Rydberg atoms in optical tweezers

Lecture 1: Dipolar interactions between atoms


Lecture 3: Many-body physics with Rydberg atoms: spin models and transport
Recap of Lecture 1

Toy Model

- $e^-$
- $\mathbf{E}_{\text{lat}}$
- $e^+$
- $g^-$
- $g^+$

$\frac{d_1 d_2}{R^3}$

$C_6 / R^6$

$C_3 / R^3$

$e^-$

$g^+$

$g^-$

$\mathbf{E}$

$\mathbf{E}_0$

DC field

Induced dipole

Note:

$\mathbf{E}_0 - \mathbf{E}$

$\mathbf{E}_0 - \mathbf{E}

\langle \mathbf{S} | d | \mathbf{S} \rangle \neq 0$

Stark
Goal: many-body physics and QIP with individual atoms

Addressable!!

\[ \omega \sim 50 \]

\[ \omega = \frac{C_6}{R^6} \]

\[ R \sim 5 \mu m \]

\[ \frac{C_6}{R^6} \sim 0.1 \ H \]

\[ \frac{C_3}{R^3} \sim \]
Vander Waals vs Resonant interaction

\[ \text{1 eV} \approx 400 \text{THz} \]

\[ \frac{C_6}{r^6} \sim \frac{C_3}{R^3} \approx \text{few Hz} \]

\[ \frac{C_6}{r^6} \sim \left( \frac{d^2}{r^3} \right)^2 \frac{1}{2\hbar \omega_0} = \frac{C_3}{R^3} \]

\[ \frac{C_3}{R^3} \ll 1 \]

\[ \text{Note: All other also have magnetic dipole interaction} \]

\[ \text{But: } \frac{\mu B^2}{R^3} \ll \frac{C_6}{r^6} \]
Outline

1. Experimental considerations: arrays of individual atoms

2. "Rydbergology": scalings, interactions, blockade...

3. Measurement of interactions between Rydberg atoms: towards many-body physics

4. Application of Rydberg blockade to QIP
Outline

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Optical dipole trap

Classical

$d = \alpha \mathbf{E}$
Optical dipole trap

Classical

\[ E \]

\[ d = \alpha E \]

Harmonic oscillator model

\[ \alpha(\omega) \]

Interaction atom - light

\[ U(x) \sim -\alpha E(x)^2 \]

\[ \Rightarrow \text{Conservative POTENTIAL} \]

Elastically bound e⁻ model:

\[ m \ddot{x} = -m\omega_0^2 x - (\frac{1}{\omega - \omega_0})^2 \]

\[ \alpha \sim \frac{q^2}{m} \frac{1}{\omega_0^2 - \omega^2 - i\omega} \]

\[ |\omega - \omega_0| \gg 1 \]

\[ \alpha \sim \frac{1}{\omega_0^2 - \omega^2} \]

\[ U \sim -\frac{1}{2} \cdot d \cdot E = -\frac{1}{2} \alpha E^2 \]

\[ |\psi > \sim U_{\text{min}} \quad E_{\text{max}} \]

\[ \alpha < 0 \Rightarrow U_{\text{min}} \quad E_{\text{max}} \text{ min.} \]
Optical dipole trap

Classical

\[ d = \alpha E \]

Quantum

\[ \hbar \Omega = d \cdot E \]
\[ d = \langle e | \hat{D} | g \rangle \]

Harmonic oscillator model

\[ \alpha(\omega) \]

Interaction atom - light

\[ U(x) \sim -\alpha E(x)^2 \]

\[ \Rightarrow \text{Conservative POTENTIAL} \]
Optical dipole trap

Classical

\[ \mathbf{d} = \alpha \mathbf{E} \]

Harmonic oscillator model

\[ \alpha(\omega) \sim \frac{1}{(\omega_0 - \omega)} \]

Interaction atom - light

\[ U(x) \sim -\alpha E(x)^2 \]

⇒ Conservative POTENTIAL

Quantum

\[ \hbar \Omega = \mathbf{d} \cdot \mathbf{E} \]

\[ \mathbf{d} = \langle e | \hat{\mathbf{D}} | g \rangle \]

\[ \omega_0 > \omega \]

\[ |e\rangle \quad \Delta = \omega - \omega_0 \]

\[ |g\rangle \quad \frac{\hbar \Omega^2}{4\Delta} \]

Trap depth \( \sim 100 \mu K - 1 mK \)

⇒ cold atoms

\[ P \sim 1 W \sim 100 \mu m \]
Optical tweezers: trapping in 3D

High field seekers $\omega < \omega_0$

Gaussian beam

$z_R = \frac{\pi w^2}{\lambda}$

$\sim w$
Optical tweezers: trapping in 3D

High field seekers $\omega < \omega_0$

Gaussian beam

Diffraction limited optics $w \sim \lambda$

Trapping volume $\sim \pi \lambda^3$

$z_R = \frac{\pi w^2}{\lambda}$

$\text{Ex: } 1 \text{ mW on } 1 \mu\text{m}$

$\text{Trap depth } = 1 \text{ mK}$

$\text{NA} = \sin \alpha$

$w \sim \lambda/\text{NA}$
Optical tweezers: trapping in 3D


Dipole trap light 850 nm

$w \sim \frac{\lambda}{2 \text{NA}}$

aspheric lens

$\text{NA} = 0.5$

$f = 10 \text{ mm}$

$w \sim 1 \mu m$

Volume $\sim 1 \mu m^3$
Single atoms in optical tweezers


Fluorescence 780nm

Dipole trap light 850 nm

Dichroic mirror

Reservoir = cold atoms $^{87}$Rb

$T \sim 100 \, \mu$K
Single atoms in optical tweezers


Dipole trap light 850 nm

Fluorescence 780 nm

Dichroic mirror
Single atoms in optical tweezers


Fluorescence 780nm

Dipole trap light 850 nm

Dichroic mirror

Non-deterministic single-atom source
Light-assisted collisions prevents 2 atoms...

1. $k \rightarrow \uparrow \rightarrow \downarrow \Rightarrow \alpha \rightarrow \pi$

2. $780\text{nm} \rightarrow 950$

$F = -\alpha \nu$

Cooling / heating

Doppler $T \sim h \nu$

$1\pi = \Delta \pi \gg \nu$

$\pi \rightarrow \uparrow \rightarrow 1\pi$

$s - s$

$s + p - p$

$780\text{nm} - p$

$s - s$
Which atoms?

- Laser cooled
- Single atom
Holographic 2D arrays of tweezers

Spatial Light Modulator (liquid crystals) Reconfigurable

SLM pattern

Iterative algorithm
[Gerchberg – Saxton (1972)]

\[ |\text{FT}[e^{i\varphi(x, y)}]|^2 \]

Bergamini, JOSA B 21, 1889 (2004)
Nogrette, PRX 4, 021034 (2014)
Holographic 2D arrays of tweezers

Spatial Light Modulator (liquid crystals) 
Reconfigurable

SLM pattern
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[Gerchberg – Saxton (1972)]

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Bergamini, JOSA B 21, 1889 (2004)
Nogrette, PRX 4, 021034 (2014)
Atom-by-atom assembling of 2D arrays

**Problem:** stochastic loading ($\rho \sim 0.5$)
Atom-by-atom assembling of 2D arrays

Problem: stochastic loading ($\rho \sim 0.5$)

Solution: sort atoms in arrays

Atom-by-atom assembling of 2D arrays

**Problem:** stochastic loading ($p \sim 0.5$)

**Solution:** sort atoms in arrays


Moving atoms with a tweezers

![Graph showing the relationship between $U/h$, MHz, and $x$, µm, with $t/T = -0.04$. The graph includes a line labeled 'SLM light' and another labeled 'Moving tweezer + SLM'. The equation $p \sim 0.993(1)$ is also shown.](image)
Gallery of assembled 2D arrays... (single-shot images...)

Initial

14 moves
15 moves
53 moves
41 moves
43 moves

Final

Gallery of assembled 2D arrays... (single-shot images...)

Initial

14 moves

15 moves

53 moves

41 moves

43 moves

Final

Now ~ 200 atoms

1D

~100 μm

Schymik, PRA 2020
Gallery of assembled 2D arrays... (single-shot images...)

Initial

14 moves 15 moves 53 moves 41 moves 43 moves

Final

Now ~ 200 atoms

1D

2D

~100 μm

Schymik, PRA 2020
Gallery of assembled 2D arrays... (single-shot images...)

Initial

- 14 moves
- 15 moves
- 53 moves
- 41 moves
- 43 moves

Final

- Now ~ 200 atoms

1D

- ~100 μm

2D

- Schymik, PRA 2020
Gallery of assembled 2D arrays... (single-shot images...)

Initial
- 14 moves
- 15 moves
- 53 moves
- 41 moves

Final
- Now ~ 200 atoms

1D
- ~100 μm

2D
- Schymik, PRA 2020

L. da Vinci
Now widely used,... with variants (1D, 2D & 3D)

Lukin (Harvard), 2016

Rb
Birkl (Germany)

Ahn (Korea)

Sr

Yb

~300 atoms
arXiv:2012.12281

Endres (Caltech) Kauffman (JILA)
2018

Thompson (Princeton)
2018
It also works in 3d!

Di Leonardo, Optics Express 15, 1913 (2007)

**Averaged fluorescence**

imaged “slice-by-slice”


It also works in 3d!

Di Leonardo, Optics Express 15, 1913 (2007)

Averaged fluorescence imaged “slice-by-slice”

Assembled Pyrochlore lattice

Plane 1
Plane 2
Plane 3

Front view
Side view

$N = T \sim 20$ s. lifetime of an atom in a tweezer

$N$ atoms $\rightarrow T/N$: lifetime of a config.

$L \sim N$ moves $\Delta t$.

$N \sim 1000$

Questions?
Outline

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References:


Special Issue on Rydberg Atomic Physics, J. Phys. B (2016) contains many reviews
Alkali: 1 external electron

\[ 1s^2 2s^2 \ldots (n - 1)p^6 ns \]
"Rydberg atom" = a highly excited atom (e.g. Rb)
“Rydberg atom” = a highly excited atom (e.g. Rb)

\[ E_n = -\frac{R_y}{(n - \delta_{nlj})^2} \]

\[ \delta_{nlj} \approx 3 \]

Quantum defects (experimental)

\[ l = 0 \]

\[ n^* = n - \delta \]

\[ E_n = -\frac{13.6}{n^*^2} \]
“Rydberg atom” = a highly excited atom (e.g. Rb)

\[ E_n = -\frac{R_y}{(n - \delta_{nlj})^2} \]

Quantum defects (experimental)

For Rb:

\[ n \geq 30 \]

<table>
<thead>
<tr>
<th>( L )</th>
<th>( J )</th>
<th>( \delta_{L,J} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/2</td>
<td>3.131</td>
</tr>
<tr>
<td>1</td>
<td>1/2</td>
<td>2.654</td>
</tr>
<tr>
<td></td>
<td>3/2</td>
<td>2.641</td>
</tr>
<tr>
<td>2</td>
<td>3/2</td>
<td>1.348</td>
</tr>
<tr>
<td></td>
<td>5/2</td>
<td>1.346</td>
</tr>
<tr>
<td>3</td>
<td>5/2</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>7/2</td>
<td>0.016</td>
</tr>
</tbody>
</table>
“Rydberg atom” = a highly excited atom (e.g. Rb)

\[ \langle r \rangle \sim n^2 a_0 \]

\[ n \gg 1 \]

Rydberg states

\[ |n, l\rangle \]

Energy (THz)

\[ n = 5 \]

\[ n = 6 \]

\[ n = 7 \]

\[ n = 8 \]

\[ n = 9 \]

\[ n = 6 \]

\[ n = 7 \]

\[ n = 8 \]

\[ n = 9 \]

\[ l = 0 \]

\[ l = 1 \]

\[ l = 2 \]

\[ \sim 100 \text{ nm} \]

\[ e^- \text{ core} \]
Rydberg atoms are huge...

The classical diameter of Rydberg atoms is given by $4a_0(n-\delta_0)^2$, where $a_0$ is the Bohr radius. For higher principal quantum numbers $n$, the diameter increases dramatically, reaching the size of red blood cells for large $n$. Red blood cells typically have a diameter of around 8 μm, leukocytes around 10 μm, and thrombocytes around 2 μm. Bacteriophages are much smaller, with diameters around 2 mm, while typical bacteria range from 1 to 5 μm. Viruses, such as the bluetongue virus, are even smaller, with diameters around 125 nm.
The Bohr model to recover quickly the scalings...

\[ \begin{align*}
\frac{m v^2}{r} &= \frac{e^2}{r^2} \\
\mu r c &= n^2 \\
\{ \text{Bohr} \}
\end{align*} \]

\[ \rightarrow \quad r = \left( \frac{m^2 \hbar^2}{e^2} \right)^{\frac{1}{2}} \]

\[ \left( e^2 = \frac{q^2}{4\pi\epsilon_0} \right) \]
“Rydberg atom” = a highly excited atom (e.g. Rb)

\[ \langle r \rangle \sim n^2 a_0 \]

\( n \gg 1 \)

\( n = 5 \)
\( n = 6 \)
\( n = 7 \)
\( n = 8 \)
\( n = 9 \)

\( l = 0 \)
\( l = 1 \)
\( l = 2 \)

Long lifetime \( \tau \sim n^3 \)
\[ \Rightarrow n > 60, \tau > 100 \mu s \]
“Rydberg atom” = a highly excited atom (e.g. Rb)

\[ \langle r \rangle \sim n^2 a_0 \]

\[ \sim 100 \text{ nm} \]

\[ n \gg 1 \]

**Long lifetime** \( \tau \sim n^3 \)

\[ \Rightarrow n > 60, \tau > 100 \mu s \]

**Large transition dipole:**

\[ \langle d \rangle (n, l) \rightarrow (n, l \pm 1) \rangle \sim n^2 e a_0 \]
“Rydberg atom” = a highly excited atom (e.g. Rb)

\[ \langle r \rangle \sim n^2 a_0 \]

\[ \sim 100 \text{ nm} \]

\[ n \gg 1 \]

\[ \Rightarrow \text{Long lifetime } \tau \sim n^3 \]

\[ \Rightarrow n > 60, \tau > 100 \mu s \]

\[ \text{Large transition dipole: } d[(n, l) \rightarrow (n, l \pm 1)] \sim n^2 e a_0 \]

\[ \Rightarrow \text{Exaggerated properties:} \]

- strong interaction
- strong coupling to fields (DC, MW)
Rydberg’s have exaggerated properties

Do it as a problem.

Table 1. Properties of Rydberg states.

<table>
<thead>
<tr>
<th>Property</th>
<th>n-scaling</th>
<th>Value for 80S_{1/2} of Rb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binding energy $E_n$</td>
<td>$n^{-2}$</td>
<td>-500 GHz</td>
</tr>
<tr>
<td>Level spacing $E_{n+1} - E_n$</td>
<td>$n^{-3}$</td>
<td>13 GHz</td>
</tr>
<tr>
<td>Size of wavefunction $\langle r \rangle$</td>
<td>$n^2$</td>
<td>500 nm</td>
</tr>
<tr>
<td>Lifetime $\tau$</td>
<td>$n^3$</td>
<td>200 $\mu$s</td>
</tr>
<tr>
<td>Polarizability $\alpha$</td>
<td>$n^7$</td>
<td>-1.8 GHz/(V/cm)^2</td>
</tr>
<tr>
<td>van der Waals coefficient $C_6$</td>
<td>$n^{11}$</td>
<td>4 THz $\cdot\mu$m^6</td>
</tr>
</tbody>
</table>

\[ \Gamma = d_{ij}^3 \omega^3 \]

\[ \Gamma = 2\alpha \left( \frac{1}{n^3} \right)^3 n^4 \]

\[ c_c \sim n^n \]
Rydberg atoms: a few historical landmarks

1975  Spectroscopy using lasers (Gallagher, Kleppner, Haroche...)

1980 – 2000  Cavity Quantum Electrodynamics using Rydbergs

High Q cavity: photon lifetime > 1ms + large dipole ⇒
1 Rydberg interacts with 1 photon!

Haroche, Walther...

1998  Rydbergs meet cold atoms P. Pillet and T. Gallagher

Anderson, PRL 80, 249 (1998)
Mourachko, PRL 80, 253 (1998)

Diffusion of excitation faster than motion ⇒ correlations between all atoms

$k_B T \ll $ Interaction energy

⇒ $T < 1$ mK

$p + p \leftrightarrow s + s'$
$p + s \leftrightarrow s + p$

“Frozen” gas
A new era: the Rydberg Blockade idea

Fast Quantum Gates for Neutral Atoms

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S. L. Rolston
National Institute of Standards and Technology, Gaithersburg, Maryland 20899

R. Côté and M. D. Lukin

Dipole Blockade and Quantum Information Processing in Mesoscopic Atomic Ensembles

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L. M. Duan, D. Jaksch, J. I. Cirac, and P. Zoller
Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria
(Received 7 November 2000; published 26 June 2001)
A new era: the Rydberg Blockade idea
A new era: the Rydberg Blockade idea

If $\hbar \Omega \ll U_{\text{vdW}}$ : no excitation of $|rr\rangle$ $\Rightarrow$ blockade
A new era: the Rydberg Blockade idea

Blockade $\Rightarrow$ entanglement and gates!!
The first blockade experiments

Atomic ensembles

Gould, PRL 2004
Martin, PRL 2004
Weidemuller, PRL 2004
Pillet, PRL 2006
Raithel, PRL 2005
Pfau, PRL 2007

Individual atoms

IO Palaiseau

Blockade + collective excitation $\sqrt{2}$
Gaétan et al., Nat. Phys. 5, 115 (2009)

U. Wisconsin

Blockade
Urban et al., Nat. Phys. 5, 110 (2009)
1) **Blockade for Na atoms:**

\[
\frac{1}{VN} \left( \sum_{g=-g}^{g} h_{gg} e^{i\Delta t_{g}} \right)
\]

2) **Influence of Black-Body radiation on lifetime of Rydb. state.**

**Questions?**

\[
\begin{align*}
\Gamma_{n \to n-1} & \sim n^5 \\
\text{BB induced: } \Gamma_{n \to n-1} \times \overline{n}_\text{ph BB} & \sim 100 \mu s.
\end{align*}
\]
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Softwares to calculate interaction energies


Coherent optical Rydberg excitation ($n = 50 – 100$)

$^8_7\text{Rb}$

$E$

$|5s_{1/2}, F = 2, M = 2\rangle$

$6p_{3/2}$

1013 nm

420 nm

Global excitation
Coherent optical Rydberg excitation \((n = 50 - 100)\)

\[ |5s_{1/2}, F = 2, M = 2 \rangle \]

Effective Rabi frequency:

Light-shift:
Coherent optical Rydberg excitation ($n = 50 - 100$)

$^8\text{Rb}$

$\Omega$

$|5s_{1/2}, F = 2, M = 2\rangle$

Optical excitation ($\Omega = 0.5 - 5$ MHz)

Effective Rabi frequency:

Light-shift:
Microwave manipulations ($n = 50 – 100$)

\[ |5s_{1/2}, F = 2, M = 2 \rangle \]

$\Omega_{\text{MW}}$ 9.1 GHz

\[ |62d_{3/2}, F = 2, M = 2 \rangle \]

D. Barredo et al., PRL 114, 113002 (2015)
Questions ?
Interactions between Rydberg atoms

van der Waals

\[ \Delta E \sim \frac{C_6}{R^6} \]

\[ |ns, ns \rangle \]

Resonant interaction

\[ \Delta E \sim \frac{C_6}{R^6} \]

\[ |n' p, ns \rangle \]

\[ V \sim \frac{C_3}{R^3} \]

Interactions between Rydberg atoms

\[
\hat{V} \sim \frac{d_A d_B}{4\pi \epsilon_0 R^3}
\]

van der Waals

\[\Delta E \sim \frac{C_6}{R^6}\]

Collective excitation of two interacting Rydberg atoms

If $\hbar \Omega \ll U_{vdW}$: no excitation of $|rr\rangle \Rightarrow$ blockade

Dynamics governed by $\Omega$ only
Collective excitation of two interacting Rydberg atoms

\[ U_{\text{vdW}} = \frac{C_6}{R^6} \]

\[ \begin{align*}
\langle gg | D_1 + D_2 | \sqrt{\frac{1}{2}} (|rg\rangle + |gr\rangle) \rangle &= 2 \Omega \sqrt{2} \\
\frac{1}{\sqrt{2}} (|rg\rangle + |gr\rangle) &\quad \text{Collective oscillation between} \ |gg\rangle \ \text{and} \ \frac{1}{\sqrt{2}} (|rg\rangle + |gr\rangle) \\
\langle g | D_1 + D_2 | |g\rangle \rangle &= 2 \Omega \sqrt{2}
\end{align*} \]

with coupling \( \Omega \sqrt{2} \quad (N \text{ atoms} \Rightarrow \Omega \sqrt{N}) \)
From independent atoms to blockade \((62d_{3/2})\)

\[ P_{rr} = \left( \sin^2 \frac{\Delta t}{2} \right)^2 \]

\[ \hbar \Omega \gg U_{vdW} \]
From independent atoms to blockade ($62d_{3/2}$)

$$\hbar \Omega \gg U_{\text{vdW}}$$

$$\hbar \Omega \ll U_{\text{vdW}}$$
From independent atoms to blockade (62d_{3/2})

\[ P_{rr} \]

Blockade regime

\[ \hbar \Omega \gg U_{\text{vdW}} \]

\[ \hbar \Omega \ll U_{\text{vdW}} \]
From independent atoms to blockade (62d_{3/2})

$$P_{rr} = \varphi_r (1 - \varphi_c) + \varphi_c$$

$$P_{rg} + P_{gr}$$

Blockade regime
From independent atoms to blockade (62d\textsubscript{3/2})

Blockade regime

Freq. ratio = 1.41 \approx \sqrt{2}
From independent atoms to blockade ($62d_{3/2}$)

Blockade regime

\[ P_{rr} \]

\[ P_{rg} + P_{gr} \]

\[ \frac{1}{\sqrt{2}} (|rg\rangle + |gr\rangle) \]

\[ |gg\rangle \]
Collective excitation of two interacting Rydberg atoms

If $\hbar \Omega \approx U_{\text{vdW}}$ : dynamics governed by $\Omega$ and $U_{\text{vdW}}$

$$|\psi(t)\rangle = \alpha |gg\rangle + \beta \sqrt{\frac{1}{2}} (|gr\rangle + |rg\rangle) + \chi |gg\rangle$$
From independent atoms to blockade ($62d_{3/2}$)

$P_{rr}$

$P_{rg} + P_{gr}$

$h\Omega \approx U_{vdW}$

R (μm)

Pulse area $\Omega t$
From independent atoms to blockade ($62d_{3/2}$)

R ($\mu$m)

$P_{rrr}$

Theory (Schrödinger eq.)

$V = \Omega = 1$ MHz

time ($\mu$s)

Fit $\Rightarrow$ extract $U_{vdW}$
Measurement of vdW interaction between 2 atoms

Theory curves: direct diagonalization (dipole-dipole interaction)

No adjustable parameter!

Many-body physics with arrays of atoms

Lecture 1: Dipolar interactions between atoms


Lecture 3: Many-body physics with Rydberg atoms: spin models and transport