$|\mathbf{k}_1| \simeq |\mathbf{k}_2| = \frac{\pi}{2d}$$

$$\Rightarrow \phi_{m,n} = \frac{\pi}{2}(m+n)$$

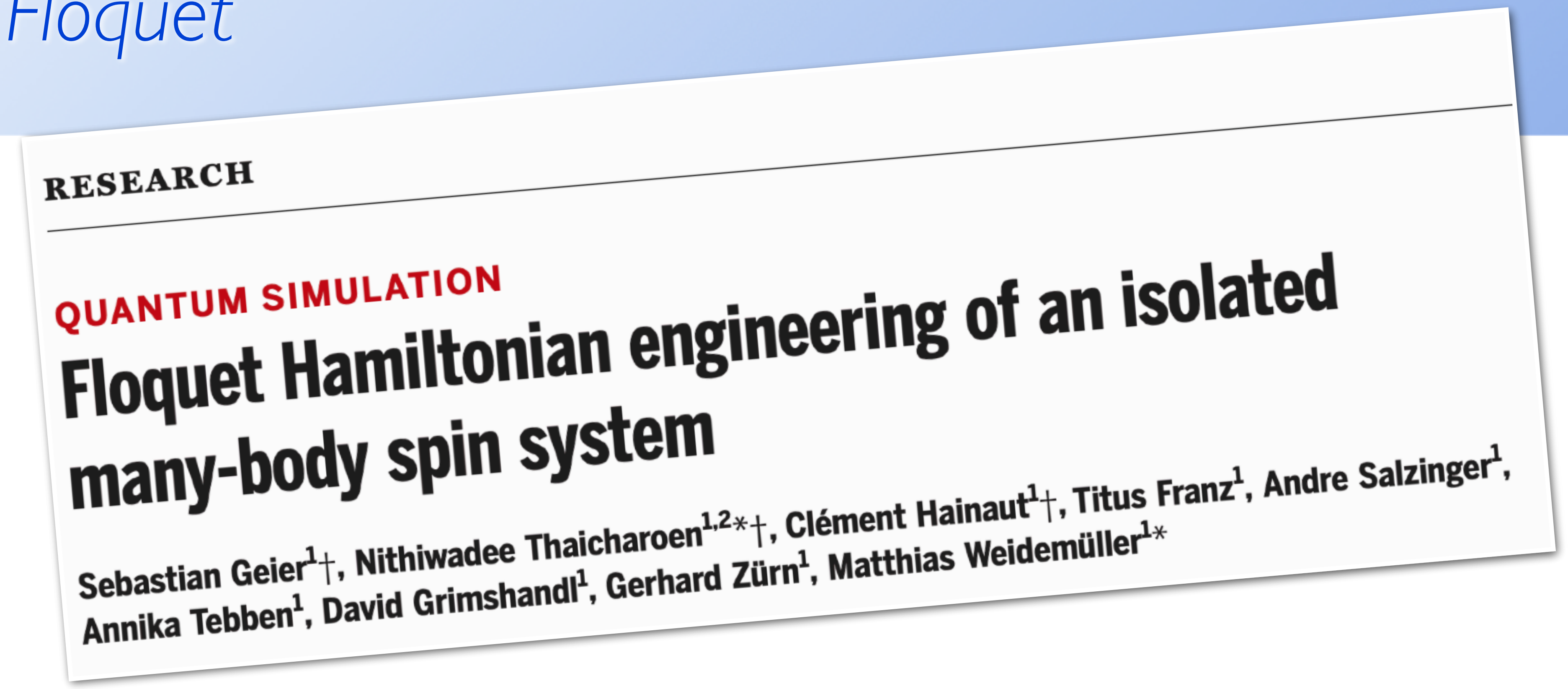
Flux through one unit cell:

$$\Phi = \phi_{m,n+1} - \phi_{m,n} = \frac{\pi}{2}$$

depends only on phase difference along y!

The value of the flux is ***fully tunable*** by changing the geometry of the driving-beams!

From XX to XYZ Spin Models

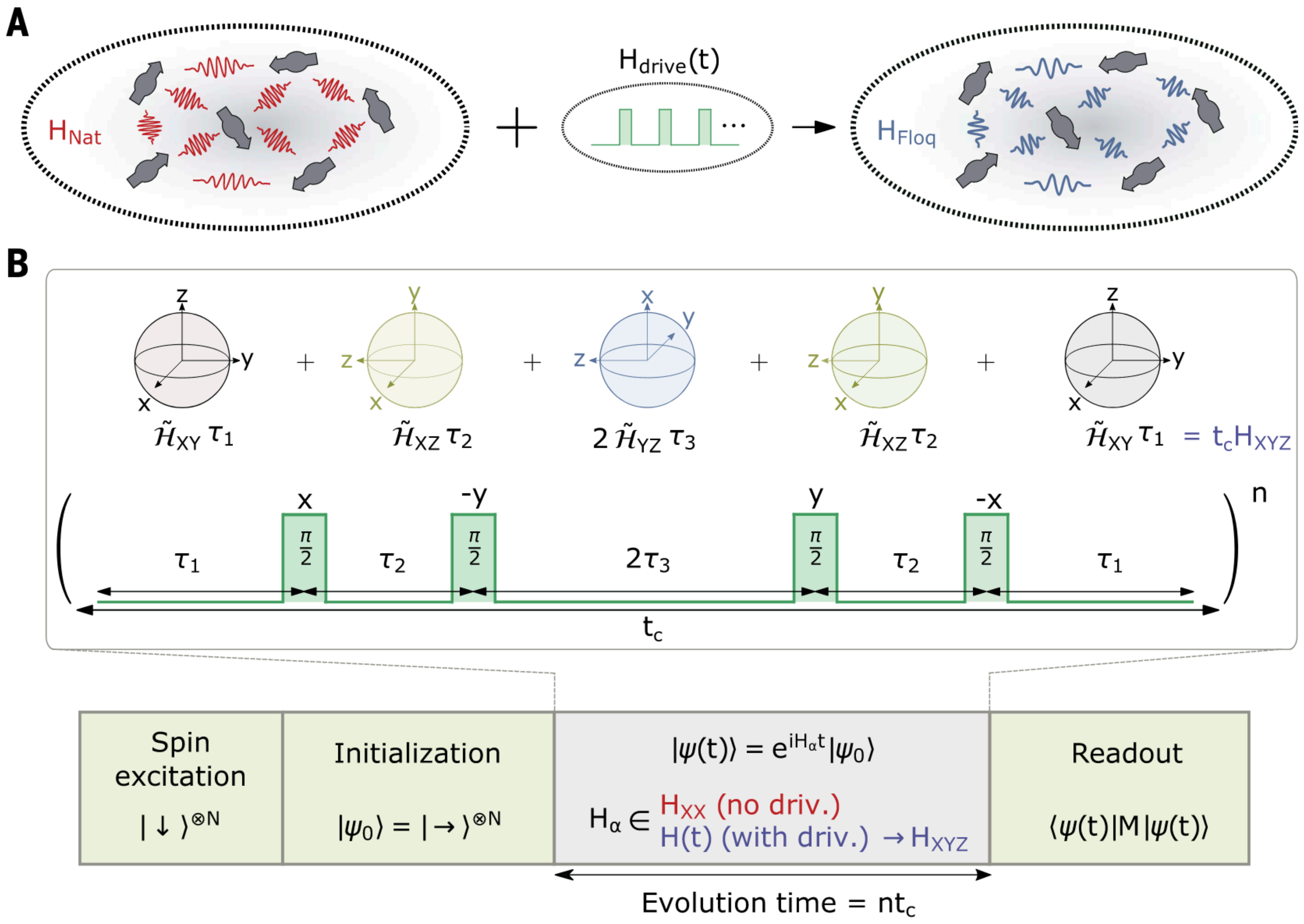

RESEARCH

QUANTUM SIMULATION

Floquet Hamiltonian engineering of an isolated many-body spin system

Sebastian Geier[1]†, Nithiwadee Thaicharoen[1,2]*†, Clément Hainaut[1]†, Titus Franz[1], Andre Salzinger[1], Annika Tebben[1], David Grimshandl[1], Gerhard Zürn[1], Matthias Weidemüller[1]*

S. Geier. *et al.* *Science* **374**, 1149 (2021)



$$H(t) = H_{XX} + H_{\text{drive}}(t)$$

$$H_{XX} = \sum_{i,j} J_{ij}/\hbar\left(S_x^i S_x^j + S_y^i S_y^j\right) \text{ and}$$

$$H_{\text{drive}}(t) = \sum_i \Omega(t)\left[\cos\varphi(t) S_x^i + \sin\varphi(t) S_y^i\right],$$

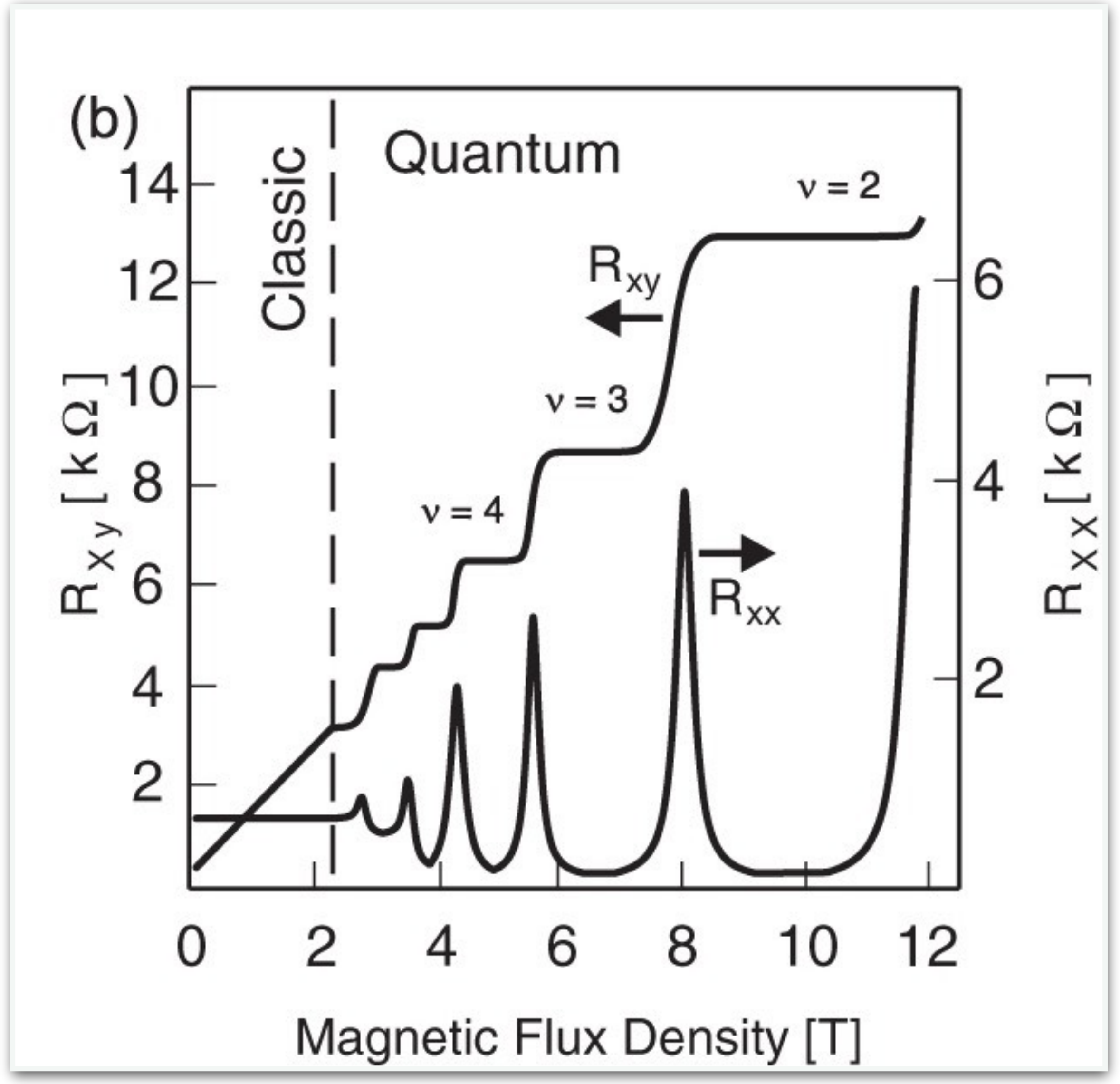
$$H_{\text{Floq}} = \frac{1}{t_c}\sum_{i=1}^{5} \tilde{\mathcal{H}}_i \tau_i$$

$$H_{XYZ} = \frac{2}{3}\sum_{i,j} J_{ij}/\hbar\left(\delta_x S_x^i S_x^j + \delta_y S_y^i S_y^j + \delta_z S_z^i S_z^j\right)$$

Anomalous Topological Floquet System

Integer quantum Hall effect

- Remarkably *stable plateaus* independent of microscopic details
- Robustness rooted in *top. properties* of the energy bands
  TKNN, Phys. Rev. Lett. (1982)

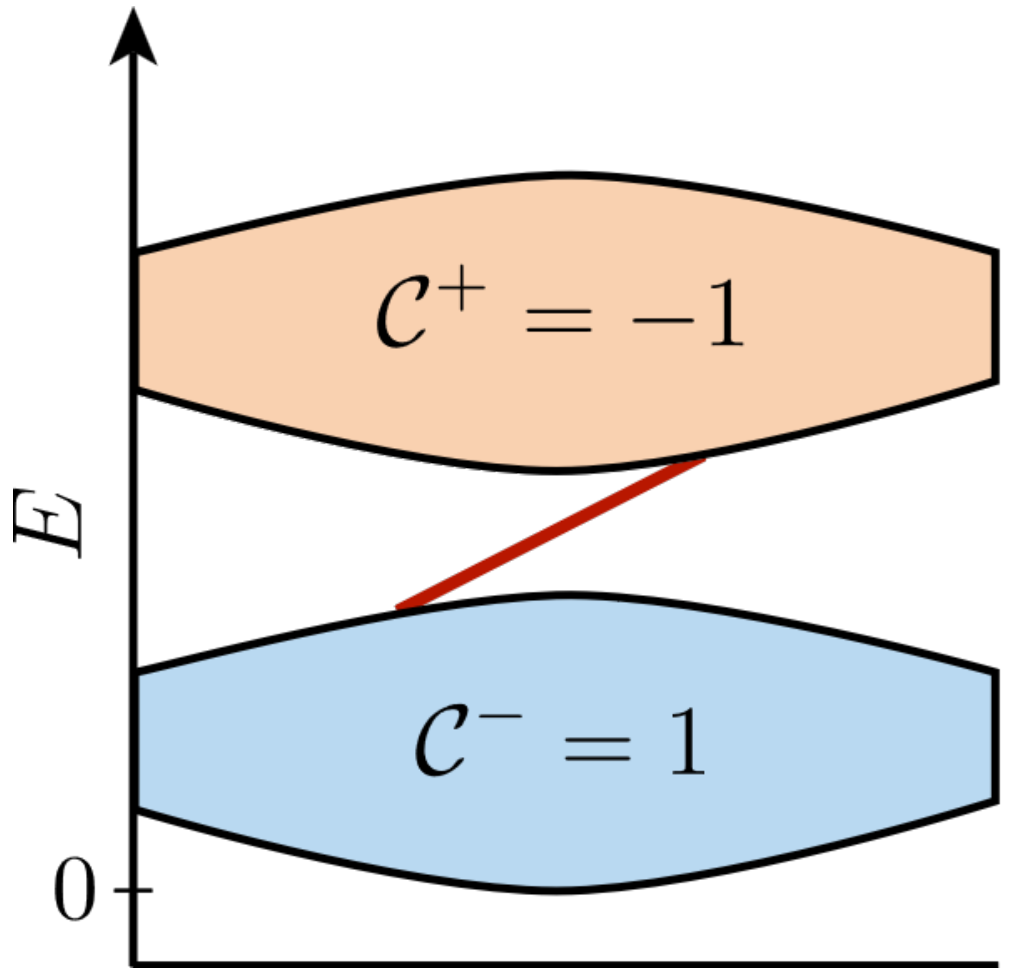


$$R_{xy} = \frac{1}{i}\frac{h}{e^2}, \qquad i = (1, 2, 3, \ldots)$$

$$i = \sum_{\text{bands}} \nu_{\text{ch}}$$

- *Chern number* uniquely determines *# net edge modes in the gap*

→ known *bulk-edge correspondence*



Chern number = difference between the number of chiral edge modes *above* and *below* the band.

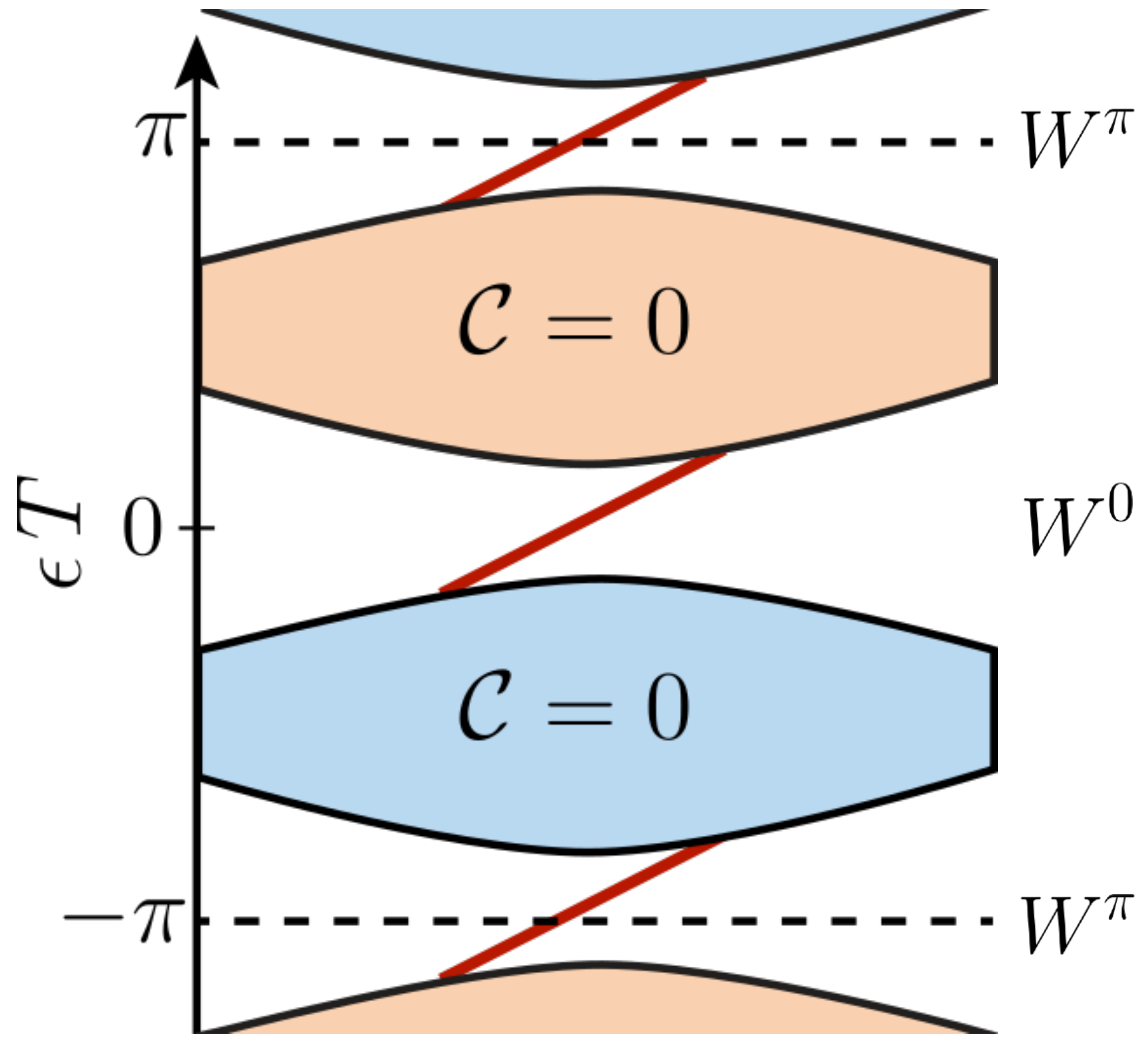
Periodically driven systems

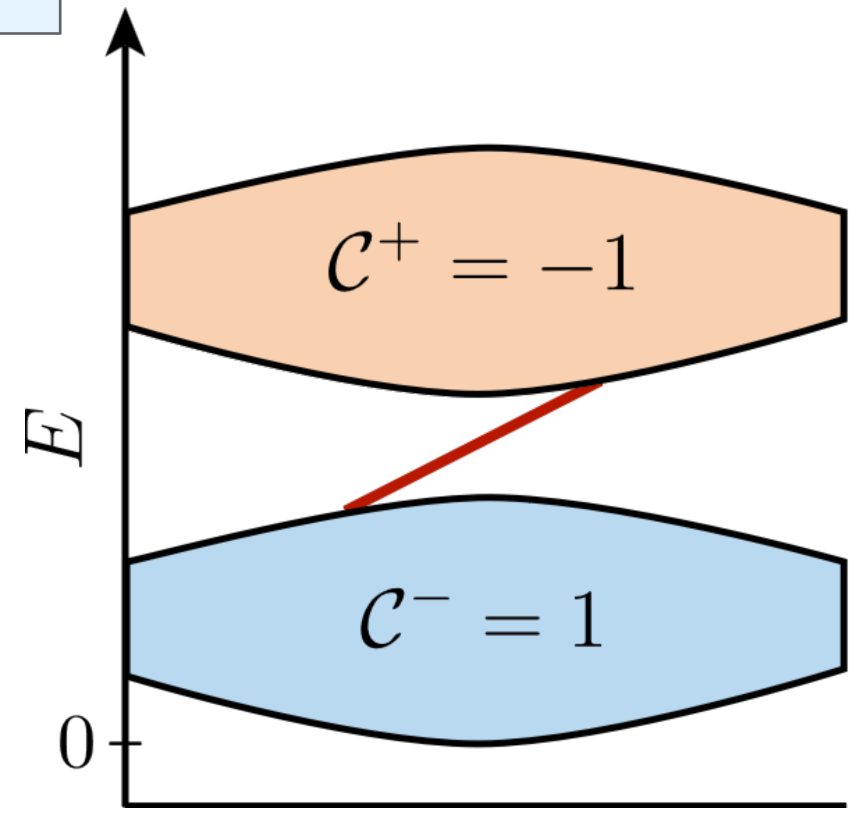
- Periodically driven system: *periodic quasienergies*
- Anomalous Floquet phases: **All Chern numbers are zero** but chiral edge modes exist

→ new set of top. invariants: *Winding numbers*

$$\mathcal{C}^{\pm} = \mp(W^0 - W^\pi)$$

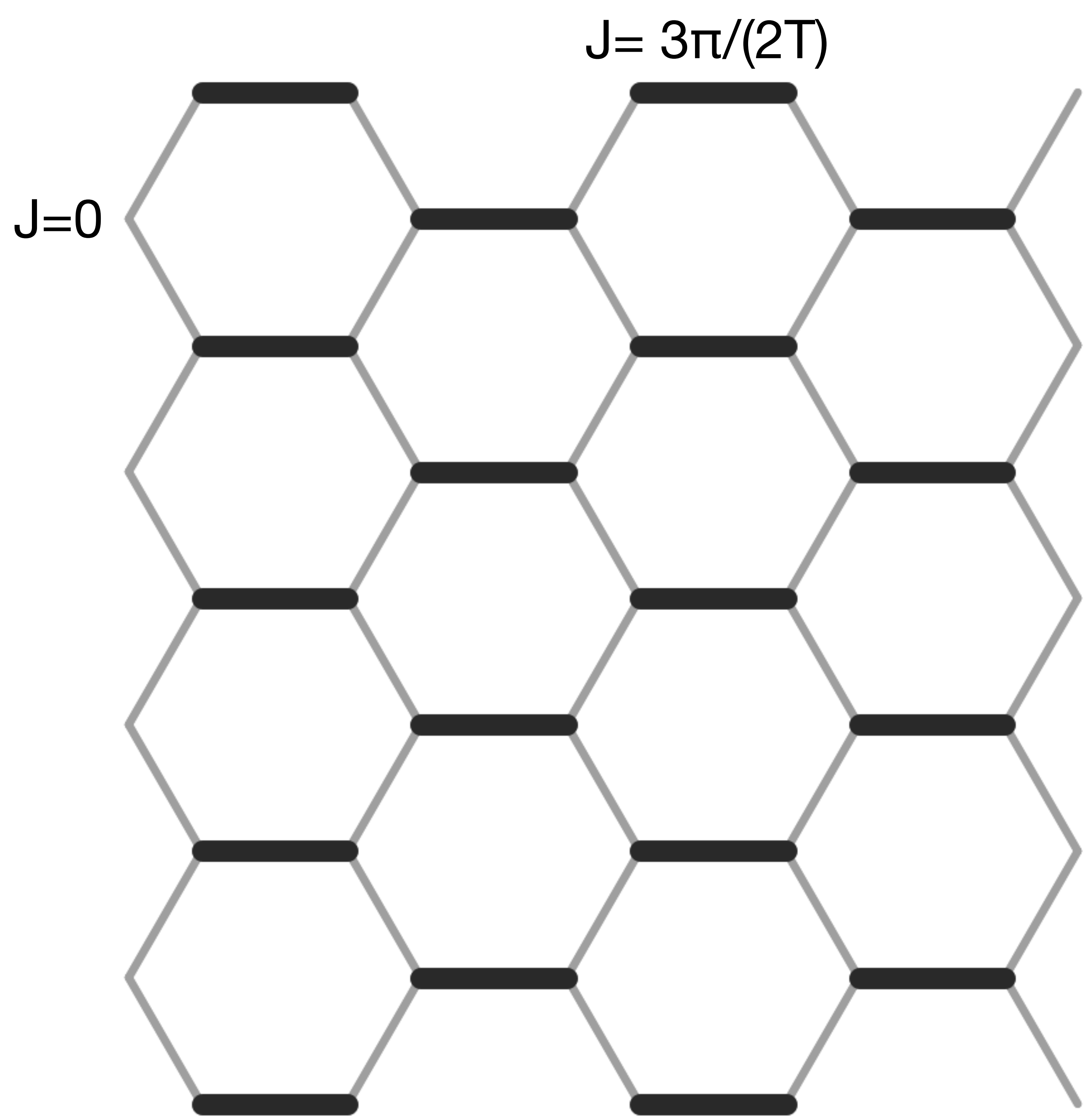
- Static system: *bounded energies*



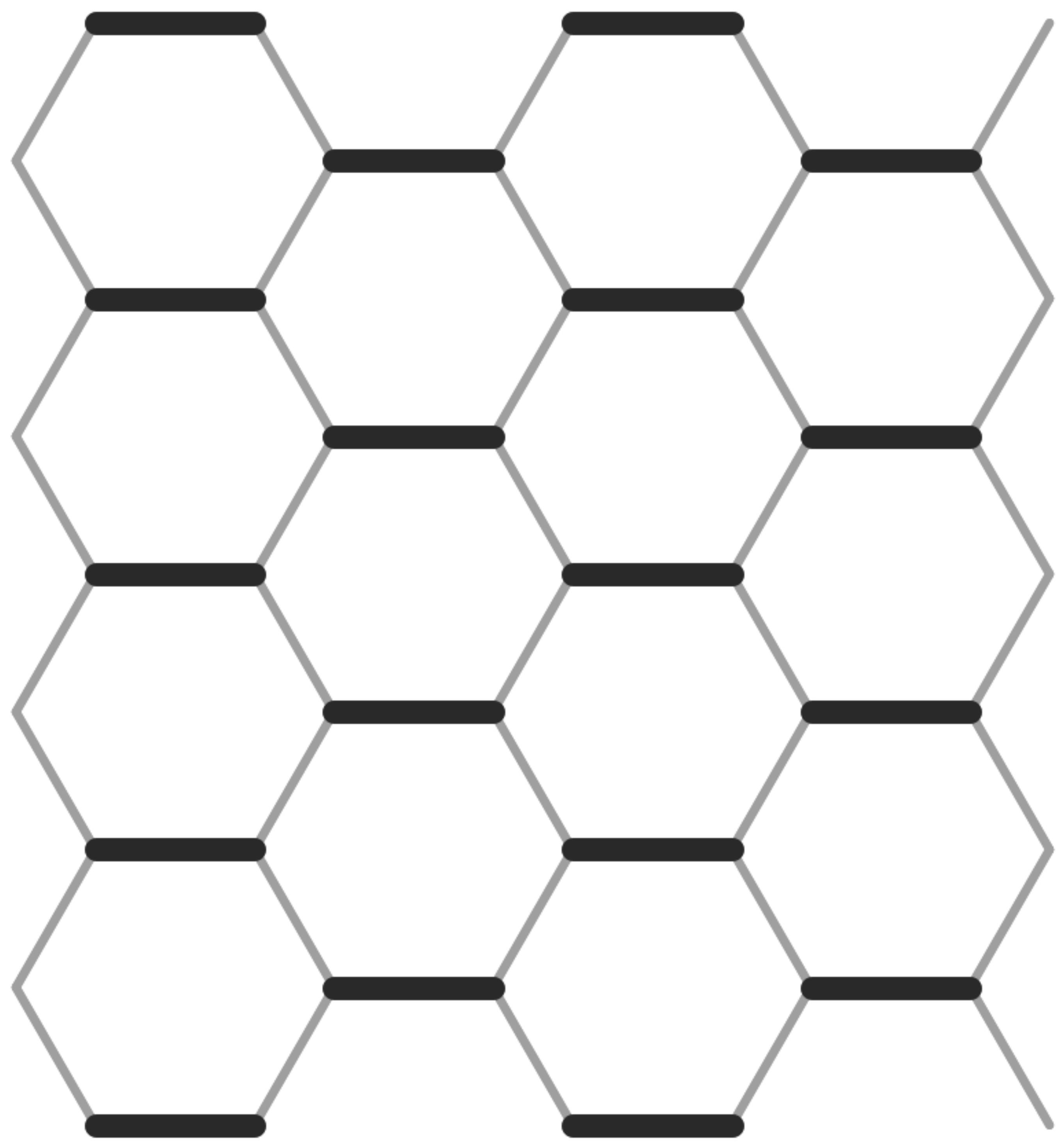


T. Kitagawa *et al.*, Phys. Rev. B **82**, 235114 (2010)
M. Rudner *et al.*, PRX **3**, 031005 (2013)

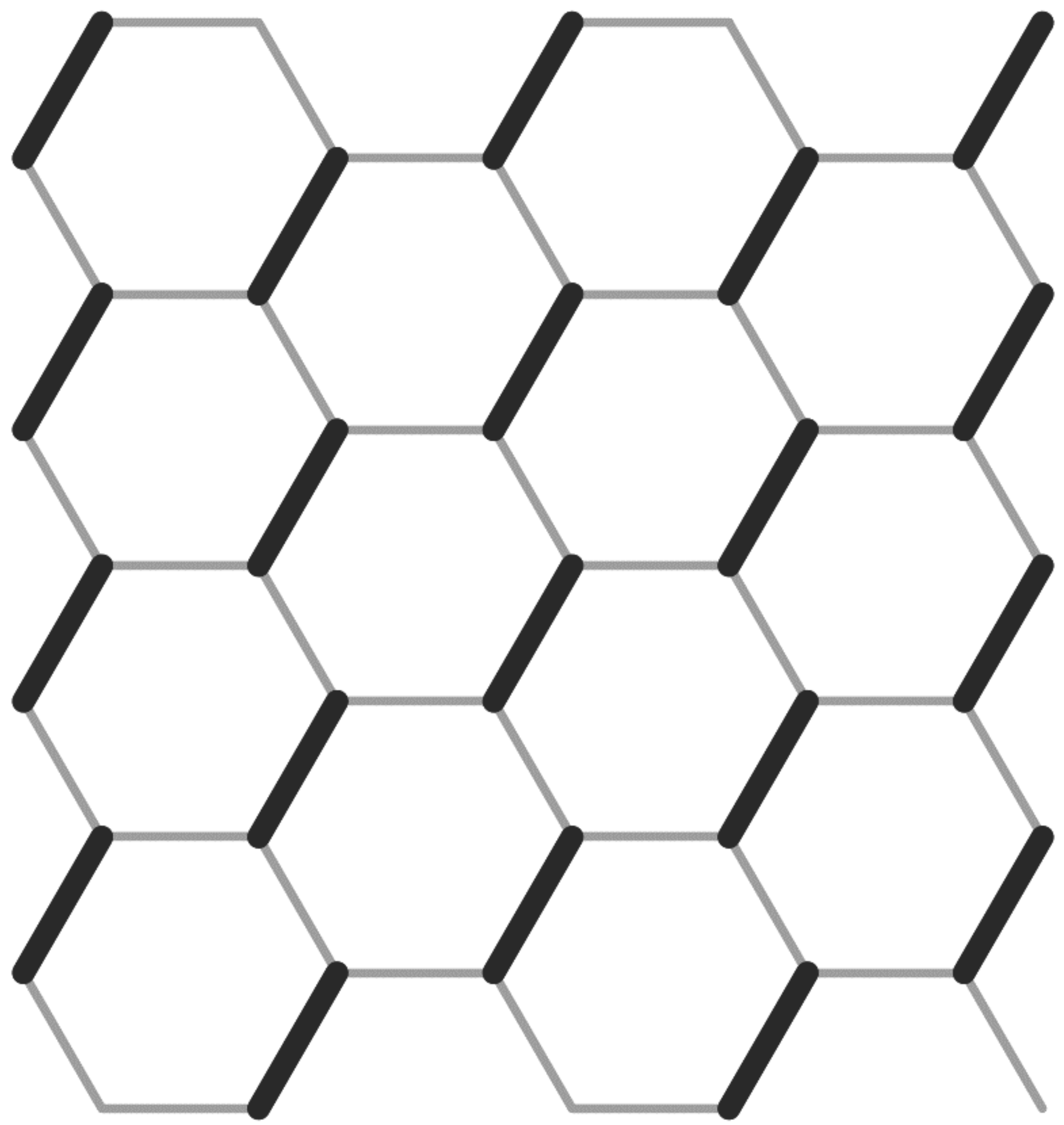
A toy model

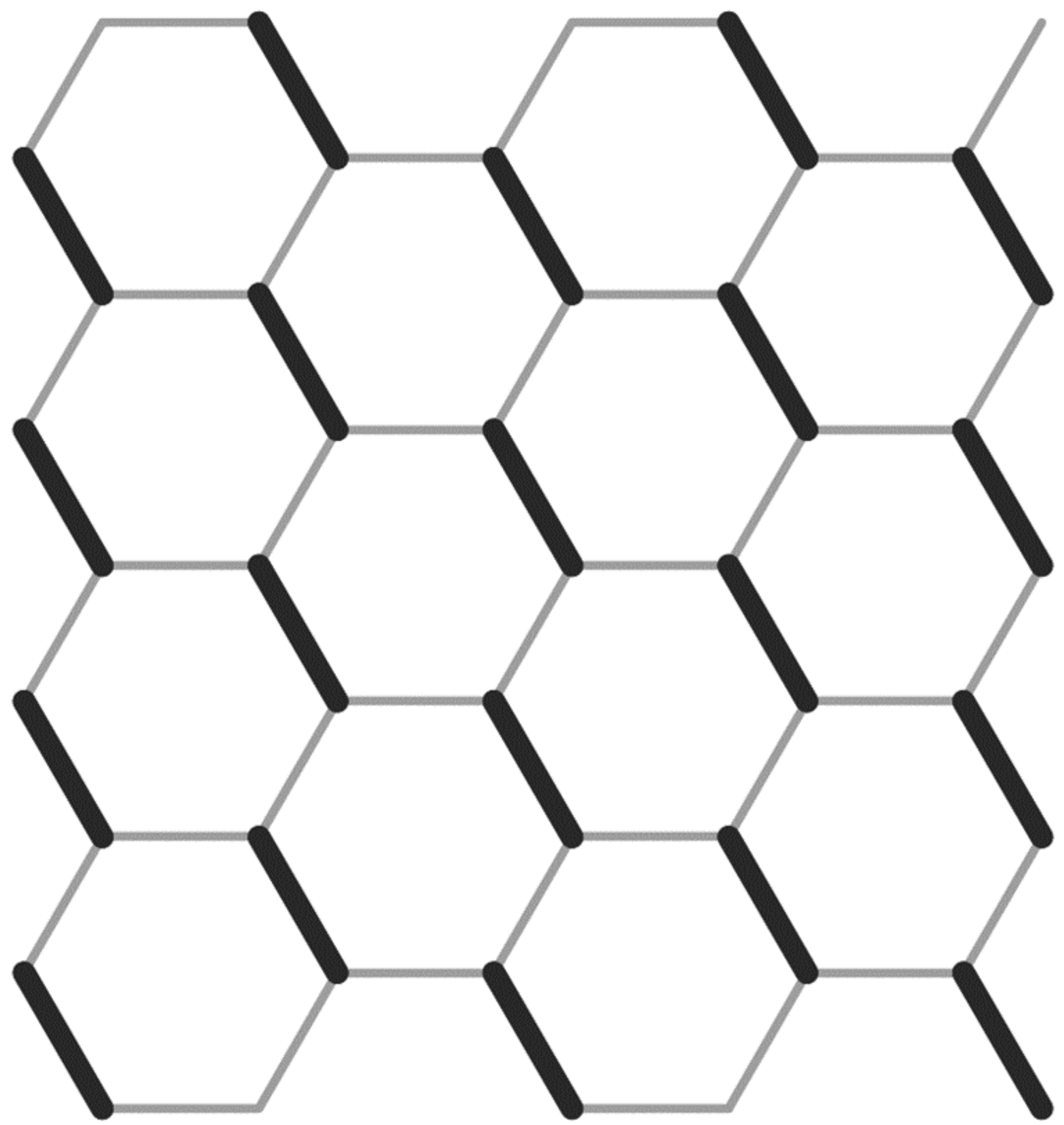


A toy model

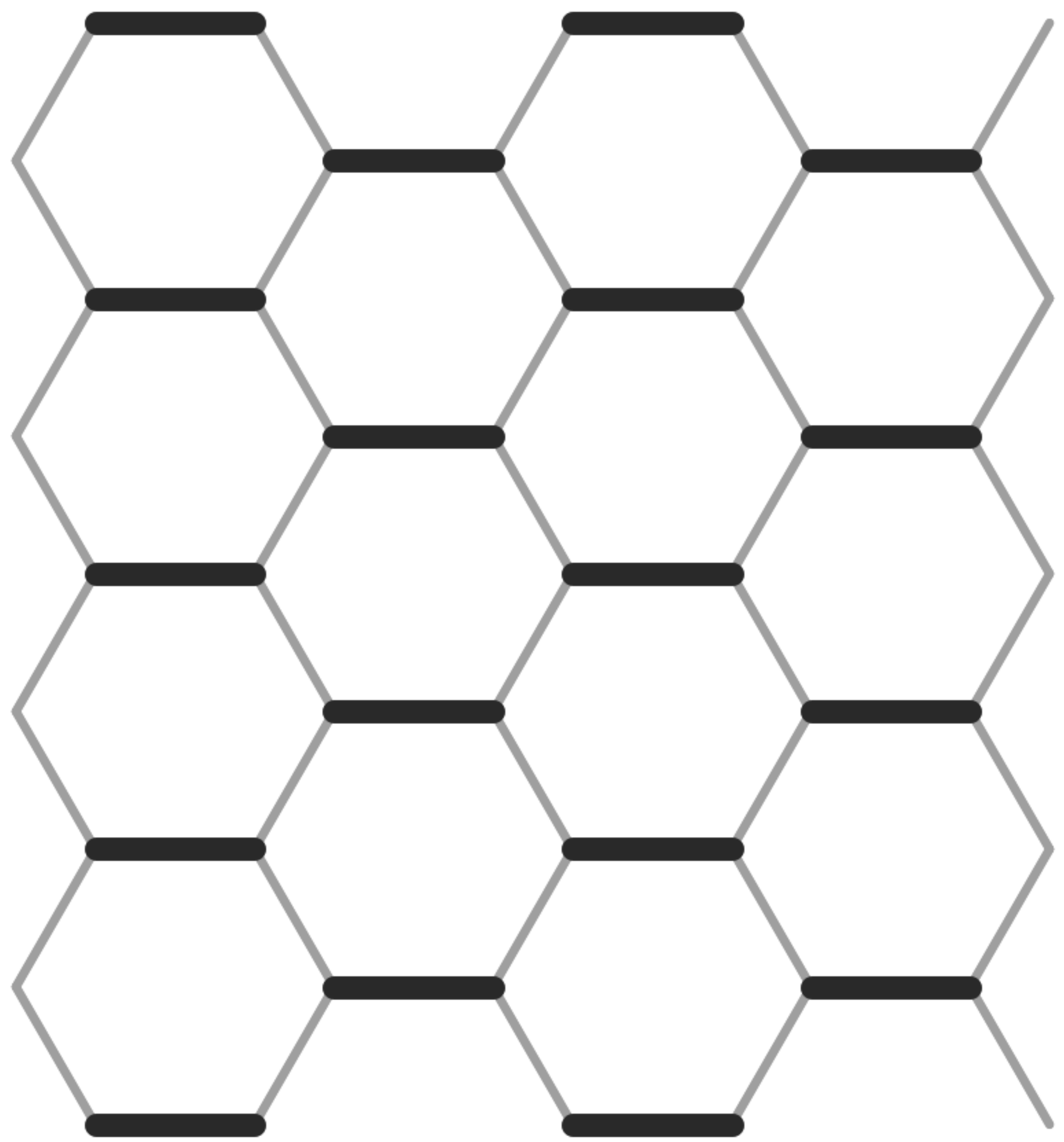


A toy model

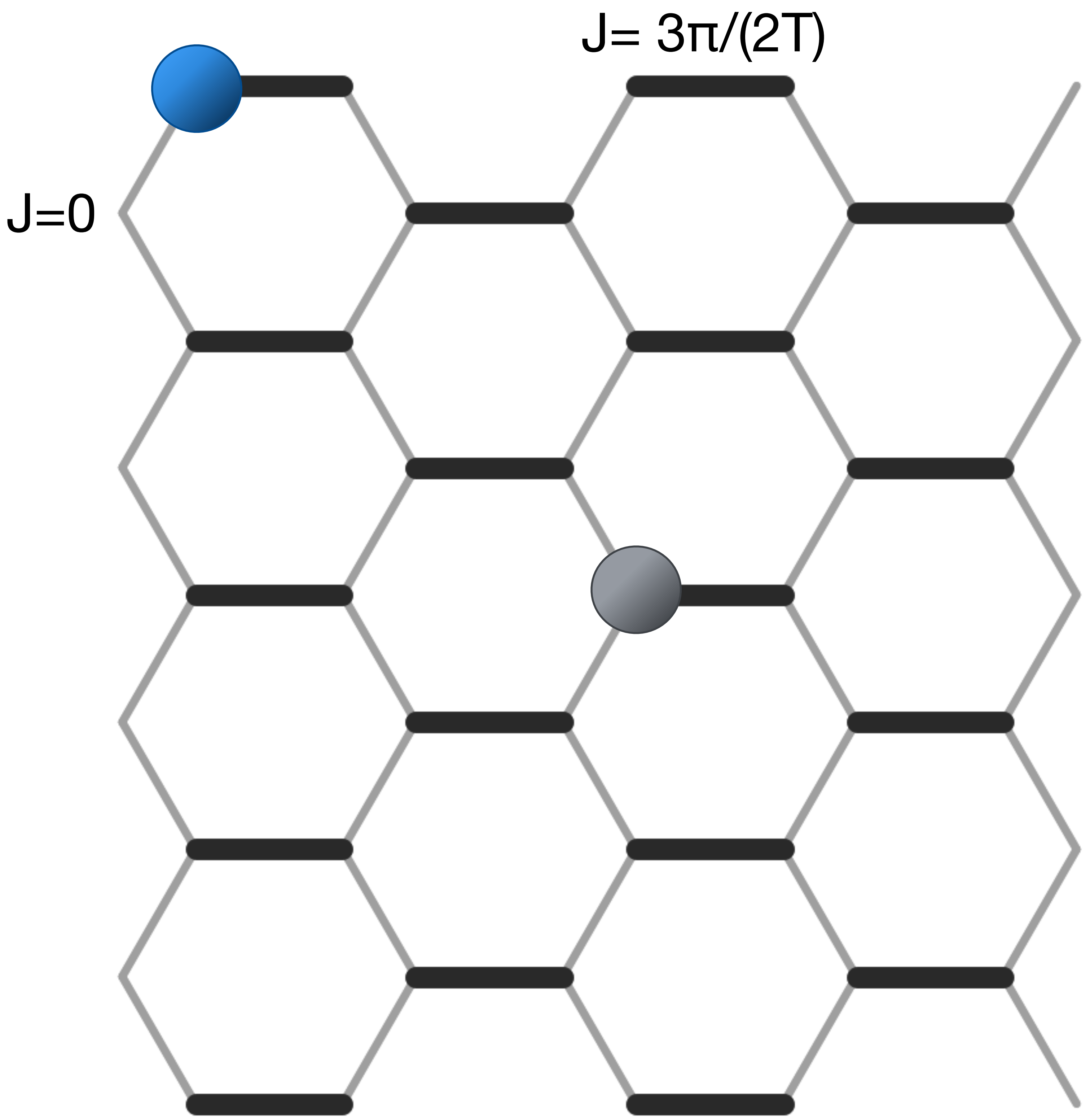




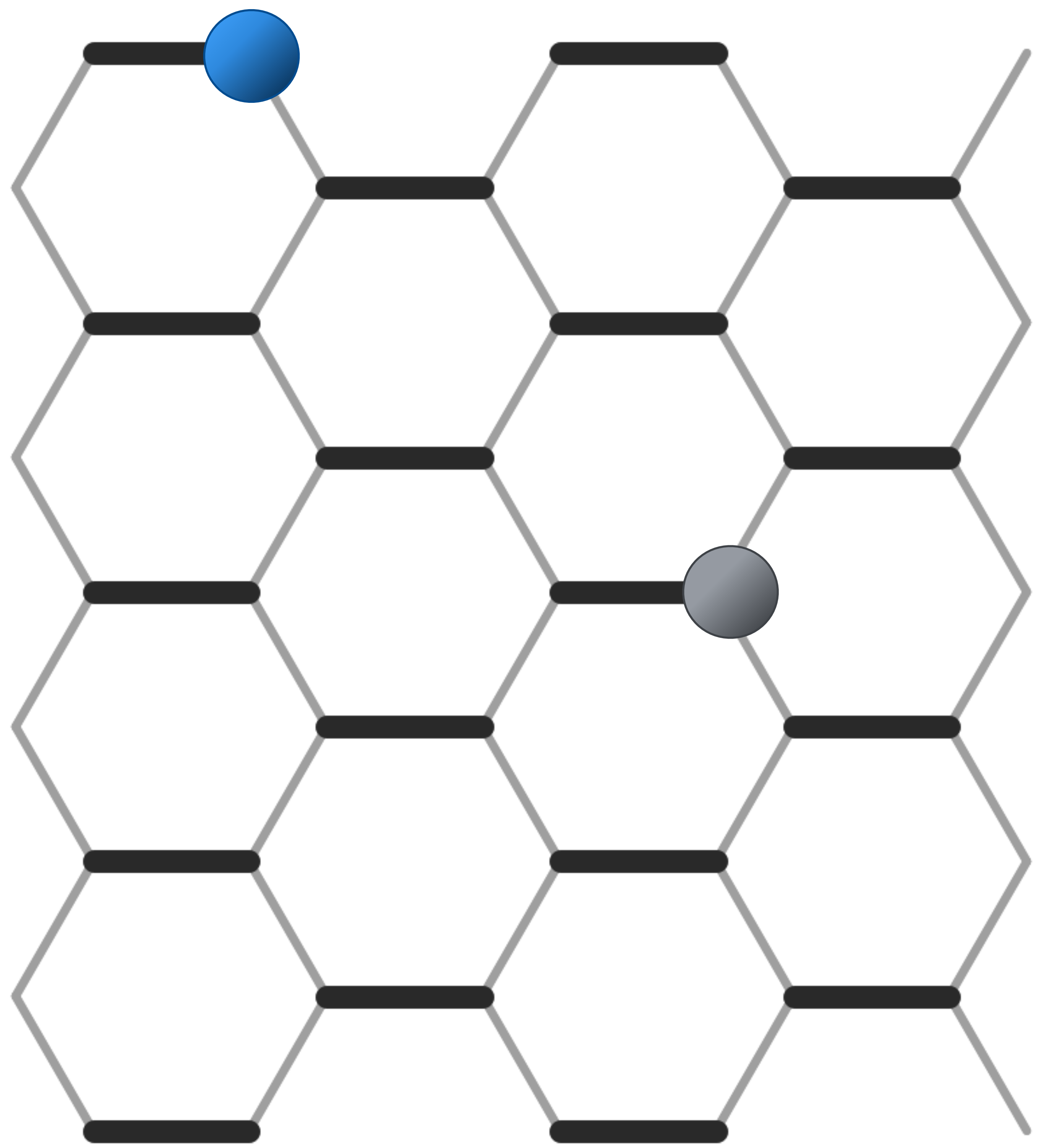
A toy model



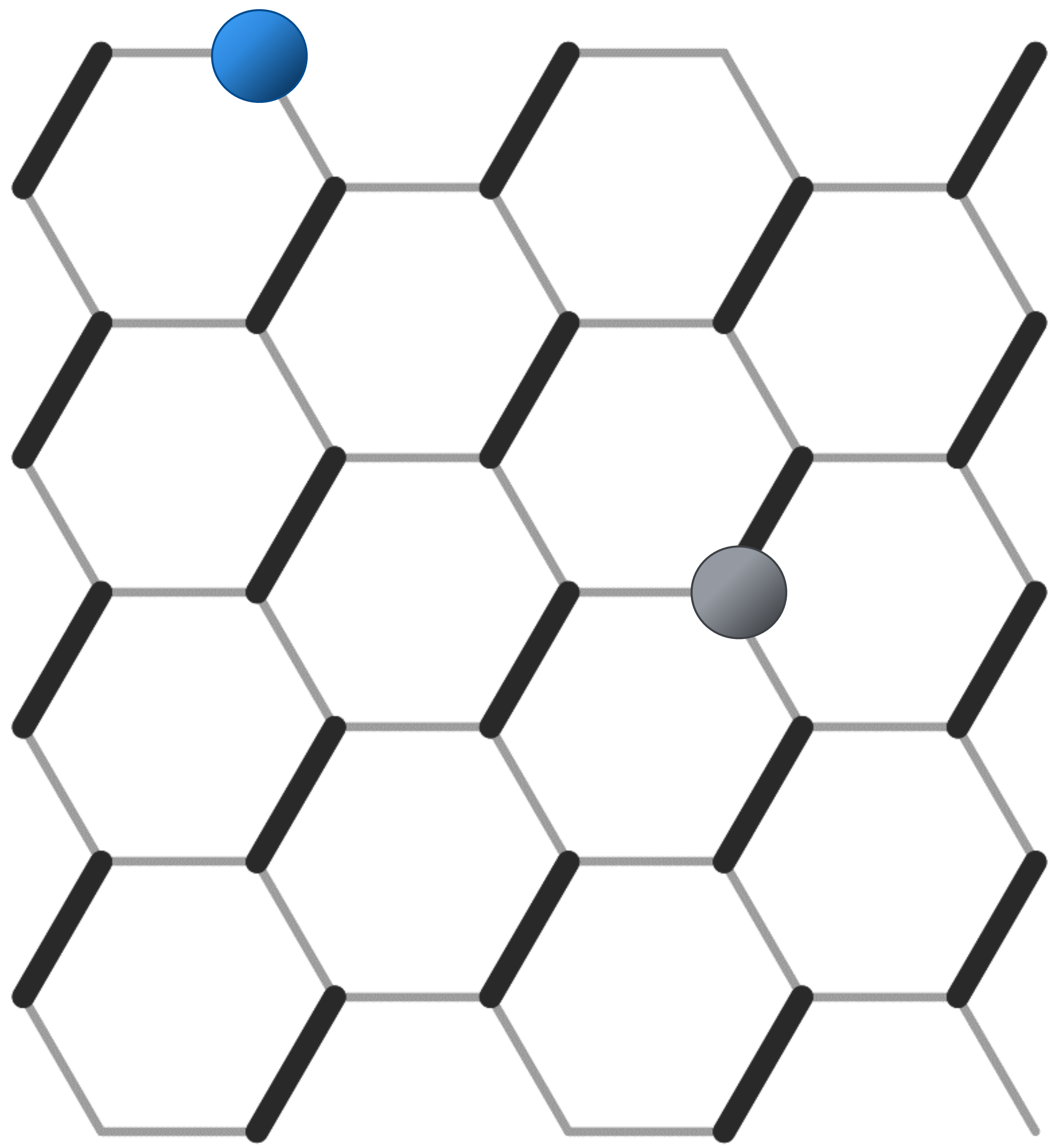
A toy model



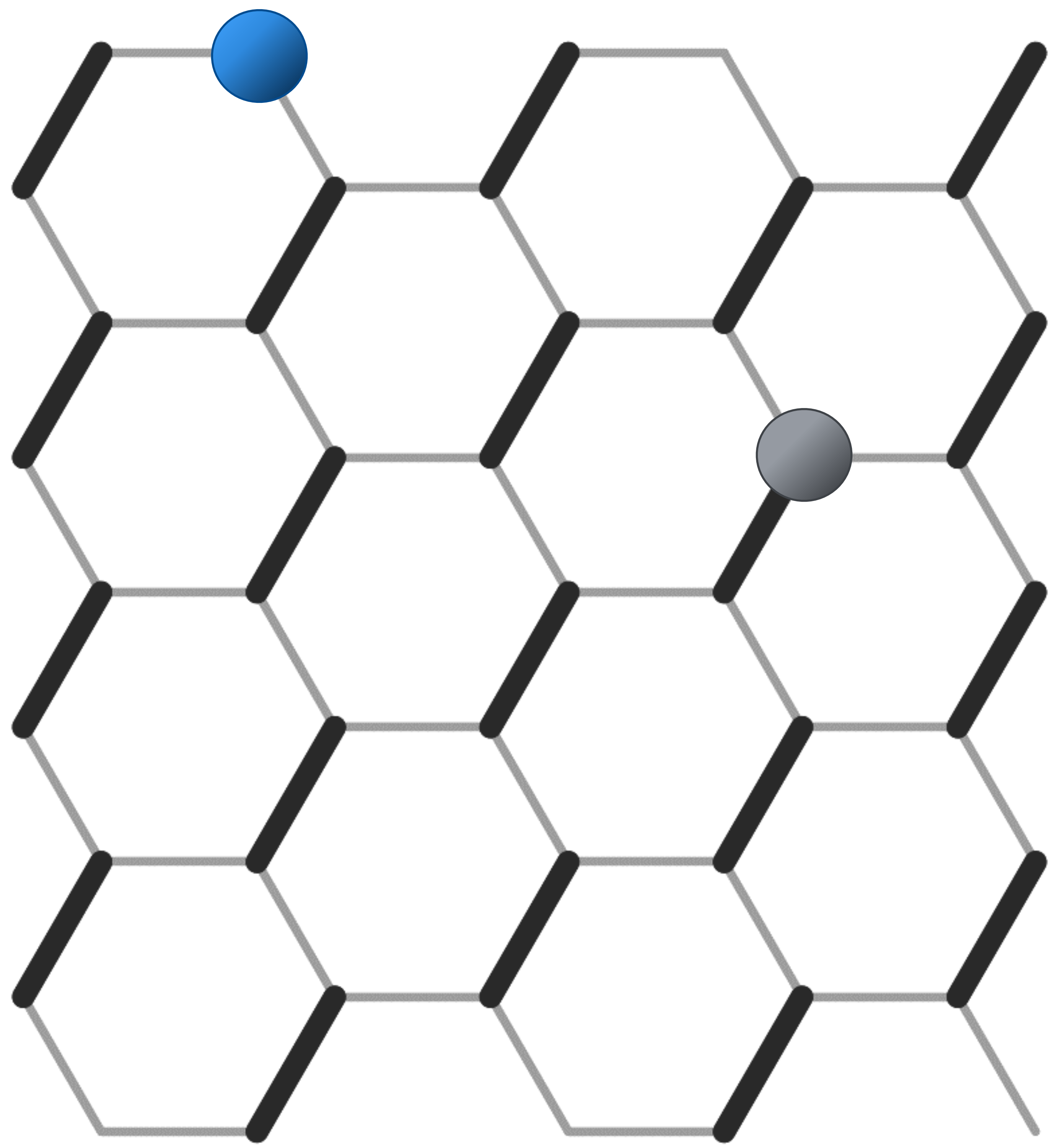
A toy model



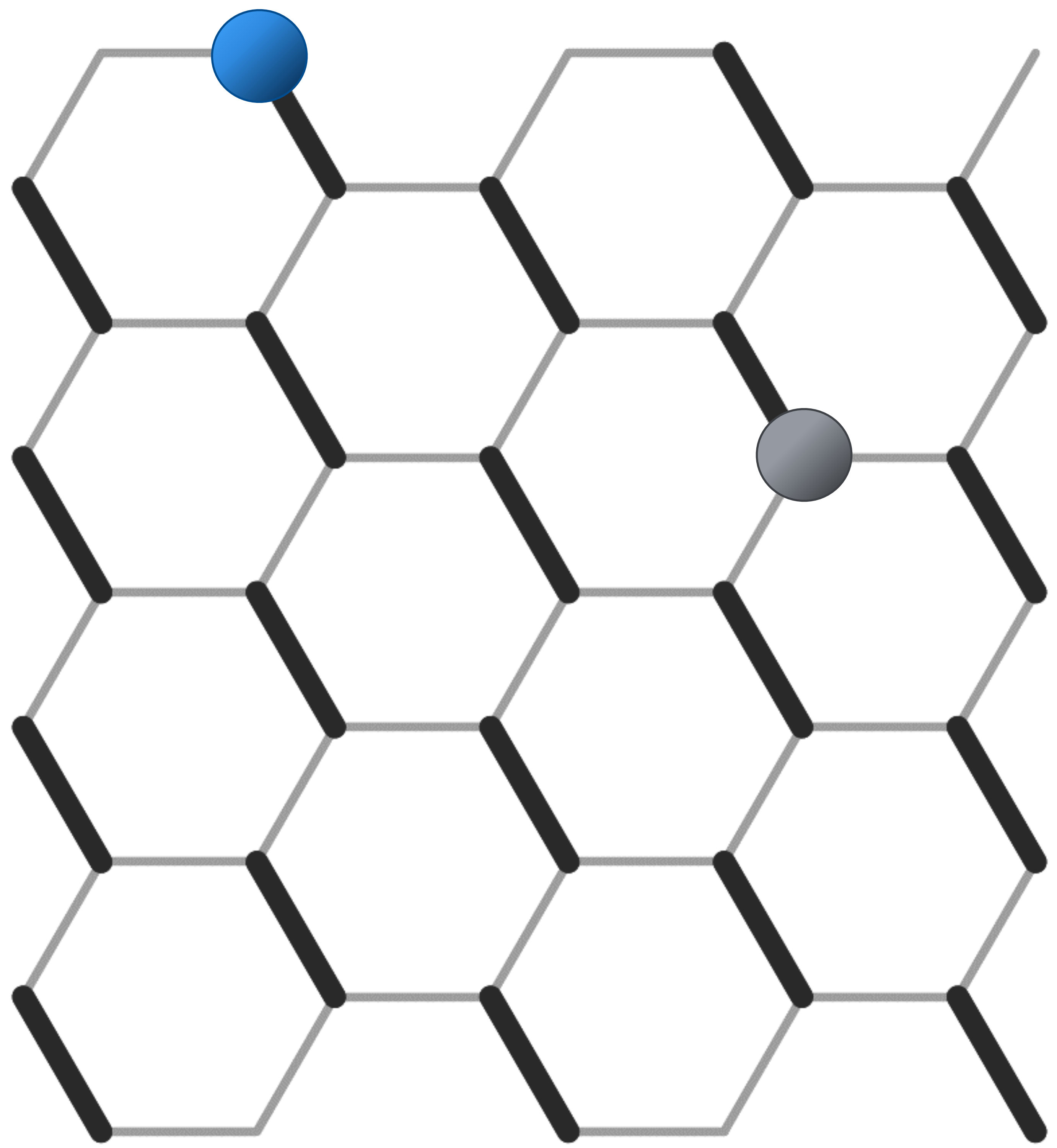
A toy model

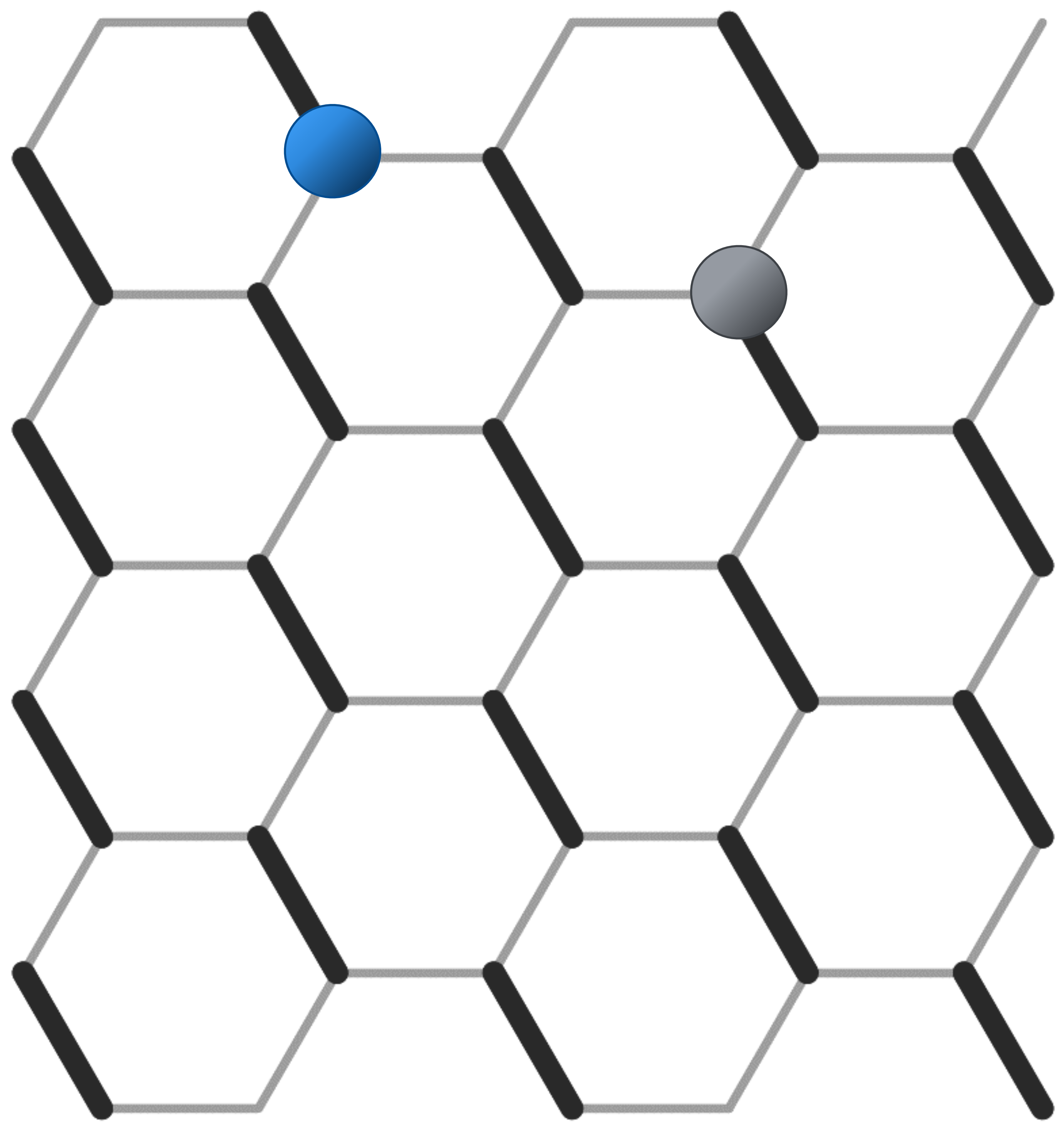


A toy model



A toy model





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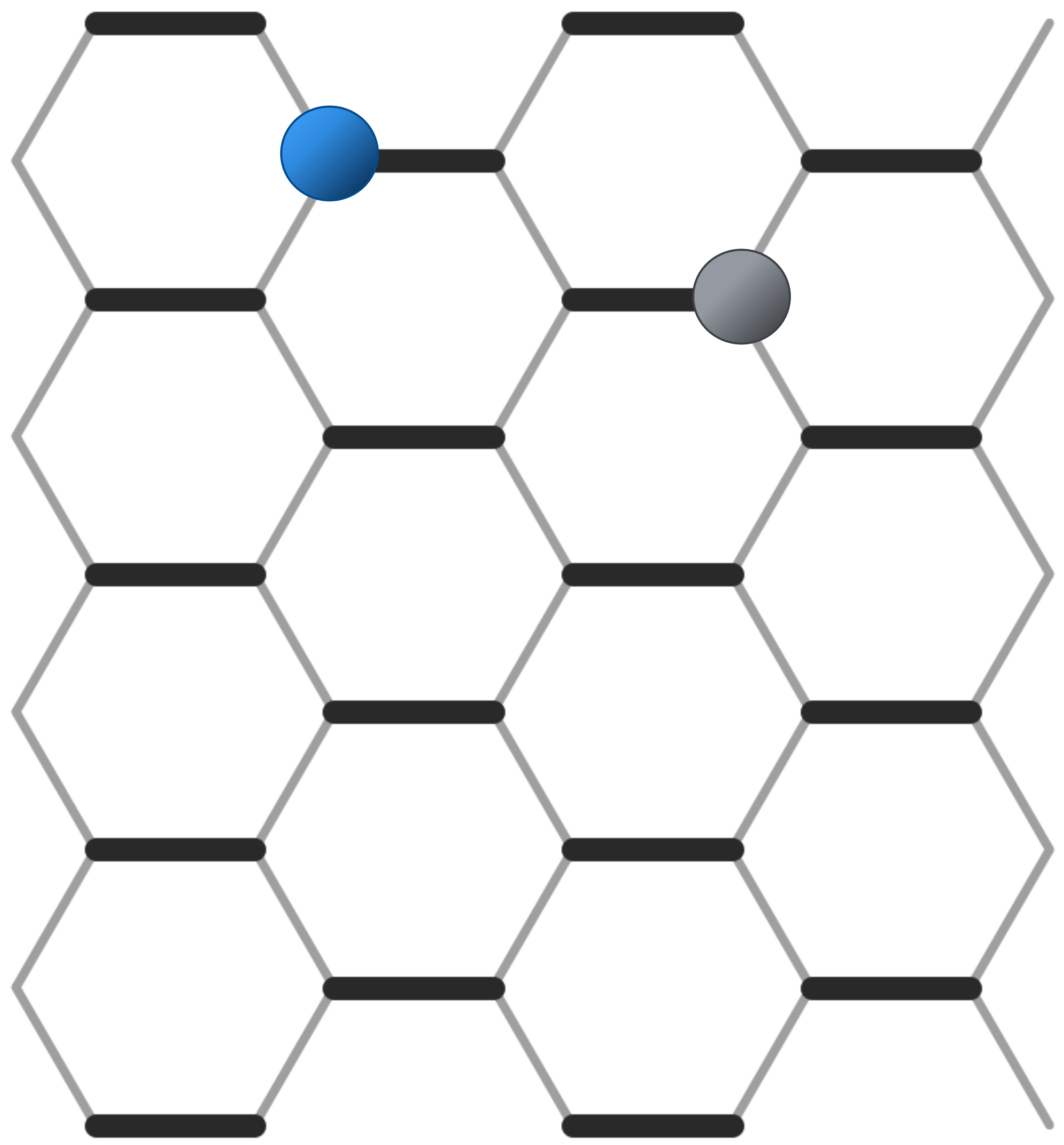

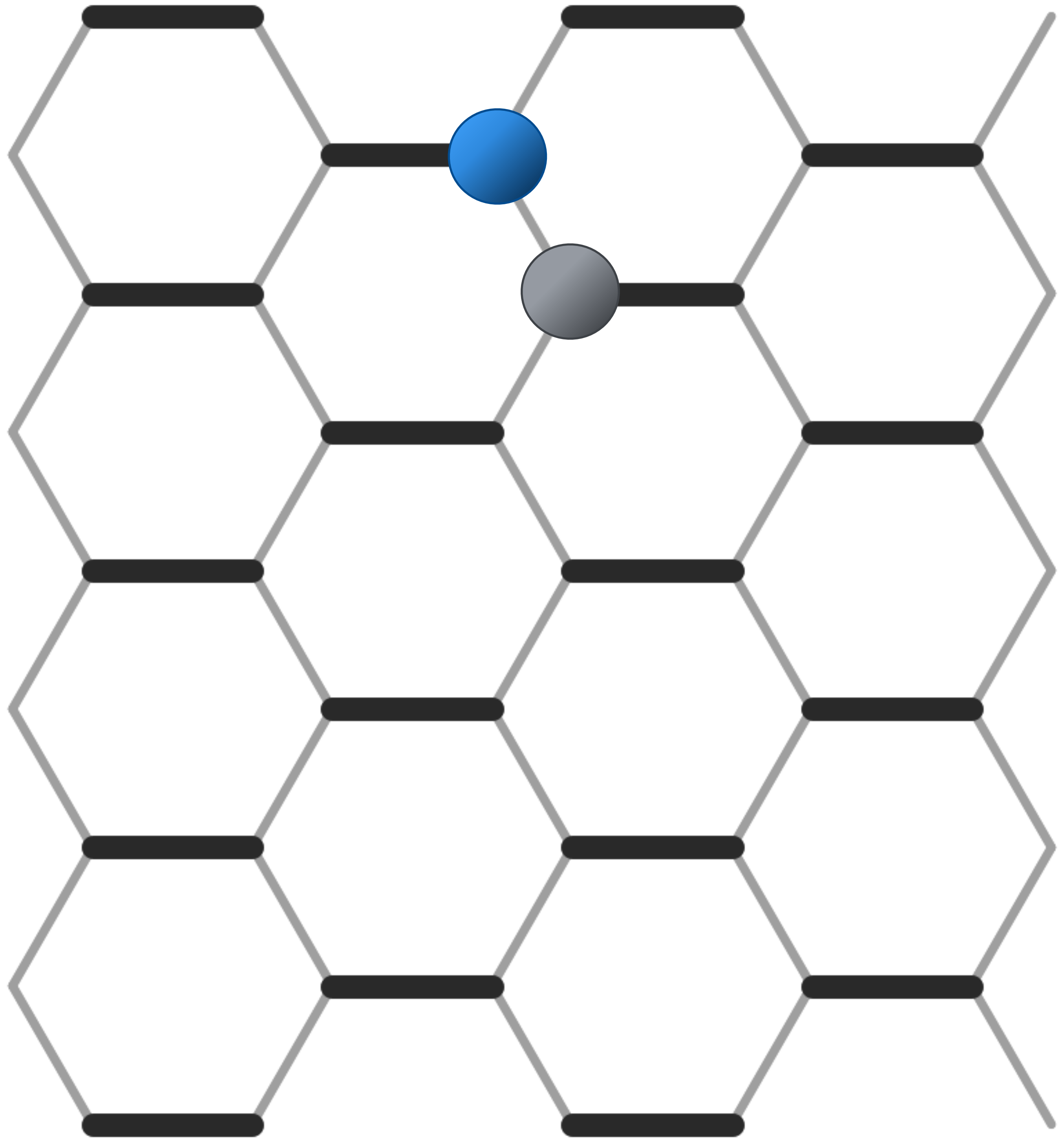

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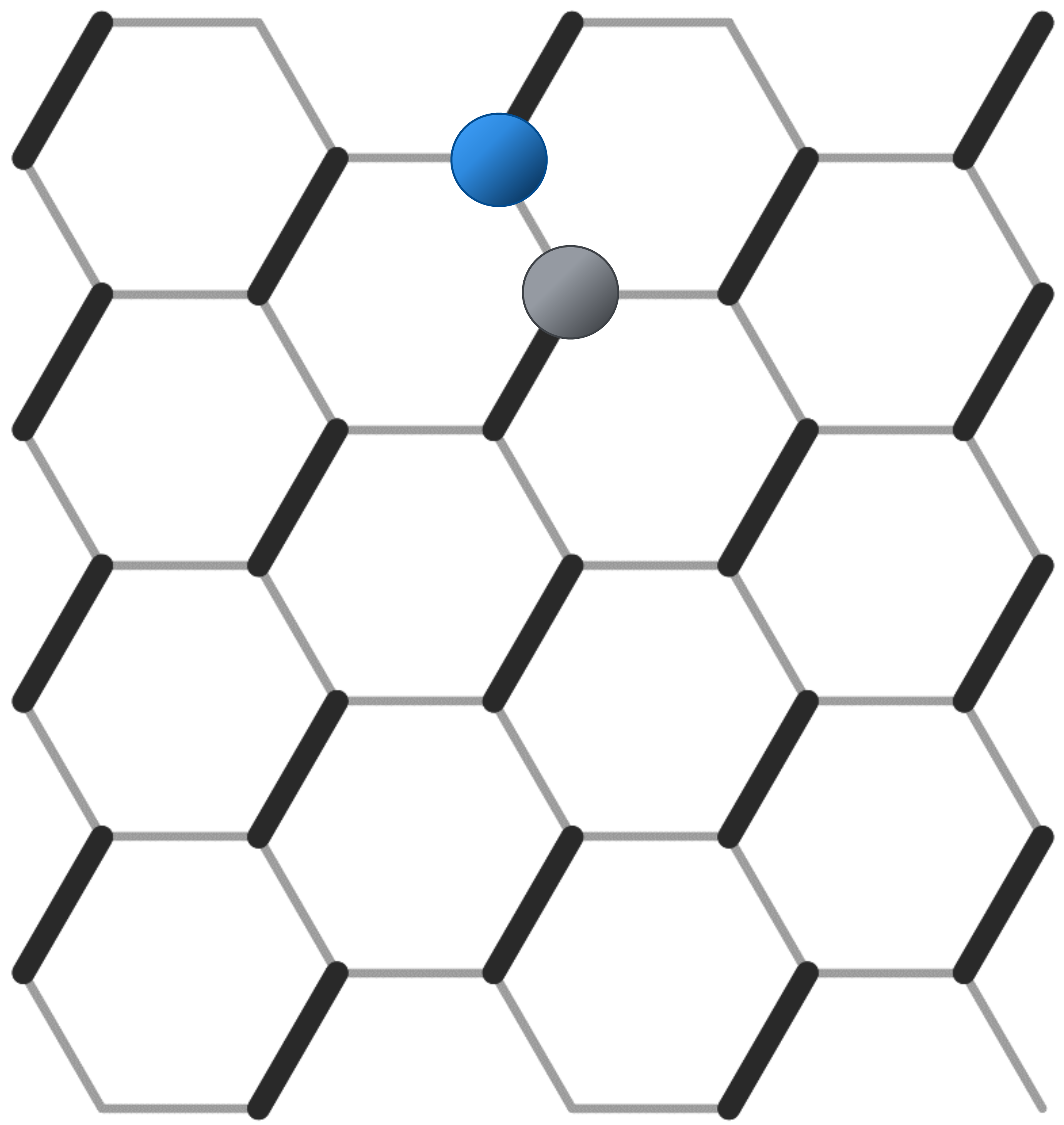

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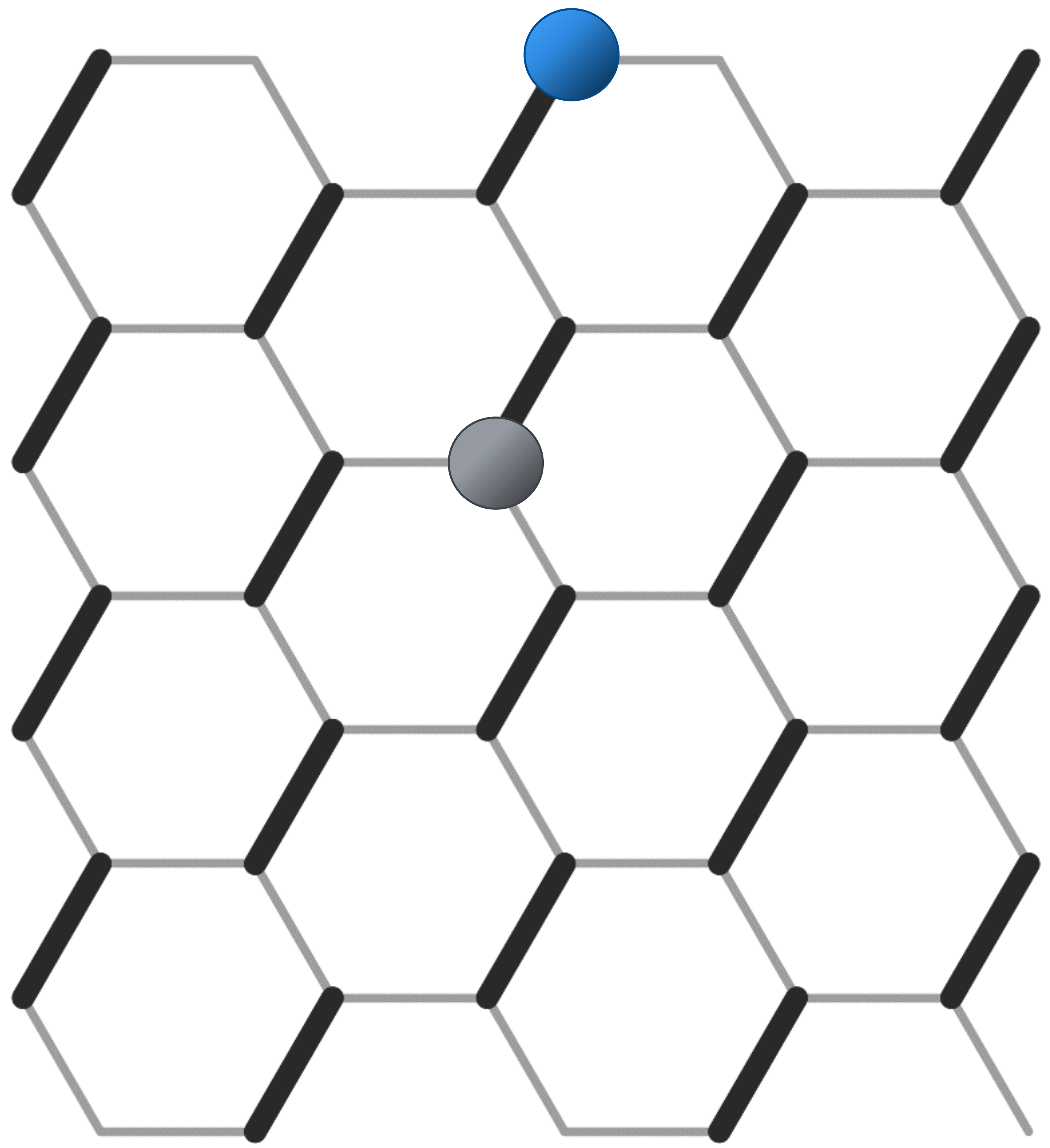

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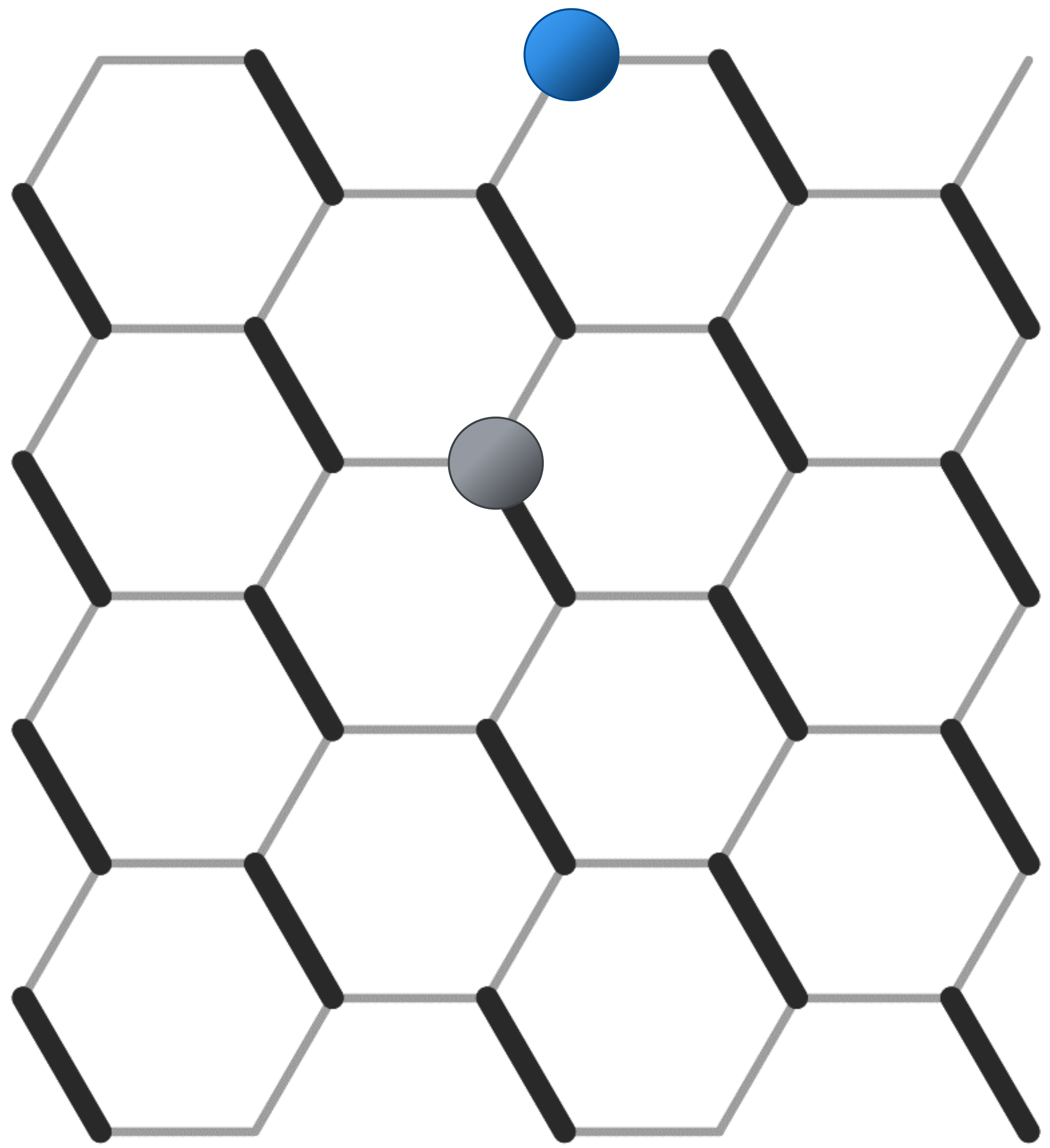
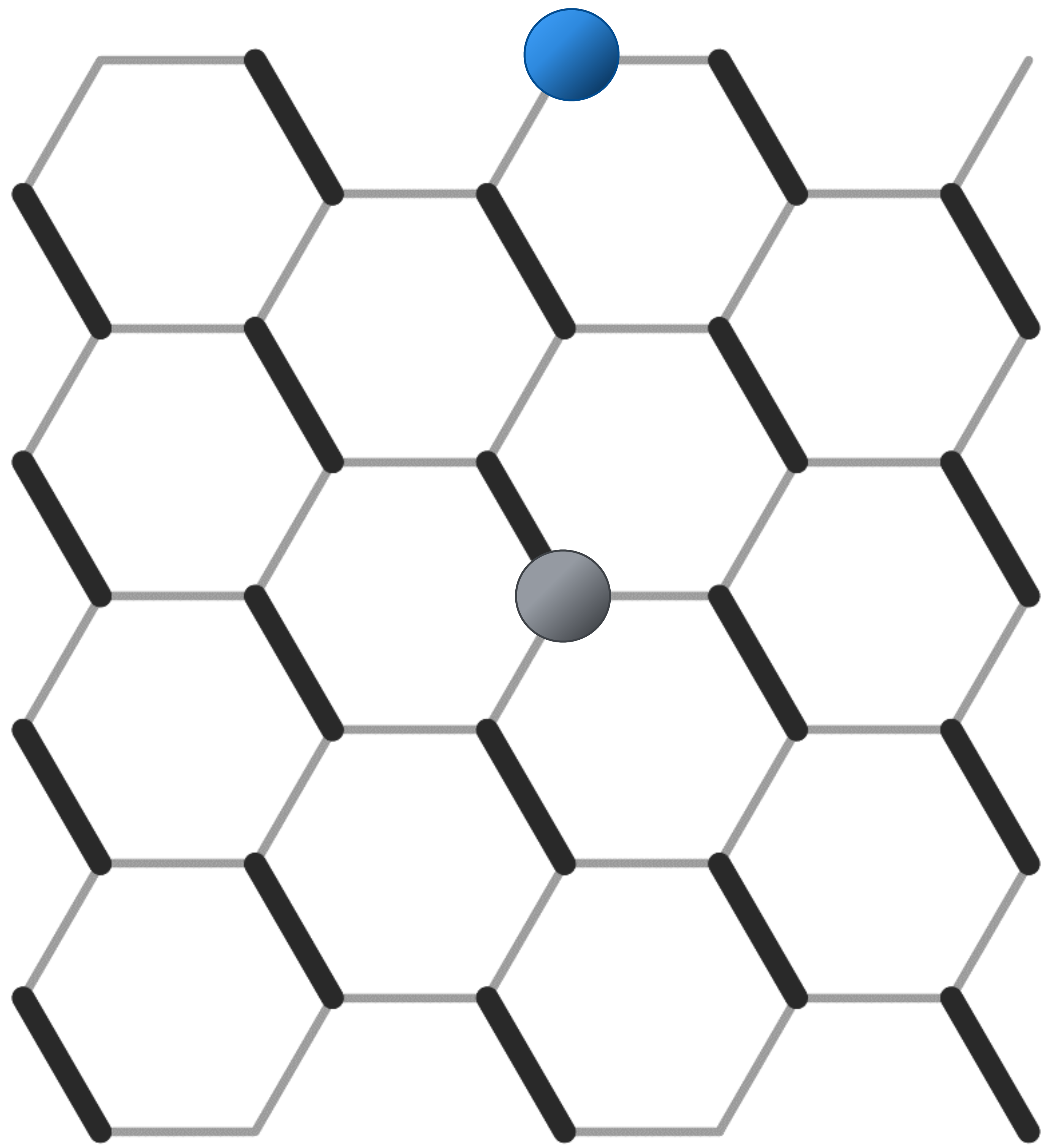
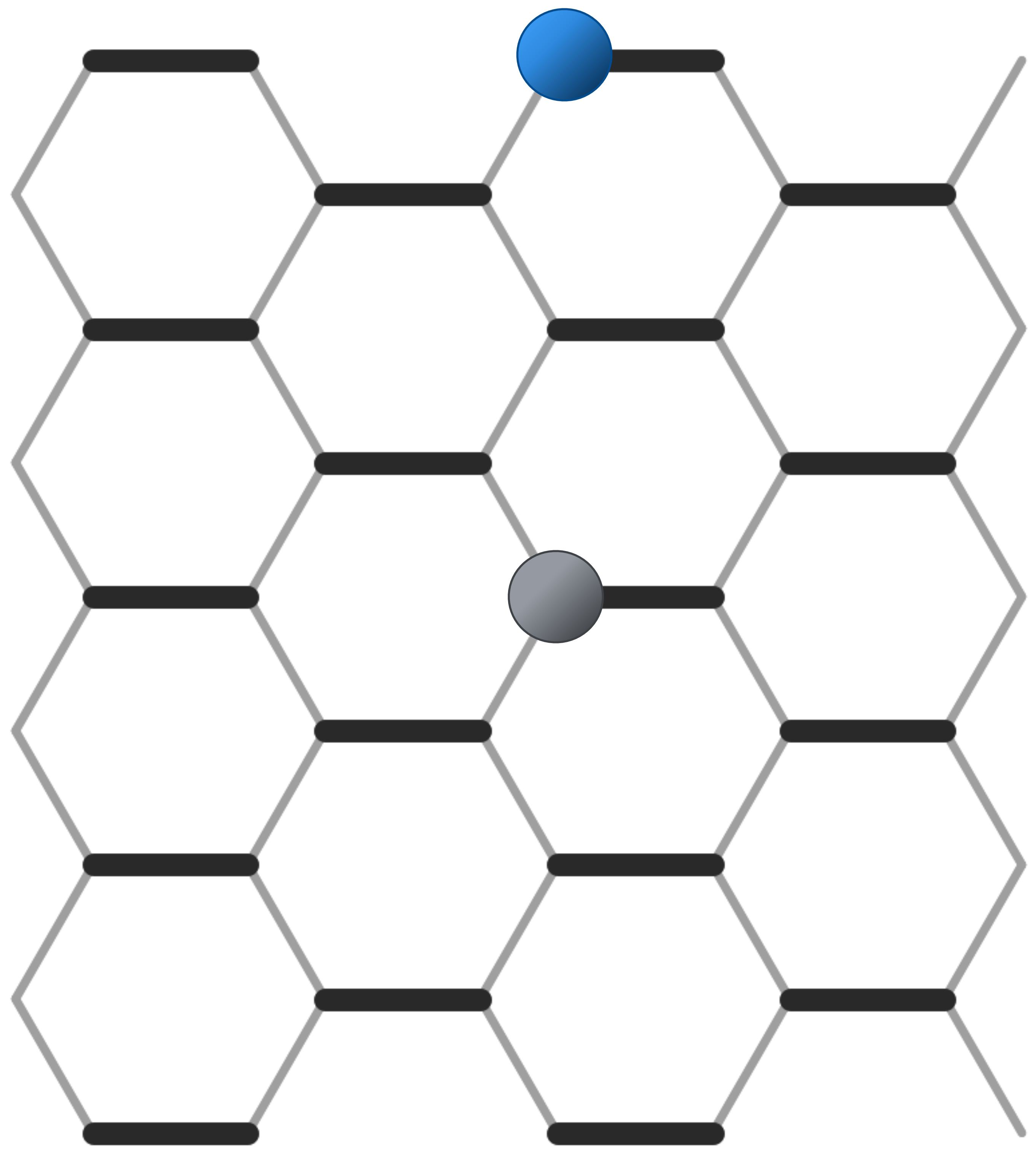


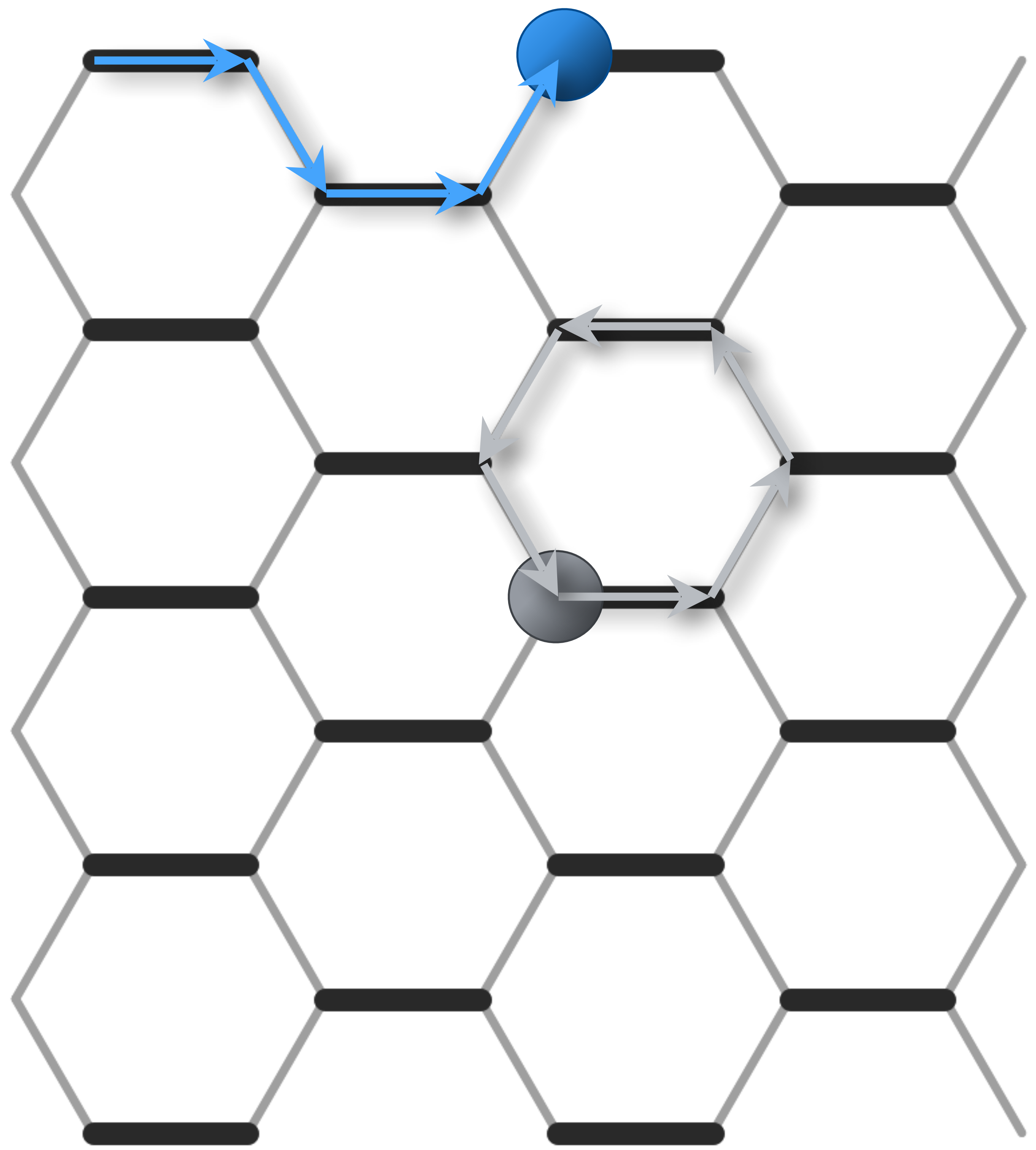
A toy model





A toy model

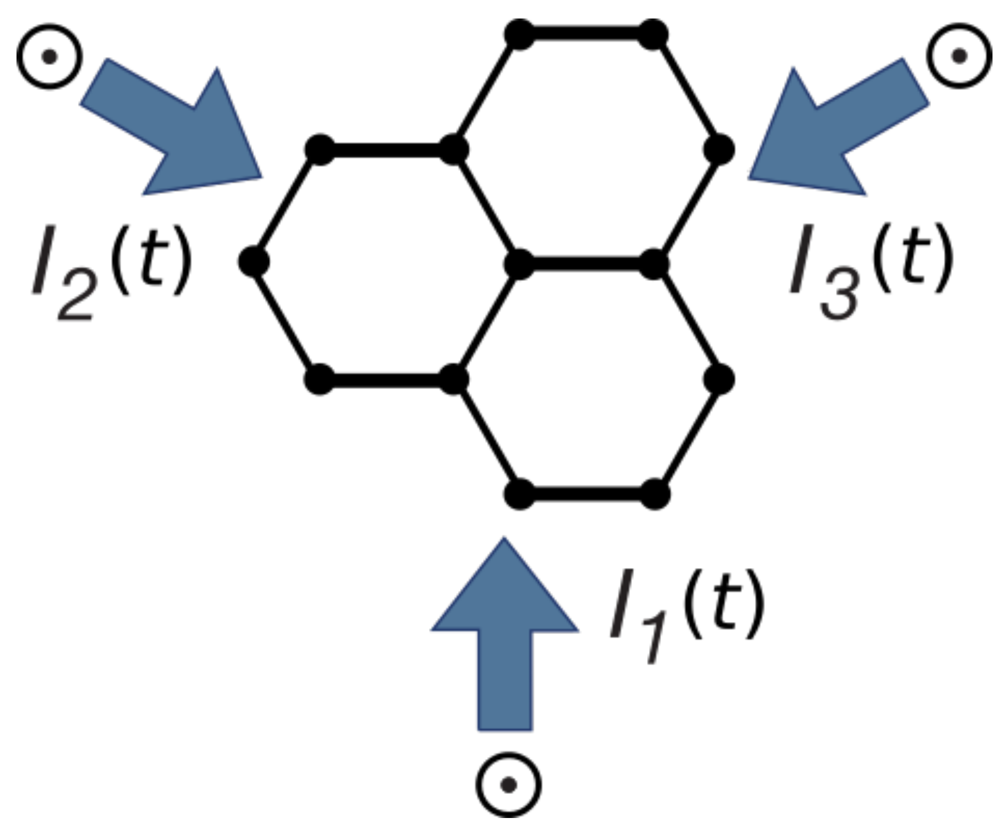
- average one period
  $\tilde{H} = 0$
  → $\mathcal{C} = 0$
- but there is transport on the edge



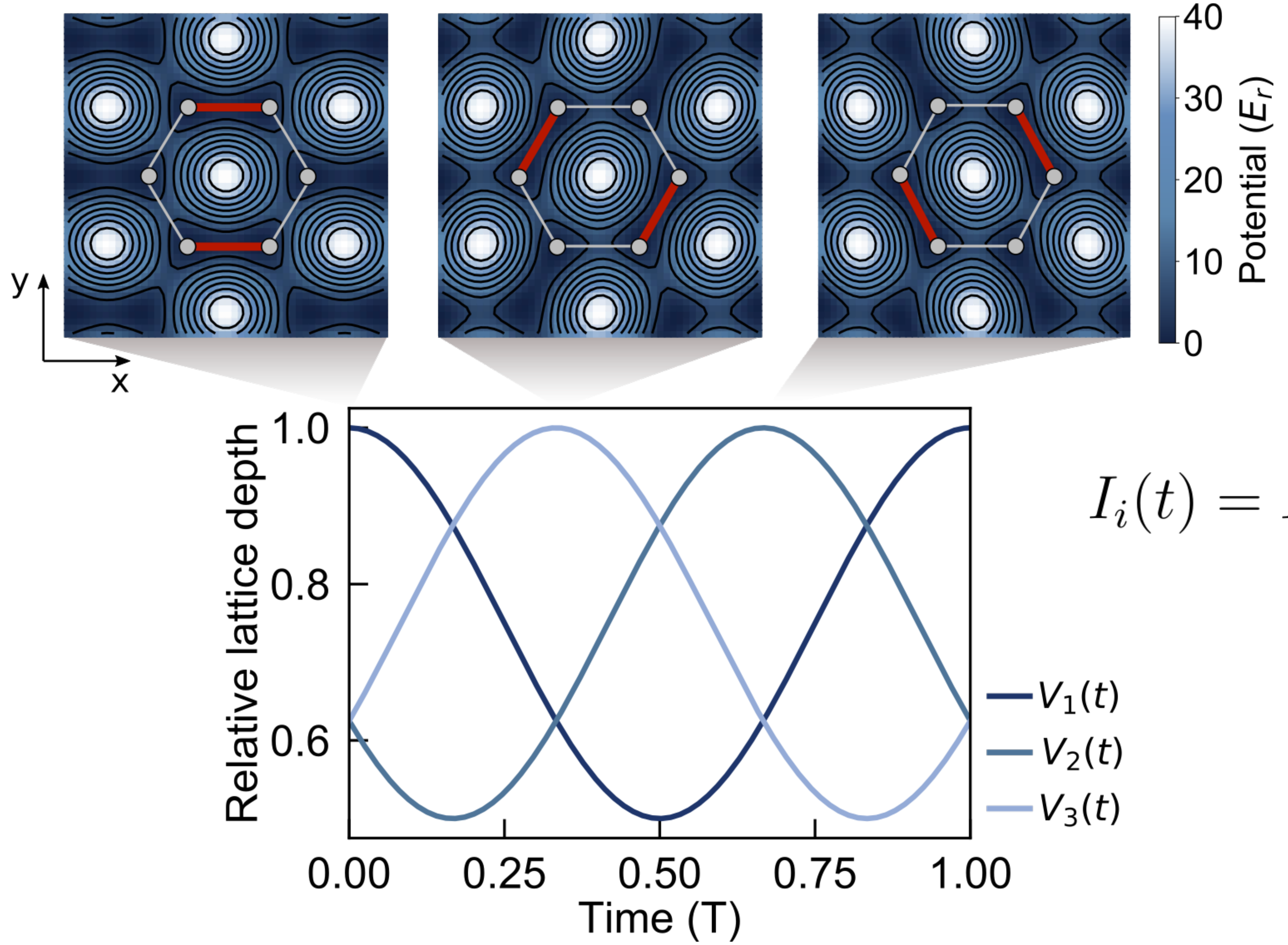
Modulated hexagonal lattice

BEC of ^{39}K atoms in optical dipole trap

Quasimomentum-resolved measurements



Continous intensity modulation:

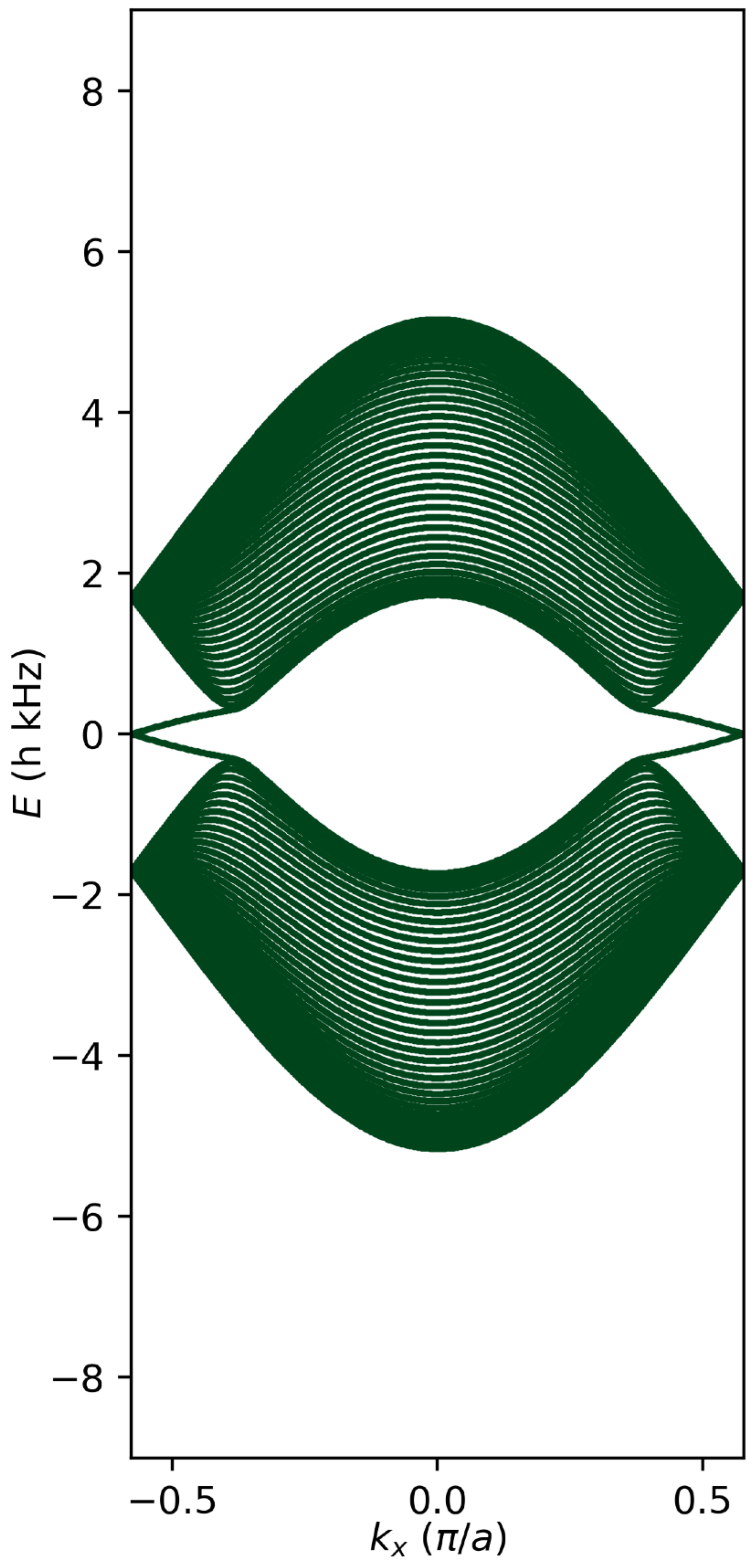


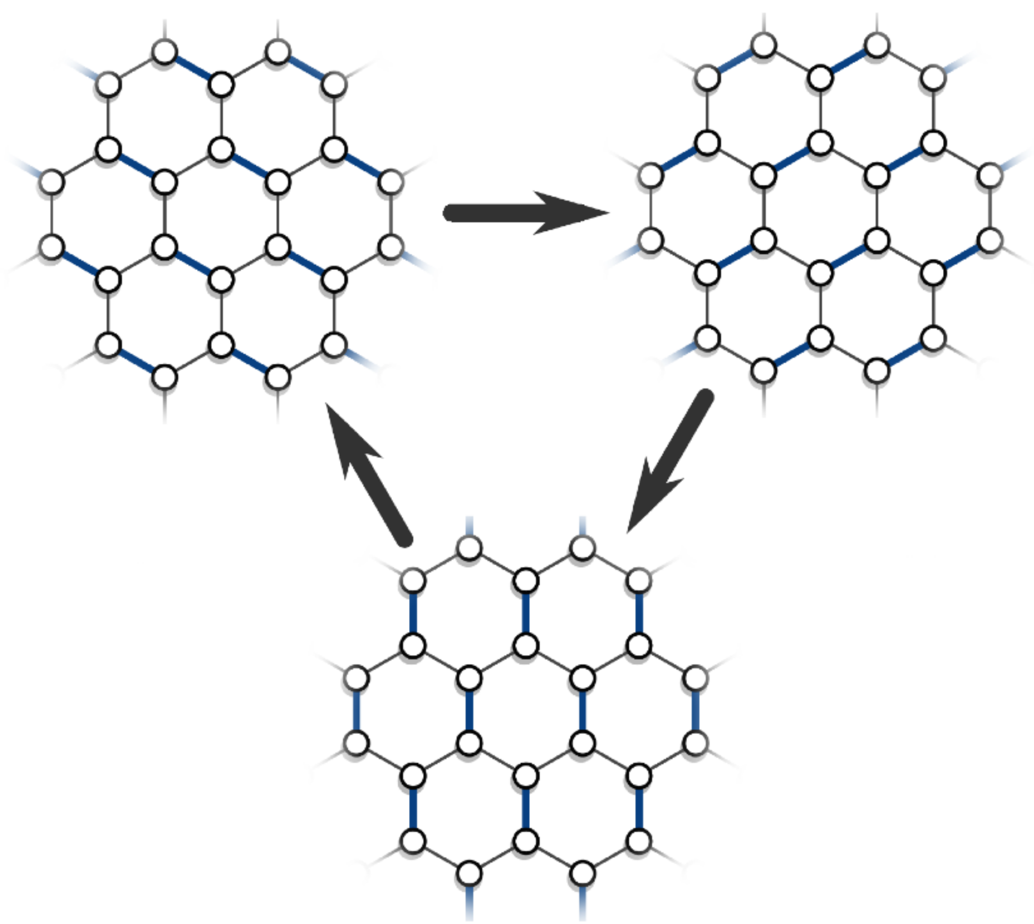
$$I_i(t) = I_0(1 - m + m\cos(\omega t + \phi_i))$$

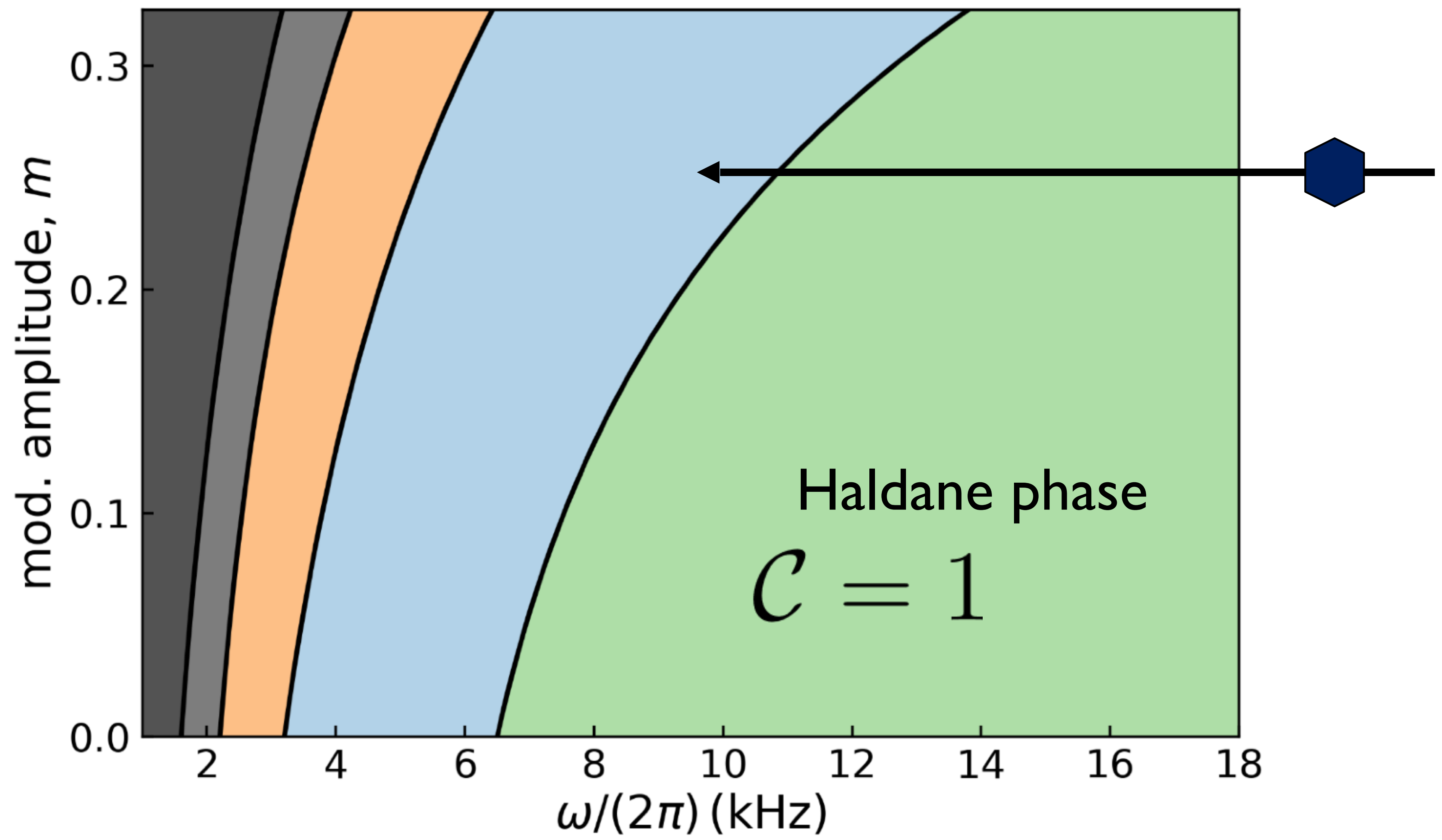
Relative amplitude	m
Modulation frequency	ω

see also: A. Quelle *et al.*, New J Phys. **19**, 113010 (2017)
T. Kitagawa *et al.*, Phys. Rev. B **82**, 235114 (2010)

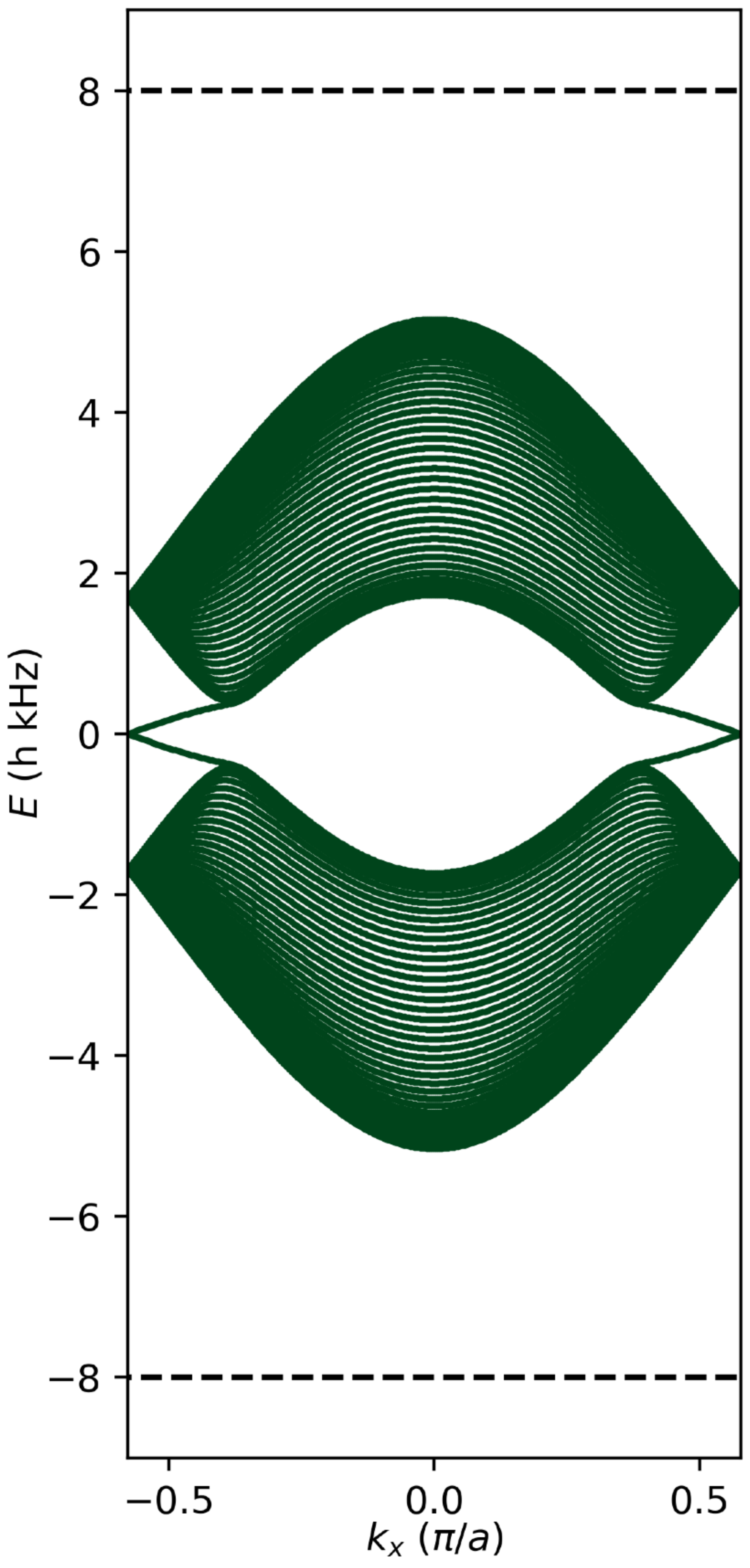
Topological Floquet phases

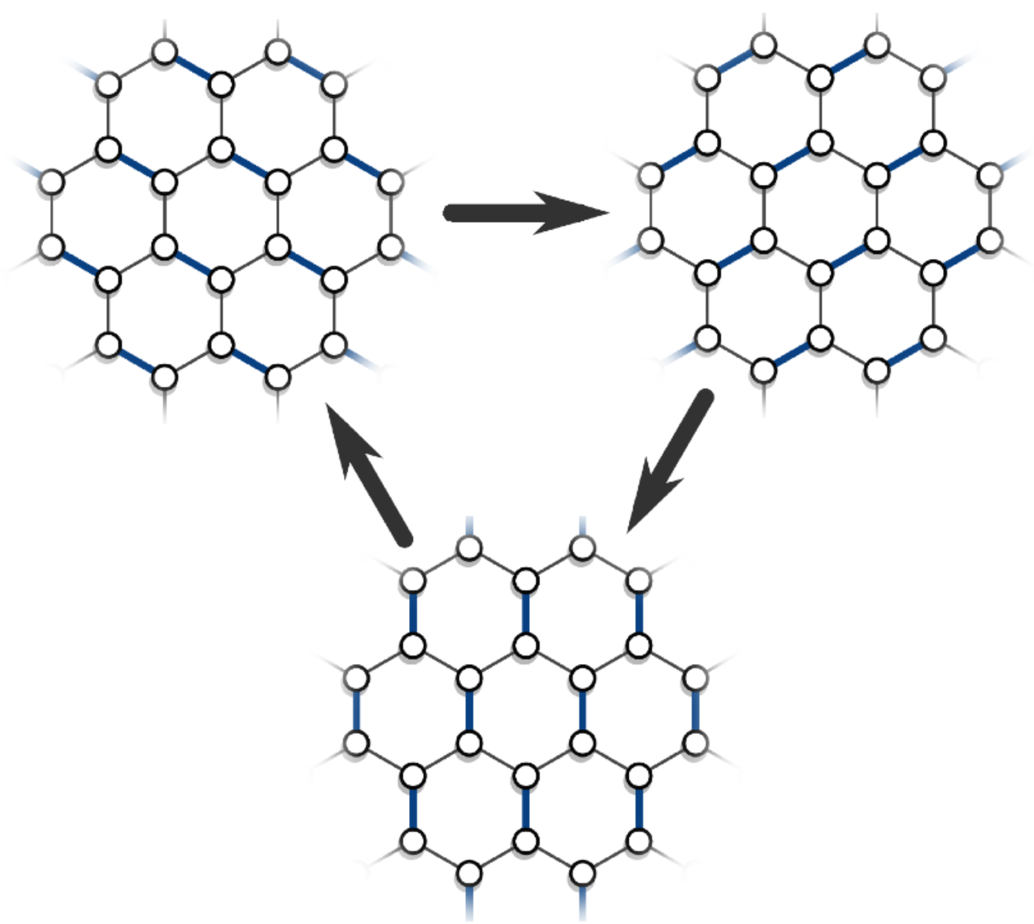


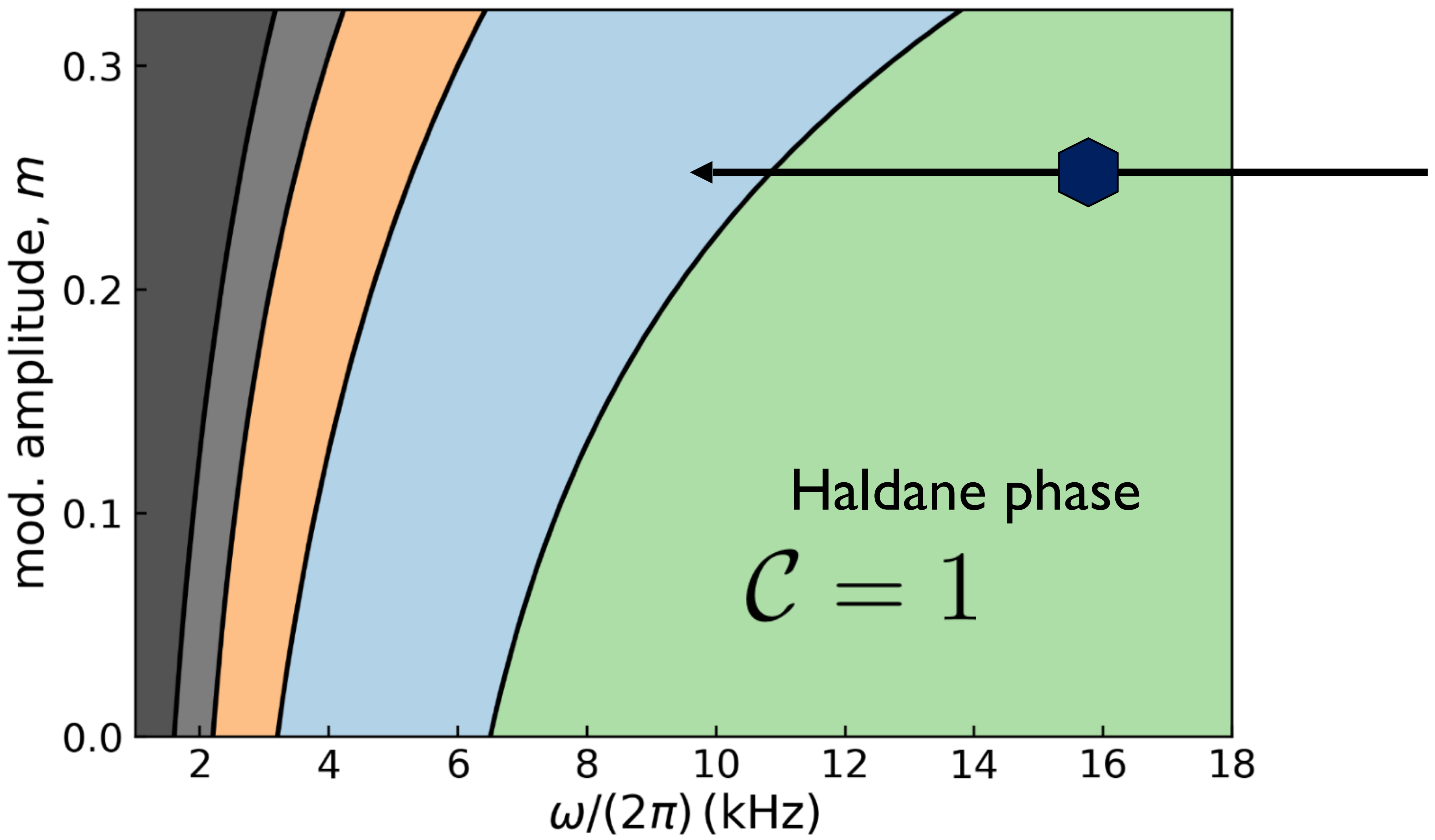




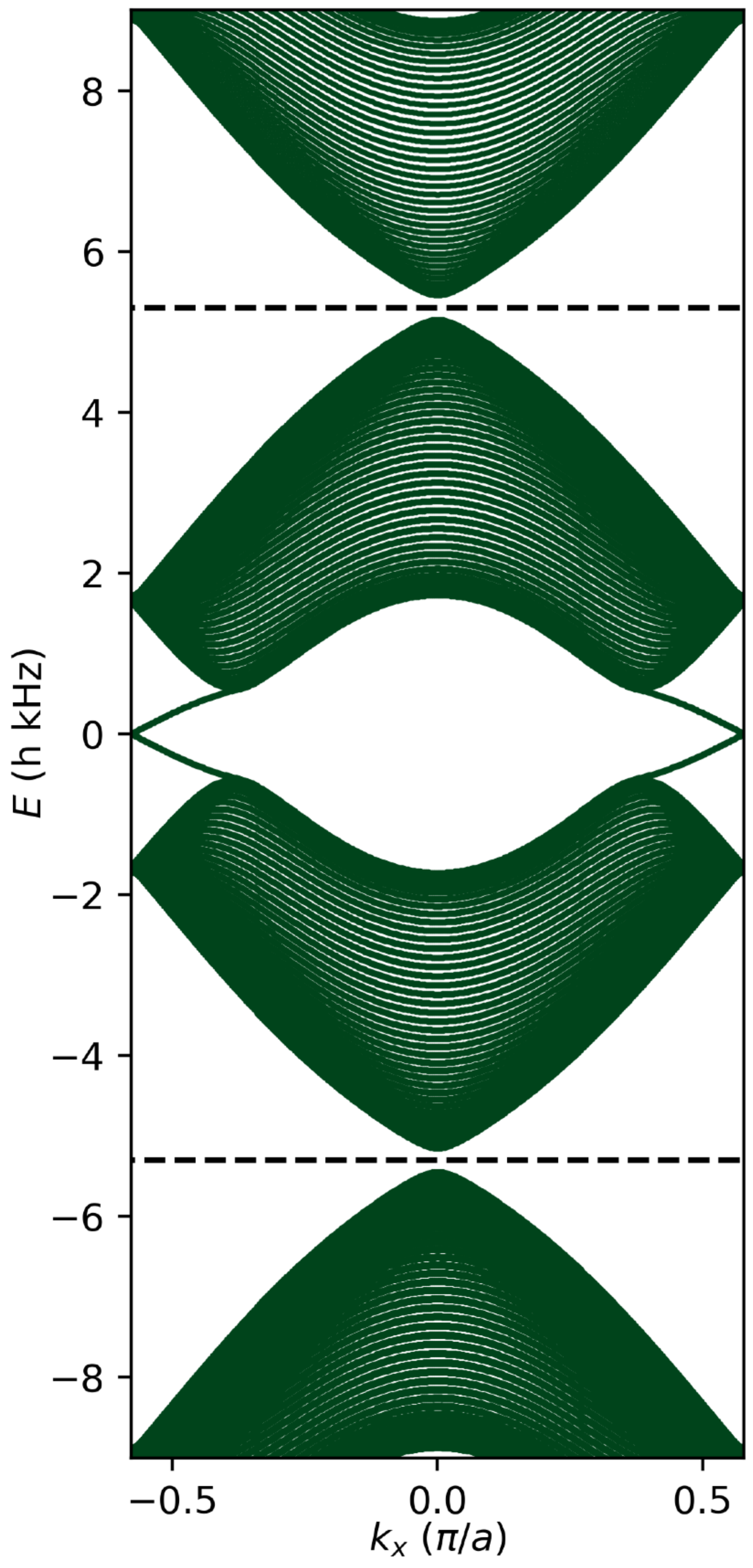
Topological Floquet phases

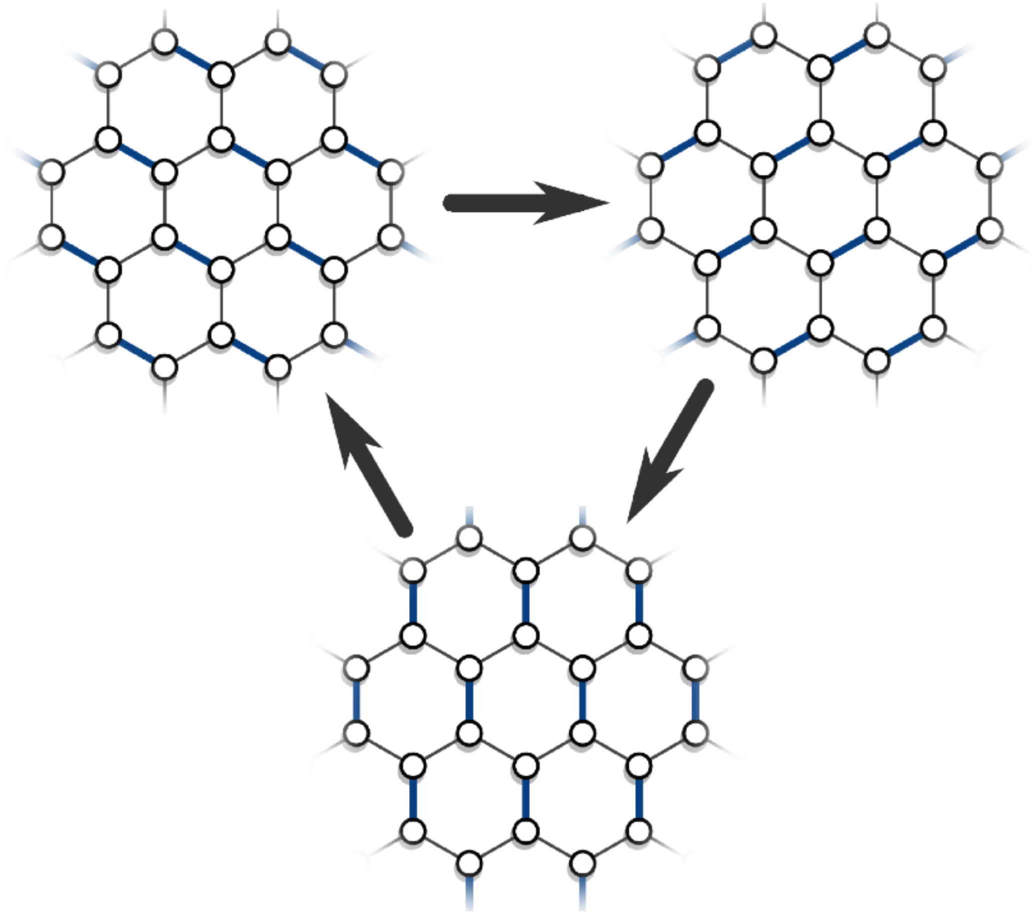


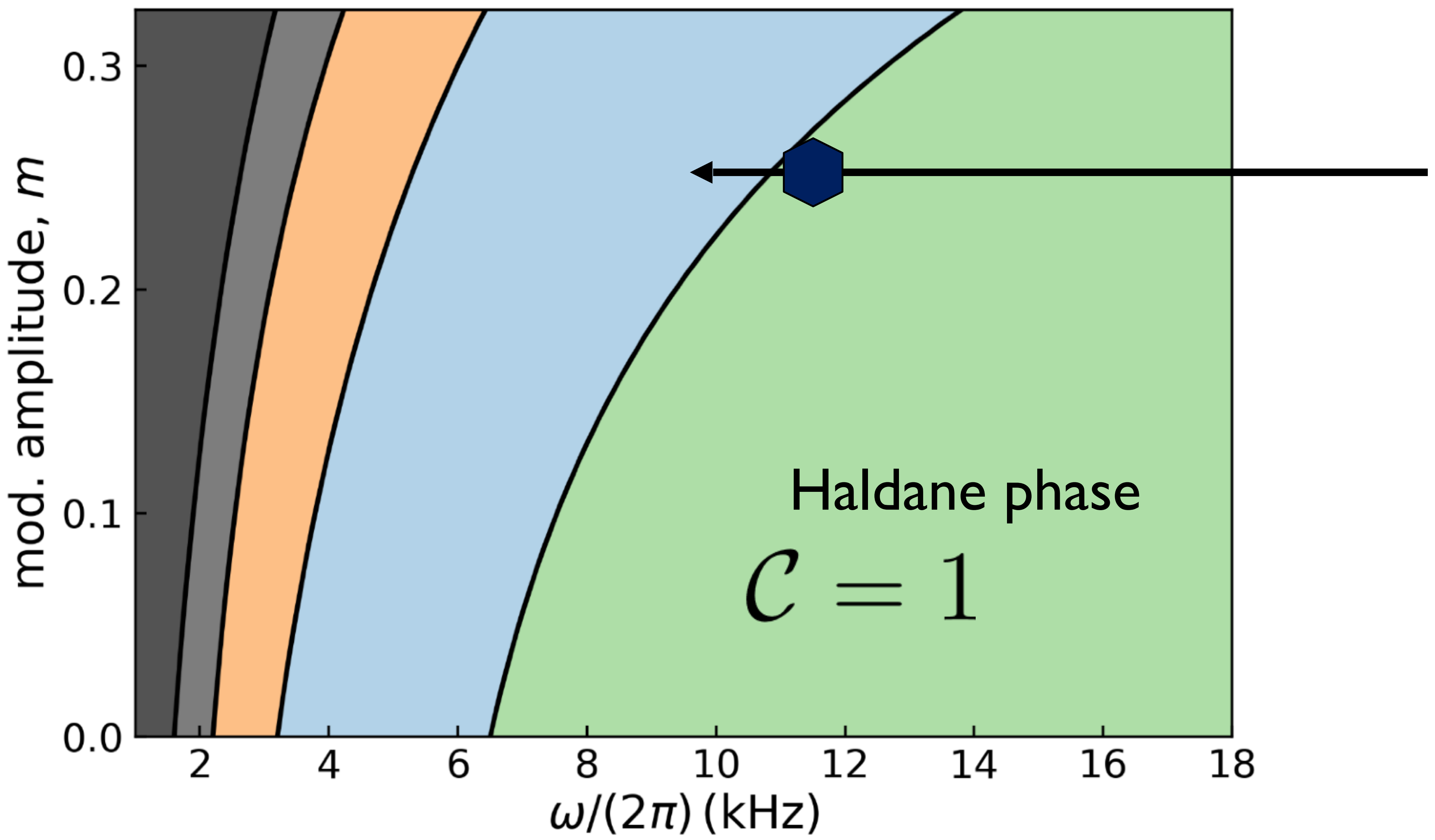




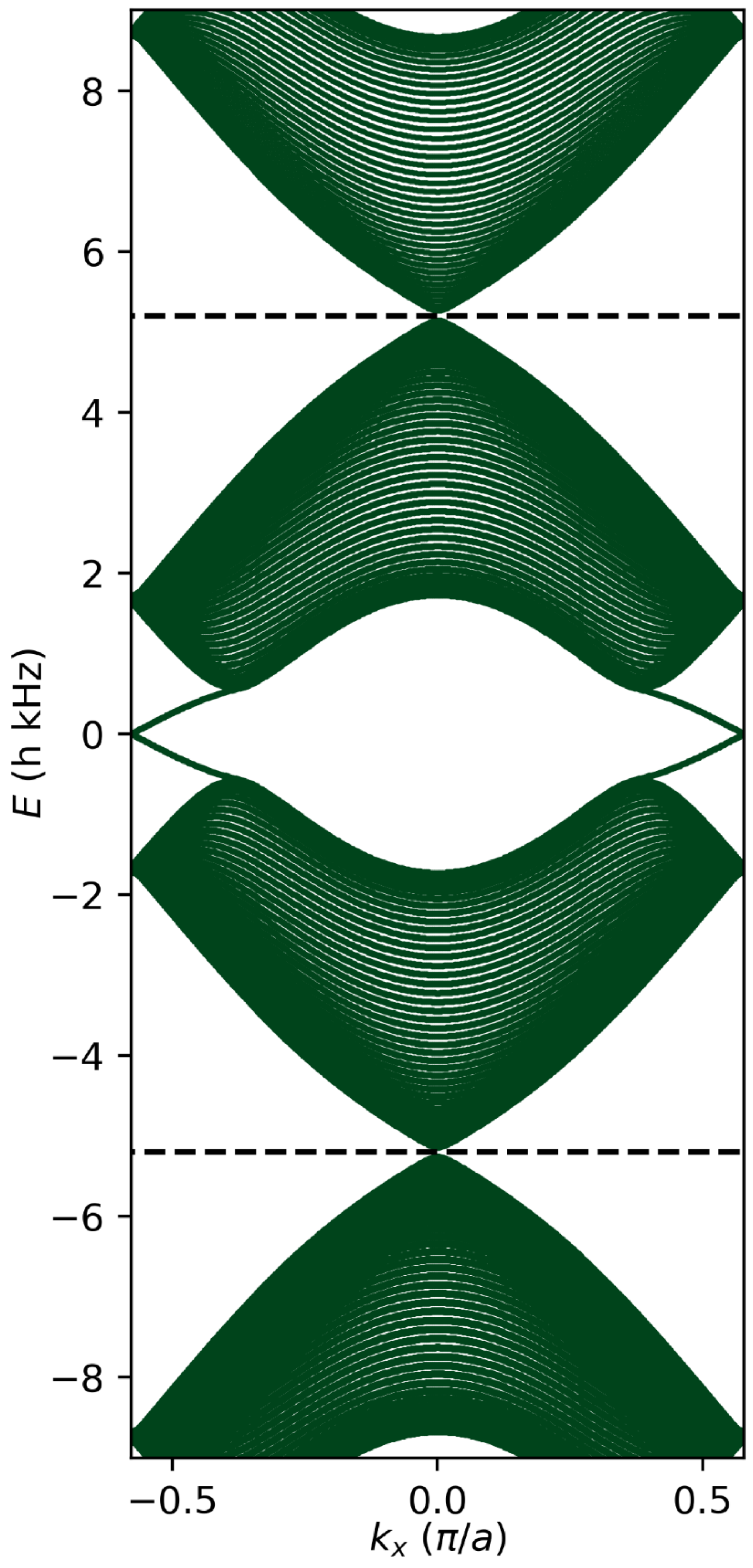
Topological Floquet phases

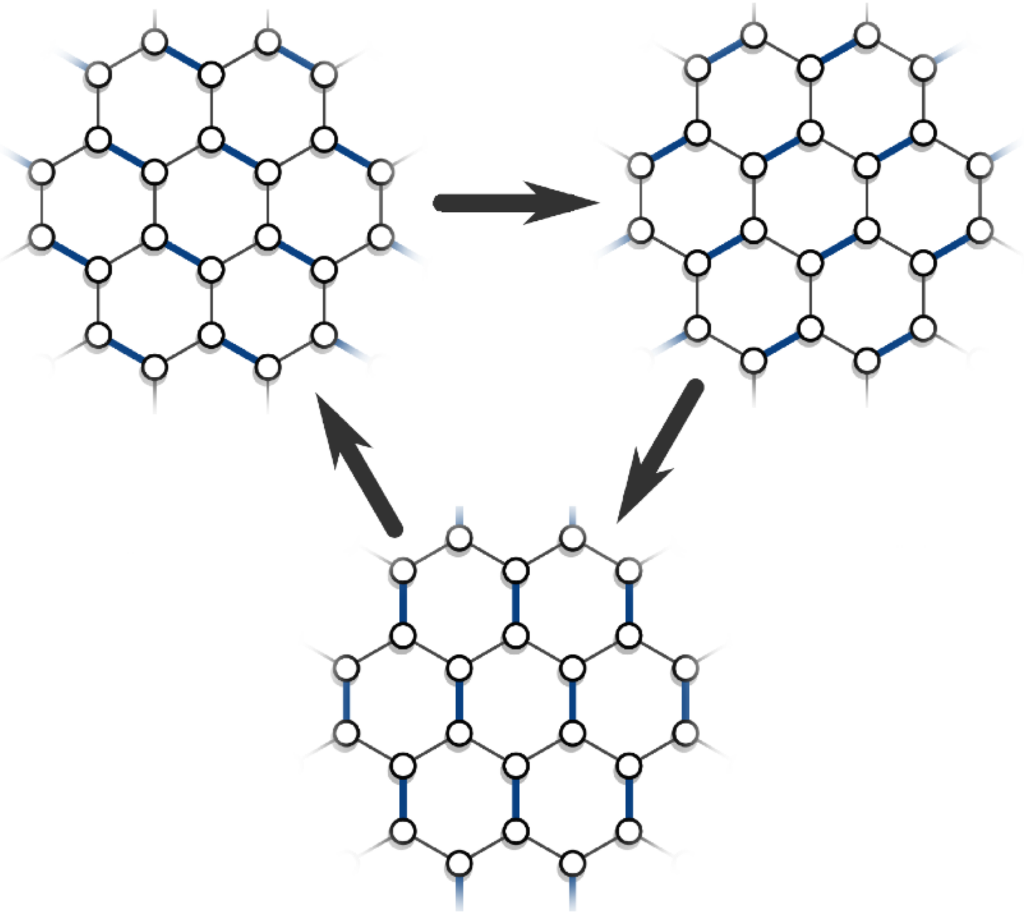


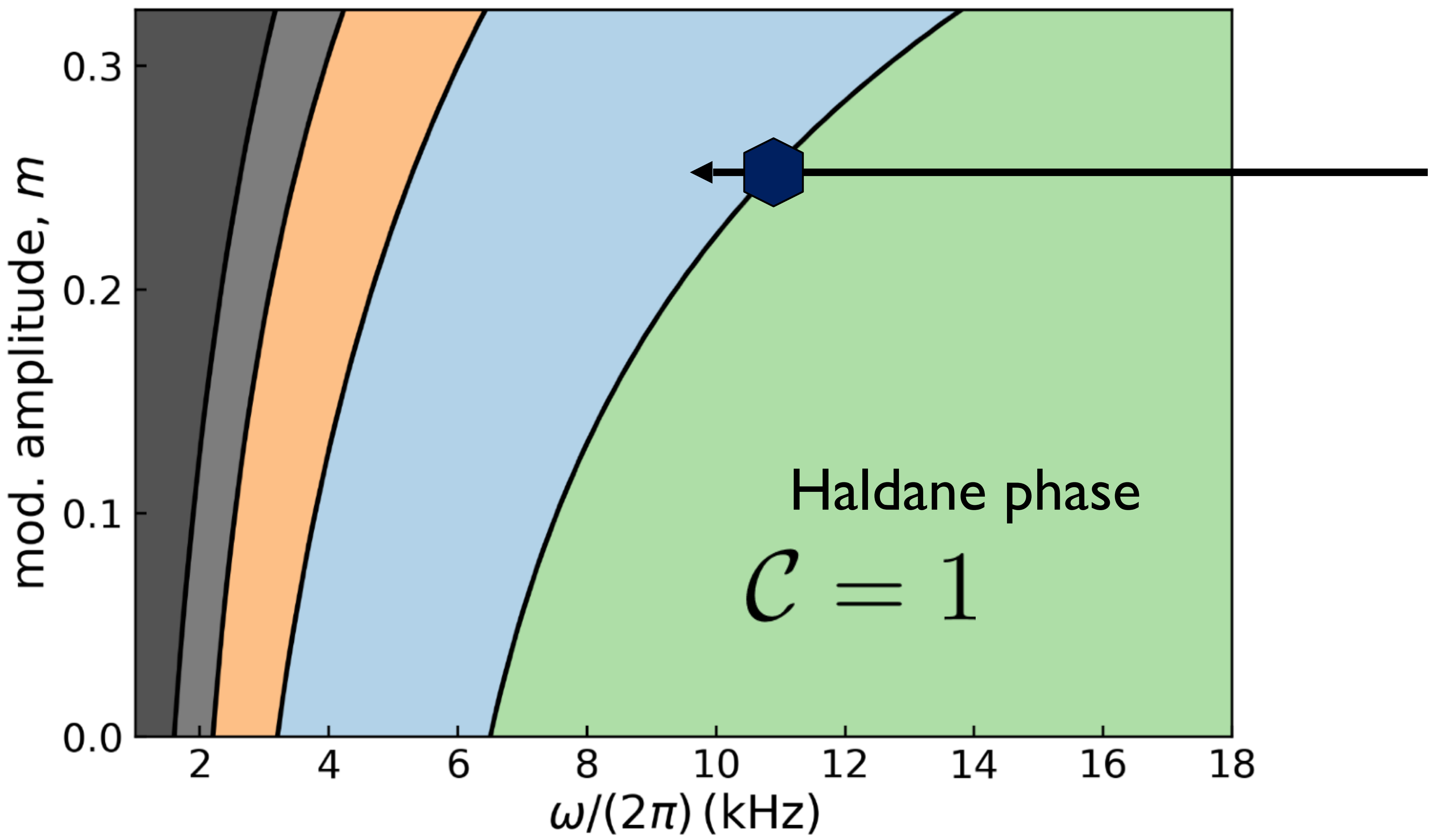




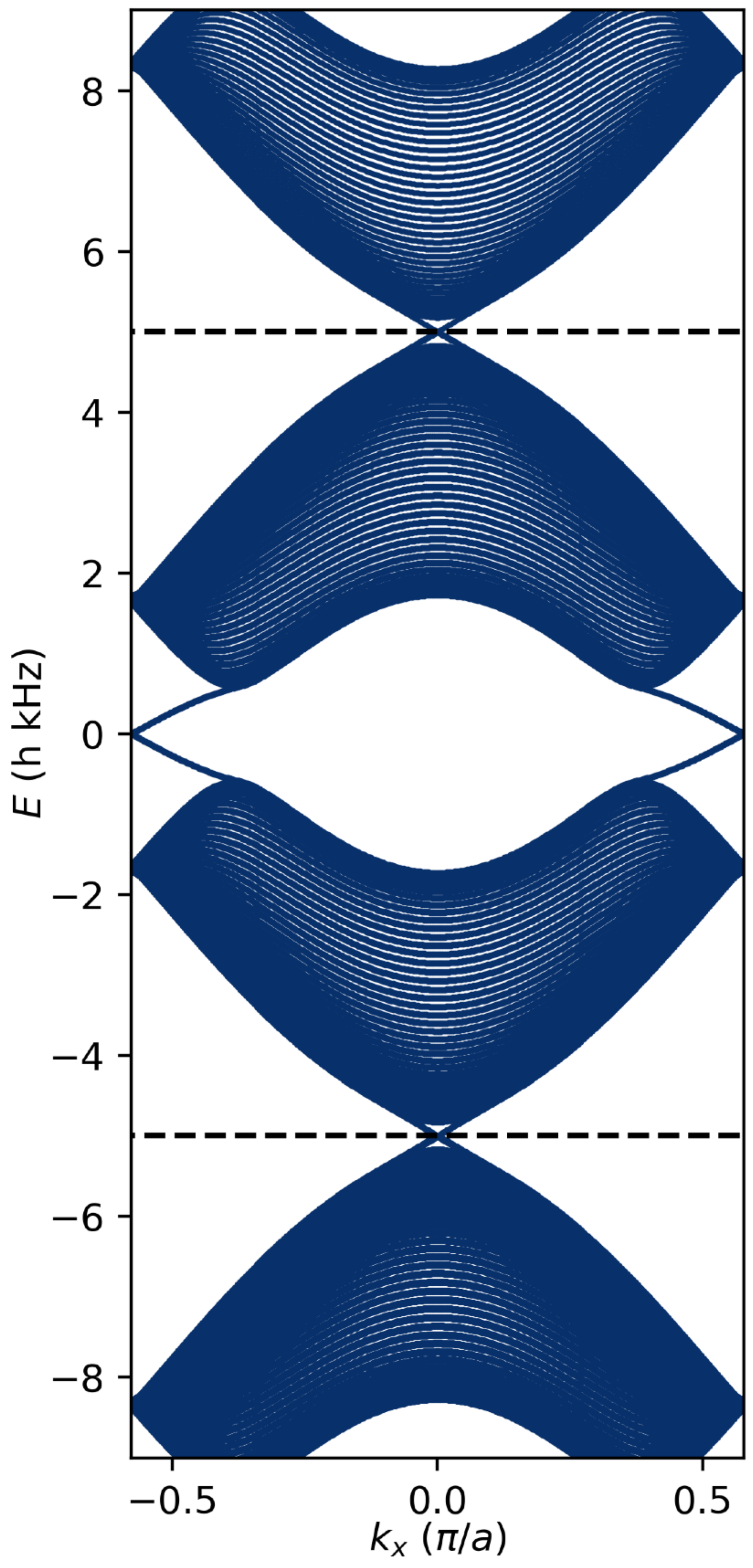
Topological Floquet phases

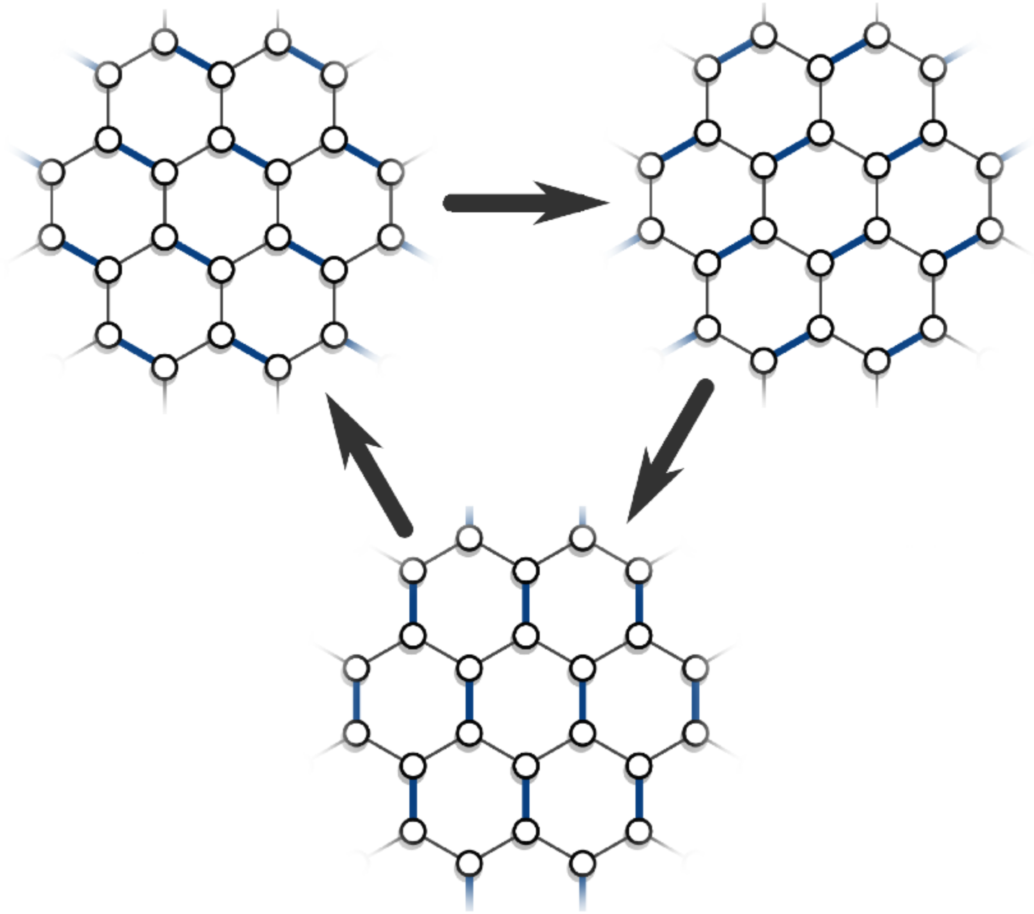


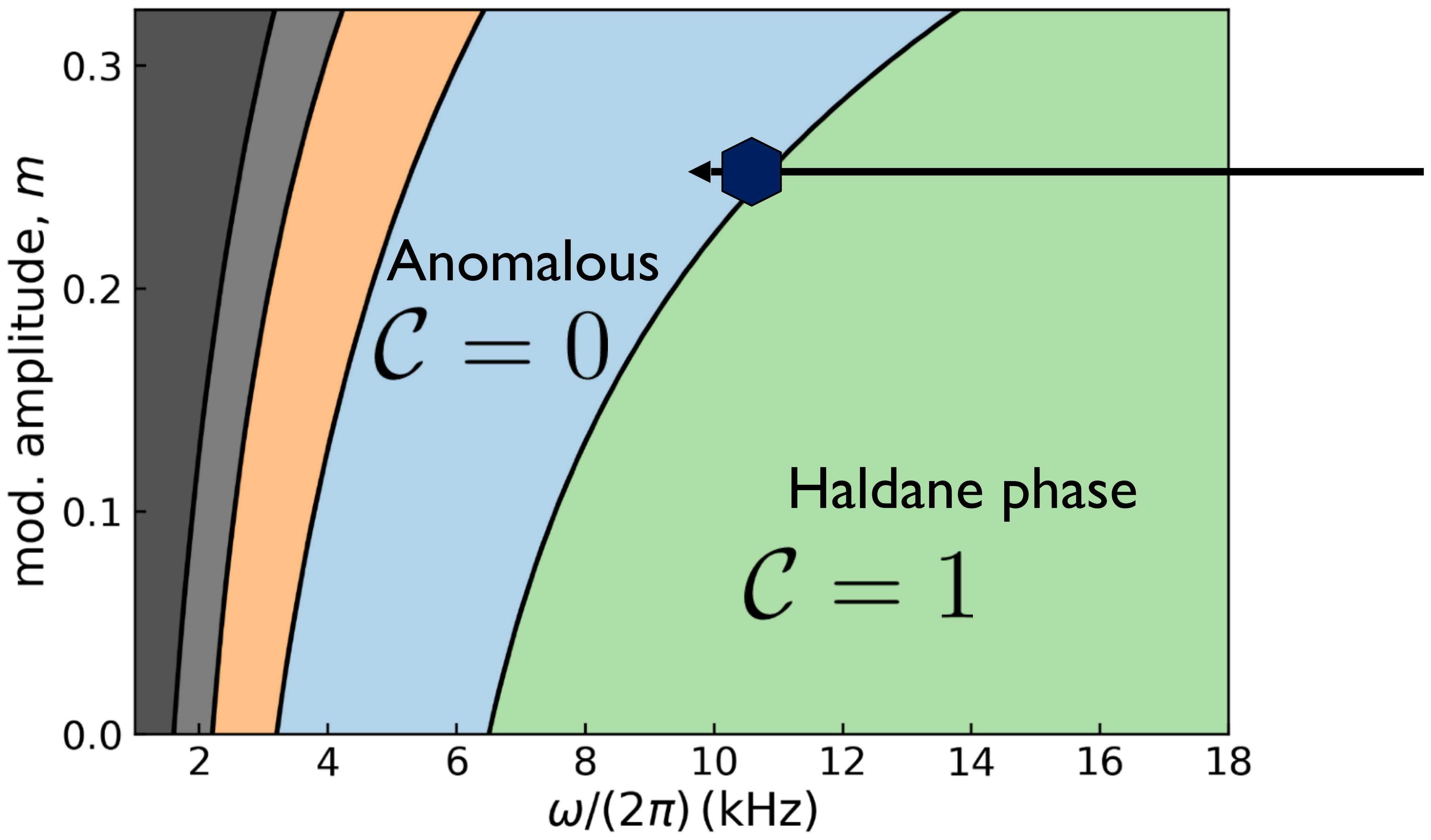




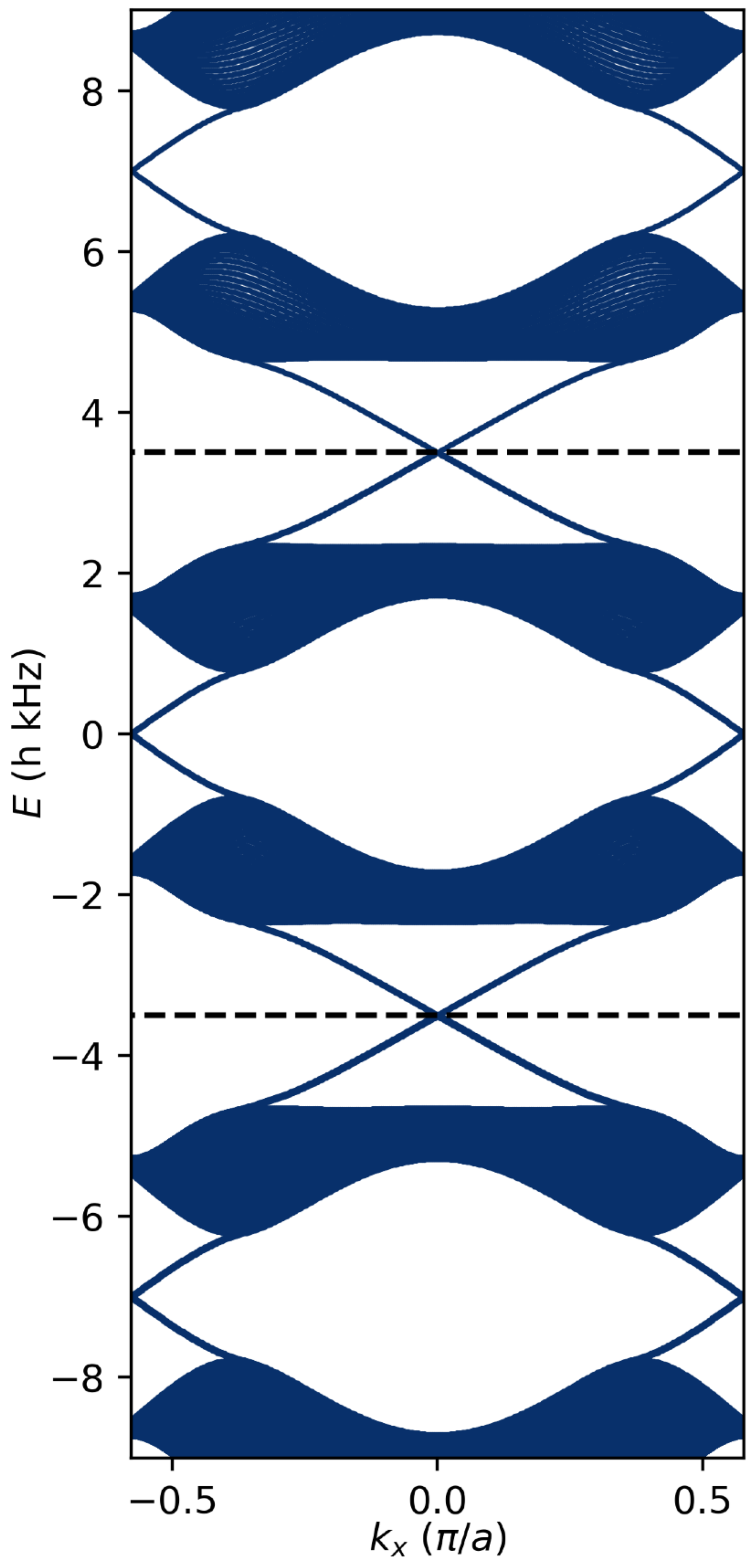
Topological Floquet phases

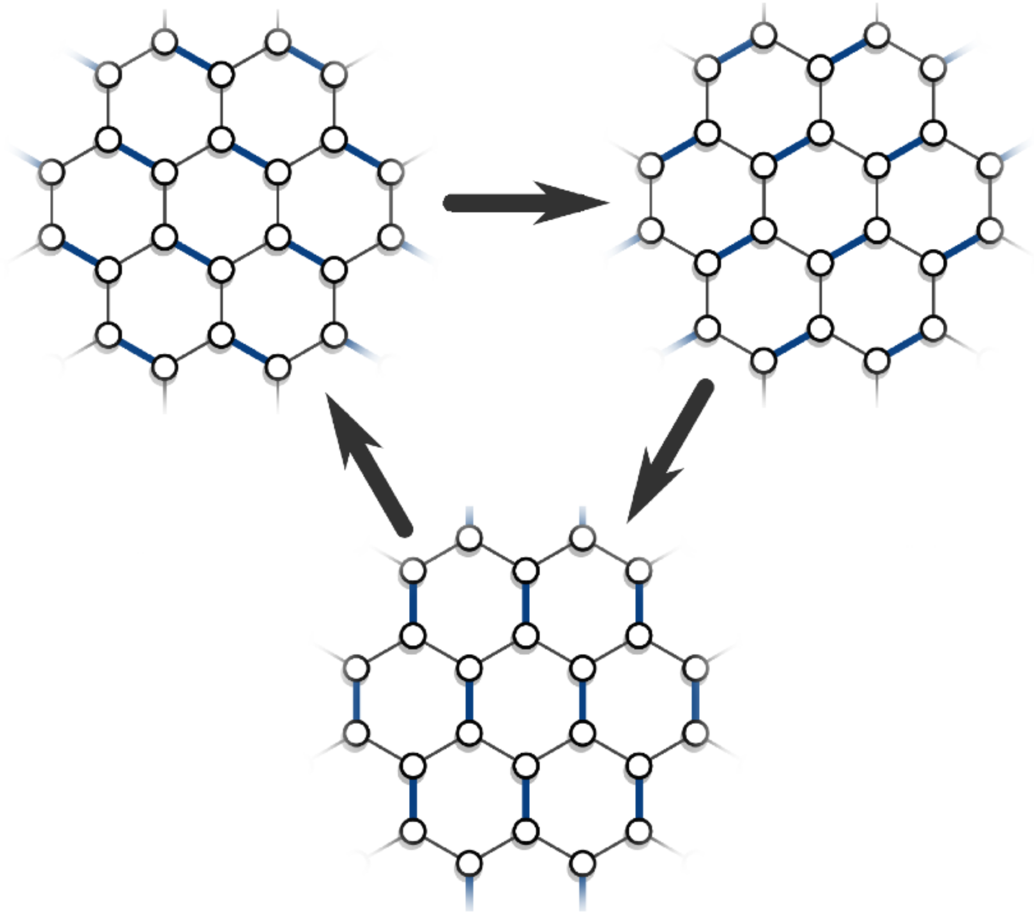


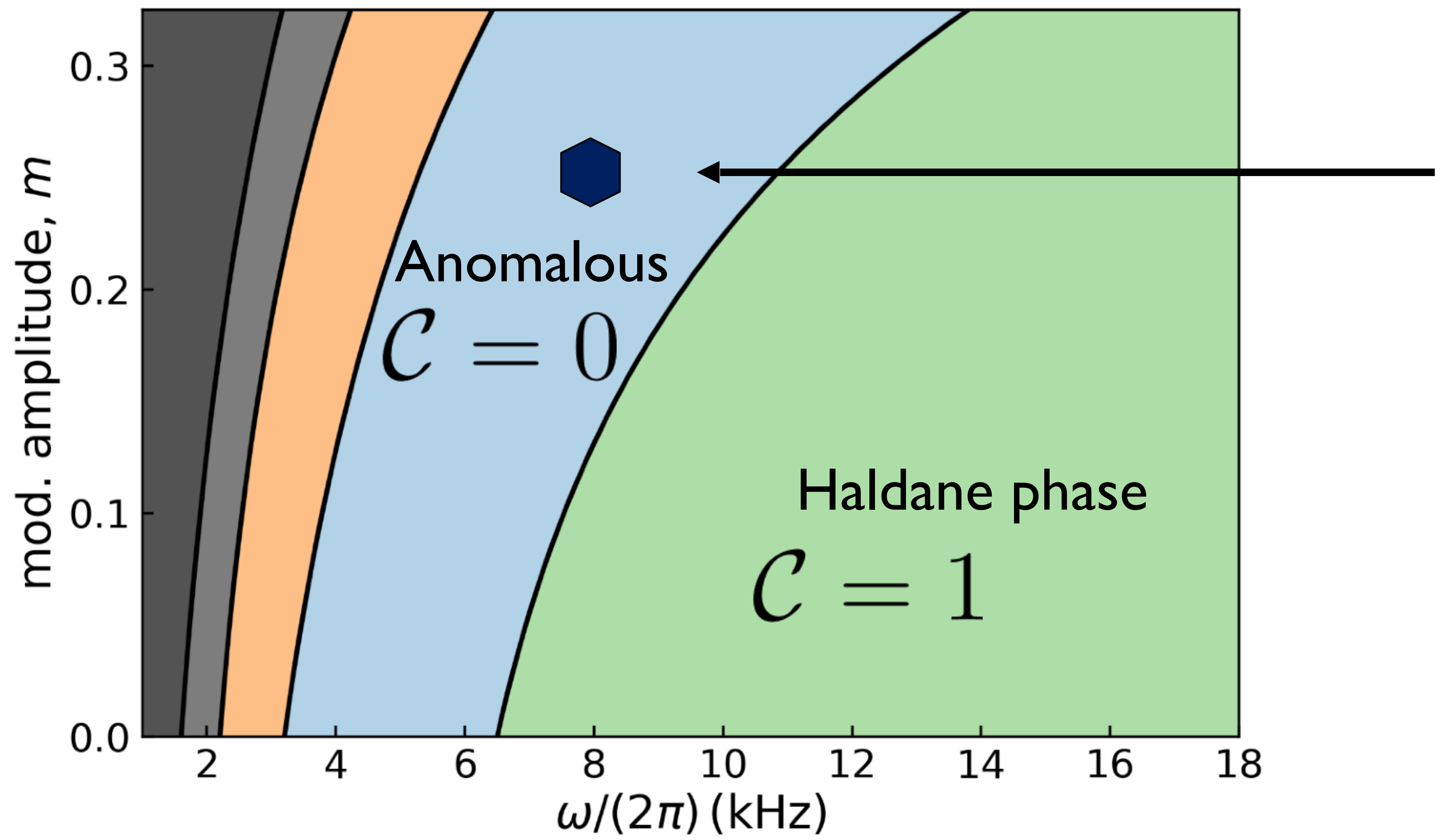




Topological Floquet phases







Bulk characterization

ARTICLES
https://doi.org/10.1038/s41567-020-0949-y
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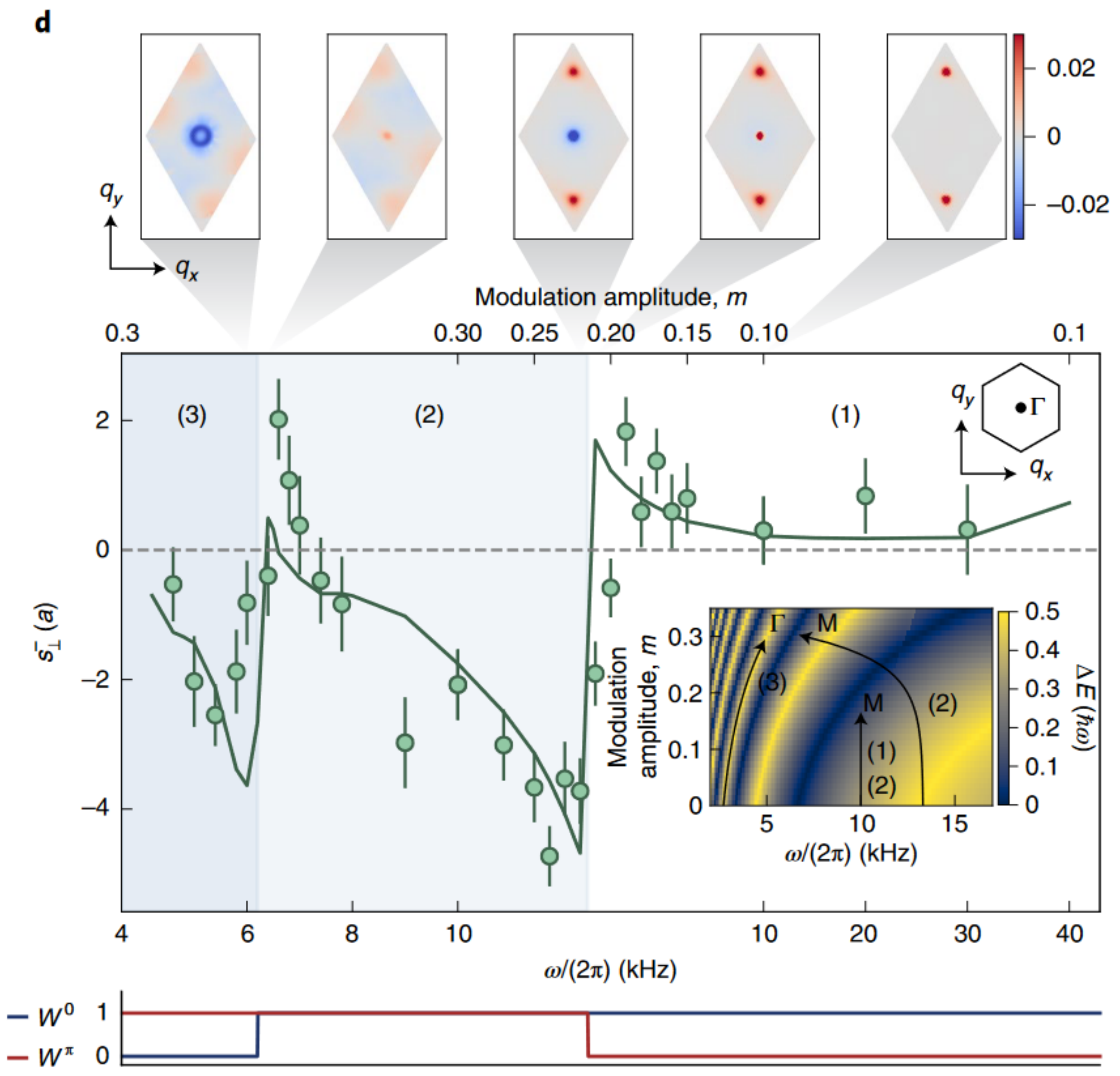
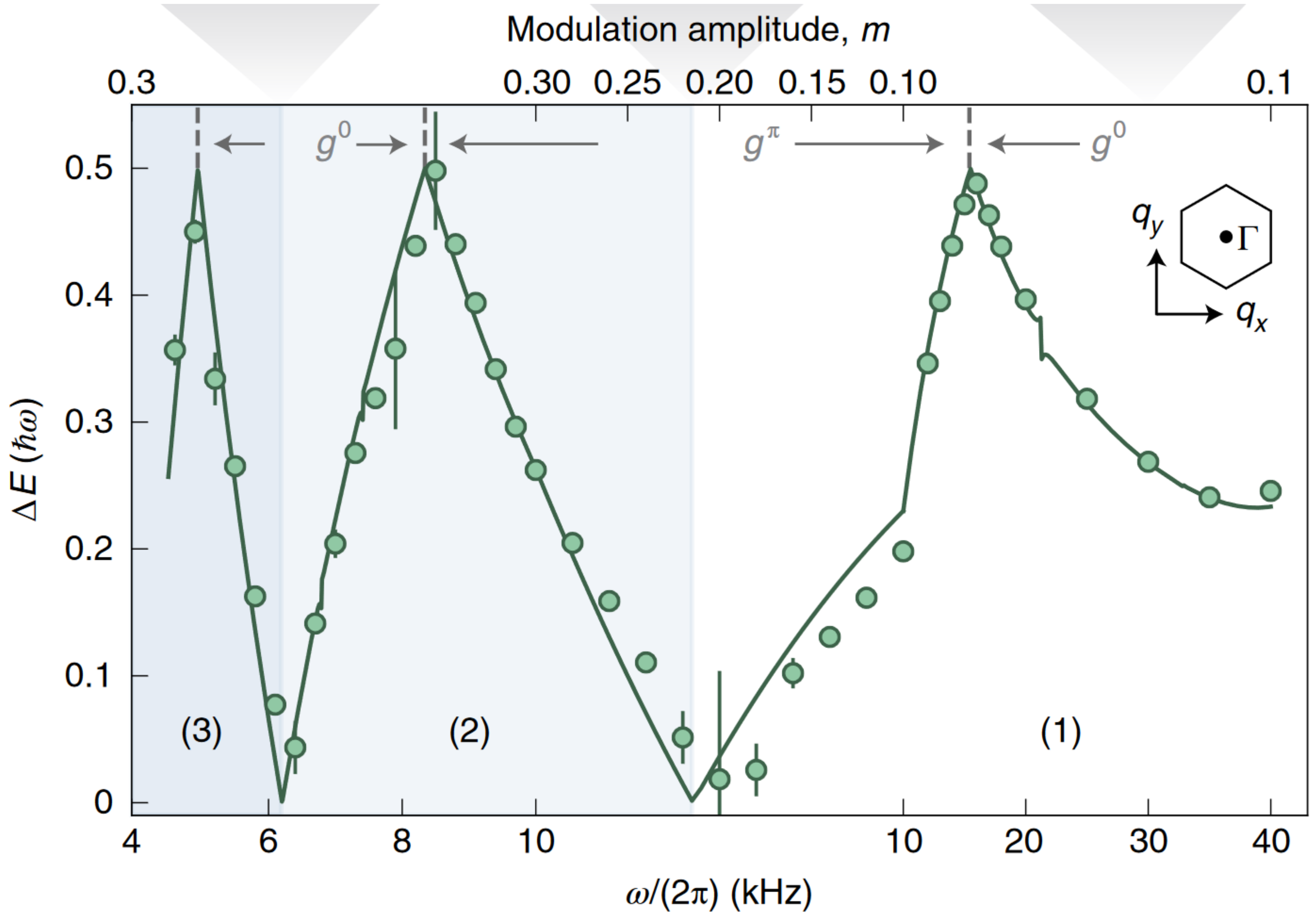
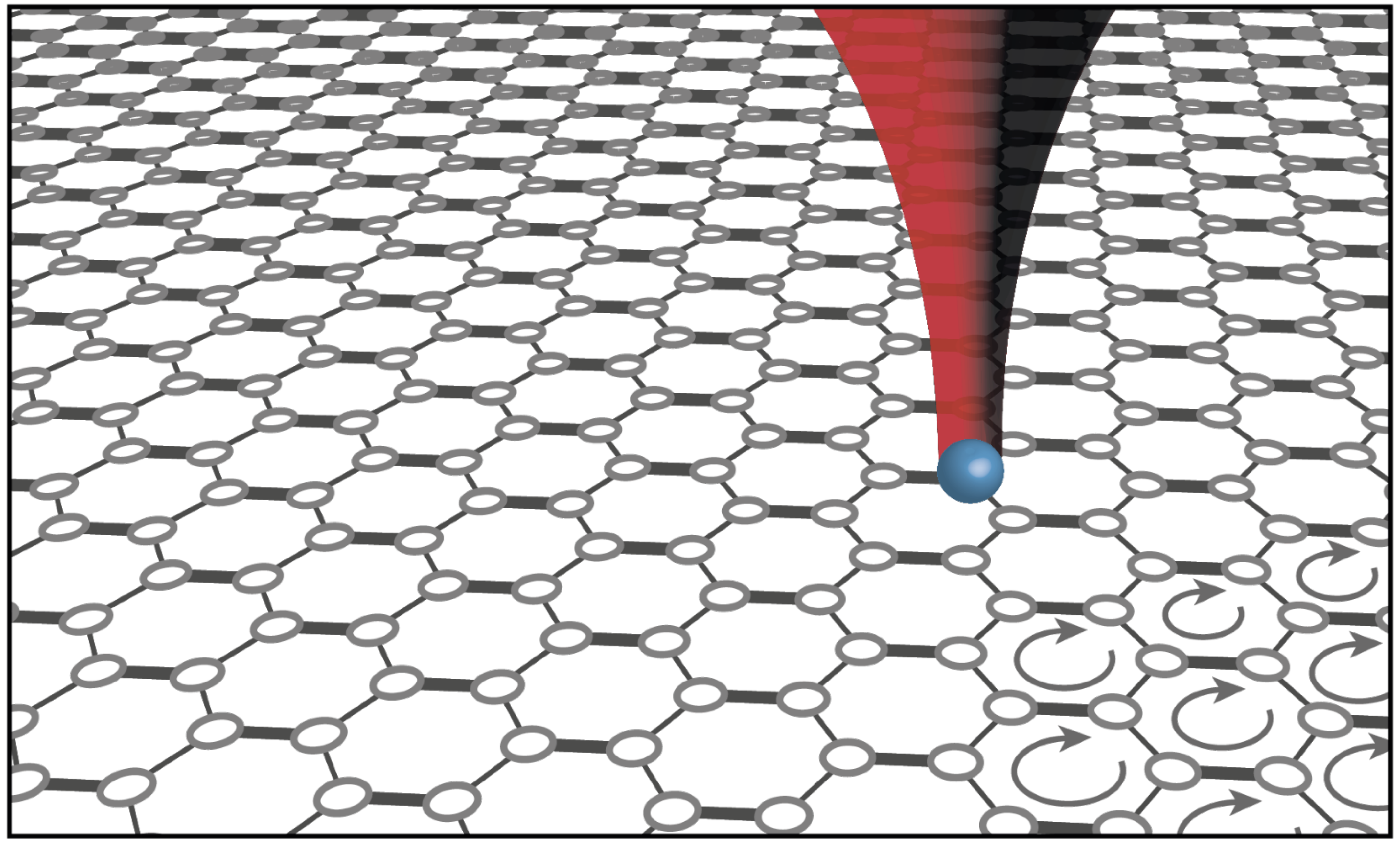
Realization of an anomalous Floquet topological system with ultracold atoms

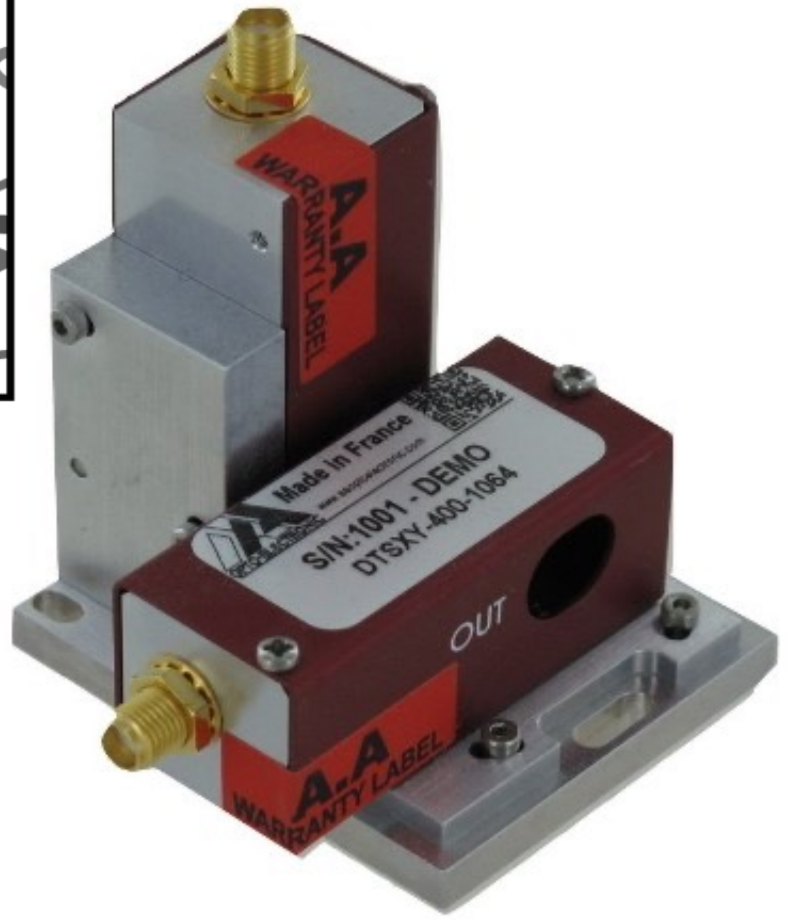
Karen Wintersperger[1,2], Christoph Braun[1,2,3], F. Nur Ünal[4,5], André Eckardt[4,6], Marco Di Liberto[7], Nathan Goldman[7], Immanuel Bloch[1,2,3] and Monika Aidelsburger[1,2] ✉

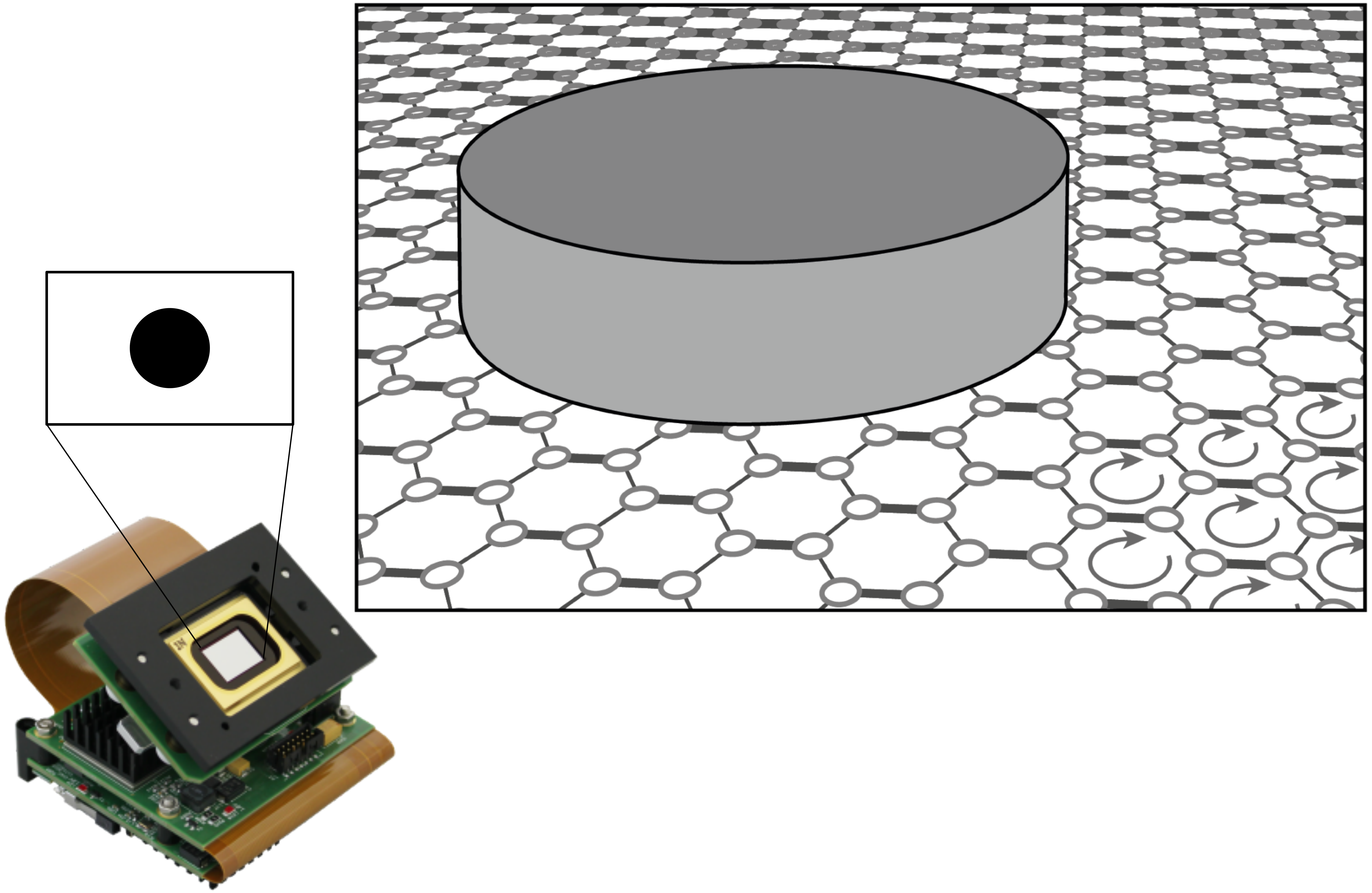




Preparation of initial state







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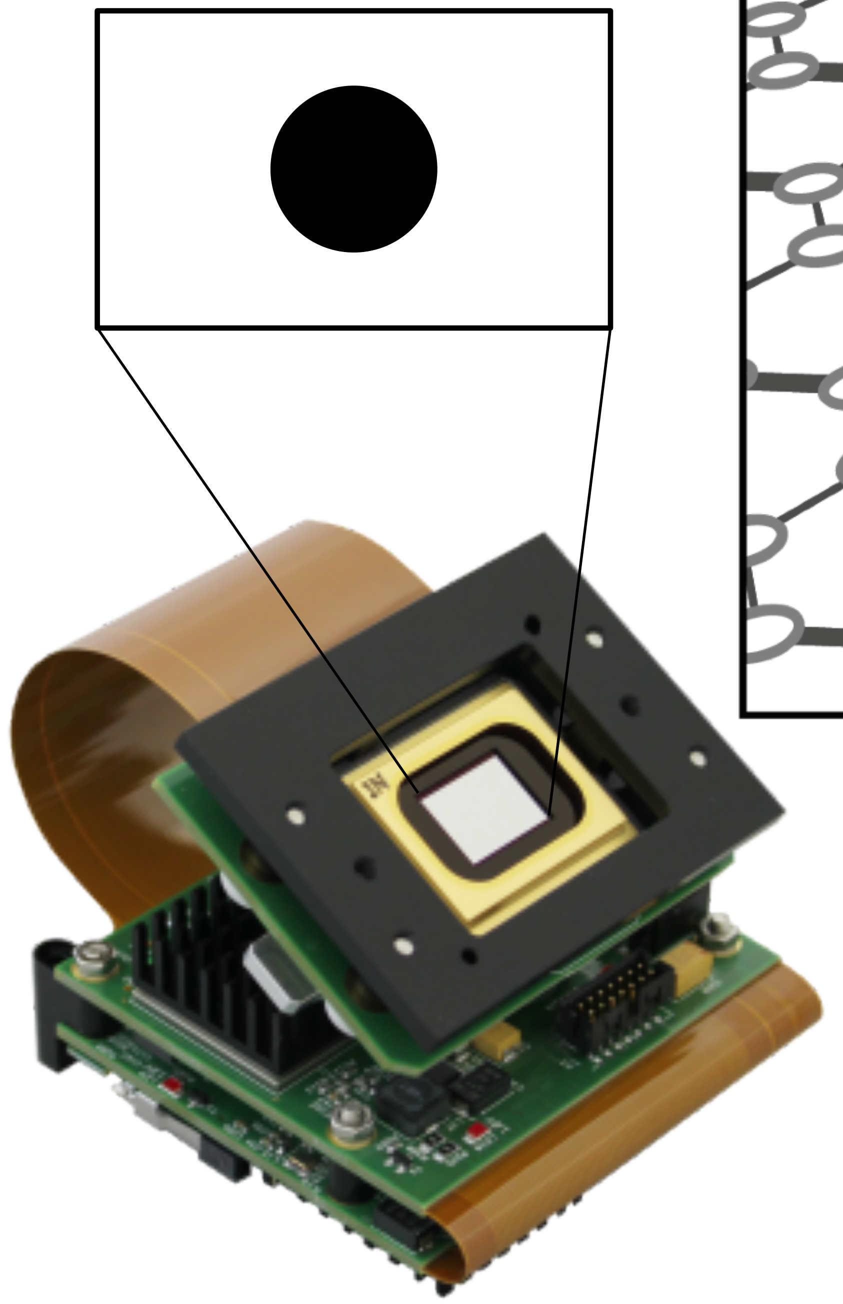
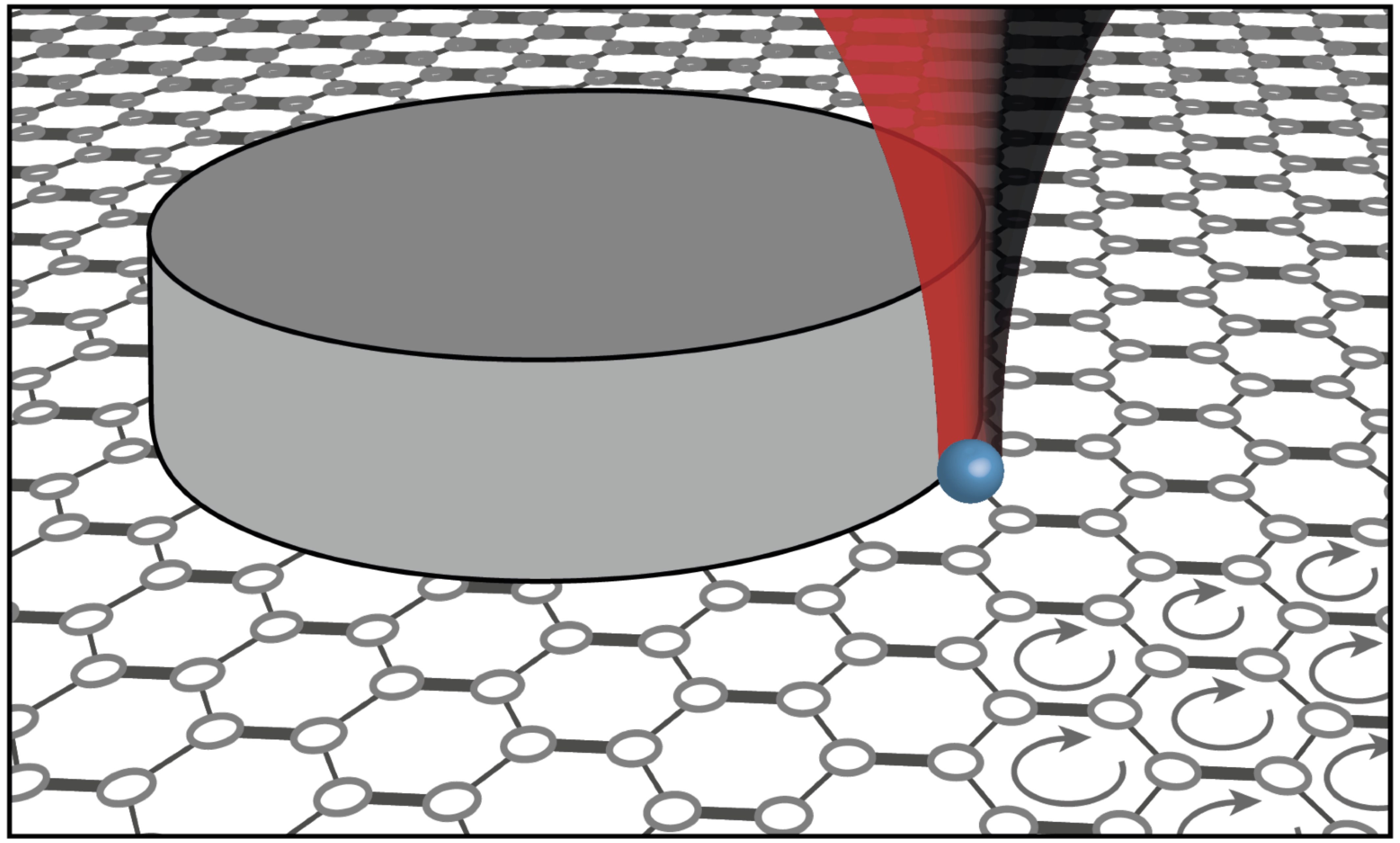
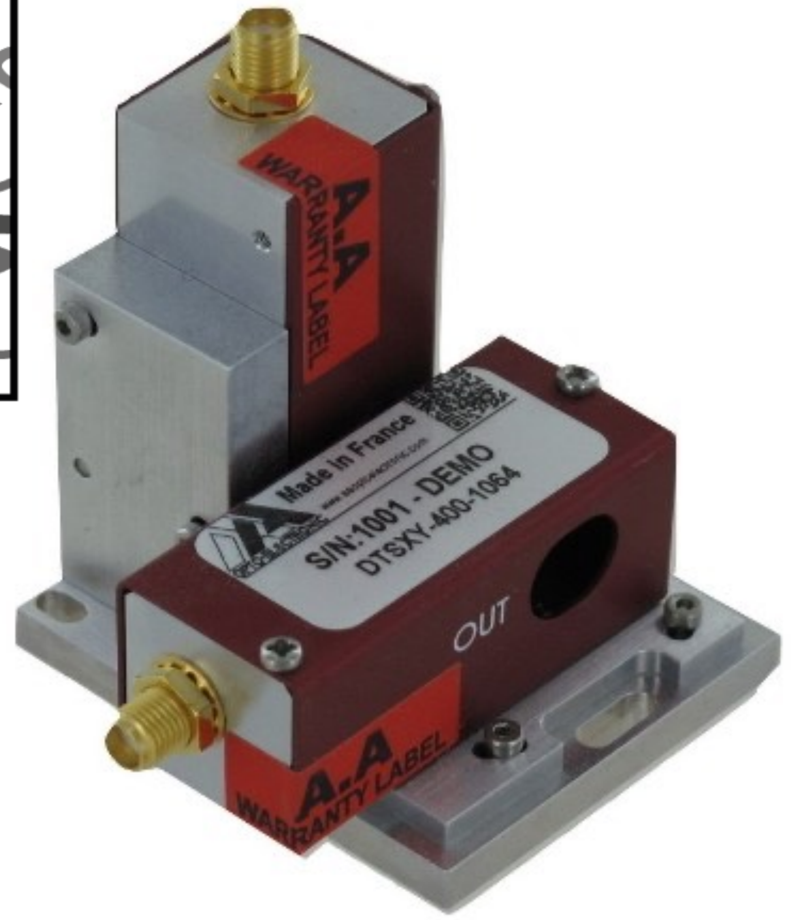
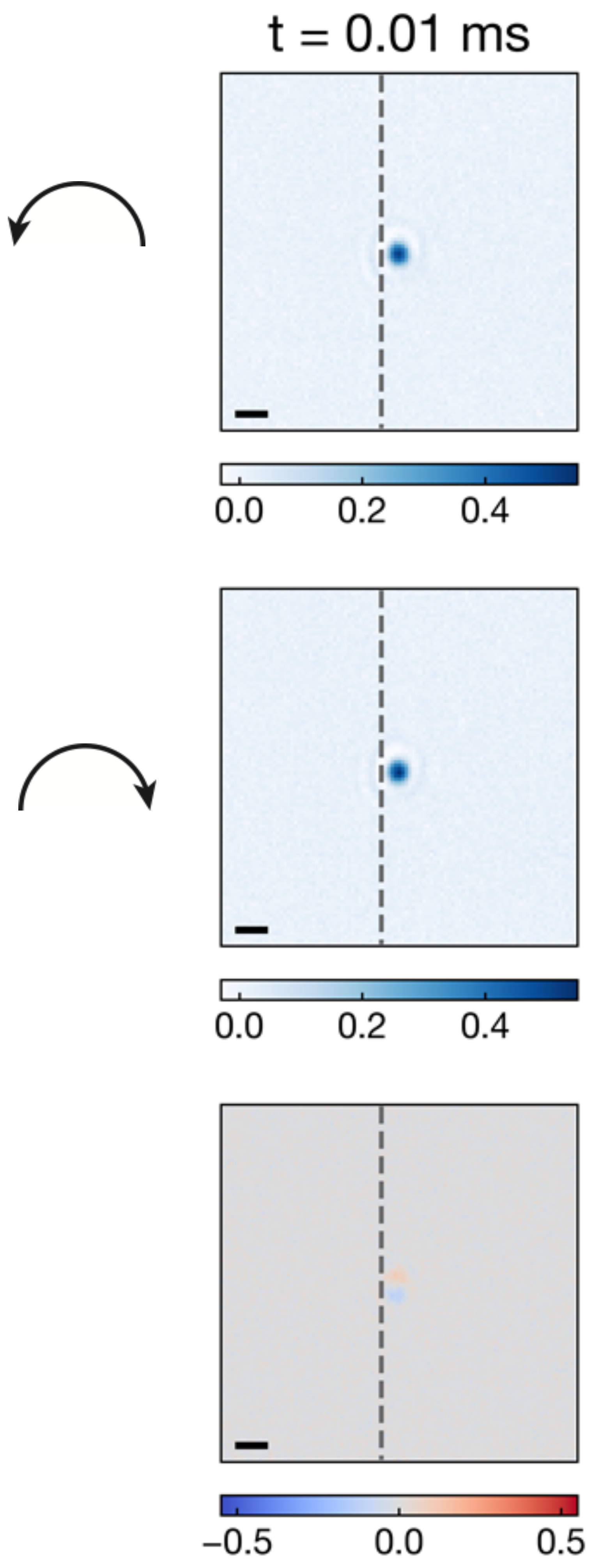






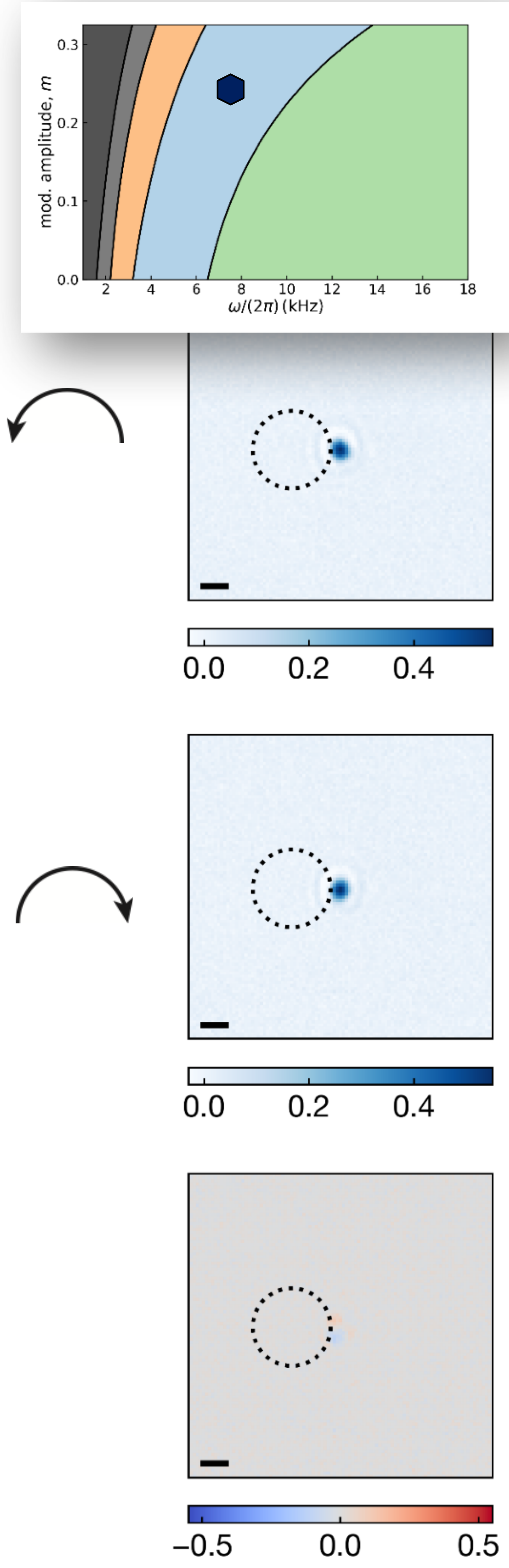
Edge state in the Anomalous regime



C. Braun et al. arXiv:2304.01980

Photonic systems: L. J. Maczewsky et al., Nat. Commun. **8**, 13756 (2017).

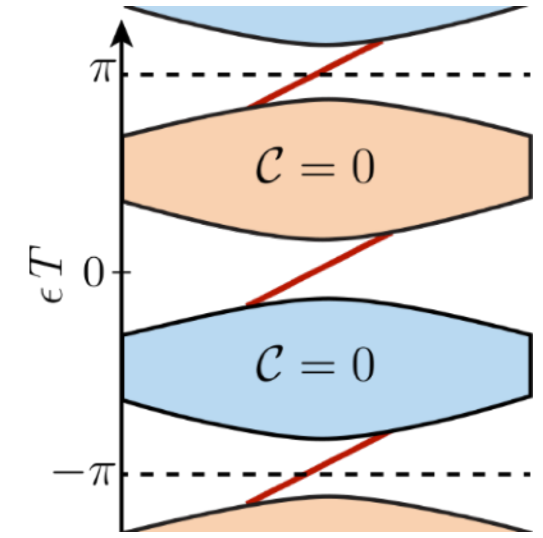
Probing Topology in real space



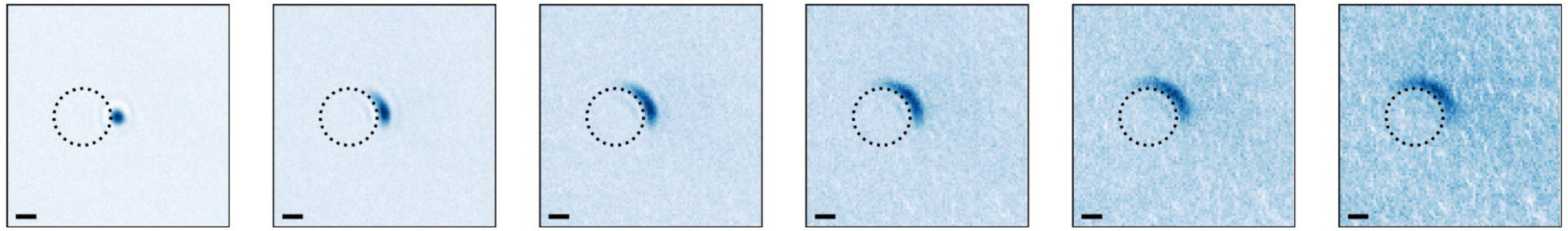
Edge state in the Anomalous regime

Summary and Outlook

- Experimental realization of an anomalous Floquet system
- Experimental observation of edge states



K. Wintersperger, et al.
Nat Phys. **16**, 1058-1063 (2020)

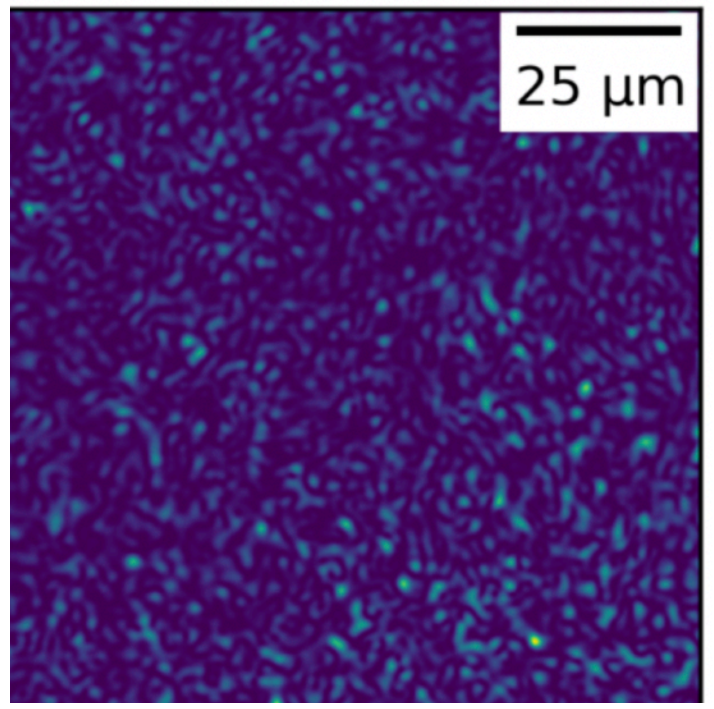


C. Braun et al. arXiv:2304.01980

Photonic systems: L. J. Maczewsky et al., Nat. Commun. **8**, 13756 (2017).

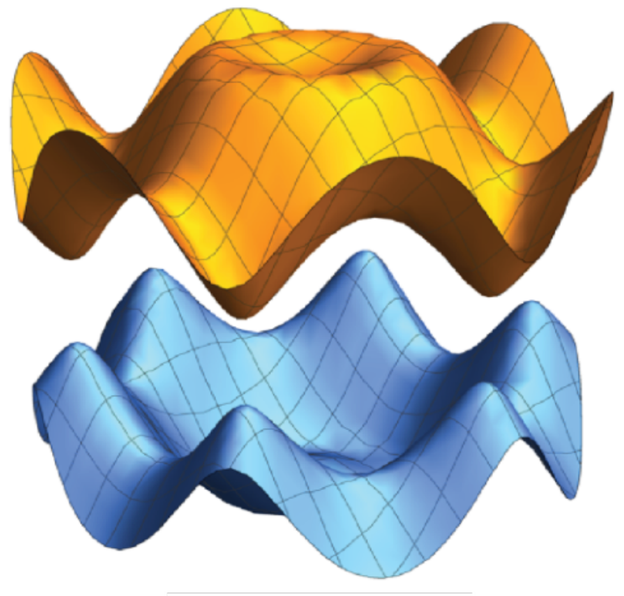
Outlook:

- Interplay of topology & disorder & interactions
- Optical speckle potential



P. Titum et al. Phys. Rev. X **6**, 021013 (2016)

- Band dispersion in anomalous regime



T. Sedrakyan et al., PRL. (2015)