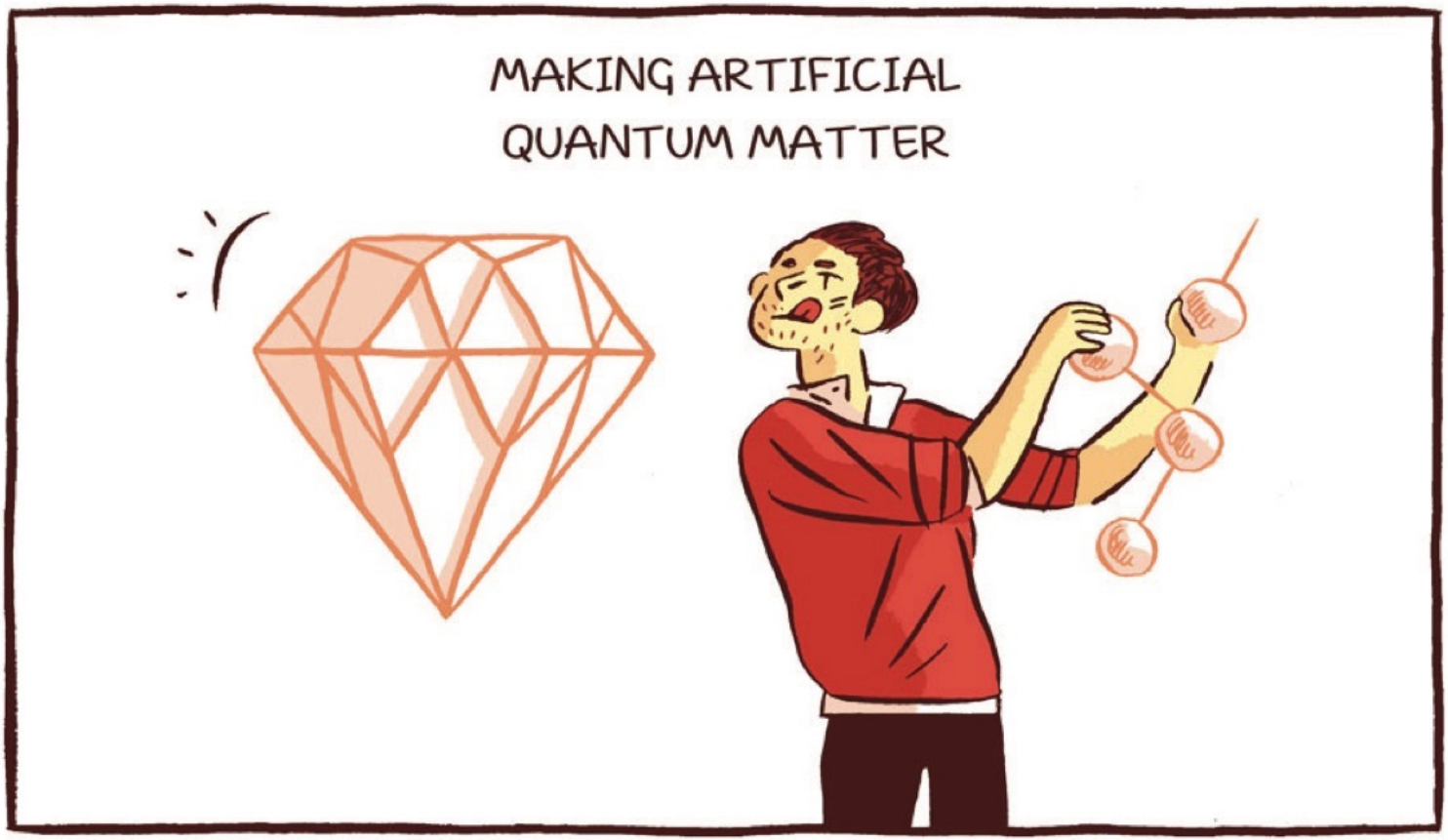
Rydberg atoms in optical tweezers:

many-body physics and QIP

Antoine Browaeys

Laboratoire Charles Fabry,
Institut d'Optique, CNRS, FRANCE

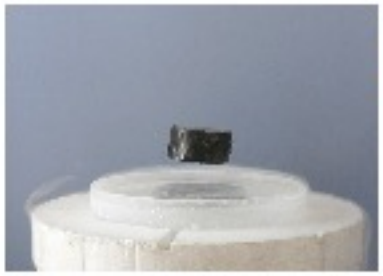
Quantum state engineering: create controlled quantum states

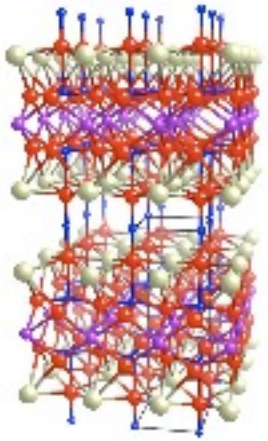


Héloïse Chochois, Labex PALM

Quantum state engineering: create controlled quantum states

Many-body physics

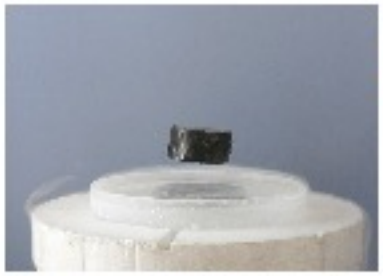


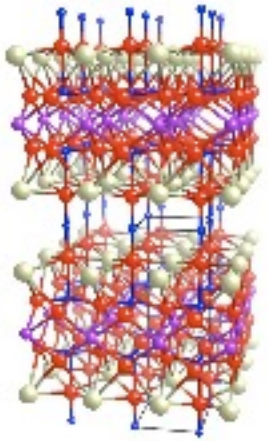


Quantum simulation.

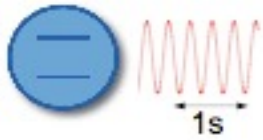
Quantum state engineering: create controlled quantum states

Many-body physics

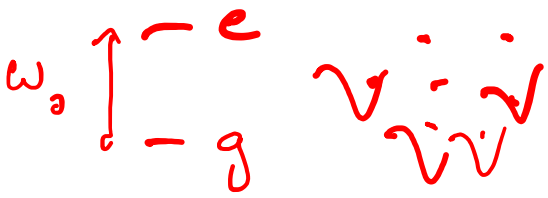




Quantum metrology

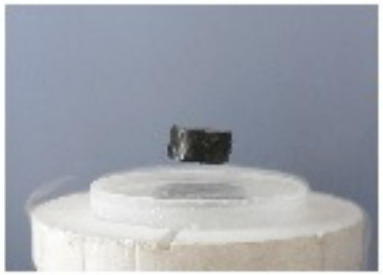


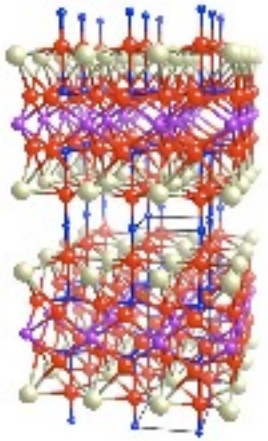
$$\frac{\Delta\nu}{\nu_0} \propto \frac{1}{\sqrt{N}} \quad \text{qu. states} \quad \frac{\Delta\nu}{\nu_0} \propto \frac{1}{N}$$



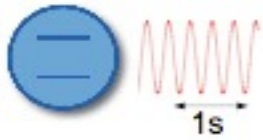
Quantum state engineering: create controlled quantum states

Many-body physics





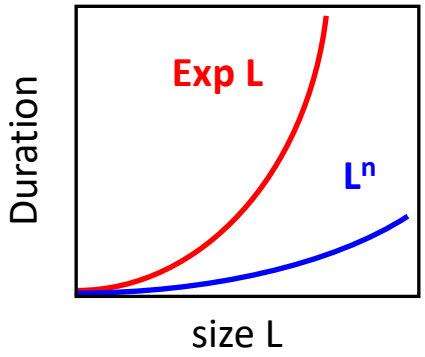
Quantum metrology



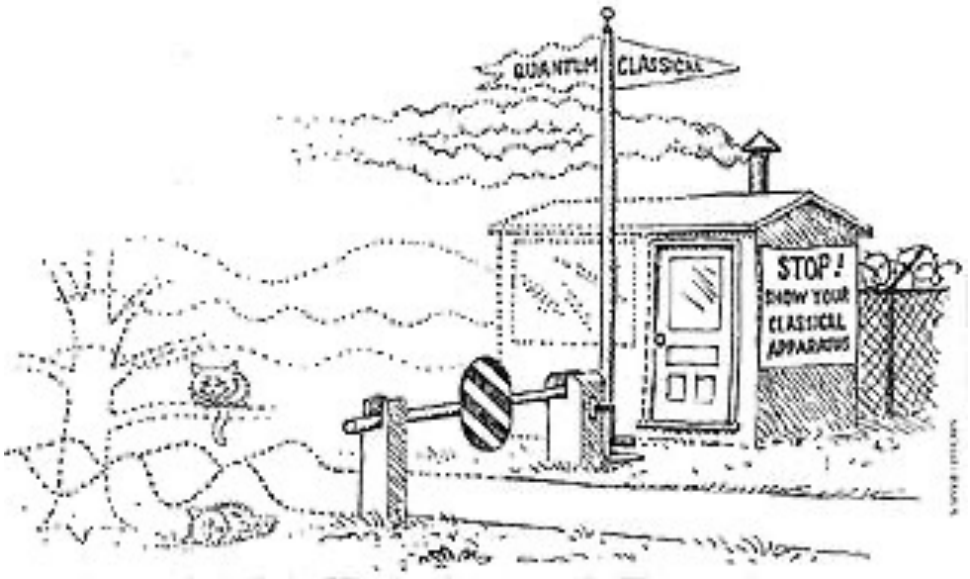
$$\frac{\Delta\nu}{\nu_0} \propto \frac{1}{\sqrt{N}} \quad \text{qu. states} \quad \frac{\Delta\nu}{\nu_0} \propto \frac{1}{N}$$

Quantum information

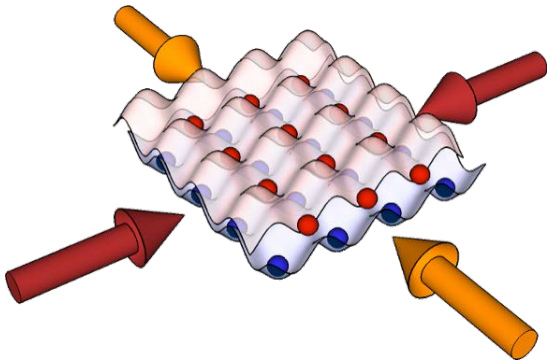
Message



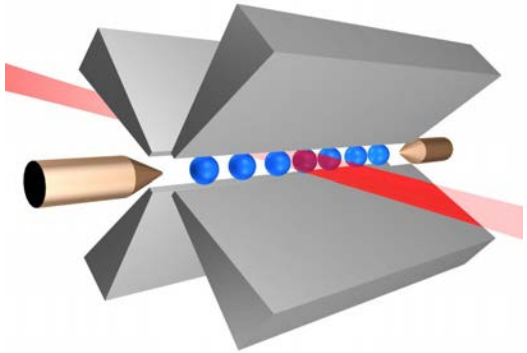
Transition quantum / classical



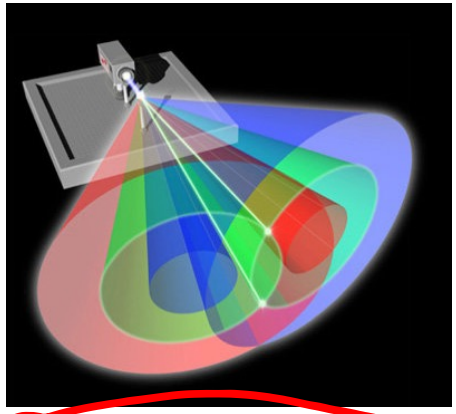
Quantum state engineering with individual systems



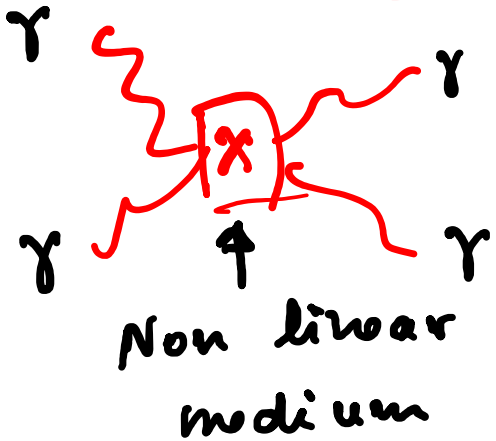
Cold atoms and molecules



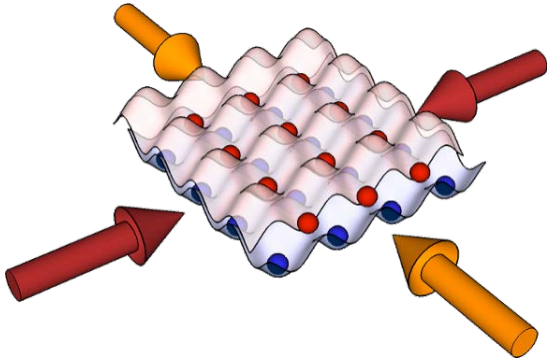
Trapped ions



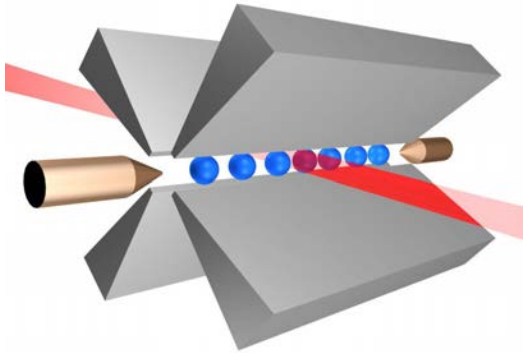
Photons in NL media



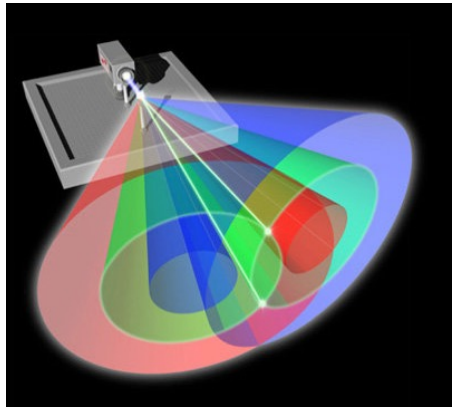
Quantum state engineering with individual systems



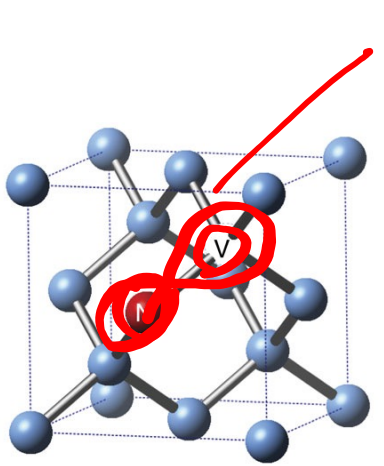
Cold atoms and molecules



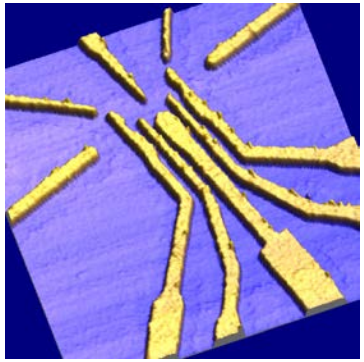
Trapped ions



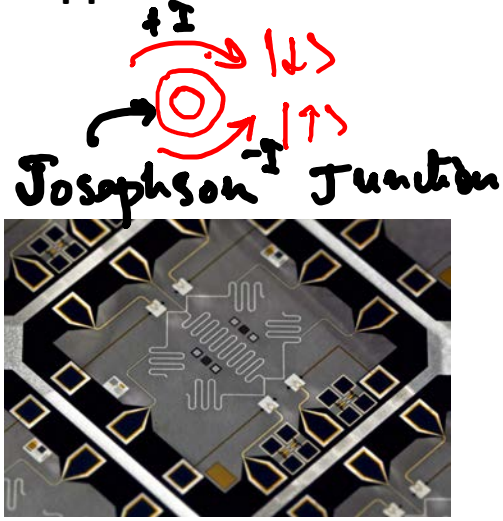
Photons in NL media



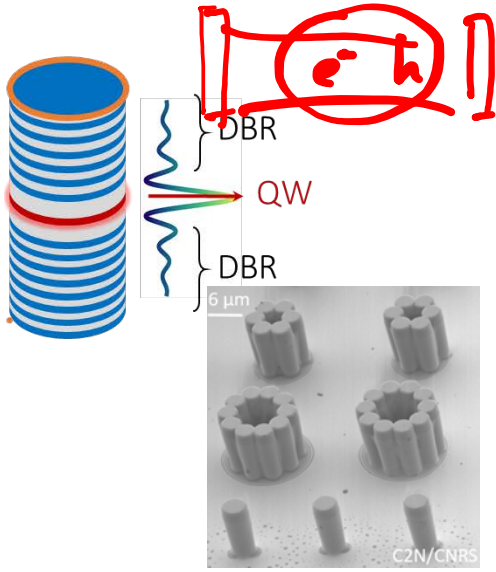
NV centers



Quantum dots

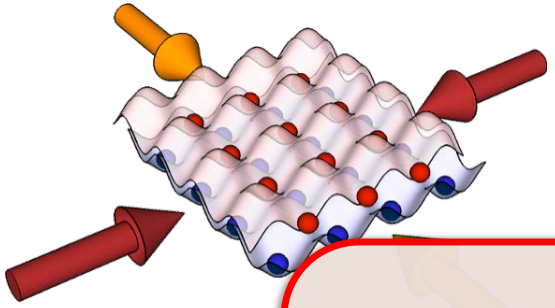


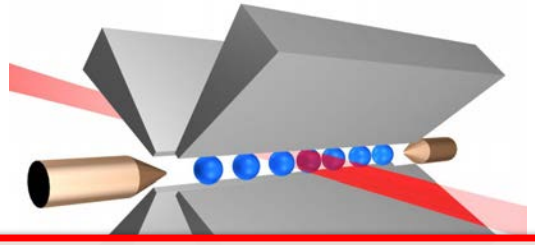
Superconducting qubits

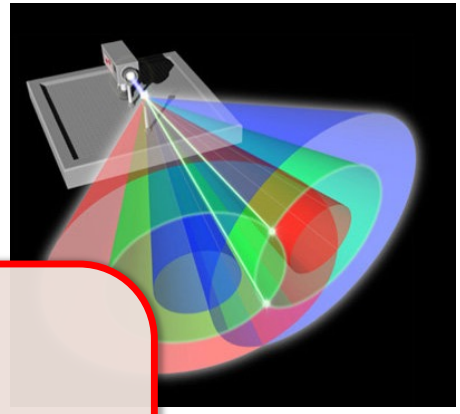


Polaritons in ½ cond.

Quantum state engineering with individual systems







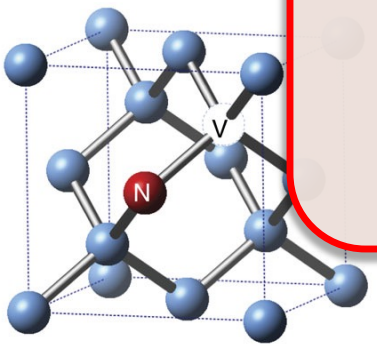
Cold atoms and molecules ions **Photons in NL media**

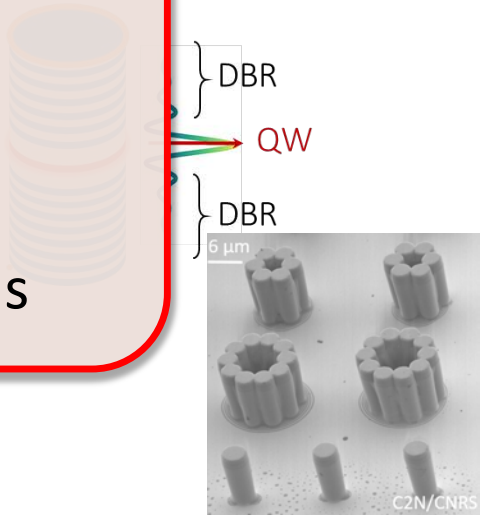
Two-level systems to encode a spin:

$|\uparrow\rangle$

$|\downarrow\rangle$

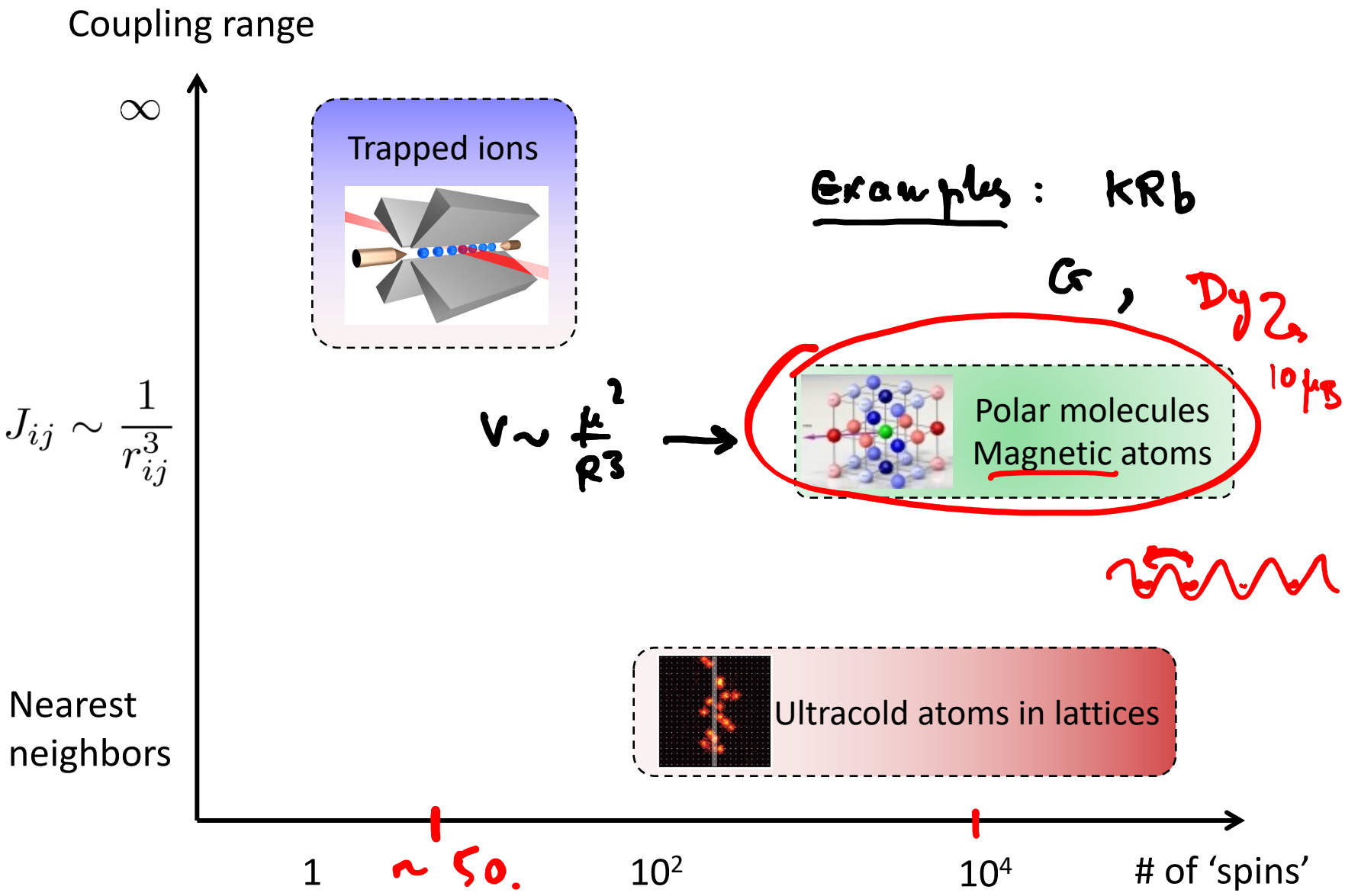
Addressable + **controlled** interactions





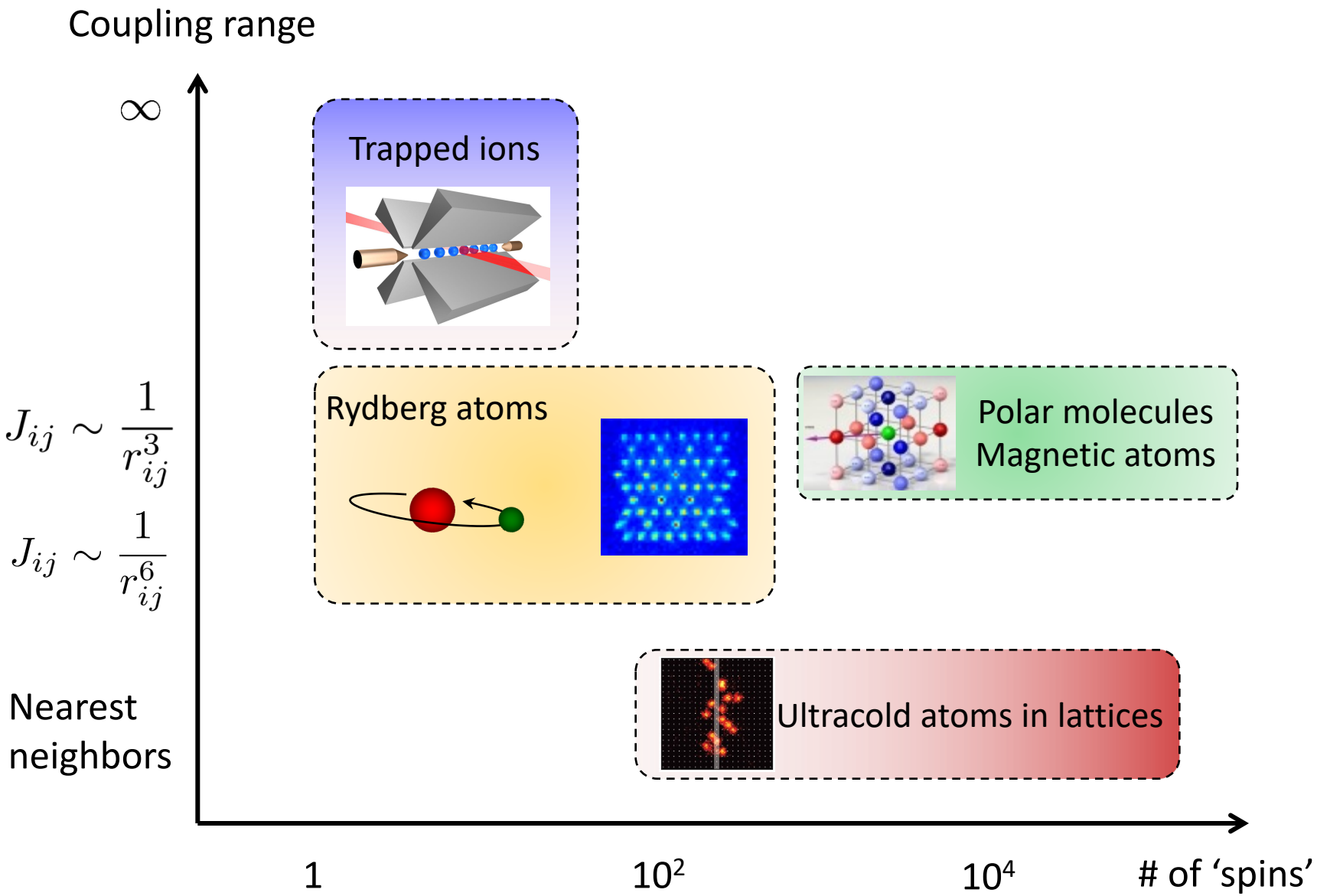
NV centers **Quantum dots** **Superconducting qubits** **Polaritons in ½ cond.**

“Atomic” arrays (ignoring bulk gases…)



See e.g. Hazzard *et al.*, PRA **90**, 063622 (2014)

“Atomic” arrays (ignoring bulk gases…)



See e.g. Hazzard *et al.*, PRA **90**, 063622 (2014)

“Atomic” arrays (ignoring bulk gases…)

Scalable: beyond 100 particles

Addressability: local manipulation and measurement $\langle \sigma_i^\alpha \rangle, \langle \sigma_i^\alpha \sigma_j^\beta \rangle, \ldots$

$\langle \sigma_i \sigma_j \sigma_k \rangle$

Coupling range

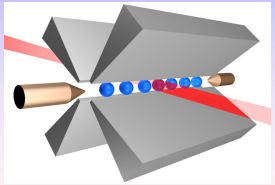
∞

$J_{ij} \sim \frac{1}{r_{ij}^3}$

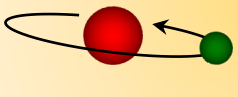
$J_{ij} \sim \frac{1}{r_{ij}^6}$

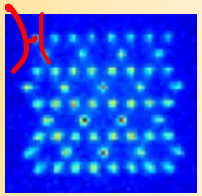
Nearest neighbors

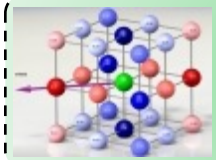
Trapped ions



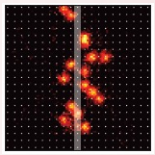
Rydberg atoms







Polar molecules
Magnetic atoms



Ultracold atoms in lattices

1 10^2 10^4 # of ‘spins’

See e.g. Hazzard *et al.*, PRA **90**, 063622 (2014)

“Atomic” arrays (ignoring bulk gases…)

Scalable: beyond 100 particles

Addressability: local manipulation and measurement $\langle \sigma_i^\alpha \rangle, \langle \sigma_i^\alpha \sigma_j^\beta \rangle, \ldots$

Programmable: controlled geometry, interactions…

Coupling range

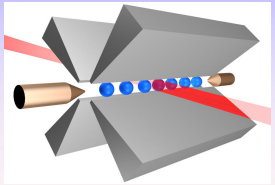
∞

$J_{ij} \sim \frac{1}{r_{ij}^3}$

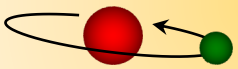
$J_{ij} \sim \frac{1}{r_{ij}^6}$

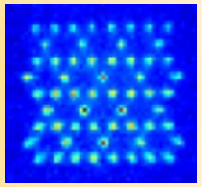
Nearest neighbors

Trapped ions

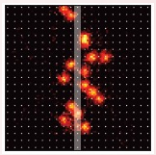


Rydberg atoms





Polar molecules
Magnetic atoms



Ultracold atoms in lattices

1 10^2 10^4 # of ‘spins’

See e.g. Hazzard *et al.*, PRA **90**, 063622 (2014)

Outline

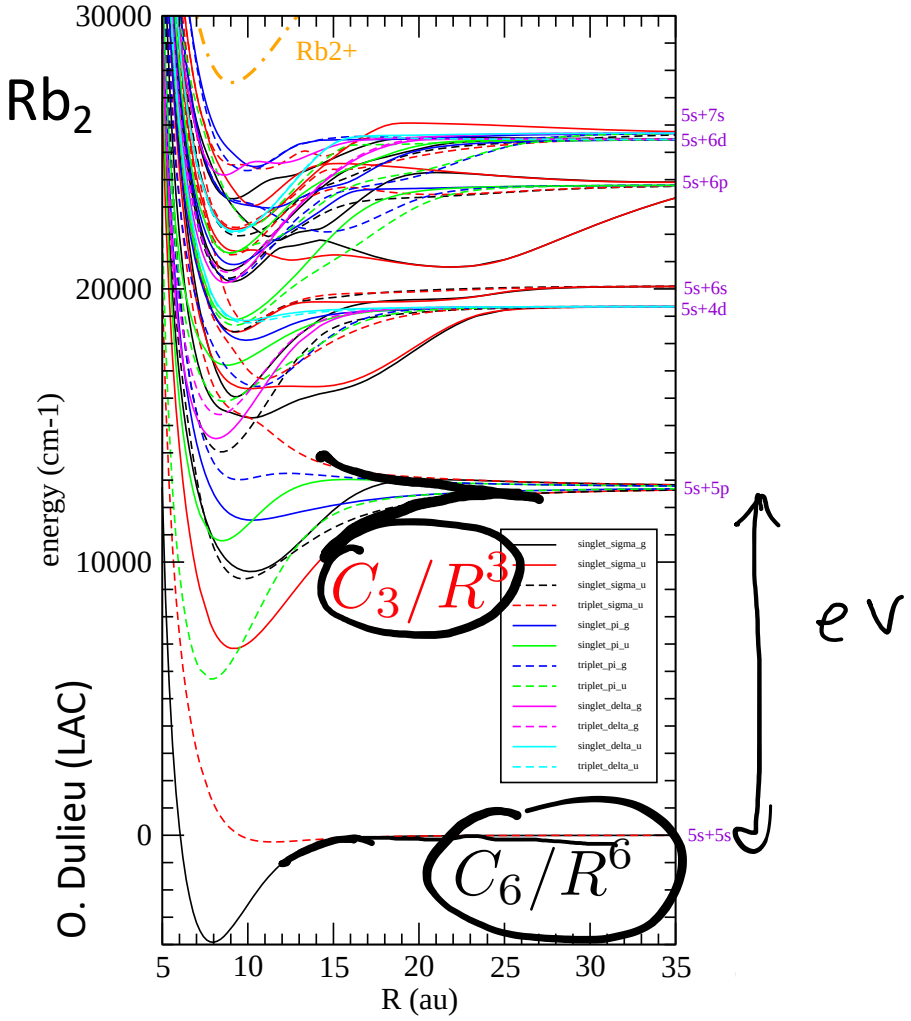
Lecture 1: Dipolar interactions between atoms

Lecture 2: Basics of Rydberg physics. Arrays of atoms. Rydberg blockade (2-body physics) & gates

Lecture 3: Many-body physics with Rydberg atoms: spin models and transport

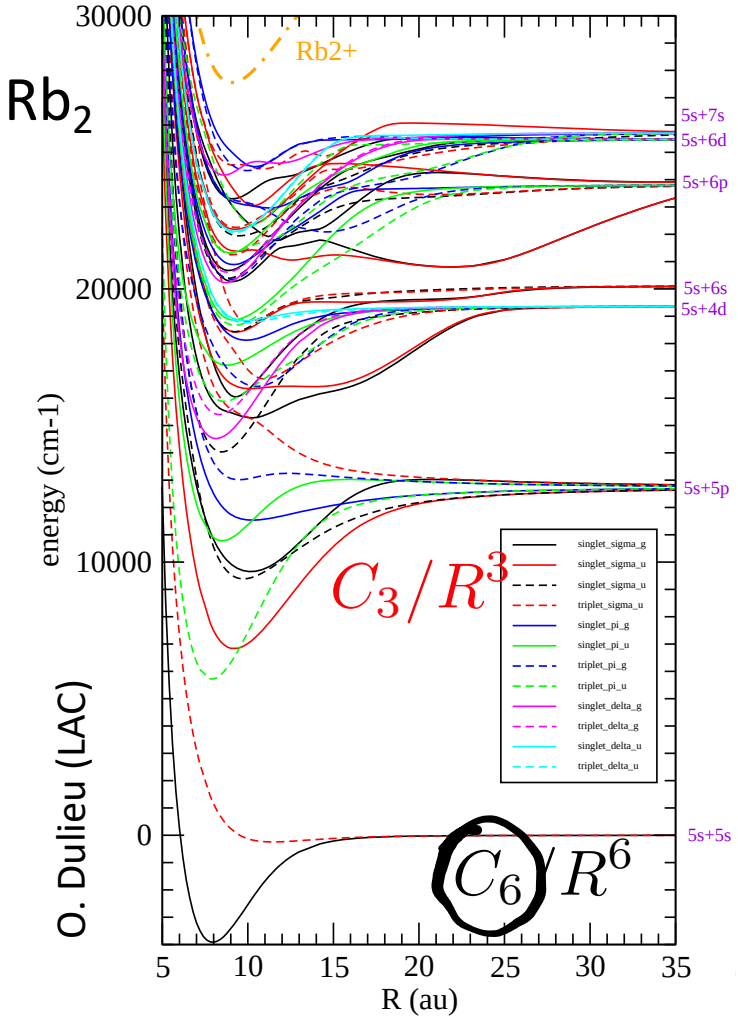
Interaction between atoms

Goal: understand *long-range* interactions between atoms



Interaction between atoms

Goal: understand *long-range* interactions between atoms



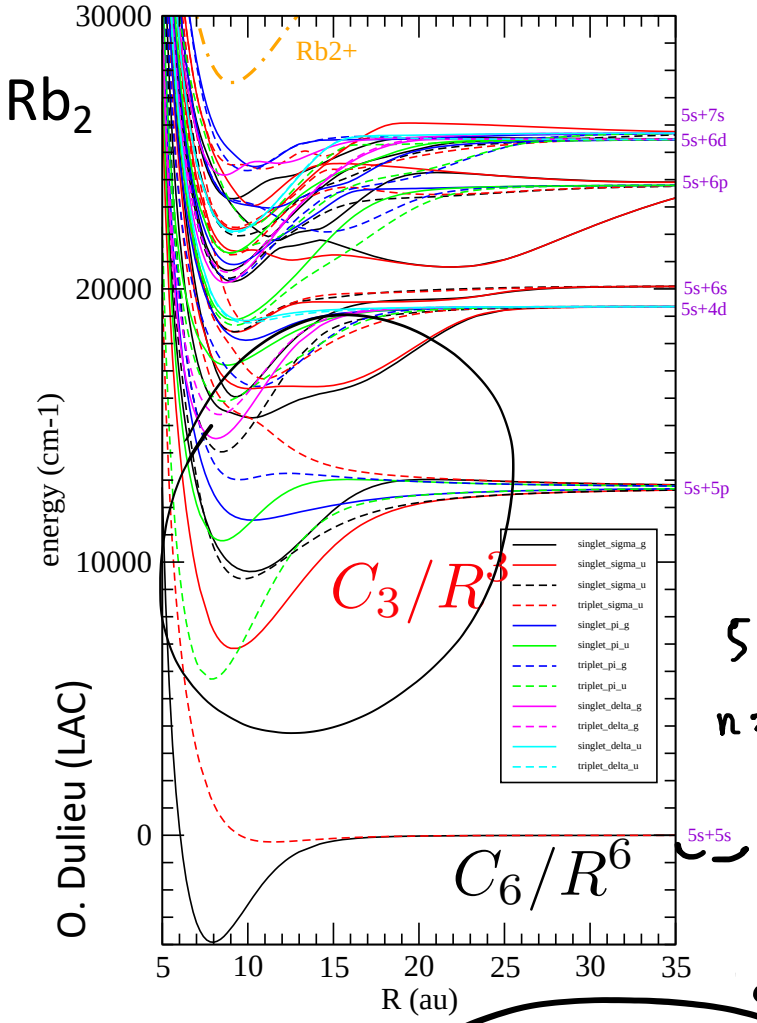
Van der Waals: C_6/R^6

système	C_6 [u.a.]	$\Gamma/2\pi$ [MHz]	λ [nm]
Li-Li	1389	5.87	671
Na-Na	1556	9.80	589
K-K	3897	6.04	767
Rb-Rb	4691	6.07	780
Cs-Cs	6870	5.22	852
Mg-Mg	627	80.9	235
Ca-Ca	2121	34.6	423
Sr-Sr	3103	32.0	461
Er-Er	1760	29.7	401
Dy-Dy	2275	32.2	421
Yb-Yb	1929	29	399

J. Dalibard, Collège de France, 2021

Interaction between atoms

Goal: understand *long-range* interactions between atoms



Van der Waals: C_6/R^6

système	C_6 [u.a.]	$\Gamma/2\pi$ [MHz]	λ [nm]
Li-Li	1389	5.87	671
Na-Na	1556	9.80	589
K-K	3897	6.04	767
Rb-Rb	4691	6.07	780
Cs-Cs	6870	5.22	852
Mg-Mg	627	80.9	235
Ca-Ca	2121	34.6	423
Sr-Sr	3103	32.0	461
Er-Er	1760	29.7	401
Dy-Dy	2275	32.2	421
Yb-Yb	1929	29	399

5 S

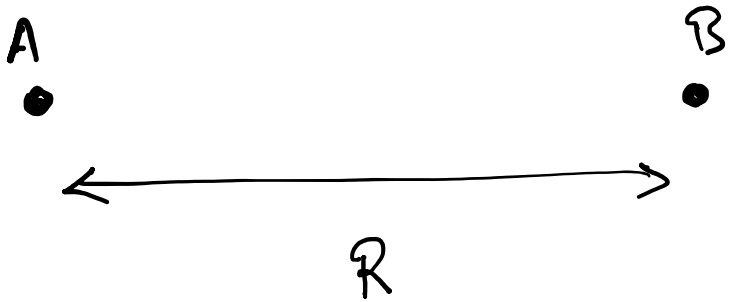
$n=5,\ l=0$

$|n=5, l=0\rangle_A \otimes |n=5, l=0\rangle_B$

J. Dalibard, Collège de France, 2021

Useful for: scattering length (quantum gases), Rydberg physics...

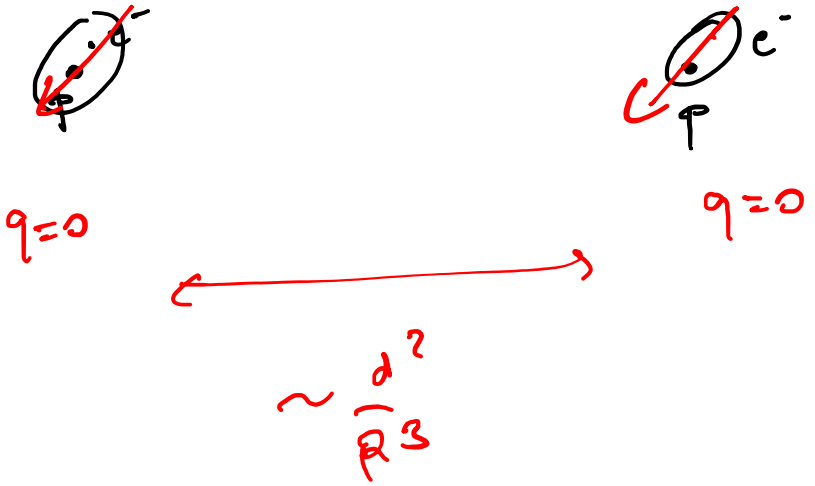
Interaction between atoms



Point-like neutral particle

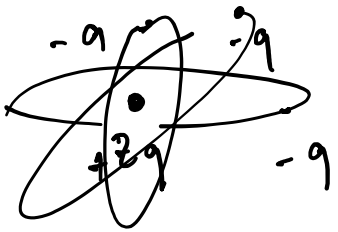
⇒ no interaction!

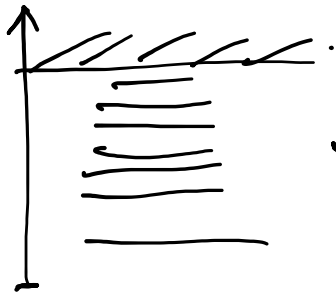
But: atoms are not point-like!



⇒ dipole

Atomic dipoles (1)





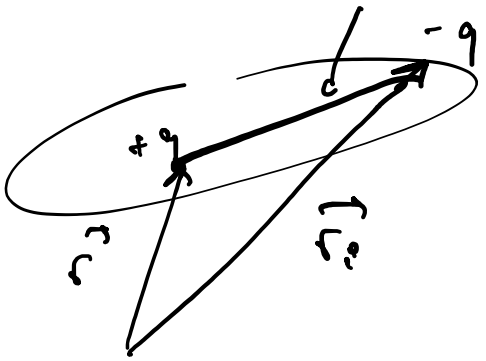
$\psi_{n\ell m}(\vec{r})$

n

ℓ

m

$|\vec{r} - \vec{r}_i|$



$$\vec{D} = \sum_{i=1}^{z} q\,(\vec{r}_i - \vec{r})$$

(1e⁻ $\vec{d} = q\vec{r}$)

Quantum : $\hat{\vec{D}}$

Matrix element of $\vec{D}$: $|n\ell m\rangle$

$\vec{D} \to -\vec{D}$

$$\langle n\ell m | \hat{\vec{D}} | n\ell m \rangle = 0$$

$$\int \psi^{*}_{n\ell m}\, q\vec{r}\, \psi_{n\ell m}(\vec{r})\, d^3r$$

$\ell = 0$

$\ell = 1$

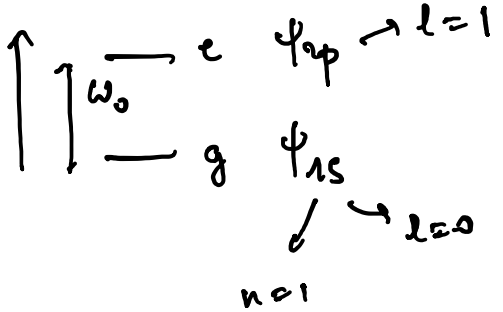
$$\int |\psi_{n\ell m}|^2 \, q\vec{r} = 0$$

$$\langle n\ell m | \vec{D} | n\ell \pm 1\, m \rangle \neq 0$$

artificial atom $[H, L^2] \neq 0$

Atomic dipoles (2)

Average dipole : $|\psi(t)\rangle$ $\langle\psi(t)|\vec{D}|\psi(t)\rangle = \langle D\rangle$



$\langle e|D_z|g\rangle = d_{eg}$ (⚠ $\langle e|D|e\rangle = \langle g|D|g\rangle = 0$)

$|\psi(t)\rangle = \alpha|g\rangle + \beta e^{-i\omega_0 t}|e\rangle$

$\alpha^2 + \beta^2 = 1$

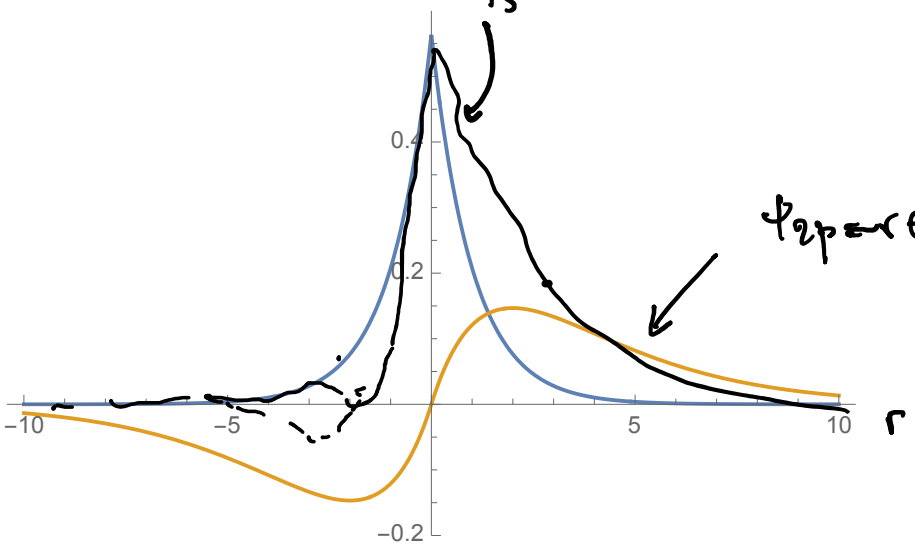
$\Rightarrow \langle\vec{D}\rangle = 2\alpha\beta\, d_{eg} \cos\omega_0 t$

⚠ Do not mix ① $\langle n l m|D|n' l' m'\rangle$ (Matrix element)

② Average dipole $\langle\psi(t)|D|\psi(t)\rangle$

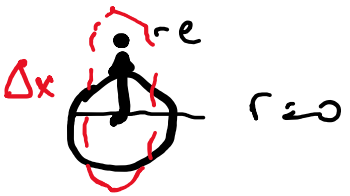
Atomic dipoles (3)

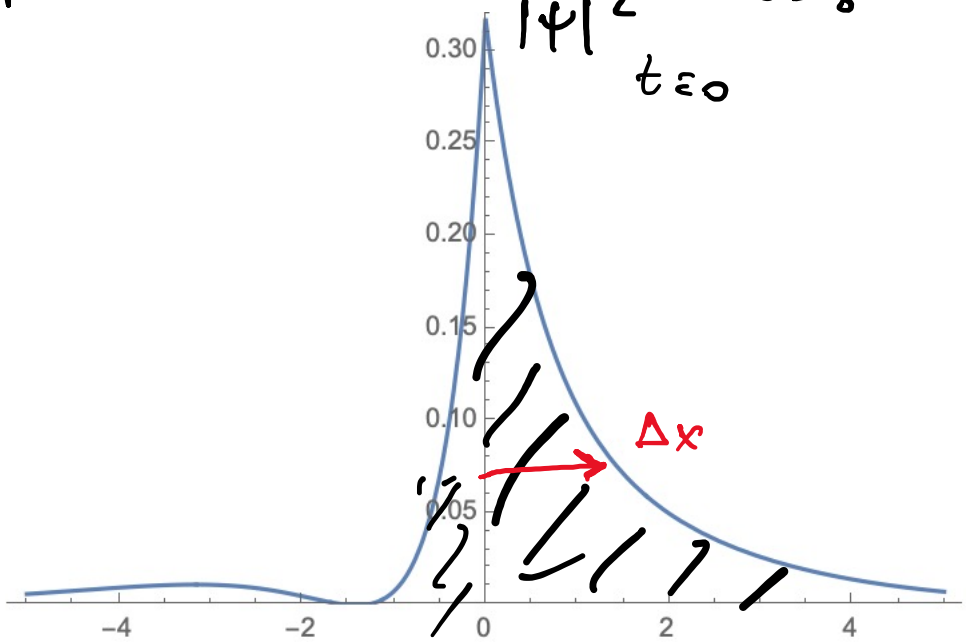
Ex: H atom



$$\psi(\vec{r},t) = \psi_{1s} + \psi_{2p} e^{-i\omega_0 t}$$

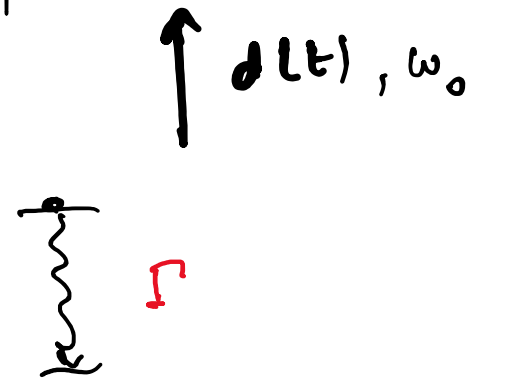
$$|\psi(r,t)|^2 = |\psi_{1s}|^2 + |\psi_{2p}|^2 + 2\psi_{1s}\psi_{2p}\cos\omega_0 t$$

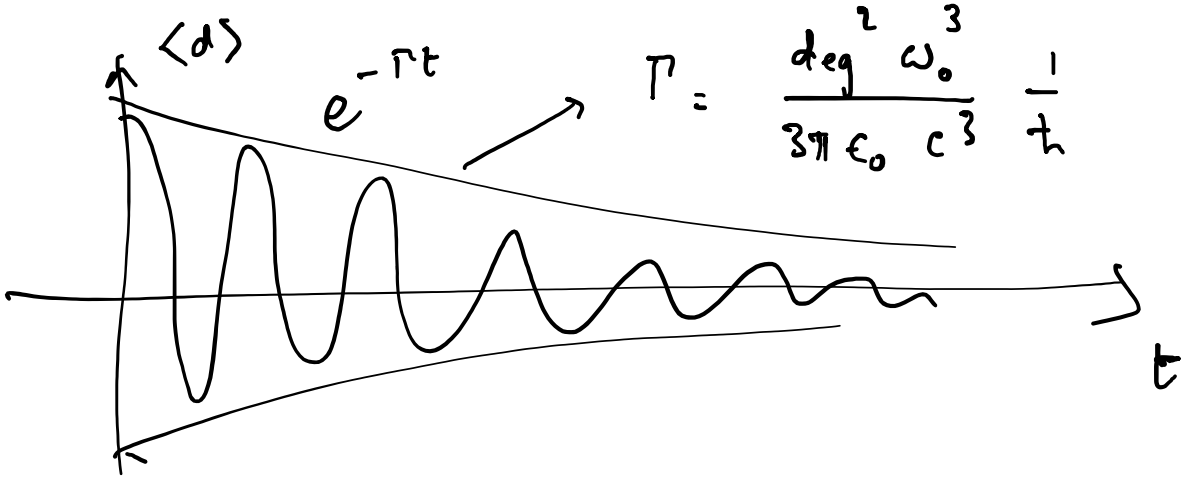




Atomic dipoles (4)

Dipole radiates a field !





① — e

— g

μw $\omega_0 \sim$ GHz

Γ

" $\Gamma \to 0$ " $\omega_0 \downarrow$

Many oscillations.

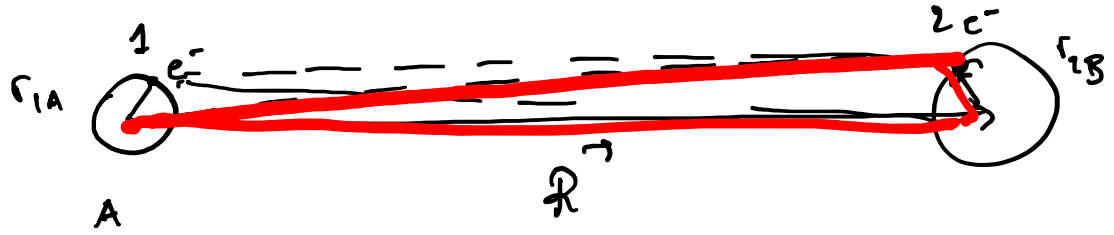
② — e

— g

$\lambda \sim \mu m$. $\omega_0 \sim$ 400 THz

$Q = \frac{\omega_0}{\Gamma} \sim 10^6$. ($\omega_0 \to 4\times10^{12}$, $\Gamma \to$ 10 MHz)

Dipolar interaction between 2 atoms



$e^2 = \frac{q^2}{4\pi\epsilon_0}$.

$$H = \frac{p_1^2}{2m} - \frac{e^2}{r_{1A}} + \frac{p_2^2}{2m} - \frac{e^2}{r_{2B}} + \frac{e^2}{R} + \frac{e^2}{r_{12}} - \frac{e^2}{r_{1B}} - \frac{e^2}{r_{2A}}$$

$\vec{r}_{12} = \vec{R} - \vec{r}_1 + \vec{r}_2$

$\vec{r}_{1B} = \vec{R} + \vec{r}_{2A}$

$$\frac{1}{|\vec{R} + \vec{\rho}|} \sim \frac{1}{R}\left(1 - \frac{\vec{\rho}\cdot\vec{R}}{2R^2} - \frac{\rho^2}{R^2} + \frac{3}{2}\left(\frac{\vec{\rho}\cdot\vec{R}}{R^2}\right)^2 + \dots\right)$$

dipole-dipole interaction. H_I

Size of atom $\sim a_0 \ll R$

$$\Rightarrow \dots \Rightarrow H_I = \frac{q^2}{4\pi\epsilon_0} \frac{\vec{r}_{1A}\cdot\vec{r}_{2B} - 3(\vec{r}_{1A}\cdot\vec{u})(\vec{r}_{2B}\cdot\vec{u})}{R^3}$$

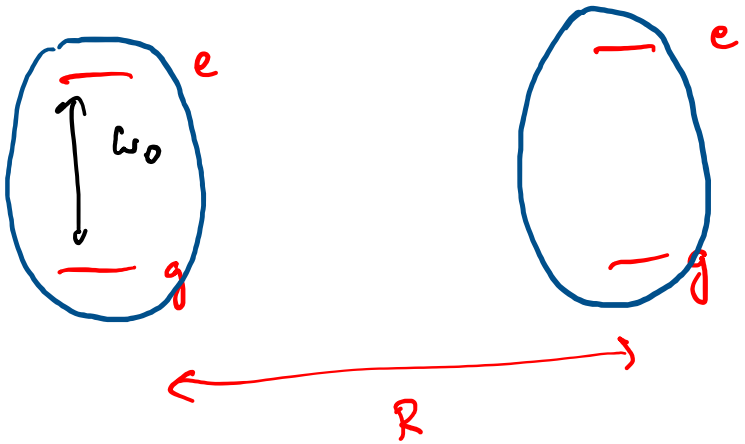
$\vec{u} = \frac{\vec{R}}{R}$.

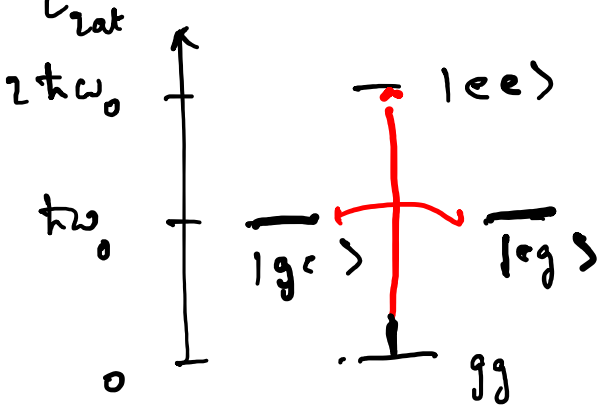
$\vec{D}_1 = q\vec{r}_1$, $\vec{D}_2 = q\vec{r}_2$

$$\hat{H}_I = \frac{\hat{D}_1\cdot\hat{D}_2 - 3(\hat{D}_1\cdot\vec{u})(\hat{D}_2\cdot\vec{u})}{4\pi\epsilon_0 R^3}$$

$\mathcal{E} = \mathcal{E}_1 \otimes \mathcal{E}_2 \rightarrow |n_1 \ell_1 m_1\rangle + |n_2 \ell_2 m_2\rangle$

A toy model for two interacting atoms





$$\langle gg|D_1 D_2|ee\rangle = \frac{d_{eg}^2}{R^3}$$

$$\langle eg|D_1 D_2|ge\rangle = \frac{d_{eg}^2}{R^3}$$

$d_{eg}\langle e|D_z|g\rangle$

$\vec{D_1}\cdot\vec{u} = 0$

$\vec{D_1}$ $\vec{D_2}$ $\vec{R}$

$$H = \frac{\hat{D_1}\,\hat{D_2}}{R^3}$$

$$H = \begin{pmatrix} 0 & \frac{d_{eg}^2}{R^3} & 0 & 0 \\ \frac{d_{eg}^2}{R^3} & 2\hbar\omega_0 & 0 & 0 \\ 0 & 0 & \hbar\omega_0 & \frac{d_{eg}^2}{R^3} \\ 0 & 0 & \frac{d_{eg}^2}{R^3} & \hbar\omega_0 \end{pmatrix} \begin{matrix} gg \\ ee \\ eg \\ ge \end{matrix}$$

2 Blocks !

The off-resonant block: van der Waals

$$H^{(1)} \begin{pmatrix} 0 & d_{eg}^2/R^3 \\ d_{eg}^2/R^3 & 2\hbar\omega_0 \end{pmatrix}$$

Recall:

$$H = \begin{pmatrix} 0 & V \\ V & \Delta \end{pmatrix} \begin{matrix} |g\rangle \\ |e\rangle \end{matrix} \quad \text{when} \quad \boxed{\Delta \gg V}$$

Δ — $+V^2/\Delta$ → $|\tilde{e}\rangle = |e\rangle - \frac{V}{\Delta}|g\rangle$

0 — $-V^2/\Delta$ → $|\tilde{g}\rangle = |g\rangle + \frac{V}{\Delta}|e\rangle$

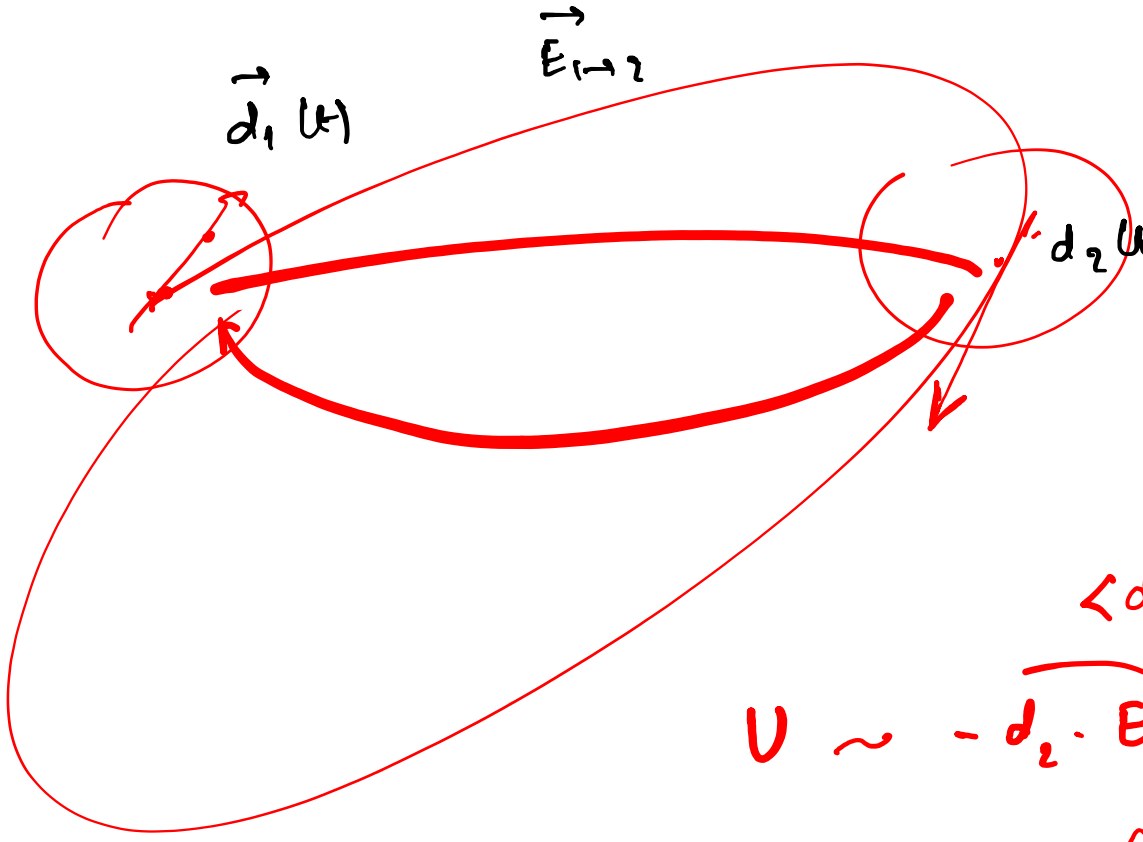
$2\hbar\omega_0$ — $\left(\frac{d_{eg}^2}{R^3}\right)^2 \frac{1}{2\hbar\omega_0}$

0 — $-\left(\frac{d_{eg}^2}{R^3}\right)^2 \frac{1}{2\hbar\omega_0} = \Delta E^{(2)}$

↳ C_6/R^6. **Van der Waals**

⚠ $\langle gg | \frac{d_1 d_2}{r^3} | gg \rangle = \Delta E^{(1)} = 0$

van der Waals from a "classical" viewpoint



Von der Waals:

correlations between dipoles!

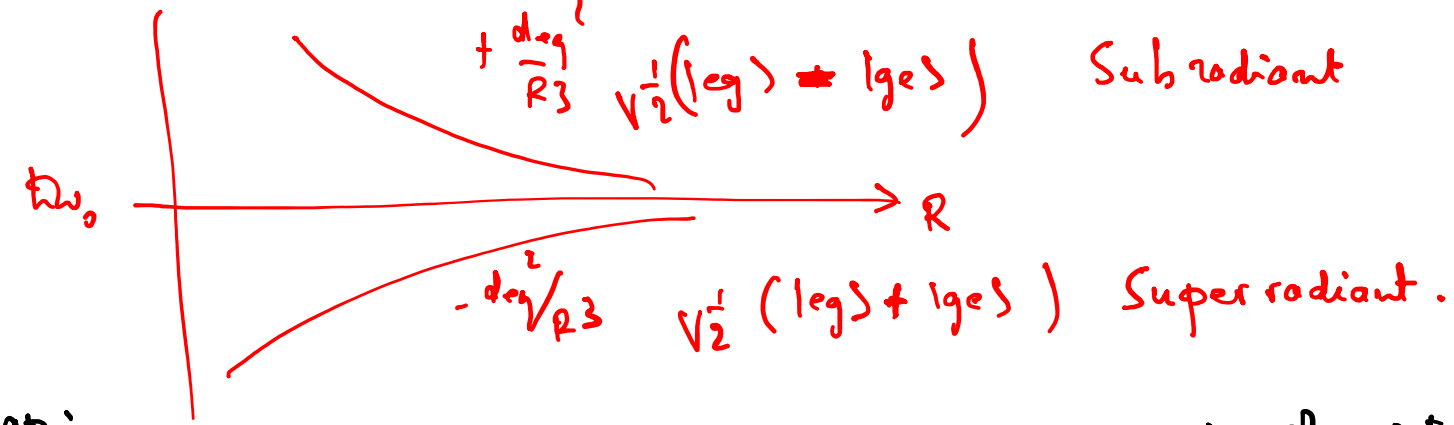
$\langle \vec{d}_1 \rangle = 0$

$\langle d_1^2 \rangle = 0$

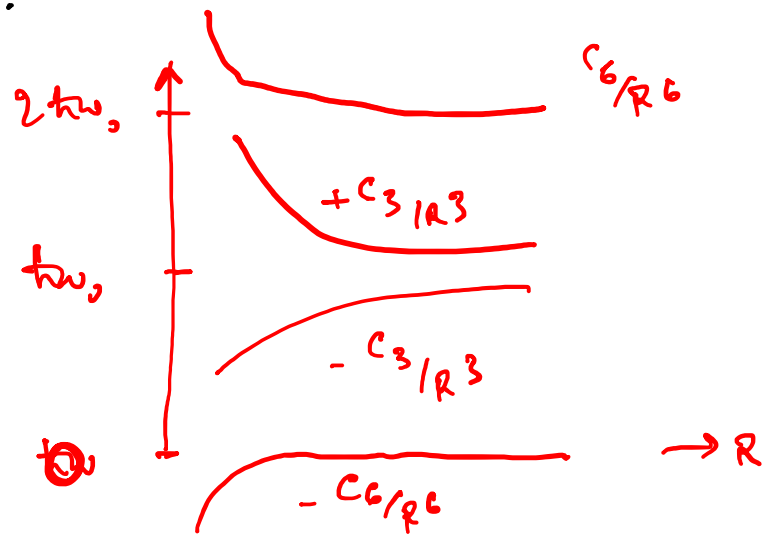
$\langle \vec{d}_1 \vec{d}_2 \rangle$

The resonant block: resonant dipole interaction

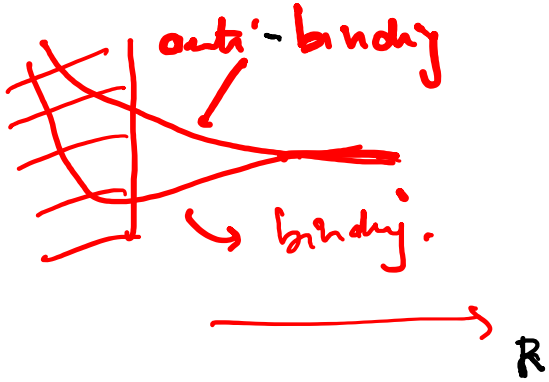
$$H^{(2)} = \begin{pmatrix} \hbar\omega_0 & \frac{d_{eg}^2}{R^3} \\ \frac{d_{eg}}{R^3} & \hbar\omega_0 \end{pmatrix} \begin{matrix} |eg\rangle \\ |ge\rangle \end{matrix} \longrightarrow \hbar\omega_0 \pm \frac{d_{eg}^2}{R^3}$$



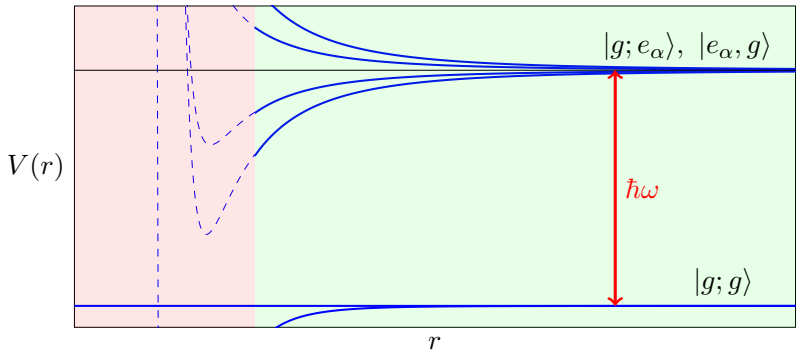
Recap:

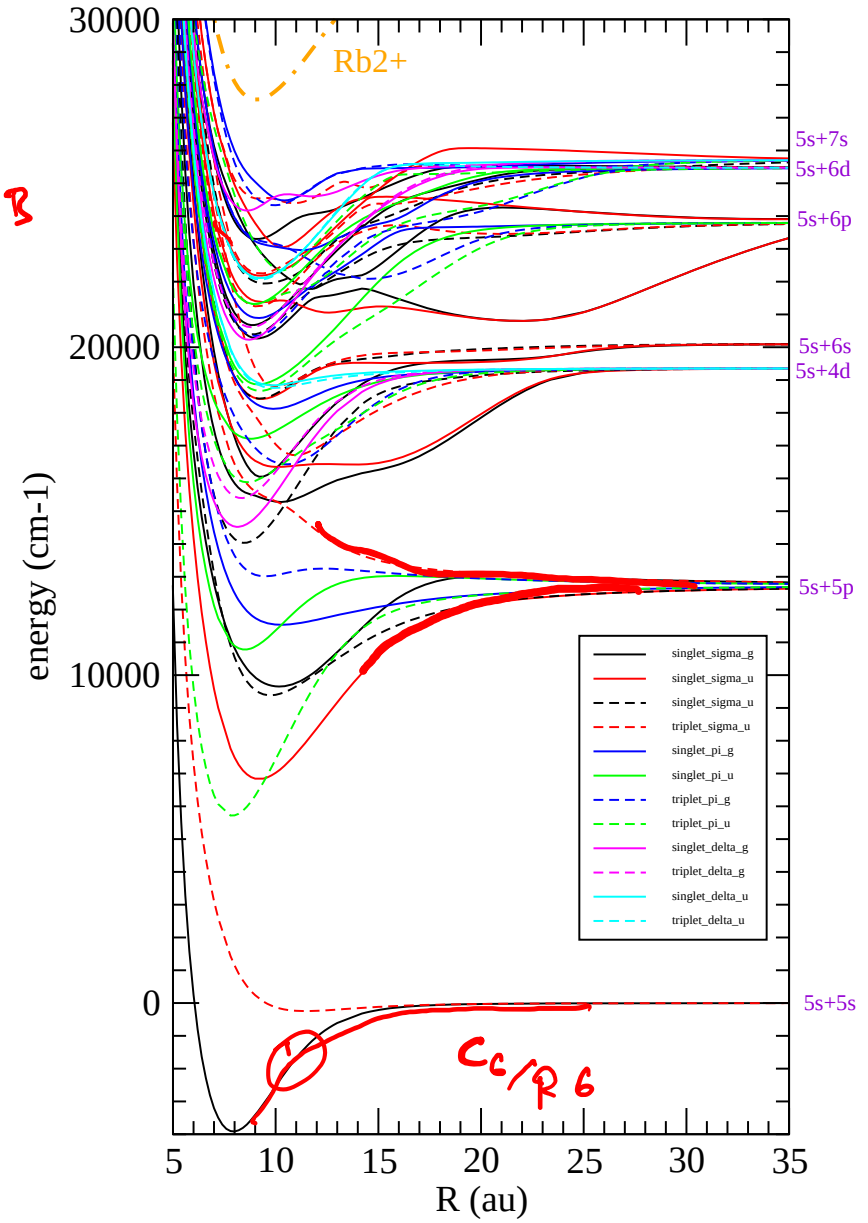


Note: in chemistry:



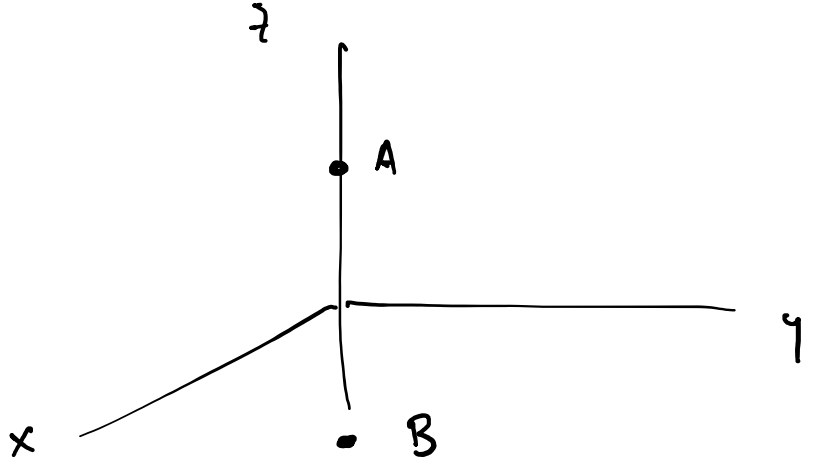
Molecular potentials in real life: example of Rb_2





O. Dulieu, LAC

Ground state of "real" atoms



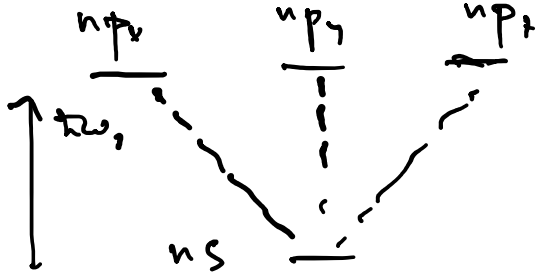
$$H = \frac{1}{4\pi\epsilon_0 r^3}\left[\hat{d}_{Ax}\hat{d}_{Bx} + \hat{d}_{Ay}\hat{d}_{By} - 2\,\hat{d}_{Az}\hat{d}_{Bz}\right]$$

$|ns \cdot ns\rangle$

$np_x \quad np_y \quad np_z$

$p \rightarrow l = 0$

$x \rightarrow m = +1,\ 0,\ -1$



$$|\langle p_x|\vec{D}|s\rangle|^2$$

$$\Delta E^{(2)}_{ns\,ns} = -\left(\frac{1}{4\pi\epsilon_0 r^3}\right)^2 \frac{1}{2\hbar\omega_0}\left[|d_{px,s}|^4 + |d_{py,s}|^4 + 4|d_{pz,s}|^4\right]$$

As: $np \rightarrow ns$

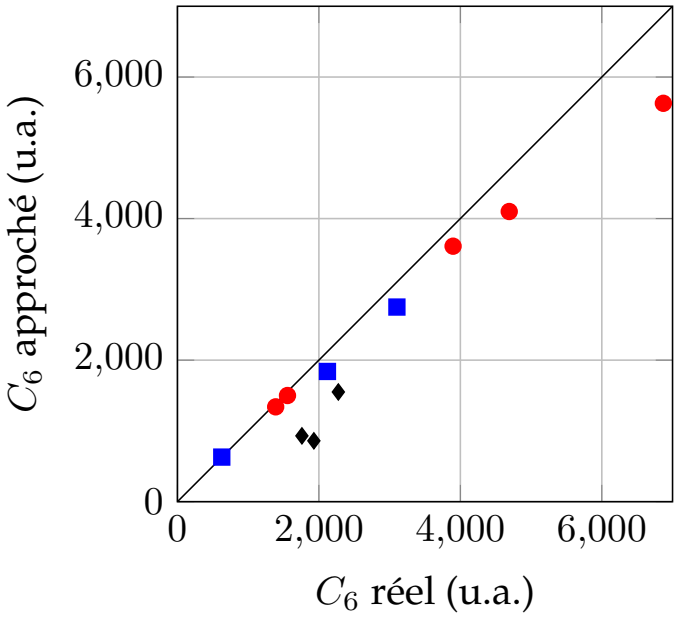
$$\hbar\Gamma = \frac{|d_{ps}|^2 k^3}{3\pi\epsilon_0}, \quad k = \frac{\omega_0}{c}$$

$$\Rightarrow \quad \Delta E_{ns\,ns} = -\frac{27}{16}\left(\frac{\hbar\Gamma}{(kr)^6}\right)\frac{\Gamma}{\omega_0}$$

Ground state of “real” atoms

$$C_6 = -\frac{27}{16}\frac{\hbar\Gamma}{k^6}\frac{\Gamma}{\omega_0}$$

système	C_6 [u.a.]	$\Gamma/2\pi$ [MHz]	λ [nm]	C_6 approché [u.a.]
Li-Li	1389	5.87	671	1340
Na-Na	1556	9.80	589	1500
K-K	3897	6.04	767	3610
Rb-Rb	4691	6.07	780	4100
Cs-Cs	6870	5.22	852	5629
Mg-Mg	627	80.9	235	630
Ca-Ca	2121	34.6	423	1840
Sr-Sr	3103	32.0	461	2750
Er-Er	1760	29.7	401	930
Dy-Dy	2275	32.2	421	1550
Yb-Yb	1929	29	399	860

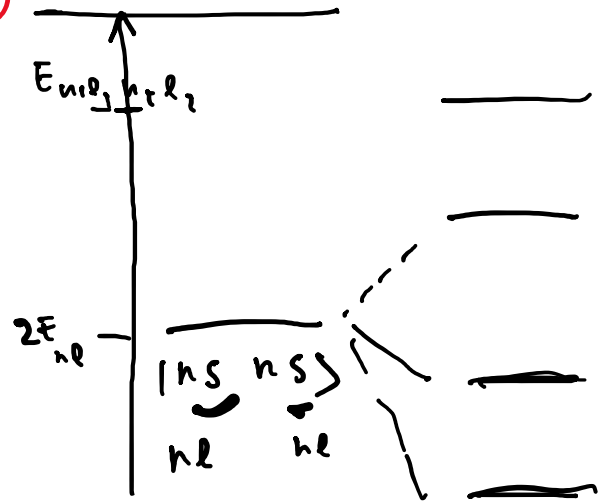


ee

gg $\left(\frac{d^2}{R^3}\right)^2 \frac{1}{2\hbar\omega_0}$

Excited states of "real" atoms: the Rydberg case

① Van der Waals



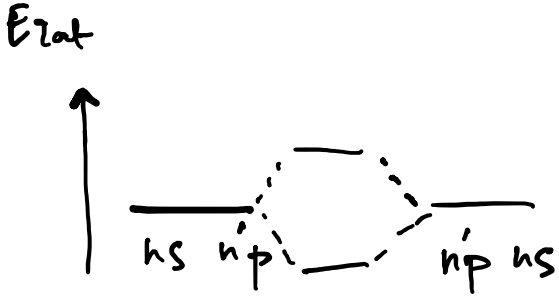
$n_1\ell_1, n_2\ell_2 \qquad \ell_1 = \pm 1$

$\langle n_1\ell_1 n_2\ell_2 | H_{dd} | n\ell\ n\ell \rangle$

$C_6 > 0$ OR $C_6 < 0$

$\Rightarrow \Delta E^{(2)}_{ns\,ns} = \sum_{\substack{n_1\ell_1 \\ n_2\ell_2}} \frac{|\langle n_1\ell_1 n_2\ell_2 | H_{dd} | n\ell\ n\ell \rangle|^2}{2E_{n\ell} - E_{n_1\ell_1} - E_{n_2\ell_2}} \propto \frac{C_6}{R^6}$ ($\frac{1}{R^3}$)

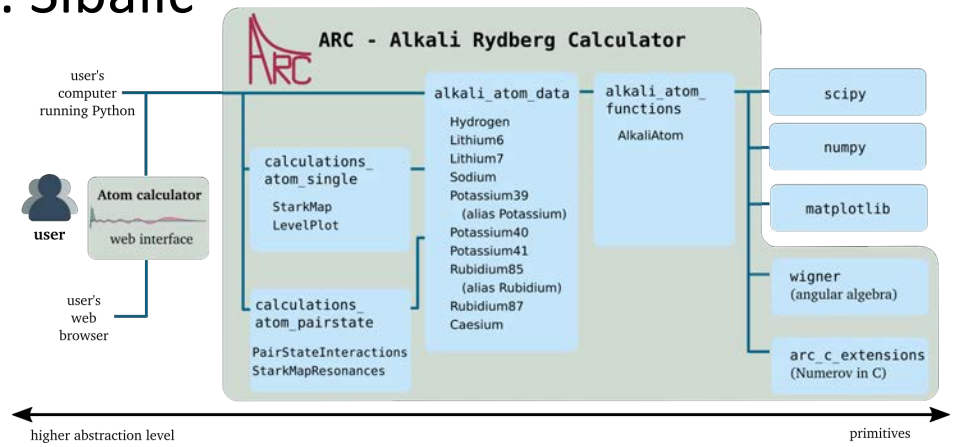
② Resonant



$\frac{C_3}{R^3}$ Resonant dip-dip interaction.

On-line interaction calculator for Rydberg atoms

N. Sibalic



https://arc-alkali-rydberg-calculator.readthedocs.io/en/latest/

S. Weber

Docs » Pairinteraction - A Rydberg Interaction Calculator

Pairinteraction - A Rydberg Interaction Calculator

build passing | build passing | codecov 67% | pypi v0.9.5a0 | arXiv 1612.08053 | License GPLv3

The *pairinteraction* software calculates properties of Rydberg systems. The software consists of a C++/Python library and a graphical user interface for pair potential calculations. For usage examples visit the tutorials section of the documentation. Stay tuned by signing up for the newsletter so whenever there are updates to the software or new publications about pairinteraction we can contact you. If you have a question that is related to problems, bugs, or suggests an improvement, consider raising an issue on GitHub.

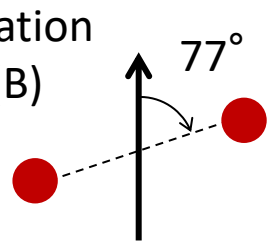
https://pairinteraction.github.io/pairinteraction/sphinx/html/index.html

On-line interaction calculator for Rydberg atoms

https://pairinteraction.github.io/pairinteraction/sphinx/html/index.html

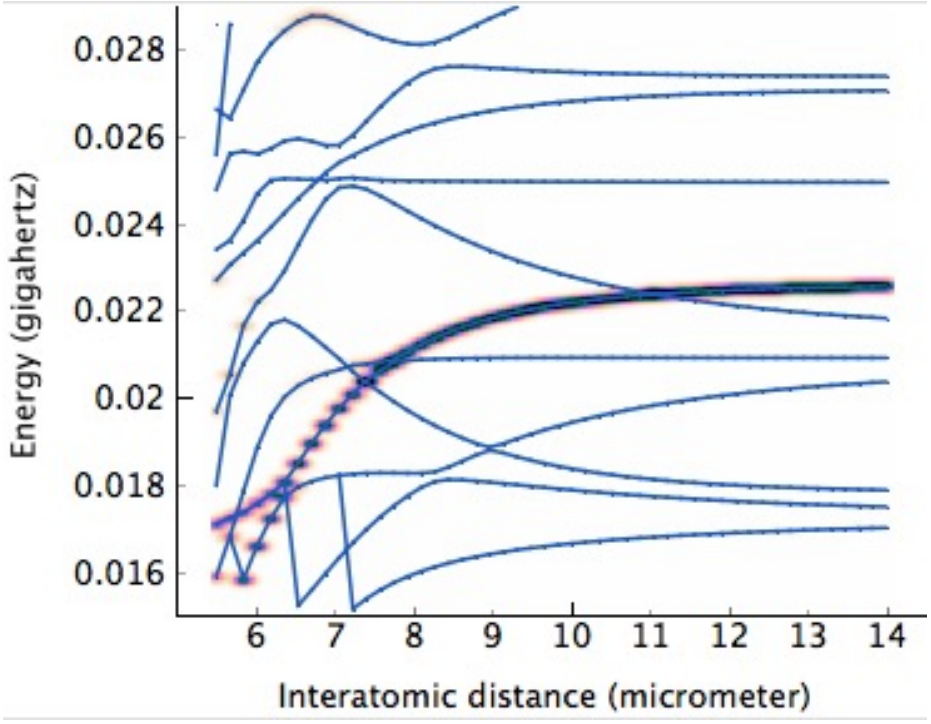
$61D_{3/2}$

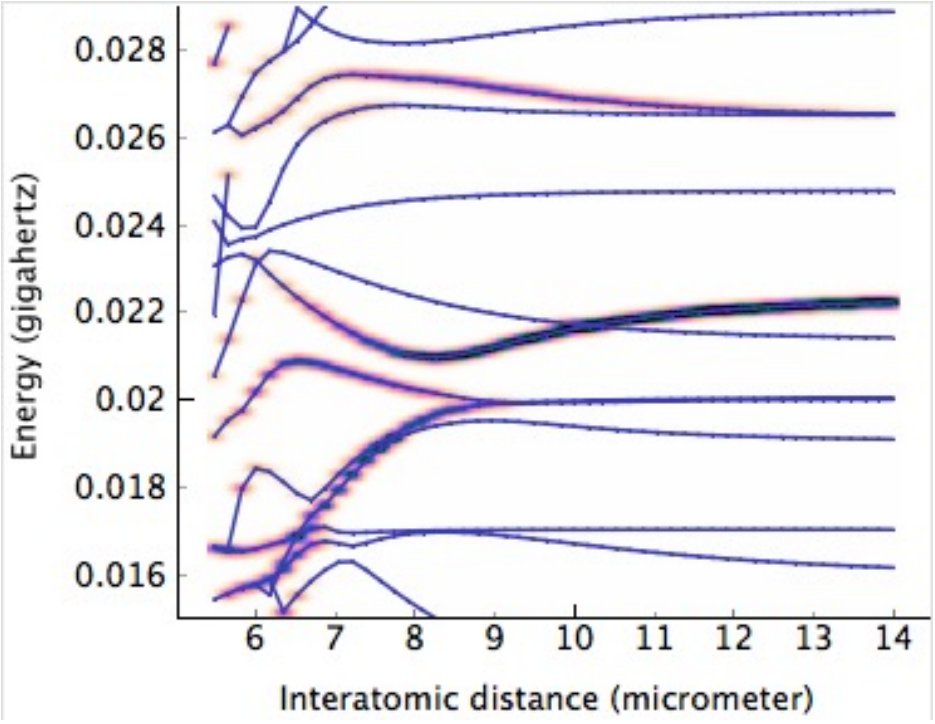
^{87}Rb



B = 6.9 G
E = 0 mV/cm

B = 6.9 G
E = 20 mV/cm





Outline

Lecture 1: Dipole interactions between atoms

Lecture 2: Arrays of atoms. Basics of Rydberg physics. Rydberg blockade (2-body physics) & gates

Lecture 3: Many-body physics with Rydberg atoms: spin models and transport