Granular Matter I

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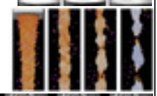
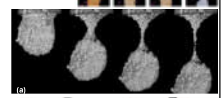
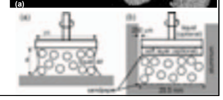
http://jfi.uchicago.edu/~jaeger/group/index.html

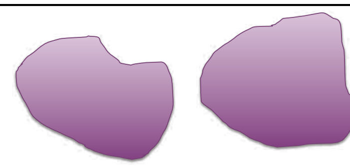
Outline

- Brief intro to granular matter
- Granular jets & freely falling streams
- Dense suspension droplet break-up
- Shear thickening in suspensions







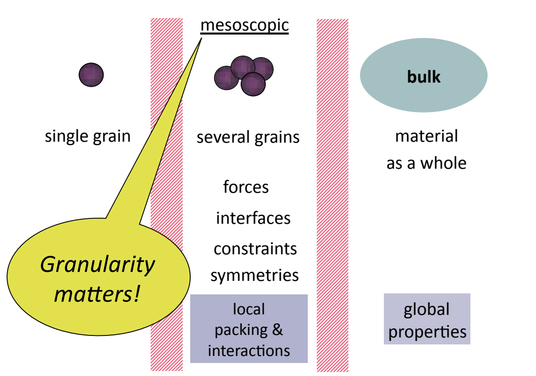


Lots of internal degrees of freedom → integrate / average /ignore

Action happens at interfaces & contacts: local, short-ranged

Overall, aggregate behavior can be strikingly different from that of individual grain

1960s – 80s:	granular systems = percolating thin metal films (M-I or I-S transitions)
1990s – today:	granular systems = aggregates of macroscopic particles; but also co-polymer domain patterns, perhaps fluctuating high-Tc puddles, and more recently (again) arrays of nanoparticles



Mesoscopic regime not tied to any particular length scale

(Macroscopic) Granular matter

Solid-, liquid-, gas-*like* states
 and transitions between them

Behavior *far* from equilibrium

- non-Brownian (kT irrelevant, mgd huge)
- friction and dissipation during collisions (cannot easily relax)
- free volume important (jamming, dilation)

numerical values for 3-D monodisperse spheres

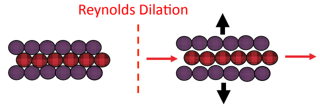
occupied volume fraction

ρ = 0.74 — crystal

ρ ≈ 0.64 — random close packing (RCP)

ρ ≈ 0.56 — random loose packing (RLP)

Reynolds Dilation



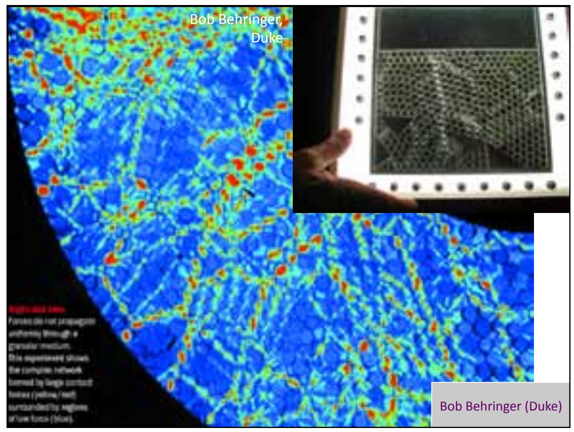
O. Reynolds, 1886

If we want to get elastic materials light we shake them up … to get these dense we squeeze them into the measure. With corn it is the reverse; … if we try to press it into the measure we make it light - to get it dense we must shake it - which … causes a rearrangement in which the grains take the closest order.

Unique consequences for granular matter

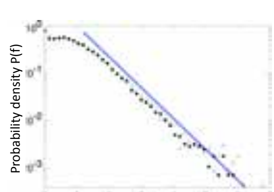
- Typically stuck in amorphous metastable state once energy is removed
- Inherently heterogeneous structure (not just in static case but even if driven)
- Friction: non-linear, due to surfaces plus particle geometry
- Dilation: can work like positive feedback that localizes shear (➔ shear band formation, avalanching)
- Time scales 10^{-6} … 10^{6} s, length scales nm … >10 particle diameters;

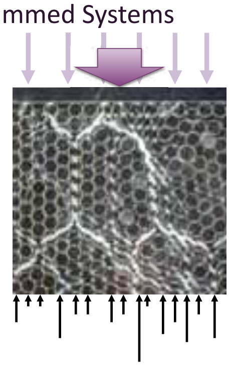
➔ Issues: How to separate relevant scales? How to perform averages?



Force Networks in Jammed Systems

- **Inhomogeneous force paths**
- **Characteristic probability distribution of forces, P(f)**





Mueth et al. PRE 57 ('98)

Blair et al., PRE 63 ('01)

Jamming

= geometrical confinement such that relative particle movement is suppressed and a yield stress develops

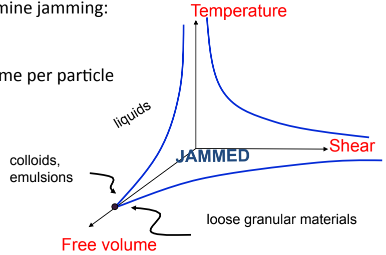
Hull is flexible!

Rigidity comes from increasing packing density by just a few %

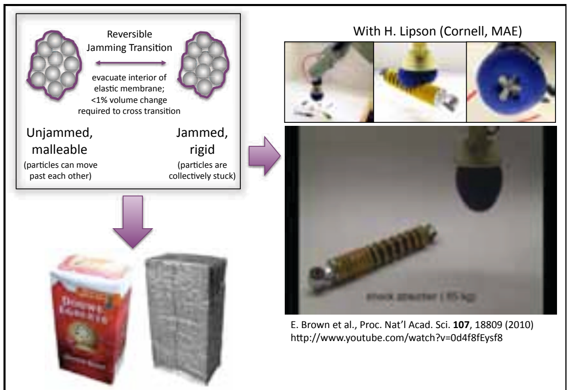
Jamming Phase Diagram

3 parameters determine jamming:

- Temperature
- Available free volume per particle
  (i.e., packing density)
- Shear stress



Sid Nagel (Chicago), Andrea Liu (UPenn)



E. Brown et al., Proc. Nat'l Acad. Sci. **107**, 18809 (2010)
http://www.youtube.com/watch?v=0d4f8fEysf8

Use jamming to increase # DoF of actuators

E. Steltz et al., Jamming as an Enabling Technology for Soft Robotics,
Proc. SPIE **7642**, 764225 (2010)

Currently Hot Research Topics for Granular Materials
(my obviously biased list)

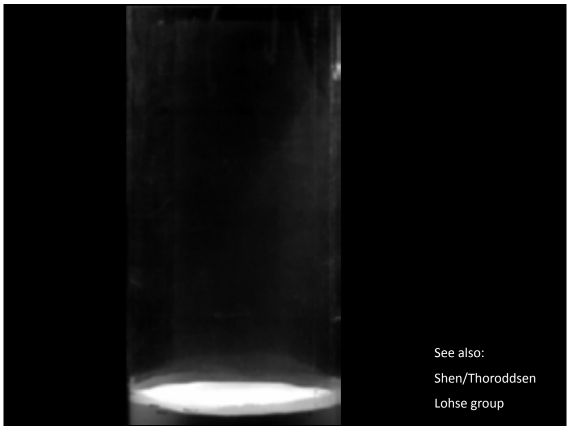
Effect of particle shape on properties of jammed state

Jamming/unjamming transition under shear, away from point J

Nature of the granular fluid state (incl. interactions of particles with interstitial medium)

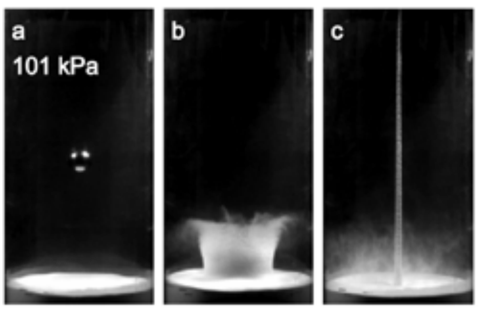
Effect of attractive interactions (cohesion, "wet" granular material)

➔ These lectures: look at examples for items 2-4



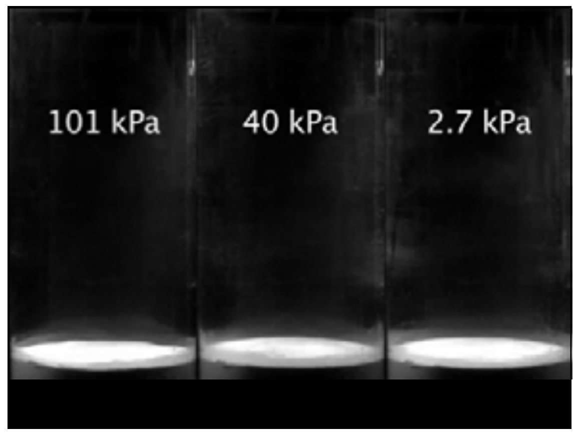
- What keeps the jet so collimated? *OR:* How does a collectively liquid-like state emerge from a bunch of macroscopic, individually solid particles that interact via short-range (contact) forces?
- For dry, freely flowing grains, how does the emerging granular liquid differ from ordinary liquids?
- What can we learn about local grain-grain interactions from analyzing the structures formed by granular liquids?

Granular Jets

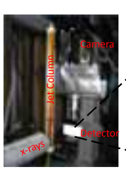


Fine loose powder (<200μm)
Impact velocity 0-10m/s

Royer et al., *Nature Physics* 1 ('05)



Looking inside the bed material….



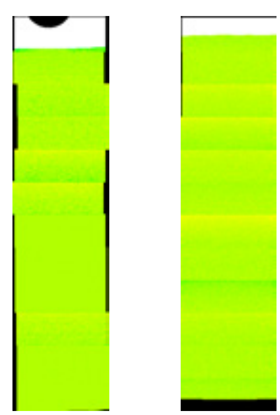
1" steel sphere
penetrating bed at ~3m/s

Stitch together 14 independent recordings

High-speed radiographs, >5000 full frames per second
Collaboration with CARS at Argonne National Lab's Advanced Photon Source

Track Bed Density Changes Quantitively

Atmospheric



3 Torr

During time scale of impact, interstitial gas is effectively trapped

➔Presence of gas opposes changes in packing fraction

➔Loose packing with air behaves almost like incompressible fluid

$$k = \frac{d^2(1-\phi)^3}{180\phi^2} \qquad D = \frac{k}{\eta}\frac{P_0}{1-\phi}$$

Fine-grained, loosely-packed bed $d \sim 50\mu m$ $\phi \sim 0.50 \ldots 0.58$

$D \sim 400 \ldots 850\ cm^2/s$ @ 101kPa

$\tau_D \sim L^2/D \sim 90 \ldots 180ms$ = time to diffuse depth of bed

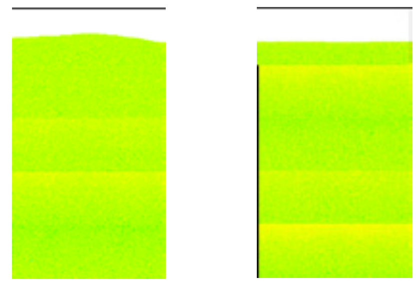
$\tau_{travel} < 60ms$ = time for sphere to travel through bed

$\tau_{rise} \sim 30ms$ = time for bed to rise

What about dense packing?

Sphere Impact Into ***Dense*** Bed

$\phi \sim 0.6$



atmospheric

3 Torr

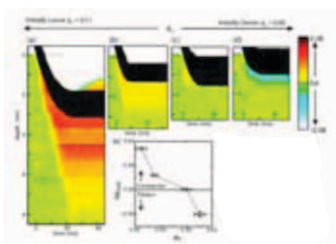
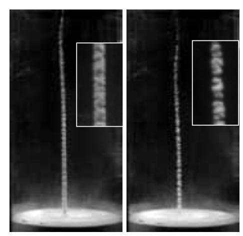


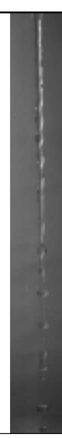
Fig. 4: Packing changes in vacuum. (a)–(d) Space-time plots of the centerline of the X-ray movies in fig. 1, showing the change in packing $\Delta\phi$ in vacuum ($P_0 < 1$ kPa) for different initial packing densities: (a) $\phi_0 = 0.51 \pm 0.01$, (b) $\phi_0 = 0.53 \pm 0.01$, (c) $\phi_0 = 0.58 \pm 0.02$, (d) $\phi_0 = 0.60 \pm 0.01$. (e)–(f) Packing density observed below the sphere after it has come to rest; (e) extremum change in packing $\Delta\phi_{extr}$ ($\Delta\phi_{extr} > 0$ for compaction, $\Delta\phi_{extr} < 0$ for dilation)

Royer et al. EPL 93 (2011)



t

t + Δt

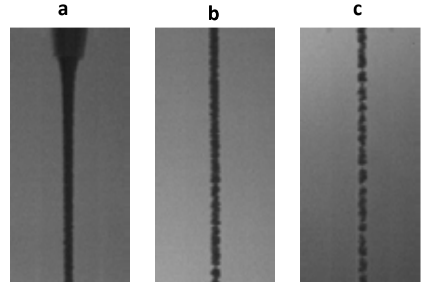


Nagel group
U of Chicago

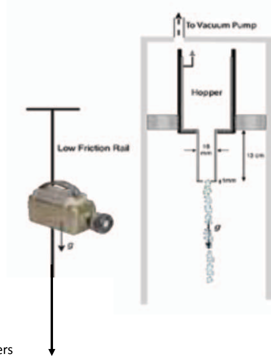
Freely falling granular stream

50 micron glass spheres flow through funnel (5mm nozzle diam.)

a b c



Matthias Möbius, PRE 2006



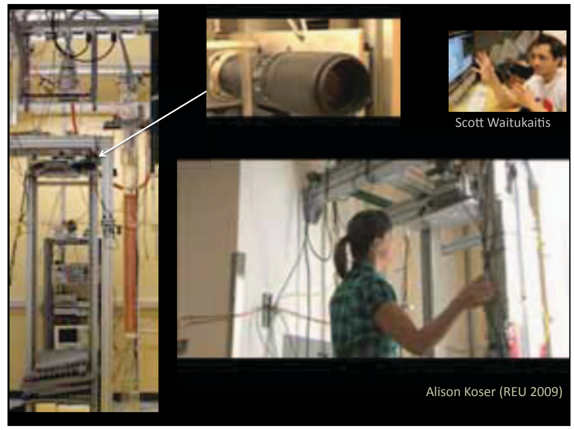
2 meters

John Royer

Track evolution of stream in co-moving frame

vary air pressure
101 kPa to 0.05 kPa (factor of 6 in drag force)

grain material:
glass 50μm to 350 μm
copper 100μm
coated glass (hydrophobic, silver, ...)



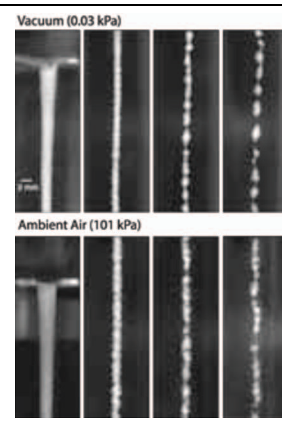
Air Drag

Increased air drag has little effect on cluster formation

Instead, rips clusters apart

Can use drag to estimate inter-particle forces:

- grains ripped off when air drag ~ cohesion
- gives **nN** cohesive forces



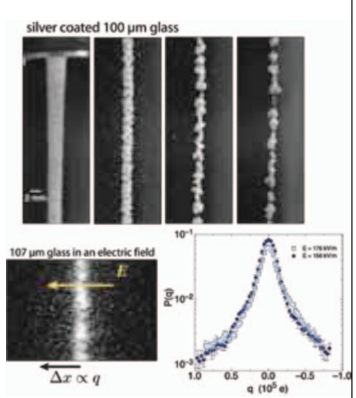
Electrostatics

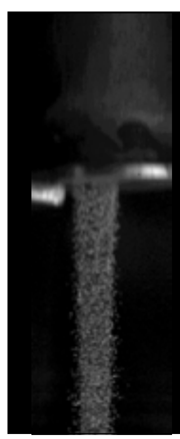
E-field 10^4 - 10^5 V/m across stream (parallel plate capacitor)

Stream = neutral, charge distribution P(q) nearly symmetric

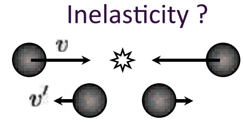
Maximum attractive force only 0.1 nN: ➔ too small

Clusters also observed with conductive, silver coated glass spheres





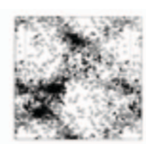
Inelasticity ?



coefficient of restitution $e = \frac{v'}{v} \leq 1$

d = 130μm copper
Otherwise identical conditions (vacuum, 4 mm nozzle)

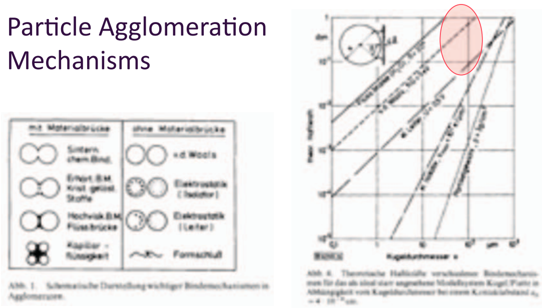
Lower coefficient of restitution (*e* ~ 0.90 vs. ~ 0.97 for glass)



McNamara, Goldhirsch, Luding, Pöschel, Zippelius, ...

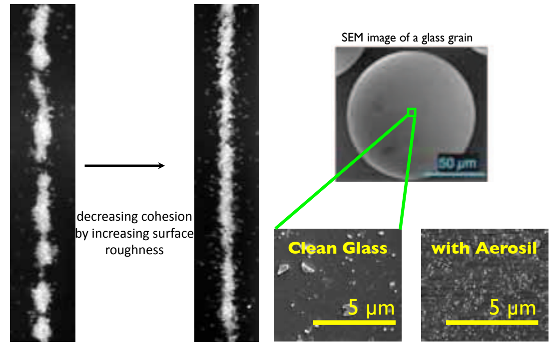
Clustering not simply due to inelasticity alone

Particle Agglomeration Mechanisms



Principles of Agglomeration, Helmar Schubert, Chem.-Ing.-Tech. 51, 266 (1979)

Controlling Clustering by Altering **Nanoscale** Surface Roughness



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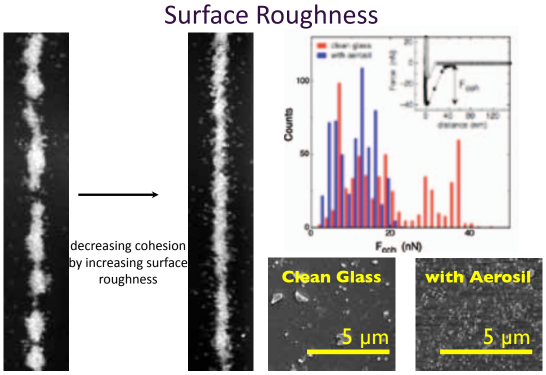
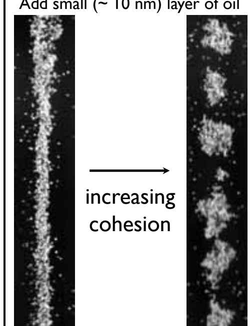
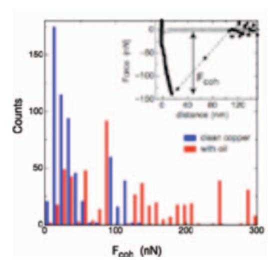


Clustering due to 'Sticky' Collisions

Add small (~ 10 nm) layer of oil



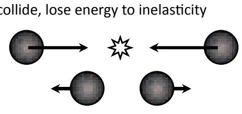


Estimate energy loss from AFM data:

$$W_{coh} = \int F(\xi)d\xi \sim 10^{-15}\ \mathrm{J}$$

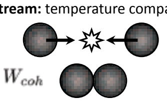
Scenario for Cluster Formation

Near nozzle: large velocity fluctuations ~ 1 cm/s (high granular temperature) particles collide, lose energy to inelasticity

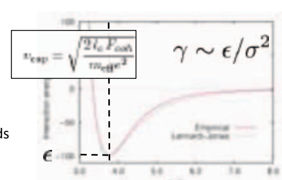


$$\frac{1}{2}m(\delta u)^2 \gg W_{coh}$$

Further downstream: temperature comparable to W_{coh}, particles collide, stick



$$\frac{1}{2}m(\delta u)^2 \sim W_{coh}$$



$$v_{esp} = \sqrt{\frac{2 l_c F_{coh}}{m_{eff}e^2}}$$

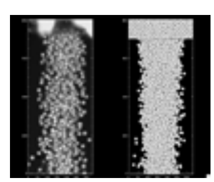
$$\gamma \sim \epsilon/\sigma^2$$

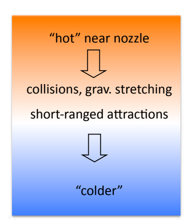
ϵ $\sim \sigma$

Clustering due to attractive potential
Analogy to surface tension in normal liquids

$$\gamma \sim W_{coh}/d^2 \sim 0.1\ \mu\mathrm{N/m}$$

Granular Temperature $T \sim m(\delta v)^2$

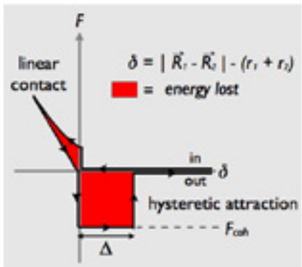




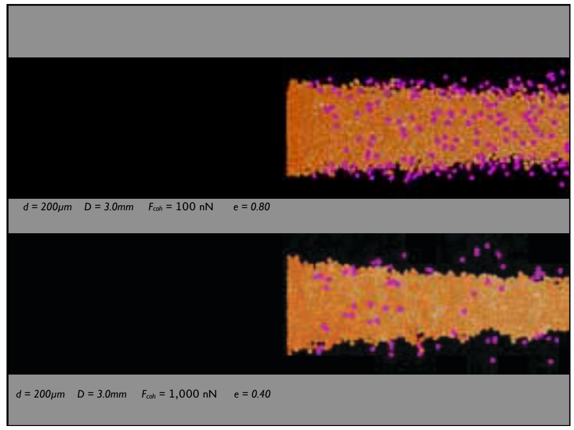
3D MD simulations with ~100,000 particles, using Itasca PFC3d
Scott Waitukaitis, Helge Grütjen

Model: Linear contact + Hysteretic Attractive Force

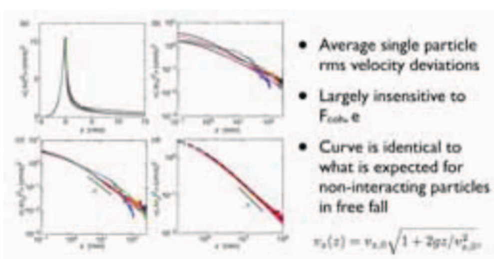
- **Repulsive part:** linear contact model for normal & tangential components
  - rotation included
  - damping yields *e*
- **Attractive part:** Hysteretic "step" force
  - commonly used to model liquid bridges
- PFC3d, ~100,000 particles



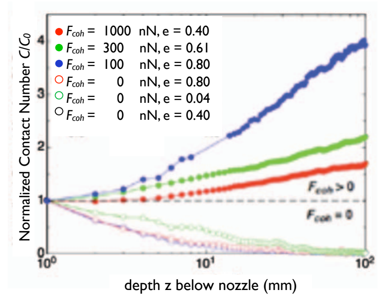
Fingerle and Herminghaus, Phys. Rev. E (2008)



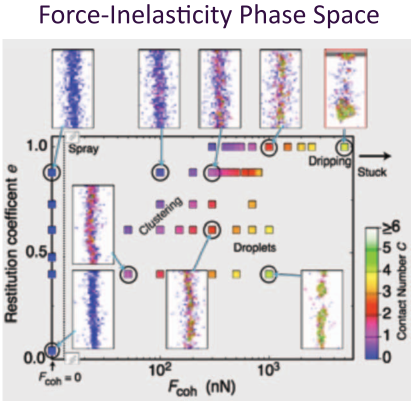
Granular temperature not a sensitive diagnostic



No Cohesion: Particles **Always** Drift Apart in velocity gradient (gravitational stretching)



Force-Inelasticity Phase Space



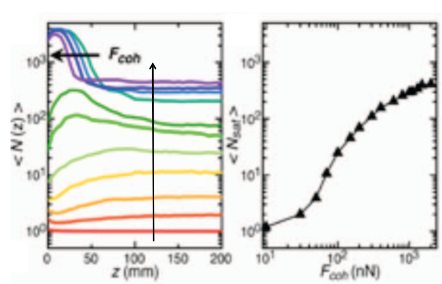
Wide range of behaviors:

- Spraying
- Clustering
- Drop-forming
- Dripping

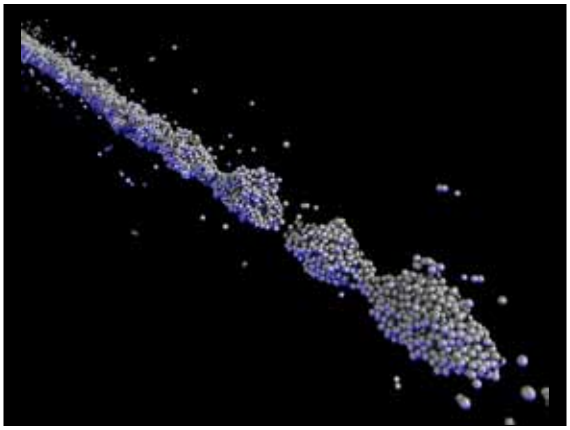
No droplets for $F_{coh} = 0$

Both e & F_{coh} affect cluster size, necking behavior

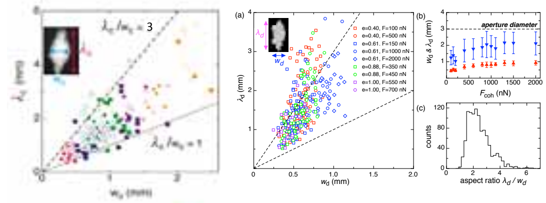
Diagnostic of grain-grain attractions



Use like molecular beams?



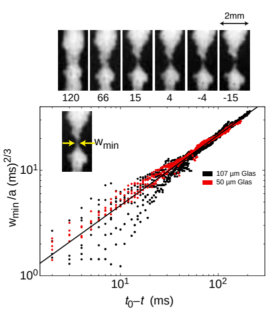
Break-up Details



Avg. droplet aspect ratio < 3
Neck shape roughly symmetric

…different from ordinary liquids

Neck Width Scaling During Breakup



Different from thermal exponent (0.418), instead > 1/2

Consistent with

$$w_{\min} = \left[a\left(t_0 - t\right)\right]^{2/3}$$

Same scaling as inviscid liquid break up (ie. water in air)

In liquid case

$$a = 1.4\sqrt{\frac{\gamma}{\rho}}$$

effective surface tension ~ μN/m
4 orders of magnitude below standard liquids

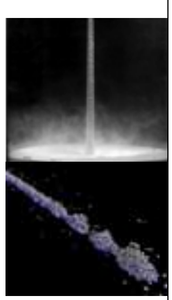
Liquid nanojet simulations

Moseler & Landman, Science **289** (2000)

See also J. Eggers and E. Villermaux, Physics of liquid jets, Reports on Progress In Physics **71**, 036601 (2008).

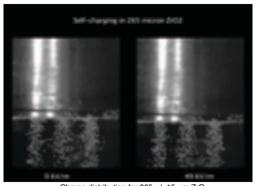
To sum up Jets & Streams:

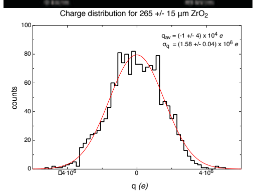
- Tiny attractive interactions (**nN, nm**) drive clustering and droplet formation…same as for molecules, but here acting between *macroscopic* constituents, therefore often masked by gravity
- Corresponding effective surface tension 4-5 orders lower than water ➔ ultra-low regime not reachable with ordinary liquids under ambient conditions; break-up neither Rayleigh-Plateau nor thermal: where do aspect ratio, neck shape and 2/3 power law come from?
- **Freely Falling Granular Streams:** can probe wide range of behaviors from gas to liquid to plastically deforming solid; granular analog of molecular beam to probe subtle grain-grain interactions *in situ*



Next: add water….

Granular Self-Charging



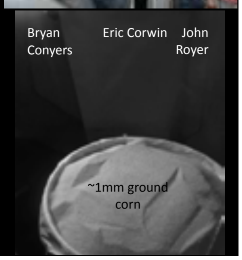


Dense Suspensions

More Permeable Bed

scale up system by factor ~20

drop shot put from several stories into 55 gallon drum filled with 1mm diameter particles



Thanks to:

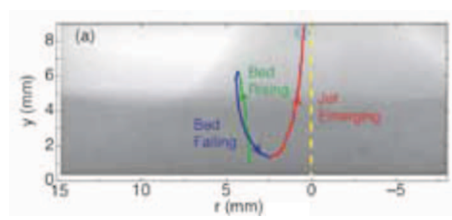
Matthias Möbius	first work on clustering freely falling streams
John Royer	jets from impact, freely falling streams
Scott Waitukaitis	stream simulations, streams as probes of granular self-charging
Helge Grütgen	stream simulations
Marc Miskin	break-up of suspensions

Sid Nagel, Wendy Zhang

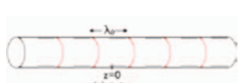
http://jfi.uchicago.edu/~jaeger/group/

Tracking the Bed Response

Bed particle flow:



Interaction of interstitial gas with grains ➔ granular bed behaves like incompressible liquid



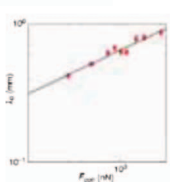
- Initial Situation: infinitely long stream with constant velocity gradient
- Final Situation: Droplets of length λ_0 with 0 COM kinetic energy
- Symmetry says COM velocity of each droplet constant
- Just looking at one droplet, energy accounting says

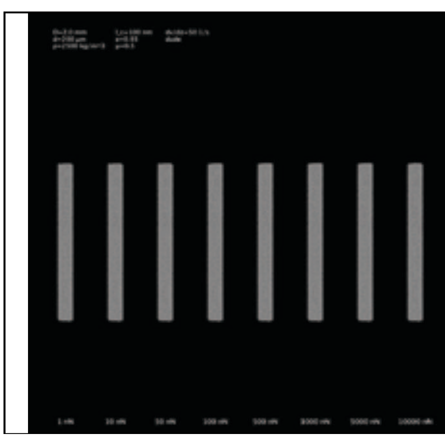
$$K_i + W - U_i = K_f + \Delta E_{int}$$

K = kinetic energy
W = work done on given droplet by neighboring drops
U_i = energy stored in bonds initially (>0)
ΔE_{int} = change in internal energy during drop formation

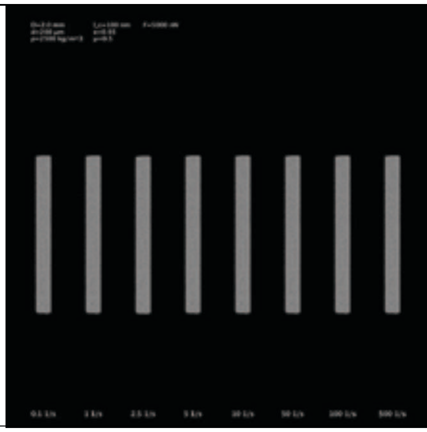
$K_f = 0$, $K_i = \lambda_0^3$, all other terms ~ λ_0

➔ $F_{coh} \sim (\lambda_0)^{1/2}$





dv/dz=50s^{-1}
Force varied

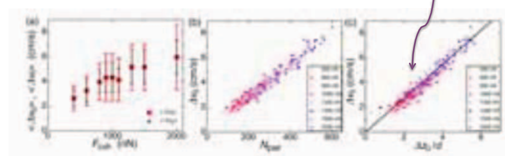


F = 5mN
Gradient varied

Droplet separation

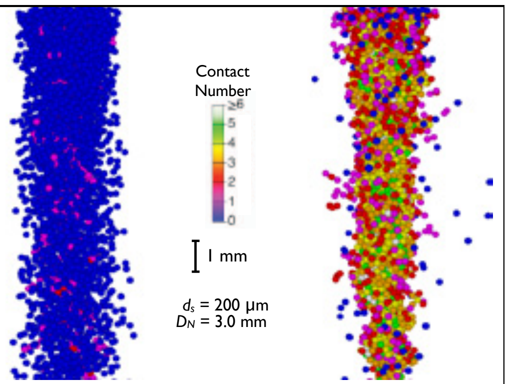
Near nozzle:

$$\Delta v_0 \simeq \frac{gd}{v_{z,0}}\left(\frac{\Delta z_0}{d}\right)$$



- Initial and final separation velocity same at a given force
- Initial separation velocity just scales with initial separation of droplets; scaling factor is just velocity gradient at nozzle

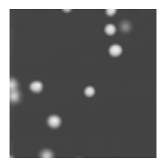
---



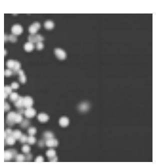
F_{coh} = 100 nN
e = *0.80*

F_{coh} = 1000 nN
e = *0.40*

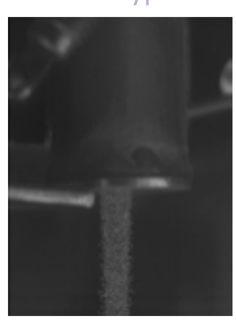
Collide and Capture



Parabolic Orbits

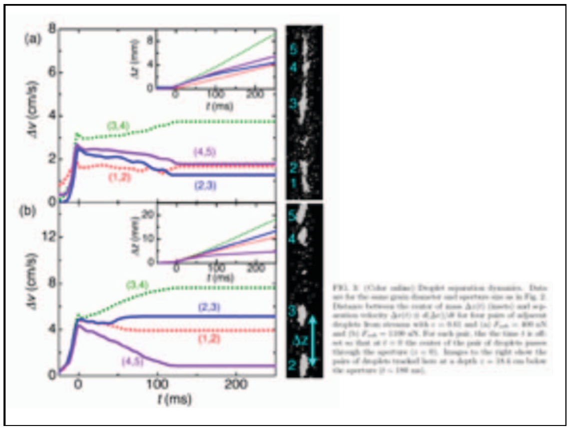


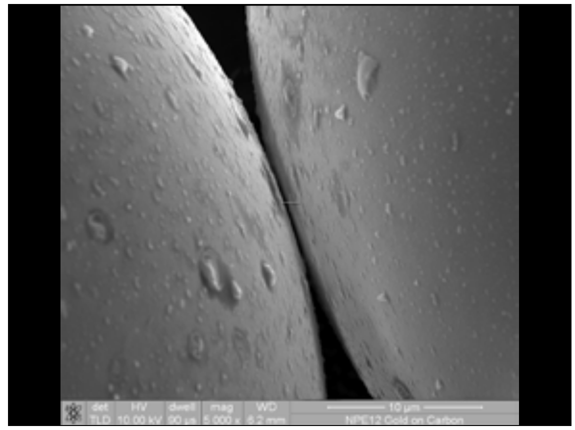
Particle Type & Interactions Important

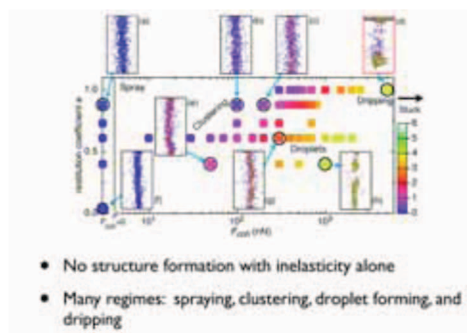


100μm copper

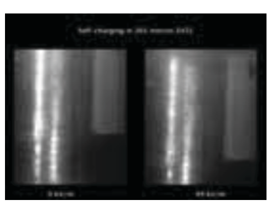
100μm copper + 10nm oil

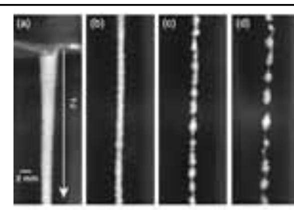

(a)
(b)
Δv (cm/s)
Δz (mm)
t (ms)
(3,4)
(4,5)
(1,2)
(2,3)
Δz



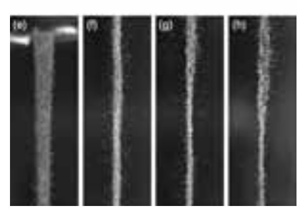


(1) The grains are Zirconium Dioxide, 265 +/- 15 microns(2) Fall from an array of three 2.5 mm diameter nozzles(3) Enter the plate region between the second and third horizontal white stripes (first white stripe is a reflection)(4) Mean of charge distribution is zero (within error) and sigma is ~1.5 million electrons.(5) Pressure ~4 mT(6) Fluidized for 1/2 hour beforehand(7) All from single batch

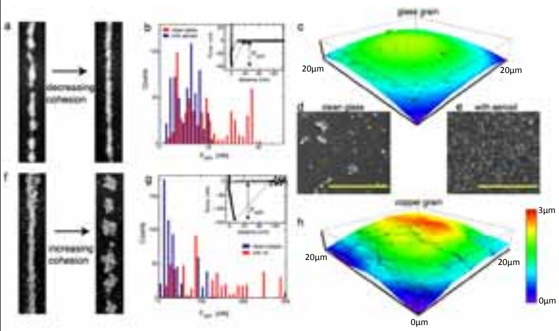


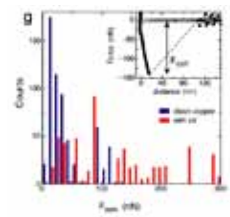


107µm glass



130µm copper





$$\frac{1}{2}m(\delta v_c)^2 = W_{coh}$$

$$W_{coh} = \int F_{coh}(x)dx$$

$W_{coh} \sim 10^{-15}$ J

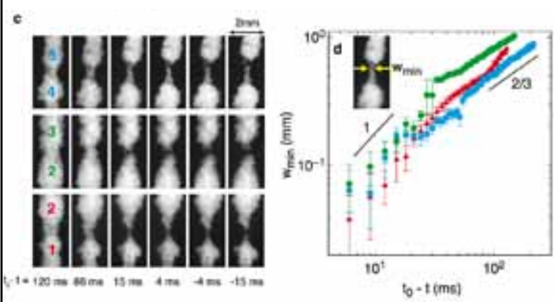
$v_c \sim 0.5$mm/s

Near nozzle, fluctuations ~10mm/s .....quench T by factor 100 in 1/4s

Effective surface tension: $\gamma \sim \varepsilon/\sigma^2 \sim W_{coh}/d^2 \sim$ 0.1µN/m

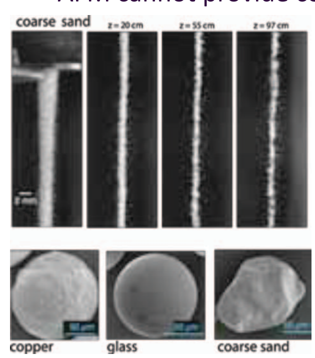
H_2O: 70mN/m

Time to break-up scaling



Inviscid regime: $w_{min} = 1.4(\gamma/\rho)^{1/3}(t_0 - t)^{2/3}$

AFM cannot provide complete picture!



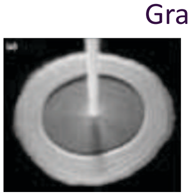
Coarse sand: F_{coh} nearly identical to glass spheres, but clustering very different

AFM: only simple, head on collisions, measure max. force

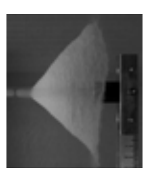
Need **total energy loss** from cohesive & dissipative forces (including sliding and rolling motion)

Simplify: “sticky” collisions where head-on collisions dominate

Granular Fluids


Boudet et al. 2007


Lohse et al. 2004


Cheng et al. 2007

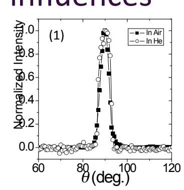
Remarkable similarities to conventional liquids.

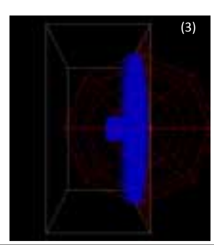
- How does liquid-like behavior emerge for an aggregate of grains?
- What features come simply from large number of rapid collisions?
- When are attractive interactions between grains important?

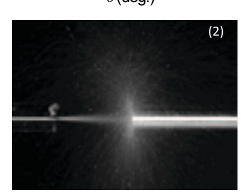
other possible influences

Role of surrounding gas?

(1) air vs, He;
(2) small particles & small # density
(4) simulations, w/o gas (T. Pöschel).



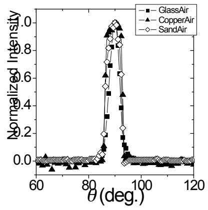


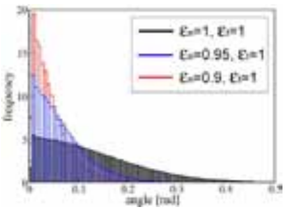


more possibilities…

Inelastic nature of grain-grain collisions

(1) copper particles;
(2) rough sand;
(3) simulations (T. Pöschel).

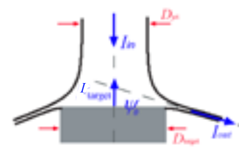




Inelastic collisions and air effects → detailed shape of emerging, post-collision sheet

Sufficiently large # or rapid collisions between particles → basic properties of sheet

Momentum Conservation → Cone Angle



$$I_{in} - I_{out}\cos\psi_0 = I_{\text{target}}$$

When $D_{target}/D_{Jet}<1$,

$$I_{\text{target}} = (A\pi/4)D^2_{\text{target}}\rho U_0^2$$

A: average glancing collision angle for a particle (A ~ 1)

When $D_{target}/D_{Jet}>>1$, entire momentum of jet reflected.

$$I_{in} = (\pi/4)\rho D^2_{Jet}U_0^2$$

$$I_{out} = CI_{in}$$

C: average coefficient of restitution, depending on coefficient of restitution for single collision and fraction of particles hitting on the target.

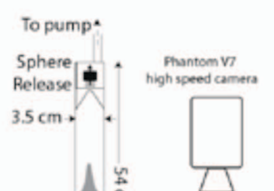
$$C = [1-\varepsilon(D_{\text{target}}/D_{Jet})^2]$$

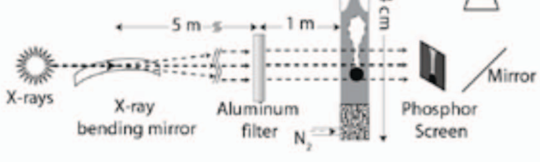
ε: coefficient of restitution for single collision

$$\psi_0 = \begin{cases} \cos^{-1}\left(1-(A-\varepsilon)\left(\frac{D_{\text{target}}}{D_{\text{jet}}}\right)^2\right); & \frac{D_{\text{target}}}{D_{\text{jet}}} << 1 \\ 90^\circ & \frac{D_{\text{target}}}{D_{\text{jet}}} >> 1 \end{cases}$$

Imaging the bed interior at Argonne's APS

John Royer *et al.*, Nature Physics ('05); PRL ('07); PRE ('08)





Permeability Determines Bed Response

$$k = \frac{d^2(1-\phi)^3}{180\phi^2} \qquad D = \frac{k}{\mu}\frac{P_0}{1-\phi}$$

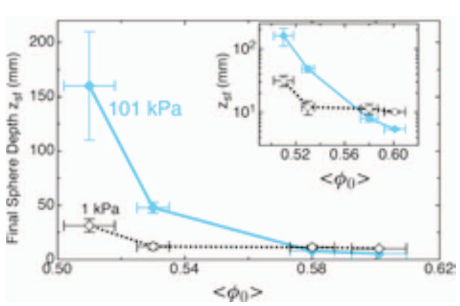
Fine-grained, loosely-packed bed

d ~ 50µm ϕ ~ 0.50…0.57
D ~ 400 … 850 cm²/s @ 101kPa
$\tau_D \sim L^2/D$ ~ 90…180ms = time to diffuse through bed depth

time for bed to rise = τ_{rise} ~ 30ms < τ_D

On short time scales → interstitial gas can become trapped; whole system behaves less compressible

Impact Depth vs. Bed Density (for fixed v_o)

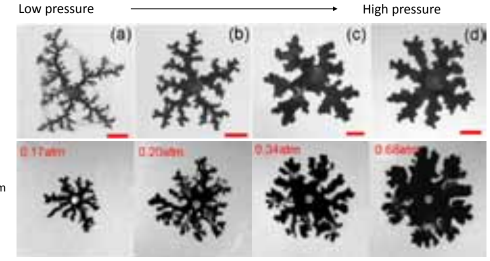


Patterns at Different Pressures

Low pressure → High pressure

d=50µm

d=100µm

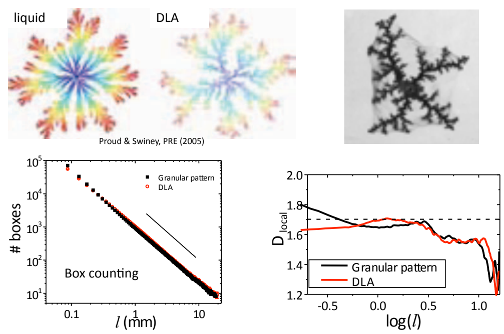


Different from ordinary liquids:

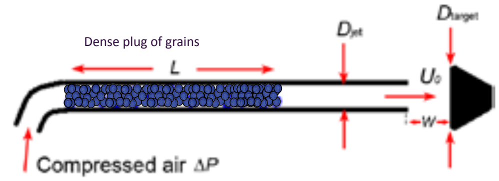
Shape of fingers & pressure dependence

Ristroph, Thrasher, Mineev-Weinstein, and Swinney, PRE '06

Near Yielding: Fractal Dimension as Expected for Zero Surface Tension Limit



Granular Analog to Water Bell



Vary:

(1) Granular material: Glass beads, Copper beads.

(2) Bead diameter, d=50μm to 2mm.

(3) Impact velocity: U_0=1.3m/s to 16m/s.

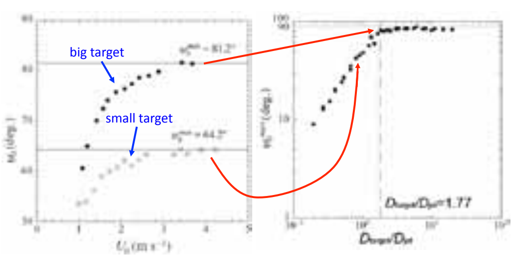
(4) Target size: D_{target}=0.13cm to 2.8cm.

Fix D_{jet}=0.75cm, L=40cm and w=2.54cm.

Cone angle depends on Weber number: $We = \rho U_0 D_{jet}/\gamma$

ρ: density of water; U_0: velocity of jet;
γ: surface-tension of water

Data for water C. Clanet, J. Fluid Mech. **430**, 111 (2001)



Granular Liquids: "Zero" Surface Tension?

- *Granular Hele-Shaw system:*
  - patterns and cusps as predicted for vanishing surface tension
- *Granular jets impacting targets:*
  - quantitative agreement with high-We results for water
  - with increasing particle # density, transition from particulate to liquid-like behavior (thin sheets, "water bells")
  - "liquid" produced by brief interval of rapid collisions right in front of target; sheet is "finger print" of this interval, but no longer a liquid (density too low)

Viscous Fingering at Liquid Interfaces

O. Praud and H. Swinney, PRE **72**, 011406 (2005).

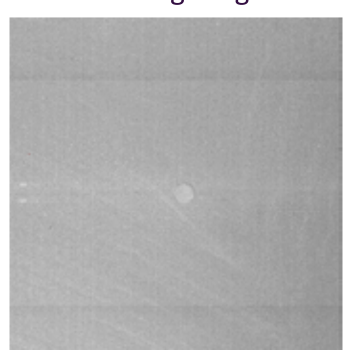
For the zero-surface-tension limit, theory predicts

• Fractal patterns & DLA-like growth • Finite time singularities

In ordinary liquids: zero tension limit approximated by increasing the flow rate

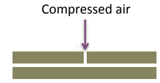
What happens for dry granular material forced by gas (air)?

Granular Fingering in a Hele-Shaw Cell



Xiang Cheng *et al.*, Nature Physics ('08)

Compressed air

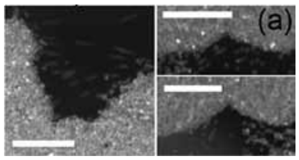


Low pressure
(ΔP=0.2atm)

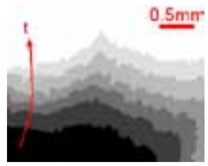
Grain diameter: d= 100µm; gap size: b = 2.5-14d Video frame rate 250 frame/s

Singular Dynamics for Vanishing Surface Tension: Cusps in Granular Fingering

Near threshold pressure for yielding:

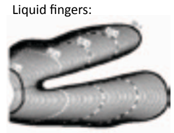


Scale bars: 0.5mm



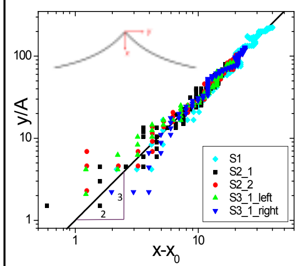
Cusps different from any finger structure found in normal liquids, where surface tension rounds off sharp features.

Liquid fingers:



Ristroph et al., PRE 74, 015201(R)

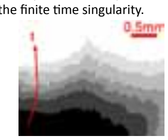
Cusp Shape Scaling



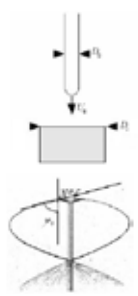
Shape predicted for Laplacian growth in limit of zero surface tension:

$$y = A(x - x_0)^{3/2}$$

Consistent with profile of finger tips as they become sharpest, i.e., near the finite time singularity.



Water Bells

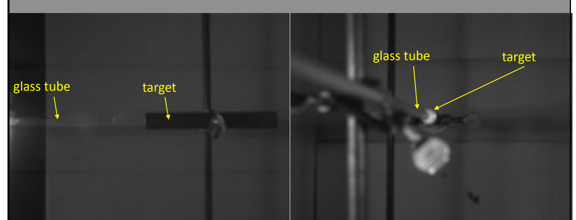


Water bell

Savart, 1833.

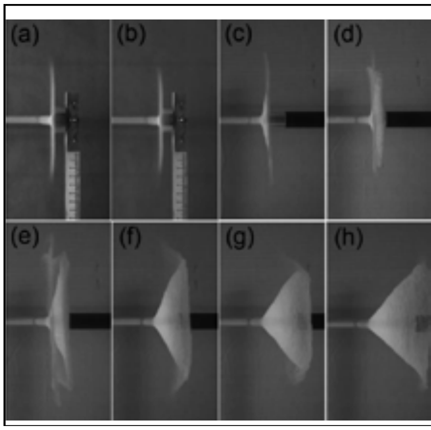
water jet from faucet hitting a cylindrical target

How about a fluid with vanishing surface tension?



Side view

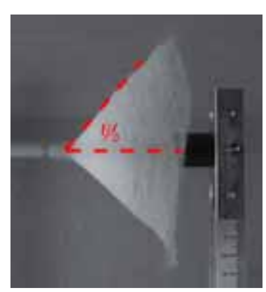
View along stream



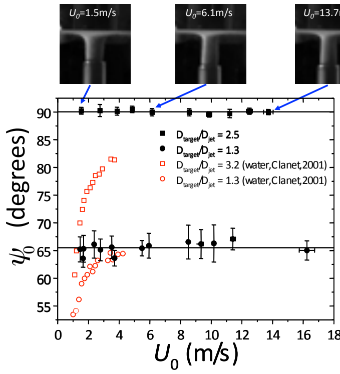
Xiang Cheng *et al.*, PRL ('08)

From (a) to (h): D_{target}/D_{jet} = 4.0, 3.7, 2.0, 1.6, 1.3, 1.2, 1.0 and 0.85.

Cone Angle



Grains: Cone Angle Independent of Velocity



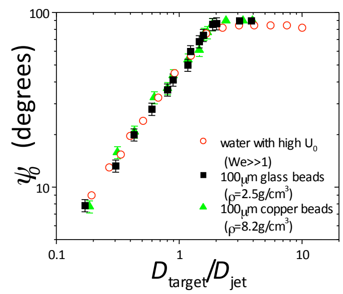
Shoot jet upward to avoid asymmetry due to gravity.

Black: grains

Red: water

$We = \rho U_0 D_{jet}/\gamma$

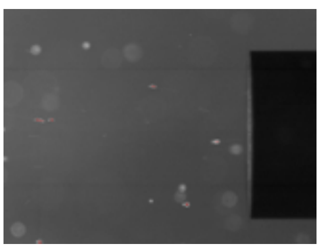
Large *We* limit: grains behave like H_2O



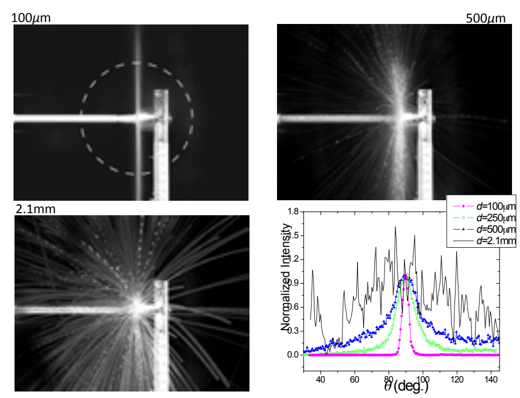
For non-cohesive granular material, $\gamma \rightarrow 0$ ➔ $We = \rho U_0 D_{jet}/\gamma >> 1$, independent of U_0

From Particles to Liquids

How does liquid-like behavior emerge from the particle nature of a granular material?



500μm diam. glass beads hitting aluminum target



What does it take to create a liquid?

- Sufficiently many / strong interactions
- Attractions? ←→ Confinement?

cold stream

↓

"hot" collision volume in front of target (~ 1 particle deep; >30 particles across)

↓

cold gas