

Boulder School, July 28,29, 2016

Spin liquid in organic materilas

K. Kanoda, UTokyo

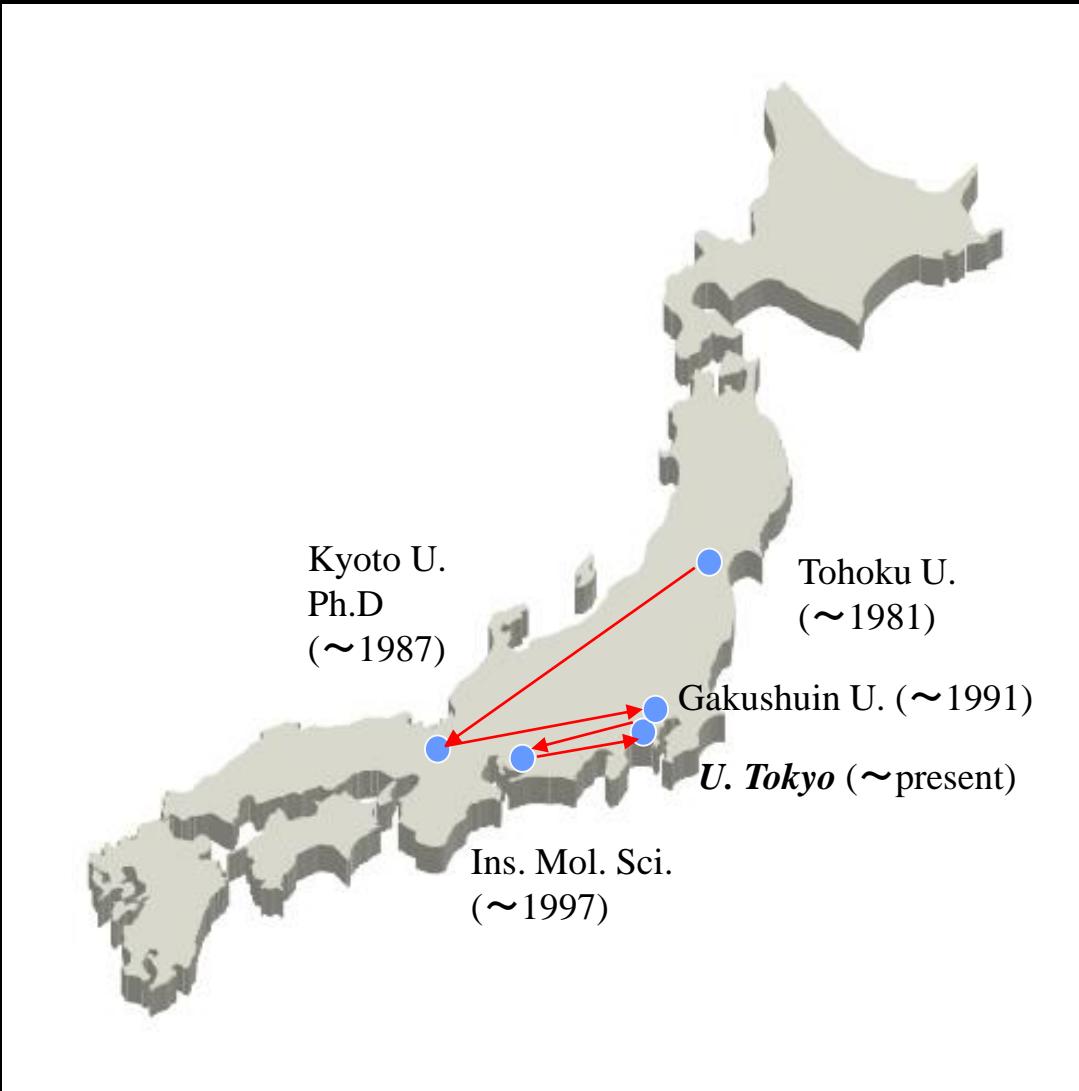
References

(review article)

Yi Zhou, K. Kanoda and Ng Tai Kai

Spin liquid states

arXiv. 1607.03228

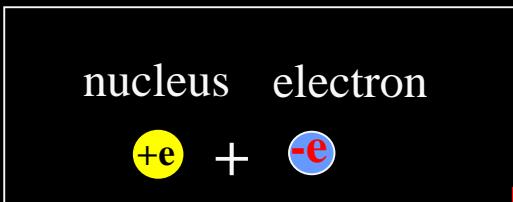


Physics of condensed matter

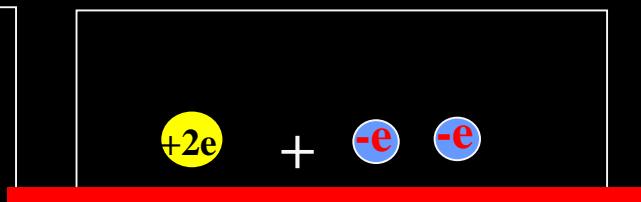
Understanding *low-energy state* of nucleus and electron assembly

More is differently different.

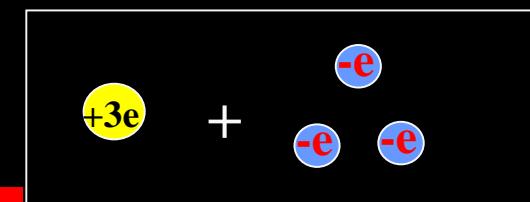
Hydrogen



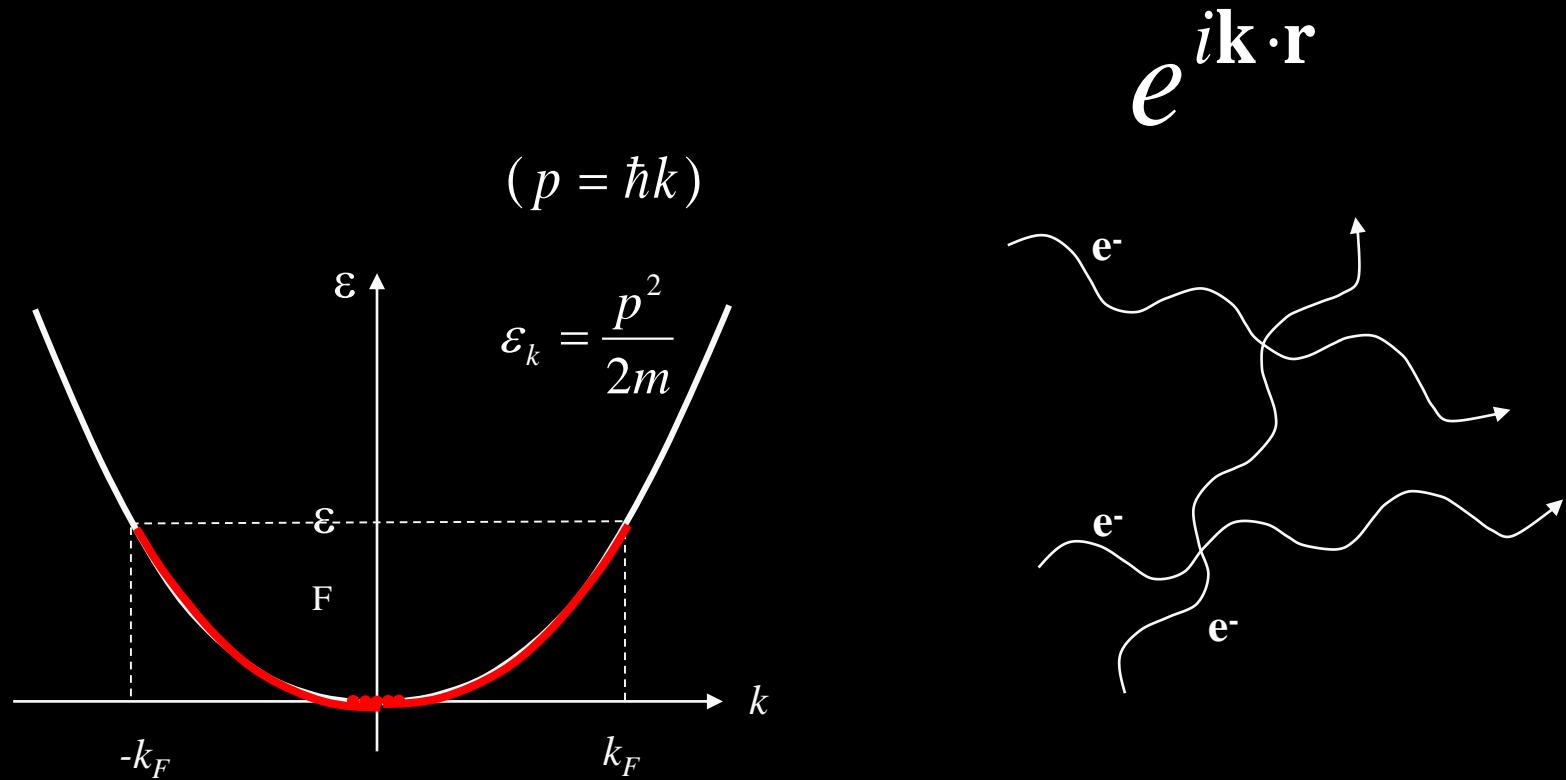
Helium



Lithium

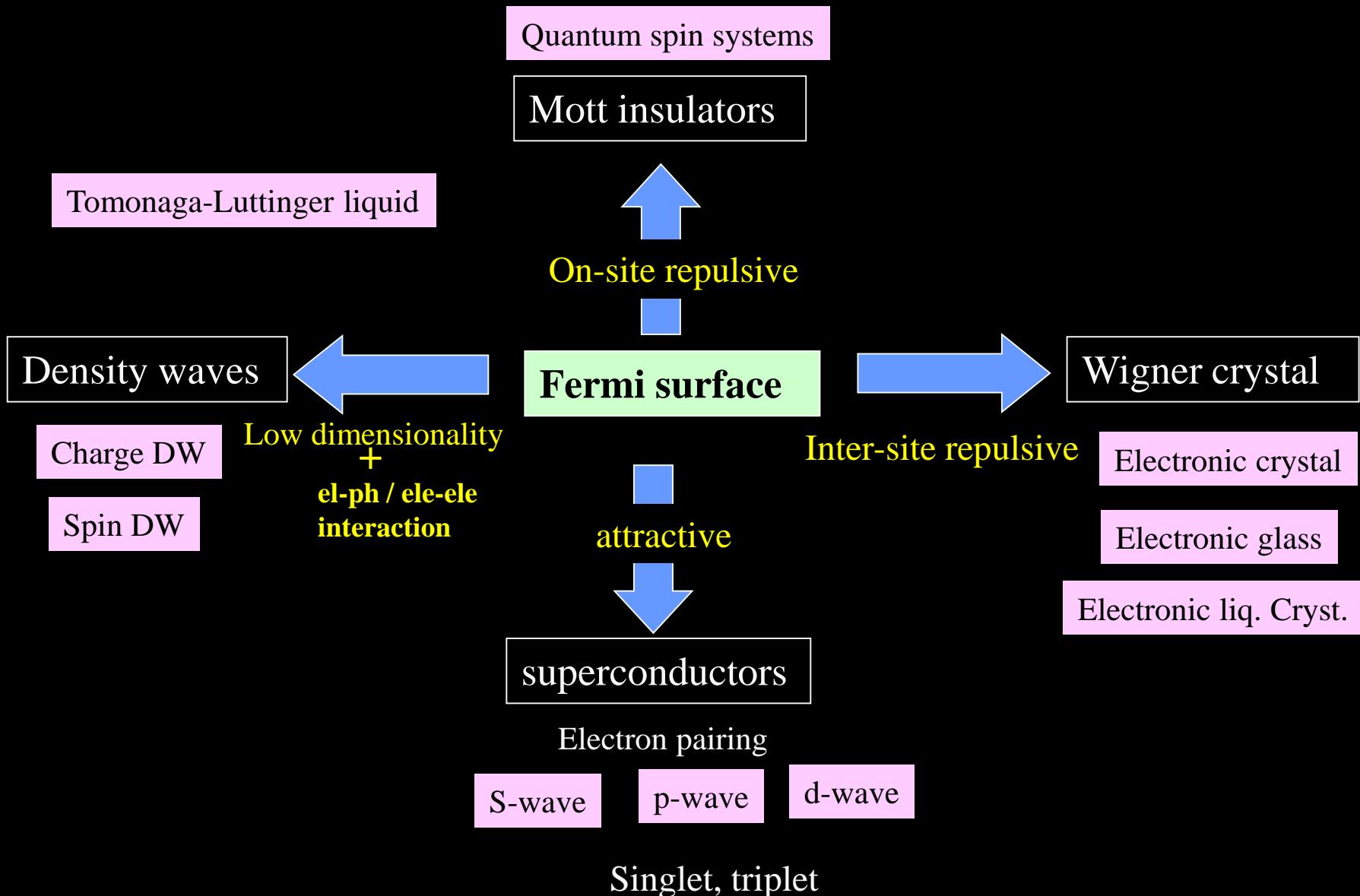


Without interaction,
electrons are free waves with Fermi
surface.



Fermi gas

With interaction, Fermi surfaceunstable



Contents

1. Fundamentals of organic materials

complex in real space, but simple in k -space

2. Electron correlation in organic materials

all-in-one systems for Mott physics

3. Spin liquid in quasi-triangular lattice

controlled frustration, correlation, disorder, doping

(Optional)

4. Massless Dirac Fermions in organic materials

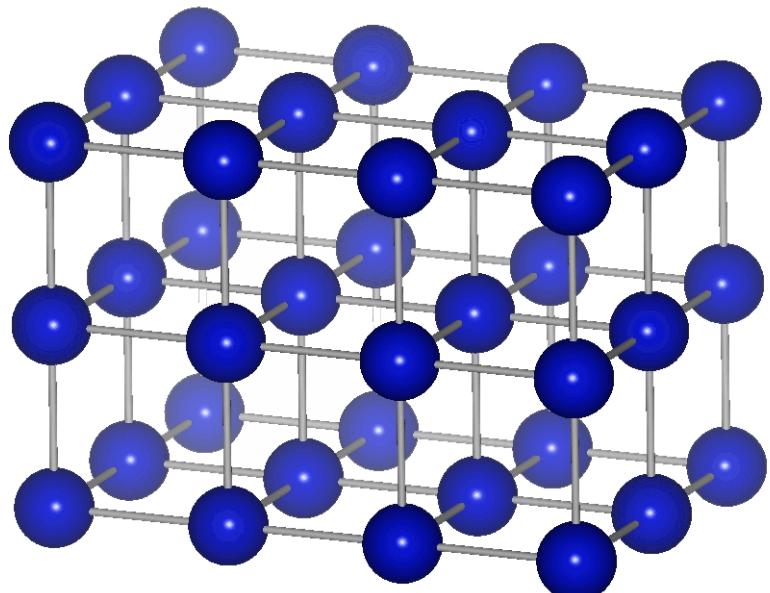
Dirac cone reshaping and ferromagnetism

1. Molecular materials and electronic structures

Keywords;

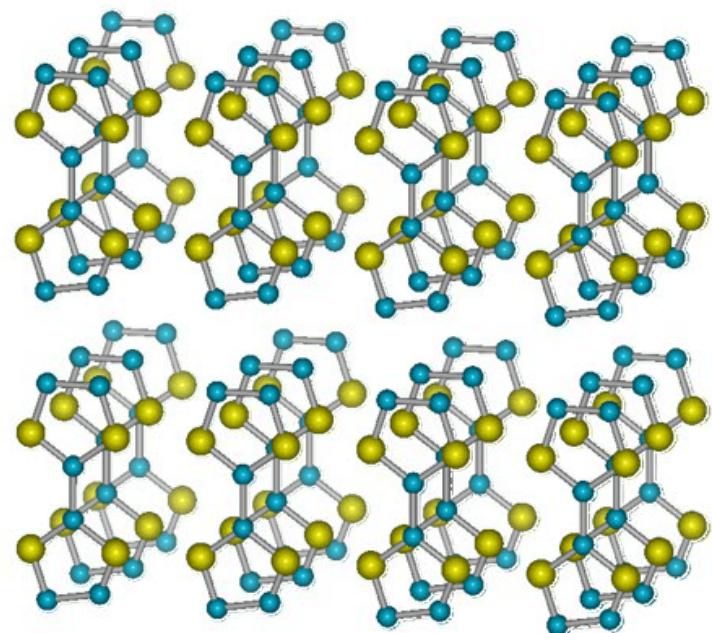
a variety of lattice structures
concept of molecular orbital
simple band structure
highly compressible system

Inorganic solid



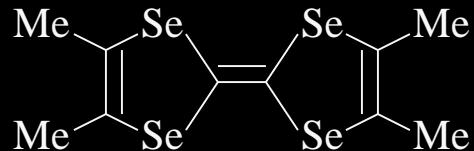
Simple metals,
oxides,....

Organic (molecular) solid

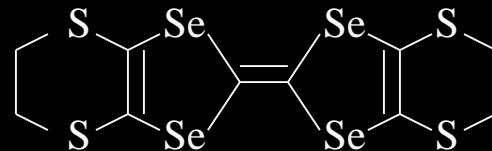


π electron systems

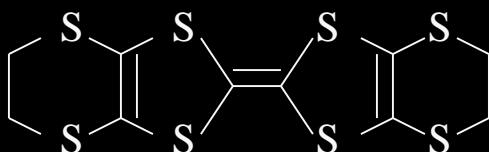
Organic molecules giving (super)conductors



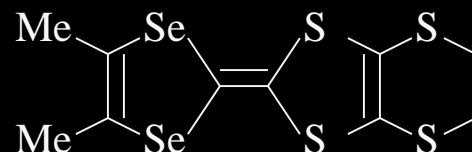
TMTSF



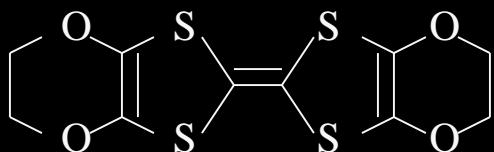
BEDT-TSF (BETS)



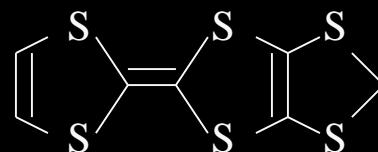
BEDT-TTF (ET)



DMET

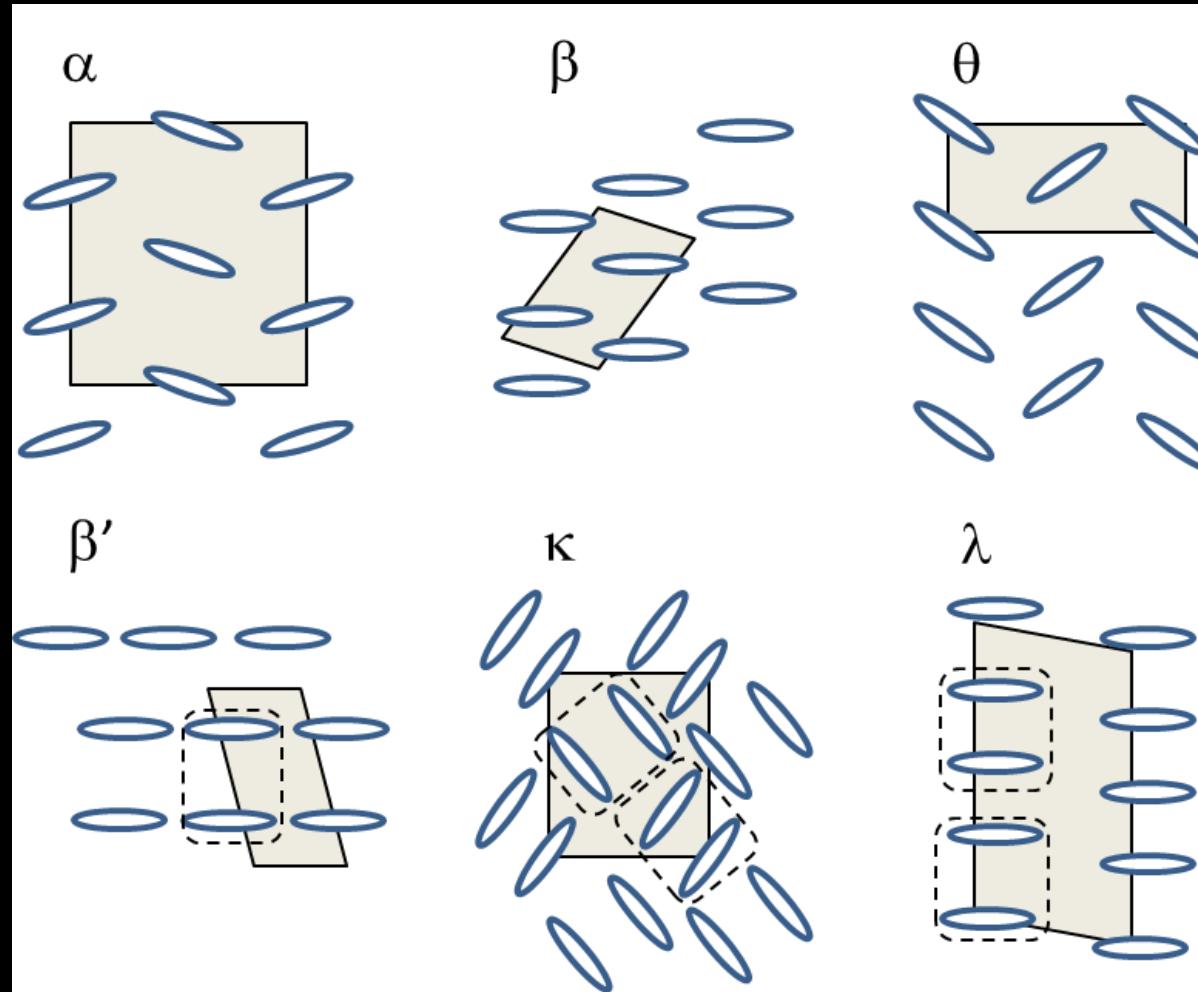


BEDO-TTF (BO)

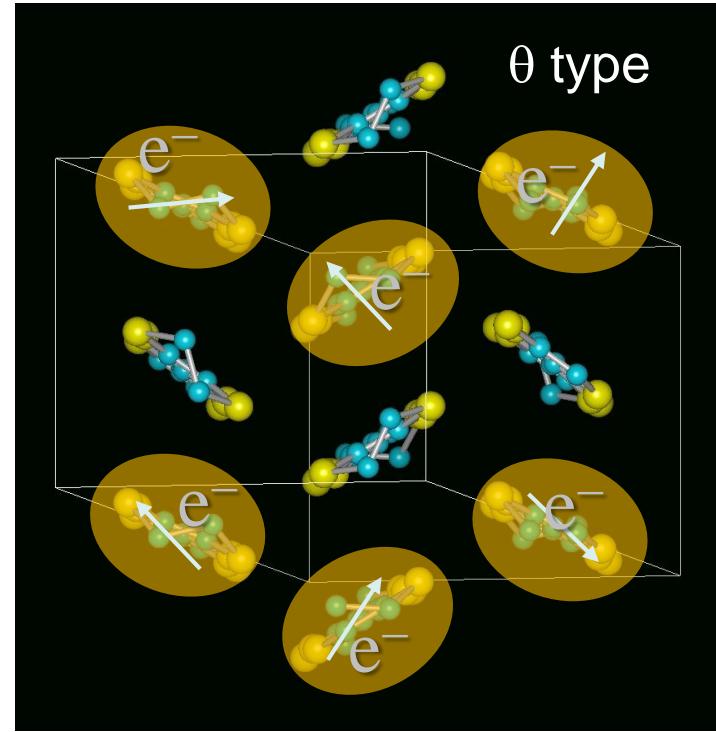
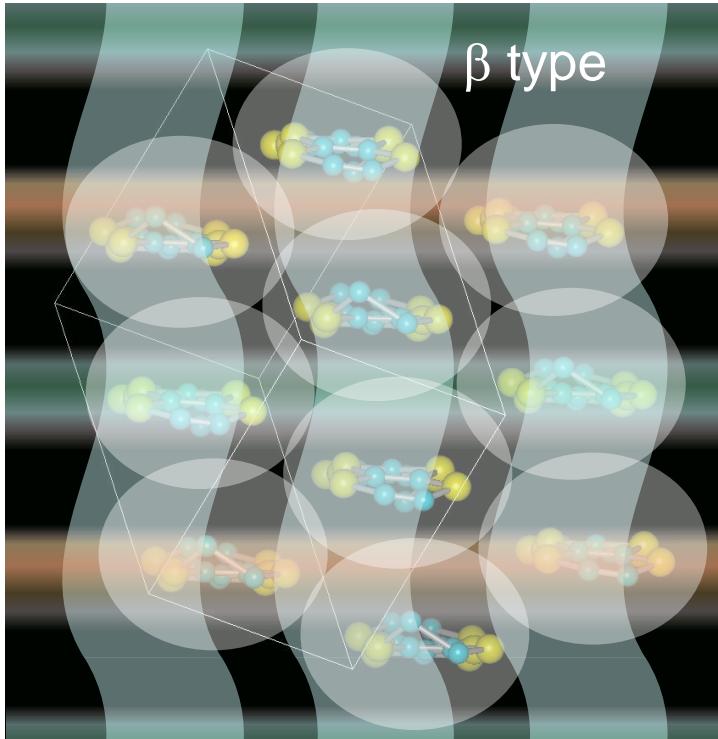


MDT-TTF

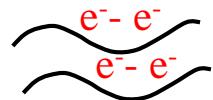
Molecular arrangement degrees of freedom



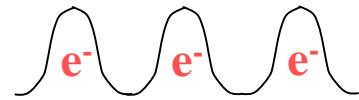
Molecular arrangement degrees of freedom



superconductivity

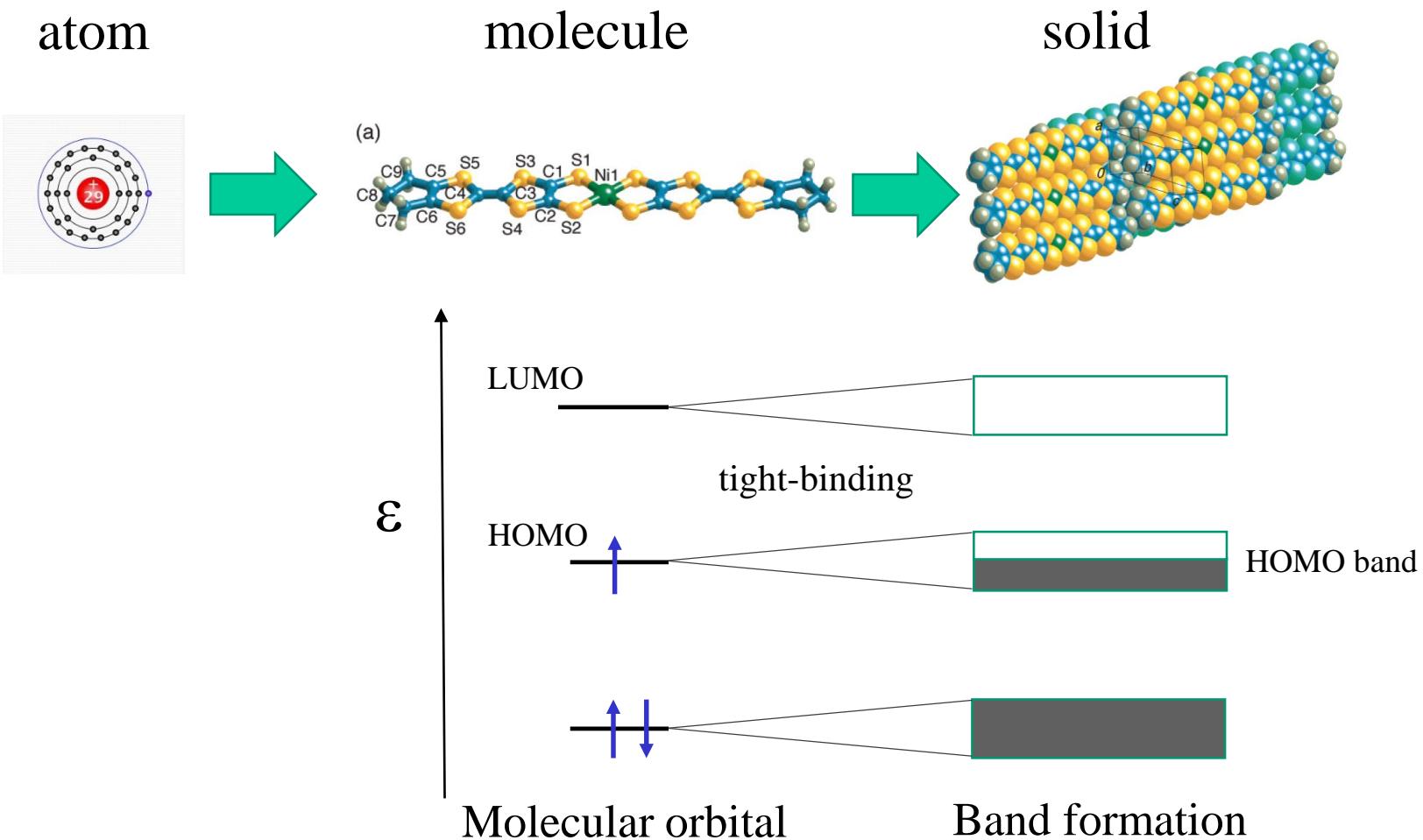


Electronic crystals



Organic conductors

complex in real space, but simple band structure



Electronic structure

Molecular orbital is a minimum entity for electrons

No need to look into atomic orbitals in a low energy scale

Molecular material; structure is complicated in real space, but electronic structure is surprisingly simple in *k*-space

It's because of the hierarchy; atomic orbital → molecular orbital → electronic band

1) The simplest non-degenerate case; hydrogen molecule

Key concept

Molecular orbital Linear combination of atomic orbitals

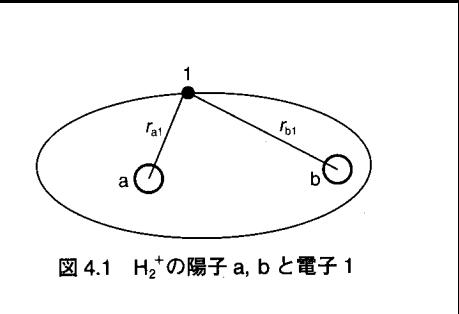
$$\varphi = c_a \phi_a + c_b \phi_b \quad H = -\frac{\hbar^2}{2m} \nabla^2 - \frac{e^2}{4\pi\epsilon_0 r_a} - \frac{e^2}{4\pi\epsilon_0 r_b} + \frac{e^2}{4\pi\epsilon_0 R}$$

$$\varphi_1 = \frac{1}{\sqrt{2(1+S)}} (\phi_a + \phi_b) \text{ bonding orbital}$$

$$\varphi_2 = \frac{1}{\sqrt{2(1-S)}} (\phi_a - \phi_b) \text{ antibonding orbital}$$

$$S = \int \phi_a * \phi_b d\tau$$

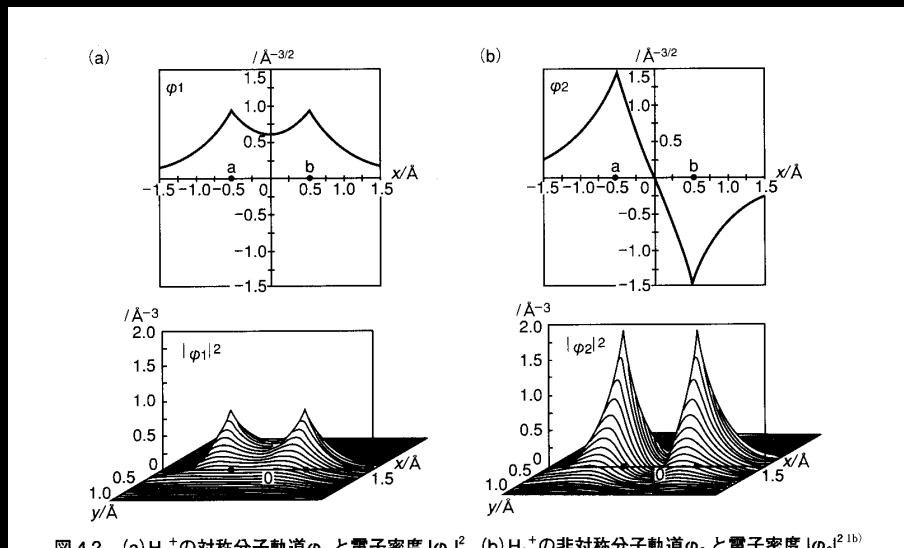
Overlapping integral



$$H_{aa} = \int \phi_a * H \phi_a d\tau$$

$$H_{ab} = \int \phi_b * H \phi_a d\tau$$

Transfer integral



2) The degenerate case: carbon atom

$$\varphi_{lmn} = R_{nl} Y_{lm}(\vartheta, \phi)$$

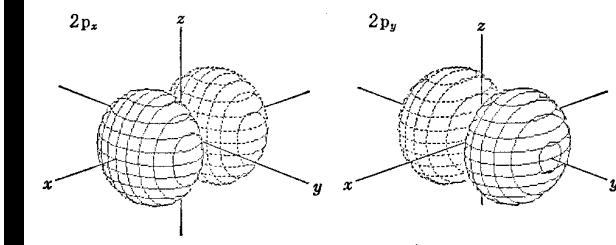
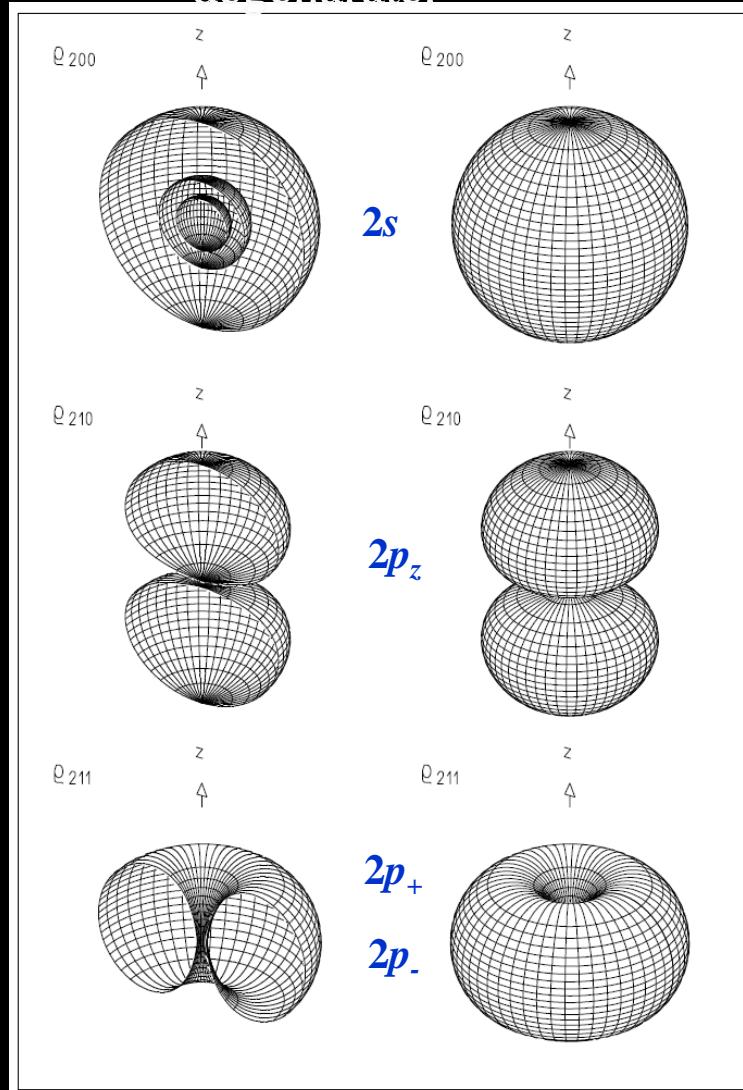
For n=2, four orbitals ($2s, 2p \times 3$) are degenerate.

$$Y_{00}(\vartheta, \phi)$$

$$Y_{10}(\vartheta, \phi)$$

$$Y_{11}(\vartheta, \phi)$$

$$Y_{1-1}(\vartheta, \phi)$$



Atomic p -orbital → molecular orbital

First, consider symmetry of coordination and reconstruct orbitals (intra-atomic hybridization)

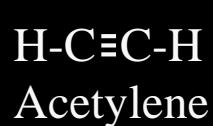
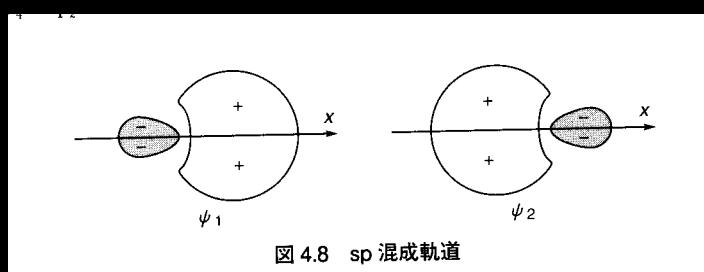


Next, reconstruct orbitals between neighbors (inter-atomic hybridization like hydrogen molecule)

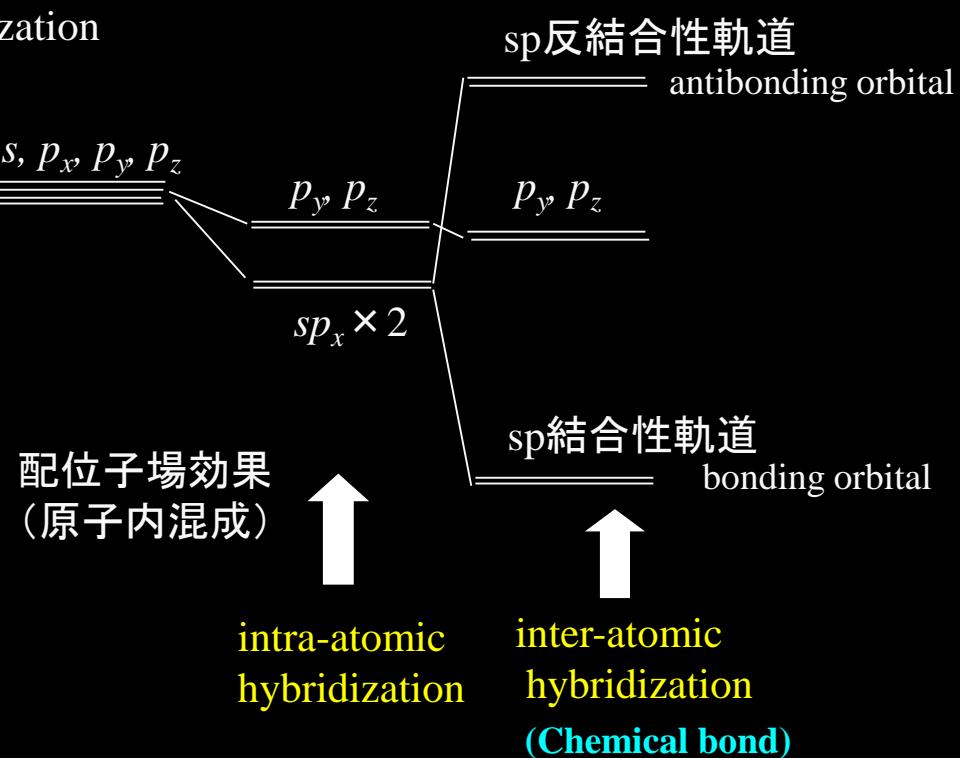


Finally, construct the overall molecular orbitals

i) Uniaxial 2-way coordination; sp hybridization



$$sp_x = \frac{1}{\sqrt{2}}(s \pm p_x)$$



Atomic p -orbital → molecular orbital

Imagine alien atoms are approaching carbon → intra-atomic hybridization

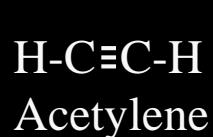
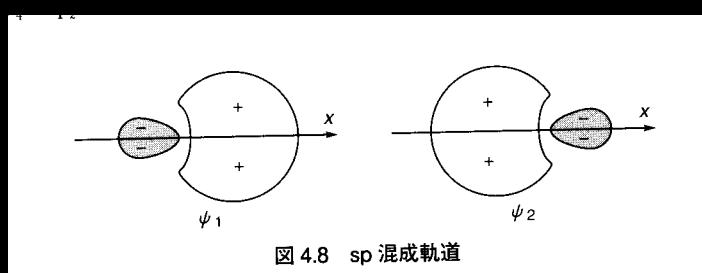


When the alien atoms get close to carbon → inter-atomic hybridization like hydrogen molecule)

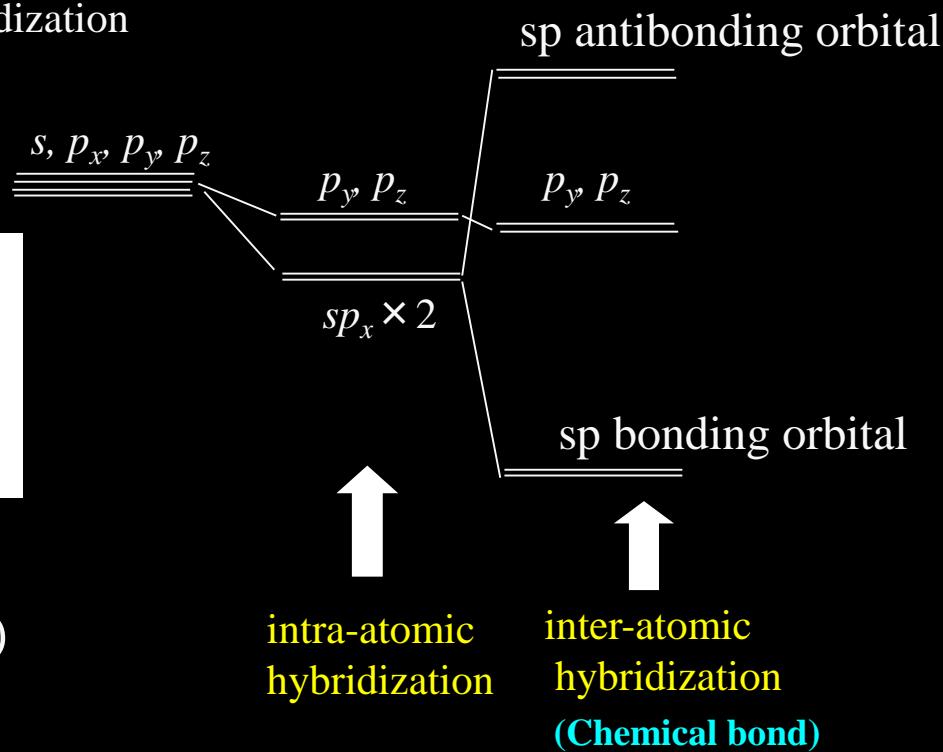


Finally, construct the overall molecular orbitals

i) Uniaxial 2-way coordination; sp hybridization



$$sp_x = \frac{1}{\sqrt{2}}(s \pm p_x)$$

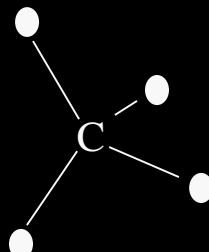


Tetrahedral 4-way coordination; sp^3 hybridization

intra-atomic
配位子場効果
(原子内混成)

inter-atomic
化学結合
(原子間混成)

Chemical bond



s, p_x, p_y, p_z



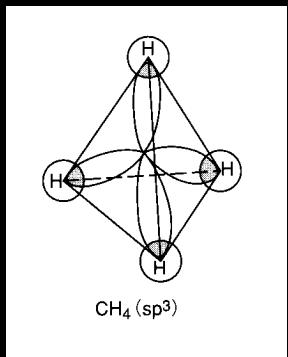
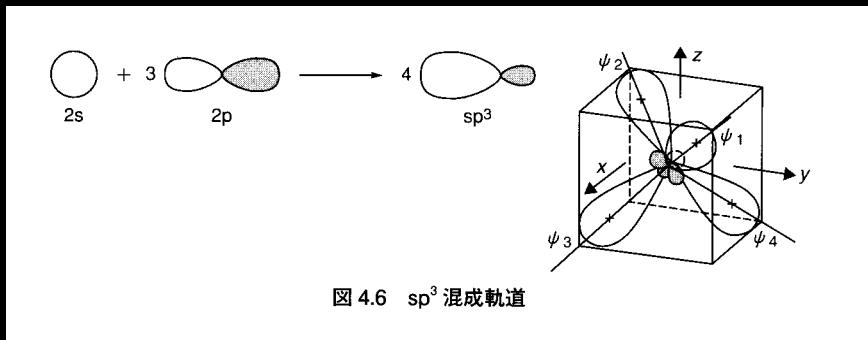
sp^3 反結合性軌道

antibonding orbital

$sp^3 \times 4$

sp^3 結合性軌道

bonding orbital



メタン、ダイヤモンド
methane, diamond

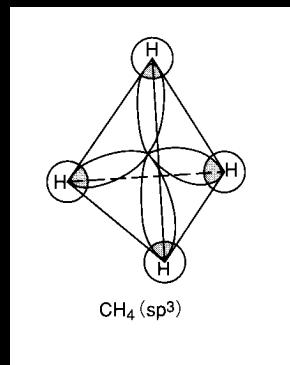
$$sp^3 = \frac{1}{2}(s + p_x + p_y + p_z)$$

$$\frac{1}{2}(s + p_x - p_y - p_z)$$

$$\frac{1}{2}(s - p_x - p_y + p_z)$$

$$\frac{1}{2}(s - p_x + p_y - p_z)$$

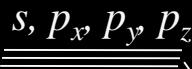
Tetrahedral 4-way coordination molecular orbital; the case of CH_4 (sp^3)



intra-atomic
hybridization

inter-atomic
hybridization

Molecular orbital



$\text{sp}^3 \times 4$

sp^3 antibonding orbital

sp^3 bonding orbital

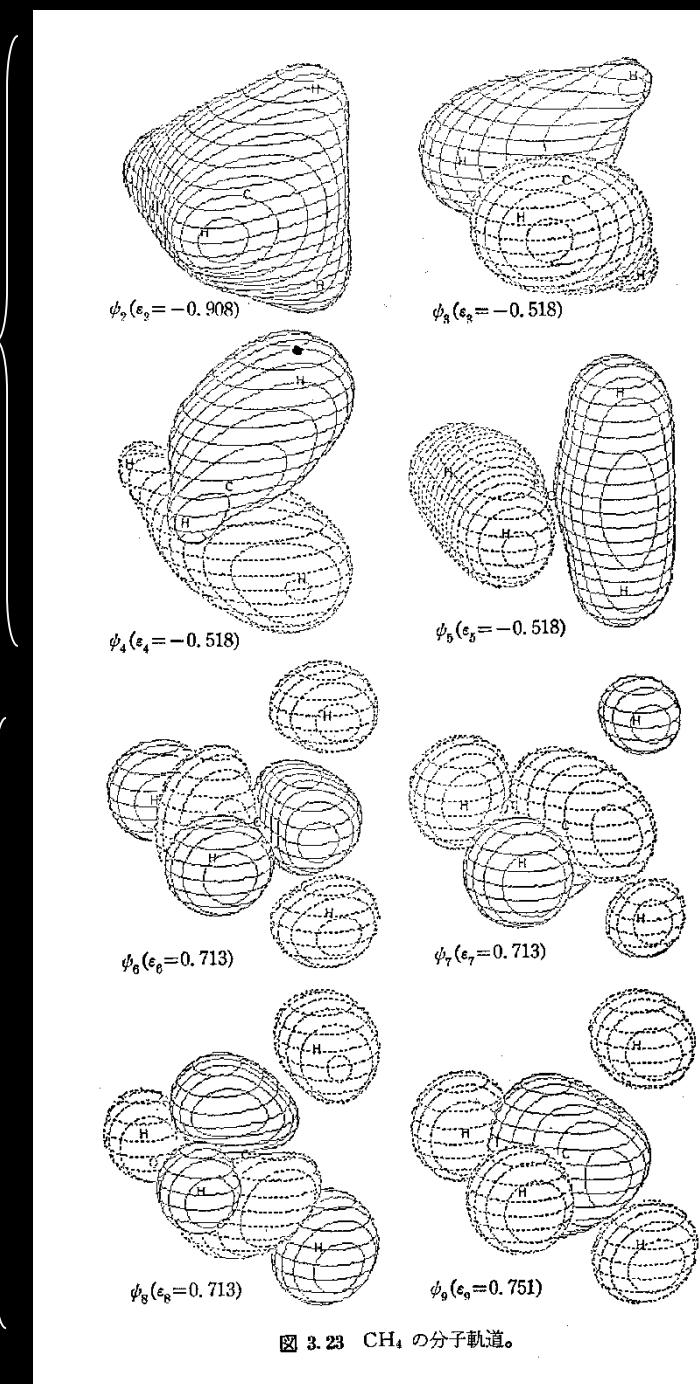
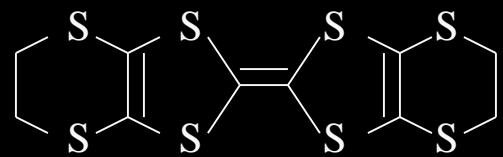
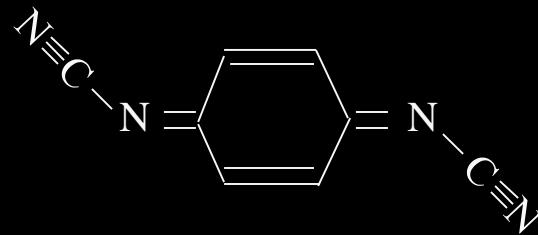


図 3.23 CH_4 の分子軌道。

Molecular orbital in molecular conductors



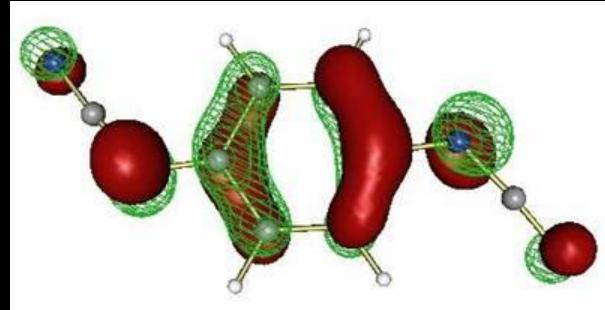
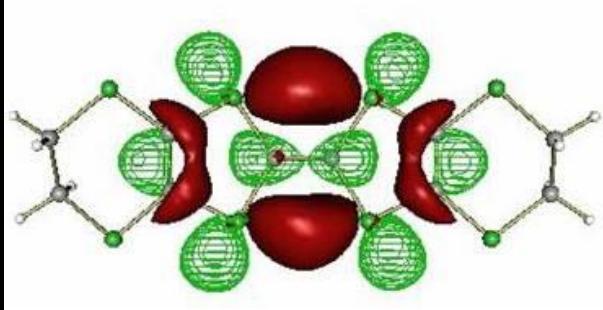
BEDT-TTF (ET)



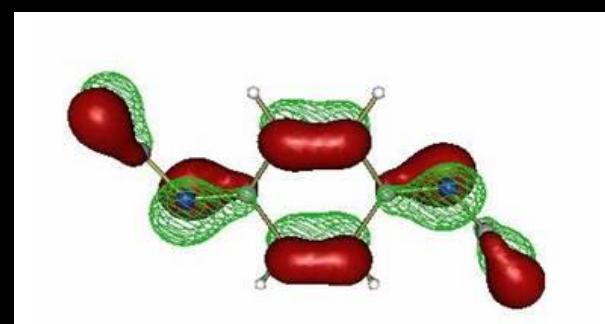
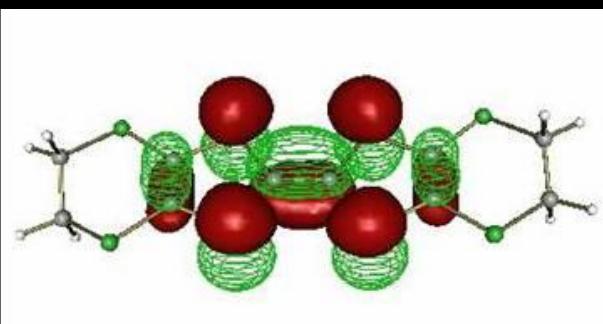
DCNQI

ϵ

Lowest Unoccupied Molecular Orbital (LUMO)



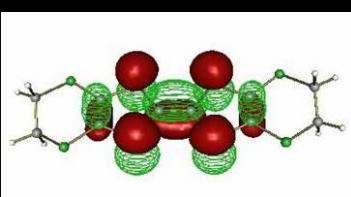
Highest Occupied Molecular Orbital (HOMO)



By Imamura
and Tanimura

Band-structure calculations I; π electronic system $\kappa\text{-}(\text{ET})_2\text{X}$

well described by tight-binding model of MO



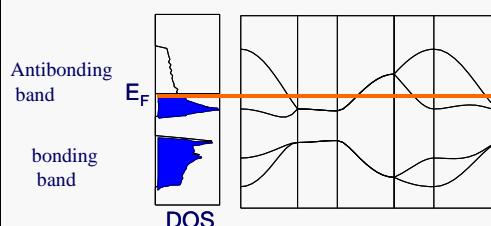
HOMO + tight-binding approx.

T. Mori et al.,

LUMO level
HOMO level

unregistered

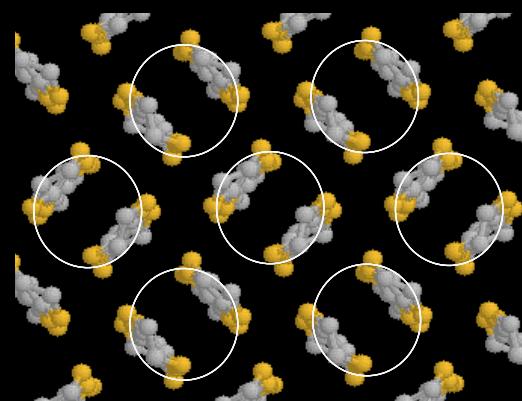
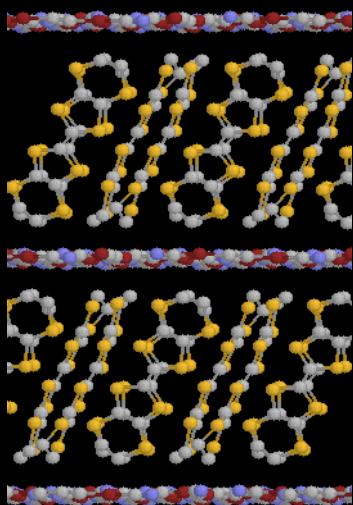
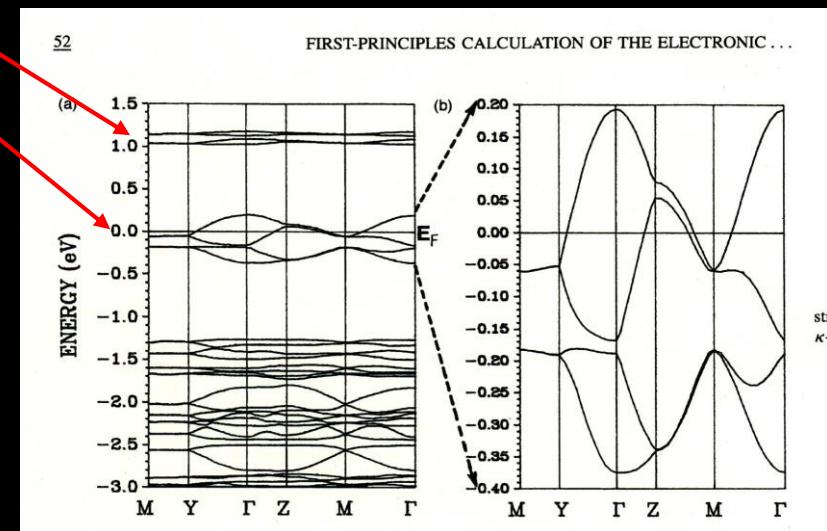
拡張Hückel法 + 強束縛近似



フェルミ面

First-principles calculations

Y.-N. Xu *et al.*, *Phys. Rev. B* 52, 12946



12 948

YONG-NIAN XU, W.Y. CHING, Y.C. JEAN, AND Y. LOU

52

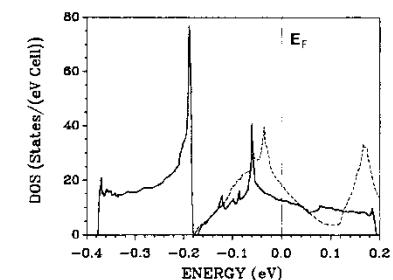
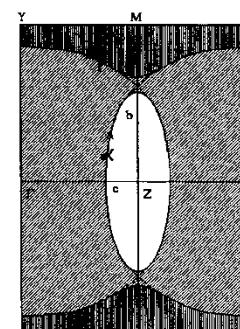
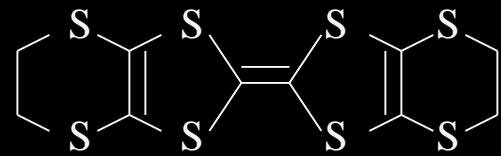


FIG. 2. Calculated Fermi surface of the $\kappa\text{-}(\text{BEDT-TTF})_2\text{Cu}(\text{NCS})_2$. The two orbits α and β are marked. The dashed lines indicate the path of electron in the magnetic breakdown.

FIG. 3. Calculated density of states of $\kappa\text{-}(\text{BEDT-TTF})_2\text{Cu}(\text{NCS})_2$ (solid line); K_3C_{60} (dashed line).

lated to the saddle point of the band at M . By resolving the DOS into atomic and orbital components, we find that the DOS at E_F are exclusively derived from the C and S atoms in the ET molecules, and their orbital components are almost entirely of p character (45% of $\text{C}-3p$, 52% of $\text{S}-3p$, and 3%

Notion of molecular orbital



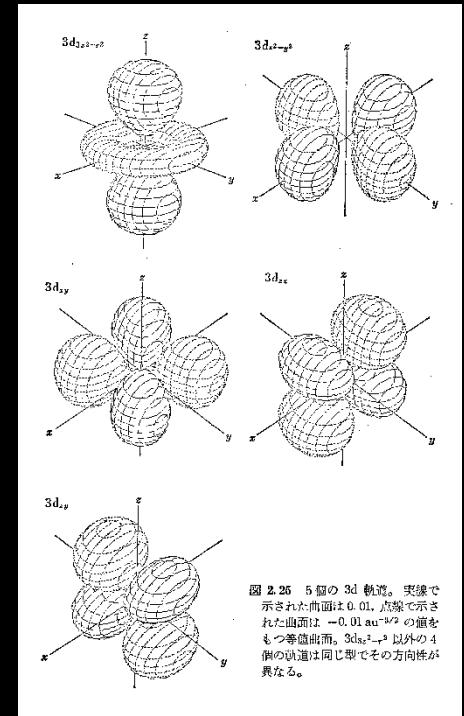
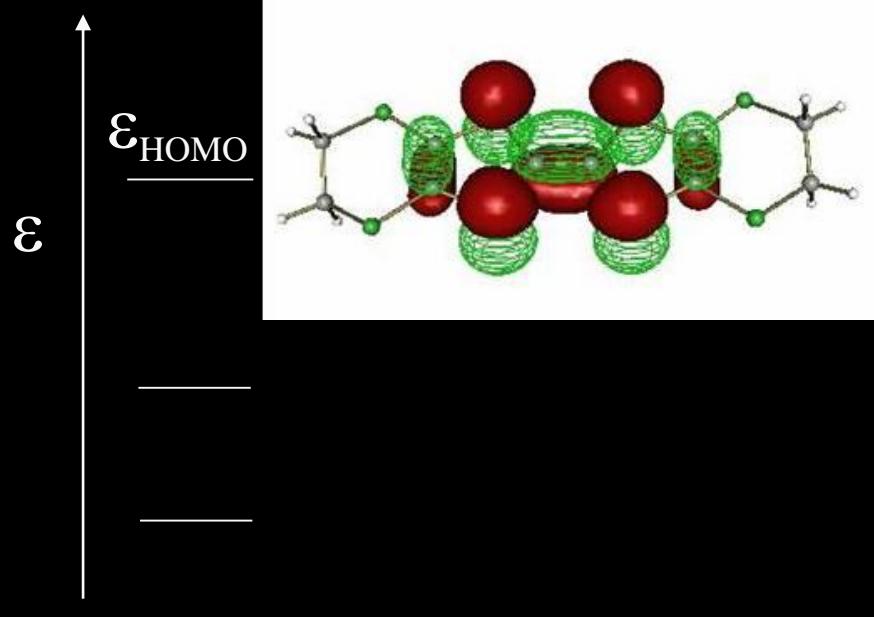
BEDT-TTF (ET)



Cu

HOMO

d orbital



Molecular conductors

Seemingly complicated structure in real space
but

Simple electronic structure in k space
(MO is a minimum electronic entity)

In many cases,
no orbital degeneracy
negligible spin-orbit interaction

Highly compressible



Model systems to look into correlation effect
in simple electronic systems

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all-in-one systems for Mott physics

3. Spin liquid in quasi-triangular lattice

controlled frustration, correlation, disorder, doping

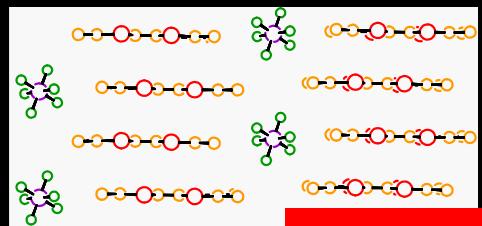
(Optional)

4. Massless Dirac Fermions in organic materials

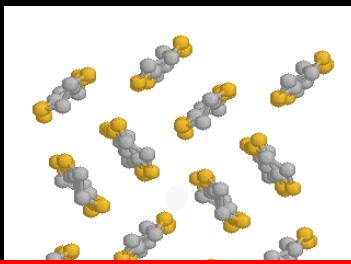
Dirac cone reshaping and ferromagnetism

Correlation-induced insulating phases everywhere in organics

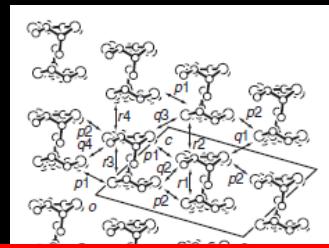
Quasi 1D 1/4-filled
 $(\text{TMTSF})_2\text{X}$



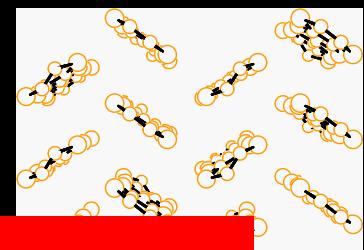
Quasi 2D 1/2-filled
 $\kappa-(\text{ET})_2\text{X}$



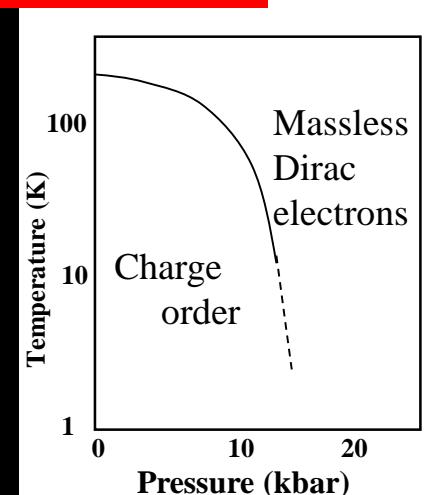
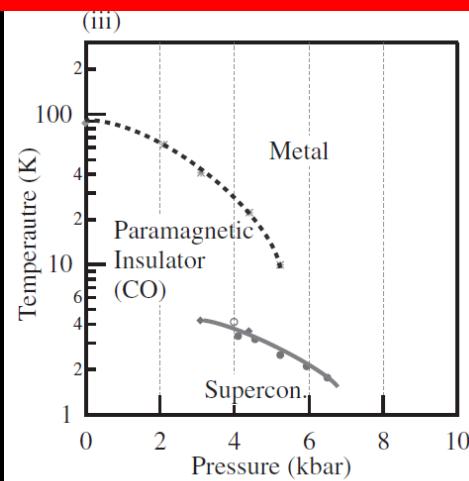
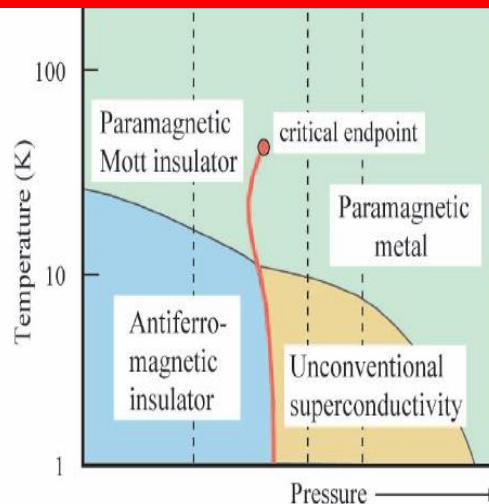
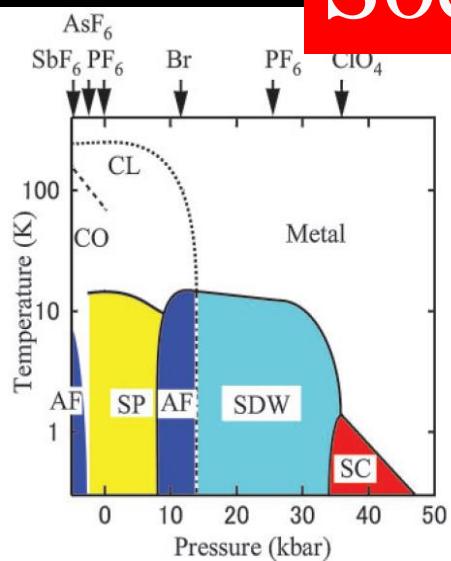
Quasi 2D 1/4-filled
 $\beta-(\text{meso}, \text{DMeET})_2\text{PF}_6$



Quasi 2D 1/4-filled
 $\alpha-(\text{ET})_2\text{X}$



Sociology of electronic system



SDW/SC
Nesting

Mott/SC
On-site repulsion

CO/SC
Inter-site repulsion

CO/DE
Inter-site repulsion

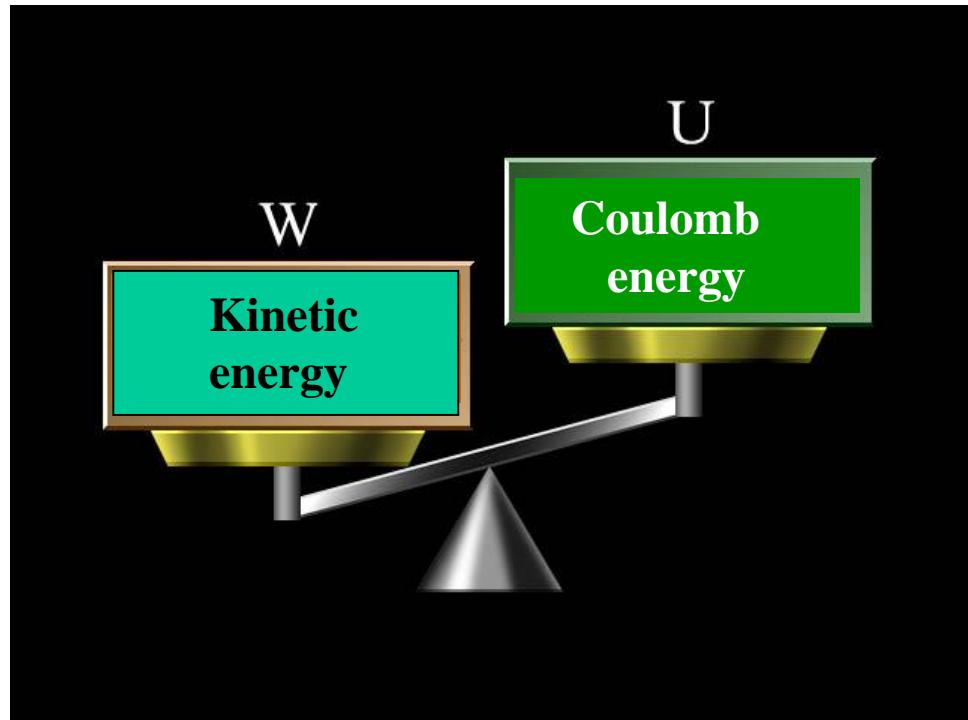
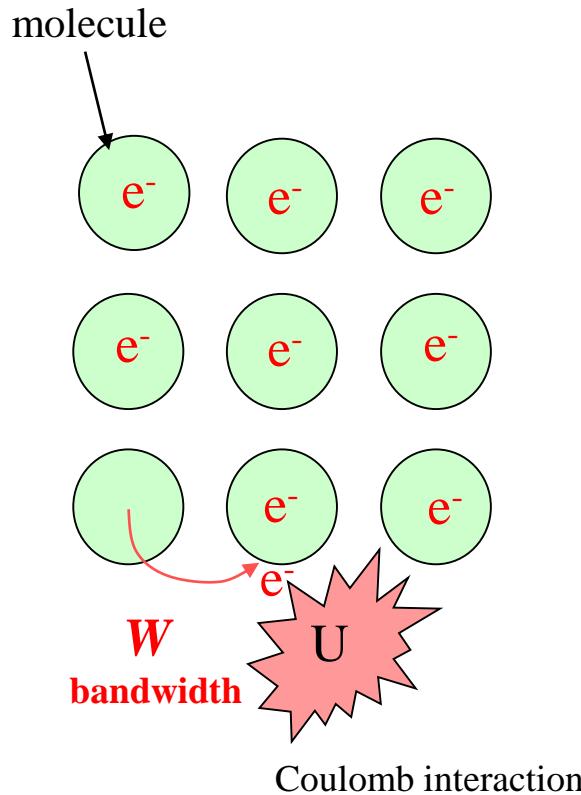
3. Mott transition

N. Mott (1949)



Mott transition

Competition between kinetic energy and Coulomb



$W > U$

metal

Wave-like

$W < U$

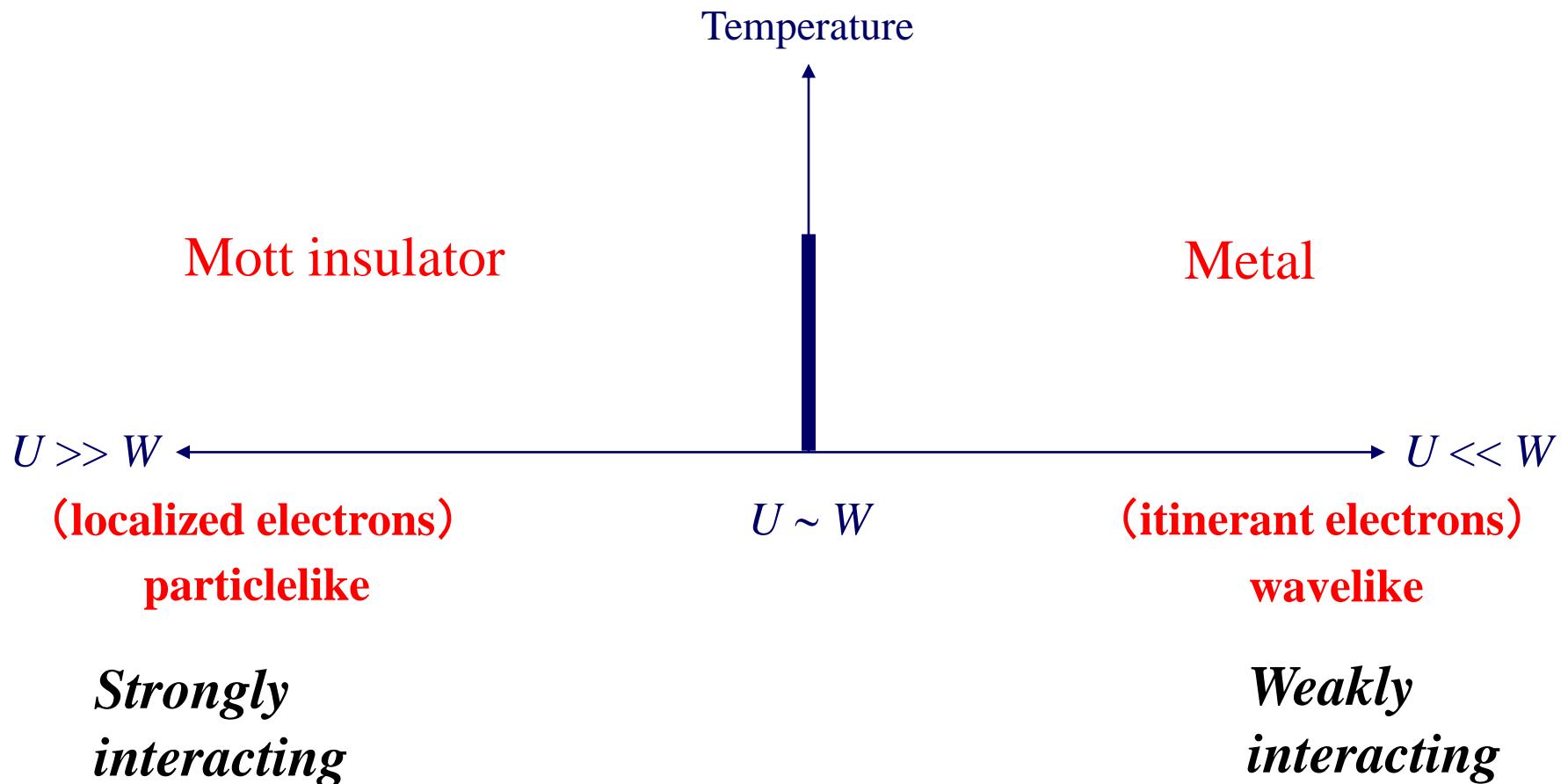
insulator

particle-like

Competition between kinetic energy and Coulomb energy

(W : bandwidth)

(U : Coulomb repulsion)



Hubbard model

Hubbard Hamiltonian

$$H$$

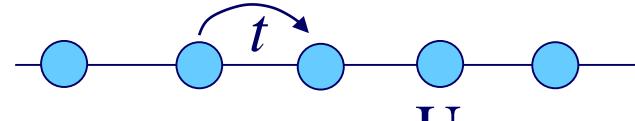
$$H = \sum_{(i,j)\sigma} (t c_{i,\sigma}^+ c_{j,\sigma} + h.c.) + U \sum_i n_{i,\uparrow} n_{i,\downarrow}$$

off-diagonal diagonal

$$H'$$

$$U$$

1 particle/site



Wannier

Bloch

$$c_{j,\sigma} = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{R}_j} c_{\mathbf{k},\sigma}$$

diagonal off-diagonal

$$H = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k},\sigma}^+ c_{\mathbf{k},\sigma} + \frac{U}{N} \sum_{k_1,k_2,k_3,k_4} c_{k_1,\uparrow}^+ c_{k_2,\downarrow}^+ c_{k_3,\uparrow} c_{k_4,\downarrow} \delta_{k_1+k_2,k_3+k_4}$$

H H'

In the weak correlation regime, $W \sim 2zt \gg U$

Hubbard Hamiltonian

Calculate life time of Bloch electron

$$H = \left[\sum_{\mathbf{k}, \sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}, \sigma}^+ c_{\mathbf{k}, \sigma} \right] + \frac{U}{N} \sum_{k_1, k_2, k_3, k_4} c_{k_1, \uparrow}^+ c_{k_2, \downarrow}^+ c_{k_3, \uparrow} c_{k_4, \downarrow} \delta_{k_1+k_2, k_3+k_4}$$

H_0

H' (perturbation)

Scattering term

$$\begin{aligned} \text{Scattering rate } \frac{1}{\tau(k_1)} &= \sum_{k_2, k_1, k_2} \frac{2\pi}{\hbar} \langle k_1, k_2 | H' | k_1, k_2 \rangle^2 \delta(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_1} - \epsilon_{k_2}) f_{k_2} (1 - f_{k_1}) (1 - f_{k_2}) \\ &= \frac{2\pi}{\hbar} \frac{1}{N^2} \sum_{k_2, k_1, k_2} U^2 \delta(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_1} - \epsilon_{k_2}) \delta_{k_1+k_2, k_1+k_2} f_{k_2} (1 - f_{k_1}) (1 - f_{k_2}) \end{aligned}$$

$$3D, \quad \frac{1}{\tau(k_1)} \propto T^2$$

$$2D, \quad \frac{1}{\tau(k_1)} \propto T^2 \log \frac{\epsilon_F}{k_B T}$$

$$1D, \quad \frac{1}{\tau(k_1)} = \frac{2\pi}{\hbar} U^2 \frac{1}{8\pi^2} \frac{a^2}{(\hbar v_F)^2} k_B T$$

At low-Temperatures

$$\frac{\hbar}{\tau} \ll k_B T$$

Fermi liquid

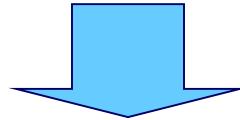
✗ Non-Fermi liquid

In the strong correlation regime, $W \sim 2zt \ll U$

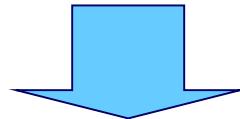
Hubbard Hamiltonian

$$H = \sum_{(i,j)\sigma} (tc_{i,\sigma}^+ c_{j,\sigma} + h.c.) + U \sum_i n_{i,\uparrow} n_{i,\downarrow}$$

H' (perturbation) H_0



Heisenberg Hamiltonian $H = \sum_{(i,j)} J \mathbf{S}_i \mathbf{S}_j$ $J = 4t^2/U$

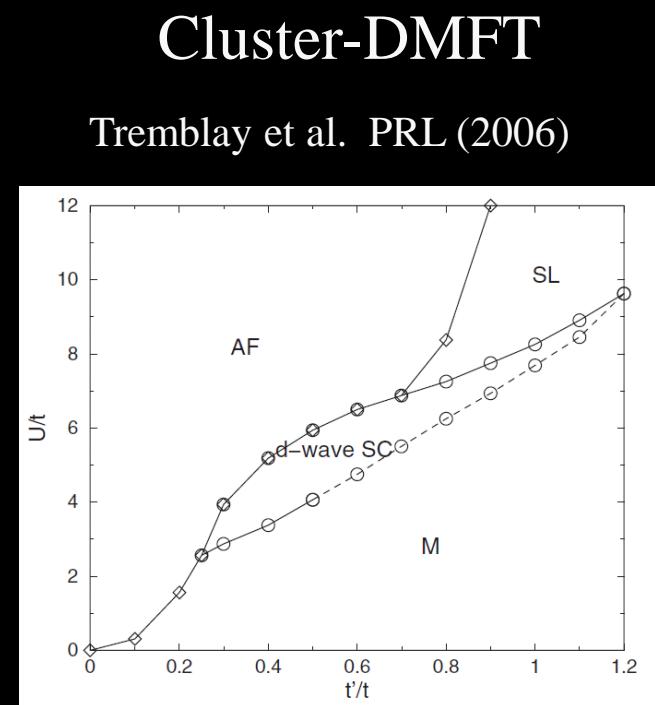
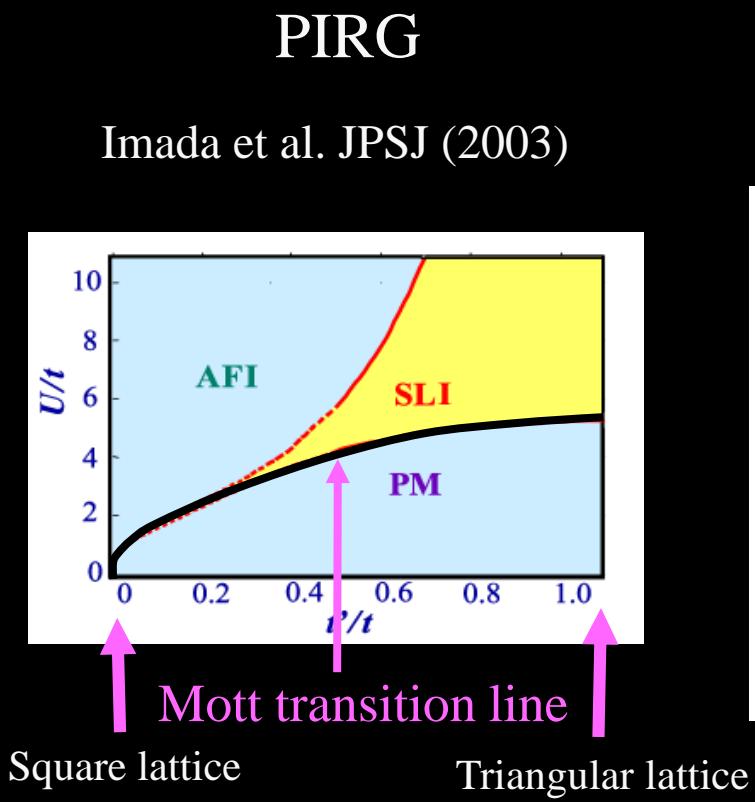
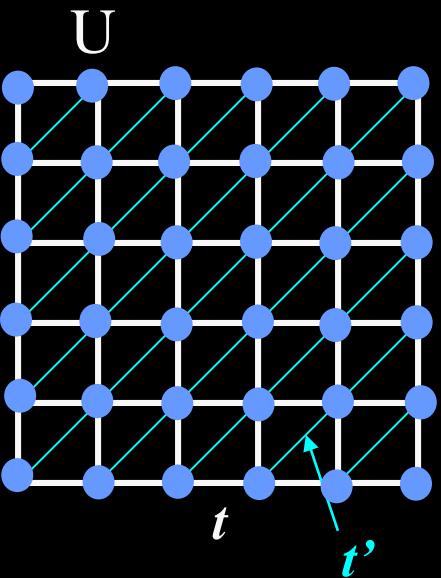


Antiferromagnetic insulator

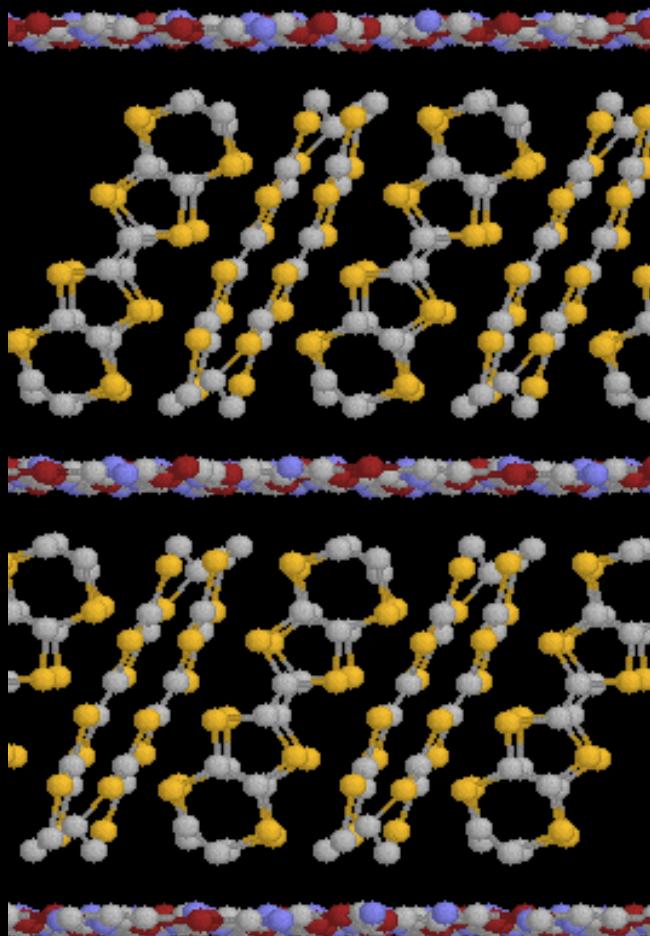
Mott transition occurs at $W \sim U$,
but depends on dimension and lattice geometry

1-D Hubbard models are always Mott insulators.

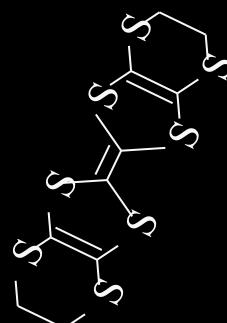
2D $\frac{1}{2}$ -filled Hubbard model on anisotropic triangular lattice



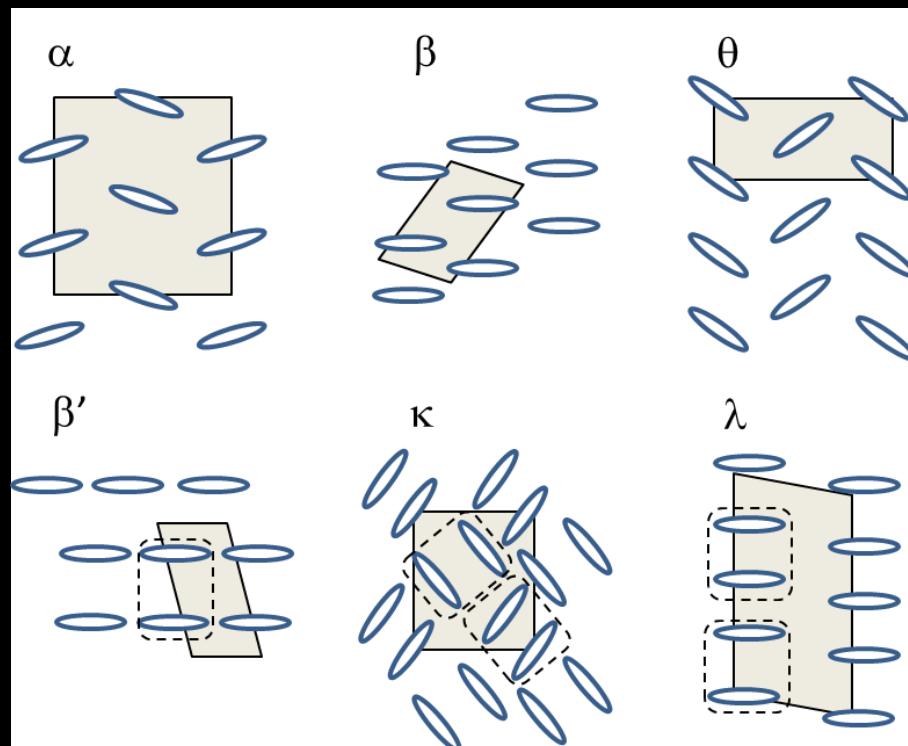
Layered molecular conductors, $(\text{BEDT-TTF})_2\text{X}$



X

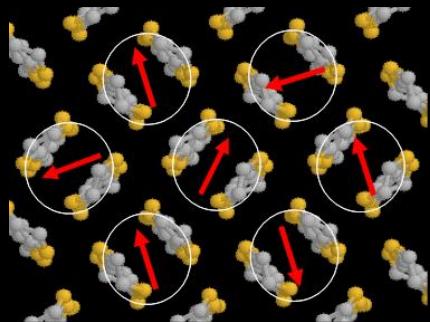


A variety of in-plane structures



κ -(ET)₂X family are on the verge of Mott transition

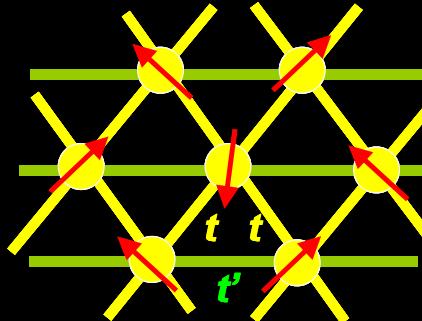
in-plane structure



dimer model

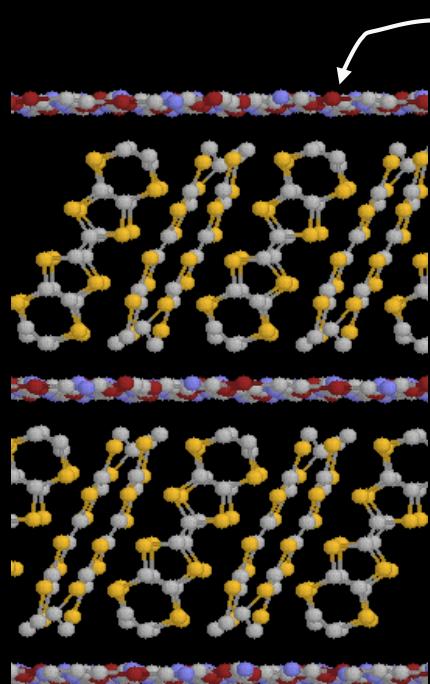


Kino & Fukuyama

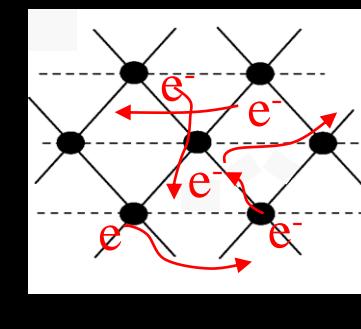
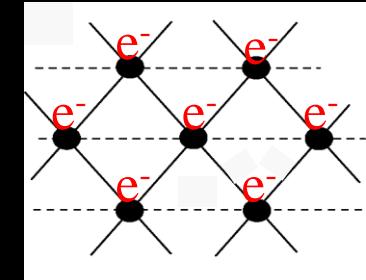


Triangular lattice
Half-filled band

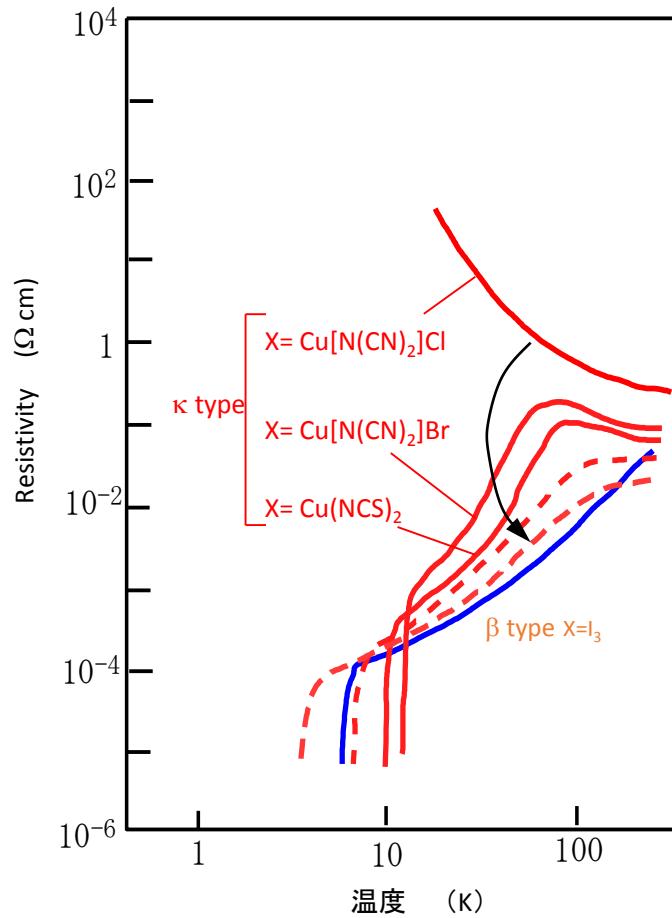
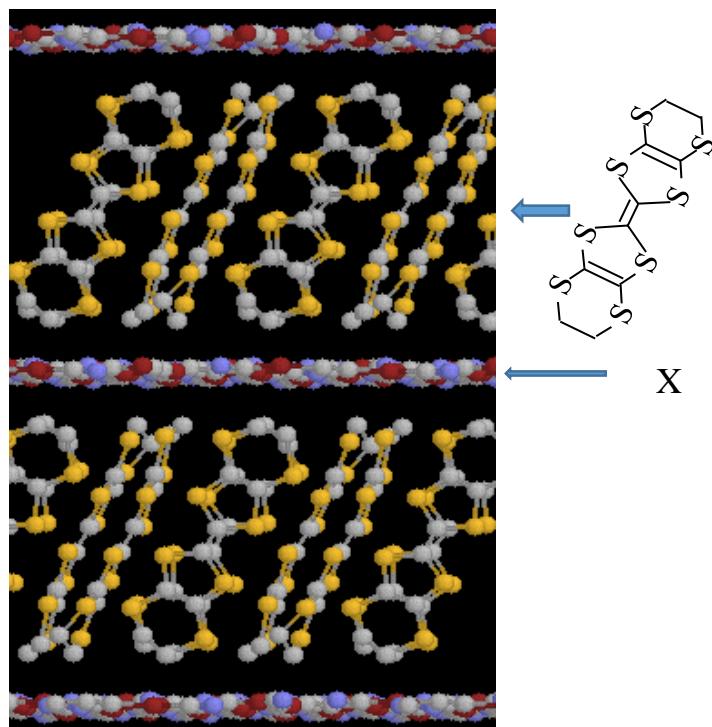
t : inter-dimer transfer integral
 U : on-site Coulomb



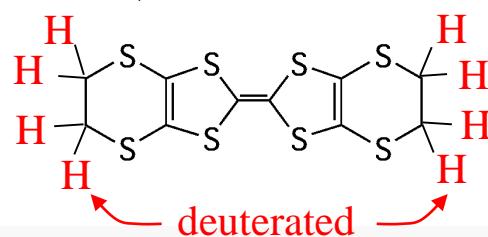
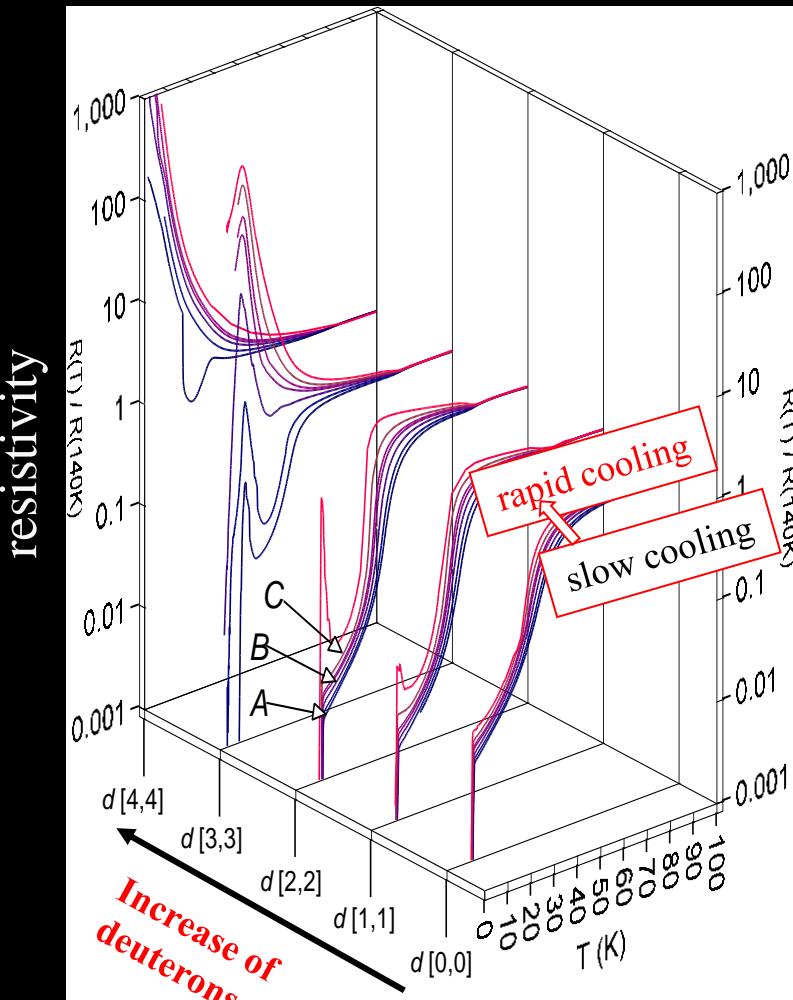
| X | Ground state | U/t | t'/t |
|--------------------------------------|----------------|-----|------|
| Cu ₂ (CN) ₃ | Mott insulator | 8.2 | 1.06 |
| Cu[N(CN) ₂]Cl | Mott insulator | 7.5 | 0.75 |
| Cu[N(CN) ₂]Br | Metal (SC) | 7.2 | 0.68 |
| Cu(NCS) ₂ | Metal (SC) | 6.8 | 0.84 |
| Cu(CN)[N(CN) ₂] | Metal (SC) | 6.8 | 0.68 |
| Ag(CN) ₂ H ₂ O | Metal (SC) | 6.6 | 0.60 |
| I ₃ | Metal (SC) | 6.5 | 0.58 |



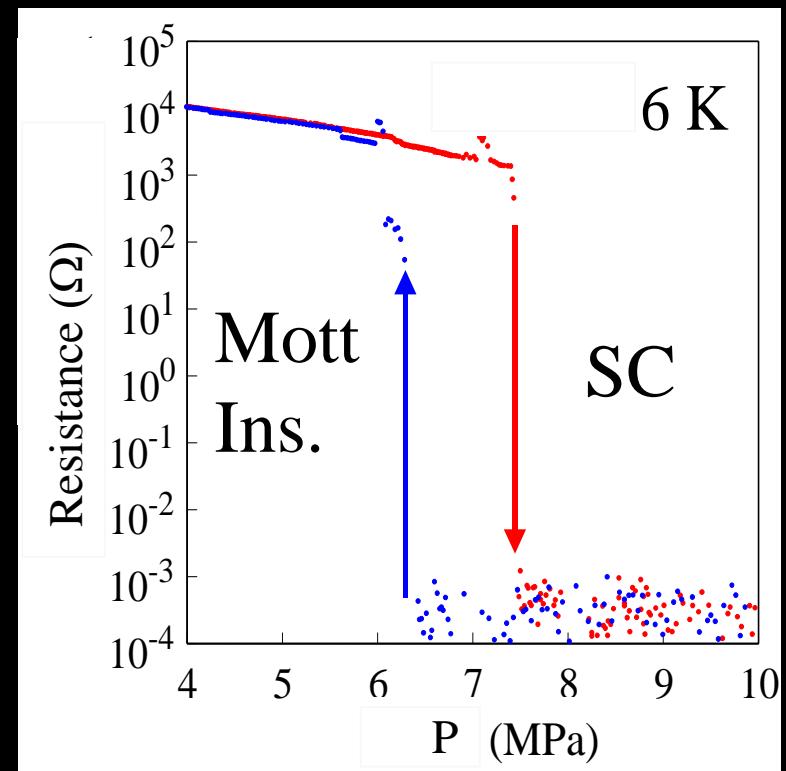
Resistivity of κ -(BEDT-TTF)₂X



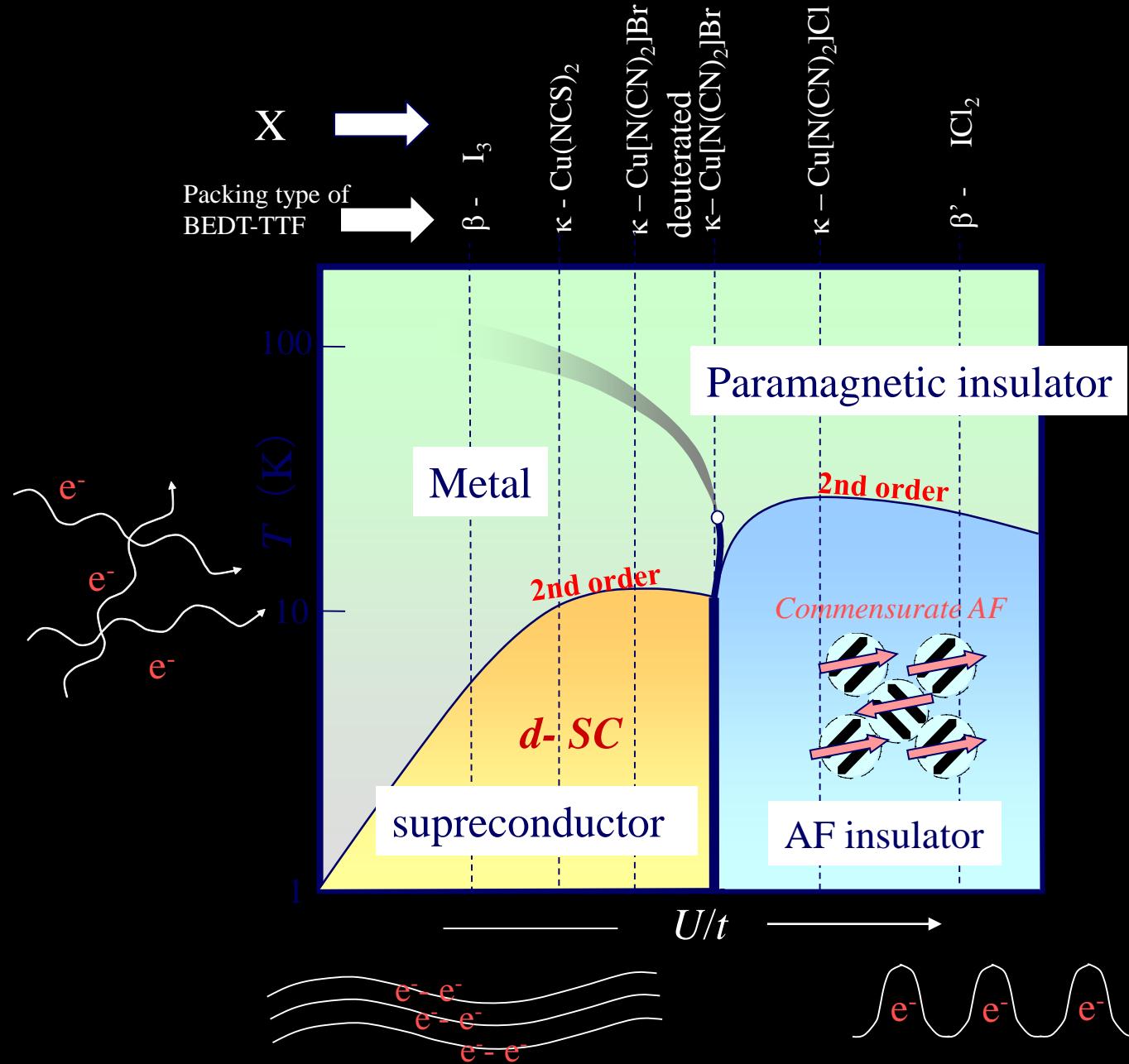
SIT by fine pressure tuning or isotope substitution



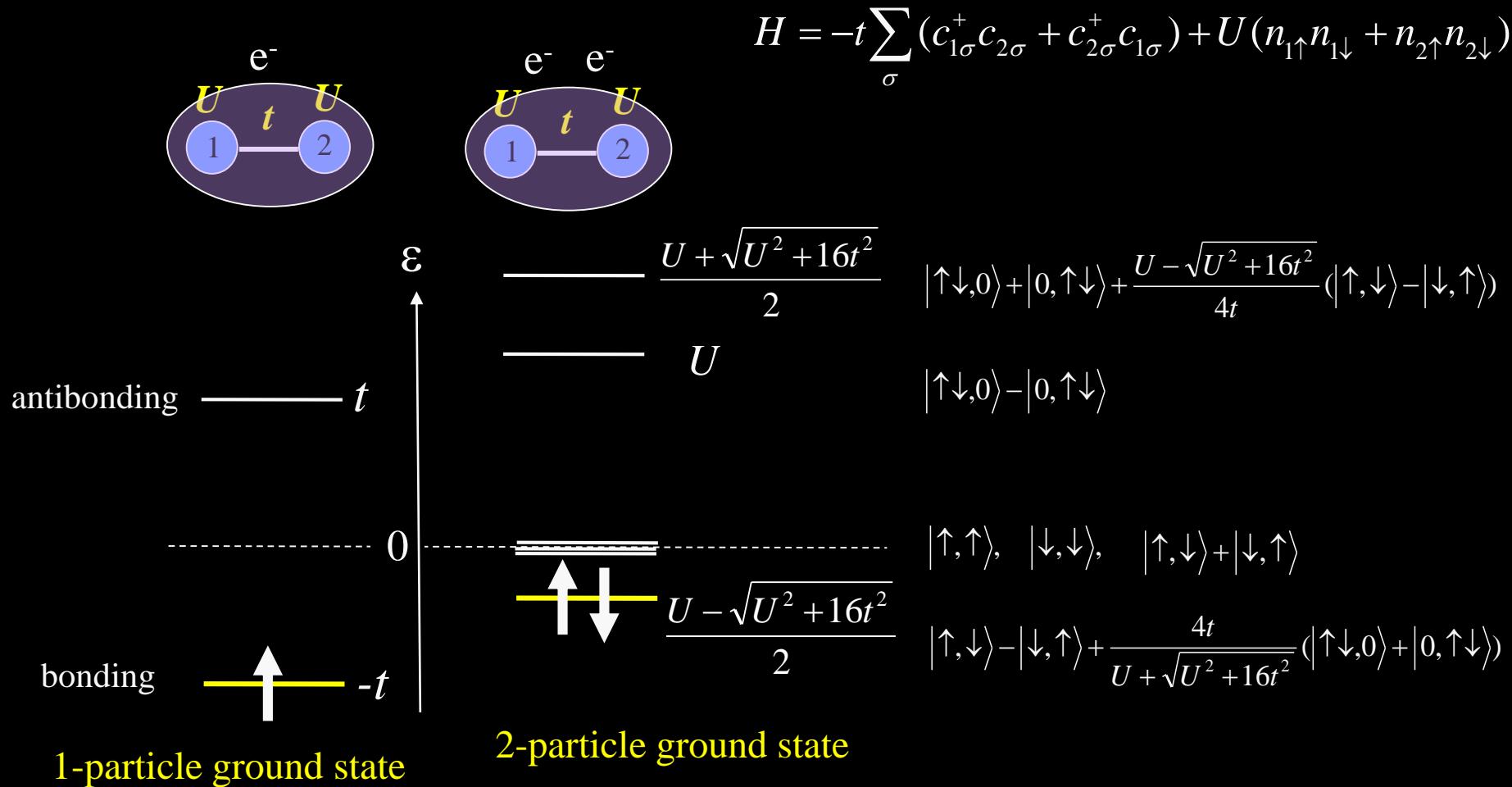
κ -(deuterated ET)₂Cu[N(CN)₂]Br



κ -(ET)₂X family are on the verge of Mott transition



2 electrons/dimer with on-site Coulomb energy \rightarrow Hubbard model of a hydrogen molecule

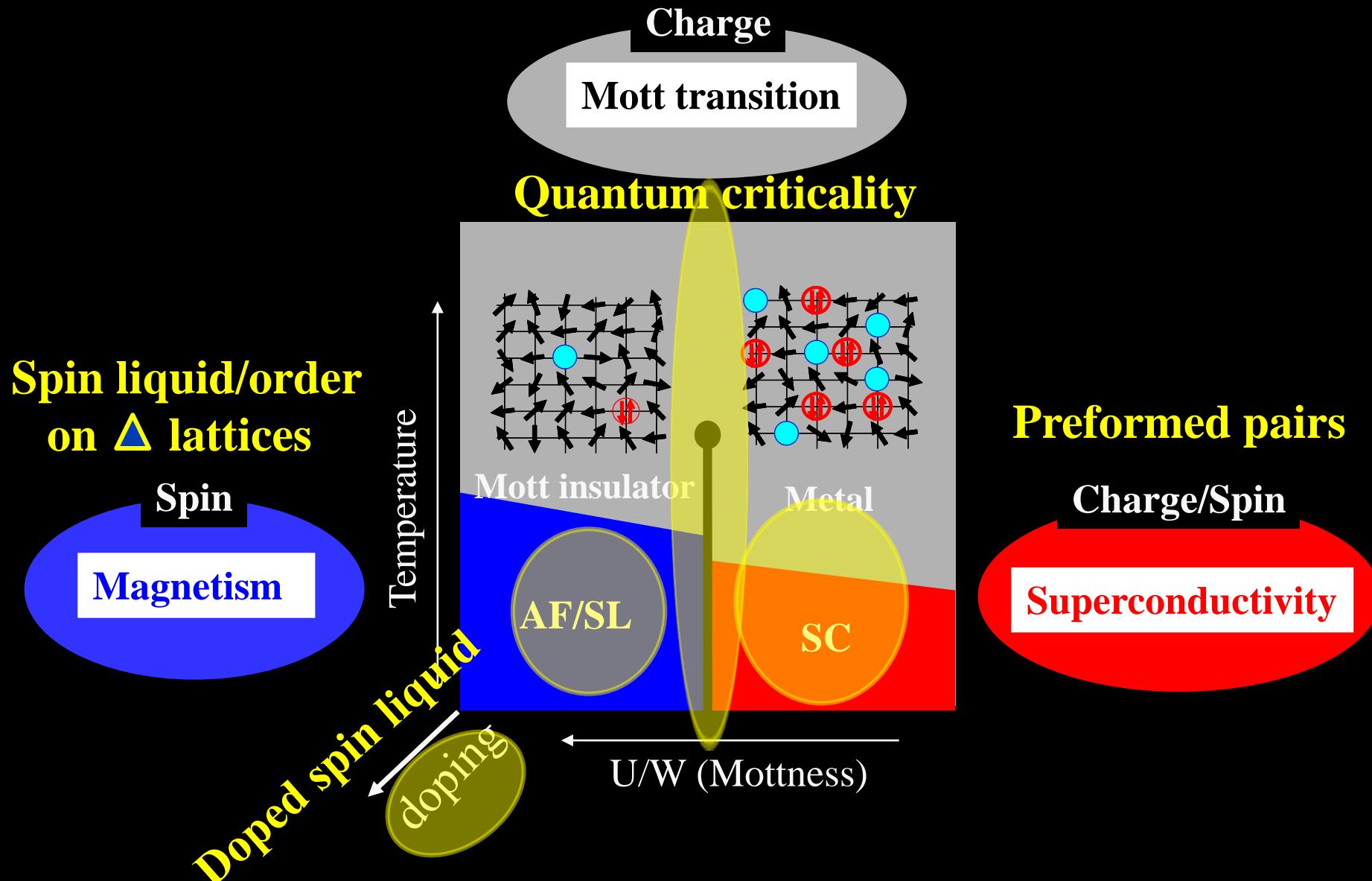


$$U_{eff} = \frac{U - \sqrt{U^2 + 16t^2}}{2} - 2(-t) = 2t + \frac{U - \sqrt{U^2 + 16t^2}}{2} \approx 2t$$

From band-structure calculations, $U_{eff} \sim 2t \sim 0.5$ eV and bandwidth $W \sim 0.4$ eV

comparable

Mott physics in 2D organics



Interacting spins → Order or not ?

1900

1950

2000

1936

1949

1973

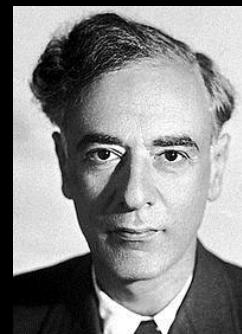
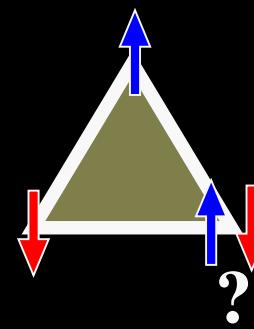
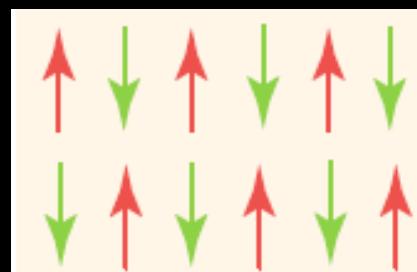
Heisenberg

$J S_i \cdot S_j$
Antiparrallel
interaction

Neel
proposed

proved
by neutron

Anderson
proposed



?

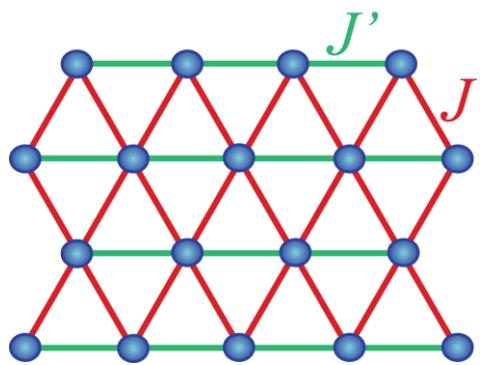
Landau



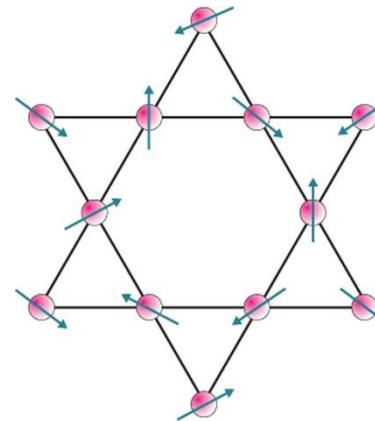
Anderson

Triangle-based lattices

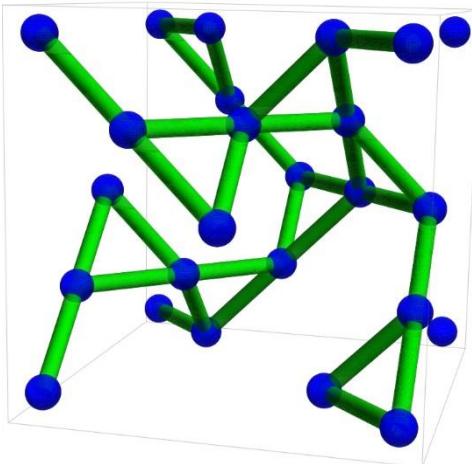
Triangular lattice



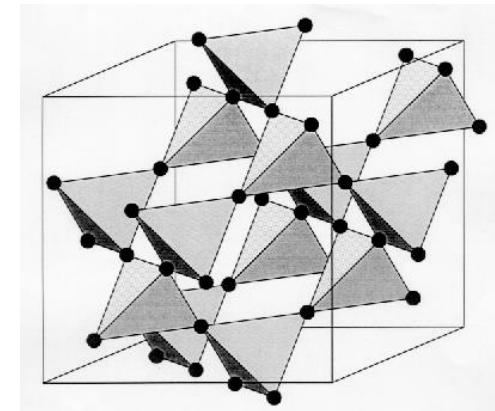
Kagome lattice



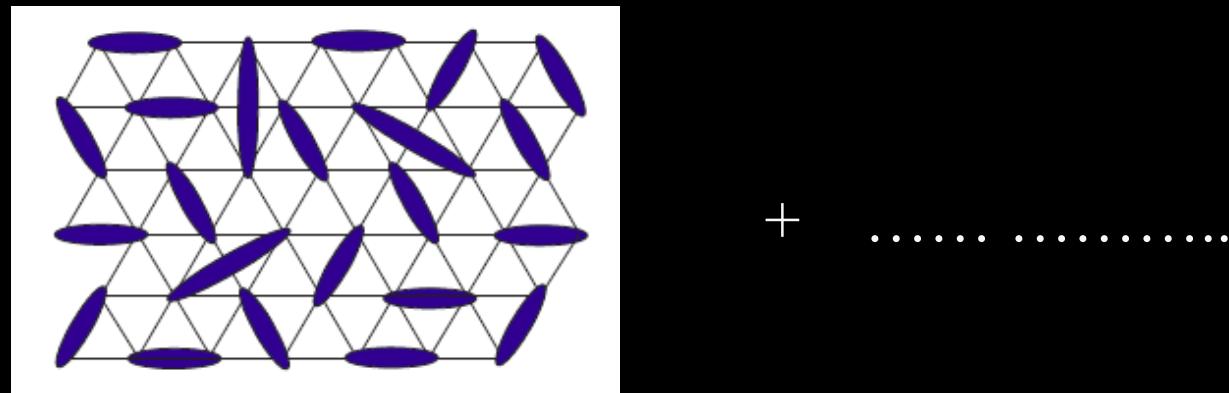
Pyrochlore lattice



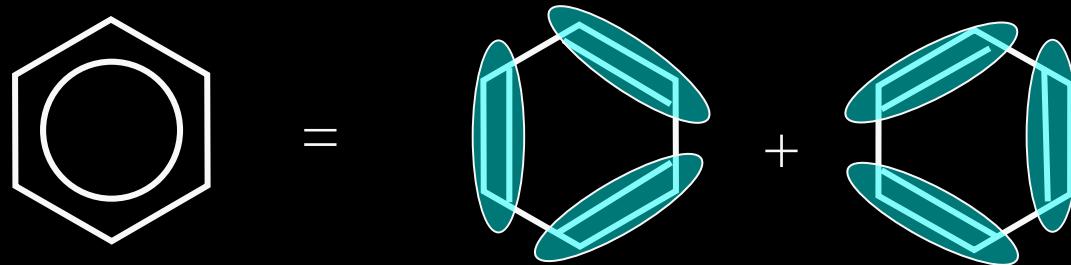
Hyper Kagome lattice



Anderson' idea of spin liquids: Resonating Valence Bond (RVB) state



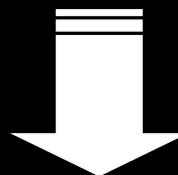
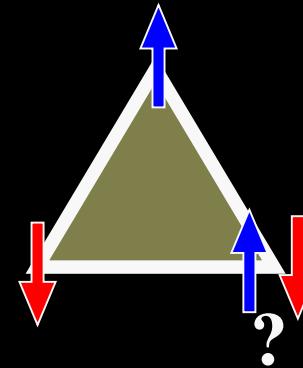
In analogy with benzene



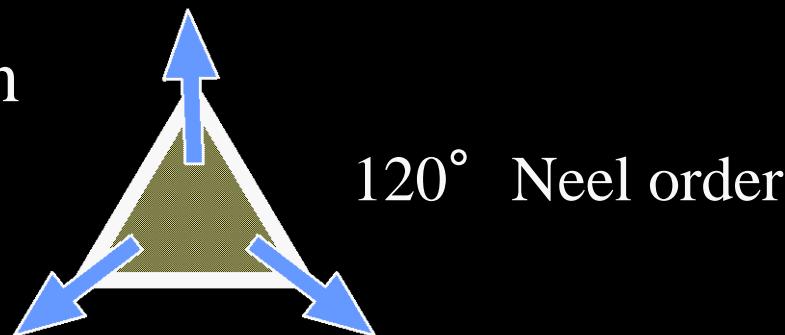
However, ~1990

Heisenberg model

$$H = 2J \sum_{(i,j)} \mathbf{S}_i \cdot \mathbf{S}_j \quad \text{on}$$

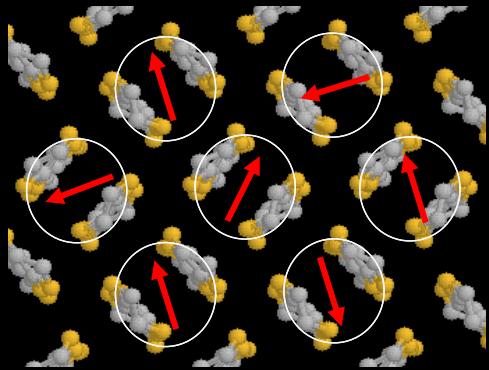


Solution



No spin liquid material in 20th centurythe end ?

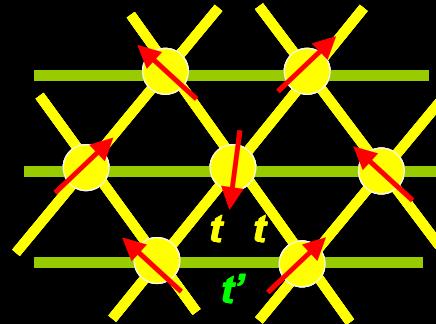
Q2D organics κ -(ET)₂X; *spin-1/2 on triangular lattice*



Kino & Fukuyama

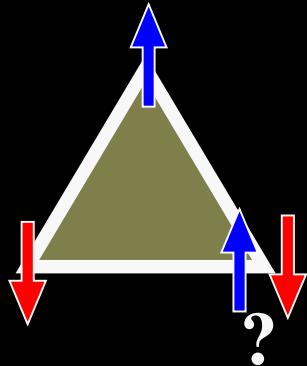


dimer model



$$t'/t = 0.4 \sim 1.1$$

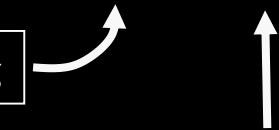
Spin frustration



Quantum spin liquid ?
Anderson (1973)

| X- | Ground State | t'/t |
|---|----------------|--------------|
| $\text{Cu}_2(\text{CN})_3$ | Mott insulator | 1.06 0.80 |
| $\text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$ | Mott insulator | 0.75 0.44 |

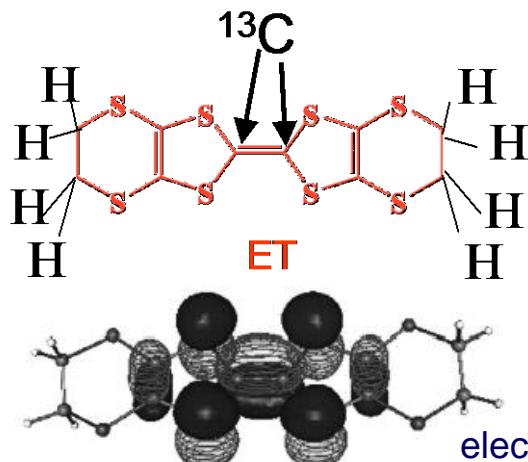
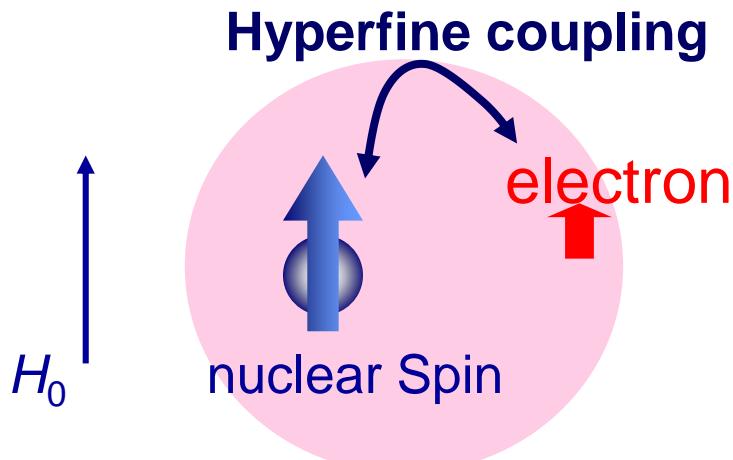
Huckel & tight binding



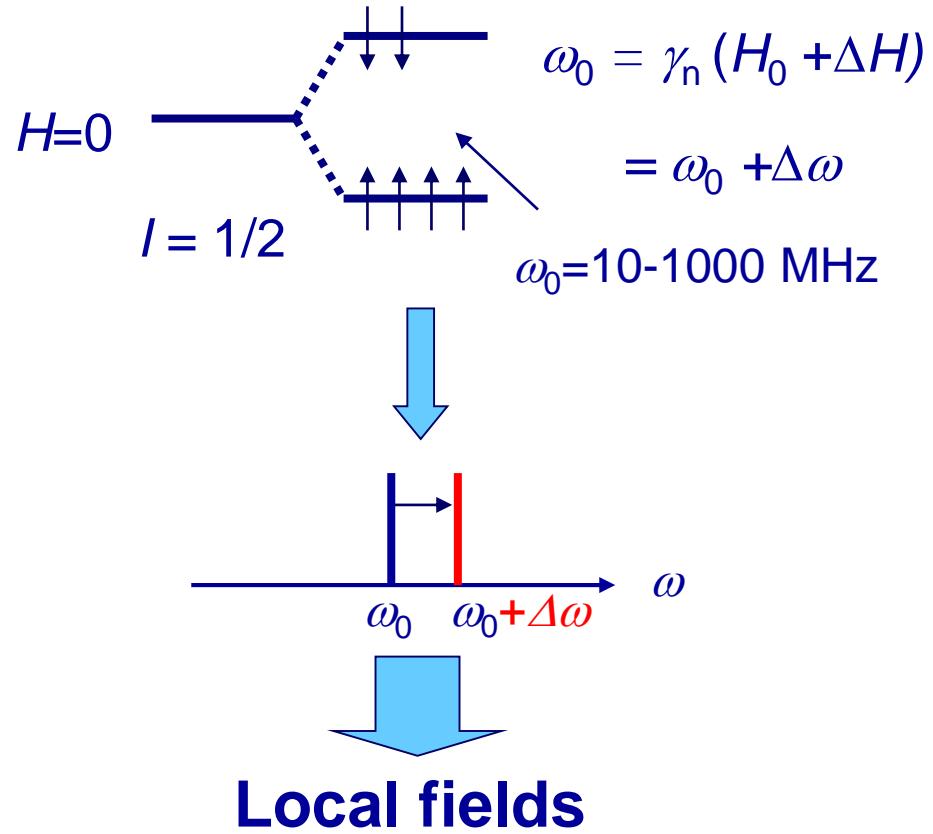
Ab initio; Kandpal et al. PRL (2009)
Nakamura et al. JPSJ (2009)

Magnetism

Nuclear Magnetic Resonance



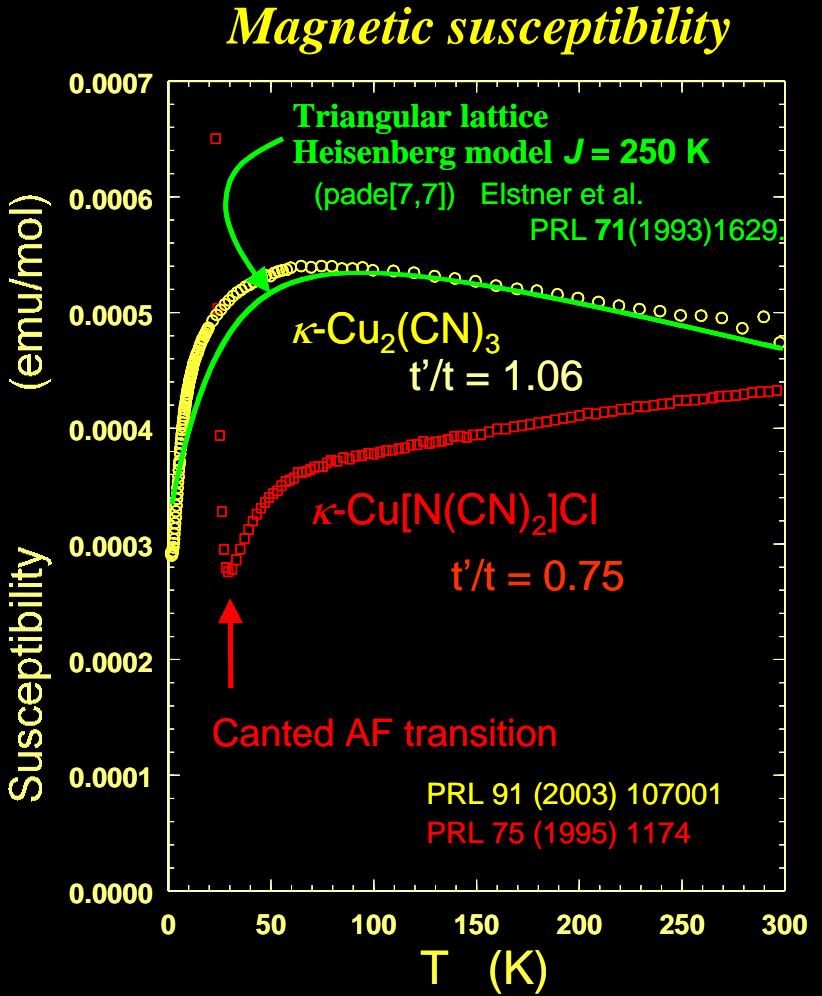
Y. Imamura, et al., JCP111(1999)5986



Knight shift, $K = \Delta\omega/\omega_0$

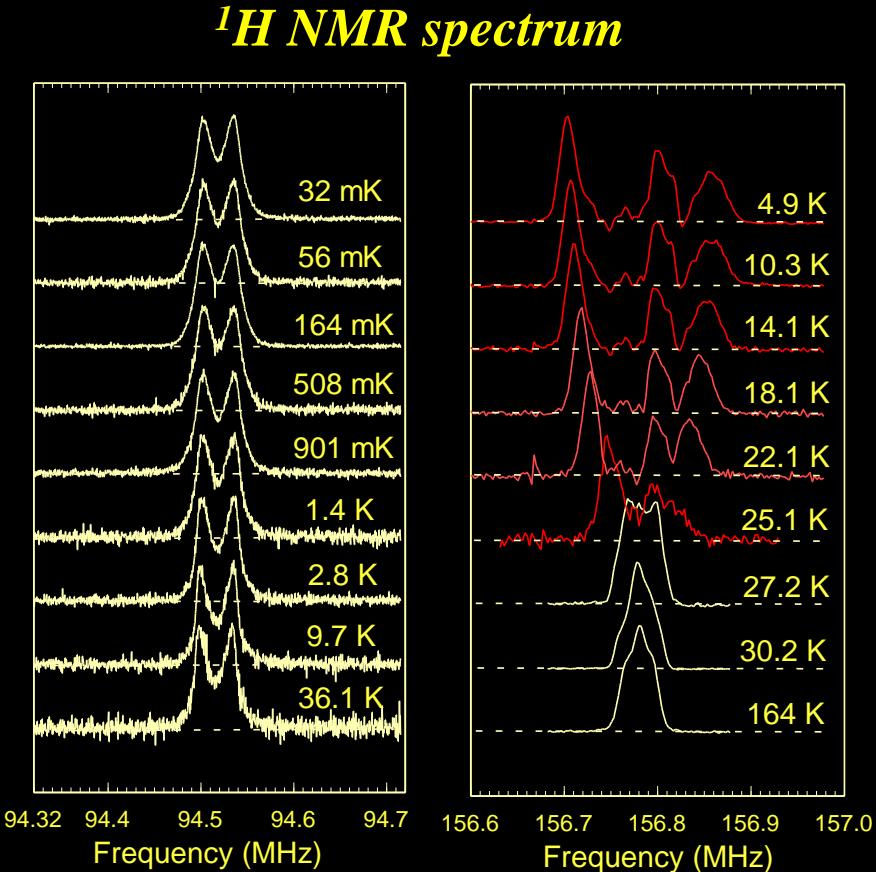
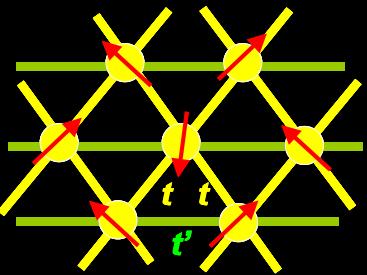
Mott insulators κ -(ET)₂X

Spin ordering or not ?

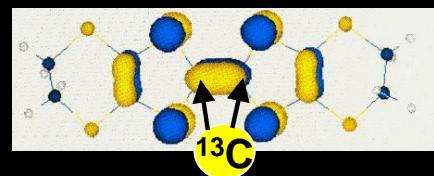


Also see Zheng et al. PRB 71 (2005) 134422

| X | t'/t |
|---|-------------|
| $\text{Cu}_2(\text{CN})_3$ | 1.06 (0.8) |
| $\text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$ | 0.75 (0.44) |

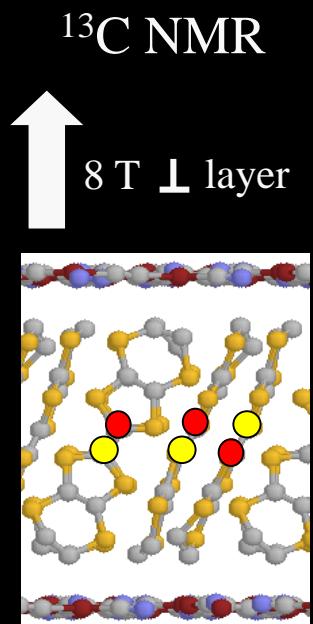
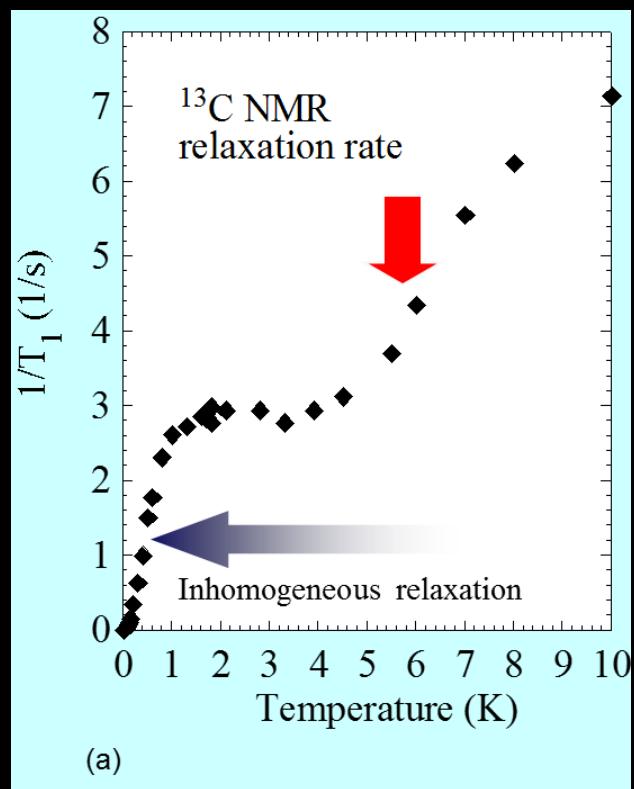


Spin anomaly around 5-6K in κ -(ET)₂Cu₂(CN)₃

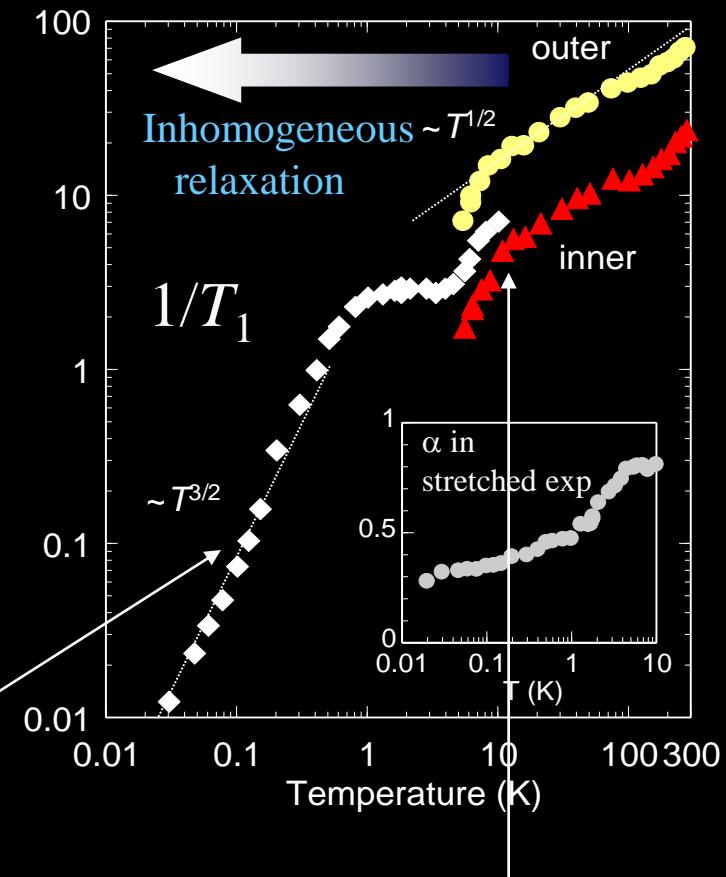


Shimizu *et al.*, PRB 70 (2006) 060510

NMR relaxation rate



Gapless



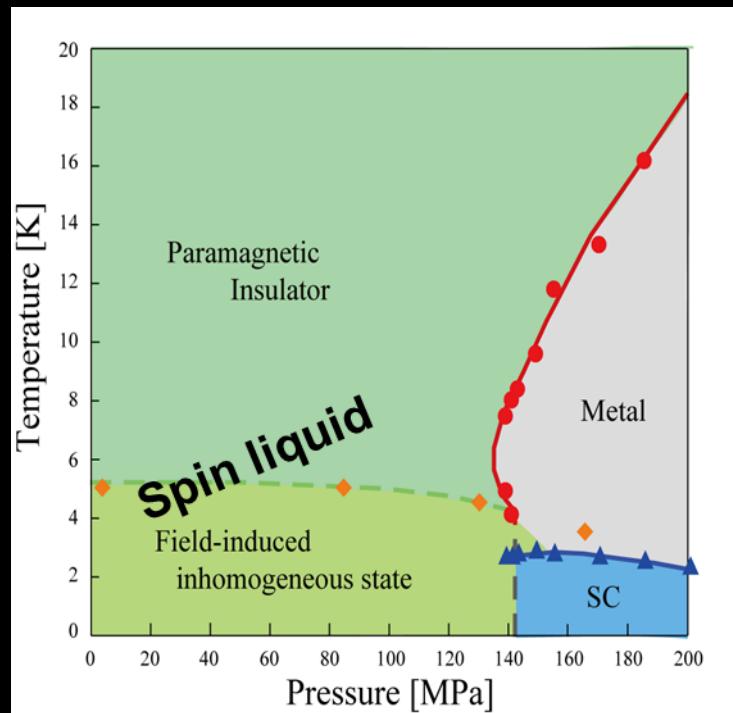
Why a spin liquid realized instead of



expected in
Heisenberg model ?

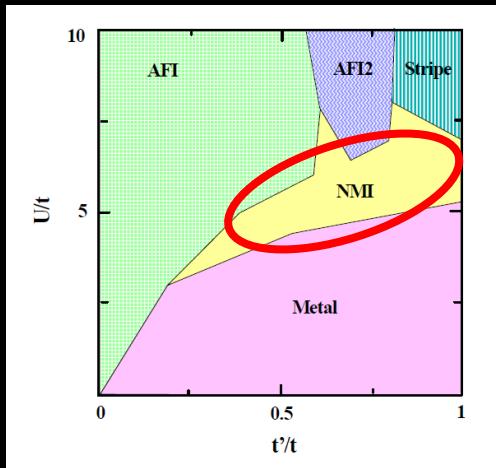
→near Mott transition

“Hubbard spin liquid”

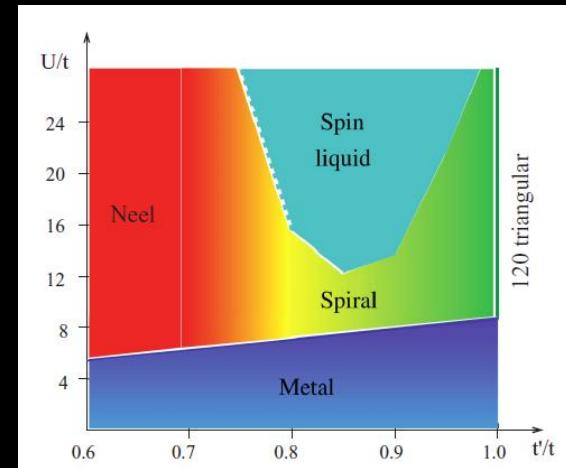


Spin liq. emerges Hubbard model ?

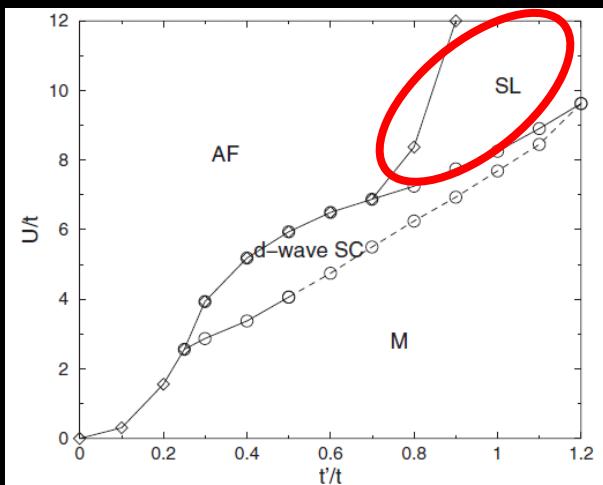
PIRG; Morita, Mizusaki, Imada (2002), (2006)



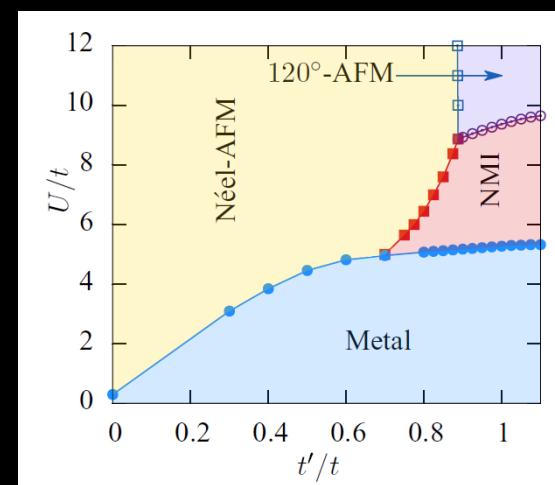
Tocchio et al. PRB (2013)



Cellular DMFT; Kyung, Tremblay (2006)



VCA + LDFA Laubach et al. PRB (2015)



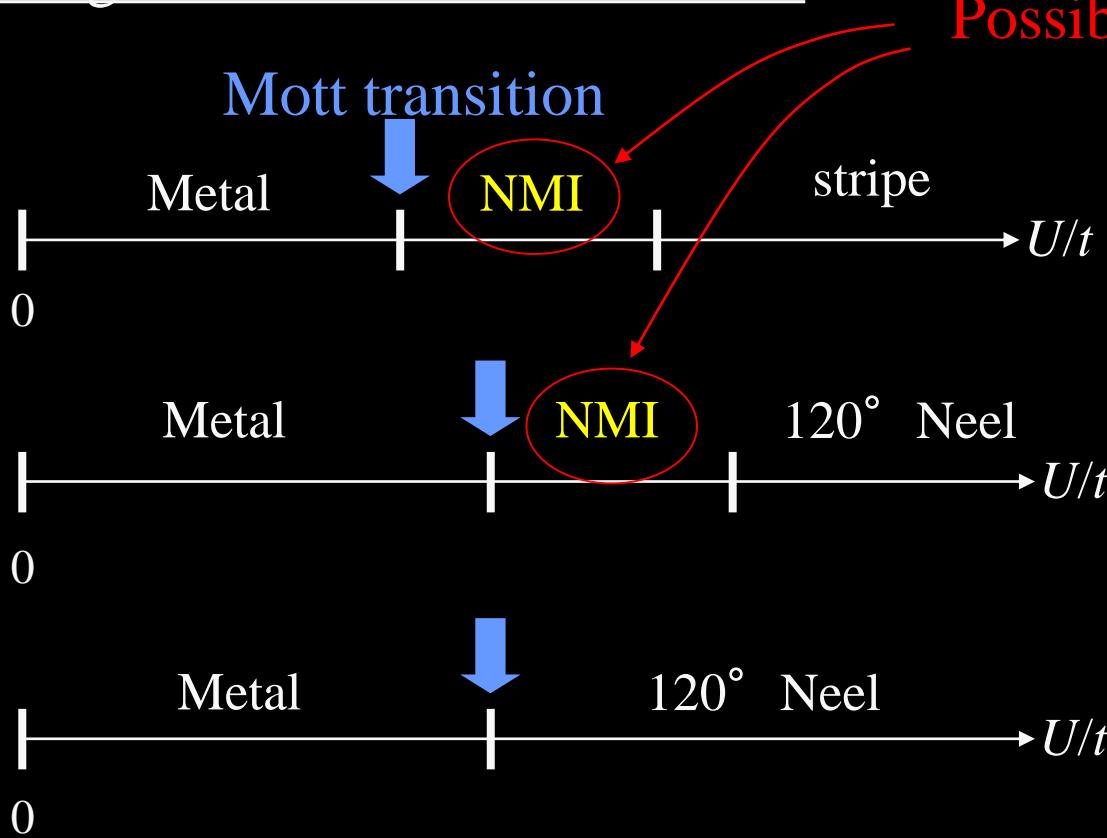
Why not



expected in Heisenberg model ?

→near the Mott transition

Triangular lattice Hubbard model

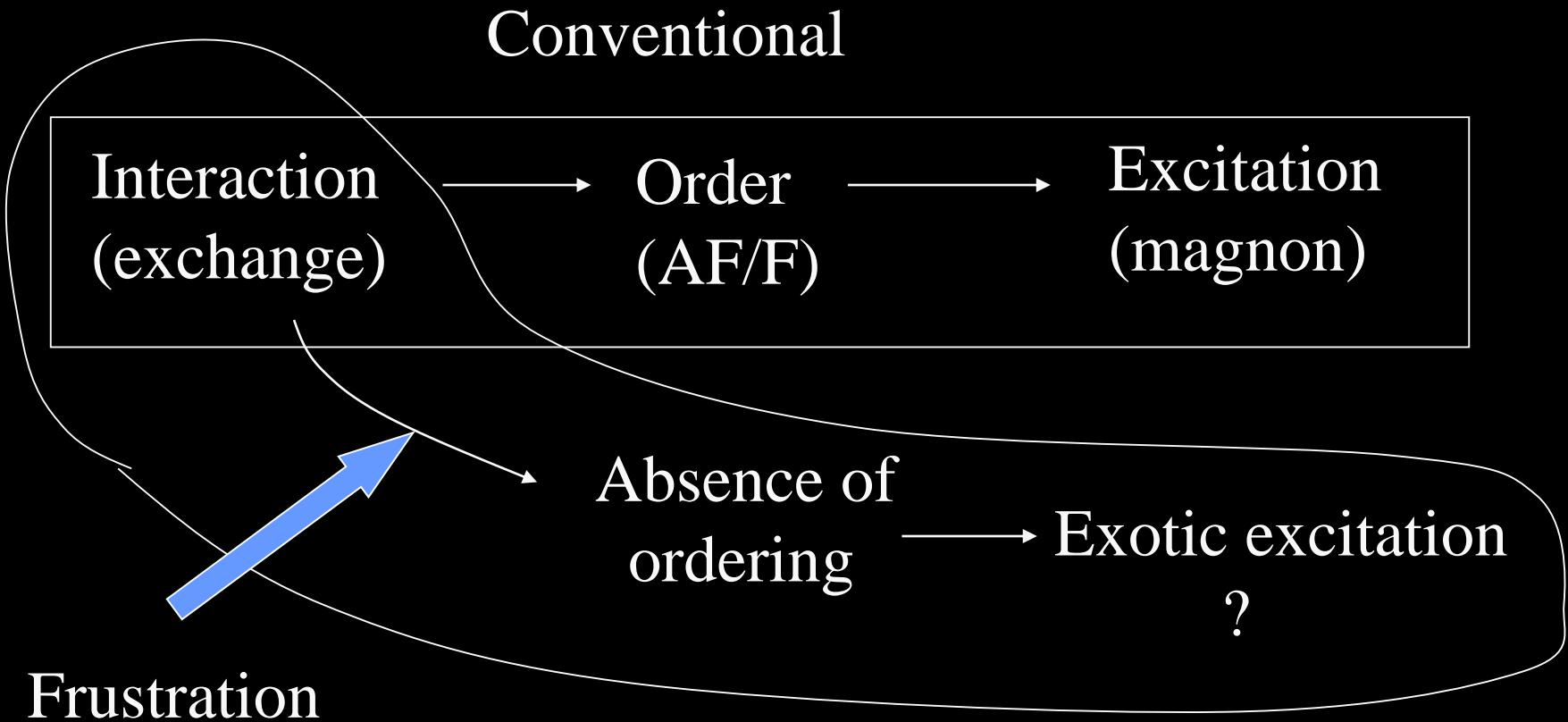


Possible spin liquid

Morita et al. (2002)
Mizusaki et al. (2006)

Sahelsara et. al. (2008)
Yoshioka et al. (2009)

T. Watanabe et. al. (2008)
Inaba et al. (2008)



Thermodynamics

Specific heat by Yamashita and Nakazawa (Osaka Univ.)

At low temperatures

Finite γ

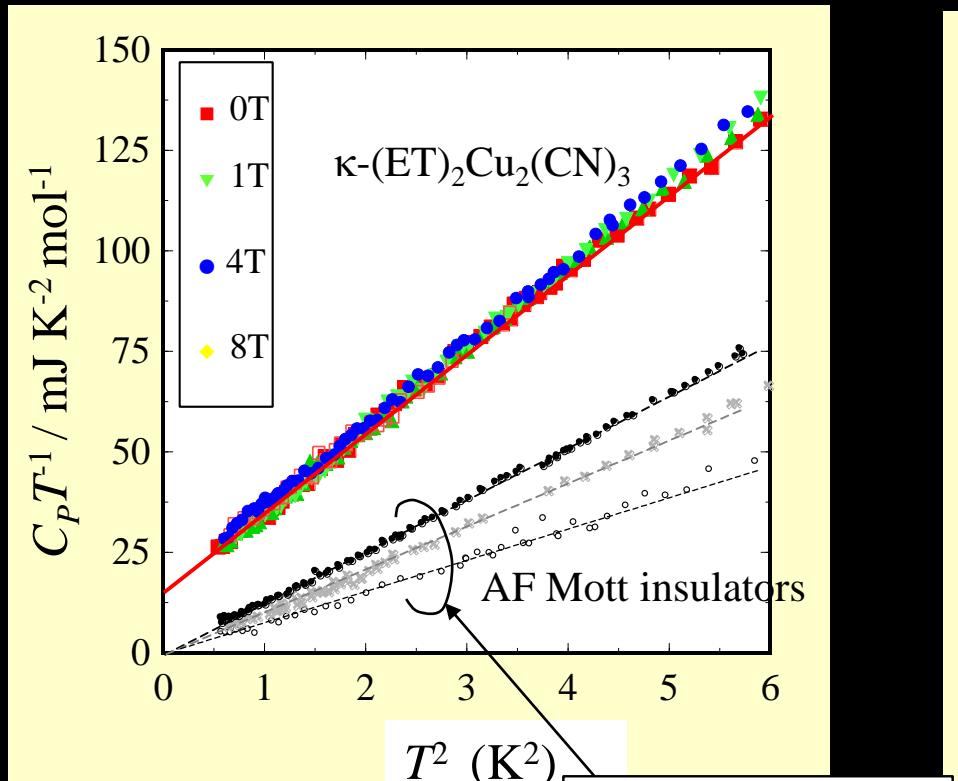
→ Low-lying spin excitations

$$C = \gamma T + \beta T^3$$

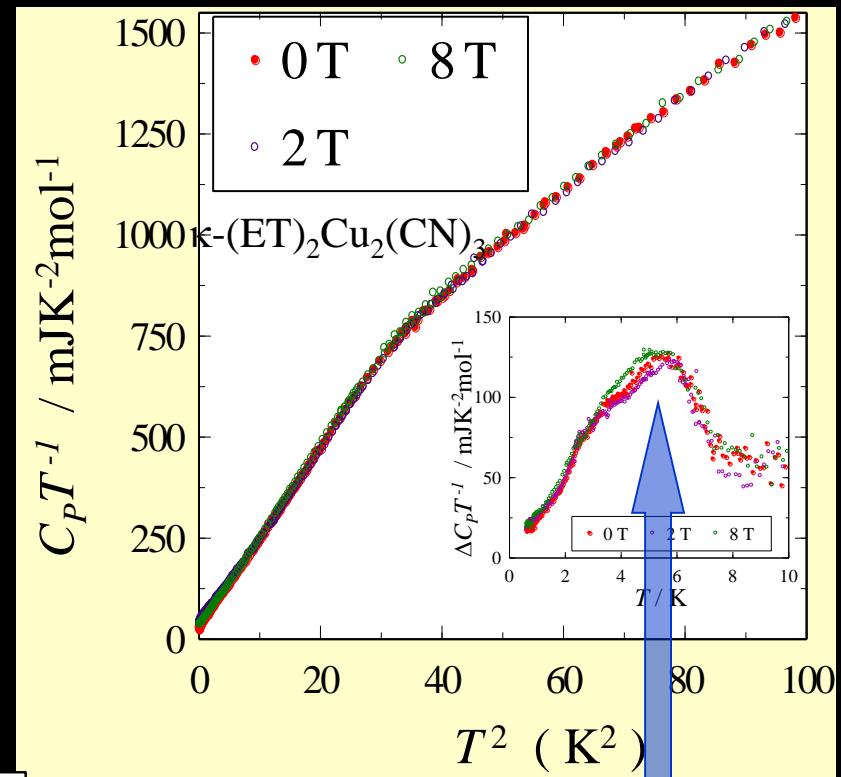
At higher temperatures;

Field-insensitive anomaly

→ Hidden order or some crossover ?



$$\gamma = (2\pi^2 k_B^2/3) D(\varepsilon_F)$$



Anomaly at 5-6 K

Wilson ratio

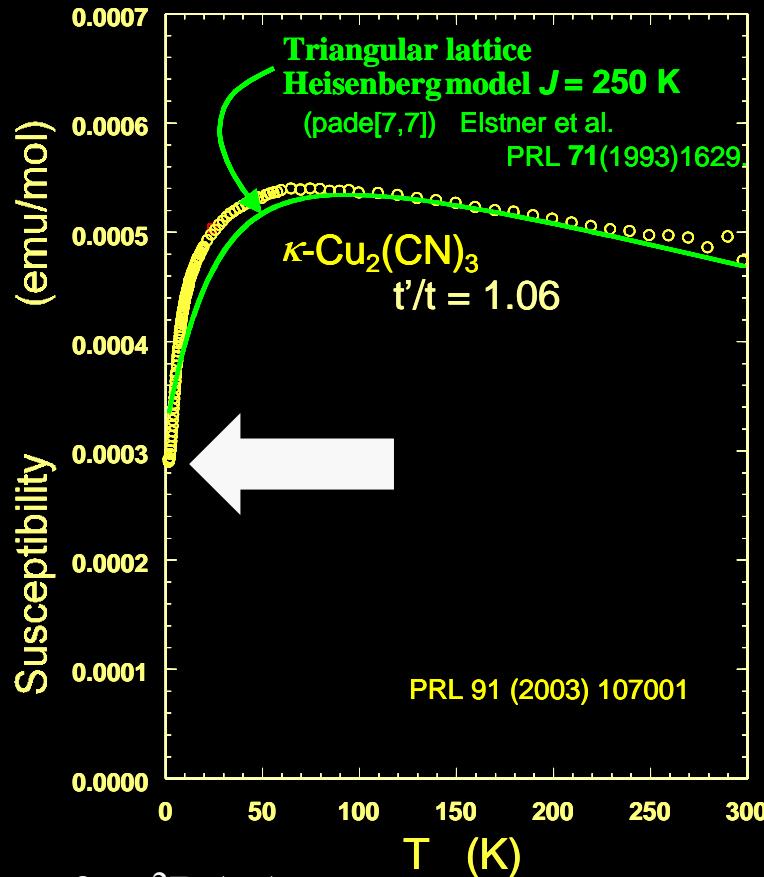
$$R_W = \frac{\{\chi/(2\mu_B^2)\}}{\{\gamma/(2\pi^2 k_B^2/3)\}}$$

~ 1.6



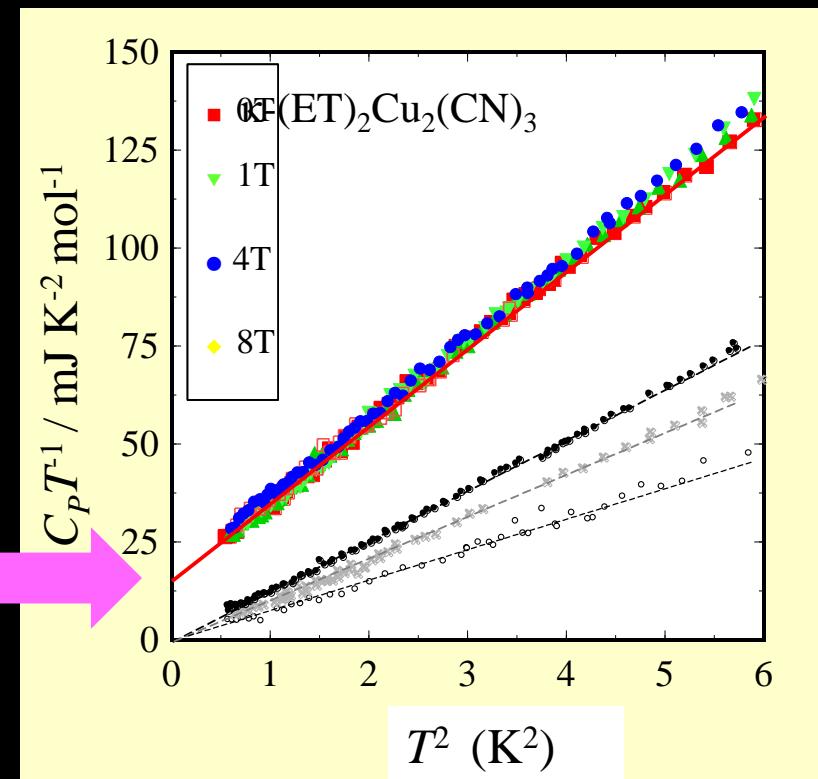
Degenerate Fermionic objects in Mott insulator

$$\chi_{\text{spin}} = 3 \times 10^{-4} \text{ emu/mol}$$



$$\chi = 2\mu_B^2 D(\varepsilon_F)$$

$$\gamma = 13 \text{ mJ/K}^2\text{mol}$$



$$\gamma = (2\pi^2 k_B^2/3) D(\varepsilon_F)$$

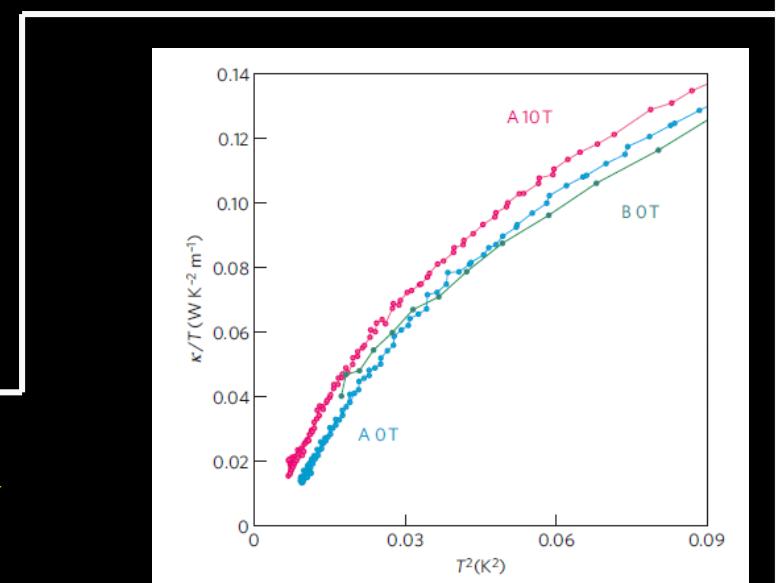
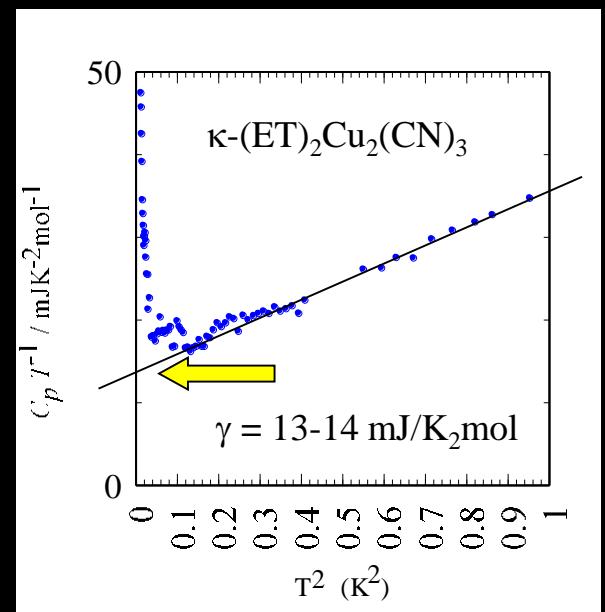
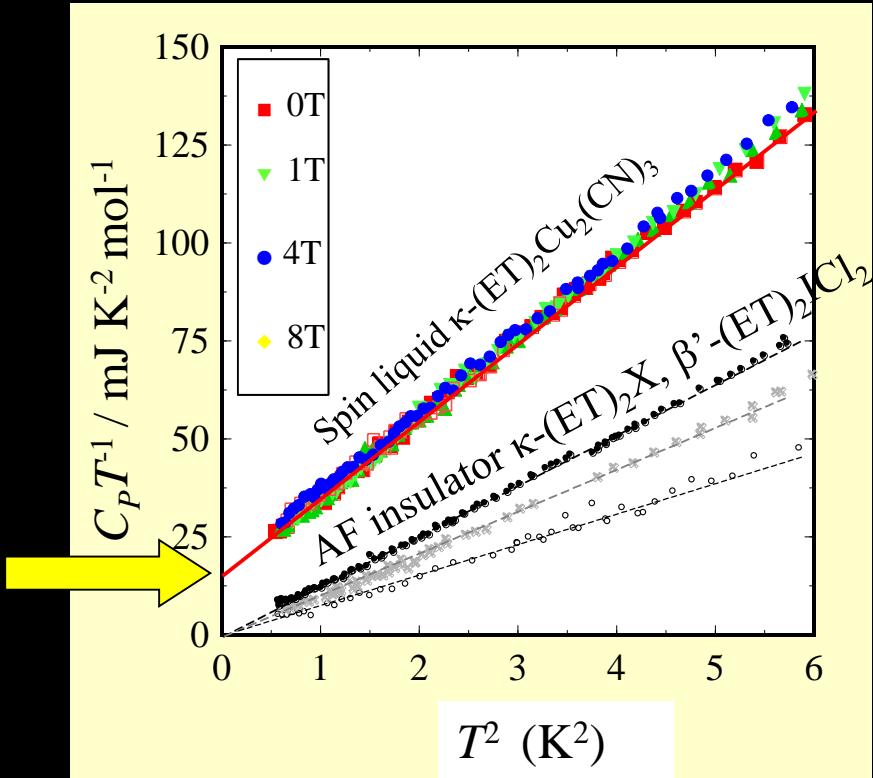
Spin liquid in κ -(ET)₂Cu₂(CN)₃; Gapless or gapped

Specific heat \rightarrow gapless ($\gamma = 13\text{-}14 \text{ mJ/K}^2\text{mol}$)

Wilson ratio ~ 1.1

\rightarrow Degenerate chargeless Fermionic objects

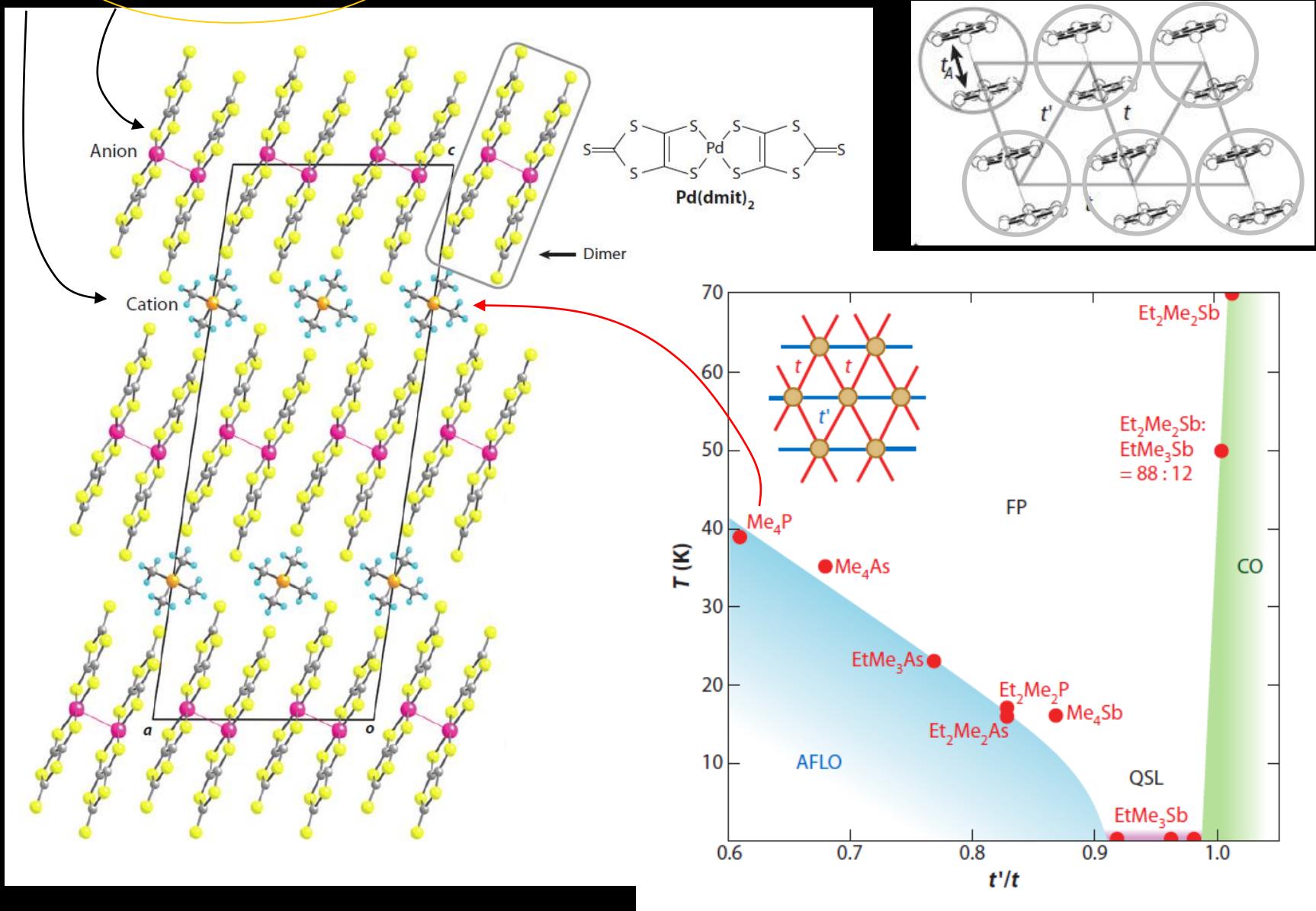
S. Yamashita *et al.*, , Nature Phys. 4 (2008) 459



Thermal conductivity \rightarrow gapped; 0.46 K

M. Yamashita et a., Nature Phys. 5 (2009) 44

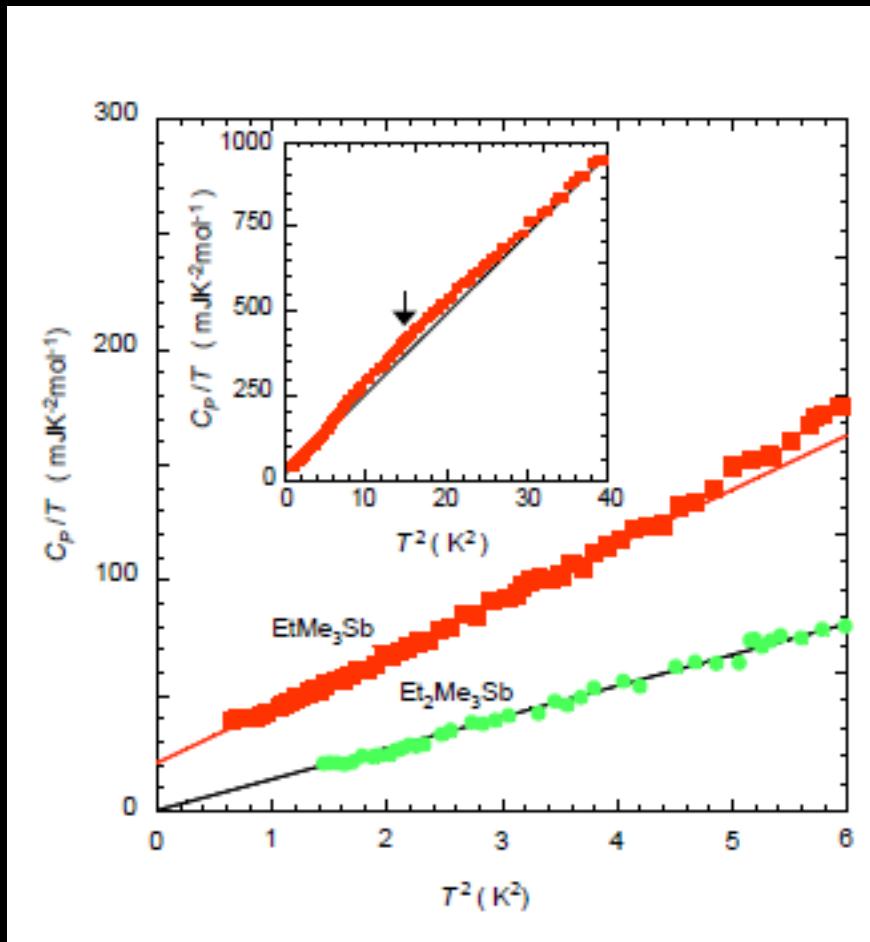
$A[Pd(dmit)_2]_2$; quasi-triangular lattice systems



Spin liquid in EtMe₃Sb[Pd(dmit)₂]₂

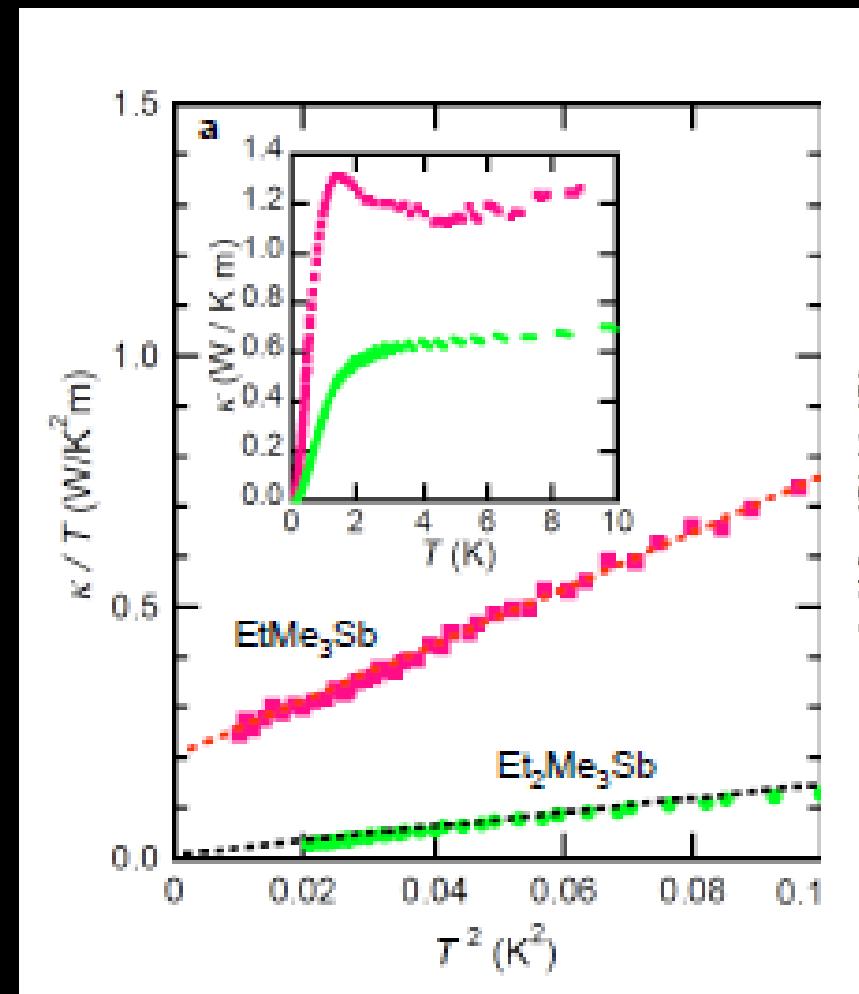
Specific heat

S. Yamashita et al., Nat. Commun. 2, 275 (2011)



Thermal conductivity

M. Yamashita et al, Science 328, 1246 (2010)

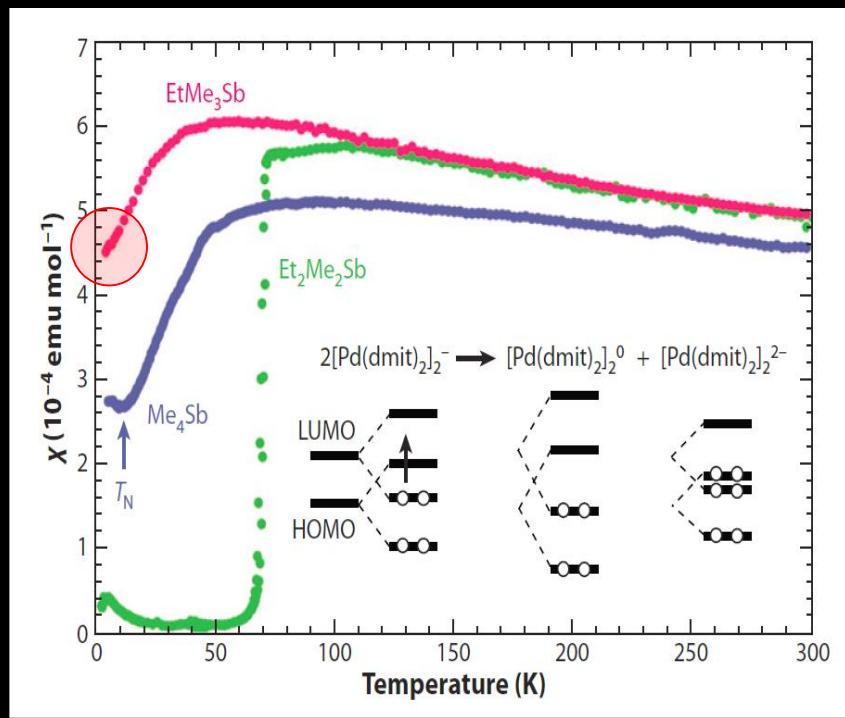


Wilson ratio

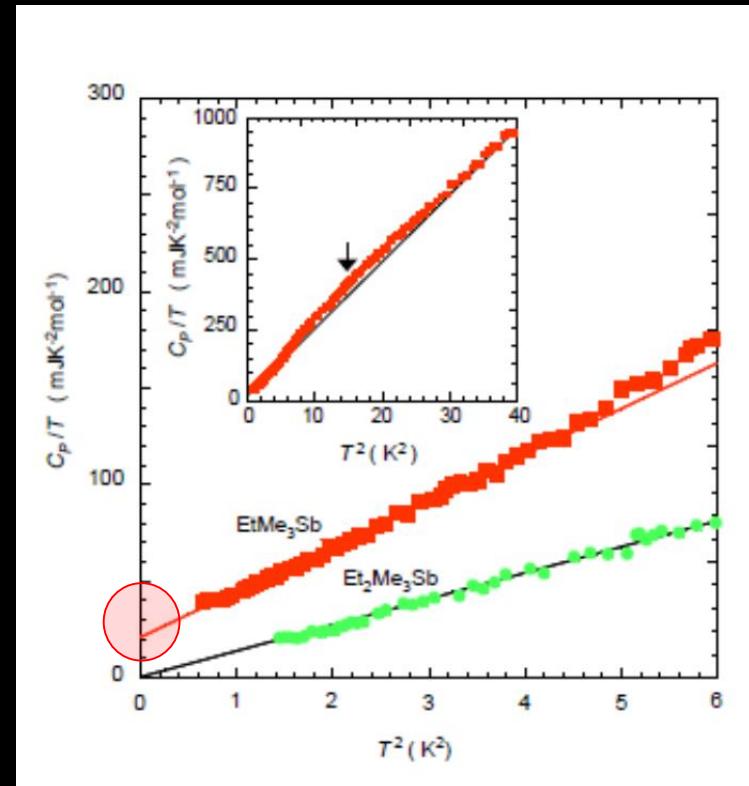
$$R_W=1.6$$

$$\chi_{\text{spin}} = 4.5 \text{ emu/mol}$$

$$\gamma = 20 \text{ mJ/mol K}^2$$



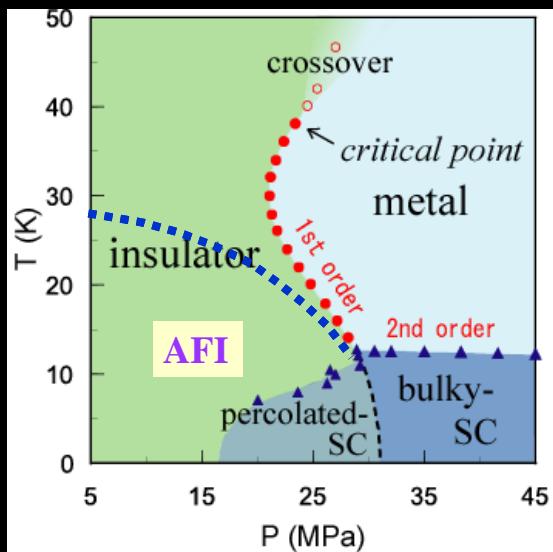
R. Kato, Bull. Chem. Soc. Jpn. 87, 355 (2014)



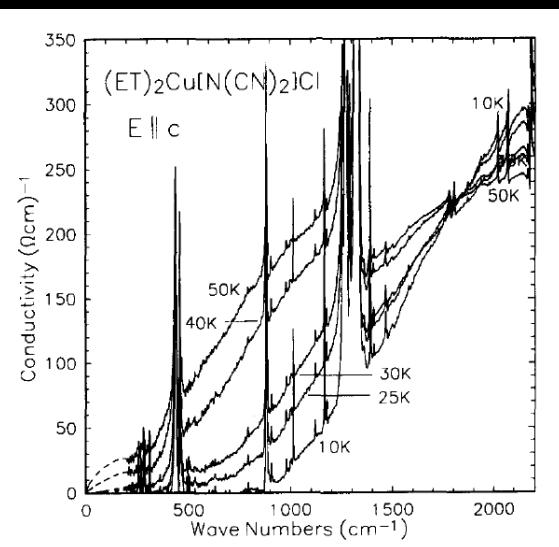
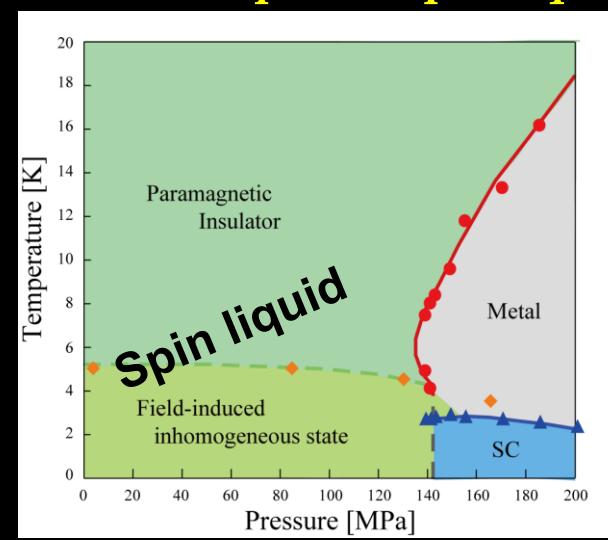
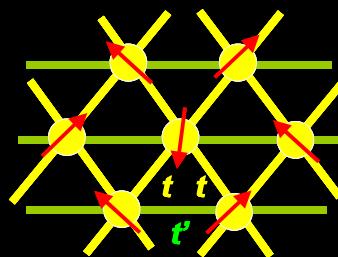
S. Yamashita et al., Nat. Commun. 2, 275 (2011)

Charge excitation in antiferromagnet and spin liquid

Charge gap is clearly opened on AF ordering, but remains undeveloped in spin liquid.



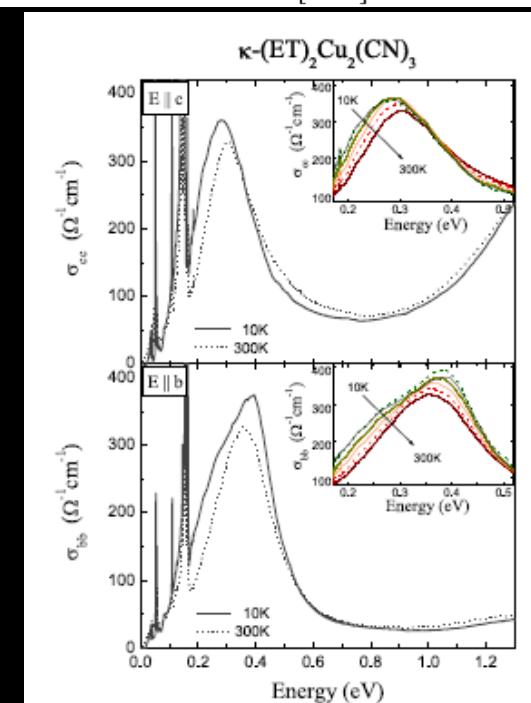
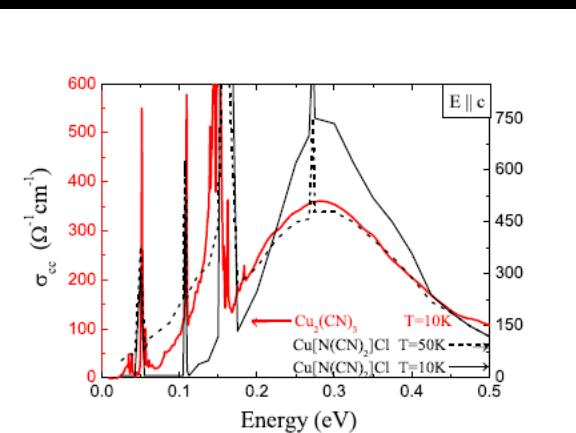
$$t'/t = 0.75 \quad t'/t = 1.06$$



Kornelsen et al., SSC 81 (1992)343

Optical conductivity

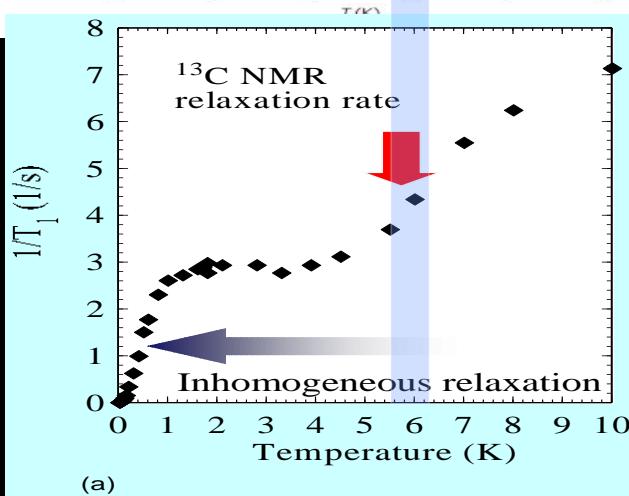
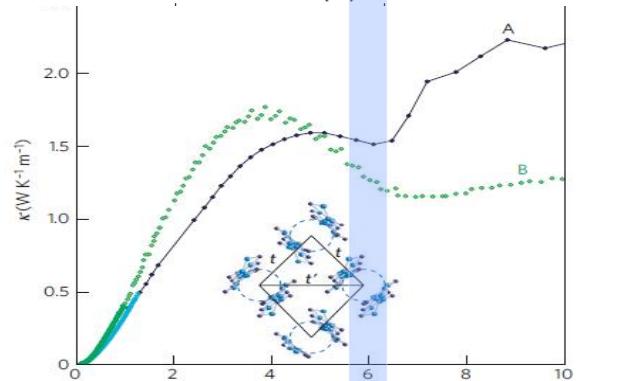
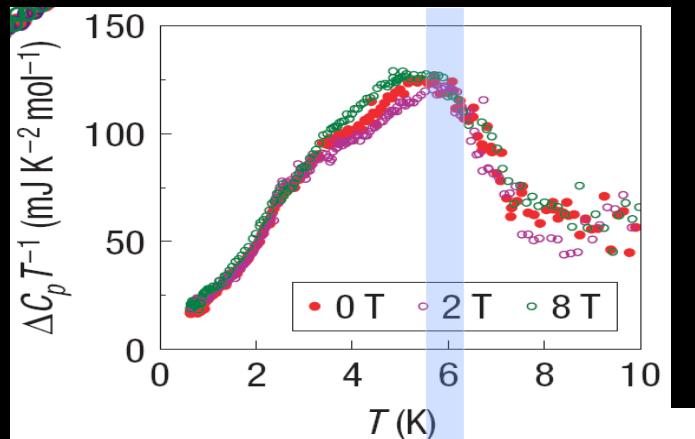
Kezsmarki et al.
PRB 74(2006)201101



Thermodynamic anomaly at 6K in κ -(ET)₂Cu₂(CN)₃

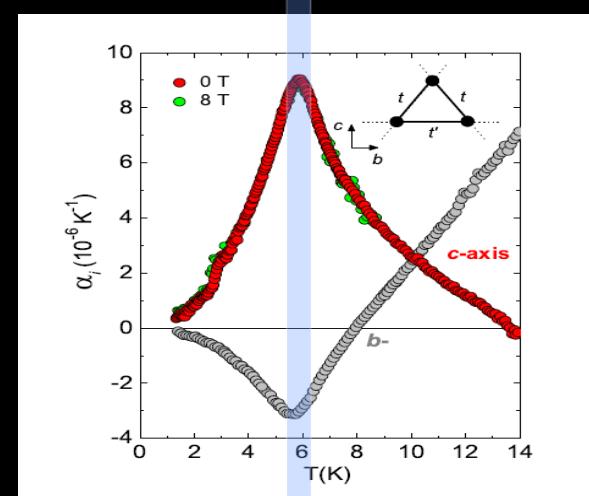
Specific heat

S. Yamashita *et al.*,
Nature Phys. **4** (2008)
459



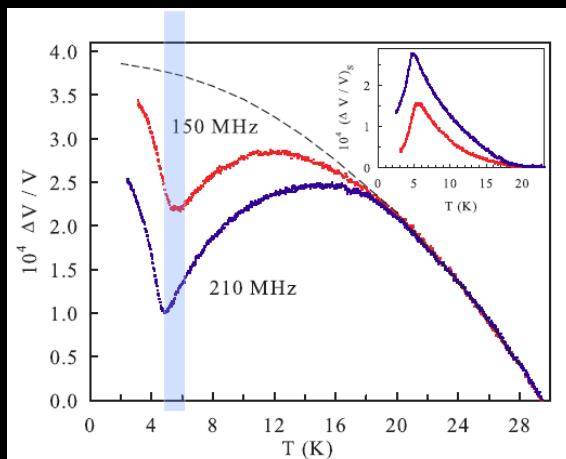
Thermal expansion coefficient

Manna *et al.*, *PRL* **104** (2010) 016403



Ultrasound velocity

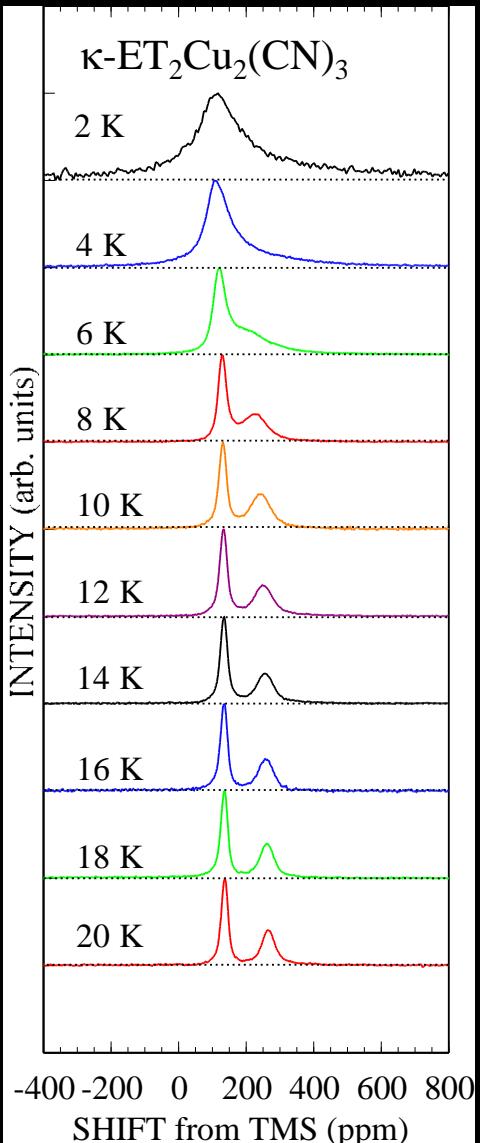
Poirier *et al.*,



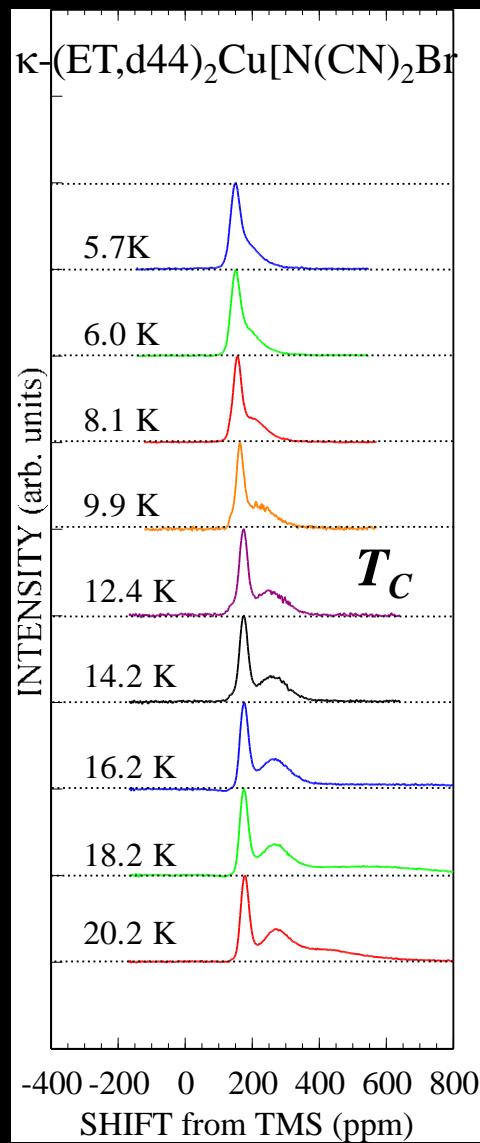
Can you distinguish SL and SC ?

NMR spectra $B \parallel a$ axis

SL

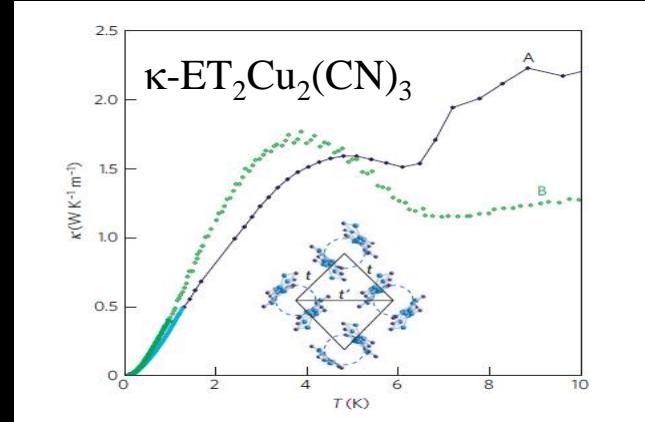


SC



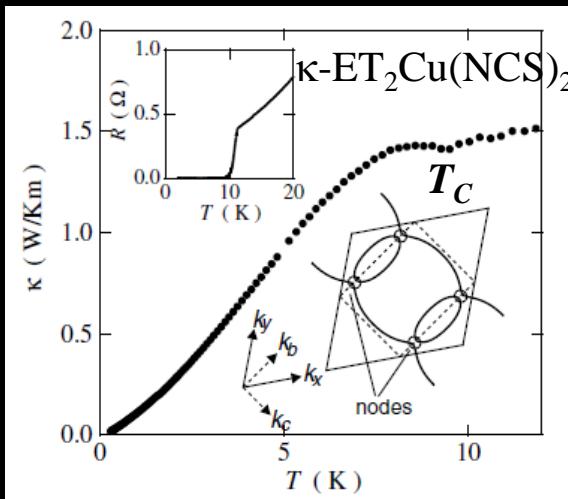
Thermal conductivity

SL



M. Yamashita *et al.*, *Nature Phys.* 5 (2009) 44

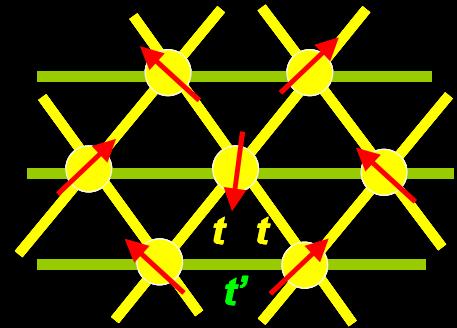
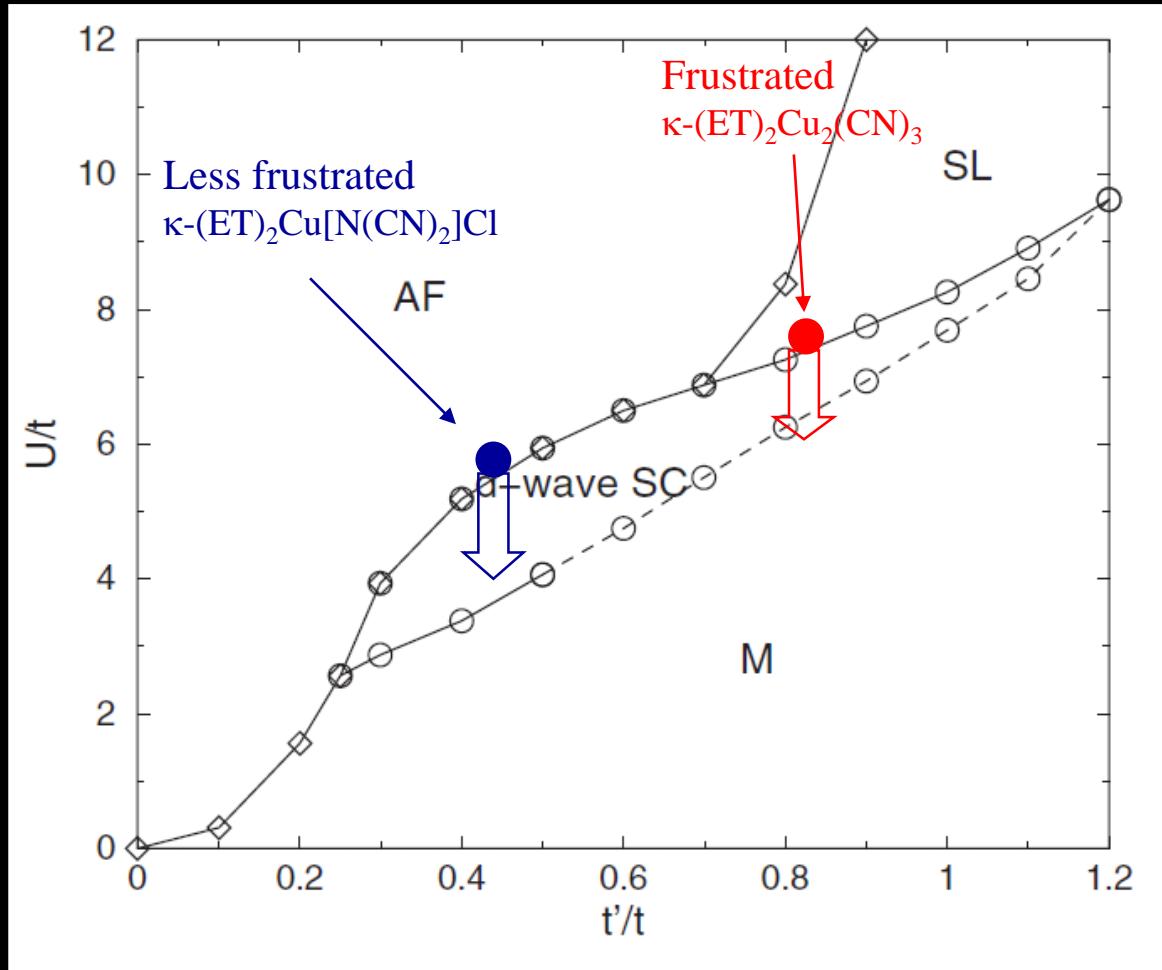
SC



Y. Matsuda *et al.*,
J. Phys: Condens. Matter 5 (2006) R705

Pressurize AFI and spin liquid

$\frac{1}{2}$ -filled Hubbard model (Cluster DMFT)
Kyung, Tremblay PRL (2006)



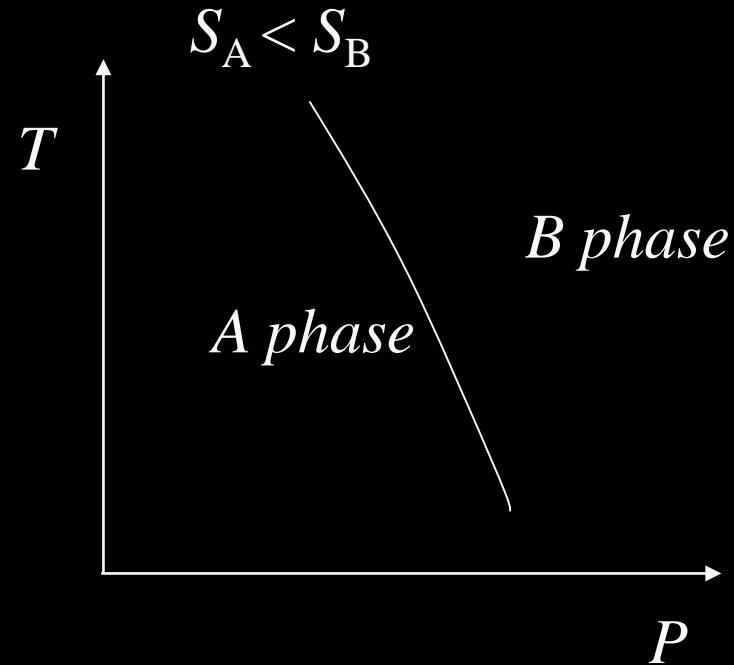
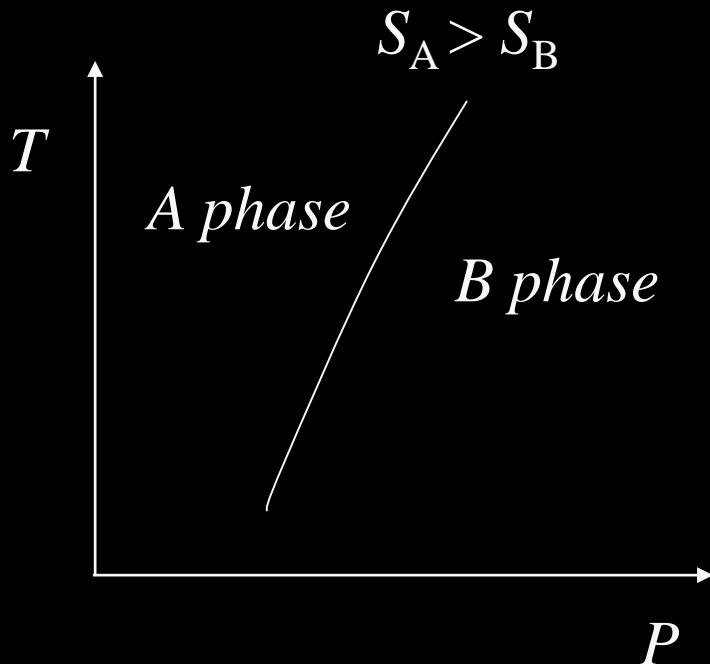
Material parameters

Kandpal et al.
PRL 103 (2009) 067004

Thermodynamics of Mott transition

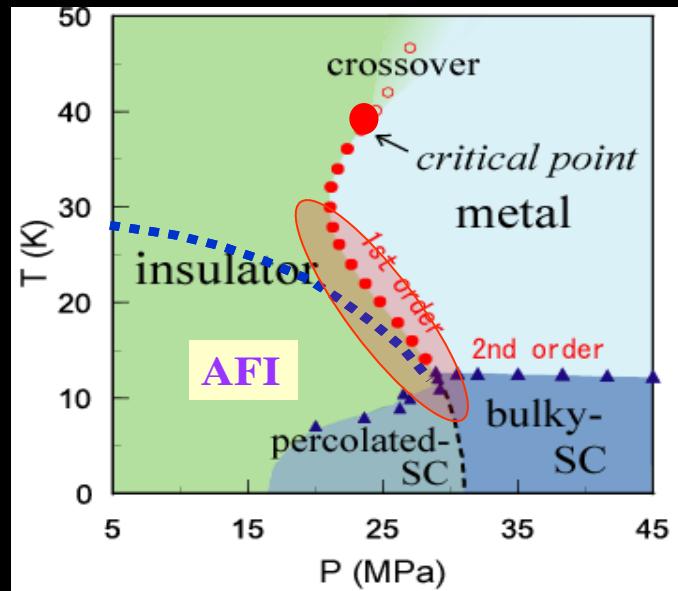
Entropy balance known from phase diagram

$$\text{Clausius Clapeyron} \quad dT/dP = (V_A - V_B)/(S_A - S_B) > 0$$



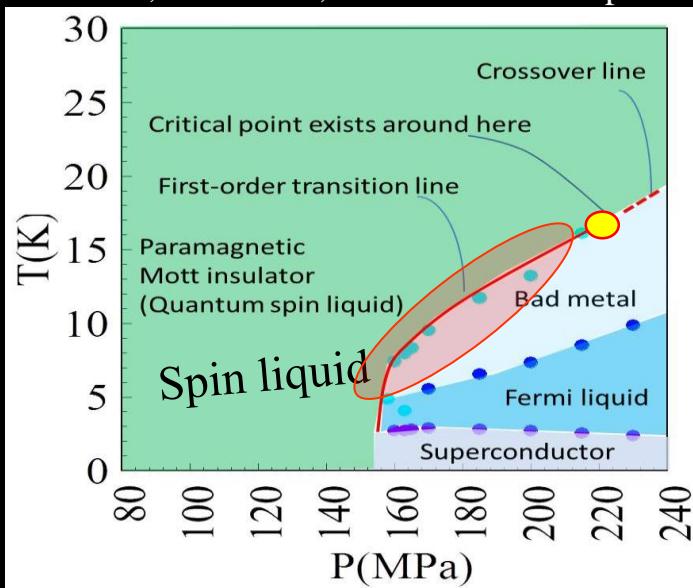
κ -(ET)₂Cu[N(CN)₂]Cl $t'/t \sim 0.44\text{-}0.75$

Kagawa *et al.*, Nature 2005, PRL 2004; PRB 2004,



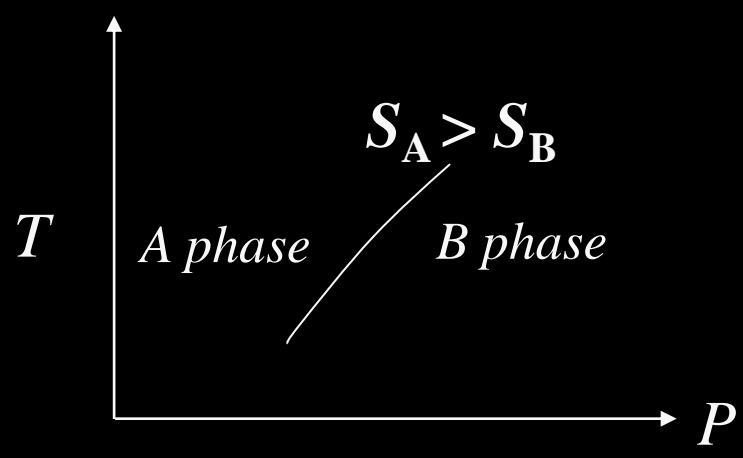
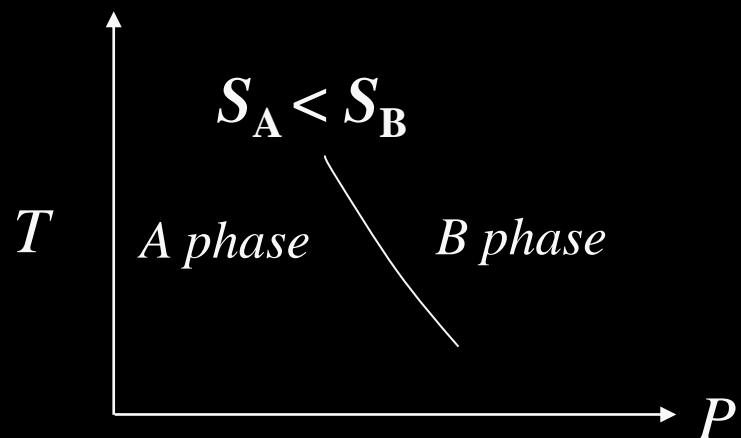
κ -(ET)₂Cu₂(CN)₃ $t'/t \sim 0.80\text{-}1.06$

Kurosaki et al., PRL 2005, Furukawa et al.unpublished



Thermodynamics of Mott transition

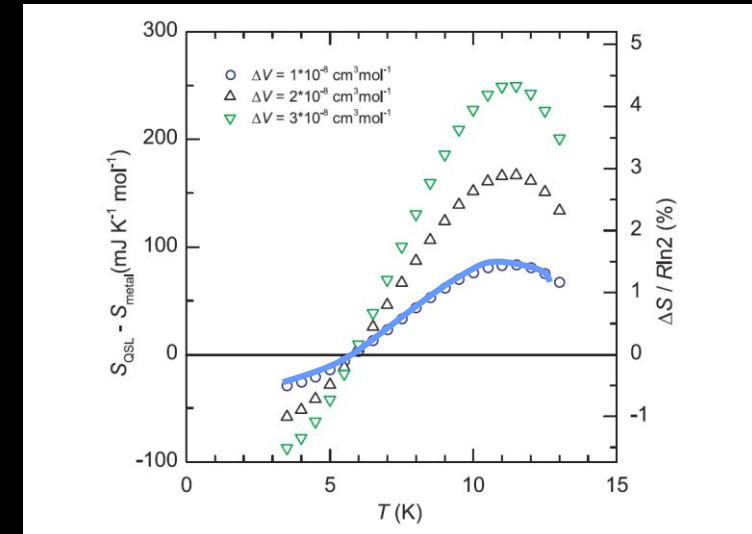
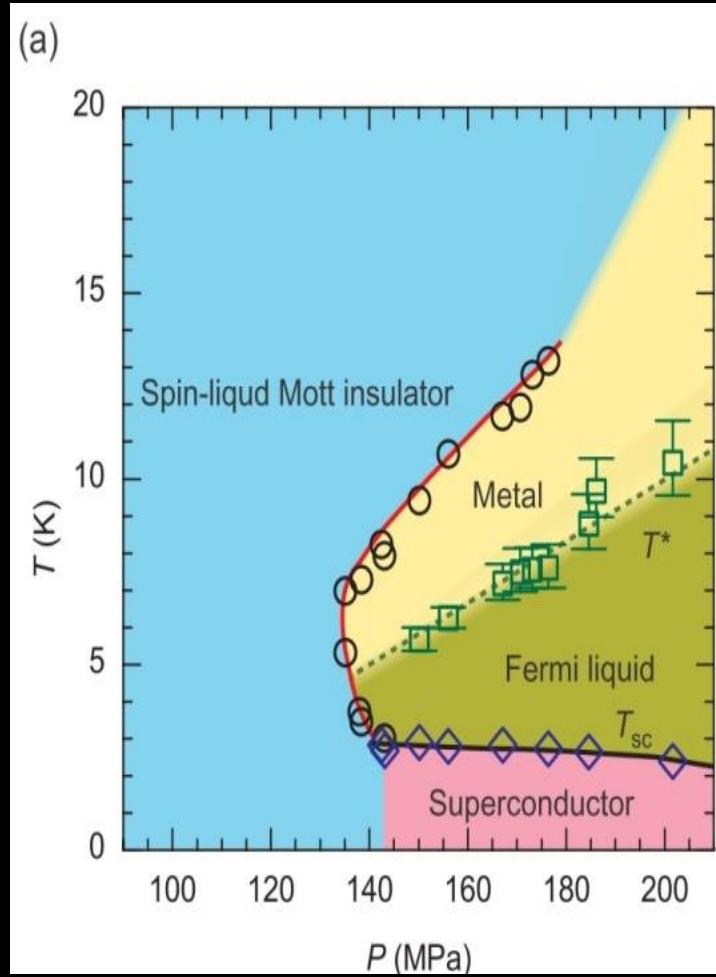
Clausius Clapeyron
 $dT/dP = \Delta V/\Delta S = (V_A - V_B)/(S_A - S_B) > 0$



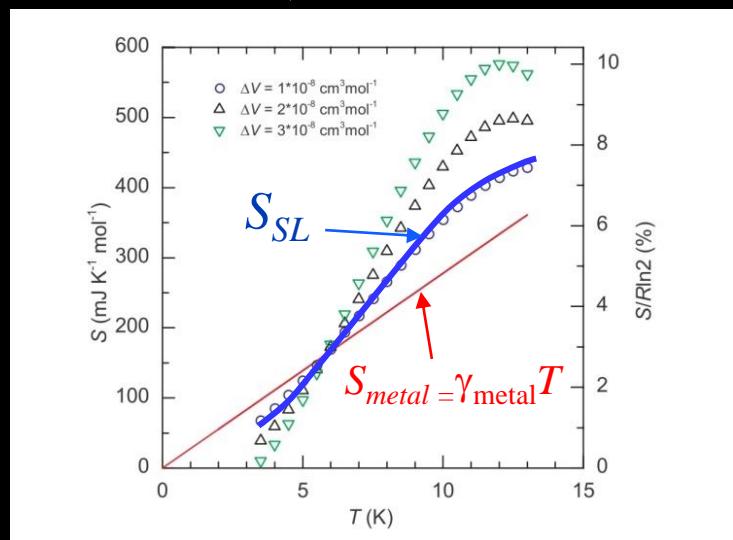
Entropy of spin liquid

$$\Delta S = S_{SL} - S_{metal} = (dP/dT) \Delta V$$

parameter

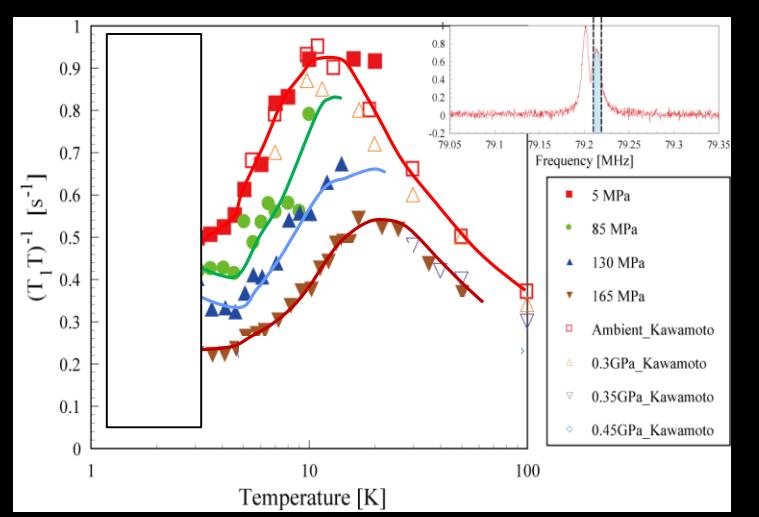
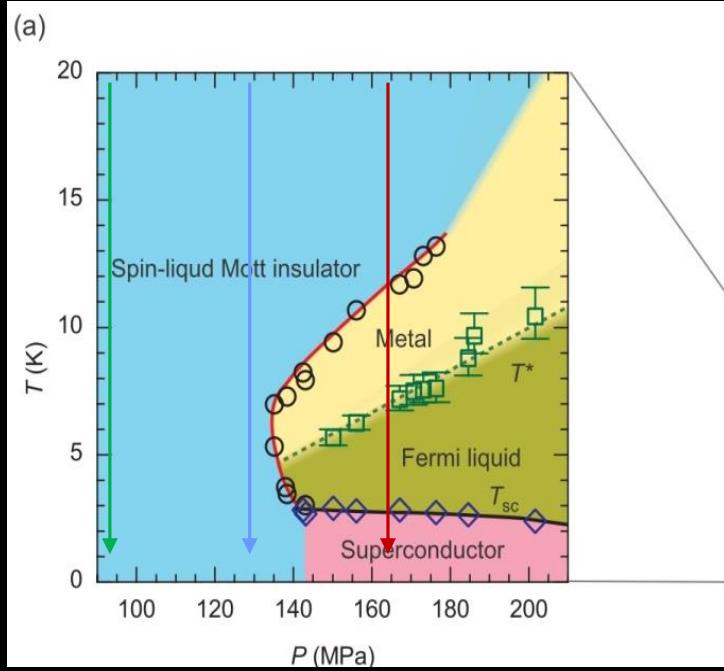


S_{SL} $\gamma_{metal} = 27.5 \text{ mJ/mol K}^2$

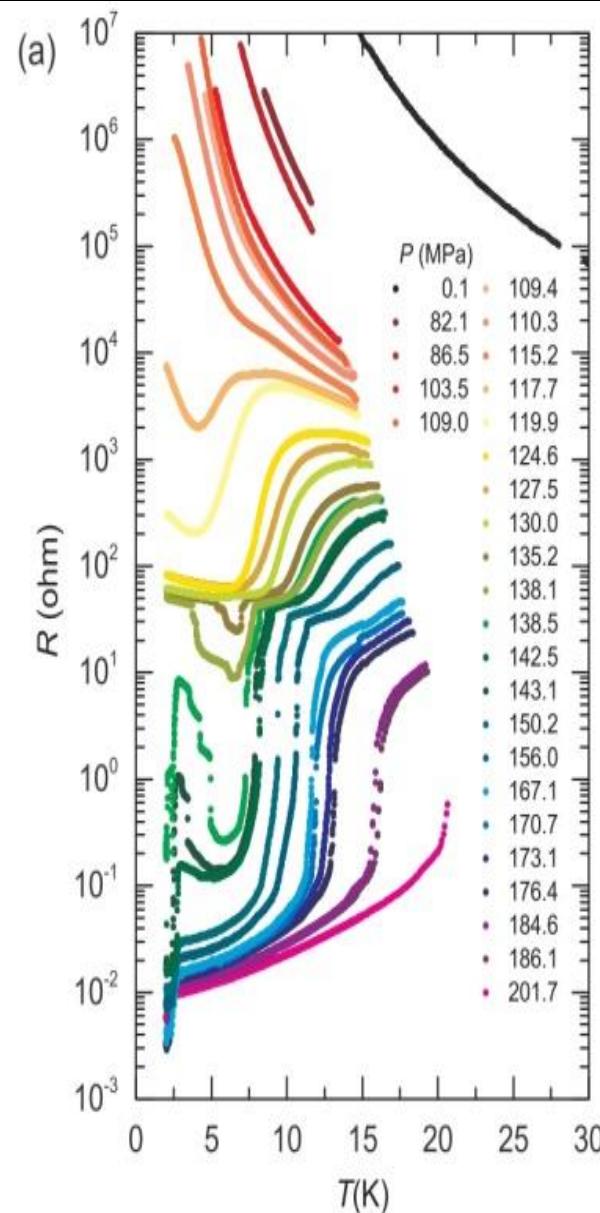


Mott transition of SL; drastic change in charge transport but not in spin

Phase diagram

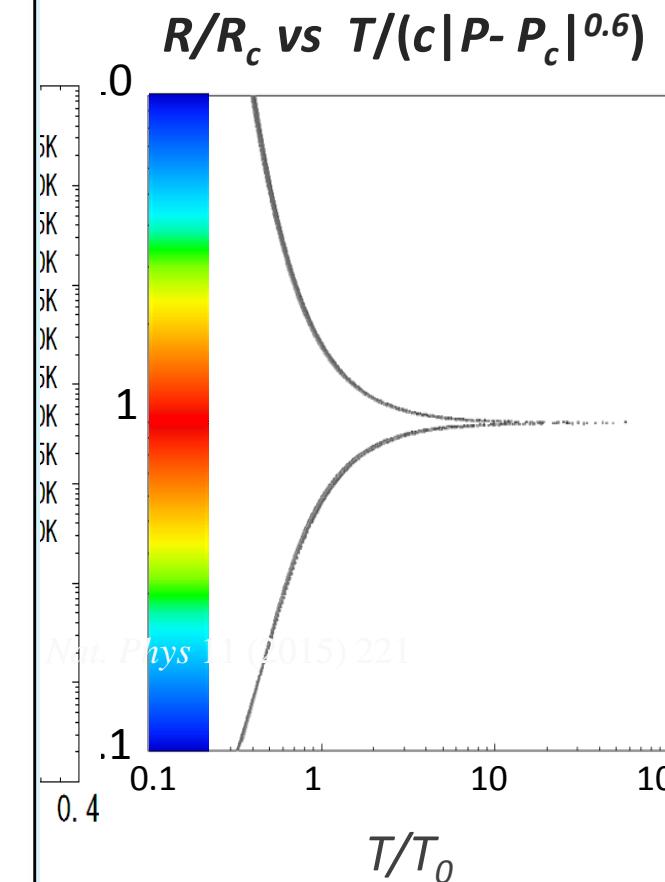
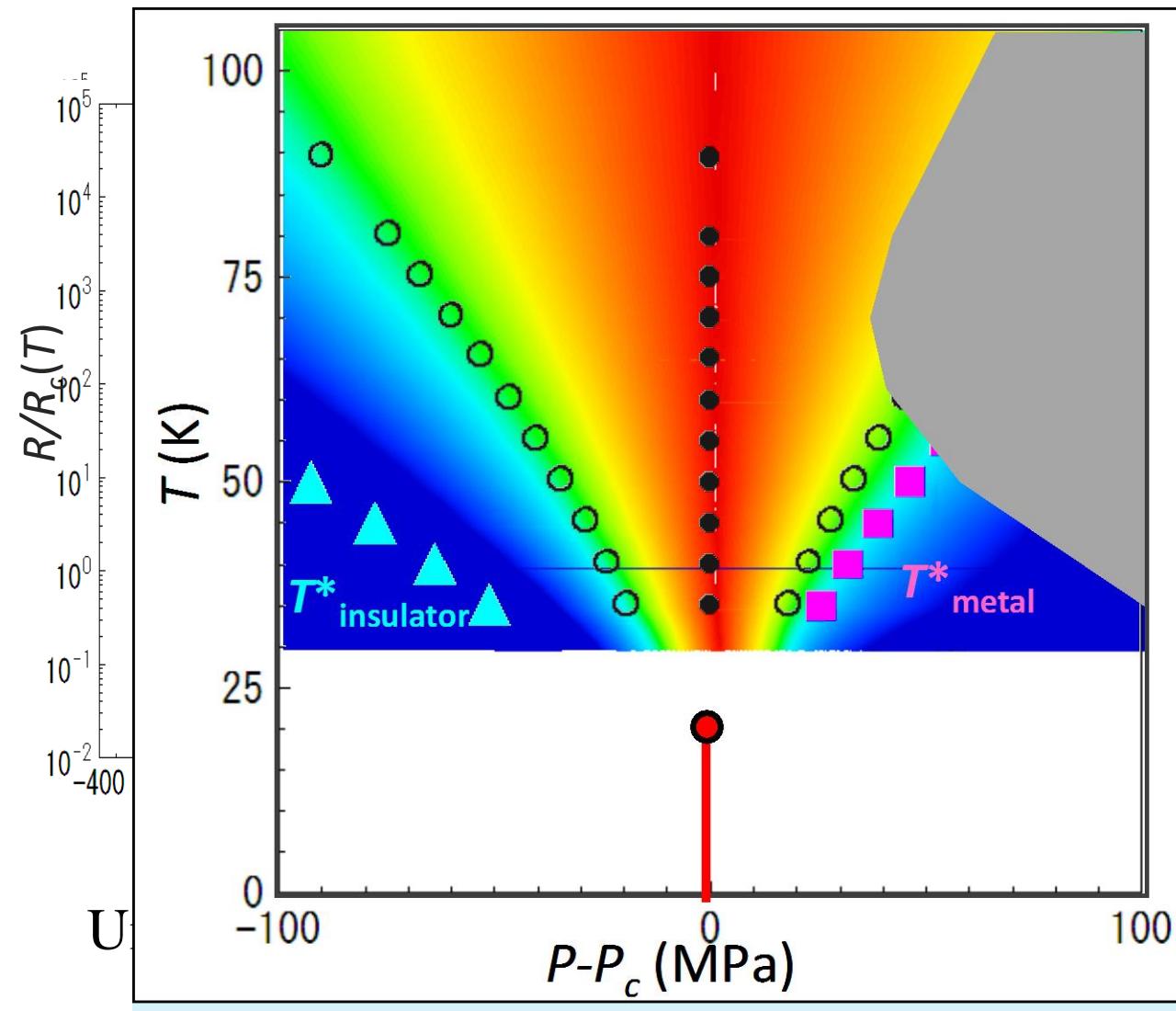


resistivity



Experimental test of scaling

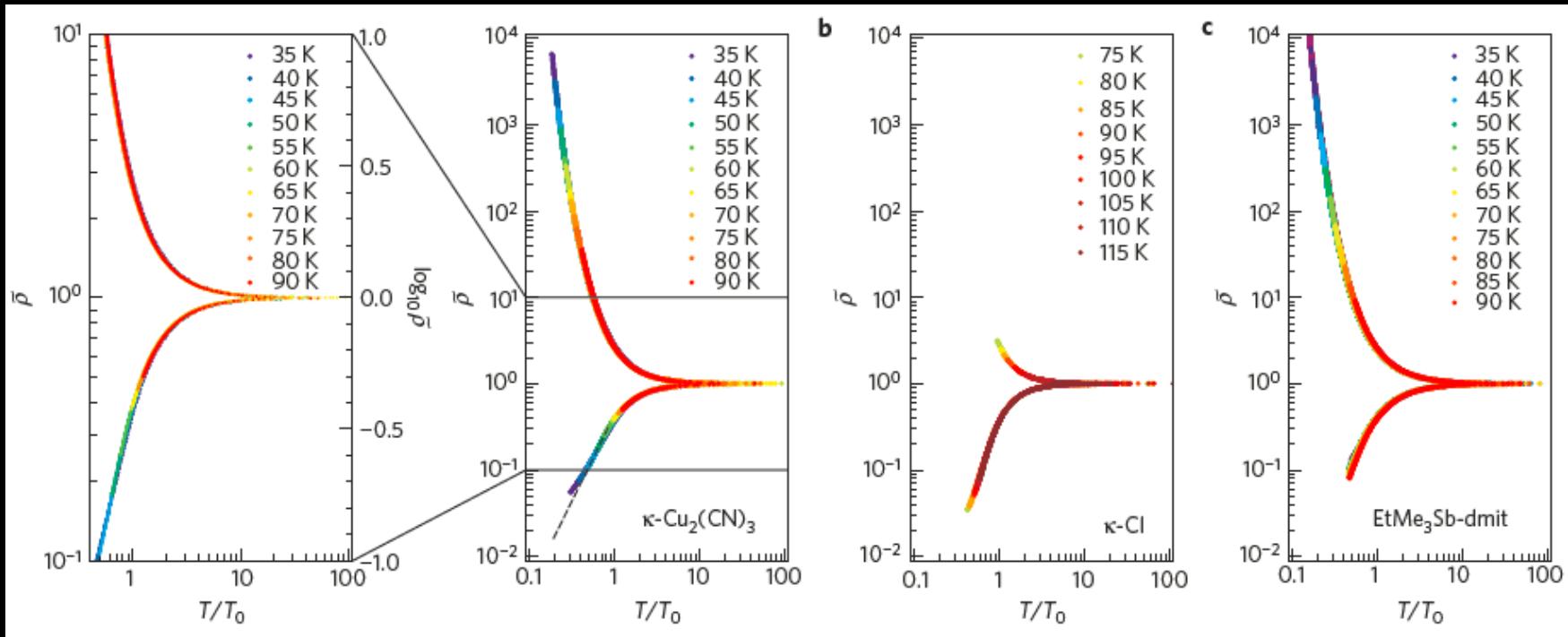
Furukawa et al.,
Nat. Phys 11 (2015) 221



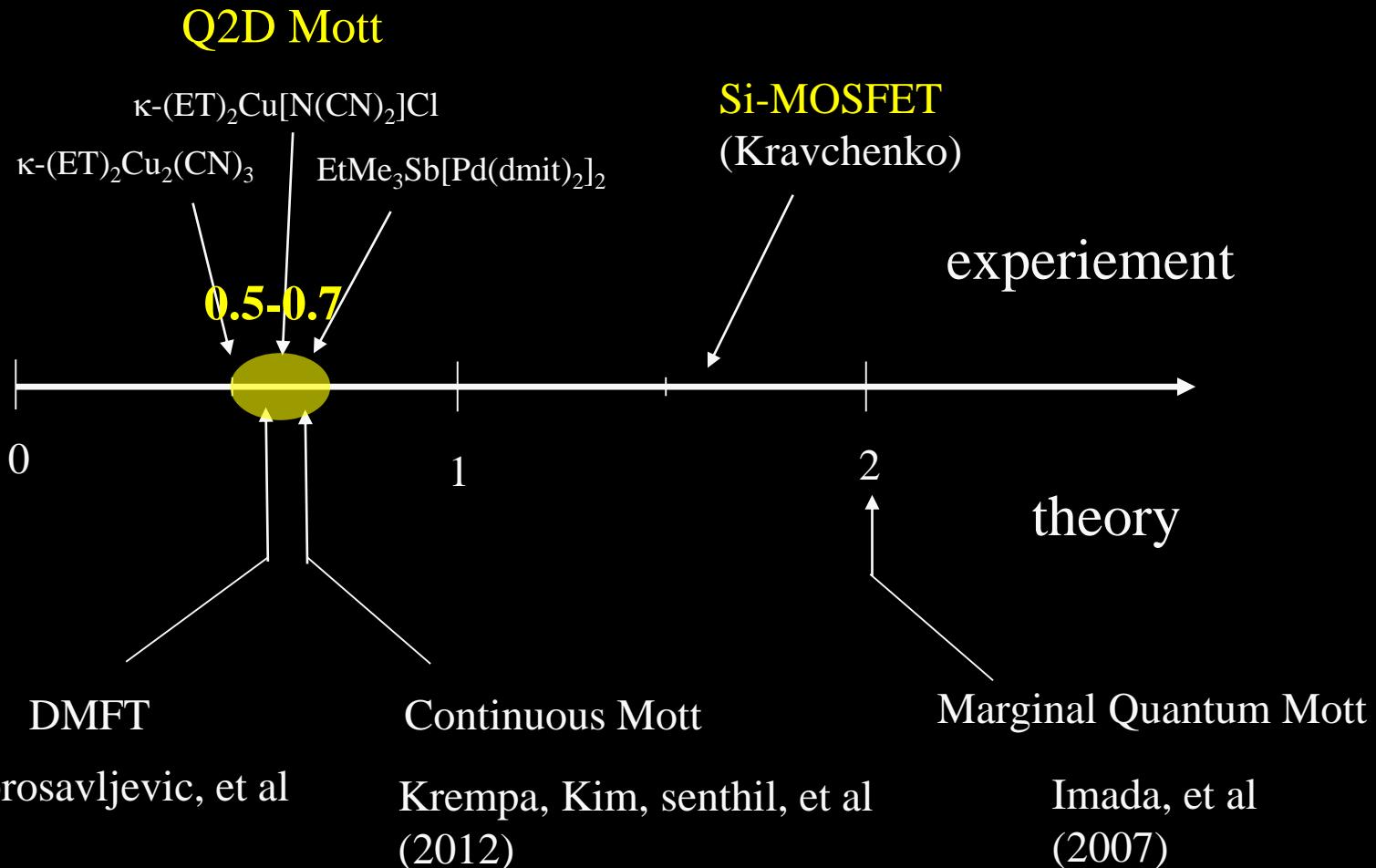
Perfect scaling
for $T > 1.5T_c$

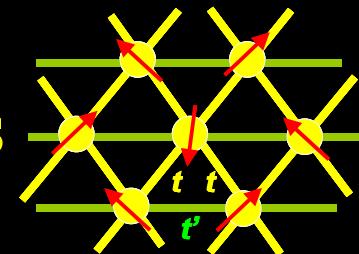
QC scaling --- nearly material -independent

Furukawa et al., Nat. Phys 11 (2015) 221



Critical exponents, $z\nu$, in metal-insulator transitions





Mott phase diagrams of quasi-triangular lattices

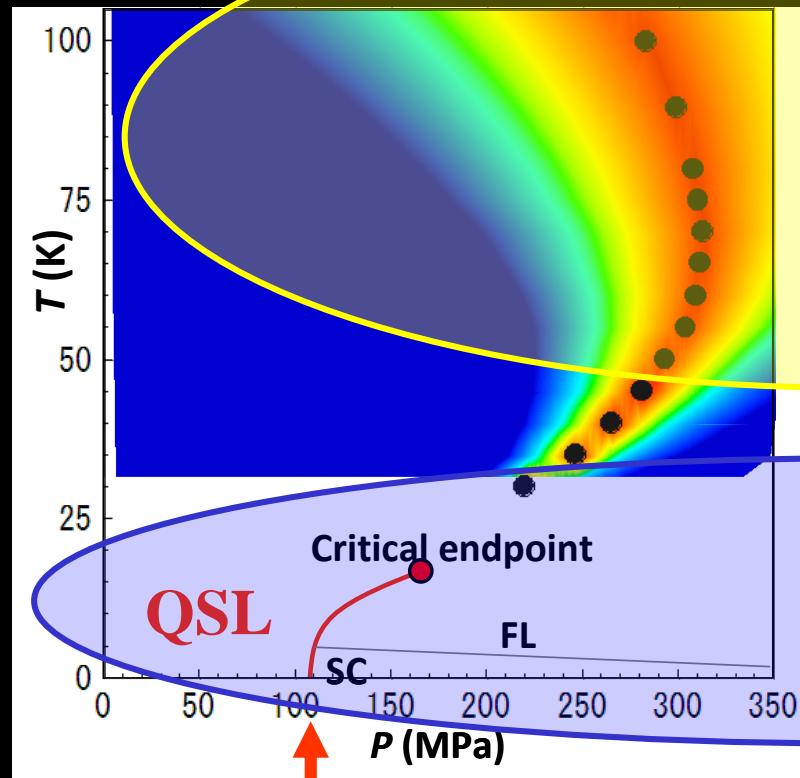
$\kappa\text{-}(\text{ET})_2\text{Cu}_2(\text{CN})_3$
 $t'/t = 0.80\text{-}1.0$

$\kappa\text{-}(\text{ET})_2\text{Cu}[\text{N}(\text{CN})_2]\text{Cl}$
 $t'/t = 0.44\text{-}0.75$

frustrated

less frustrated

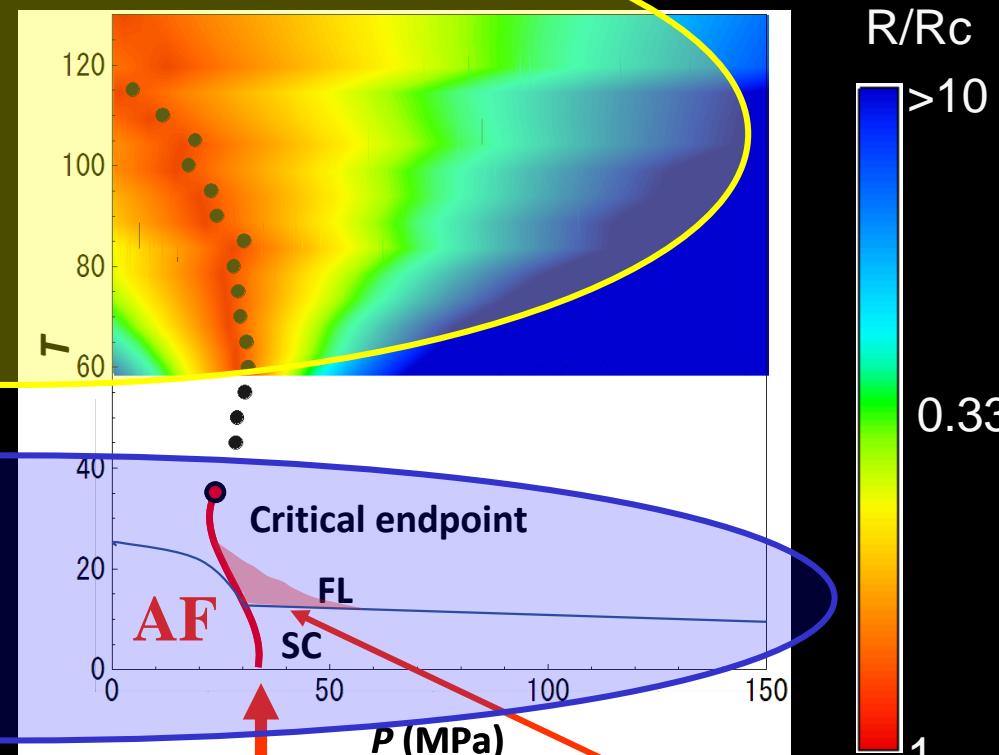
Similar QC behavior at high T



Weak Mott, Low T_c

Dissimilar at low T

Strong Mott, High T_c , Pseudo-gap



R/R_c

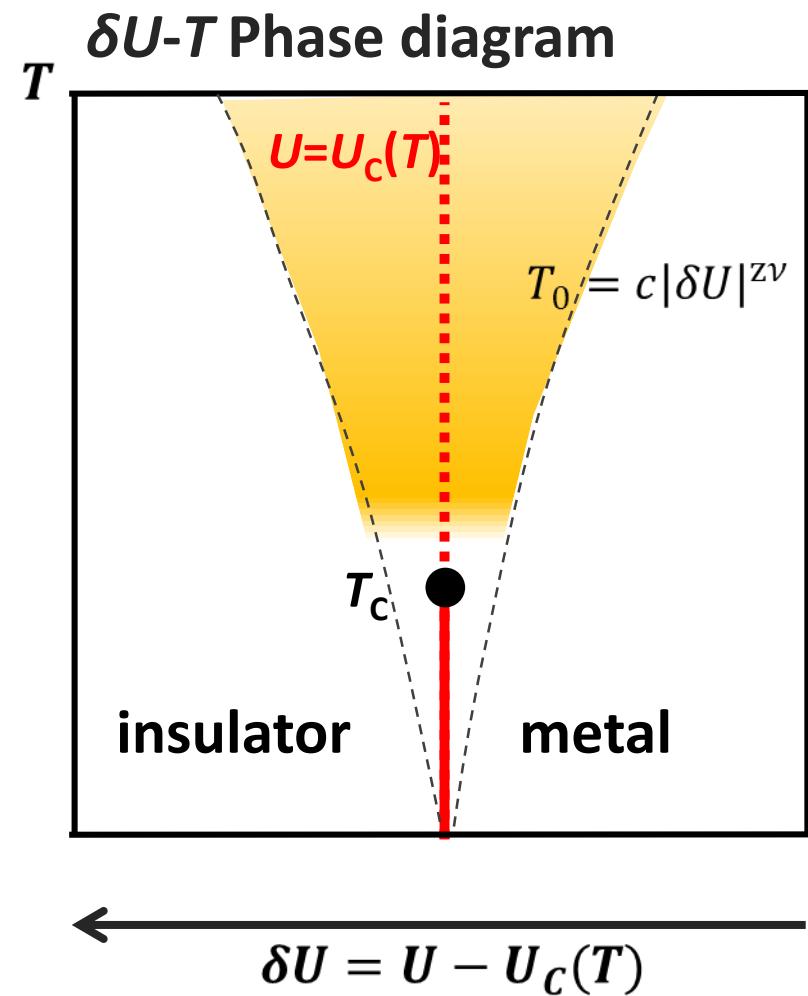
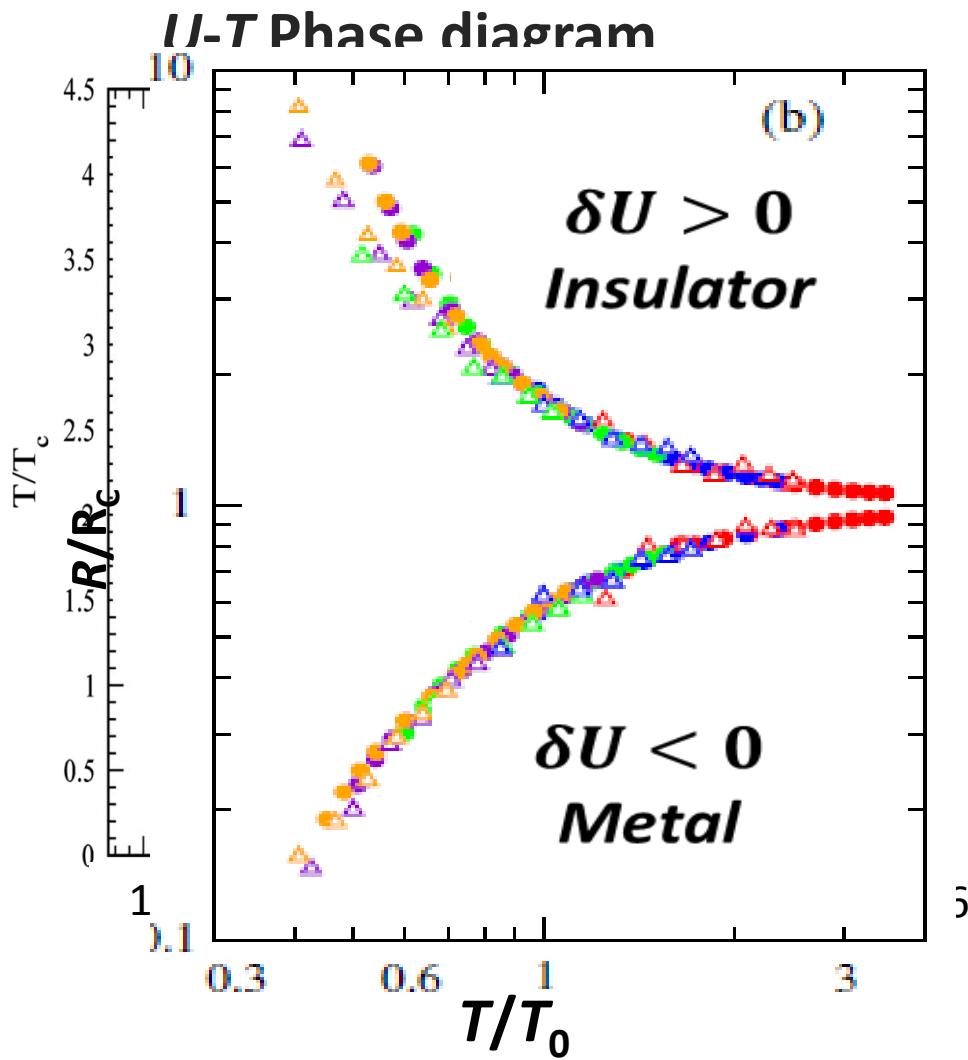
>10

0.33

1

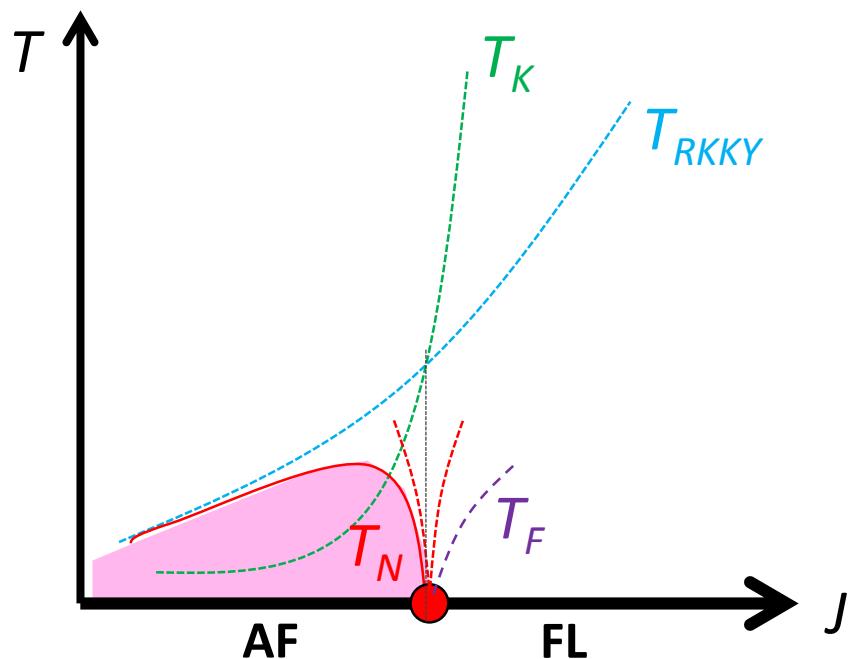
Single-site DMFT of Hubbard model

H.Terletska , V.Dobrosavljevic *et al.*, Phys. Rev. Lett **107**, 026401(2011)

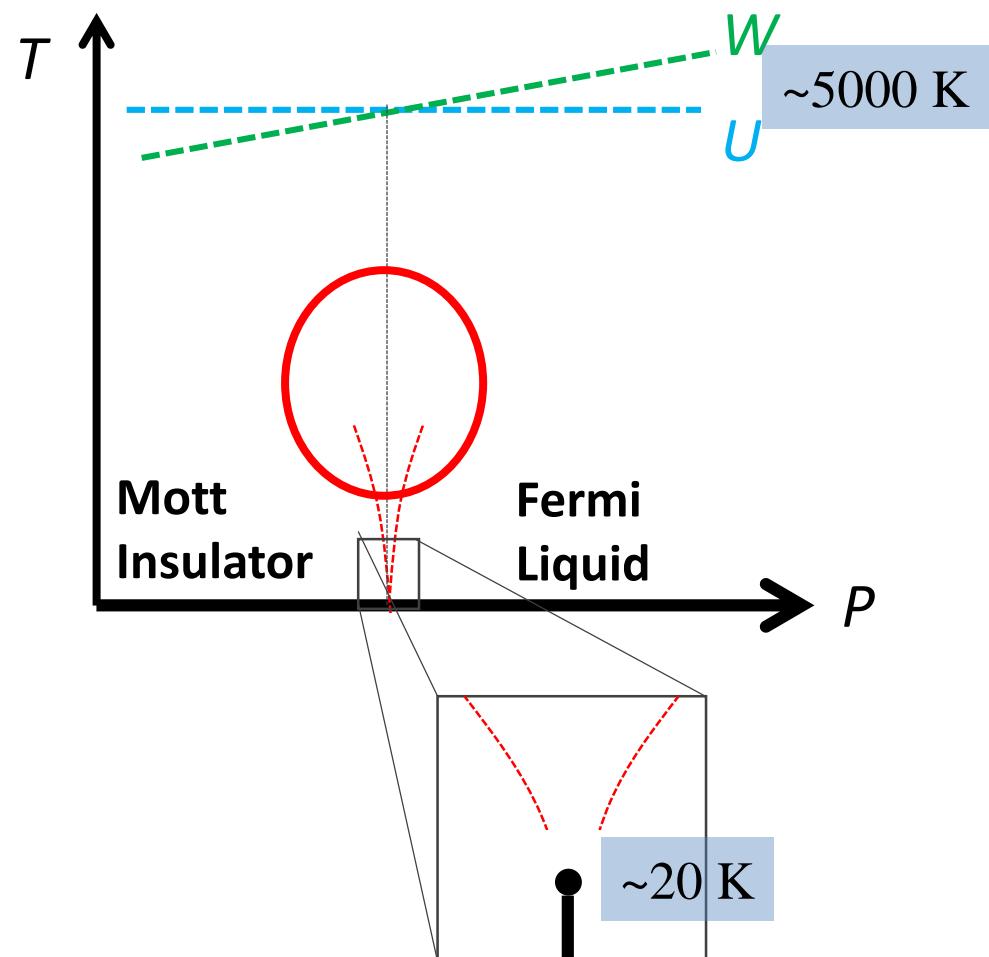


Possible quantum critical behavior in an intermediate energy range

Heavy fermion system

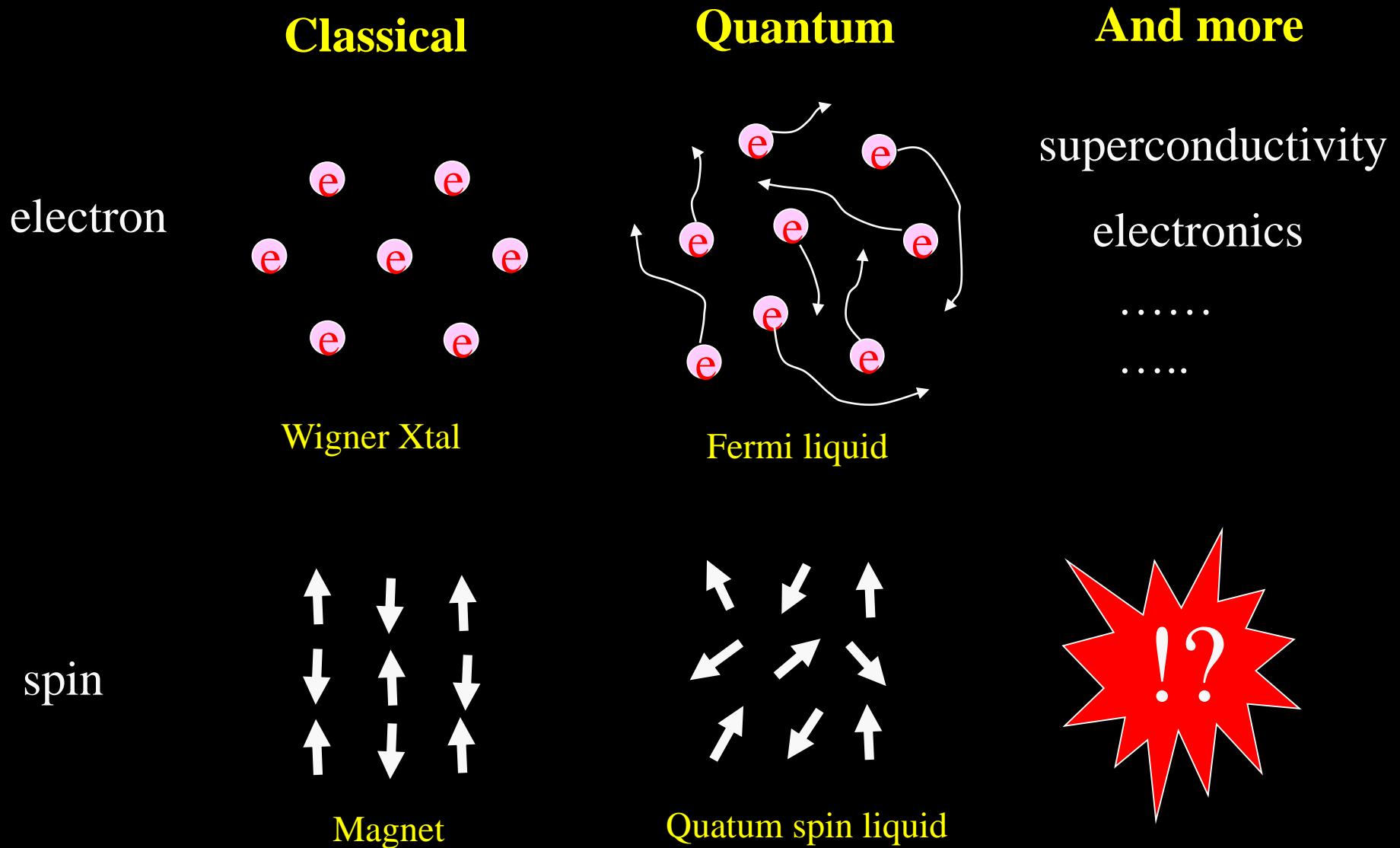


Organic Mott system



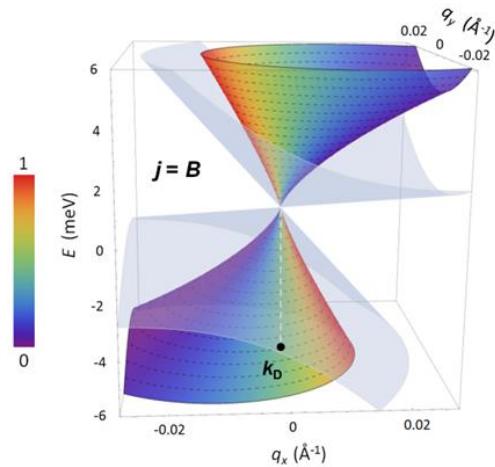
Quantum criticality ($T_c < T \ll t, U$)

Why eager for spin liquid ?

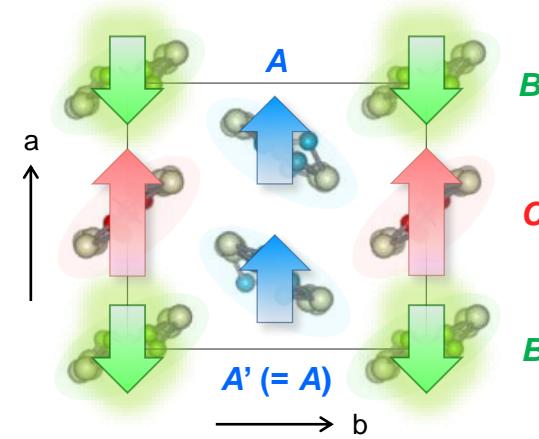


Electron correlation in massless Dirac fermions

Dirac cone reshaping



Ferrimagnetism



- M. Hirata *et al.*, Nat. Commun. (2016) in press
D. Liu *et al.*, PRL (2016)
K. Miyagawa *et al.*, JPSJ (2016)
K. Ishikawa *et al.*, PRB (2016)

NMR



Michihiro Hirata

NMR



Kyohei Ishikawa

NMR



Kazuya Miyagawa

Sample preparation



Masafumi Tamura

NMR



Claude Berthier

RG calculation
(continuum model)



Denis Basko

Mean-field calculation
(lattice model)



Akito Kobayashi



Genki Matsuno



LNCMI



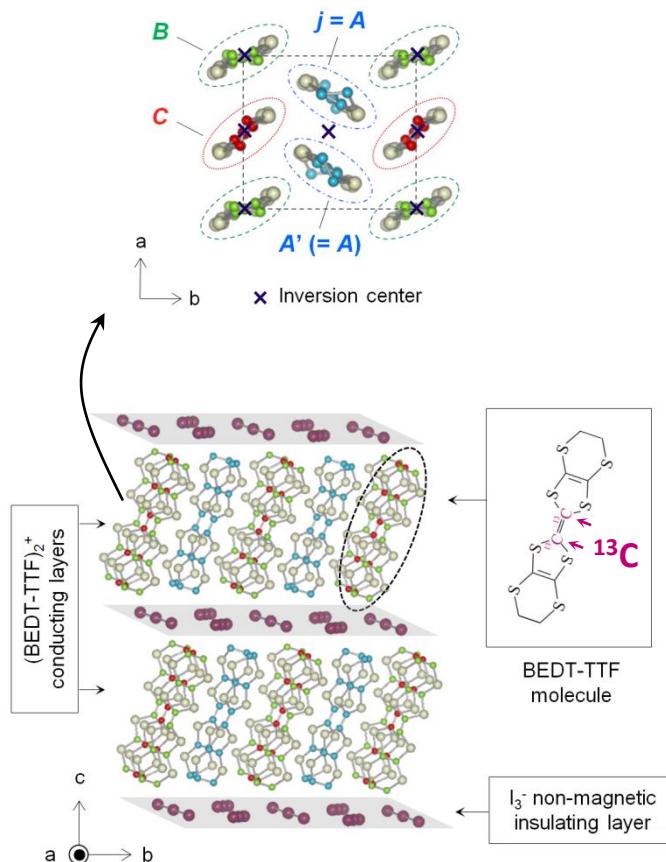
laboratoire
de physique et
de modélisation
des milieux condensés

Université
Joseph Fourier
GRENOBLE

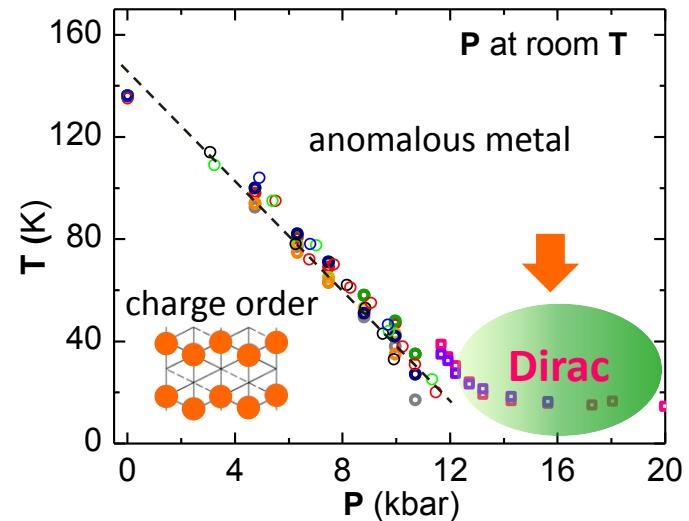


名古屋大学
NAGOYA UNIVERSITY

Organic Conductor α -(BEDT-TTF)₂I₃



Phase diagram

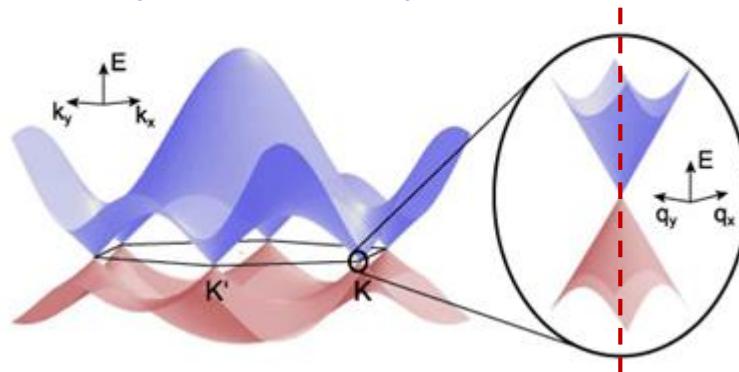
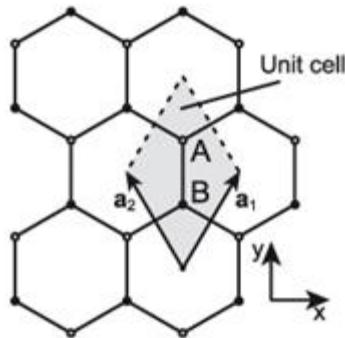


D. Liu *et al.*, PRL, in press
H. Schwenk *et al.*, Mol. Cryst. Liq. Cryst. **119** (1985)

Charge order is suppressed by pressure and a Dirac semimetal emerges !

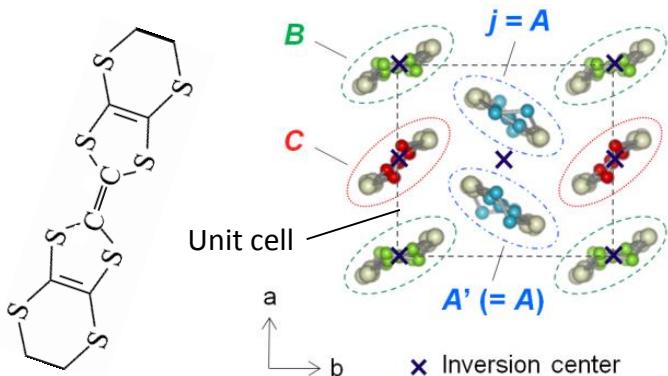
1. Dirac Cones Everywhere

◆ Vertical cone in Graphene ← Atomic Orbs. (A & B sublat.)



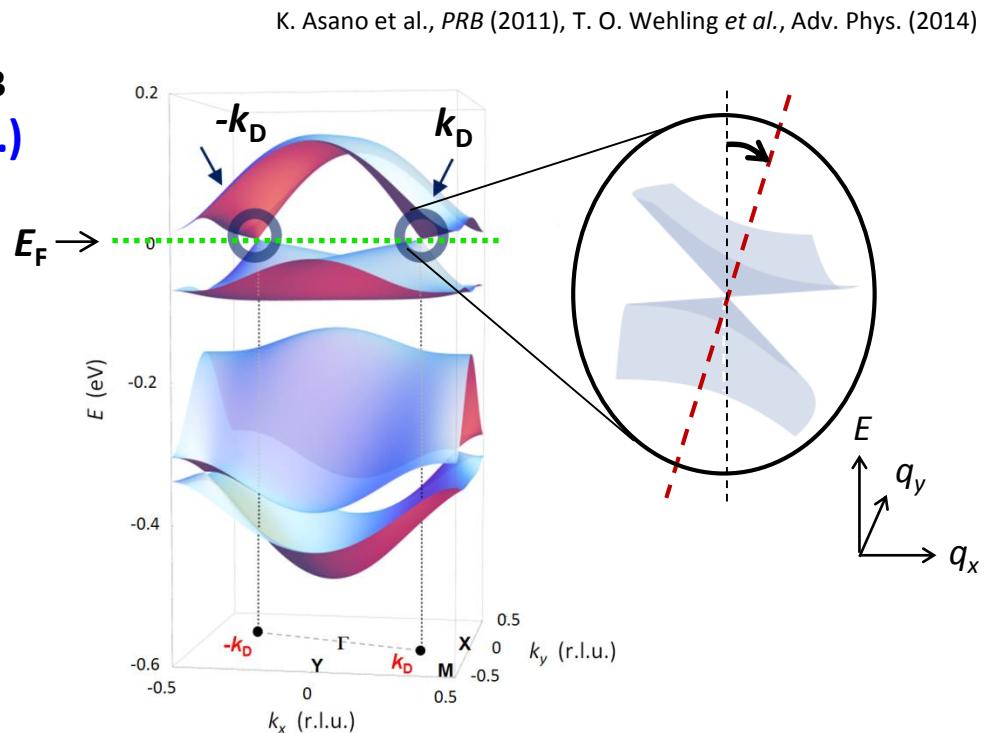
General cones exist in various systems: d-wave SC, ^3He , Topological Ins., **Organic Solids**

◆ **Tilted** cone in $\alpha\text{-}(\text{BEDT-TTF})_2\text{I}_3$
← Molecular Orbs. (A, A', B, C sublat.)

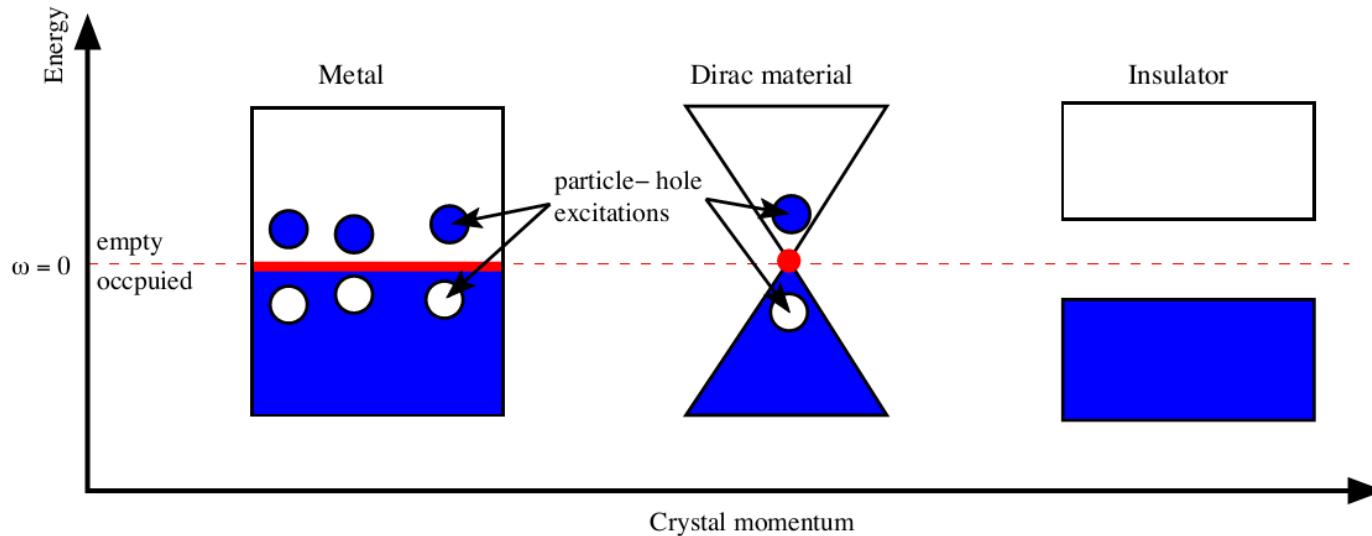


BEDT-TTF

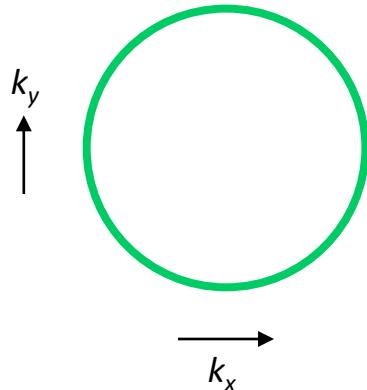
- S. Katayama *et al.*, Eur. Phys. J. B **67** (2009)
M.O. Goerbig *et al.*, PRB **78** (2008)
K. Kajita *et al.*, JPSJ **83** (2014)



2. Short-ranged or Long-ranged?



Fermi surface



Fermi point



Gapped

Unscreened →
Long-ranged

Coulomb interaction is

Short-ranged

Unscreened Long-range Coulomb Interaction

SOVIET PHYSICS JETP

VOLUME 32, NUMBER 4

APRIL, 1971

POSSIBLE EXISTENCE OF SUBSTANCES INTERMEDIATE BETWEEN METALS AND
DIELECTRICS

A. A. ABRIKOSOV and S. D. BENESLAVSKII

L. D. Landau Institute of Theoretical Physics

Submitted April 13, 1970

Zh. Eksp. Teor. Fiz. 59, 1280–1298 (October, 1970)

The question of the possible existence of substances having an electron spectrum without any energy gap and, at the same time, not possessing a Fermi surface is investigated. First of all the question of the possibility of contact of the conduction band and the valence band at a single point is investigated within the framework of the one-electron problem. It is shown that the symmetry conditions for the crystal admit of such a possibility. A complete investigation is carried out for points in reciprocal lattice space with a little group which is equivalent to a point group, and an example of a more complicated little group is considered. It is shown that in the neighborhood of the point of contact the spectrum may be linear as well as quadratic.

The role of the Coulomb interaction is considered for both types of spectra. In the case of a linear dispersion law a slowly varying (logarithmic) factor appears in the spectrum. In the case of a quadratic spectrum the effective interaction becomes strong for small momenta, and the concept of the one-particle spectrum turns out to be inapplicable. The behavior of the Green's functions is determined by similarity laws analogous to those obtained in field theory with strong coupling and in the neighborhood of a phase transition point of the second kind (scaling). Hence follow power laws for the electronic heat capacity and for the momentum distribution of the electrons.

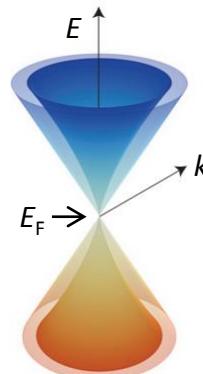
- ✓ Long-range part preserved
- ✓ Logarithmic *divergence* of v_F



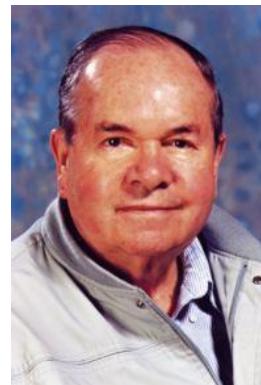
$$v(k) = v \left(1 + \frac{\alpha}{4} \ln(\Lambda/k) \right), \quad \Lambda/k \gg 1$$

A. A. Abrikosov *et al.*, JETP 32 (1971)

- Reshaping of *vertical* axes cones in graphene



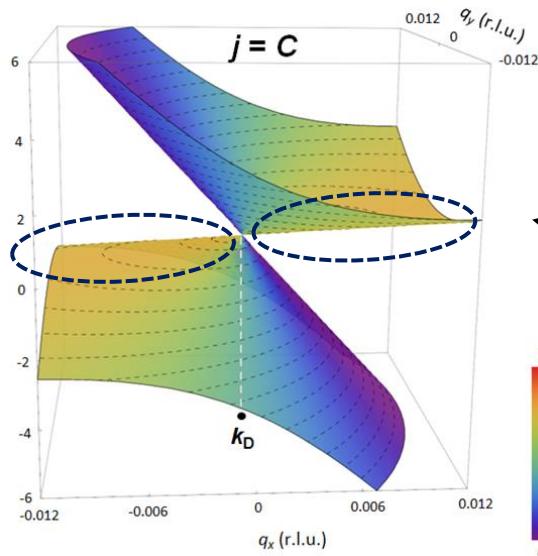
D. C. Elias *et al.*, Nat. Phys. 7 (2011)
C. Faugeras *et al.*, PRL 114 (2015)
V. N. Kotov *et al.*, Rev. Mod. Phys. 84 (2012)



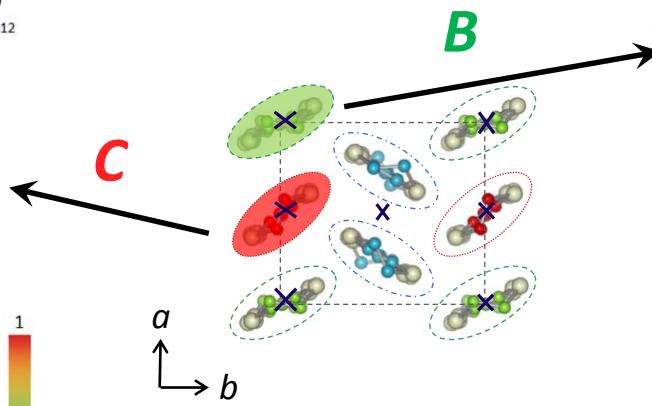
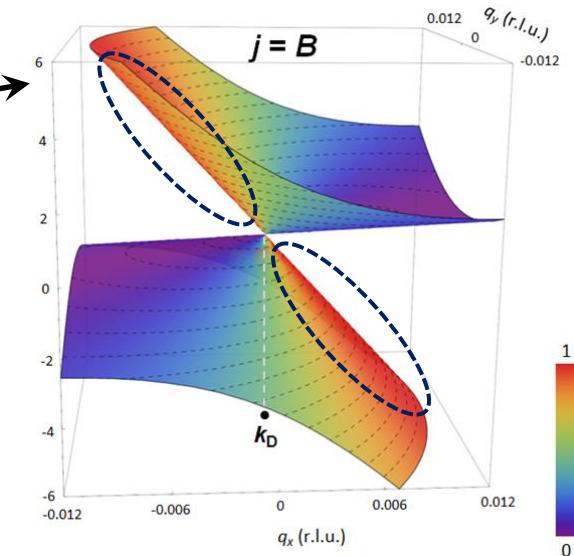
A. A. Abrikosov

Molecular site to k -space correspondence

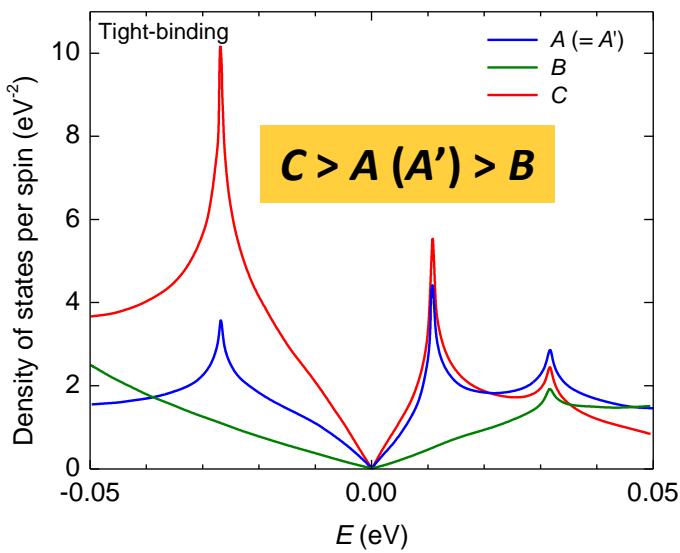
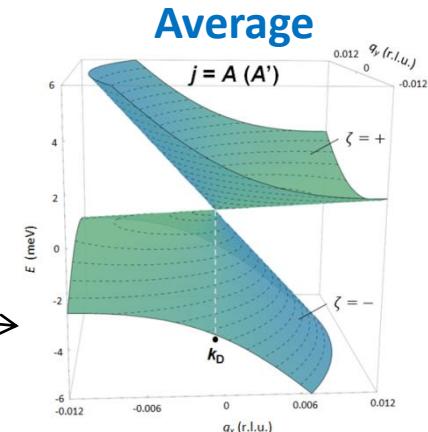
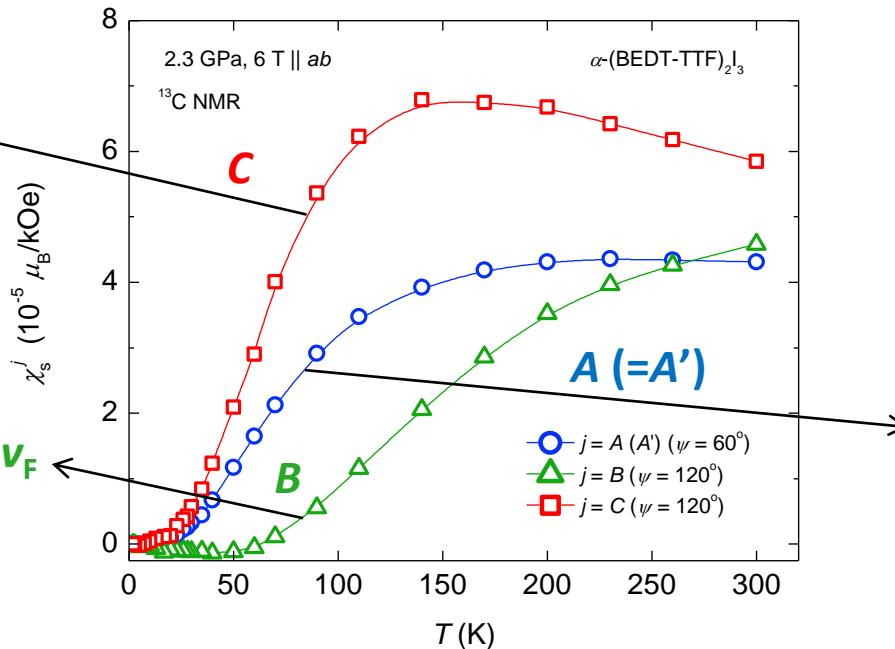
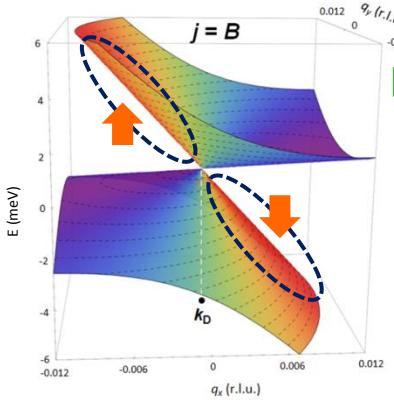
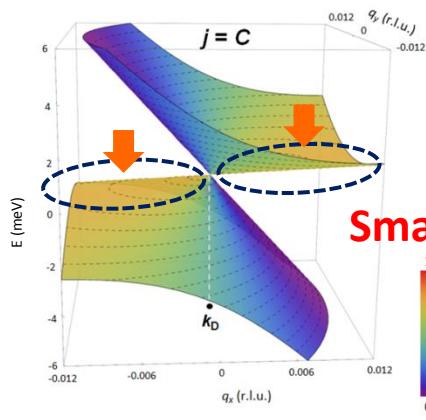
Gentle slope (Small v_F)



S. Katayama *et al.*, Eur. Phys. J. B 67 (2009)
Steep slope (Large v_F)

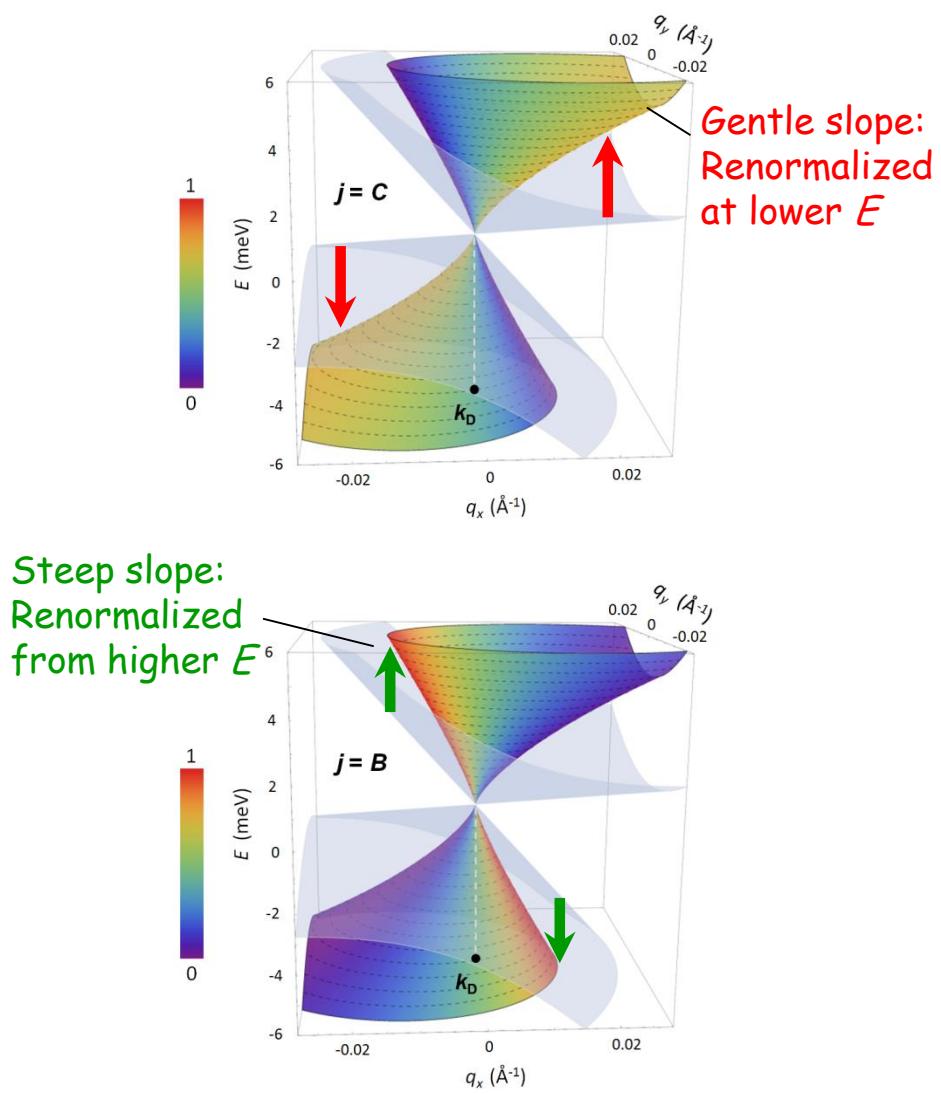
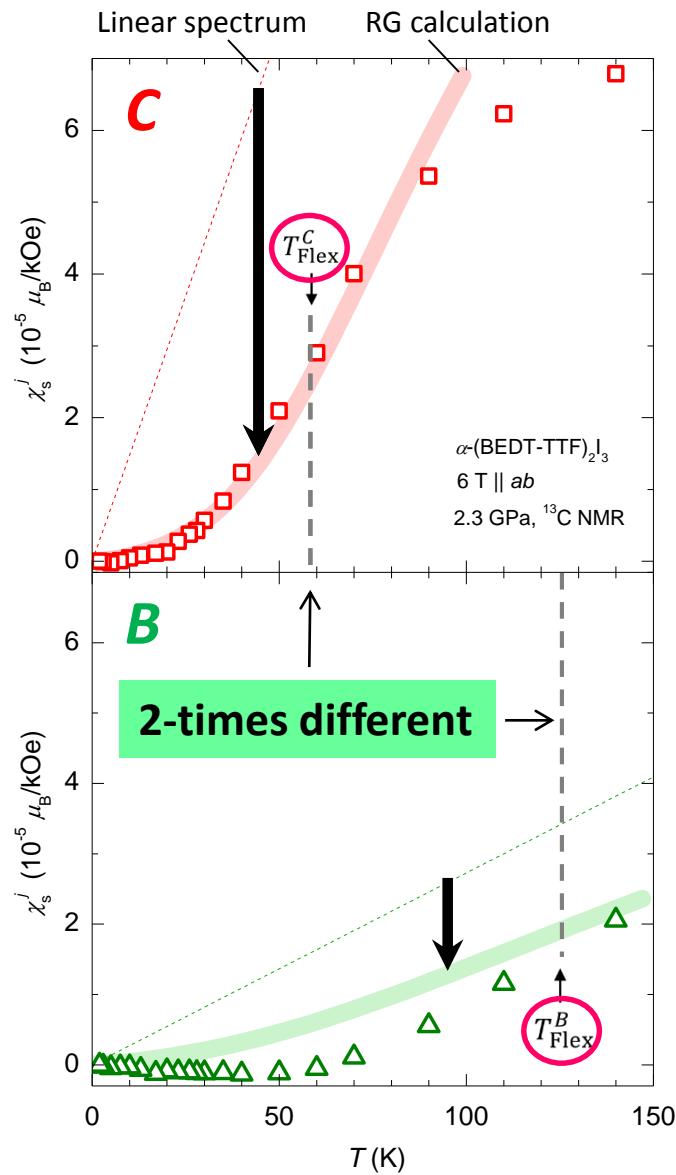


J -sublattice Electron Spin Susceptibility



Kino *et al.*, *J. Phys. Soc. Jpn.* **75** (2006);
 Katayama *et al.*, *Eur. Phys. J. B* **67** (2009)
 Kobayashi *et al.*, *J. Phys. Soc. Jpn.* **82** (2013)

A Non-uniform v_F Renormalization



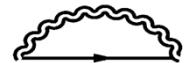
Renormalization Group (RG) Calculation

Anisotropic tilted cone

$$E_\zeta(\mathbf{q}) = \hbar \left(\mathbf{w}_0 \cdot \mathbf{q} + \zeta \sqrt{v_x^2 q_x^2 + v_y^2 q_y^2} \right)$$

S. Katayama *et al.*, Eur. Phys. J. B **67** (2009)

RPA self energy



$$\text{wavy} = \text{wavy} + \text{solid loop}$$

(cf. H. Isobe *et al.*, J. Phys. Soc. Jpn. **81** (2012): using HF exchange term

RG equations (leading order in $1/N$; $N \gg 1$)

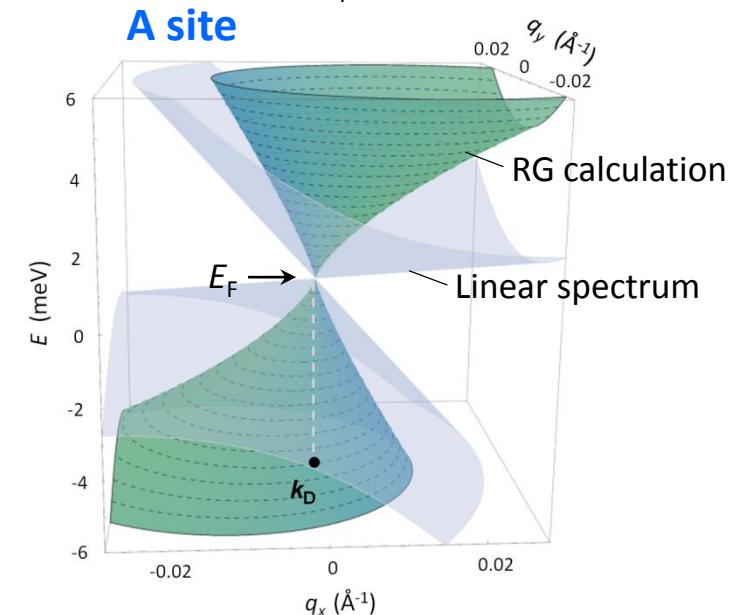
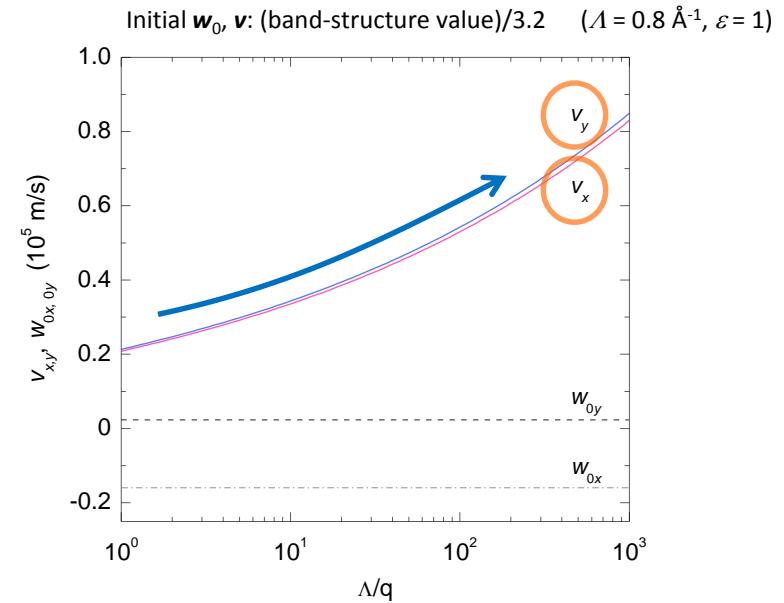
$$\frac{1}{v_x} \frac{dv_x}{dl} = \frac{8}{\pi^2 N} \int_0^{2\pi} \frac{d\varphi}{2\pi} 2 \cos^2 \varphi F(g_\varphi)$$

$$\frac{1}{v_y} \frac{dv_y}{dl} = \frac{8}{\pi^2 N} \int_0^{2\pi} \frac{d\varphi}{2\pi} 2 \sin^2 \varphi F(g_\varphi)$$

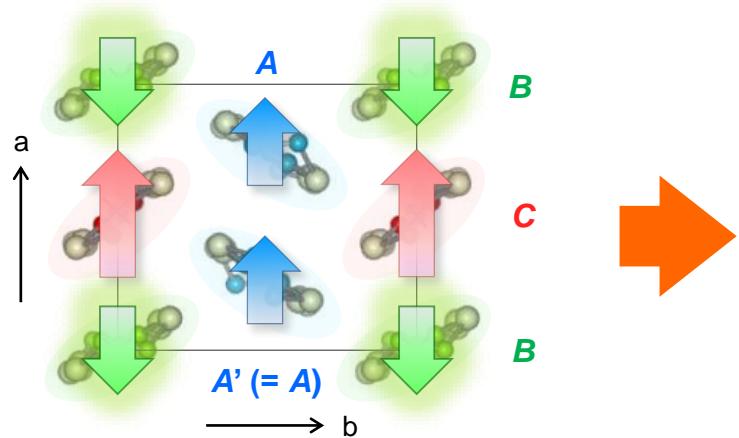
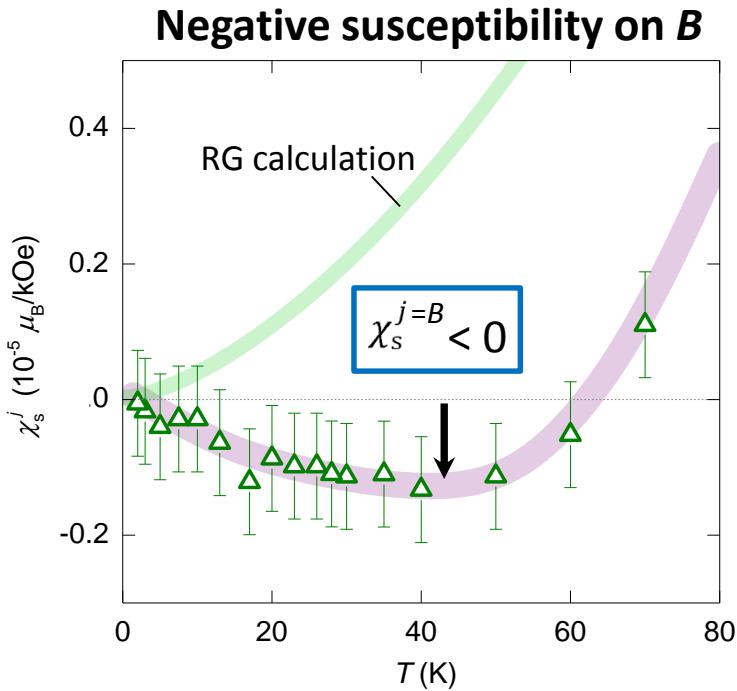
$N = 4$: fermion flavor, $\mathbf{q} = (\mathbf{k} - \mathbf{k}_D) = q(\cos \varphi, \sin \varphi)$, $l = \ln(\Lambda/q)$

$$F(g_\varphi) = (-\pi/2 + g_\varphi + \arccos g_\varphi / \sqrt{1 - g_\varphi^2}) / g_\varphi$$

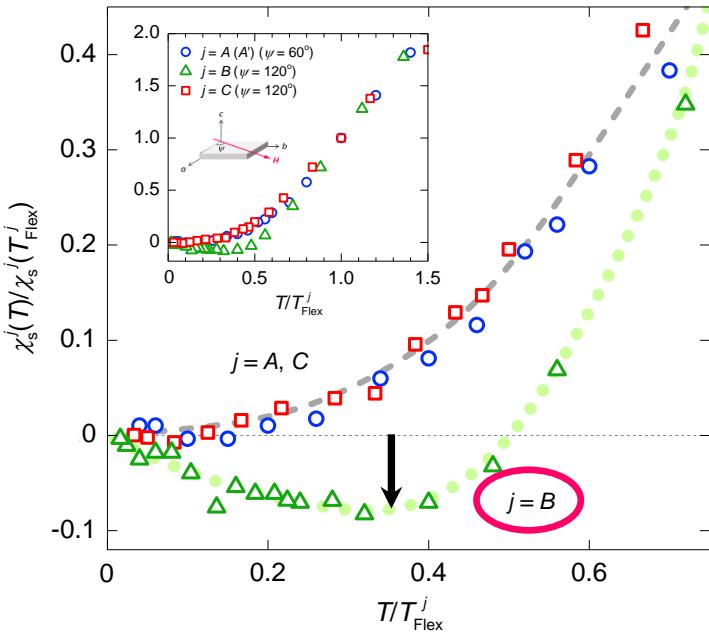
$$g_\varphi = 2\pi e^2 N / 16 \varepsilon \sqrt{v_x^2 \sin^2 \varphi + v_y^2 \cos^2 \varphi}$$



Negative Susceptibility on the *B* Sublattice



Anomaly observed uniquely on *B*



**Ferrimagnetic spin
Polarization exist on top
of the v_F renormalization**

Hubbard model; Mean-field calculation

A. Kobayashi *et al.*, J. Phys. Soc. Jpn. **82** (2013)

Hubbard model (nearest neighbor)

$$H = \sum_{(i\alpha:j\beta),\sigma} (t_{i\alpha:j\beta} a_{i\alpha\sigma}^\dagger a_{j\beta\sigma} + \text{h.c.}) + \sum_{j\alpha} U a_{j\alpha\uparrow}^\dagger a_{j\alpha\downarrow}^\dagger a_{j\alpha\downarrow} a_{j\alpha\uparrow}$$

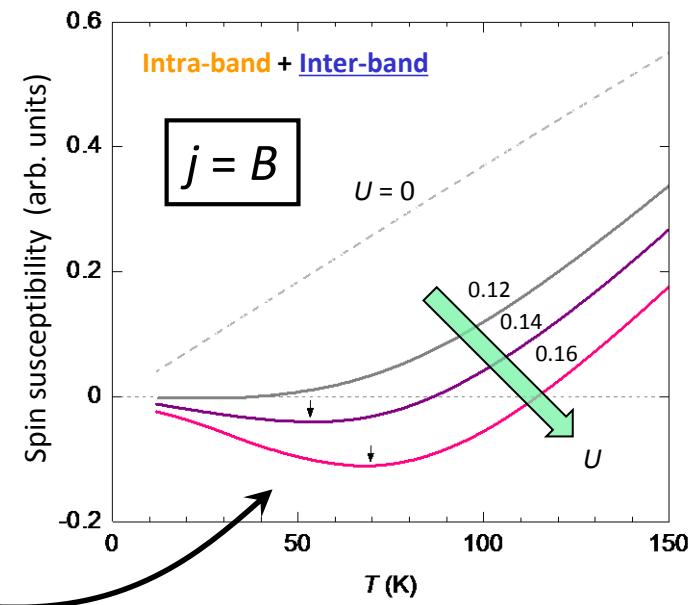
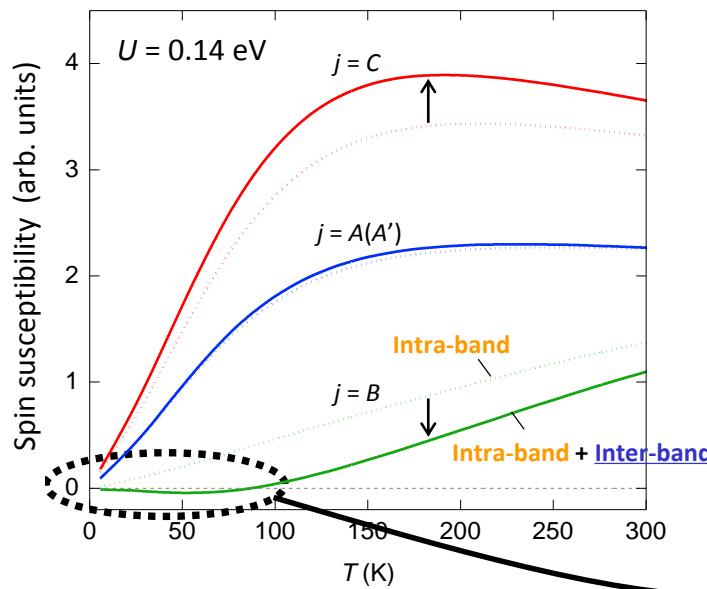
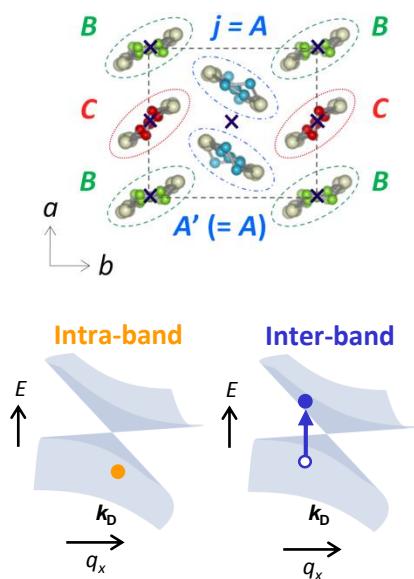
RPA spin susceptibility ($\mathbf{Q} = 0$)

$$\chi_{ij, \text{RPA}}(\mathbf{Q}, \omega) = (\hat{\chi}_{\text{RPA}})_{ij}(\mathbf{Q}, \omega) = \left[(\hat{I} - \hat{\chi}^{(0)} U \hat{I})^{-1} \hat{\chi}^{(0)} \right]_{ij}(\mathbf{Q}, \omega),$$

$$\chi_{\text{RPA}}^j = \sum_{i=1}^4 \chi_{ij, \text{RPA}}(\mathbf{0}, 0)$$

$$\left. \begin{aligned} \sum_{j=1}^4 \tilde{\epsilon}_{ij\sigma}(\mathbf{k}) d_{j\eta\sigma}(\mathbf{k}) &= E_{\eta\sigma}(\mathbf{k}) d_{i\eta\sigma}(\mathbf{k}), \\ \tilde{\epsilon}_{ij\sigma}(\mathbf{k}) &= \epsilon_{ij}(\mathbf{k}) + U \langle N_{i\sigma} \rangle \delta_{ij}, \\ \langle N_{j\sigma} \rangle &= \frac{1}{N_{\text{u.c.}}} \sum_{\mathbf{k}} \sum_{\eta=1}^4 d_{j\eta,-\sigma}^*(\mathbf{k}) d_{j\eta,-\sigma}(\mathbf{k}) f(E_{\eta,-\sigma}(\mathbf{k}) - \mu), \end{aligned} \right\}$$

$$\chi_{ij}^{(0)}(\mathbf{Q}, \omega) = -\frac{1}{N_{\text{u.c.}}} \sum_{\mathbf{k}} \sum_{\eta, \eta'=1}^4 \mathcal{F}_{ij}^{\eta\eta'}(\mathbf{k}, \mathbf{Q}) \frac{f(E_{\eta}(\mathbf{k}+\mathbf{Q})) - f(E_{\eta'}(\mathbf{k}))}{E_{\eta}(\mathbf{k}+\mathbf{Q}) - E_{\eta'}(\mathbf{k}) - \hbar\omega - i\delta},$$



Hubbard model; Mean-field calculation

A. Kobayashi *et al.*, J. Phys. Soc. Jpn. **82** (2013)

Hubbard model (nearest neighbor)

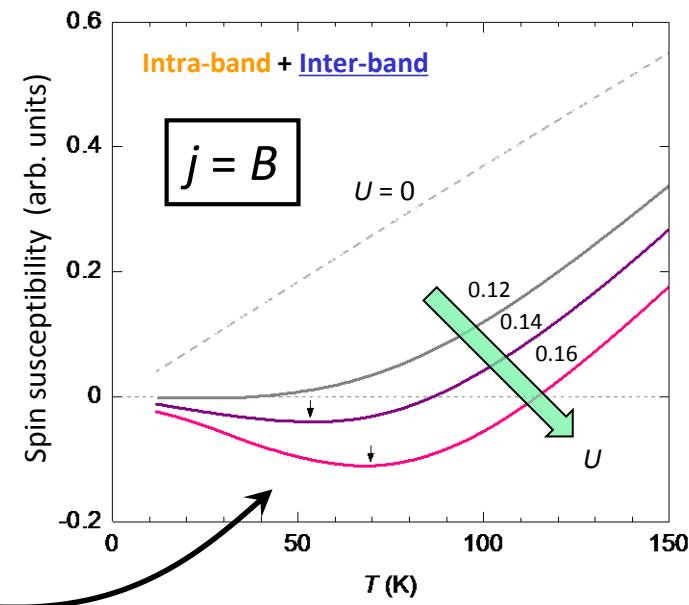
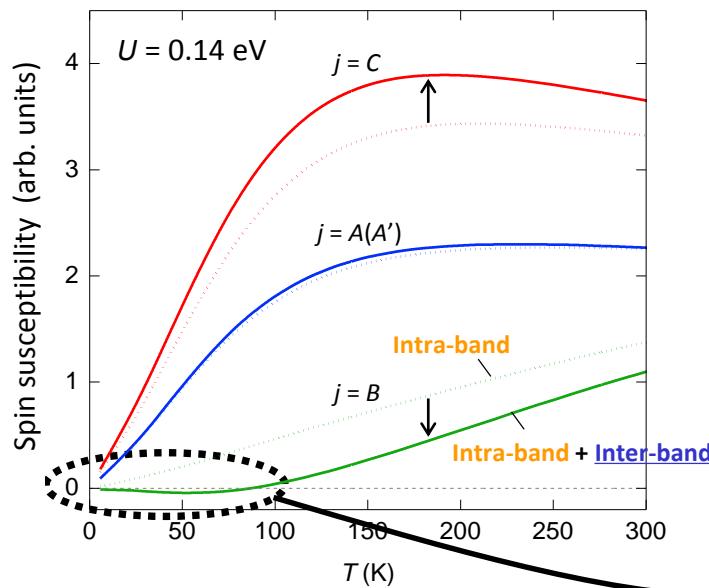
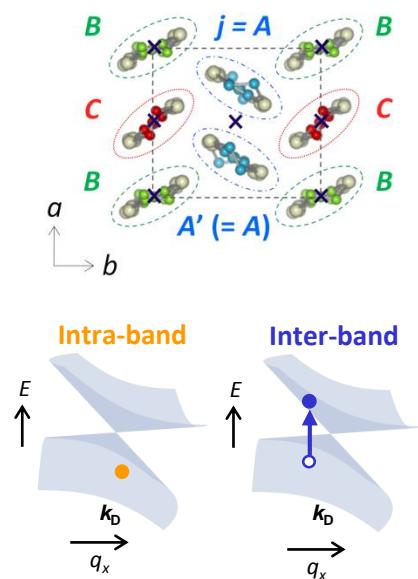
$$H = \sum_{(i\alpha:j\beta),\sigma} (t_{i\alpha:j\beta} a_{i\alpha\sigma}^\dagger a_{j\beta\sigma} + \text{h.c.}) + \sum_{j\alpha} U a_{j\alpha\uparrow}^\dagger a_{j\alpha\downarrow}^\dagger a_{j\alpha\downarrow} a_{j\alpha\uparrow}$$

RPA spin susceptibility ($Q = 0$)

$$\chi_{ij, \text{RPA}}(\mathbf{Q}, \omega) = (\hat{\chi}_{\text{RPA}})_{ij}(\mathbf{Q}, \omega) = \left[(\hat{I} - \hat{\chi}^{(0)} U \hat{I})^{-1} \hat{\chi}^{(0)} \right]_{ij}(\mathbf{Q}, \omega),$$

$$\left. \begin{aligned} \sum_{j=1}^4 \tilde{\epsilon}_{ij\sigma}(\mathbf{k}) d_{j\eta\sigma}(\mathbf{k}) &= E_{\eta\sigma}(\mathbf{k}) d_{i\eta\sigma}(\mathbf{k}), \\ \tilde{\epsilon}_{ij\sigma}(\mathbf{k}) &= \epsilon_{ij}(\mathbf{k}) + U \langle N_{i\sigma} \rangle \delta_{ij}, \\ \langle N_{j\sigma} \rangle &= \frac{1}{N_{\text{u.c.}}} \sum_{\mathbf{k}} \sum_{\eta=1}^4 d_{j\eta,-\sigma}^*(\mathbf{k}) d_{j\eta,-\sigma}(\mathbf{k}) f(E_{\eta,-\sigma}(\mathbf{k}) - \mu), \end{aligned} \right\}$$

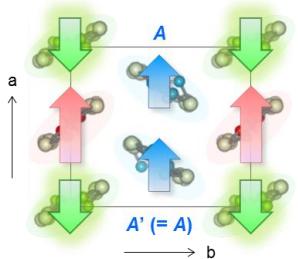
Ferrimagnetic Polarization due to SR Coulomb Int.



Conclusions

Short-range Coulomb, U, V

Ferrimagnetism



**Collapse of band structure
into charge order state**

Long-range Coulomb, $1/r$

Cone reshaping

