Dynamics of disordered systems

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Plan of Lectures

1. Introduction
2. Coarsening processes
3. Formalism
4. Dynamics of disordered spin models

First lecture

Plan of the 1st Lecture

Plan

1. Equilibrium *vs.* out of equilibrium classical systems.
2. How can a classical system stay far from equilibrium ?

   From single-particle to many-body.

   Diffusion

   Phase-separation & domain growth

   Glasses

   Driven systems

   Active matter
3. Details on the non-equilibrium behaviour

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Introduction

General setting

- Closed & open systems
- Equilibrium & out of equilibrium
  - Long time scales
  - Forces & energy injection
- Individual & collective effects

Introduction

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- Equilibrium & **out of equilibrium**
  - Long time scales
  - Forces & energy injection
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Isolated systems

Dynamics of a classical isolated system

Foundations of statistical physics.

Question : does the dynamics of a particular system reach a flat distribution over the constant energy surface in phase space ?

Ergodic theory, $\in$ mathematical physics at present.

Dynamics of a (quantum) isolated system :

a problem of current interest, recently boosted by cold atom experiments.

Question : after a quench, i.e. a rapid variation of a parameter in the system, are at least some observables described by thermal ones ?

When, how, which ? some comments in 4th lecture

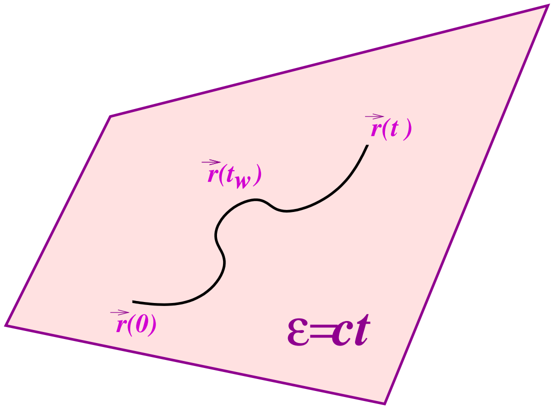
Introduction

General setting

- Closed & **open** systems
- **Equilibrium** & out of equilibrium
  - Long time scales
  - Forces & energy injection
- Individual & **collective effects**

Statistical mechanics

Equilibrium ensembles



- Isolated system $\Rightarrow$ conserved energy $\mathcal{E}$
- Ergodic hypothesis

$$S = k_B \ln \mathcal{N} \qquad\qquad \beta \equiv \frac{1}{k_B T} = \left.\frac{\partial S}{\partial \mathcal{E}}\right|_{\mathcal{E}}$$

Microcanonical definition

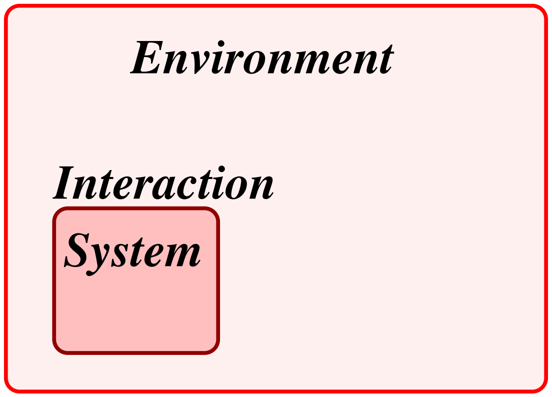
$$\mathcal{E} = \mathcal{E}_{syst} + \mathcal{E}_{env} + \mathcal{E}_{int}$$

Neglect $\mathcal{E}_{int}$ (short-range int.)

$$\mathcal{E}_{syst} \ll \mathcal{E}_{env}$$

$$p_{eq}(\mathcal{E}_{syst}) = g(\mathcal{E}_{syst}) e^{-\beta \mathcal{E}_{syst}} / Z$$

Canonical ensemble



Environment

Interaction

System

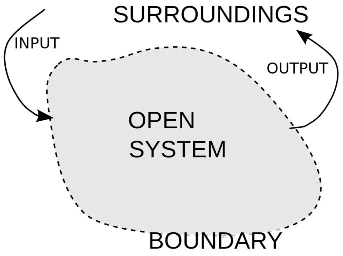
Open systems

Aim

Our interest is to describe the statics and dynamics of a classical (or quantum) system coupled to a classical (or quantum) environment.

The Hamiltonian of the ensemble is

$$\boxed{H = H_{syst} + H_{env} + H_{int}}$$



The dynamics of all variables are given by Newton (or Heisenberg) rules, depending on the variables being classical (or quantum).

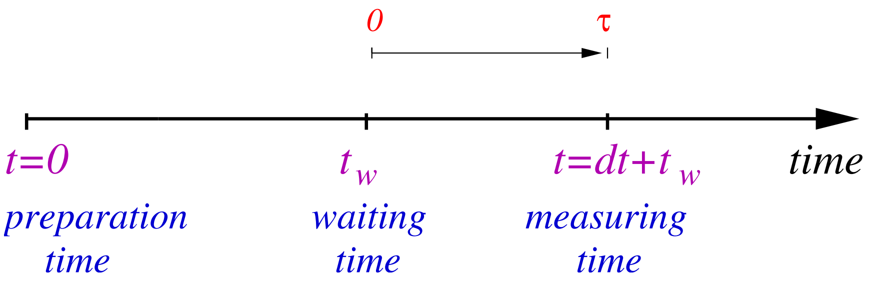
The total energy is conserved, $\mathcal{E} =$ ct but each contribution is not, in particular, $\mathcal{E}_{syst} \neq$ ct, and we'll take $e_0 \ll \mathcal{E}_{syst} \ll \mathcal{E}_{env}$.

In and out of equilibrium

Take a **mechanical point of view** and call $\{\vec{\zeta}_i\}(t)$ the variables

e.g. the particles' coordinates $\{\vec{r}_i(t)\}$ and momenta $\{\vec{p}_i(t)\}$

Choose an initial condition $\{\vec{\zeta}_i\}(0)$ and let the system evolve.

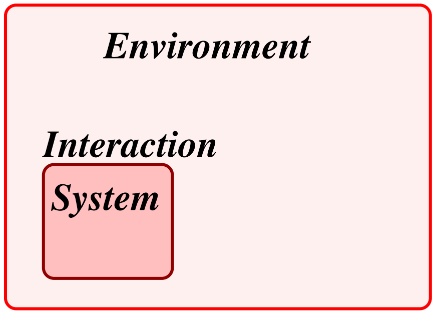


- For $t_w > t_{eq}$: $\{\vec{\zeta}_i\}(t)$ reach the equilibrium pdf and thermodynamics and statistical mechanics apply (e.g., **temperature** is a well-defined concept).
- For $t_w < t_{eq}$: the system remains out of equilibrium and thermodynamics and (Boltzmann) statistical mechanics **do not** apply.

Dynamics in equilibrium

Conditions

Take an open system coupled to an environment



Necessary :

— The bath should be in equilibrium

same origin of noise and friction.

— Deterministic force

conservative forces only, $\vec{F} = -\vec{\nabla} V$.

— Either the initial condition is taken from the equilibrium pdf, or the latter should be reached after an equilibration time t_{eq} :

$$\boxed{P_{eq}(\vec{v}, \vec{r}) \propto e^{-\beta(\frac{mv^2}{2} + V(\vec{r}))}}$$

Dynamics in equilibrium

Two properties

- One-time quantities reach their equilibrium values:

$$\langle\, A(\{\vec{r}\}_\xi)(t)\,\rangle \;\to\; \langle\, A(\{\vec{r}\})\,\rangle_{eq}$$

[the first average is over realizations of the thermal noise (and initial conditions) and the second average is taken with the equilibrium (Boltzmann) distribution]

- All time-dependent correlations are stationary

$$\langle\, A_1(\{\vec{r}\}_\xi)(t_1)A_2(\{\vec{r}\}_\xi)(t_2)\cdots A_n(\{\vec{r}\}_\xi)(t_n)\,\rangle = \langle\, A_1(\{\vec{r}\}_\xi)(t_1+\Delta)A_2(\{\vec{r}\}_\xi)(t_2+\Delta)\cdots A_n(\{\vec{r}\}_\xi)(t_n+\Delta)\,\rangle$$

for any n and Δ. In particular, $C(t,t_w) = C(t-t_w)$.

Proof: 3rd lecture

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Too long equilibration time

Environment in but system away from equilibrium

- The equilibration time goes beyond the experimentally accessible times.

$$\boxed{t_{eq} \gg t_{exp}}$$

Microscopic system with no confining potential, $\boxed{t_{eq_x} = \infty}$

e.g., **Diffusion processes**.

Macroscopic systems in which the equilibration time grows with the system size, $\boxed{\lim_{N \gg 1} t_{eq}(N) \gg t}$

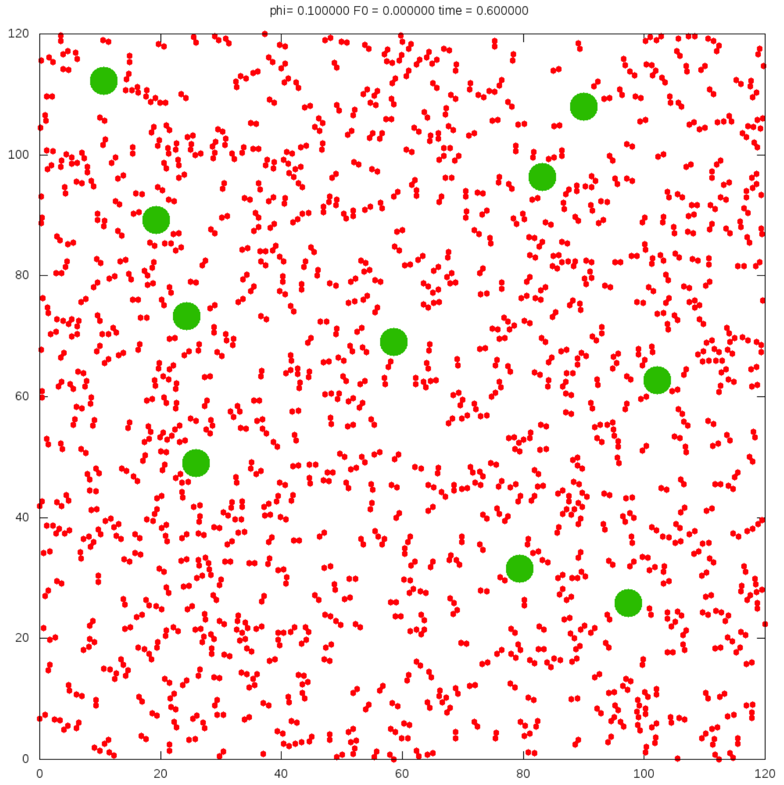
e.g., **Critical dynamics, coarsening, glassy physics**.

- **Driven systems** $\boxed{\vec{F} \neq -\vec{\nabla} V(\vec{r})}$

e.g., **Sheared liquids, vibrated powders, active matter**.

Microscopic system

Brownian motion : diffusion



First example of dynamics of an *open system*

The system : the Brownian particle

The bath : the liquid

Interaction : collisional or potential

Canonical setting

A few Brownian particles or tracers ● embedded in, say, a molecular liquid.

Late XIX, early XX (Brown, Einstein, Langevin)

Langevin approach

Stochastic Markov dynamics

From Newton's equation $\vec{F} = m\vec{a} = m\dot{\vec{v}}$ and $\vec{v} = \dot{\vec{x}}$

$$\boxed{m\dot{v}_a = -\gamma_0 v_a + \xi_a}$$

with $a = 1, \ldots, d$ (the dimension of space), m the particle mass,
γ_0 the friction coefficient,
and $\vec{\xi}$ the time-dependent thermal noise with Gaussian statistics,
zero average $\langle \xi_a(t) \rangle = 0$ at all times t,
and delta-correlations $\langle \xi_a(t) \xi_b(t') \rangle = 2\,\gamma_0 k_B T\, \delta_{ab}\, \delta(t - t')$.

Dissipation for $\gamma_0 > 0$ the averaged energy is not conserved,

$$2\langle \mathcal{E}_{syst}(t) \rangle = m\langle v^2(t) \rangle \neq 0.$$

Brownian motion

Normal diffusion

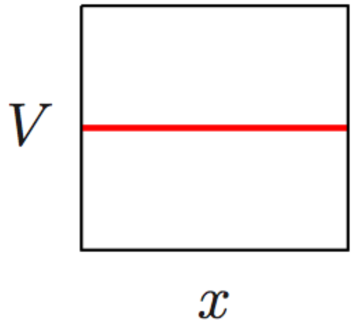
For simplicity, take a one dimensional system, $d = 1$.

The relation between friction coefficient γ_0 and amplitude of the noise correlation $2\gamma_0 k_B T$ ensures equipartition for the velocity variable

$$\boxed{m\langle v^2(t)\rangle \to k_B T} \quad \text{for} \quad t \gg t_r^v \equiv \frac{m}{\gamma_0}$$

Langevin 1908

But the position variable x diffuses since $e^{-\beta V}$ is not normalizable.



$$\boxed{\langle x^2(t)\rangle \to 2D\,t} \quad (t \gg t_r^v = m/\gamma_0)$$

$$\boxed{D = k_B T/\gamma_0}$$ diffusion constant.

The particle is out of equilibrium !

Brownian motion

Normal diffusion

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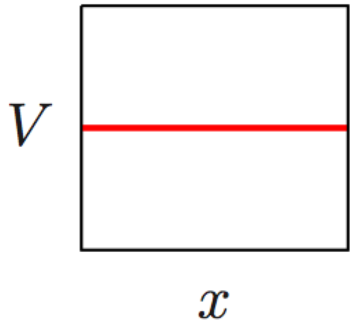

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Coexistence of equilibrium (v) and out of equilibrium (x) variables

Macroscopic systems

Discussion of several macroscopic systems with slow dynamics due to

$$\boxed{\lim_{N\gg 1} t_{eq}(N) \gg t}$$

Examples :

Ordering processes

Domain growth, phase separation

Systems with quenched disorder

Random ferromagnets, spin-glasses

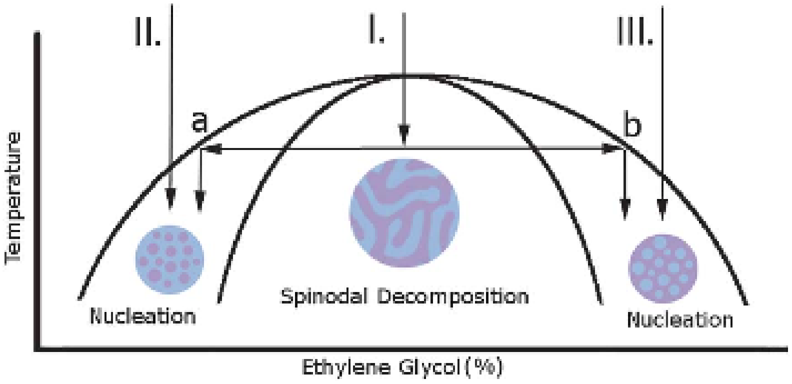
Systems with frustrated interactions

Spin ices

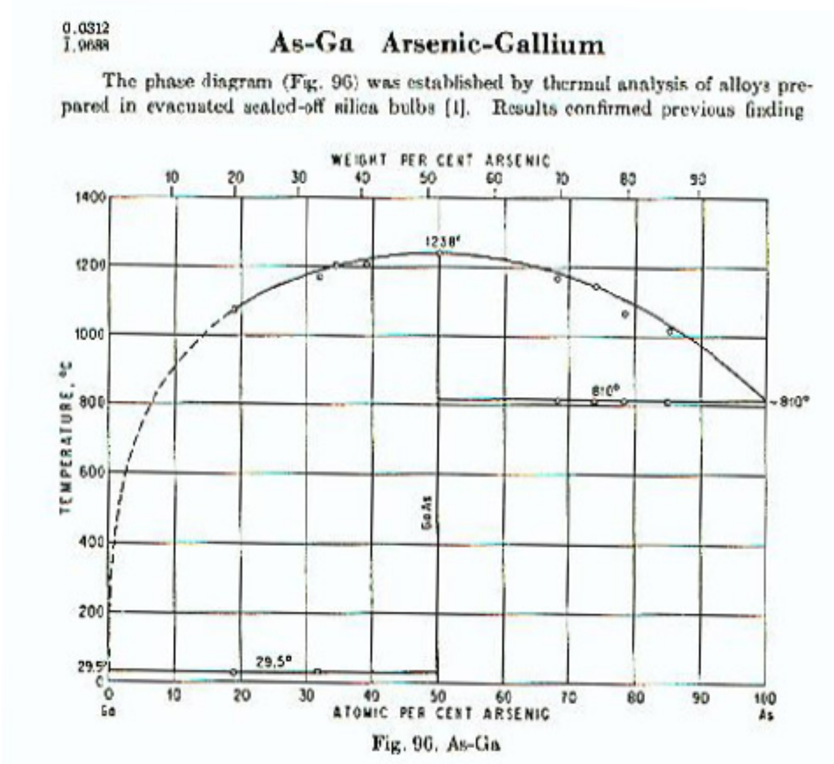
Phase separation

Demixing transitions

Two species ● and ●, repulsive interactions between them.



Sketch



0.0312
1.9688

As-Ga Arsenic-Gallium

The phase diagram (Fig. 96) was established by thermal analysis of alloys prepared in evacuated sealed-off silica bulbs [1]. Results confirmed previous finding

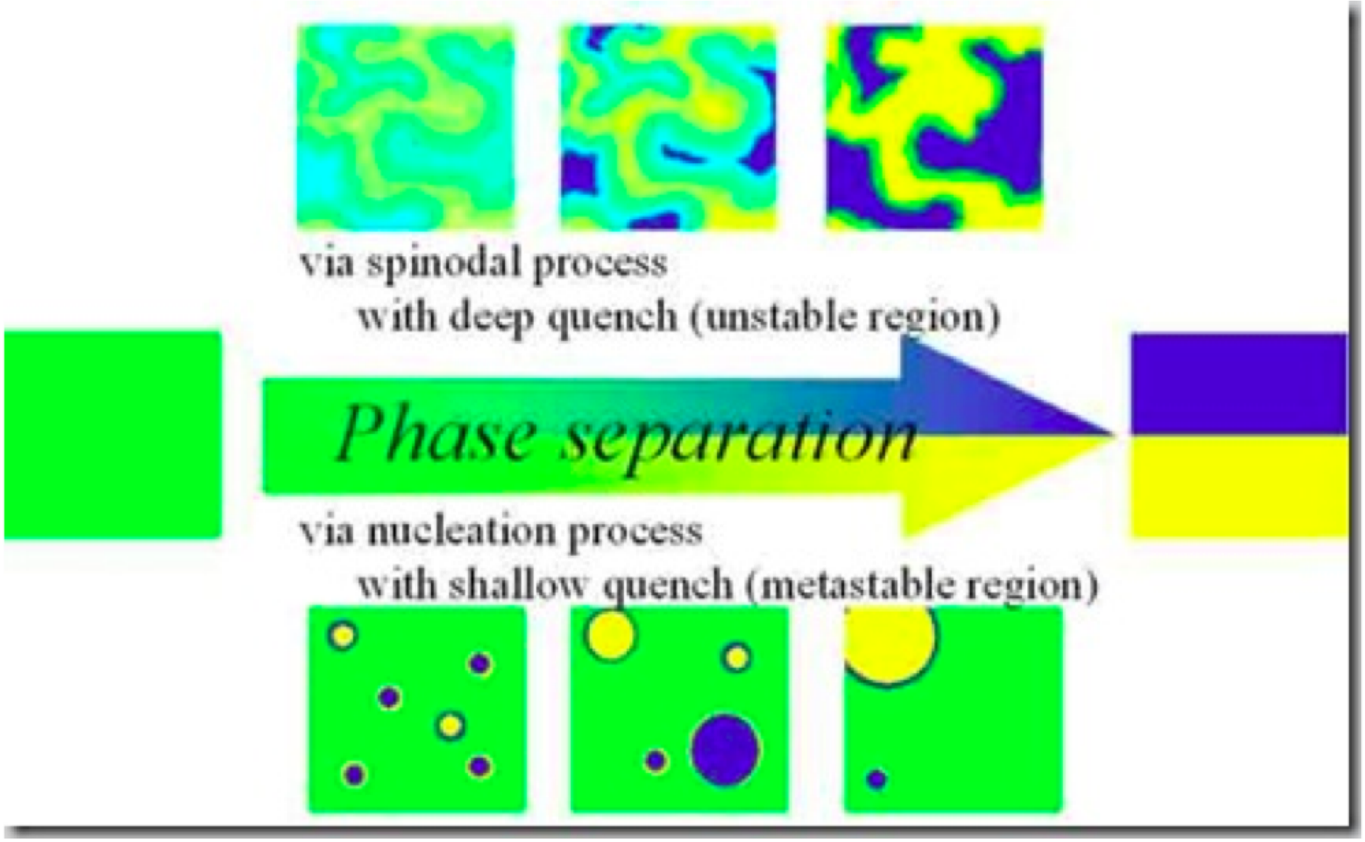
Fig. 96. As-Ga

Experimental phase diagram

Binary alloy, **Hansen & Anderko, 54**

Phase separation

Phase ordering kinetics



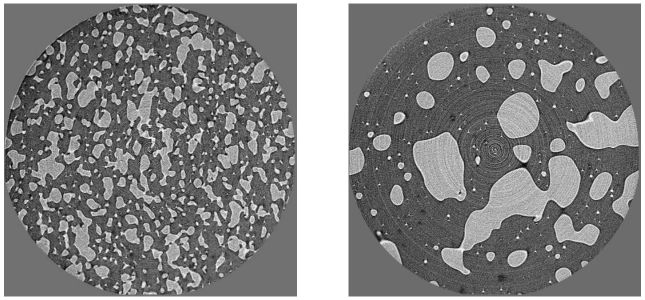
Phase ordering kinetics

Are these quench dynamics fast processes ? Can we simply forget what happens during the transient, t_{eq}, and focus on the subsequent *equilibrium* behaviour ?

No !

It turns out that this is a very slow regime. Its duration grows with the size of the system and it diverges in the thermodynamic limit $N \to \infty$.

We understand the mechanisms for relaxation : interface local curvature driven dynamics and matter diffusion.



The domains get rounder

The regions get darker and lighter

Quenched disorder

Quenched variables are frozen during time-scales over which other variables fluctuate.

Time scales $\boxed{t_{micro} \ll t \ll t_q}$

t_q could be the diffusion time-scale for magnetic impurities, the magnetic moments of which will fluctuate in a **magnetic system** or ;

the flipping time of impurities that create random fields acting on other magnetic variables.

Weak disorder (modifies the critical properties but not the phases) *vs.* **strong disorder** (modifies both).

E.g., **random ferromagnets** ($J_{ij} > 0$) *vs.* **spin-glasses** ($J_{ij} \overset{>}{<} 0$).

Spin-glasses

Magnetic impurities (spins) randomly placed in an inert host

Quenched random interactions

Interacting via the RKKY potential

$$V(r) \propto \frac{\sin 2\pi k_F r}{r^3}$$

very rapid oscillations (change in sign) and slow power law decay

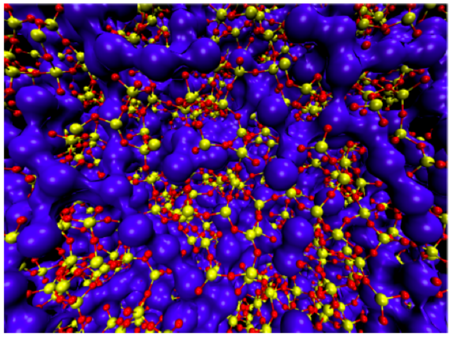
Sandard lore : there is a 2nd order static phase transition at T_s

separating a paramagnetic from a spin-glass phase.

No dynamic precursors above T_s.

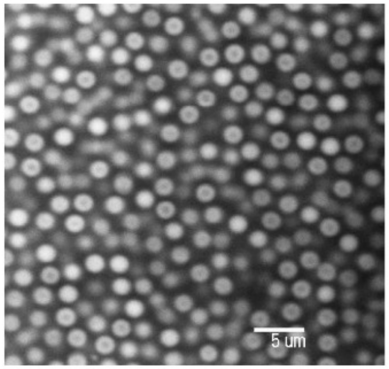
Glassy dynamics below T_s with aging, memory effects, etc.

What do glasses look like ?



Simulation

Molecular (Sodium Silicate)

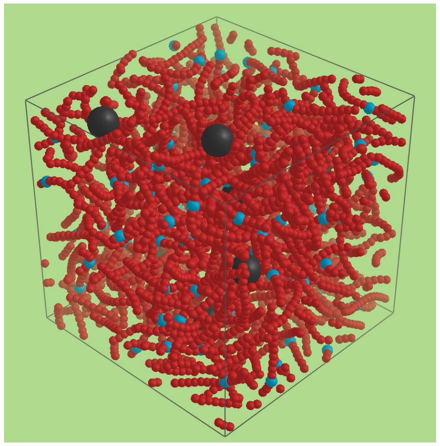


Confocal microscopy

Colloids (e.g. $d \sim 162$ nm in water)

Experiment

Granular matter



Simulation

Polymer melt

Structural glasses

Characteristics

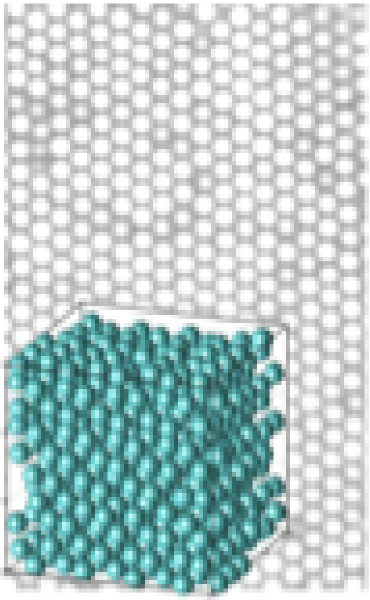
- Selected variables (molecules, colloidal particles, vortices or polymers in the pictures) are coupled to their surroundings (other kinds of molecules, water, etc.) that act as thermal baths in equilibrium.
- There is no quenched disorder.
- The interactions each variable feels are still in competition, e.g. Lenard-Jones potential, frustration.
- Each variable feels a different set of forces, time-dependent heterogeneity.

Sometimes one talks about **self-generated disorder**.

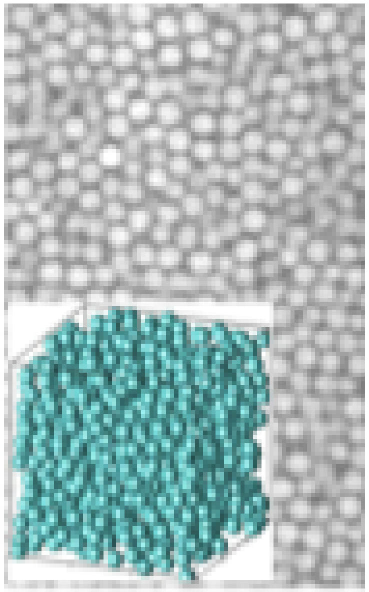
Structural Glasses

e.g., **colloidal ensembles**

Micrometric spheres immersed in a fluid



Crystal



Glass

In the glass : no obvious growth of order, slow dynamics with, however, scaling properties.

What drives the slowing down ?

We do not know

Long time-scales for relaxation

Systems with competing interactions remain out of equilibrium and it is not clear

- whether there are phase transitions,
- which is the nature of the putative ordered phases,
- which is the dynamic mechanism.

Examples are :

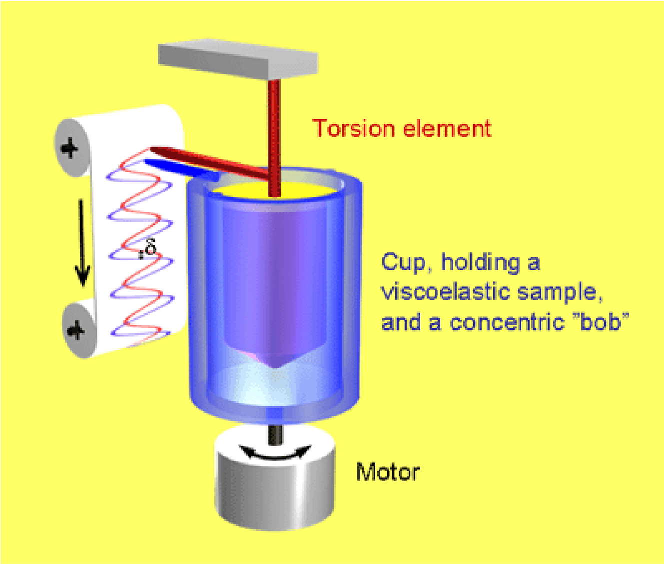
- systems with quenched disorder,
- systems with geometric frustration,
- glasses of all kinds.

Static and dynamic mean-field theory has been developed – both classically and quantum mechanically – and they yield new concepts and predictions.

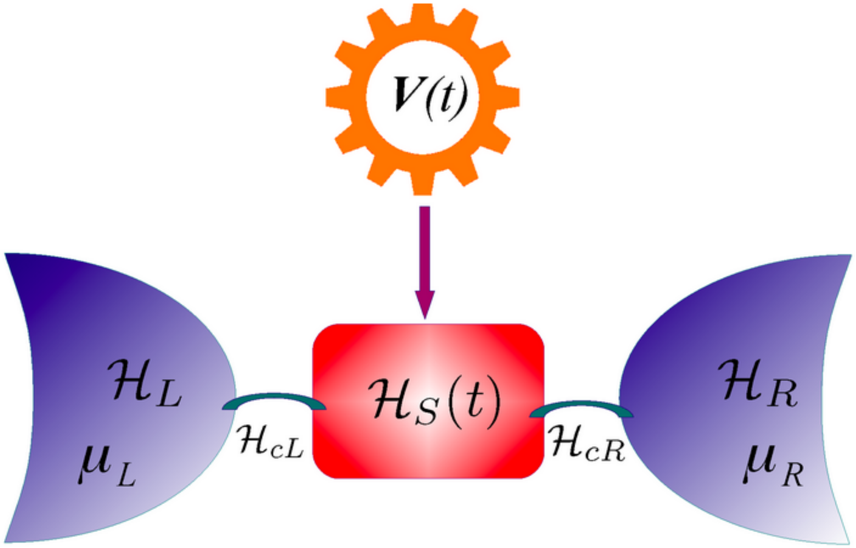
Extensions of the RG have been proposed and are currently being explored.

Energy injection

Traditional : from the borders (outside)



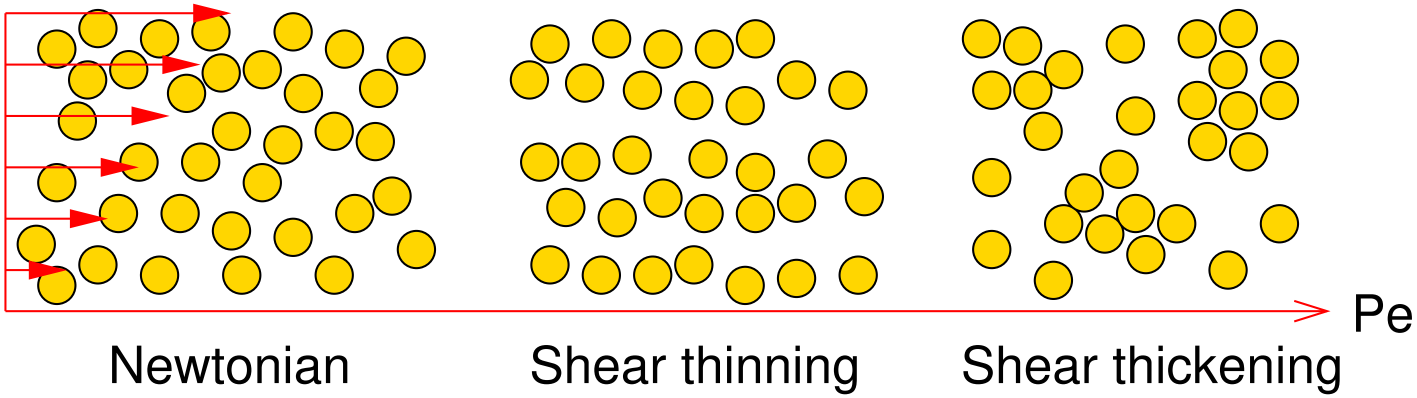
Rheology



Transport

Drive & transport

Rheology of complex fluids



Rheology of complex fluids

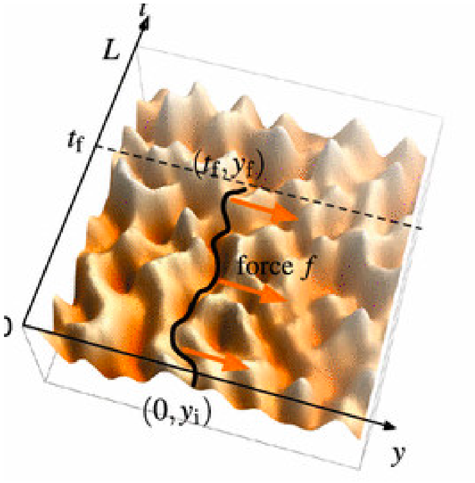
Shear thinning τ_{relax} decreases, *e.g.* paints

Shear thickening τ_{relax} increases, *e.g.* cornstarch & water mix

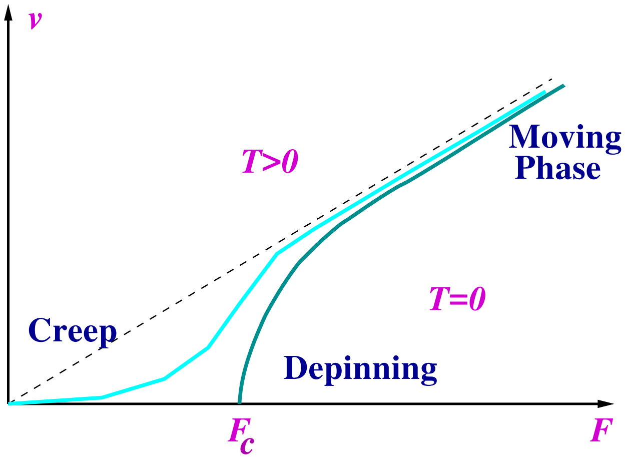
e.g. review **Brader 10**

Drive & transport

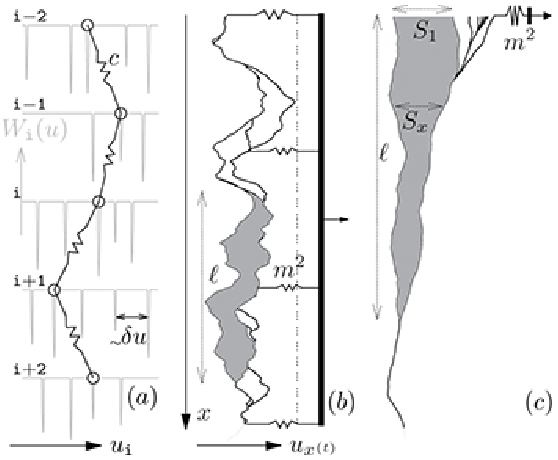
Driven interface over a disordered background



A line



Depinning & creep



avalanches

e.g. review **Giamarchi et al 05**, connections to earthquakes **Landes 16**

Active matter

Definition

Active matter is composed of large numbers of active "agents", each of which consumes energy in order to move or to exert mechanical forces.

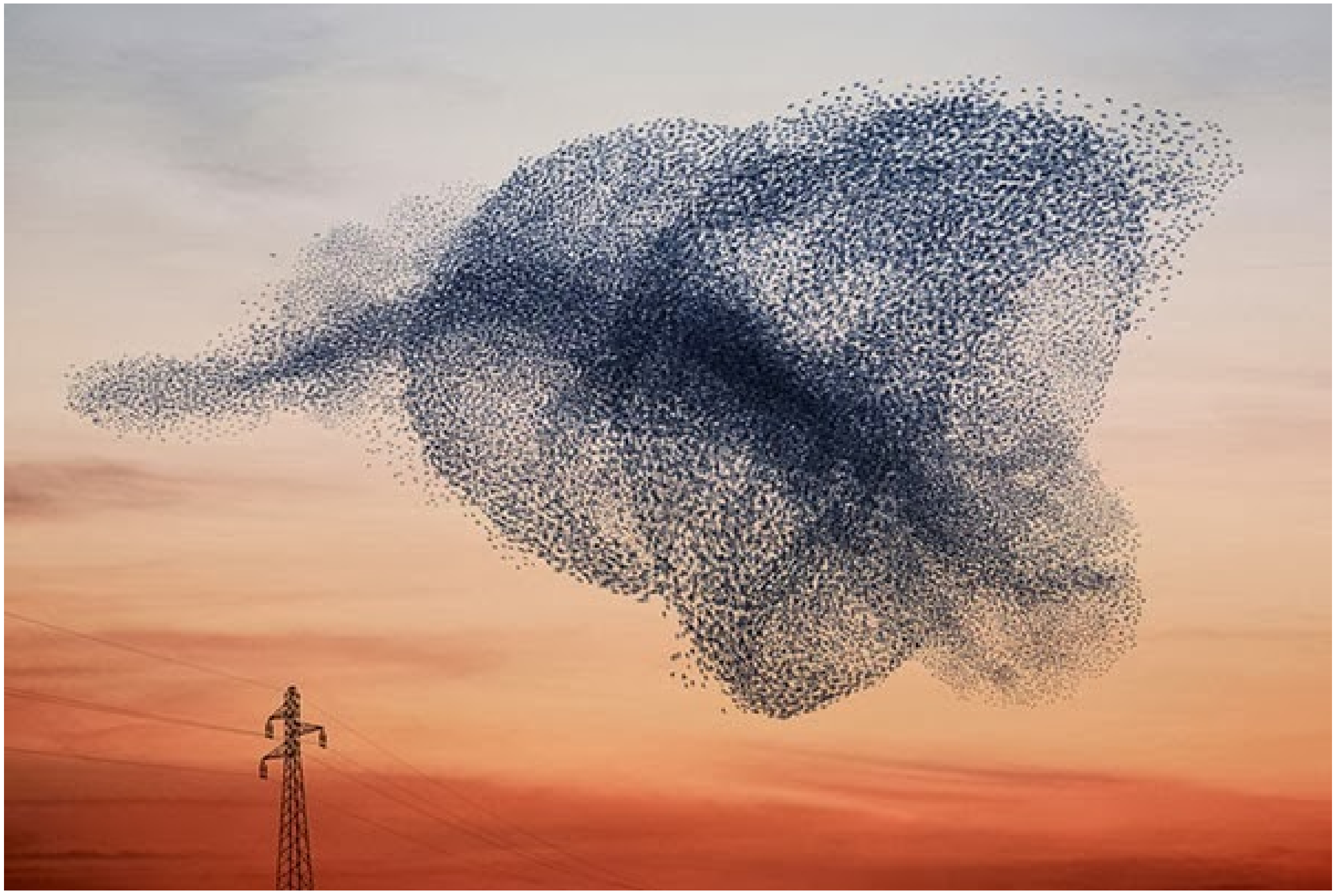
Due to the energy consumption, these systems are intrinsically out of thermal equilibrium.

Energy injection is done “uniformly” within the samples (and not from the borders).

Coupling to the environment (bath) allows for the dissipation of the injected energy.

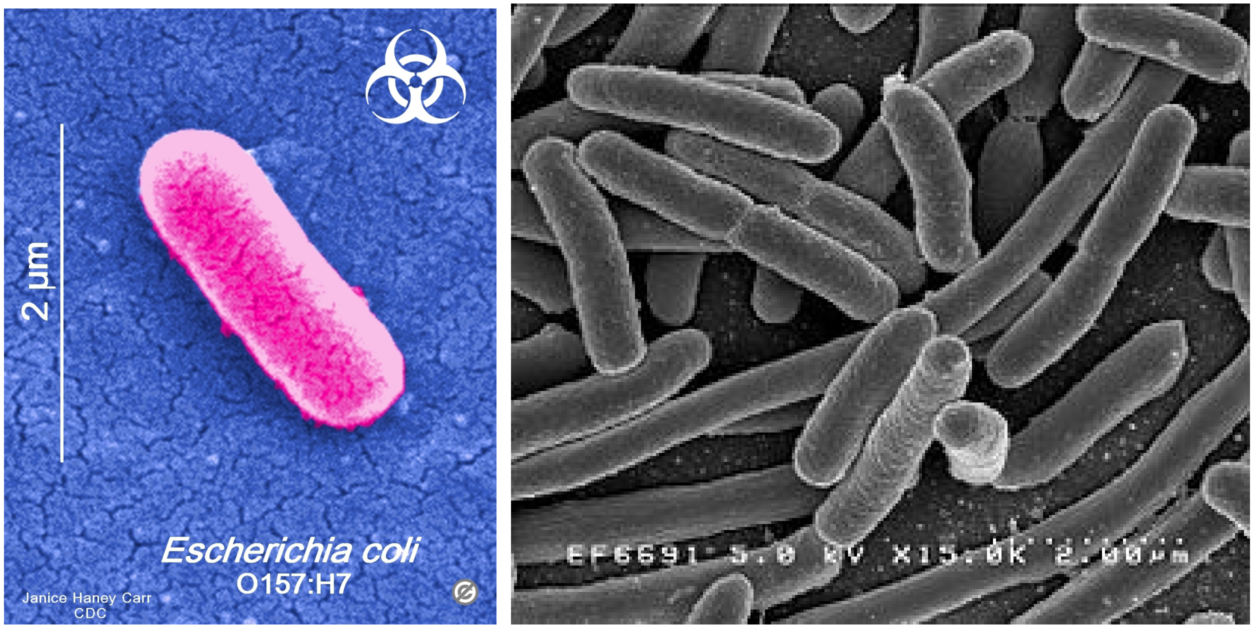
Natural systems

Birds flocking



Natural systems

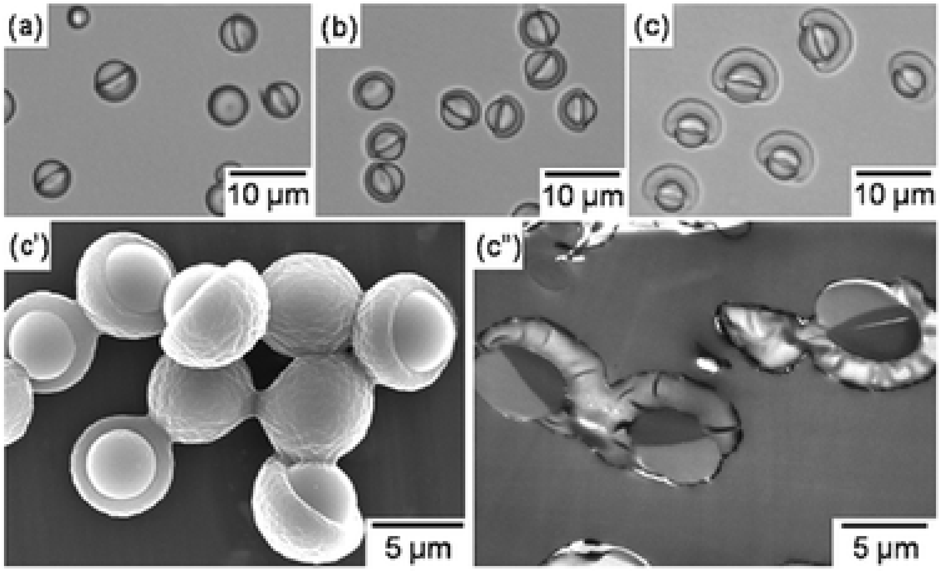
Bacteria



Escherichia coli - Pictures borrowed from the internet.

Artificial systems

Janus particles



Particles with two faces (Janus God)

e.g. **Bocquet group** ENS Lyon-Paris, **di Leonardo group** Roma

In these lectures:

Focus on relaxational cases (no weird forces)

Plan of the 1st Lecture

Plan

1. Equilibrium *vs.* out of equilibrium classical systems.
2. How can a classical system stay far from equilibrium ?
   From single-particle to many-body.
   Diffusion
   Phase-separation & domain growth
   Glasses
   Driven systems
   Active matter
3. **More details on the non-equilibrium behaviour**
   Dynamic classes

Dynamical classes

Distinction

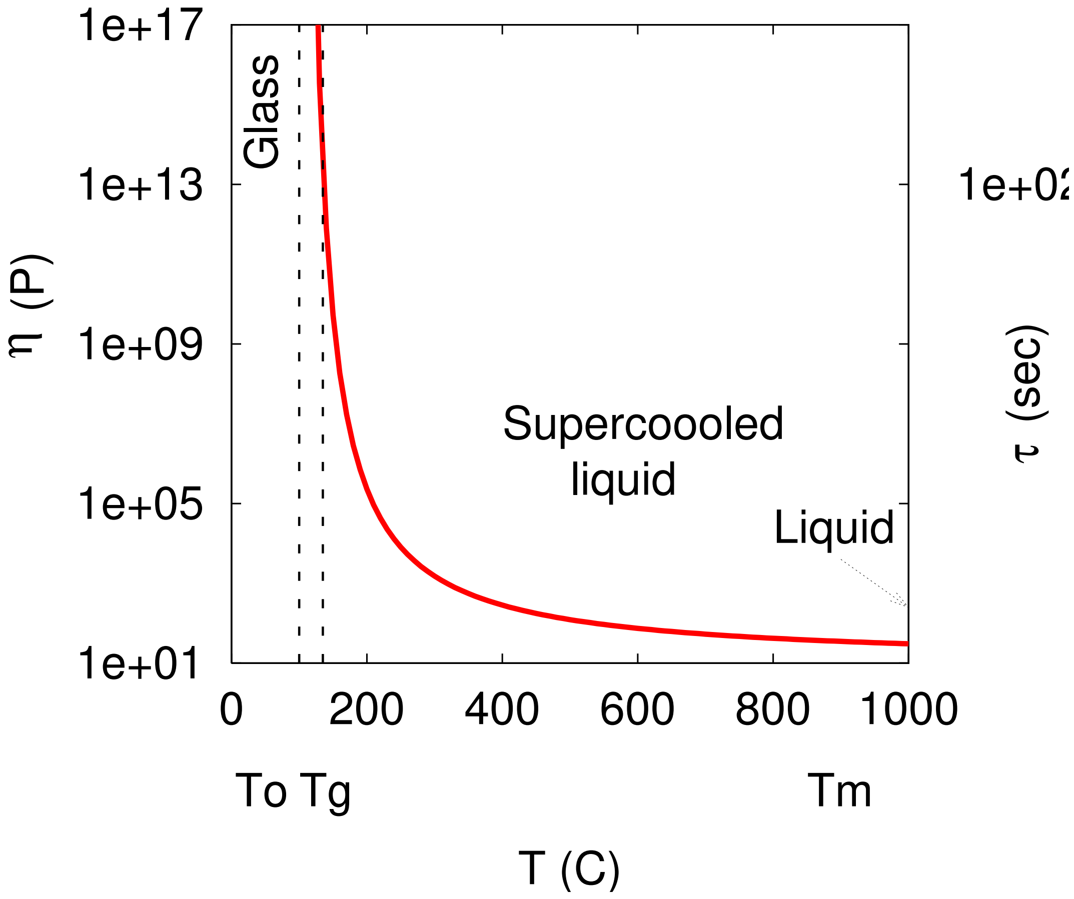
- Let us review the phenomenology of each kind of glass in more detail.
- T_g in structural glasses looks like a mixed 1st-2nd order transition.
- Spin-glass transition is seen as a conventional 2nd order transition.
- A family of models that capture this phenomenology.

Kirkpatrick, Thirumalai & Wolynes, late 80s

- Static and dynamic analytic methods to study them.
- Results & open questions.

Glassy dynamic arrest

Observable (viscosity) *vs* control parameter (temperature)



Characteristic temperatures

Fragile glasses

$T > T_m$ (equilibrium) liquid

At T_m, melting transition (avoided)

$T_g < T < T_m$ (metastable) supercooled liquid

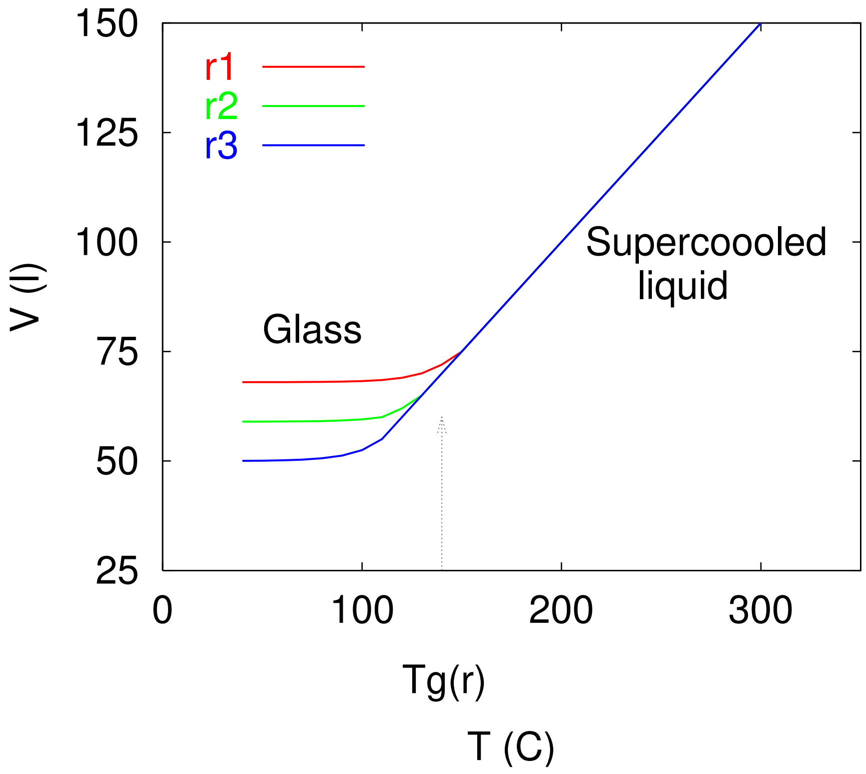
At T_g, $\tau_{relax} = 100$ sec Dynamic crossover

$T < T_g$ Glassy non-equilibrium dynamics

At T_0, $\tau_{relax} \to \infty$? Phase transition to ideal glass ?

Cooling rate effects

$r \equiv \frac{\Delta T}{\Delta t}$ $\quad$ $r1 > r2 > r3$



There is no single value of V at fixed $T < T_g(r)$

The system is out of equilibrium below $T_g(r)$.

Observables

Positional order

The (fluctuating) local particle number density

$$\rho(\vec{r}_0) = \sum_{i=1}^{N} \delta(\vec{r}_0 - \vec{r}_i)$$

with normalisation $\int d^d\vec{r}_0 \, \rho(\vec{r}_0) = N$.

The density-density correlation function $C(\vec{r} + \vec{r}_0, \vec{r}_0) = \langle \rho(\vec{r} + \vec{r}_0)\rho(\vec{r}_0) \rangle$ that, for homogeneous (independence of $\vec{r}_0$) and isotropic ($\vec{r} \mapsto |\vec{r}| = r$) cases, is simply $C(\vec{r} + \vec{r}_0, \vec{r}_0) = C(r)$.

The double sum in $C(\vec{r} + \vec{r}_0, \vec{r}_0) = \langle \sum_{ij} \delta(\vec{r} + \vec{r}_0 - \vec{r}_i)\delta(\vec{r}_0 - \vec{r}_j) \rangle$ has contributions from $i = j$ and $i \neq j$: $C_{\rm self} + C_{\rm diff}$

Observables

Positional order

The density-density correlation function

$$C(\vec{r}+\vec{r}_0,\vec{r}_0) = \langle \rho(\vec{r}+\vec{r}_0)\rho(\vec{r}_0)\rangle = \sum_{ij}\langle \delta(\vec{r}+\vec{r}_0-\vec{r}_i)\delta(\vec{r}_0-\vec{r}_i)\rangle$$

is linked to the structure factor

$$S(\vec{q}) \equiv N^{-1}\langle \tilde{\rho}(\vec{q})\tilde{\rho}(-\vec{q})\rangle = \frac{1}{N}\langle \sum_{i=1}^{N}\sum_{j=1}^{N} e^{-\mathrm{i}\vec{q}\cdot(\vec{r}_i-\vec{r}_j)}\rangle$$

by

$$N\, S(\vec{q}) \;=\; \int d^d r_1 \int d^d r_2\, C(\vec{r}_1,\vec{r}_2)\, e^{-\mathrm{i}\vec{q}\cdot(\vec{r}_1-\vec{r}_2)}$$

Observables

Positional order

In isotropic cases, i.e. liquid phases, the pair correlation function

$\frac{N}{V}\, g(r) =$ average number of particles at distance r

from a tagged particle at $\vec{r}_0$

is linked to the structure factor

$$S(\vec{q}) = \frac{1}{N} \langle \sum_{i=1}^{N} \sum_{j=1}^{N} e^{-\mathrm{i}\vec{q}\cdot(\vec{r}_i - \vec{r}_j)} \rangle$$

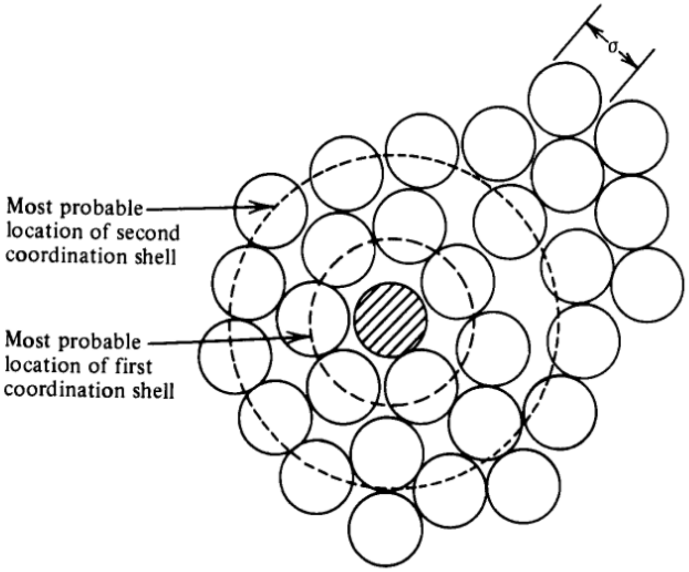
by

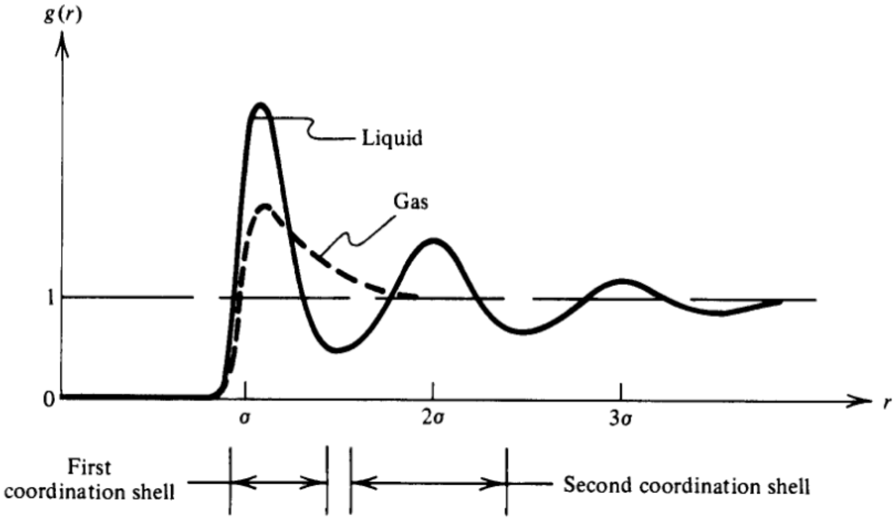
$$S(\vec{q}) = 1 + \frac{N}{V} \int d^d r \; g(r) e^{\mathrm{i}\vec{q}\cdot\vec{r}}$$

Peaks in $g(r)$ are related to peaks in $S(q)$. The first peak in $S(q)$ is at $q_0 = 2\pi/\Delta r$ where Δr is the distance between peaks in $g(r)$ (that is close to the inter particle distance as well).

Observables

Liquid

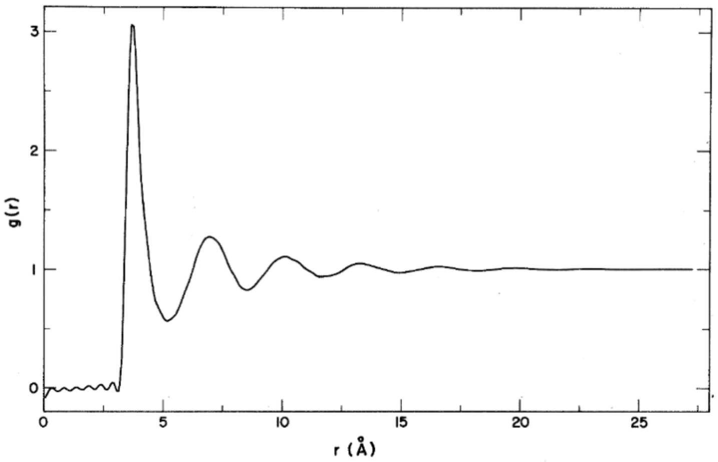


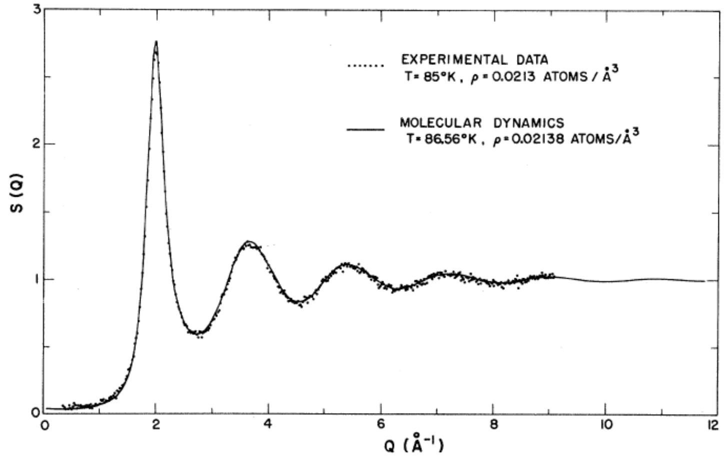


“Introduction to Modern Statistical Mechanics”, **Chandler** (OUP)

Observables

Experiments & simulations of liquids





Inter-peak distance between the peaks in $g(r)$ is $\Delta r \simeq \sigma \simeq 3$Å

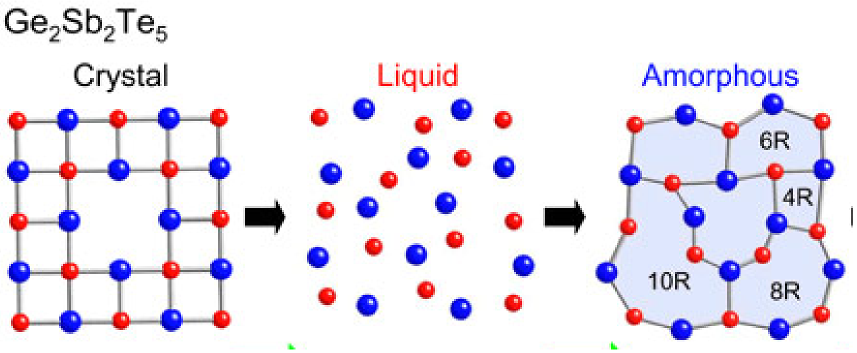
Position of the first peak in $S(q)$ is at $q_0 \simeq 2\pi/\Delta r \simeq 2$ Å^{-1}

“Structure Factor and Radial Distribution Function for Liquid Argon at 85K”,

Yarnell, Katz, Wenzel & König, Phys. Rev. Lett. 7, 2130 (1973)

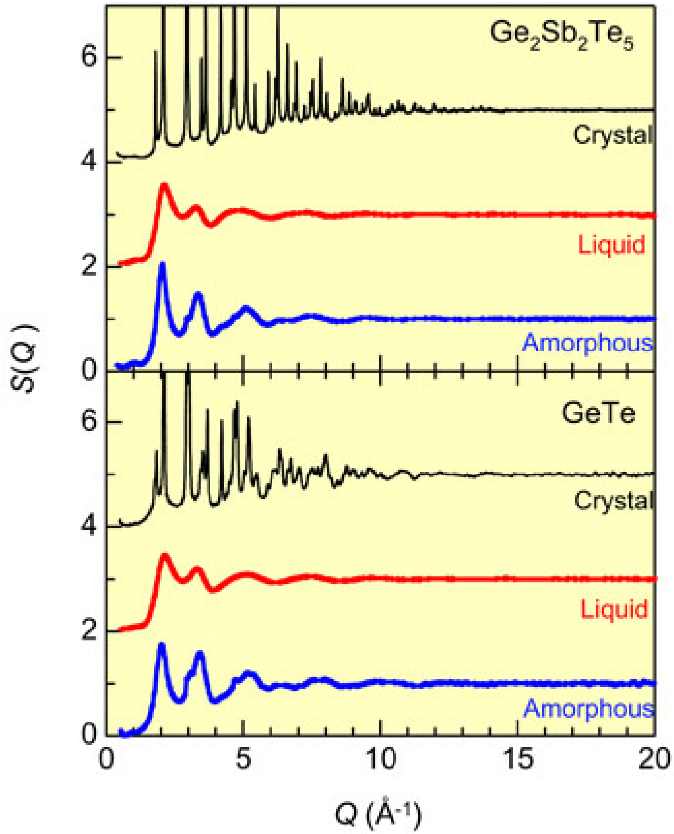
Observables

Structure factors



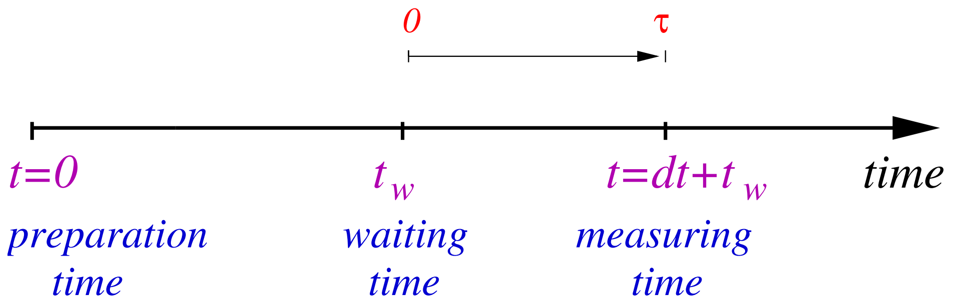
"RMC Analyses Solve High-Speed Phase-Change Mechanism"

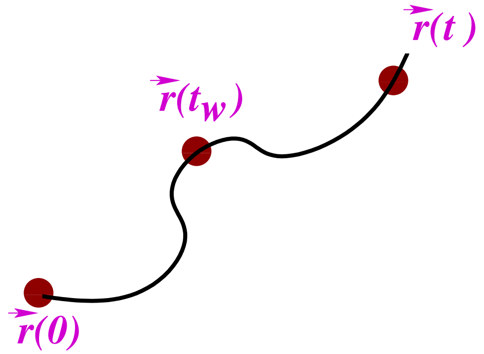
Matsunaga, Kojima, Yamada, Kohara, Takata (2006)



Two-time observables

Correlations





t_w not necessarily longer than t_{eq}.

The two-time correlation between $A[\{\vec{r}_i(t)\}]$ and $B[\{\vec{r}_i(t_w)\}]$ is

$$C_{AB}(t, t_w) \equiv \langle\, A[\{\vec{r}_i(t)\}] B[\{\vec{r}_i(t_w)\}]\, \rangle$$

average over realizations of the dynamics (initial conditions, random numbers in a MC simulation, thermal noise in Langevin dynamics, etc.)

Correlation functions

One can define a two-time dependent density-density correlation

$$\langle\, \delta\rho(\vec{x},t)\delta\rho(\vec{y},t_w)\,\rangle$$

The angular brackets indicate a “thermal” average ; i.e.

over different dynamical histories (runs of simulation/experiment)

Upon averaging one expects :

isotropy (all directions are equivalent)

invariance under translations of the reference point $\vec{x}$.

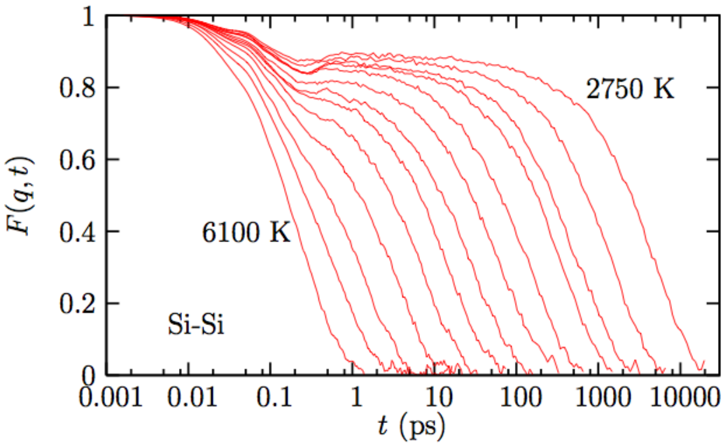
Thus, $\langle\, \delta\rho(\vec{x},t)\delta\rho(\vec{y},t_w)\,\rangle = g(r;t,t_w)$, with $r = |\vec{x}-\vec{y}|$. Its Fourier transform is $F(q;t,t_w)$ and it has a self part $F_s(q;t,t_w)$

Super-cooled liquids

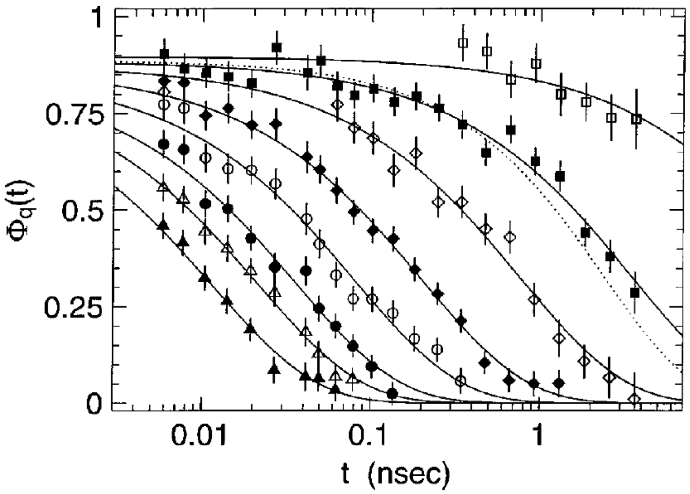
Equilibrium decay above T_g

The intermediate or self correlation

$$F_s(q;t,t_w) = N^{-1} \sum_{i=1}^{N} \langle \, e^{i\vec{q}(\vec{r}_i(t)-\vec{r}_i(t_w))} \, \rangle$$



MD simulations of silica



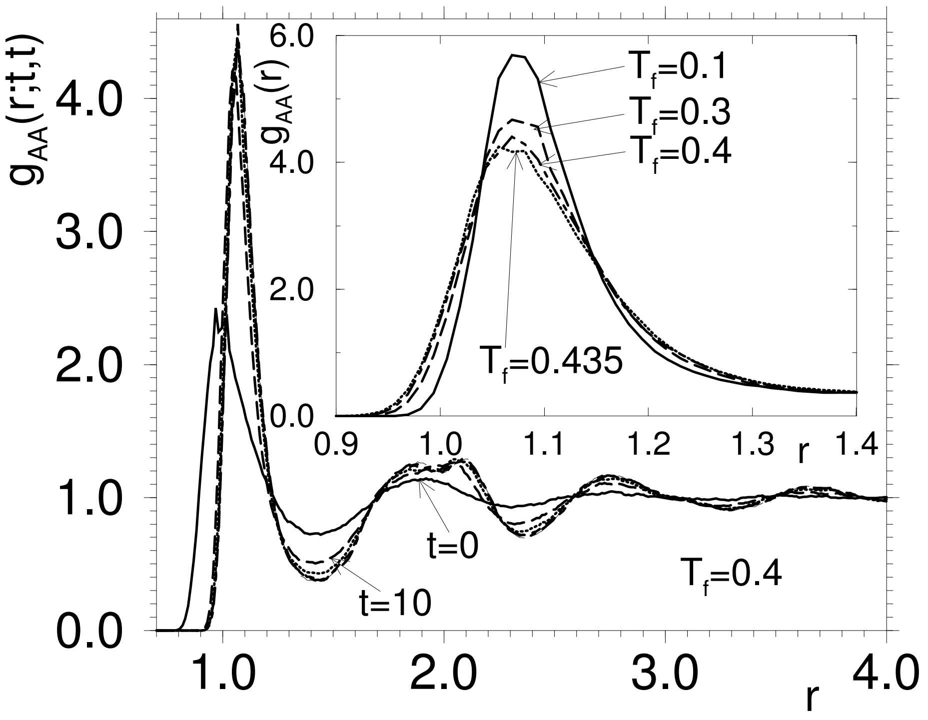
Experiments in glycerol

Note the plateau

The relaxation time τ_α increases by 5 orders of magnitude

Super-cooled liquids

but the structure is always the one of a liquid !



No important change in structure in the full range of temperatures in which the relaxation time, τ_α varies by 5-10 orders of magnitude.

The plateau

here, in a binary Lennard-Jones mixture

First stationary relaxation towards the plateau : ‘cages’

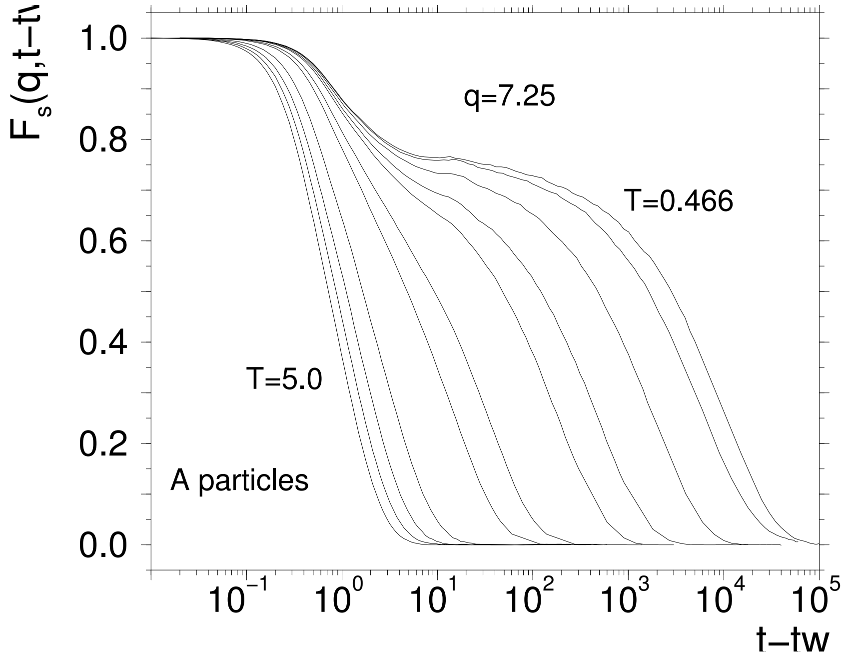
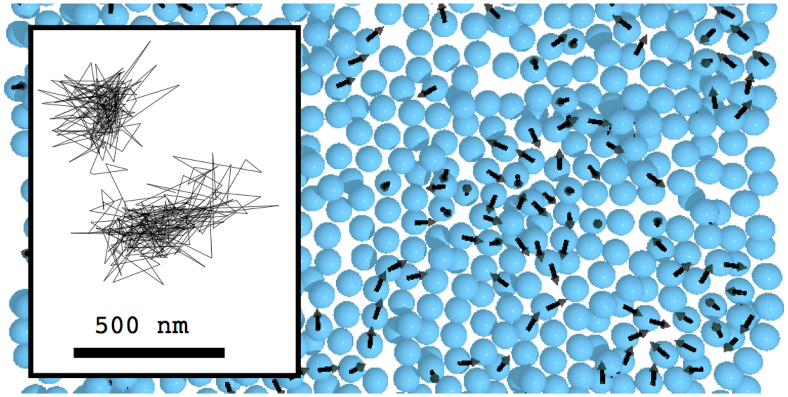


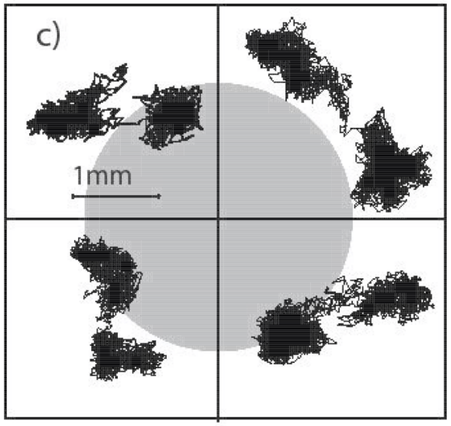
Figure from **J-L Barrat & Kob 99**

Is the plateau an order parameter ?

Real space view

First stationary relaxation towards the plateau : ‘cages’





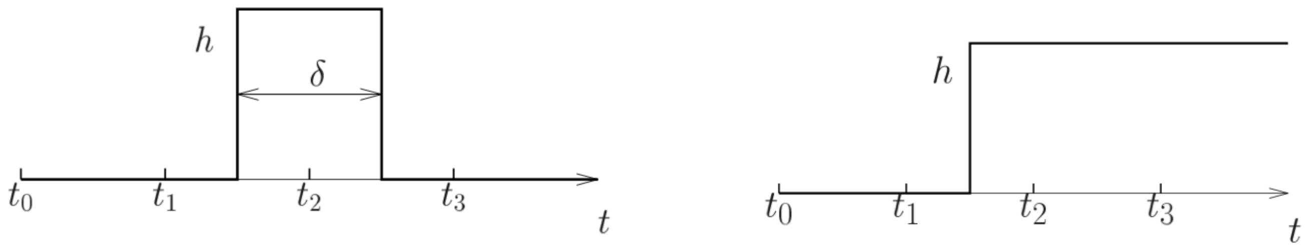
Colloids **Weeks et al. 02**

Powders **Pouliquen et al. 03**

The particles’ displacement is much smaller than the particle radius.

Second non-stationary relaxation below the plateau : ‘structural’

Response to perturbations



The perturbation couples linearly to the observable $B[\{\vec{r}_i\}]$

$$E \quad \to \quad E - hB[\{\vec{r}_i\}]$$

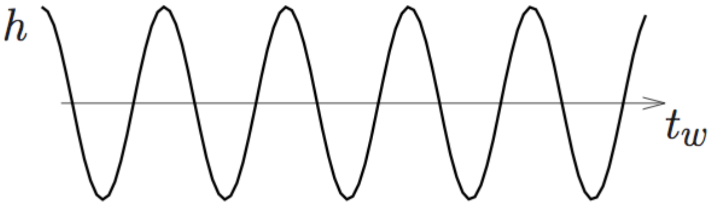
The linear instantaneous response of another observable $A(\{\vec{r}_i\})$ is

$$R_{AB}(t,t_w) \equiv \left\langle \left. \frac{\delta A[\{\vec{r}_i\}](t)}{\delta h(t_w)} \right|_{h=0} \right\rangle$$

The linear integrated response or dc susceptibility is

$$\chi_{AB}(t,t_w) \equiv \int_{t_w}^{t} dt'\, R_{AB}(t,t')$$

ac response to perturbations



$$\chi(\omega,t_w) = \int_0^{t_w} dt'\, R(t_w,t')h(\omega,t') = \int_0^{t_w} dt'\, R(t_w,t')he^{i\omega t'}$$

$\chi'(\omega,t_w) = \mathrm{Re}\chi(\omega,t_w)$ (in phase)

$\chi''(\omega,t_w) = \mathrm{Im}\chi(\omega,t_w)$ (out of phase)

are related by Kramers-Krönig $\chi''(\omega,t_w) = -\pi^{-1}P\int d\omega'\, \frac{\chi'(\omega,t_w)}{\omega'-\omega}$

In equilibrium $\chi(\omega,t_w) \to \chi(\omega)$

Dynamics in super-cooled liquid

Propylene carbonate $T_{VF} = 132$ K, $T_g = 159$ K, $T_m = 218$ K

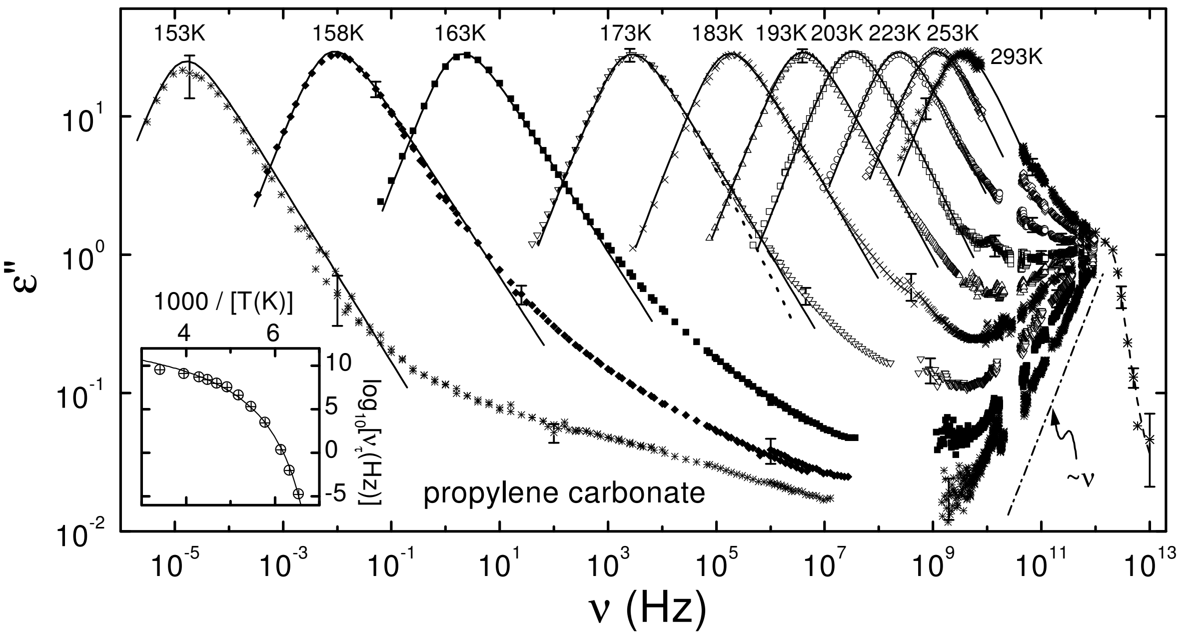


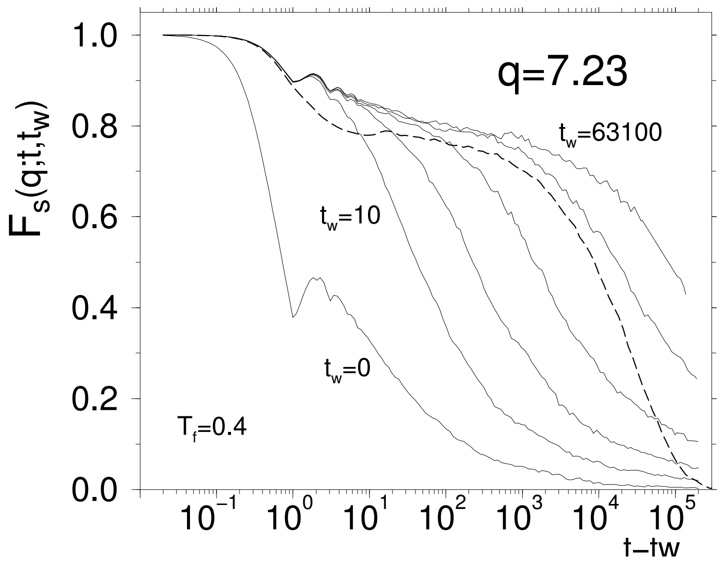
Fig. 2, U. Schneider et al.

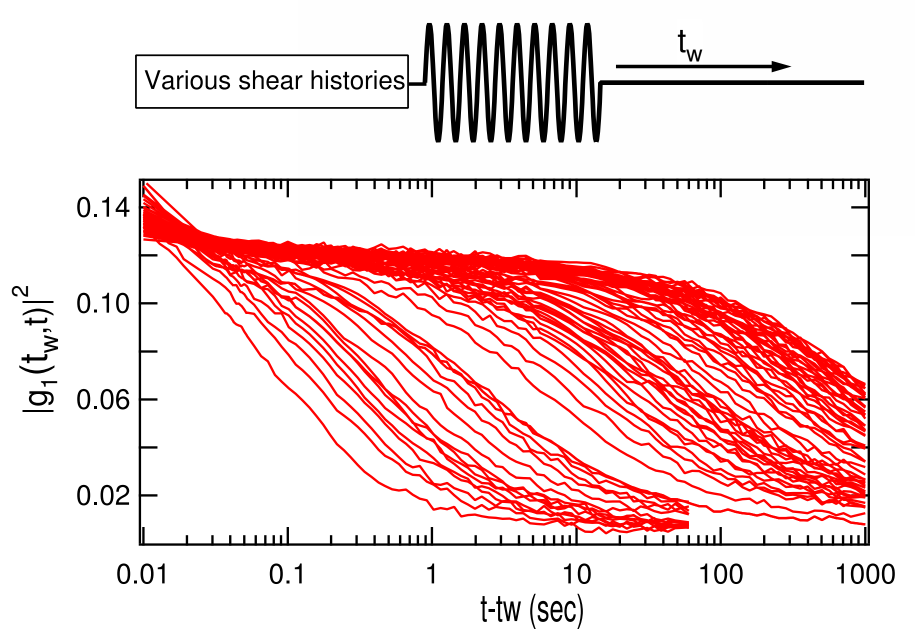
Low frequency peak ≈ Cole-Davison function or FT of stretched exp.

Schneider, Lunkenheimer, Brand, & Loidl 98

Still lower temperature

Out of equilibrium relaxation





L-J mixture **J-L Barrat & Kob 99**

Colloids **Viasnoff & Lequeux 03**

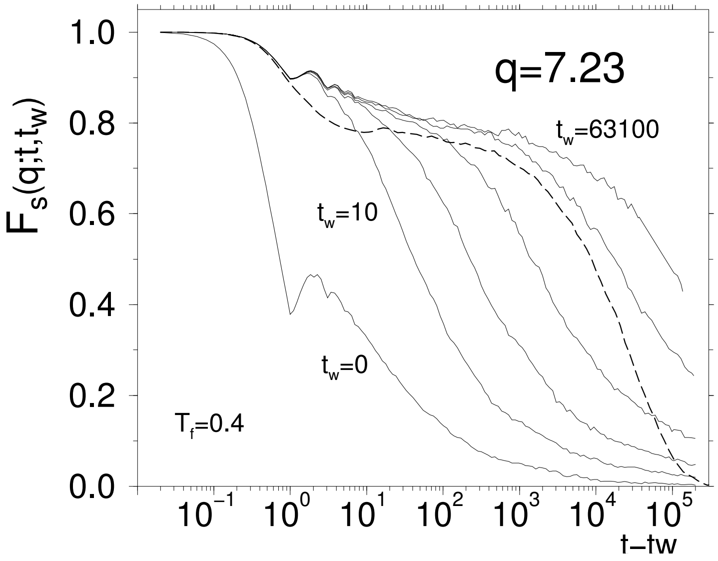
$$\boxed{t_{micro} \ll t \ll t_{eq}}$$

The equilibration time goes beyond the experimentally accessible times

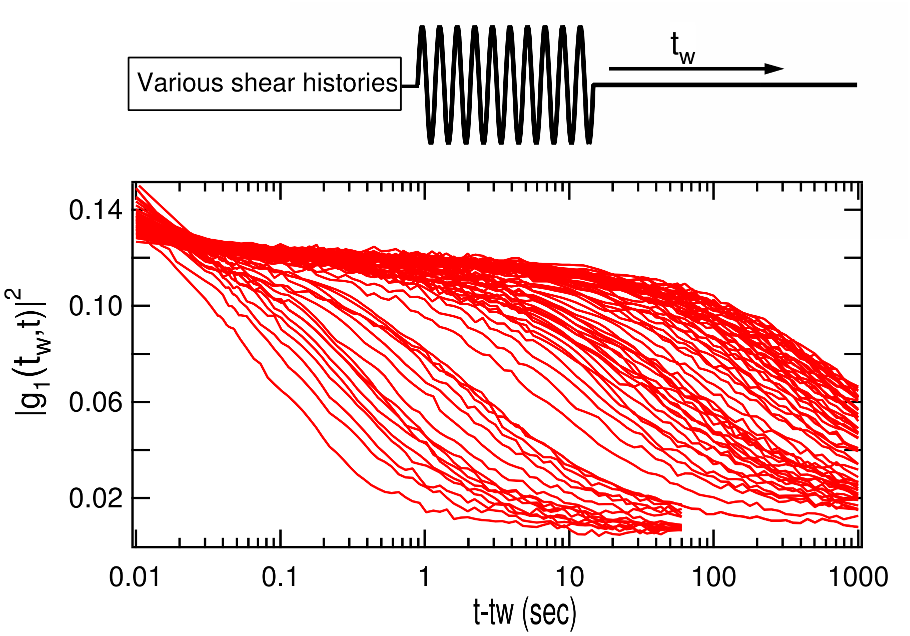
The same is observed in all other glasses.

Still lower temperature

Ageing effects



L-J mixture **J-L Barrat & Kob 99**



Colloids **Viasnoff & Lequeux 03**

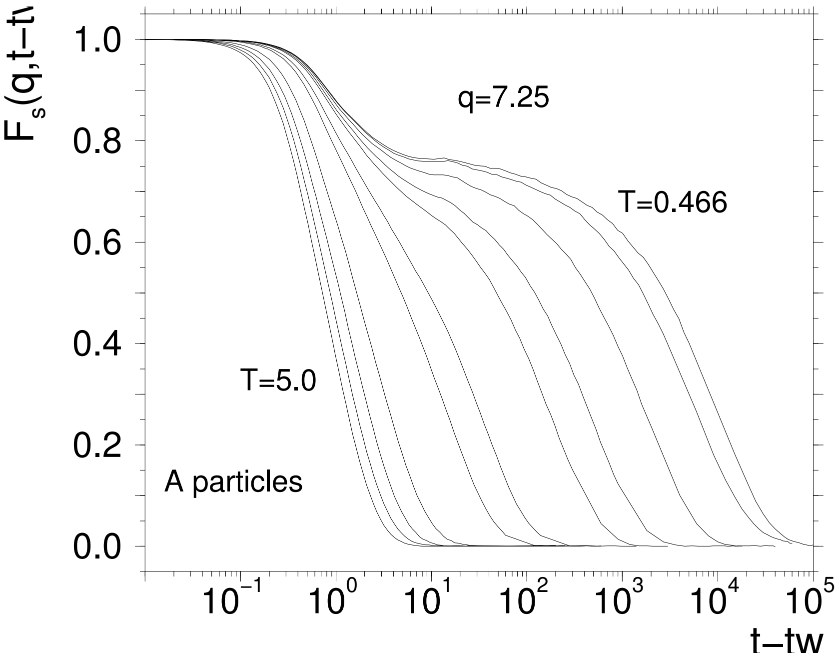
$$\boxed{t_{micro} \ll t \ll t_{eq}}$$

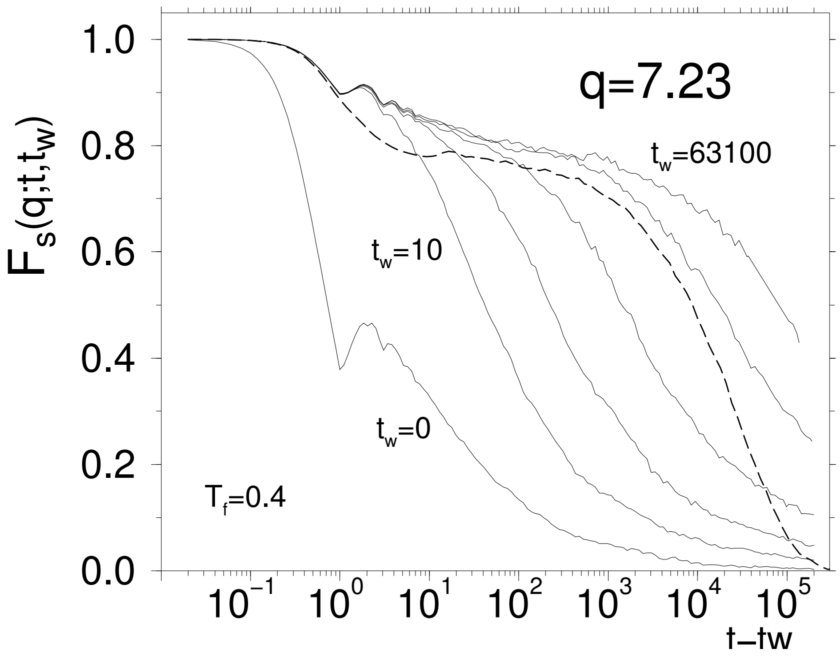
Ageing the relaxation is slower for older systems

The plateau

Binary Lennard-Jones mixture

First stationary relaxation towards the plateau : ‘cages’



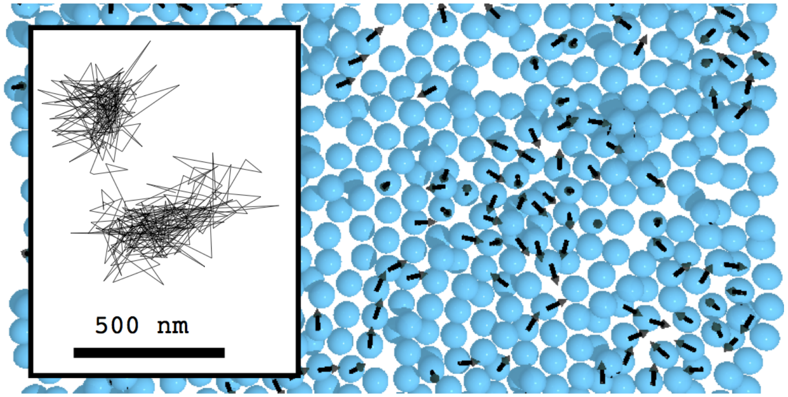


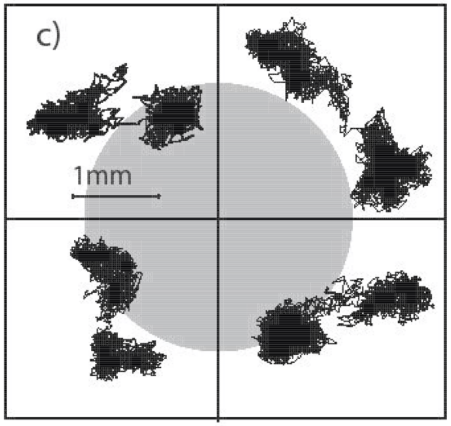
J-L Barrat & Kob 99

Note that the structural relaxation is stationary at $T > T_g$ (left) and non-stationary $T < T_g$ (right)

Real space view

First stationary relaxation towards the plateau : 'cages'





Colloids **Weeks et al. 02**

Powders **Pouliquen et al. 03**

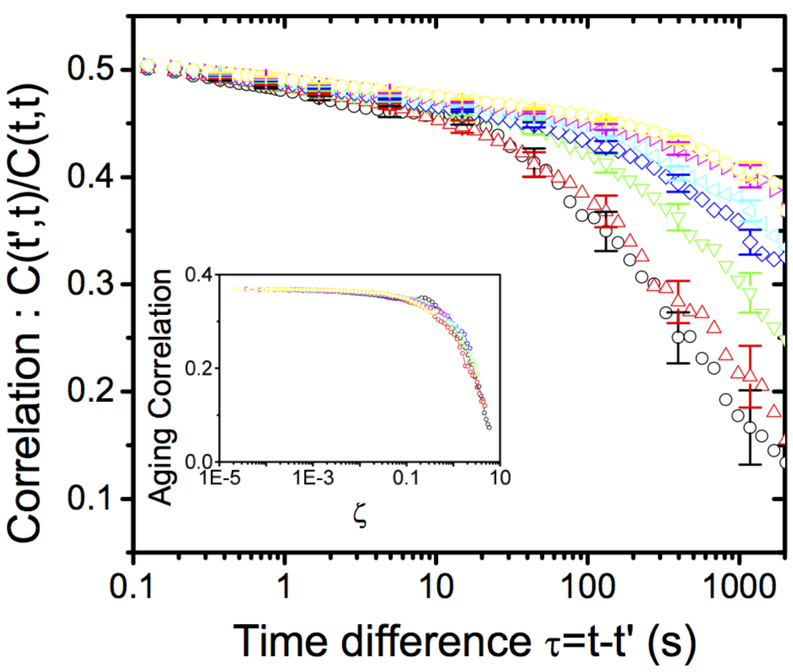
The particles' displacement is much smaller than particle radius.

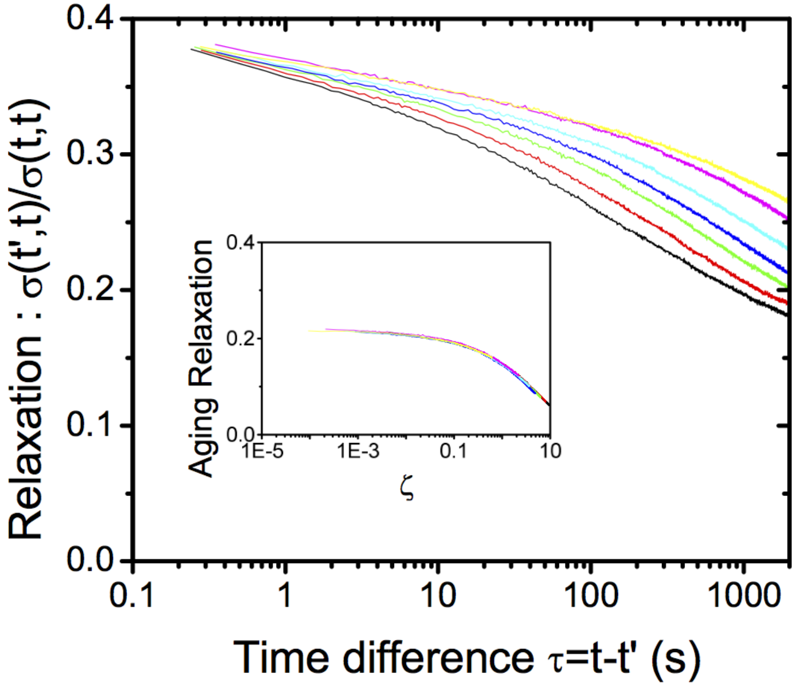
Second non-stationary relaxation below the plateau : 'structural'

The one that demonstrates the out of equilibrium physics

An insulating spin-glass

Thiospinel





Self-correlation

spontaneous

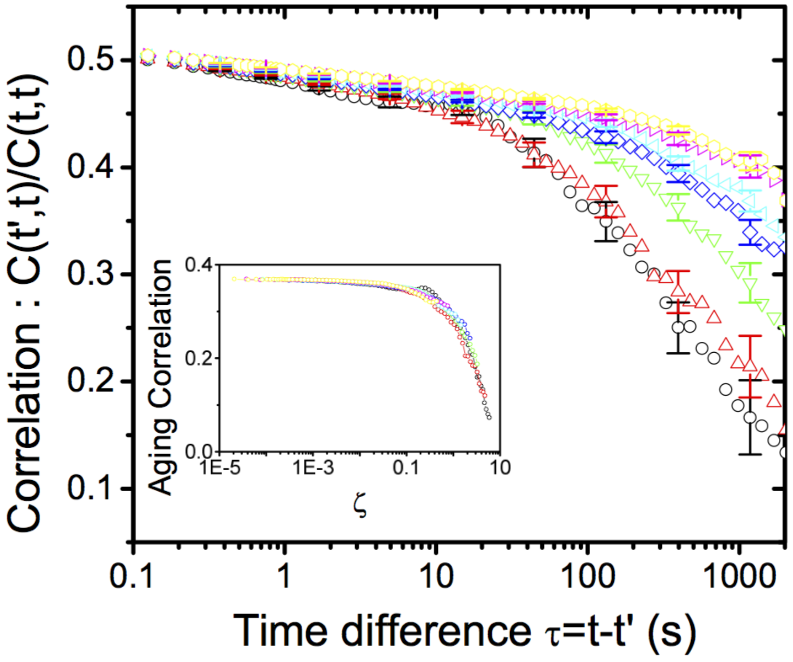
Thermo-remanent magnetisation

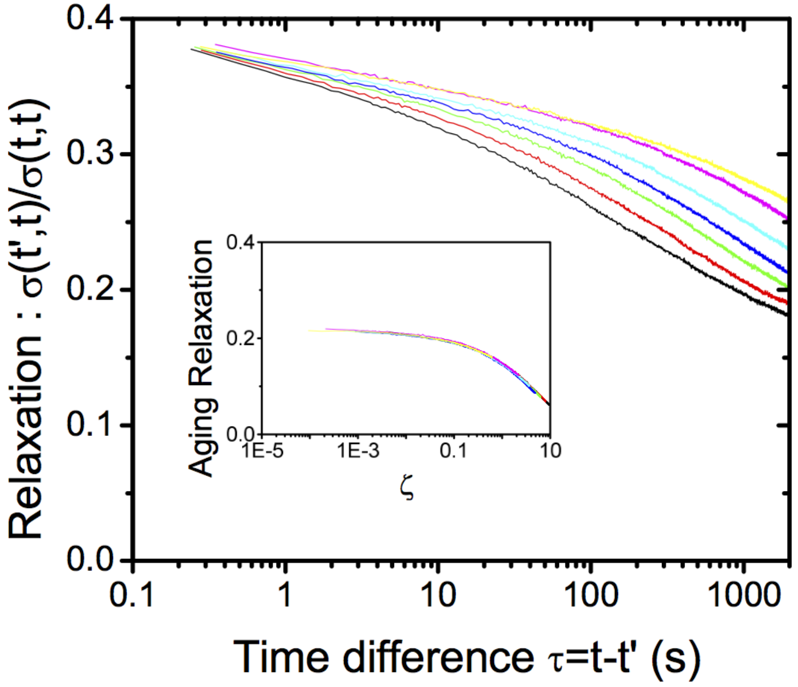
induced (a response)

Herisson & Ocio 01

An insulating spin-glass

Thiospinel



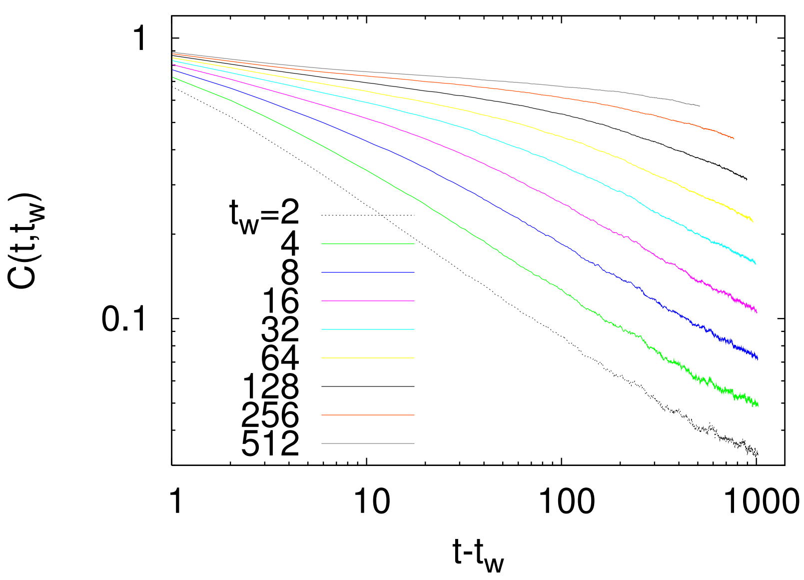


There seems to be a plateau (maybe inclined) separating a stationary from a non-stationary regime

Herisson & Ocio 01

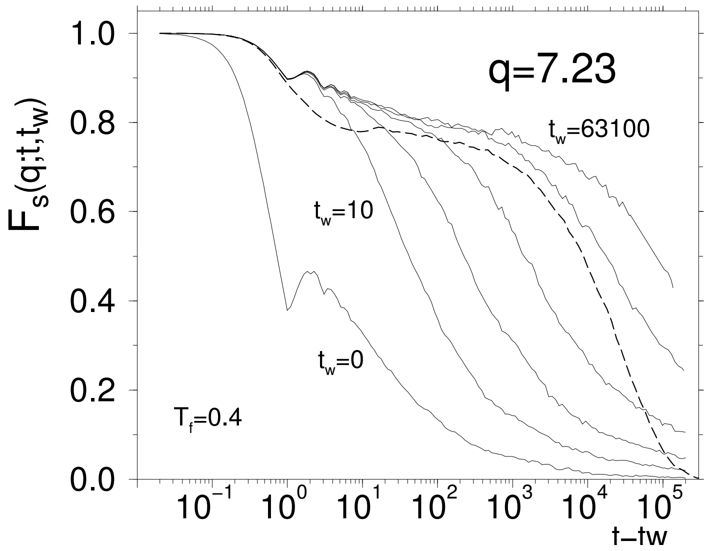
Ferromagnet vs glass

Not so different as long as correlations are concerned



2d Ising model - spin-spin

Sicilia *et al.* **07**



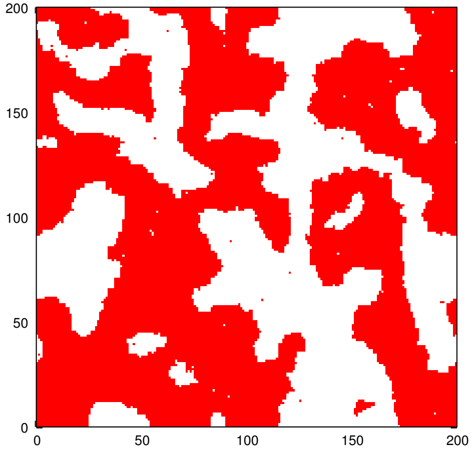
Lennard-Jones - density-density

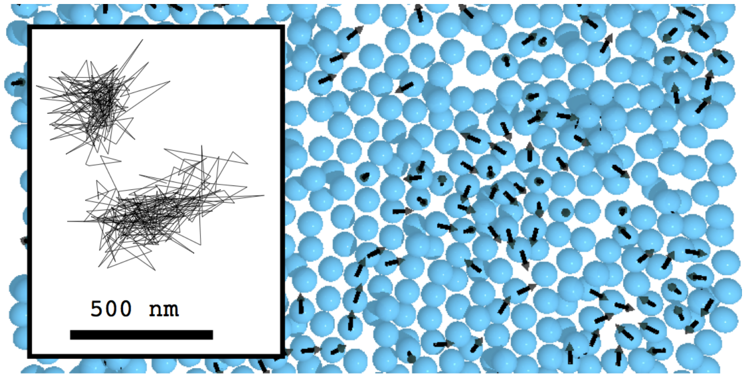
Kob & Barrat 99

Different two-time regimes

Interpretation

- In phase ordering kinetics, thermal fluctuations within domains *vs.* domain wall motion.
- In particle systems, rattling within cages *vs.* structural relaxation.



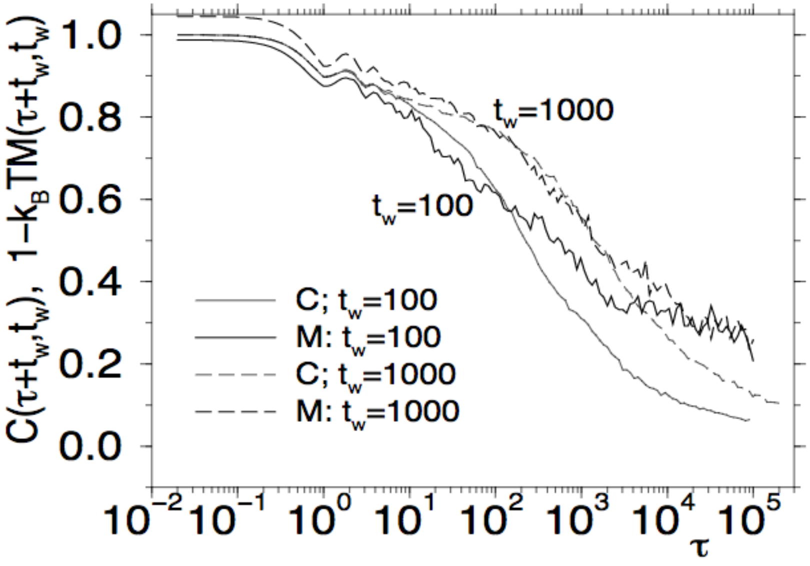


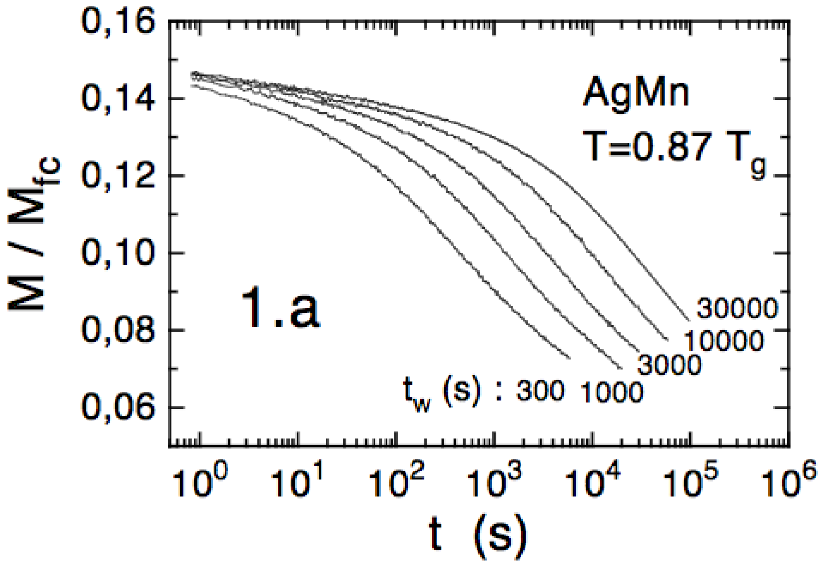
Cages in colloidal suspensions

Domain growth in the 2d Ising model.

Response functions

Glasses and spin-glasses





Lennard-Jones mixture

Kob & J-L Barrat 98

A metallic spin-glass

Vincent et al. 96

Summary

Spin-glasses & coarsening systems

2nd order phase transition at T_c

Paramagnet Critical slowing down **Spin-glass/ferromagnet**

Exponential relax Non-exponential relax

Equilibrium Long-relaxation to equilibrium **Non-equilibrium**

Separation of time-scales

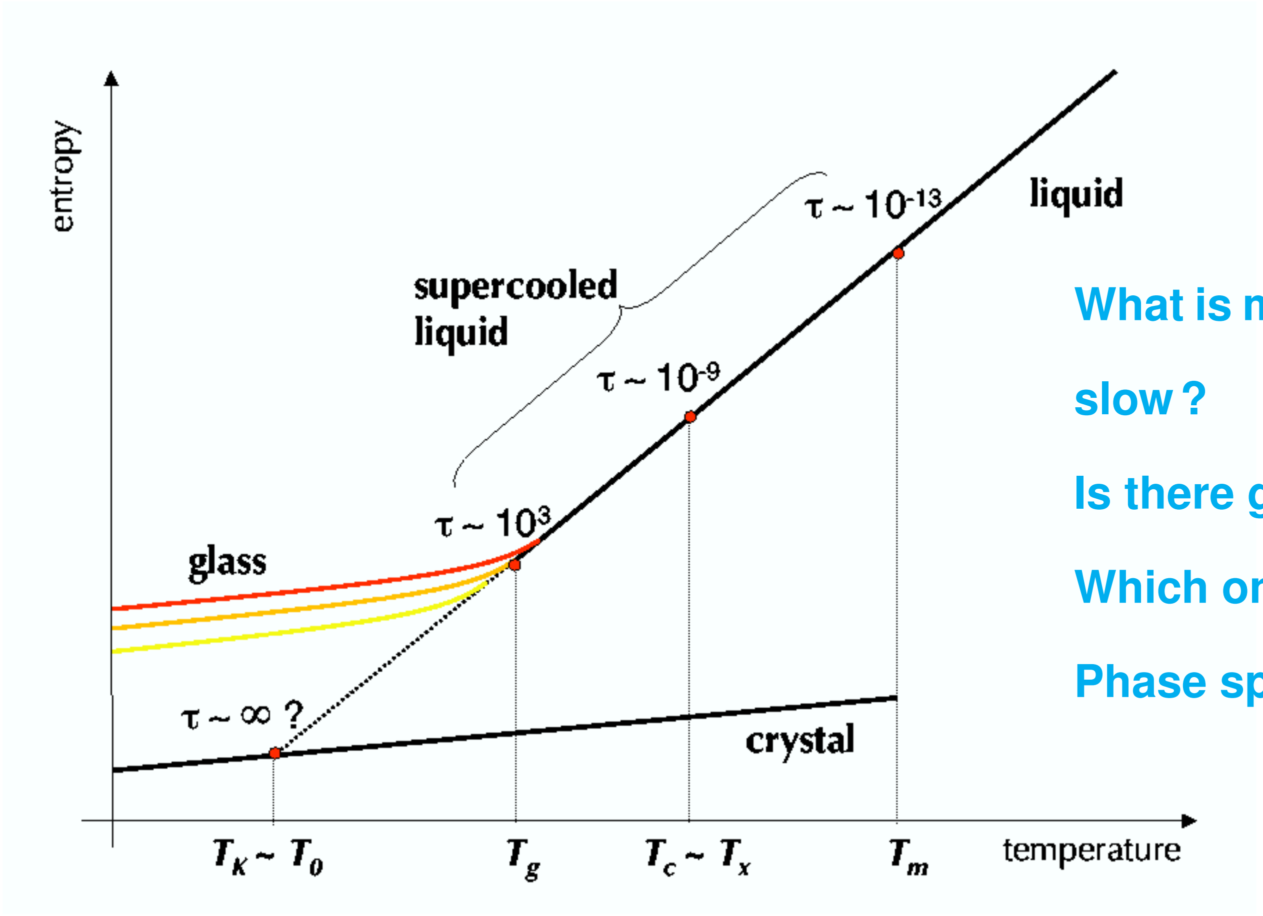
Stationary **Aging**

Aging means that correlations and reponses depend on t *and* t_w

ac susceptibilities depend on ω *and* t_w

Fragile glasses

Time-scales from calorimetric measurement of entropy



What is making the relaxation so slow ?

Is there growth of static order ?

Which one ?

Phase space picture ?

Summary

Structural glasses

Crytallization at T_m is avoided by cooling fast enough.

Liquid Supercooled liquid **Glass**

Exponential relax Non-exponential relax

Equilibrium Metastable equilibrium **Non-equilibrium**

Separation of time-scales &

An exponential number of metastable states !

Stationary **Aging**

Aging means that correlations and reponses depend on t *and* t_w

ac susceptibilities depend on ω *and* t_w

There might be an **equilibrium transition** to an **ideal glass** at T_s.

Challenges

in classical non-equilibrium macroscopic systems

- Coarsening

  The systems are taken across *usual phase transitions*.

  The *dynamic mechanisms* are well-understood :

  competition between equilibrium phases & topological defect annihilation.

  The difficulty lies in the calculation of observables in a time-dependent non-linear field theory.

- Glasses

  Are there phase transitions ?

  The dynamic mechanisms are not well understood.

  The difficulty is conceptual (also computational).

- General question

  Do these, as well as sheared liquids or active matter, enjoy some kind of thermodynamic properties ?

Methods

Many body systems

- **Coarsening phenomena**

  Identify the order parameter $\phi(\vec{x}, t)$ (a field). Write Langevin or Fokker-Planck equations for it and analyse them. A difficult problem. Non-linear equations. Neither perturbation theory nor RG methods are OK. Self-consistent resummations tried.

- **Glassy systems**

  The "order parameter" is a composite object depending on two-times. Spin models with quenched randomness yield a mean-field description of the dynamics observed. Classes of systems (ferromagnets, spin-glass and fragile glasses) captured.

Disordered spin systems

Classical p-spin model

$$\boxed{H_{syst} = - \sum_{i_1 < \dots < i_p}^{N} J_{i_1 i_2 \dots i_p} s_{i_1} s_{i_2} \dots s_{i_p}}$$

Ising, $s_i = \pm 1$, or spherical, $\sum_{i=1}^{N} s_i^2 = N$, spins.

Sum over all p-uplets on a complete graph : fully-connected model.

Random exchanges $P(J_{i_1 i_2 \dots i_p}) = e^{-p! \, J^2_{i_1 i_2 \dots i_p}/(2N^{p-1} J^2)}$

Extensions to random graphs possible : dilute models.

$p = 2$ Ising : Sherrington-Kirkpatrick model for spin-glasses

$p = 2$ spherical $\approx$ mean-field ferromagnet

$p \geq 3$ Ising or spherical : models for fragile glasses

Disordered spin systems

Random K-sat problem

A clause is the 'logical or' between K requirements imposed on Boolean variables x_i chosen randomly from a pool of N of them.

A formula is the 'logical and' between M such clauses, $F = \bigwedge_{\ell=1}^{M} \bigvee_{i=1}^{K} x_i^{(\ell)}$. It is satisfied when all M clauses are.

The search for a solution can be set as the search for the spin configuration(s) with vanishing energy

$$\boxed{H_{syst} = \alpha 2^{-K} N + \sum_{R=1}^{K} (-1)^R \sum_{i_1 < \cdots < i_R}^{N} J_{i_1 i_2 \dots i_R} \, s_{i_1} s_{i_2} \dots s_{i_R}}$$

with $\alpha = M/N$, Ising spins, $s_i = \pm 1$, and interactions

$J_{i_1 \dots i_R} = 2^{-K} \sum_{\ell=1}^{M} C_{\ell, i_1} \dots C_{\ell, i_R}$

with $C_{\ell, i_k} = +, -$ for the condition $x_{i_k}^{(\ell)} =$ T,F and $C_{\ell, i_k} = 0$ otherwise.

Sum of classical dilute $p \le K$-spin models

Methods

for classical and quantum disordered systems

Statics

TAP Thouless-Anderson-Palmer Replica theory	fully-connected (complete graph) Gaussian approx. to field-theories
Cavity or Peierls approx.	dilute (random graph)
Bubbles & droplet arguments functional RG	finite dimensions

Dynamics

Generating functional for classical field theories (MSRJD).

Schwinger-Keldysh closed-time path-integral for quantum dissipative models (the previous is recovered in the $\hbar \to 0$ limit).

Perturbation theory, renormalization group techniques, self-consistent approx.

Methods

for classical and quantum disordered systems

Statics

TAP Thouless-Anderson-Palmer Replica theory	fully-connected (complete graph) Gaussian approx. to field-theories
Cavity or Peierls approx.	dilute (random graph)
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Dynamics

Generating functional for classical field theories (MSRJD).

Perturbation theory, renormalization group techniques, self-consistent approximations

Some references

Spin-glasses

Slow Dynamics and Aging in Spin Glasses, E. Vincent, J. Hammann, M. Ocio, J-P Bouchaud and L. F. Cugliandolo, arXiv:cond-mat/9607224 (Sitges Conference Proceedings, published by Springer-Verlag).

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Dynamics of glassy systems, L. F. Cugliandolo, arXiv :cond-mat/0210312 (Les Houches Summer School 2002, published in the Les Houches collection).

and unpublished notes (see webpage & www.lpthe.jussieu.fr/˜leticia)

Growing lengths

Growing length scales in aging systems, F. Corberi, L. F. Cugliandolo, and H. Yoshino, arXiv :1010.0149 (Leiden work-shop, published by Oxford University Press)